

Edge detection

Grad: in first 't' its the max in " zero, **Img not continuous** so fit surf, compute grad for surf.
Description: **Normal:** direction of the max intensity. **direction:** perpendicular to the normal
Strength: speed of intensity variation across the edge (derivative in normal direction)

Img.grad: gradient magnitude unaffected by orientation, not directly usable in real
 $I = |\partial I / \partial y|, \partial I / \partial x| / G = \sqrt{(\partial I / \partial y)^2 + (\partial I / \partial x)^2}$ | $\theta = \arctan(\|\partial I / \partial y\| / \|\partial I / \partial x\|)$

Noise: more noise derivative less detect edges

Fourier: Differentiating emphasizes high freq therefore noise/(derivative) **Magnified Noise**,

derivative of F is not recognisable) **DFT** is discrete equivalent of 2D Fourier

2D function f is written as sum of sinusoids **-DFT** of f with g is product of g&IDFTs.

$e(2\pi i(u+xv))$ **Frequency** = $\sqrt{u^2+v^2+2}$ larger=more lines

- Removing high freq for smooth result:
- $u + v^2 \geq T$ detect high frequencies.

Convolution: Faster because dg/dx precomputed (Kernel=gaussian, mask)

$$m * f(x,y) = \sum_{i=0}^w \sum_{j=0}^h m(i,j)f(x-i, y-j)$$

[-1 0 1]	[-1 -1 0]	[-1 0 1]	[-1 0 -1]
[0 1 0]	[0 1 0]	[0 1 0]	[0 1 0]
[1 0 -1]	[1 0 1]	[1 0 1]	[1 0 -1]

Pad with zeros

Filter operator

Gaussian: Fast kernel separable small / integral=1. **big sigma:** broader, up robustness, degrade precision **sigma=0** approximate Dirac function DFT of Gaussian: DFT of a Gaussian is a Gaussian. **finite support:** Its width is inversely proportional to the original Gaussian

Gaussians as Low-Pass Filters: The Fourier transform of a convolution is the product of their Fourier transforms: $F(g * f) = F(g)F(f)$ • If g is a Gaussian, so is F(g). if g is broad, the support of F(g) is small. **support of (g*f)** • no high-frequencies in g * f Convolving with a Gaussian suppresses the high frequencies. **separable**(1D cov fast)

$$\iint g_1(u,v)f(x-u, y-v)dudv = \int g_1(u)\left(\int g_2(v)f(x-u, y-v)dv\right)du \quad g_2(x, y) = \frac{1}{2\pi\sigma^2} \exp(-x^2+y^2/2\sigma^2)$$

Continuous Gaussian Derivatives: Img ' computed by convolving with the ' of Gaussian

Canny (Sdg,T1,T2): 1)Conv Grad strength/direction. 2)T Non Maxima

Suppression: Find local max pixel on grad direction, require check interpolated pixels p&r.

4)Hysteresis Thresholding-T: two T: high & low-pixel with edge strength above high

T is edge, below low T is not above the low threshold and next to edge is edge.

disadvantages: Choosing the right scale(sigma) is a difficult(small=more details)

advantages: Fast, Not data training needs

Texture: LIMIT / Non-local / Non trivial to measure/ Subject to deformations/Hard to characterize

Note: used in conjunction with effective Machine Learning technique.

/scale dependent phenomenon /Assign pixels whose texture $f(x,y) = \frac{1}{\sqrt{M * N}} \sum_{\mu=0}^{M-1} \sum_{\nu=0}^{N-1} F(\mu,\nu)e^{-2\pi i(\mu x/\lambda + \nu y/\lambda)}$ is similar the same values to textural image.

texel represents the smallest graphical element repeat on texture, **Microstructures** define reflectance properties **quantities/statistics** of texture can be computed from gray/ color/ statistical less intuitive, more effective. **Creating Textural:** Each pixel, compute a feature vector using image patch or filters set. **Run** classification to assign texture value to each pixel. **Textural Metrics:** Spectral metrics: Fourier transform, Angular and radial bins in the Fourier domain capture the directionality and fluctuation speed of an image texture

