Constant hazard after t_{max}

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1 Motivation

In the mNRM, a function h is used to map internal time intervals ΔT (typically sampled from an exponential distribution) to real-world time intervals Δt:

$$\Delta T = \int_{t_0}^{t_0+\Delta t} h(s)\mathrm{d}s$$

This function h is typically called the propensity function or the *hazard*. If a primitive function for h is known, evaluating ΔT for a given Δt is straightforward. To calculate Δt from a specific value of ΔT, the inverse of this primitive function would then be needed.

For some hazards, calculating ΔT from Δt is possible, but calculating Δt for a given ΔT is not. This is the case if the integral itself has an upper bound: because ΔT can be any positive number, mapping it to a finite value of Δt cannot be done.

Simulations that we are interested in involve people, which are not expected to live indefinitely. For this reason it is possible to define a particular t_{max} after which the hazard just stays constant. If t_{max} is chosen so that the person involved would be 200 years old at that point, it will not make a difference for the simulation outcome as the person will be deceased long before this t_{max}. The major benefit however is that the integral will no longer have a fixed upper limit, and therefore a mapping from ΔT to Δt should always be possible.

2 Using the modified hazard

So instead of performing calculations with the function h that we really want to use, we'll be working with a function g defined as follows:

$$g(t) = \left\{ \begin{array}{ll} h(t) & \text{if } t < t_{max} \\ h(t_{max}) & \text{if } t \geq t_{max} \end{array} \right.$$

The mapping between ΔT and Δt is then done using this modified function g:

$$\Delta T = \int_{t_0}^{t_0+\Delta t} g(s)\mathrm{d}s$$

We always know the value of t_0, so if $t_{max} < t_0$ we know that we're in the regime where we're basically using a constant hazard:

$$\Delta T = \int_{t_0}^{t_0+\Delta t} h(t_{max})\mathrm{d}s = h(t_{max})\Delta t,$$

allowing a very straightforward mapping between the two time measures. If, on the other hand, $t_{max} > t_0$, we have to take care to use the modified hazard correctly.

2.1 Mapping Δt to ΔT

We already know that $t_{max} > t_0$, and in this direction, we also have $t_0 + \Delta t$ as an input. If $t_0 + \Delta t < t_{max}$, then we simply need to calculate the same thing we would have calculated using the unmodified hazard:

$$\Delta T = \int_{t_0}^{t_0+\Delta t} g(s)\mathrm{d}s = \int_{t_0}^{t_0+\Delta t} h(s)\mathrm{d}s$$

On the other hand, if $t_0 + \Delta t > t_{max}$, we need to split the calculation in two:

$$\Delta T = \int_{t_0}^{t_0+\Delta t} g(s)\mathrm{d}s = \int_{t_0}^{t_{max}} h(s)\mathrm{d}s + \int_{t_{max}}^{t_0+\Delta t} h(t_{max})\mathrm{d}s$$

$$\Leftrightarrow \Delta T = \int_{t_0}^{t_{max}} h(s)\mathrm{d}s + h(t_{max})(t_0 + \Delta t - t_{max}) = \Delta T_{max} + h(t_{max})(t_0 + \Delta t - t_{max})$$

2.2 Mapping ΔT to Δt

In this case we have ΔT as input and we know t_0, but we don't know yet if t_{max} is larger or smaller than $t_0 + \Delta t$. However we can calculate the value of the internal time interval that corresponds precisely to t_{max}, and this is in fact ΔT_{max} from before:

$$\Delta T_{max} = \int_{t_0}^{t_{max}} h(s)\mathrm{d}s$$

If this value is larger than the input value ΔT, then we've actually not yet reached t_{max} and simply need to invert

$$\Delta T = \int_{t_0}^{t_0+\Delta t} h(s)\mathrm{d}s.$$

On the other hand, if this value is smaller than ΔT, the relevant equation again is

$$\Delta T = \int_{t_0}^{t_{max}} h(s)\mathrm{d}s + h(t_{max})(t_0 + \Delta t - t_{max}) = \Delta T_{max} + h(t_{max})(t_0 + \Delta t - t_{max}),$$

which can be solved for Δt:

$$\Delta t = \frac{\Delta T - \Delta T_{max}}{h(t_{max})} + t_{max} - t_0$$