(a) Rs. 330 (b) Rs. 1665
(c) Rs. 440 (d) Rs. 365

15. Find the equation of the line on which length of the perpendicular from the origin is 5 and the angle which this perpendicular makes with the x-axis is 60° :
(a) $x\sqrt{3}+2y+8=0$ (b) $x+\sqrt{2}y-7=0$
(c) $x+\sqrt{3}y=10$ (d) none of (a),(b), (c)

16. Find the equation of the line which passes through the point (3, – 4) and makes an angle of 60° with the positive direction of x-axis :
(a) $x\sqrt{2}+y\sqrt{3}=0$ (b) $\sqrt{3}x-y=4+3\sqrt{3}$
(c) $x\sqrt{3}+y=3\sqrt{2}+5$ (d) none of (a), (b),(c)

17. Find the equation of the line joining the points of intersection of $2x+y=4$ with $x-y+1=0$ and $2x-y-1=0$ with $x+y-8=0$:
(a) $2x+3y+6=0$ (b) $3x+2y+12=0$
(c) $3x-2y+1=0$ (d) none of (a), (b),(c)

18. Find the equation of one of the two lines which pass through the point (4, 5) which make an acute angle 45° with the line $2x-y+7=0$:
(a) $x-2y=0$ (b) $7x+5y-3=0$
(c) $3x+y+8=0$ (d) $x-3y+11=0$

19. Find the equation of the straight line which passes through the point (5, – 6) which is parallel to the line $8x+7y+5=0$:
(a) $3x-5y+8=0$ (b) $7x+8y+5=0$
(c) $7x-8y+2=0$ (d) $8x+7y+2=0$

20. Find the equation of the straight line which passes through the point of intersection of the straight lines $x+y=8$ and $3x-2y+1=0$ and is parallel to the straight line joining the points (3, 4) and (5, 6) :
(a) $x-y+2=0$ (b) $x+y-2=0$
(c) $3x-4y+8=0$ (d) none of these

21. Find the length of the perpendicular from the point (3, – 2) to the straight line $12x-5y+6=0$:
(a) 5 (b) 4
(c) 6 (d) 8

22. Find the distance between two parallel lines $5x+12y-30=0$ and $5x+12y-4=0$
(a) 3 (b) 7
(c) $\frac{5}{2}$ (d) 2

23. Find the equation of the line through the point of intersection of $2x-3y+1=0$ and $x+y-2=0$ which is parallel to the y-axis.
(a) $x=1$ (b) $8x=7$
(c) $x+3=0$ (d) $x=6$

24. Find the equation of the line which passes through the point of intersection of the lines $x+2y-3=0$ and $4x-y+7=0$ and is parallel to the line $y-x+10=0$.
(a) $2x+2y+5=0$ (b) $3x-3y=10$
(c) $3x+2y-8=0$ (d) none of these

25. Find the equation of the line which passes through the point of intersection of the lines $2x-y+5=0$ and $5x+3y-4=0$ and is perpendicular to the line $x-3y+21=0$
(a) $2x+y+8=0$ (b) $3x+4y-7=0$
(c) $3x+y=0$ (d) none of these

26. Find the equation of the line through the intersection of the lines $3x+4y=7$ and $x-y+2=0$ having slope 3.
(a) $4x-3y+7=0$ (b) $21x-7y+16=0$
(c) $8x+y+8=0$ (d) none of these

27. Find the equation of the straight line which passes through the point of intersection of the straight lines $3x-4y+1=0$ and $5x+y-1=0$ and cuts off equal intercepts from the axis.
(a) $32x+32y+11=0$ (b) $23x+23y=11$
(c) $9x+18y+5=0$ (d) none of these

28. A straight line $\frac{x}{a}-\frac{y}{b}=1$ passes through the point (8, 6) and cuts off a triangle of area 12 units from the axes of coordinates. Find the equations of the straight line.
(a) $3x-2y=12$ (b) $4x-3y=12$
(c) $3x-8y+24=0$ (d) both (a) and (c)

