

Naïve Algorithm: Given x to classifies and K finds its K nearest neighbor in the training set (K smallest $d(x, x_i)$) • **Classify** according to the majority for the nearest neighbor. • **Supervised Learning**

Dis: Euclidean(x, x_i) = $\sqrt{\sum_{d=1}^D (x_d - x_i^d)^2}$ / **Manhattan**(x, x_i) = $\sum_{d=1}^D |x_d - x_i^d|$

Assumption: Training set and test set drawn (look) from the same **statistical distribution** If not the **decision boundary** not be useful on test set • **Training set** must be representative all objects the system is likely to encounter.

Notes: Test set: Use the test set to gauge its performance. • **small k:** Overfitting • **Big K:** smoother boundary, training error increases • **misclassifying** points near the decision boundary • **more training examples** of one kind than other represented this class unduly (no fair) favored • **Real data often lies in smaller subspaces.**

Kn Limitations: • Performance issues: Must load all of the training data and calculate distances to all training samples. It can be done in a naive way or using fancier data structures such as **K-D trees**. However, this is still slow for large datasets. • **Distance metric:** The **vanilla** version is used with the simple **Euclidean distance**, which is a **problematic** distance metric in high dimensions as well as with **noisy features** or features of **different type**. **Improve:** • **Weighing neighbors** by the inverse of their class size converts neighbor counts into the fraction of each class that falls in your K nearest neighbors. • **Under-sampling** the dominant class. • **Augmenting** the other class by generating synthetic examples.

Choosing k/ cross-validation: • **Split** training set to training set and validation set • **Choose k** that minimizes the classification error on the validation set (min on the graph(error/complexity) of the validation set) • **Train** for different K 's and pick k that yields the highest **accuracy** on the validation set

graph(error/complexity): • **training curve** decreases when the degree/ K increases • **test curve** eventually increases again • **relevant** for all all ML algorithms

Validation graph: accuracy percentage / k • **one single** meta-parameter k so easy to be tuned in cross-validation.

Rosenbrock Function: banana, $r(x, y) = 100(y - x^2)^2 + (1 - x)^2$, $f(x, y) = -1$ if $r(x, y) < 1$ otherwise

Data Reduction: Condensed Nearest Neighbors Let X be the set of training samples and P the set of prototypes. • **Initialize** • **Repeat 1.** Look for x in X such that its nearest prototype in P has a different label than itself. 2. Remove x from X and add it to P . • **Note:** Fast computation at inference time

Greedly k-NN Graph Construction: Parallel-iterative algorithm: • **Given** a random graph • **Each** node looks for potential new neighbors: 1. Among random nodes (optional). 2. Among "friends of friends". • **Note: Gossip Based Computing:** Highly parallel. • **Creates** a random graph. • **Robust** to churn, partition, breakdowns. • **Well** adapted to peer-to-peer networks.

Number of Neighbors: • More neighbors: More coverage • **Fewer** Neighbors: Better accuracy.

2D Voroni Diagram: For N training set divided the diagram for N cells each present the closest points for sample

• **Note** depends on distance calculation kernel • **Decision Boundary:** divide the graph for sector of classification

• **Note** can be formed by selected edges of the Voroni Diagram.

Multi Class: • **high-dimensional** space, everything far from everything. **Euclidean** becomes less meaningful.

effectiveness estimator: distance between neighboring training samples $< d$ that depends on the problem.

Linear Regression: supervised • **Given** N pairs $\{(x_i, t_i)\}$, find the separations for two classes • **Line fitting**

• **Line:** y-intercept w_0 The slope $w_1 = \Delta y / \Delta x$, $y(x; w) = w_0 + w_1 x$ • **positive/Negative emotions:** up/down • **2-D** define a plane, $y = w_0 + w_1 x_1 + w_2 x_2$ • **N-D** $y = w^T x$, minimize $W \{ 1/N \cdot \sum (W^T x_i - t_i)^2 \}$

Algorithm: USE for Predict quantities: • **Collect** data with continuous target variable.

Preprocess: Clean data, handle missing values, scale features, and split into training and test sets. • **Initialize** linear model parameters (weights and bias). • **Train:** Use gradient descent to minimize the mean squared error. • **Evaluate:** Test the model and measure performance using metrics like RMSE (Root Mean Squared Error).