LINIT: DFT on small patches create bad boundary effects. **Only applicable** for uniform over large areas/ can be improved by using wavelets, but only up to a point. **More** local metrics required

Statistical Metrics: intensity and color in a region /

First order gray-level statistics: • Statistics of single pixels in terms of histograms. • Inensitive to neighborhood relationships/orientation of the underlying plane. **Edge Density** and **Direction:** Edge detection is first step . • number of edge pixels in a fixed-size region tell how busy that region. • directions of the edges also help characterize the texture Edges per unit area. • $\zeta = \text{gradient_magnitude}(p) \geq \text{threshold} \} / N$, N =unit area or region. **Edge magnitude/direction** histograms: • (HG,HT)

Second Order Measures: Histogram of the co-occurrence of particular intensity values in the image. • Specified in terms of geometric relationships between pixel pairs: **Distance**• Orientation / Contrast / Dissimilarity /

Homogeneity / Energy / Uniformly / Angular second moment / P(i,j,d,θ) Frequency / Entropy With a pixel with value j occurs distance d and orientation θ from a pixel with value i.

Parameters choosing: • window size. • direction of offset, • offset distance, what channels to use, • what measures to use. • Can be addressed using Machine Learning.

Filter Based Measures: Represent textures using responses of filters collection. **appropriate** filter bank extract useful information such as spots and edges. **Traditionally** one or two spot filters and several oriented bar filters./

Gaussian: respond to horizontal and vertical edges, can be used to compute higher-order image derivatives.

For every pixel, compute responses vector to each filter / **small scale** =local details/ **large scale** = larger details. **Gabor Filters**: products of a Gaussian filter with oriented sinusoids, pairs of a symmetric filter and an anti-symmetric filter / **filter bank** is produced by varying the frequency, the scale, and the filter orientation

$G_{xy}(x,y) = \sin(k_x x + k_y y) \exp\left(-\frac{x^2}{2\sigma_x^2}\right) \quad G_{xy}(x,y) = \cos(k_x x + k_y y) \exp\left(-\frac{y^2}{2\sigma_y^2}\right)$ where k_x and k_y determine the spatial frequency and the orientation of the filter and σ determines the scale. Perform classification on output of Gabor: **Decision Forests**, **ConvNets** for every pixel a feature vector containing output of intermediate layers., **Feature Maps**(look very Gabor like, requires a large training), **U-Net**

Shape from shading:

I. Reflectance Maps A. Lambertian Reflectance

reflectance between intensity and the dot product of the light direction and the normal vectors. **Lambertian Reflectance model**, this relationship linear, perfectly diffuse reflecting surface.

II. Variational Methods

inverse 3D modeling, including the recovery of surface normals and the 3D surface itself.

A. Inverse Variational optimizes data fidelity, smoothness, integrability terms. It recover normals and recover the 3D surface.

B. Direct Recovery direct recover 3D instead first recovering normals. necessitates solving 2-order partial differential equation.

Both need certain boundary conditions to function properly.

III. Variational to Deep: Normal Map Stream allowed generation of depth and normal maps/ data-driven approach provides a means to handle complex behaviors and non-linearities more effectively.

Ambiguities in 3D Modeling 3D modeling, while powerful, comes with its share of challenges and ambiguities.

I. Bas-Relief Ambiguity Transform normals, not affect image , result in normals being non-integrable,good for Monge surfaces,

II. Convex/Concave Ambig : by known light ,3D shape ambiguous. convex/concave produce identical img, ambiguity surface.

Making the Shape-from-Shading (SfS): SfS made well-posed by leveraging perspective projection /radiance models /deep

I. Perspective Projection and Radiance

By using a perspective projection model and considering the distance to the light , the ambiguities in the SfS problem can be mitigated. makes the problem well-posed, albeit more computationally complex .also makes assumption of constant albedo.