29. Find the equations of the bisectors of the angle between the straight line $3x+4y+2=0$ and $5x-12y-6=0$.
(a) $8x+y+7=0$ (b) $16x-12y-1=0$
(c) $x+8y+4=0$ (d) both (b) and (c)

30. Find the area of the triangle formed by the lines whose equations are $2y-x=5$, $y+2x=7$ and $y-x=1$.
(a) $\frac{3}{10}$ (b) 10
(c) 6 (d) $\frac{2}{5}$

31. Find the coordinates of the orthocentre of the triangle whose vertices are (1, 2), (2, 3) and (4, 3).
(a) (2, 5) (b) (3, 4)
(c) (1, 6) (d) none of these

32. Two vertices of a triangle ABC are $B(5,-1)$ and $C(-2,3)$. If the orthocentre of the triangle is the origin, find the third vertex.
(a) $\left(\frac{7}{2},\frac{13}{2}\right)$ (b) $\left(\frac{3}{2},\frac{11}{2}\right)$
(c) $(-4,-7)$ (d) none of these

33. The area of a triangle is 5. Two of its vertices are (2, 1) and (3, – 2). The third vertex lies on $y=x+3$. Find the third vertex :
(a) $\left(\frac{2}{7},\frac{13}{5}\right)$ (b) $\left(\frac{7}{2},\frac{13}{2}\right)$
(c) $\left(\frac{9}{2},\frac{13}{2}\right)$ (d) $\left(\frac{7}{2},\frac{13}{2}\right)$ or $\left(-\frac{3}{2},\frac{3}{2}\right)$

34. A straight line L is perpendicular to the line $5x-y=1$. The area of the triangle formed by the line L and coordinate axes is 5. Find the equation of the line :
(a) $x+5y=\pm 5\sqrt{2}$ (b) $x-3y=0$
(c) $2x+y=0$ (d) $x+4y=5\sqrt{2}$

35. (1, 2) and (3, 8) are a pair of a opposite vertices of square. Find the diagonals of the square passing through (1, 2) :
(a) $x-2y=1$ (b) $2x+7y=0$
(c) $3x+2y+7=0$ (d) $3x-y=1$

Answers

INTRODUCTORY EXERCISE- 21.1

1 (b)	2. (d)	3. (d)	4. (b)	5. (c)	6. (d)	7. (a)	8. (a)	9. (b)	10. (a)
11. (b)	12. (b)	13. (c)	14. (d)	15. (c)	16. (b)	17. (a)	18. (c)	19. (c)	20. (c)
21. (c)	22. (b)	23. (a)	24. (a)	25. (b)	26. (c)	27. (b)	28. (c)	29. (a)	30. (d)

INTRODUCTORY EXERCISE- 21.1

1 (a)	2. (d)	3. (c)	4. (b)	5. (b)	6. (c)	7. (d)	8. (b)	9. (c)	10. (c)
11. (a)	12. (d)	13. (d)	14. (c)	15. (c)	16. (b)	17. (c)	18. (d)	19. (d)	20. (a)
21. (b)	22. (d)	23. (a)	24. (b)	25. (c)	26. (b)	27. (b)	28. (d)	29. (d)	30. (a)
31. (a)	32. (c)	33. (d)	34. (d)	35. (d)					

Hints & Solutions

Introductory Exercise 21.1

1. The point $(-2, 3)$ lies in the second quadrant.
2. The point $(2, -3)$ lies in the fourth quadrant
3. Distance between two points $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Let $(x_1, y_1) \equiv (-5, 3)$ and $(x_2, y_2) \equiv (3, 1)$

$\therefore$ Required distance $= \sqrt{(3+5)^2 + (1-3)^2}$

$= \sqrt{64 + 4} = \sqrt{68} = 2\sqrt{17}$ unit

4. Let $A \equiv (4, 3), B \equiv (7, -1), C \equiv (9, 3)$

$$AB = \sqrt{(7-4)^2 + (-1-3)^2} = \sqrt{25} = 5$$
$$BC = \sqrt{(9-7)^2 + (3+1)^2} = \sqrt{20} = 2\sqrt{5}$$
$$CA = \sqrt{(4-9)^2 + (3-3)^2} = \sqrt{25} = 5$$

$\therefore \quad AB = CA = 5$

Hence ABC is an isosceles triangle.