• **Regression:** Now you can predict an estimate of the corresponding y as $y_t = w^T x + w^T x_t$

Train: • **Given** X_n data • **Find** w_0 & w_1 by **min** $W \{ 1/N \cdot \sum (y_i - t_i)^2 \}$ (Leas squares)

• **Evaluation:** • **compares** the predictions of model with true annotations of test data: **fixed** parameters

• **testing** data must be **separated!**

Mean Squared Error (MSE): • **loss** function for N_t test samples

$MSE = 1/N_t \cdot \sum (y_i - t_i)^2$ • where y_i = prediction for test sample i and t_i = corresponding ground-truth value

• **Root Mean Squared Error (RMSE):** • **Square-root** of the MSE

Evaluation metrics regression:

• **Mean Absolute Error (MAE):** $MAE = (1/N_t) \sum |y_i - t_i|$

• **Mean Absolute Percentage Error (MAPE):**

$MAPE = (1/N_t) \sum |(y_i - t_i)/t_i| \cdot 100$

Minimization problem: • Gradient descent (applicable), • **Closed-form solution:** algebraic form, less applicable, it does apply linear regression, $w^* = (X^T X)^{-1} X^T t = X^+ t$

Interpreting a linear model: • Warning: coefficient (коэффициент) magnitude (мagnitude) depend on feature/attribute magnitude

• **Coefficient** might **small** to compensate that the feature range large • **Normalizing** the data can addressed it

Maximum Margin Classifier: Linear Support Vector Machine (SVM): Finds a **hyperplane** separating classes maximally by maximizing the margin between closest points (support vectors).

Algorithm: • Optimizes $\|w\|$ subject to $t^T (w \cdot x_n) \geq 1 - \xi_n$, • **minimizes** $(1/2) \cdot \|w\|^2 + C \cdot \sum \xi_n$ with constraints $\ln (w \cdot x_n) \geq 1 - \xi_n$ and $\xi_n \geq 0$.

Large C: Prioritizes a **smaller margin** and **fewer training errors**, emphasizing fitting the training data closely, potentially at the cost of overfitting.

Small C: Promotes a **larger margin** but is more lenient on **misclassifications** in the training data, which might lead to better generalization on unseen data by reducing overfitting.

Advantages: • **Effective** in high-dimensional spaces, • **efficient** with memory as it uses only support vectors, • **can** handle non-linearly separable classes using the kernel trick.

Disadvantages: • **Choosing** correct C is critical, sensitive to noisy data (outliers), • **struggles** when the number of features vastly outnumber samples, • **not** inherently suited to multi-class problems, • **requires** separable data

Usage: in applications such as face detection, handwriting recognition, image classification, and bioinformatics.

Slack Variables: Allows flexibility in separating data by introducing ξ_n to soften the margin. Points can either lie inside the margin or be misclassified, depending on the value of ξ_n .

Convex Optimization Problem: The problem is **convex**, and efficient algorithms are available for finding the **global optimum**. feature range large • **Normalizing** the data can addressed it

Support Vector Machine (SVM) Find the optimal hyperplane that maximizes the margin between two classes in a dataset. **minimize:** $(1/2) \cdot \|w\|^2$ subject to $y_i(w \cdot x_i + b) \geq 1$ for all i in the training set.

Here, w is the weight vector, b is the bias term, x_i is the feature vector, and y_i is the class label (+1 or -1).

Regularization: Introduce slack variables ξ_i to allow some misclassifications.

minimize $(1/2) \cdot \|w\|^2 + C \cdot \sum \xi_i$ subject to $y_i(w \cdot x_i + b) \geq 1 - \xi_i$ for all i .

C is a **regularization parameter** controlling the **trade-off** between maximizing the margin and minimizing the classification error. **Higher C** values lead to less margin violation but may overfit.

Lagrangian Formulation & Dual Problem: Convert the constrained optimization problem to **unconstrained** one using **Lagrange multipliers**.

$L(w, b, \alpha) = (1/2) \cdot \|w\|^2 - \sum \alpha_i (y_i (w \cdot x_i + b) - 1)$

Dual problem: Maximize the dual objective wrt α subject to $\alpha_i \geq 0$.