II. Leveraging Deep Learning potential to overcome the limitations posed by the variational approach and the assumptions about albedo. This technology uses understand and learn from the data, offering a more robust solution to the SfS problem.

Definition: $h(x, x_2) = -\sum g(x_k) + \sum (x_{k,x} x_{k+1}) / r(x_{k,x}, x_{k+1}) = \delta(x_{k,x})$, $\delta(x_{k,x})$

Live Wire: 1)define start/end points / Dynamic programming(N^2) / 1-D 1) Find min h(x, x_2..) where : $h(x, x_2..) = r(x, x_1) + \sum (r(x_i) + r(x_{i+1}))$

2) $T_1(x_2)=\min(r(x, x_1)+r(x_1, x_2), T_2(x_3)=\min(r(x_2, x_3)+r(x_3, x_4)) \dots \min(h(x_1, x_2..) = \min(r(x, x_1) + fn-1(x))$

2-D: (start point), L (List active nodes), C(u,v) (local cost u>v), (d(v)) (Total cost s>v): 1) init: d(s)<0 and d(u)< inf, for u,s, T=0, v=s

2) Loop: T< TU(v), for all v>u edges such u not in T, if d(v)+C(u,v) < d(u) then (d(u) < d(u) + C(u,v)) end end, v= argmin_w notin T(d(w)

-Can be integrated for user correct mistakes. **Limits:** optimal path not the best, hard impose global constraints, cost grows exp with dim

Must often look for local as opposed to global optimum using gradient decent tools

Snake: Max the grad along curve, Min Energy ->can generalizes to handle sophisticated models/ **Embed curve in viscous mid to solve**

Weighting coefficient

$E = E_0 + 1/2X^t K X + 1/2Y^t K Y$

$X = [x_1, \dots, x_N]^t$

$Y = [y_1, \dots, y_N]^t$

$K = \begin{bmatrix} 1 & -1 & 0 & \dots & 0 \\ -1 & 2 & -1 & \dots & 0 \\ 0 & -1 & 2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$

Ribbon Snake: the wrie don't change quickly $E = E_0 + 1/2X^t K X + 1/2Y^t K Y + 1/2W^t K_W W$

Level Set: define T, $x \in Z \subset \Omega$. Consider the as zero level set of a surface. $f(x)$ is speed at surface deforms

$0=(x, y) \in \partial \Omega$ properties:Converges towards circles. Total curvature decreases. Number of curvature extrema and zeros of curvature decreases. Relationship with Gaussian smoothing: Analogous to Gaussian smoothing of boundary over the short run, but does not cause self-intersections or overemphasize elongated parts. Can be implemented by Gaussian smoothing the function of a region.

Hugo: input:Canny edge points. Gradient magnitude and orientation. Given a parametric model of a curve. Map each contour point onto the set of parameter values for which the curves passes through it. Find the intersection for all parameter sets thus mapped.

Algorithm: Canny R-Table,possible reference points initialized to zero. For each edge point,Compute possible centers: for each table entry, compute $x_{\text{center}} = x + r^2 \cos(\theta)$ and Increment the accumulator array / **R-table:**Set of potential displacement vectors α given the boundary orientation f. Notes: can perform least squares / Instead of indexing displacements by gradient orientation, index by "visual codeword" / Limitations: Computational cost grows exponentially with parameters number / works Only on shape that can be defined by small parameter number / Approach is robust but lacks flexibility.

Graph:Generate min distance graph $O(N \log N)$,works in different situations, **Node:** Pixel/ **Edges** between to pixels /**Min Span Tree**

Deep U-Net / AlexNet / Use same network to progressively refine the results keeping the number of parameters constant

Delineation Steps 1. Compute a probability map. 2. Sample and connect the samples. 3. Assign weight to paths. 4. Retain the best paths. Human annotations Correcting: jointly train the network and adjust the annotations while preserving their topology.