5. Let $A \equiv (4, 4), B \equiv (3, 5), C \equiv (-1, -1)$

Then $AB = \sqrt{(3-4)^2 + (5-4)^2} = \sqrt{2}$

$$BC = \sqrt{(-1-3)^2 + (-1-5)^2} = \sqrt{52}$$
$$AC = \sqrt{(-1-4)^2 + (-1-4)^2} = \sqrt{50}$$

$\therefore \quad AB^2 + AC^2 = BC^2$

Hence by the Pythagorus Theorem, ABC is a right angled triangle.

6. Let $A \equiv (7, 9), B \equiv (3, -7)$ and $C \equiv (-3, 3)$

Then $AB = \sqrt{(3-7)^2 + (-7-9)^2} = \sqrt{272}$

$$BC = \sqrt{(-3-3)^2 + (3+7)^2} = \sqrt{136}$$
$$AC = \sqrt{(7+3)^2 + (9-3)^2} = \sqrt{136}$$

$\therefore \quad AB^2 = BC^2 + AC^2$

Hence by Pythagorus Theorem, triangle ABC is a right angled triangle.

Also, since $BC = CA$, hence ABC is an isosceles triangle.

Thus ABC is a right angled isoceles triangle.

7. The coordinates of the mid points of AB

$$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-3+5}{2}\ \frac{2+4}{2}\right) = (1, 3)$$

8. Here $(x_1, y_1) = (-3, 2)$ and $(x_2, y_2) = (5, 4)$

The coordinates of the point which divides AB in the ratio $2:3$

$$= \left(\frac{2 \times 5 + 3 \times -3}{2+3}, \frac{2 \times 4 + 3 \times 2}{2+3}\right) = \left(\frac{1}{5}, \frac{14}{5}\right)$$

9. The required coordinates of the point which divides the join of $(2, 4)$ and $(6, 8)$ externally in the ratio $5:3$ are $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$

Here, $m:n = 5:3, \quad (x_1, y_1) = (2, 4), \quad (x_2, y_2) = (6, 8)$

Hence the required co-ordinates $= (12, 14)$.

10. Let the co-ordinate of the centroid of ΔABC be (x, y) then

$$(x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$
$$= \left(\frac{3+2-2}{3}, \frac{1+3+2}{3}\right) = (1, 2)$$

11. Let $A \equiv (4, -2), B \equiv (5, 5)$ and $C \equiv (-2, 4)$

Then $a = BC = \sqrt{(-2-5)^2 + (4-5)^2} = 5\sqrt{2}$

$$b = AC = \sqrt{(4+2)^2 + (-2-4)^2} = 6\sqrt{2}$$
$$c = AB = \sqrt{(5-4)^2 + (5+2)^2} = 5\sqrt{2}$$

and $(x_1, y_1) \equiv (4, -2), (x_2, y_2) \equiv (5, 5),$

$(x_3, y_3) \equiv (-2, 4)$

$\therefore$ The coordinates of the incentre of the ΔABC are

$$\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

$= \left(\frac{5}{2}, \frac{5}{2}\right)$ {Substitute the values of (a, b, c)

x_1, x_2, x_3, y_1, y_2 and y_3}

12. Let $S(x, y)$ be the circumcentre, then $AS = BS = CS = R$, where R is the circumradius

Now $(AS)^2 = (BS)^2$ where $S \equiv (x, y), A \equiv (8, 6), B \equiv (8, -2)$.