Solution: $w^* = \sum (\alpha_i^* \cdot y_i \cdot x_i)$

Kernel Trick: Transform the input space into a higher-dimensional feature space.

• **Use** a kernel function

$K(x, x') = \phi(x) \cdot \phi(x')$ to compute dot products in the transformed space without explicitly computing the transformation ϕ .

• **Common kernels:** Linear ($K(x, x') = x \cdot x'$), Polynomial ($K(x, x') = (1 + x \cdot x')^d$), Radial Basis Function (RBF) or Gaussian ($K(x, x') = \exp(-\gamma \|x - x'\|^2)$).

Support Vectors: • **Data** points that lie on the margin (or violate the margin) are the support vectors. • **Decision** function only depends on support vectors: $f(x) = \sum (\alpha_i^* \cdot y_i \cdot K(x, x_i)) + b$.

Parameter Selection: Use **cross-validation** to select the best parameters for the SVM (e.g., C , kernel parameters).

Training the SVM: Use optimization algorithms like **Sequential Minimal Optimization (SMO)** to solve the dual problem and find the Lagrange multipliers (α_i) .

Making Predictions: For a new input x , classify based on the sign of the decision function $f(x) = \sum (\alpha_i^* \cdot y_i \cdot K(x, x_i)) + b$.

Multi-class Classification: For multi-class classification, use one-vs-one or one-vs-all strategies.

Scaling and Preprocessing: Scale input features for better performance. • **Perform** dimensionality reduction if needed.

Model Evaluation: Evaluate the SVM model using metrics like accuracy, precision, recall, or F1-score.

Supervised Learning: Train using an annotated training set • **Unsupervised Learning:** training set is not annotated and the system must also learn the clusters.

K-Means: • Group the samples into K clusters • **Minimize:** $\sum_{k=1}^K \sum_{i \in C_k} \|x_i - \mu_k\|^2$

mean $\mu = 1/N \cdot \sum_{i=1}^N x_i$ • **Always converge** (might not for best solution) • **Solution depends** on the initialization • **Best on homogeneous data** • **Compactness** of clusters

Algorithm: • **Given** K, X data points • **Initialize** $\{\mu_k\}$ randomly • **Until convergence:** 1) **Assign** each point x_i to the nearest center μ according to Euclidean dis 2) **Recomputing** the means centers μ

Stop criterion: • **Fixed number** of iterations (arbitrary, small number lead to bad results) • **Difference** of center locations between two iterations • **Converge**

Note: • **Try** different random initialization and keep the best result in terms of the sum of square distances. • **Bigger K:** Average within-cluster distance decreases, **too big** clusters make the results meaningless, **elbow curve** is where the drop in within-cluster distances becomes less significant

Inhomogeneous Data: all the entries/ features are of the same type(all being numbers/Or only text). • **Euclidean** distance is most appropriate • **2D** toy data, both dimensions commensurate (один) magnitude. • **Color image**, each dimension represents a color channel and varies in the range. • **In practice:** **Different data** dimensions may have different magnitudes, **They** can encode different types of information. **Heterogeneous data:** • **Large** values boost Euclidean distance and small ones much less. • **It does** not mean that they are more or less meaningful for clustering • **Solutions:** Scale each dimension by subtracting (нормировка) the smallest value and scaling the result to be between 0 and 1. • **Use** a different metric such as the Manhattan distance.

Clustering Connectivity: clusters we observe arise from the connectivity of the points.

Graph-based: Clustering Connectivity clusters we observe arise from the connectivity of the points.

• **Group** the points based on edges in a graph • **Strong connections** indicate points that should be clustered • **Weaker** suggest that the graph can be cut into pieces/ partitions • **Graph** can be **restricted** to K -nearest or similarity (or affinity) matrix neighbor of each point **Algorithm:** • **Input:** graph, nodes = data points, edge weights = similarities.

• **Split** and Evaluate: Divide graph for two clusters and calculate cut cost

• **Find** the Best Split: Iterate through different divisions and choose the one that minimizes the cut cost. Output: The two clusters that result from the best division.