Short: Edge is noisy / combination of graph-based techniques, machine learning, and semi-automated tools is required.

Segmentation: Same statistical properties -> same object / Help recognition ,tracking ,compression

Hard: basic image operations not suffice/ basic image operations do not suffice /not a Single Answer

Homogeneity:Uniform gray-level/RGB/TEX/motion/depth statistics, parametric surface can fitted.

Region growing: Algorithm:Pre: Labeled/ Unlabeled/ T , Given a set of regions A1, ... , An, let T = { $x \in \cup A_i$: $N(x) \cap \cup A_j \neq \emptyset$ }, the set of unlabeled pixels that are neighbors of already labeled ones, d be a metric, such as $\delta(x) = |g(x) - \text{mean}(\{A_i(x)|g(y)\}|$.Until all pixel labeled:

1. Represent T as a sorted list, the SSL 2. Label the first point in T.3. Add the neighbors to the SSL.

Limit: 1. Region depends in which pixels taken 2. homogeneity is noise sensitive

Histograms: similar pixels appear as bumps/ Split histogram at local-min / Label pixels according which bump

Recursive Splitting: Compute histogram/Smooth histogram/less peaks/find peaks separated by deep valleys/ Group pixels into connected regions/Smooth these regions/iterate(until no clear mins) **Limit:** T hard to choose

T Not always can be find in the histogram

Local Histograms: Compute local histograms on a coarse grid. • Use them instead of a global one.

limit not neighborhood relation / T=2, / gray- both side belong to same peak so **boundaries not found**

Saturation His: RGB,HSV, CIE LAB Color Space: three coordinates, one "brightness" two chromaticity.

Alg: Compute multiple his for each segment. • Use one to split each segment..• Repeat on result smaller segs.

Segmentation Ke Clustering: Each pixel has 2 spatial coordinates and 1 gray level or 3 color components.

• 1D space (G); 3D space (x,y,G), (H,S,V), (L,a,b); • 5D space (x,y,L,a,b).• each cluster compact as possible.

Given a set of input samples: • Group the samples into K clusters. • K is assumed to be known/given.

cluster has centroid(μ_k)=mean of points in that cluster. min sum squared distances to each point to centroid.

ALG: Randomly initialize centroids. Repeat until convergence: Assign points to nearest. Update the centroids.

Note: Works in any dimensional space. Sensitive to initial conditions; multiple initializations are advised.

Application in Image Processing: GRAY-LEVEL ONLY (1D): Clustering based on pixel intensity.

Color Only (3D): Clustering based on pixel colors/ XY + Color (5D): Spatial and color information combined to form superpixels. **Superpixels:** Homogeneous segments of an image. / Useful for reducing img complexity

assigned probability: statistics each superpixel. Train classifier to determine probability of superpixel

Use probabilities to generate segmentations using graph-based techniques or other methods.

Graph: Images -> graphs (G,V,E) with pixels = vertices (V) / edges (E) = weighted based pixel similarity.

Algorithm: **Graph Representation**

Parse img into a pixel value matrix. Treat pixel as a node in a graph. Draw edges between adjacent nodes.

Graph-Cut: total edge weight needed to cut graph into two parts. **Min-Cut:** cut of minimum weight

-Algorithm: **Min-Cut/Max-Flow:** Initialize flow to 0. Select two regions in the img define: source (S) & sink (T).

Treat each pixel as node, and connect them to S and T. Find a path from the source to the sink.

Add the path flow to the total flow. Repeat until no path from the source to the sink remains in the residual graph.

-Note The set of edges whose removal disconnects the source from the sink constitutes the Min-Cut.

This segmentation divides the image into two distinct regions based on pixel similarity.

Trivial Cut: This approach favors shorter cuts, potentially leading to bad solutions, must therefore be controlled.