$\therefore \quad (x-8)^2 + (y-6)^2 = (x-8)^2 + (y+2)^2$

$\Rightarrow \quad 36 - 12y = 4 + 4y$

$\Rightarrow \quad y = 2$

Again $(BS)^2 = (CS)^2$, where $S \equiv (x, y) B \equiv (8, -2), C \equiv (2, -2)$

$\therefore \quad (x-8)^2 + (y+2)^2 = (x-2)^2 + (y+2)^2$

$\Rightarrow \quad x^2 + 64 - 16x + y^2 + 4 + 4y$

$= x^2 + 4 - 4x + y^2 + 4 + 4y$

$\Rightarrow \quad -12x + 60 = 0$

$\Rightarrow \quad x = 5$

$\therefore$ The circumcentre $S \equiv (x, y) = (5, 2)$

13. Let $P(1, 1), Q(2, -3), R(3, 4)$ be the midpoints of sides AB, BC and CA respectively triangle ABC.

Let $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of triangle ABC.

Then, P is the mid point of AB

$\Rightarrow \quad \frac{x_1 + x_2}{2} = 1, \frac{y_1 + y_2}{2} = 1$

$\Rightarrow \quad x_1 + x_2 = 2$ and $y_1 + y_2 = 2$ ...(1)

Q is the mid point of BC

$\Rightarrow \quad \frac{x_2 + x_3}{2} = 2, \frac{y_2 + y_3}{2} = -3$

$\Rightarrow \quad x_2 + x_3 = 4$ and $y_2 + y_3 = -6$

R is the mid point of AC

$\Rightarrow \quad x_1 + x_3 = 6$ and $y_1 + y_3 = 8$

From (1), (2) and (3), we get

$$(x_1, y_1) \equiv (2, 8), \quad (x_2, y_2) \equiv (0, -6)$$

and $$(x_3, y_3) \equiv (4, 0)$$

Then the coordinates of the centroid

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$= \left(\frac{2 + 0 + 4}{3}, \frac{8 - 6 + 0}{3}\right) = \left(2, \frac{2}{3}\right)$$

14. Since $(x_1, y_1) \equiv A(2, 8)$, $(x_2\ y_2) \equiv B(0, -6)$

and $(x_3, y_3) \equiv C(4, 0)$

Now, $a = BC = 2\sqrt{3}$, $b = AC = 2\sqrt{17}$, $c = AB = 10\sqrt{2}$

The coordinates of the in-centre of the triangle ABC are $\left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c}\right)$

$\therefore$ Required coordinates of incentre are

$$\left(\frac{2\sqrt{13} + 20\sqrt{2}}{\sqrt{13} + \sqrt{17} + 5\sqrt{2}}, \frac{2\sqrt{13} - 6\sqrt{17}}{\sqrt{13} + \sqrt{17} + 5\sqrt{2}}\right)$$

15. Let A, B, C and D be the vertices of the quadrilateral whose coordinates are $(-2, -1)$ $(1, 0)$, $(4, 3)$ and $(1, 2)$ respectively.

Now, $AB = \sqrt{10}$, $BC = \sqrt{18}$, $DC = \sqrt{10}$, $AD = \sqrt{18}$

$\therefore$ $AB = CD$ and $BC = AD$ *i.e.*, the opposite sides are equal.

Hence $ABCD$ is a parallelogram.

16. Let $A(-1, 0)$, $B(3, 1)$, $C(2, 2)$ and $D(x, y)$ be the vertices of a parallelogram $ABCD$ taken in order. Since the diagonals of a parallelogram bisect each other.