• **Similarity** between points $W_{ij} = \exp(-\|x_i - x_j\|^2 / \sigma^2)$ (fully connected)

• **Graph Cut:** dividing a graph into two parts, A and B $cut(A, B) = \sum W_{ij}$ the cost

• **Normalized Cut:** modification the graph cut that considers the size of each partition to avoid favoring imbalanced partitions.

Ncut(A, B) = (cut(A, B) / Vol(A)) + (cut(A, B) / Vol(B))

• **Volume Partition (Vol(A)):** Sum of weights of all edges connected to nodes in partition A.

$Vol(A) = \sum W_{ij}$ where W_{ij} in A/B and W_{ij} is the weight of edge between nodes i and j .

• **Similarity Matrix (W):** An $N \times N$ matrix containing similarity scores between all pairs of points. W_{ij} represents the similarity between nodes i and j .

• **Degree Matrix (D):** An $N \times N$ diagonal matrix where D_{ii} is the sum of the weights of edges connected to node i . $D_{ii} = \sum W_{ij}$ for all j .

• **Graph Laplacian (L):** Defined as $(D - W)$, it's used in solving the **normalized cut problem** $L = D - W$

• **Relaxation:** • **Transform** normalized cut problem to an eigenvalue problem: $(D - W)y = \lambda Dy$, where D = degree matrix (sum of edge weights for each node) and W = similarity matrix (weights between nodes). • $(D - W)$ known as Graph Laplacian (L). • **Solve** for eigenvector associated with the **second smallest eigenvalue**. • **Eigenvector:** positive value suggests the node belongs to one group, negative value suggests it belongs to the other group.

• **Note:** Separation isn't perfect. Use a **threshold** (often the median of the eigenvector values) to assign nodes to groups more clearly.

Algorithm: K-way (K-2) partitioning options: • **Recursively** apply 2-way partitioning - not efficient and unstable. • **Use K eigenvectors** to represent each point as a K -dimensional vector, then apply K -means clustering to these vectors (dimensionality reduction followed by K -means)

Linear Classification: • **supervised learning** • **1 Class (binary)** • **Goal** Find w^* such that: • for all most positive samples $w \cdot x > 0$ • for all most negative samples $w \cdot x < 0$.

Algorithm: • **Define** a linear decision boundary.

• **Train** weights and bias by minimizing a loss function.

• **Classify** new data points based on the sign of $(w^T x + b)$.

Uses: • **Classifying** data into two categories (spam vs. non-spam)

• **Can be extended** for multi-class classification (e.g., categorizing text into multiple topics).

Minimize: $E(w) = \sum N \{ \text{Sign}(w^T x - t_n) \}$

Hyperplane: $x \in \mathbb{R}^n, w^T x = 0$ with $x = [1 \ x]$. **Signed distance:** $w^T x$, with $w = [w_0 w]$ and $\|w\| = 1$

Algorithm Decision boundary: • **Set** w_1 to 0. • **Iteratively:** pick a random index n . 1) If x is correctly classified, do nothing. 2) Otherwise, $w_1 + 1 = w_1 + t_n \cdot x_n$. **Test:** $y(x; w) = 1$ if $W \cdot X \geq 0$ / -1 otherwise

Notes: Randomizing helps.

Centered Perceptron: decision boundary goes through the origin. • **Algorithm:** Center the x 's so that $w_0 = 0$... same **Problem:** no difference between close and far from decision boundary. • **We** want positive/negative examples to be far as possible from it. • **Two different solutions** among infinitely many perceptron has no way to favor one over other

Convergence Theorem: If $\gamma > 0$ and a parameter vector w^* , with $\|w^*\| = 1$, such that $\forall n, t_n(w^T x_n) > \gamma$ the perceptron algorithm makes at most R^2 / γ^2 errors, like **Linear Classification** just instead

Perceptron pass into **Sigmoid** (differentiable "derivatives easy to compute - asymptotically=0/1)

Logistic Regression: • **supervised** • **GLM** • **Algorithm:** • **Compute** a linear combination of features. • **Pass** through the sigmoid function to get probabilities. • **Train** weights and bias by minimizing cross-entropy loss (gradient descent) • **Classify** new points based on probability threshold (e.g., 0.5).