Interactive ST Min-Cut: guessing initial linkages between S and T. $E(y|x, \lambda) = \sum_i \psi(y_i|x_i) + \lambda \sum_{(i,j) \in \mathcal{E}} \phi(y_j, y_i|x_j, x_j)$

-Object Classification and Loss Function Minimization given set of supervoxels [y1,...,yn], E= negative log-likelihood of the labels being correct.

Supervoxel Classification: strong edges from supervoxels to: source high probabilities / weak edges low prob

Classifier Training: Develop classifiers to weight edges according to supervoxel class similarities.

Loss Function Minimization: Reduce negative log-likelihood for correct labels in a supervoxel set.

Tri-class Modeling: Achieve global optimum by categorizing objects as 'Inside', 'Boundary', or 'Everything else'.

Graph-Cut on Standard Images: Utilize 'Graph-Cut' for insights in regular images.

Interactive Foreground Extraction: Use K-means for color distributions and Graph cuts for iterative segmentation of foreground and background.

I. Photometric Stereo: technique utilized for estimating the shape of a surface by observing it under known lighting conditions. **II. Influence of Light Source** Quantity in Photometric Stereo

III count of light sources significantly affects the process of surface shape determination:

A. Single Light Source many potential normals for each image point, leading to ambiguities.

B. Two Light Sources: presence may decrease ,not completely resolve ambiguities.

C. Three Light Sources: good eliminate ambiguities, even when the albedo (reflectivity) unknown.

III. Mathematical Formulation in Photometric Stereo Lambertian model: represents intensity (I) as the product of albedo (α) and the dot product of light source direction (L) and normal vectors (N).

3-vector(M) estimated by solving a 3x3 linear system, transform this to over-constraint problem,

adding more light sources can make the solution more robust against noise.

IV. Considerations for Shadows/Specularities: **A. Handling Shadows** ,Shadowed pixels for a specific light bad for Lambertian model and reducing contributions because lower intensities.

B. Accommodating Specularities Specularities,mirror-like, reflectance map accommodating specularities more complex due to incorporation of both diffuse(scattered) &specular(mirror-like).

V. Inferring Shape from Specularities utilized to infer surface normals. effective for highly

specular or shiny objects ,specularities do not fully constrain all the degrees of freedom.can used to additional info/alternatively remove noise.

assumptions: constant albedo/ absence of interreflections/ shadows/ specularities/ In single

image shape-from-shading most pronounced when it's used alongside other info sources.

VII. Key Angles in Photometric Stereo

Angle of Incidence (i): angle between the surface normal and the direction of the incident light ray.

Angle of Emissance (e): angle between the emitted light ray and the surface normal.

Phase Angle (g): Denotes the angle between the incident and emitted light rays.

More than three light increasing robustness against noise,each light providing further info solution

IX. Lambertian + Specular Reflectance Map: surfaces that exhibit both Lambertian and specular, representing a complex scenario.

S-Specular if object has glossy finish/mirrored surface, specularities give infer normals help info.

Shape from Shading Steps: Image: Use an image of a scene with known lighting conditions. **Model:** Assume a reflectance model for the objects in the scene. **Estimate:** Derive depth and surface orientation information from the intensity of the pixels.

Reflectance Map: Create: Generate a map relating surface orientations to intensities.

Use: Use map to estimate orientations in the scene.

Specular Model: Assumption: Surfaces reflect light in mirror-like manner. **Challenge:** Hard to estimate surface information due to high variability.

Lambertian Model: Assumption: Surfaces scatter light uniformly in all directions.

Benefit: Simplifies the estimation of surface information.

Body Reflection: Concept: Reflection depends on surface orientation and viewing angle. **Challenge:** Requires complex modelling for accurate surface estimation.