$\therefore$ Coordinates of the mid point of AC

= Coordinates of the mid point of BD

$$\Rightarrow \left(\frac{-1 + 2}{2}, \frac{0 + 2}{2}\right) = \left(\frac{3 + x}{2}, \frac{1 + y}{2}\right)$$

$$\Rightarrow \left(\frac{1}{2}, 1\right) = \left(\frac{3 + x}{2}, \frac{y + 1}{2}\right)$$

$$\Rightarrow \frac{3 + x}{2} = \frac{1}{2} \quad \text{and} \quad \frac{y + 1}{2} = 1$$

$$\Rightarrow x = -2 \quad \text{and} \quad y = 1$$

Hence the fourth, vertex of the parallelogram is $(-2, 1)$

17. Solve as the question no. 13 has been solved. The required vertices of the triangle are $(1, -4)$, $(3, 2)$, $(-1, 2)$.

18. Let the vertices be $A(4, -1)$, $B(6, 0)$, $C(7, 2)$, $D(5, 1)$ then the coordinates of the mid point of AC are

$$\left(\frac{4 + 7}{2}, \frac{-1 + 2}{2}\right) = \left(\frac{11}{2}, \frac{1}{2}\right)$$

Coordinates of the mid point of BD are

$$\left(\frac{6 + 5}{2}, \frac{0 + 1}{2}\right) = \left(\frac{11}{2}, \frac{1}{2}\right)$$

Thus AC and BD have the same mid point. Hence $ABCD$ is a parallelogram.

Now, $AB = \sqrt{5}$, $BC = \sqrt{5}$ $\therefore AB = BC$

So, $ABCD$ is a parallelogram whose adjacent sides are equal. Hence, $ABCD$ is a rhombus.

Also, we have $AC = 3\sqrt{2}$ and $BD = \sqrt{2}$

Clearly, $AC \neq BD$. So $ABCD$ is not a square.

19. Let A, B, C, D be the points $(-4, 2)$, $(1, -1)$, $(4, 1)$ and $(2, 5)$ respectively. Then the area of the quadrilateral $ABCD$

$$= \frac{1}{2}\{-4 \times -1 - 2 \times 1 + 1 \times 1 - 4 \times -1 + 4 \times 5 - 2 \times 1 + 2 \times 2 \times - 5 \times - 4\}$$

$$= \frac{1}{2}(4 - 2 + 1 + 4 + 20 - 2 + 4 + 20)$$

$$= \frac{49}{2} = 24 \cdot 5 \text{ square units.}$$

HINT $\begin{matrix} -4 & & 1 & & 4 & & 2 & & -4 \\ & \times & & \times & & \times & & \times & \\ 2 & & -1 & & 1 & & 5 & & 2 \end{matrix}$

NOTE Area of quadrilateral $= \frac{1}{2}\{(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_4 - x_4y_3) + (x_4y_1 - x_1y_4)\}$

20. Let A, B, C, D be the points $(3, 2)$, $(6, 3)$, $(x\ y)$ and $(6, 5)$ respectively.

Since $ABCD$ is a parallelogram, the diagonals AC and BD must bisect each other *i.e.*, the mid points of AC and the mid point of BD must coincide and hence the coordinates of the two mid points are the same

$$\therefore \quad \frac{3 + x}{2} = \frac{6 + 6}{2} \quad \text{and} \quad \frac{2 + y}{2} = \frac{3 + 5}{2}$$

or $$3 + x = 12 \quad \text{and} \quad 2 + y = 8$$

or $$x = 9 \quad \text{and} \quad y = 6$$

Hence $$(x, y) = (9, 6)$$

21. Let the line $y - x + 2 = 0$ divide the join of $(3, -1)$ and $(8, 9)$ at the point P in the ratio $k : 1$. Then the coordinates of P are

$$\left(\frac{k \cdot 8 + 1.3}{k + 1}, \frac{k \cdot 9 + 1(-1)}{k + 1}\right) = \left(\frac{8k + 3}{k + 1}, \frac{9k - 1}{k + 1}\right)$$