Uses: • **Estimating** probabilities for **Binary classification** (e.g., medical diagnosis, customer churn).

• **Extendable** to multi-class via softmax

(e.g., digit recognition, classifying images).

• **Minimize cross-entropy loss**

$E(w) = \sum \{ t_n \cdot \ln(y_n) + (1 - t_n) \cdot \ln(1 - y_n) \}$, where y_n is prediction for input x_n .

t_n is the true label of the n -th data point (= 0 or 1 binary)

• **Minimizing E(w)** pushes y_n close to 1 when $t_n = 1$, and close to 0 when $t_n = 0$.

• **Conversely,** if y_n and t_n are mismatched (e.g. $y_n < 0.5$ when $t_n = 1$), the loss is larger.

• **E(w) is convex:** its gradient with respect to w is $\sum (y_n - t_n) \cdot x_n$. • **allows efficient optimization** to find global optimum. • **y(x; w)** can be interpreted as the **probability** of x belonging to one class. It equals 0.5 on the decision boundary, and approach Activation: $\sigma^a(x) = \tilde{w}_f^T x$

Logistic regression essentially finds maximum like solution assuming **Gaussian** noise. Probability that x belongs to class k : $y^k(x) = \frac{\exp(a^k(x))}{\sum_j \exp(a^j(x))}$

• **minimize** the error-rate at **training time**.

Multi-Class Logistic Regression: • **linear** problem

Multi-class entropy: $E(\tilde{w}_1, \dots, \tilde{w}_K) = - \sum_{n=1}^N \sum_k \tilde{y}_n^k \ln(\tilde{y}_n^k)$

• **sigmoid** function is **monotonic**, the formulation is almost **unchanged**. • **Only the objective function** being minimized need to be reformulated.

Gradient of the entropy: $\nabla E_{w_j} = \sum_n (y_n^k - \tilde{y}_n^k) x_n$

• **Multi-c Cross-En Loss: Convex/Easy compute**

Problems: • **should** accept to misclassify a few training samples. • **tries** to minimize the error-rate at training time. • **Can** result in poor classification rates at test time • **does** not guarantee the largest margin

AdaBoost (Adaptive Boosting): Boosts weak classifiers (models slightly better than random guessing) to create a strong classifier through combination.

Algorithm: • **Input:** Training data set $\{(x_n, t_n)\}$ where $t_n \in \{-1, 1\}$ for $n = 1$ to N ; number of iterations T .

1) **Initialize** data weights: for each n , $w_1 = 1/N$. • **For T = 1 to T:** 1) Find classifier y_t that minimizes weighted error: $\epsilon_t = \sum [w_t \cdot I(t_n \neq y_t(x_n))]$. 2) Evaluate classifier weight: $\alpha_t = 0.5 \cdot \log((1 - \epsilon_t) / \epsilon_t)$. 3) **Update weights:** $w_{t+1} = w_t \cdot \exp(\alpha_t \cdot I(t_n \neq y_t(x_n)))$.

Final Classifier: $Y(x) = \text{sign}(\sum (\alpha_t \cdot y_t(x))$ from $T = 1$ to T). • **Weight Adjustment:** Misclassified points gain higher weights, prioritizing them in subsequent iterations.

Training error decreases exponentially if weighted errors (ϵ_t) of component classifiers are < 0.5 : $\sum (t_n \neq h(x_n)) \leq \sum (\epsilon_t + (1 - \epsilon_t) / 4)$ for $t = 1$ to T .

• **Testing error** may rise due to overfitting.

Versatility: AdaBoost is generic and can work with various weak classifiers, not just linear.

Cascade Method: Enables efficient object detection by swiftly eliminating negative instances. Vital for real-time applications prioritizing runtime speed.

Validation Set: Vital for tuning the number of iterations and averting overfitting.

Advantages: • **Augments** accuracy via classifier combination. • **Automated** classifier weight handling. • **Adaptable** to various classifier types. • **Conducts** feature selection, emphasizing critical features.

Disadvantages: • **Sensitive** to noise and outliers. • **Susceptible** to overfitting if weak classifiers are too complex or parameters aren't well-tuned.

Applications: Prevalent in face detection, spam filter, medical diagnostics, classification tasks.