Variational Method: Approach: Optimizes a cost function relating intensity and surface orientation. **Benefit:** Can handle more complex scenarios and lighting conditions. **Advantages of Shape from Shading:** **Single Image:** Can infer 3D information from a single image. **Detail:** Can capture fine details not visible in stereo or motion-based methods. **Disadvantages of Shape from Shading:** Assumptions: **Makes**

strong assumptions about light and surface properties. **Illumination:** Sensitive to changes in illumination and reflectance properties.

$$\int \int \left(I(u, v) - Ref(p, q) \right)^2 + \lambda \left[\left(\frac{\partial p}{\partial u} \right)^2 + \left(\frac{\partial p}{\partial v} \right)^2 + \left(\frac{\partial q}{\partial u} \right)^2 + \left(\frac{\partial q}{\partial v} \right)^2 \right] + \mu \left[\frac{\partial p}{\partial v} - \frac{\partial q}{\partial u} \right]^2 \epsilon$$

Data term	Smoothness term	Integrability term
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Shape from Texture

Shape from X: Deals with surface orientation or shape recovery from image texture, based on texture deformation due to surface curvature.

Structural Shape Recovery: Assumes texture resides on surface with no thickness, computation Perspective, Paraperspective, Orthographic projections.

Perspective Projection: Involves pinhole geometry without image, distortion is anisotropic, depends on depth and surface orientation.

Foreshortening: Depth versus orientation principle, with object scaling and foreshortening as special cases.

Orthographic Projection: Includes Tilt and Slant derived from image direction and compression extent.

Paraperspective Projection: A generalization of orthographic projection involving parallel projection followed by scaling.

Texture Gradient: Central to statistical shape recovery process.

Statistical Shape Recovery: Measures texture density or the number of textual primitives per unit surface, assumes homogeneous texture.

Machine Learning: Involves training a regressor for depth prediction, which may result in noisy predictions.

Markov Random Field (MRF): Graph-based technique to enforce consistency.

Deep Learning with MRF: Combines deep learning with MRF for consistency.

Enforcing Task Consistency: Training networks to predict multiple things to increase robustness.

Diverse Training Database: Very diverse training databases improve results.

Transformer Architecture: Good at modeling long-range relationships, but flattening patches loses some information.

Strengths and Limitations: Emulates an important human ability with limitations including regular texture requirement, strong assumptions, though deep learning can help weaken them.

Ambiguities: Repetitive patterns/ textureless/ occlusions cause problems. **Occlusions:** Some points are only visible in one image. They not matched. **Ignoring Occluded Pixels:** pixels haven't corresponding pixel in other img.

Left-right consistency test Disparity Map: Black pixels indicate no disparity.

Combine Disparity Maps: Merge disparity maps and smoothing result map.

Solving Variational Problem: Discretize integral and solve linear problem.

Real-T Implementation: duplicated computations can be implemented (faster)

window size: Speed is independent to w.s. **Small windows:** Good precision

Sensitive to noise. **Large windows:** Diminished precision. Increased noise

Replace Normalized Cross Correlation: Siamese nets designed to return a similarity score for potentially matching patches.

Comparative Results: Improved performance on test data but how it generalizes to unseen imgs. no necessity for this test (not clear yet)

3D Point Cloud: Disparity map transform each triplet (u, v, d) into 3D point (x, y, z) , resulting in a 3D point cloud.

Merge the 3D point clouds: dense models when have enough imgs.

Scale-Space Revisited: Gaussian pyramid. Difference of Gaussians.

Fronto-Parallel Assumption: disparity assumed be same over correlation window, equivalent to constant depth. **Multi-view reconstruction setup:**

Adjust correlation window shapes to handle orientation.

Scene Flow: Correspondences across cameras and across time.

Refining using Shape from Shading: Shape-from-shading can be used to refine shape and provide high-frequency details.

Precision vs Baseline: Beyond a certain depth stereo stops being useful. Precision is proportional to baseline length.

Short baseline: good matches /few occlusions /poor precision

Long baseline: harder match /more occlusions /better precision.