Since, this point lies on the line $y - x + 2 = 0$, we have

$$\frac{9k - 1}{k + 1} - \frac{8k + 3}{k + 1} + 2 = 0$$

or $$k = \frac{2}{3}$$

Hence the required ratio is $k : 1 = \frac{2}{3} : 1 = 2 : 3$

22. Let the points be $A(1, 2)$, $B(-5, 6)$, $C(7, -4)$, $D(h, -2)$

Given, area of the quadrilateral $ABCD = 0$

or $$\frac{1}{2}\{(6 + 10) + (20 - 42) + (-14 + 4h) + (2h + 2)\} = 0$$

or $$h = 3$$

23. Area of $\Delta PBC = \frac{1}{2}\{(5x + 3y) + (6 - 20) + (4y + 2x)\}$

$$= \frac{1}{2}(7x + 7y - 14)$$

$$= \frac{7}{2}(x + y - 2)$$

and area of $\Delta ABC = \frac{1}{2}\{(30 + 9) + (6 - 20) + (12 + 12)\}$

$$= \frac{49}{2}$$

Hence $$\frac{\Delta PBC}{\Delta ABC} = \frac{x + y - 2}{7}$$

24. Slope of the line $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-5}{9-7} = 1$

Here $(x_1, y_1) \equiv (7, 5)$ and $(x_2, y_2) \equiv (9, 7)$

25. Let m_1 and m_2 be the slopes of BA and BC respectively. Then

$$m_1 = \frac{3-1}{2-(-2)} = \frac{1}{2} \quad \text{and} \quad m_2 = \frac{-4-3}{-2-2} = \frac{7}{4}$$

Let θ be the angle between BA and BC. Then

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{\frac{7}{4} - \frac{1}{2}}{1 + \frac{7}{4} \times \frac{1}{2}}\right| = \pm\frac{2}{3}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{2}{3}\right)$$

26. If m_1 be the slope of the line passing through the points $A(3, 5)$ and $B(-4, 2)$, then

$$m_1 = \frac{2-5}{-4-3} = \frac{3}{7}$$

If m_2 be the slope of the perpendicular line CD then $m_1 \cdot m_2 = -1$ or $\frac{3}{7} m_2 = -1$

or $$m_2 = -\frac{7}{3}$$

27. Let m_1 be the slope of line AB

$$\therefore \quad m_1 = \frac{3-(-3)}{6-2} = \frac{3}{2}$$

If m_2 be the slope of a line parallel to AB, then

$$m_2 = m_1 = \frac{3}{2}$$

28. Let m_3 be the slope of line perpendicular to AB, then

$$m_1 \cdot m_3 = -1$$

$$\Rightarrow \quad \frac{3}{2} \cdot m_3 = -1$$

$$\Rightarrow \quad m_3 = -\frac{2}{3}$$

29.

$$m_1 = \text{Slope of } AB = \frac{3-6}{2-6} = \frac{3}{4}$$

$$m_2 = \text{Slope of } BC = \frac{7-3}{4-2} = 2$$

and $$m_3 = \text{Slope of } AC = \frac{7-6}{4-6} = -\frac{1}{2}$$

$$\therefore \quad m_2 \cdot m_3 = 2 \times -\frac{1}{2} = -1$$

This shows that BC is perpendicular to AC. Hence, ABC is a right-angled triangle.

30.

$$m_1 = \text{Slope of } AB = \frac{4-(-2)}{3-1} = 3$$

and $$m_2 = \text{Slope of } BC = \frac{7-4}{4-3} = 3$$

$$\therefore \quad m_1 = m_2$$

$\therefore AB$ is parallel to BC and B is common to both AB and BC. Hence, the point $A(1, -2)$, $B(3, 4)$ and $C(4, 7)$ are collinear.

