DATA STRUCTURE

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ASSIGNMENT: 1

Q.1.) $t_{n=10} = 5sec.$

$t_{n=25} = ?$

$$kn^2 = t$$

$$k \times 100 = 5$$

$$k = \frac{1}{20}$$

$$t_{n=50} = \frac{1}{20} \times 50 \times 50$$

$$= 125 sec.$$

Q2.) $T_A = n^3$ $\quad T_B = 2n^2$

break pt:

$$n^3 = 2n^2$$

$$n = 2$$

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Q3) $f(n) = n2^n$, $g(n) = 4^n$

Applying limit rule,

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} \rightarrow \lim_{n\to\infty} \frac{n2^n}{4^n} \Rightarrow \lim_{n\to\infty} \frac{n2^n}{2^{2n}}$$

Applying L'Hospital $\Rightarrow \lim_{n\to\infty} \frac{1}{2^n \log 2}$

Hence, $f(n)$ is $O(g(n))$

Q4) Let polyno. f^n $P(n)$ and $\log(n)$ f^n,

using limit rule, $\lim_{n\to\infty} \frac{\log(n)}{P(n)}$ $\left[\frac{\infty}{\infty}\right]$

Applying L'Hosp. Rule,

$$\Rightarrow \lim_{n\to\infty} \frac{1}{n(P_n)}$$

$\Rightarrow 0$

thus $\log n$ grows slower than all func^n

let 2 log func^n $\frac{\log_a n}{\log_b (n+1)}$ and.

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Applying limit rule,

$$\lim_{n\to\infty} \frac{\log_a n}{\log_b (n+1)} = \lim_{n\to\infty} \frac{\log_a n}{\log_b n + \log\left(1+\frac{1}{n}\right)}$$

$$= \lim_{n\to\infty} \frac{\log_a n}{\log_b n}$$

$$= \lim_{n\to\infty} \frac{\ln b}{\ln a}$$

Hence all log. funn grow rate, ie, constant.

Q5.) a.) Average case (Θ)	Worst case (O)
* Performs average no. of steps on input data of n elements.	* Performs maximum no. of steps on input data of n elements.
* ~~Averaged~~	* Input is arbitrary size & output is upper bound
* Averaged over all possible inputs.	eg:- $O(n^2)$ → Quicksort.
eg:- $\Theta(n\log(n))$ → Quicksort	

Q5) b.)

Worst Case (O)	Asymptotically Bounded (Ω)
* I/P is arbitrary size & O/P is upper bound.	* Growth of a running time to within const. factors below.

Q6.)

a.) $f(n) = n^4 + \log n + 17$, $g(n) = n^4$

$$\lim_{n\to\infty} f(n) \Rightarrow \lim_{n\to\infty} \frac{n^4 + \log n + 17}{n^4}$$

$$\Rightarrow \lim_{n\to\infty} \frac{4n^3 + \frac{1}{n}}{4n^3}$$

$$\Rightarrow \lim_{n\to\infty} \frac{\cancel{4n^3}}{\cancel{4n^3}} + \frac{1}{4n^4} \to 0$$

$$\Rightarrow 1 \ \text{const.}$$

Hence funⁿ is $O(g(n))$

Q7.)

a.) $K=1$

$K=2$

$K=3$

$\vdots$

$K=n-1$

$O(n-1)$

$\rightarrow O(n)$

b.) For $i=1$ $\quad j = 2,3,\dots n$ $\quad (n-1)$

$i=2$ $\quad j=3,4,\dots n$ $\quad (n-2)$

$i=n-1$ $\quad n$ $\quad (n-i)$

$n-\{(n-1)+1\}$

$(n-1)+(n-2)+(n-3)+(n-4)\dots 3+2+1$

$\frac{n(n-1)}{2}$

$O(n)$

Q8.) Quadratic Function $\Rightarrow f(n) = n^2$

$n*n = n^2$

$1+2+3 \dots (n-1)+n = n*(n+1)/2$

Quadratic algo. are practical for relatively small problems, whenever n doubles, the running time increases fourfold

1, 4, 16, 64, 256, 1024, 4096

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Q9) $T_A = 100^n$

$T_B = n^4$

Using limit rule, $\lim \frac{n^4}{100^n}$ $\left[\frac{\infty}{\infty}\right]$

$$\lim_{n\to\infty} \frac{4n^3}{100^n \log 100}$$

$$\lim_{n\to\infty} \frac{24}{100^n (\log(100))^4} = 0$$

Hence, T_A grows faster when $n \to \infty$

Q10) $f(n) = n\log n$ $\quad g(n) = \log(n!)$

$$= \log(1.2.3.4. \ldots\ldots (n-1)n)$$

$$= \log\left(\frac{n}{2}^{n/2}\right)$$

$$= \frac{n}{2} \log \frac{n}{2}$$

$$\Rightarrow \frac{n}{2} \log n - \frac{n}{2} \log 2$$

$$\Rightarrow \frac{n}{2} (\log n - \log 2) \simeq \Theta(n \log n)$$

Hence, $f(n) \in \Theta\, g(n)$

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Q1(i) a.) $2^{n+1} + 4^{n+1}$

for Θ $c_1 g(n) \le f(n) \le c_2 g(n)$

$2^{n+1} + 2^{2n+2}$

$\Rightarrow \frac{2^n}{2} + 4 \cdot 2^{2n}$

$\Rightarrow 2^{2n}\left(4 + \frac{1}{2^{n+1}}\right)$

Higest order term $2^{2n} \Rightarrow \Theta(4^n)$

b) $(n^2+6)^8$

higest order term $= (n^2)^8$

$= n^{16}$

$\Rightarrow \Theta(n^{16})$

12.) $T_A = n^2$

$T_B = n+2$

Breaking Pt.

$n^2 = n+2$

$n^2 - n - 2 = 0$

$n^2 - 2n + n - 2 = 0$

$n(n-2) + 1(n-2) = 0$

$n = 2$