Multi-Camera: 3 cam: robustness/precision. 4 cam: more redundancy.

Structured Light: R pattern and measures depth from its distortion.

Faces Low Resolution Videos: No calibration data/little texture/difficult light.

Model Based Bundle Adjustment: Adjusting PCA to minimize objective func

func to yield accurate face reconstruction from lowResolution images (also Old Movie).

Energy Minimization: Minimize the disparity continuity using graph-based seg.

Graph Cut for Stereo: Stereo is a labeling problem, use graph cut.

Assigning Edge Weights: Assign a weight that is proportional $|I(x+1,y)-I(x,y)|$.

Minimizing the Objective Function: Graph cut algorithm. Guarantees absolute minimum only when there are only two possible disparities.

α -Expansion: Nodes having label different than α can either keep or switch to α .

Graph Cut Algorithm: Start with an arbitrary labeling. Find the α -Expansion that minimizes the function. Update graph by adding /erasing edges.

Quit when no expansion improves the cost. Induce pixel labels.

NCC vs Graph Cut: Normalized correlation Graph Cut.

NCC vs Graph Cut: Left image true disparities. Normalized correlation Graph Cut.

NCC vs Graph Cut - Strengths and Limitations: method for depth recovery (real-time). Requires multiple views. Only to reasonably textured objects).

Shape from Motion Steps:

• Capture: Take multiple images of a scene from different viewpoints.

• Detect Features: Identify key features in each image.

• Match Features: Match features across images to track movement.

• Estimate Structure: Use matched features and camera movement to estimate the 3D

1. **Advantages:** Flexible: Works with unstructured image sets.

◦ Automatic: Estimates camera parameters autonomously.

◦ Cost-Effective: Cheaper 3D mapping option.

2. **Disadvantages:** Dependent: Quality relies on identifiable image features.

◦ Scale Ambiguity: Absolute scale usually unknown.

3. **Additional Points:**

◦ Photogrammetry Type: Constructs 3D from 2D images.

◦ Movement: Uses object or/and camera motion.

◦ Applications: Useful in archaeology, architecture, robotics.

◦ Bundle Adjustment: Optimization method for refining 3D positions.

Depth Prediction:

• **Capture:** Get two images, stereo cameras. (Known: camera parameters)

• **Match:** Identify same features in both images (Known: image features)

• **Calculate Disparity:** Positional difference for features. (Known: feature positions)

• **Compute Depth:** Disparity and camera parameters gives depth.

Advantages:

• **3D Perception:** Understand depth in scenes

• **Matching Difficulty:** Identifying same feature across images is challenging.

Occlusions: Some areas appear in only one image.