Introductory Exercise 21.2

1. The equation of the line with slope 2/3 and intercept on the y-axis 5 is $y = \frac{2}{3}x + 5$ ($\because y = mx + c$)

2. We have $\sqrt{3}x + 3y = 6$

or $$3y = -\sqrt{3}x + 6$$

or $$y = -\frac{1}{\sqrt{3}}x + 2$$

Comparing the above equation with $y = mx + c$

we get $m = -\frac{1}{\sqrt{3}}$ and $c = 2$

Hence slope is $\left(-\frac{1}{\sqrt{3}}\right)$ and intercept on the y-axis is 2.

3. We have $m = \frac{5}{4}$ and $(x_1, y_1) = (2, -3)$

$\therefore$ The equation of the line as point slope form is

$$y - y_1 = m(x - x_1)$$

or $$y - (-3) = \frac{5}{4}(x - 2)$$

or $$y + 3 = \frac{5}{4}(x-2)$$

or $$5x - 4y = 22$$

4. Here $a = 2$ and $b = 3$

$\therefore$ The required equation of the line is $\frac{x}{2} + \frac{y}{3} = 1$

$$\Rightarrow \quad 3x + 2y = 6$$

5. We have $3x + 4y - 12 = 0$

$$\Rightarrow \quad 3x + 4y = 12$$

$$\Rightarrow \quad \frac{3x}{12} + \frac{4y}{12} = 1 \Rightarrow \frac{x}{4} + \frac{y}{3} = 1$$

Which is of the form $\frac{x}{a} + \frac{y}{b} = 1$

Thus the required intercepts on the axes are 4 and 3.

6. The equation of the line through the points $(-1, -2)$ and $(-5, 2)$ is $(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

where $(x_1, y_1) \equiv (-1, -2)$

and $(x_2, y_2) \equiv (-5, 2)$

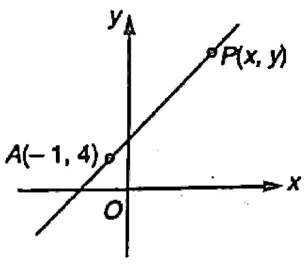
$\therefore$ Required equation is

$$y - (-2) = \frac{2-(-2)}{-5-(-1)}[x - (-1)]$$

or $$y + 2 = \frac{4}{-4}(x+1)$$

or $$x + y + 3 = 0$$

7. Let $(-1, 4)$ be the point as shown in figure and let $P(x, y)$ be any point on the line. Then the gradient (or slope) of the line is



$$\frac{y-4}{x-(-1)} = 2.5$$

$$\Rightarrow \quad \frac{y-4}{x+1}=\frac{5}{2}$$

$$\Rightarrow \quad 5x-2y+13=0$$

8. Let the equation of the straight line in the intercept form be

$$\frac{x}{a}+\frac{y}{b}=1 \quad \ldots(1)$$

Since the intercepts are equal, therefore $a=b$

$\therefore$ From equation (1)

$$x+y=a \quad \ldots(2)$$

Since this line passes through the point $(3,-5)$

$$\therefore \quad 3+(-5)=a$$

$$\Rightarrow \quad a=-2$$

$\therefore$ From equation (2), the required equation of the straight line is $x+y=-2$ or $x+y+2=0$

9. Let the equation of the straight line be

$$\frac{x}{a}+\frac{y}{b}=1 \quad \ldots(1)$$

Since intercepts a, b are equal in magnitude but opposite in sign.

$$\therefore \quad b=-a$$

$\therefore$ From eq. (1) $\frac{x}{a}+\frac{y}{(-a)}=1$

$$\text{or} \quad x-y=a \quad \ldots(2)$$

Since this line passes through the point $(-5,-8)$

$$\therefore \quad -5-(-8)=a$$

$$\Rightarrow \quad a=3$$

Hence, from (2) the required equation of the line is $x-y=3$

10. Let m_1 = slope of the line 'joining' (1, 2) and (5, 6)

$$\therefore \quad m_1=\frac{6-2}{5-1}=\frac{4}{4}=1$$

If m_2 be the slope of the perpendicular line, then $m_1m_2=-1$

$\Rightarrow m_2=-1$ ($\because m_1=1$)

