1)

$$I(x;Y) = H(x) - H(X|Y) - \sum_x P(x) \cdot \cdot \ln(P(x)) + \sum_{x,y} P(x,y) = \ln(P(x,y))$$

$$1 \longrightarrow \sum_x P(x) \ln_2 P(x) + \sum_{x,y} \ln \frac{p(x,y)}{p(y)} = -\sum_x P(x) \ln P(x) + \sum_{x,y} \ln \frac{P(x,y)}{p(y)}$$

Hermetic probability rule $\longrightarrow = \sum_x \left(\sum_y P(x,y) \right) \ln P(x) + \sum_{x,y} P(x,y) \ln \frac{p(x,y)}{p(y)}$

$$= -\sum_{x,y} P(x,y) \ln(p(x)) + \sum_{x,y} P(x,y) \ln \left(\frac{P(x,y)}{p(y)} \right)$$

$$= -\sum_{x,y} P(x,y) \ln \left(\frac{P(x) - P(y)}{P(x,y)} \right)$$

1)B)

if x and y are not depends $\rightarrow$ $P(x \cap y) = P(x) \cdot P(y)$

$$H(X|y) = -\sum_{x,y} P(x,y) \ln P(X|y)$$

$$= -\sum P(x) \cdot P(y) \ln P(x) = \sum_x P(x) \ln(P(x)) \left(\sum_y P(y) \right)$$

$$= \sum_x P(x) \ln(P(x)) = H(x)$$

$C)$

כאשר $f(x)$ חח״ע ועל אז קיימת $f^{-1}(x)$

נסמן $y = f^{-1}(x)$ כעת קיבלנו;

$$P(Y=y) = p\left(f\left(x\right)\right) = f\left(x'\right) = P(x=x')$$

$$\rightarrow H\left(x,y\right) = -\sum_{x,y} P(X=x, Y=y)\ln(P(X=x, Y=y)$$

$$= -\sum_{x\neq x}, P(X=x, X=x\cdot)\ln(P(X-x, X=x'))$$

$$if x \neq x'' \rightarrow p(x=x, X=x') = 0$$

$$= -\sum_{x} P(x=x, x=x)\ln(P(x=x|x=x))$$

$$= -\sum P(x=x)\cdot\ln\left(1\right) = -0$$

$$I\left(x;y\right) = H\left(x\right) - 0 = H\left(x\right)$$

$2)A)$

2ב) Y הוא משתנה מקרי מייצג

$$Y = \begin{cases} 0 & \text{המטבע בכיס} \\ 1 & \text{המטבע לא בכיס} \end{cases}$$

$X = \{0, 1, 2, \ldots\ldots 100\}$ ← 0 ← בכיס , 1 ← במקום 1 ... 100 ← במקום 100

$2)B)$

$P(x=0)=0.99$ $P(Y=0)=0..99$	$P(X=x)=0.0001$ $P|Y=1)=0.01$	$p(y|x)$ $x \longrightarrow$ בכל מקום אחר $y \longrightarrow$ בכיס

$P(x,y)$

$$P(x=0|Y=0)=1 \wedge P(x=0|Y=1)=0$$

$$P(x=x|Y=0)=0 \wedge P(X=x,Y=1)=0.01; x \in (100]$$

$p(y|x)$

$$p(y=0,X=0)=1 \wedge P(Y=0|X=x)=0$$

$$p(y=1|x=0)=0 \wedge p(y=1|x=x)$$

$P(x,y):$

$$P(x=0,y=0)=0.99$$

$$P(x=x|y=0)=0, x \in (100]$$

$$p(x=0,y=1)=0$$

$$P(X=X,Y=1)=0.0001$$

3)

$$H(x) = -\sum_{x} p(x)\,|n\,(P\,(x))$$

$$= -P\,(x-0)\ln(P(x=0)) - \sum_{x=2}^{100} P(x=x)\ln(P(x=x))$$

$$= -P(X=0)\ln P(x=0) - (100)\cdot P\,(x-1)\ln Px-1)$$

$$\simeq -\,(0.99)\,(-0.0145) - 100\,(0.0001)\,(-13.2877) \simeq 0.1472$$

$$H\,(Y) = -\sum_{y} P\,(y)\ln P\,(y) = -\,(0.99)\cdot\ln\,(0.99) - (0.01)\cdot(\ln 0.01) \quad \simeq 0.0808$$

$$H\,(x|\,y) = -\sum_{x} P(X=x, Y=y)\ln(P(X=x|Y=1))$$

$$= -P(X=0,Y=1)\ln((X=0|Y=1)) - (100)\,P(x=1,Y=1)\ln(P(x=1,Y=1))$$

$$\simeq 0.001\ln\,(0.01) \simeq 0.0664$$

$$H\,(Y|x) = -\sum_{x}(p(x=x,Y=0))\ln(P(Y=0|x=x))$$

$$-\sum_{x}(P(x=x,Y=1))ln(p(Y=1|X=x))$$

$$= -P(X=0,Y=0)\ln(P(Y=0|x=0) - (100)\cdot P(x=1,y=0)P(y=0|x=1)$$

$$= -0.00\ln\,(1) - (100)\cdot 0 = 0 = H\,(Y|X)$$

According the definition

$$I\,(x;y) = H\,(x) - H\,(X|y) = 0.1472 - 0.0664$$

$$I\,(Y;x) = H\,(Y) - H\,(Y|x) = 0.0808 - 0 = 0.0808 = 0..0808$$

5)A)

$$(X, 0 \oplus 0) = (X, 0) \leftarrow (x, 0) \leftarrow f\left(\overline{X}\right) = 0$$

$$(x, 1 \oplus 0) = (x, 1)$$

5)B)

$$f(x) = 1 \rightarrow \begin{array}{l} (x,0) \rightarrow (x, 0 \oplus 1) = (x, 1) \\ (x,1) \rightarrow (x, 1 \oplus 1) = (x, 0) \end{array}$$

5)c)

$$Uf\left(|x\rangle \otimes |-\rangle\right) = \frac{1}{\sqrt{2}} Uf\left(|x\rangle \otimes 10\rangle\right) - \frac{1}{2} Uf(|x\rangle \otimes 11>)$$

$$\frac{1}{\sqrt{2}}\left(|x\rangle \otimes |0\rangle \cdot f(x)\right) - \frac{1}{\sqrt{2}}\left(|x\rangle \otimes |1 \otimes F(x)\rangle\right) = \#$$

$$\# = \frac{1}{\sqrt{2}}\left(|x\rangle \otimes |0 \oplus 0\rangle\right) - \frac{1}{\sqrt{2}}\left(|x\rangle \otimes |1 \oplus 0\rangle\right) \quad \leftarrow \quad |f(\overline{x}) = 0$$

$$\frac{1}{\sqrt{2}}(|x> \otimes |0>) - \frac{1}{\sqrt{2}}\left(|x\rangle \otimes |1\rangle\right) \quad \leftarrow \quad f(\overline{x} = 1)$$

$$\# = \frac{1}{\sqrt{2}}\left(|x\rangle \otimes |1 \oplus 0\rangle\right) - \frac{1}{\sqrt{2}}\left(|x\rangle \otimes |1 \oplus 1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}(|x> \otimes |1\rangle) - \frac{1}{\sqrt{2}}\left(|x\rangle \otimes 10\right) = -|x\rangle \otimes 1->$$

$$U_f\left(|x\rangle \otimes |-y\right) = (-1)^{f(\overline{x})}\left(|\overline{x}\rangle \otimes |->\right) \rightarrow (-1)^{f(\overline{x})}$$