1. **Triangulation:** A process of determining the location of a point by measuring angles to it from known points at either end of a fixed baseline.
 2. **Geometric Stereo:** A technique used in computer vision to estimate depth information from two or more images taken from different points of view.
 3. **Epipolar Geometry:** Refers to the geometric constraints between two views of a 3D scene. It defines the term **epipole**, which is the line on which the corresponding point must lie, and the **epipolar**, which is the intersection of line joining optical centers and image plane.
 4. **Calibration Grid:** A tool used to infer projection matrices and compute epipolar lines, often taking pictures of a grid with each camera to achieve this.
 5. **Rectification:** A process that involves reprojecting images onto a common image plane, typically used to simplify the process of finding matching points between images.
 6. **Disparity:** Disparity refers to the difference in the horizontal position of a point in two images, i.e., the left image and the right image. This difference is due to the slightly different viewing angles of the two cameras. **Disparity Proportional to depth** →
 - It can be represented mathematically as $d = x - x'$, where x and x' are the locations of the same point in the left and right images, respectively. 7. **Depth:** Depth, or depth map, is the distance from the camera to each pixel (point) in the image. It gives a sense of the third dimension (z-direction) in 2D images. In other words, depth provides information about how far an object is from the viewer or the camera.
 - The depth of a point can be calculated from its disparity, given the baseline distance (B) between the two cameras and the focal length (f) of the camera. The formula is $Z = (B * f) / d$.
 8. **Correlation and Normalized Cross Correlation:** Measure of similarity between two signals, in this case, image patterns. Normalized cross correlation involves normalization to make the measure more robust to variations in intensity.
 9. **Occlusions:** Areas in images that become hidden in another view. These regions pose a challenge as they cannot be matched in stereo vision.
 10. **Window-Based Approach to Establishing Correspondences:** Involves comparing pixel windows between two images and selecting the best matching window based on a cost function.
 11. **Variational Approach:** This is used for solving an optimization problem where the aim is to find the disparity map that minimizes a certain energy function.
 12. **3D Point Cloud:** Collection of data points in 3D space, which are typically produced by stereo imaging or lidar.
 13. **Multi-view Reconstruction:** An approach that uses multiple images of a scene to reconstruct 3D information. It adjusts correlation window shapes to handle orientation.
 14. **Short vs Long Baseline:** Short baselines offer good matches with few occlusions but poor precision, whereas long baselines are harder to match with more occlusions but offer better precision.
 15. **Kinect: Structured Light:** A technique used by Kinect to determine depth information. It projects an infrared pattern and measures depth based on its distortion.
 16. **Energy Minimization:** Method used to find the disparity map that results in the best match between images while maintaining smoothness. It often involves Graph Cut or other optimization techniques.
 17. **Face Reconstruction:** Techniques used to construct a 3D model of a face from images, typically involving fitting a model to the image data.
- Sum of Squared Differences (SSD):**
- 1/Calculate SSD: $SSD = \sum (I_1(x) - I_2(x))^2$ // Used for similarity measure in image//3 Lower SSD = Higher similarity. // 4 Minimize SSD over local window for disparity.
- Knowns:** Two images: I_1, I_2 . SSD formula. // **Unknowns:** Pixel location similarity. // **Goal:** Find image similarity via SSD. Minimize SSD locally for disparity map.
- Fundamental Matrix F**
- Epipolar constraint:** $[e_{11} \ e_{12} \ e_{13}] [x_r] [e_{21} \ e_{22} \ e_{23}] [y_r] = 0$
- Rewriting in terms of image coordinates:
- Fundamental Matrix F**
- $E = K_i^T F K_r$ $E = T_x R_y$
- $\bar{u} = M_{ext} \bar{x}_r$
- Combining the above two equations, we get the full projection matrix P : $\bar{u} = M_{intr} M_{ext} \bar{x}_w = P \bar{x}_w$
- Projection Matrix P**
- Camera to Pixel World to Camera
- $\begin{bmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \end{bmatrix} = \begin{bmatrix} f_x & 0 & x_c \\ 0 & f_y & y_c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$
- $\begin{bmatrix} \bar{u}_c \\ \bar{v}_c \\ \bar{w}_c \end{bmatrix} = \begin{bmatrix} r_1 & r_2 & r_3 & t_x \\ r_4 & r_5 & r_6 & t_y \\ r_7 & r_8 & r_9 & t_z \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix}$
- $\bar{x}_w = M_{ext} \bar{x}_c$
- Left Camera Right Camera
- $\bar{x}_w = M_{ext} \bar{x}_c$
- repetitive patterns, textureless occlusions can cause problems
- $X = RX + T$ so $X^T \cdot (RX) = 0$
- Essential Matrix: $E = T_x R$, rank=2, free=5.
- Fundamental Matrix: $F = K^{-T} E K^{-1}$, rank=2, free=7 and we have $P^T F P = 0$.
- Alg. for finding F: collect 8+ constraints, normalize, solve using SVD, rearrange and find a rank 2 approximation. A constrain is: $[u' \ v' \ 1] F [u \ v \ 1]^T = 0$