$\therefore$ The required line has slope $m_2=-1$ and passes through the point $(-4,-5)$

Hence, the required equation of the line in the point slope form is

$$(y-y_1)=m_2(x_2-x_1)$$

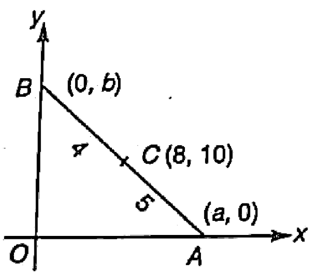
$$\text{or} \quad y-(-5)=-1\{x-(-4)\}$$

$$\text{or} \quad x+y+9=0$$

11. Let the equation of the line AB be $\frac{x}{a}+\frac{y}{b}=1 \quad \ldots(1)$

Then the coordinates of A and B are respectively $(a, 0)$ and $(0, b)$.

Since $C(8, 10)$ divides AB in the ratio 5 : 4, we have



$$\frac{5\times 0+4\times a}{5+4}=8$$

$$\text{and} \quad \frac{5\times b+4\times 0}{5+4}=10$$

$$\text{or} \quad a=18 \quad \text{and} \quad b=18$$

Hence from (1), the required equation of the line AB is

$$\frac{x}{18}+\frac{y}{18}=1 \quad \text{or} \quad x+y=18$$

12. Let the equation of the line in the intercept form be $\frac{x}{a}+\frac{y}{b}=1$, where a and b are intercepts on the axes.

Then $\quad a+b=14$ or $b=14-a$

Since the line $\frac{x}{a}+\frac{y}{b}=1$ passes through the point $(3, 4)$;

$$\therefore \quad \frac{3}{a}+\frac{4}{b}=1 \quad \text{or} \quad \frac{3}{a}+\frac{4}{14-a}=1$$

$$\text{or} \quad a^2-13a+42=0$$

$$\text{or} \quad (a-6)(a-7)=0$$

$$\therefore \quad a=6, 7$$

If $a=6$ then $b=8$

If $a=7$ then $b=7$

Hence the required equation of the line are

$$\frac{x}{6}+\frac{y}{8}=1 \quad \text{and} \quad \frac{x}{7}+\frac{y}{7}=1$$

$$\text{or} \quad 4x+3y=24 \quad \text{and} \quad x+y=7$$

13. Since the line passes through $A(a, 0)$ and $B(0, b)$, it makes intercepts a and b on the axes of x and y. Let the equation of the line be

$$\frac{x}{a}+\frac{y}{b}=1 \quad \ldots(1)$$

By the given conditions,

$$AB=13 \Rightarrow a.b=60 \quad \ldots(2)$$

From (1) $\sqrt{a^2+b^2}=13$

$$\Rightarrow \quad a^2+b^2=169$$

$$\Rightarrow \quad a+b=\pm 17$$

Again $(a-b)^2=(a+b)^2-4ab=289-240=49$

$\therefore$ we get $a=12$, $b=5$ and $a=-12$, $b=-5$

$\therefore$ The required equations of the straight line are

$$\frac{x}{12}+\frac{y}{5}=1 \quad \text{and} \quad \frac{x}{-12}+\frac{y}{-5}=1$$

i.e., $5x+12y=60$ and $5x+12y+60=0$

14. Let the equation of the cost curve as a straight line be

$$y=mx+c \quad \ldots(1)$$

where x = number of units of a good produced and y = cost of x units in rupees.

Given, when $x=50$, $y=320$ and when $x=80$, $y=380$

$$\text{from (1)} \quad 320=50m+c \quad \ldots(2)$$

$$\text{and} \quad 380=80m+c \quad \ldots(3)$$

Subtracting (2) from (3), we get $m=2$

Substituting $m=2$ in equation (2), we get $c=220$

$\therefore$ From (1) $y=2x+220$

when $x=110$, $y=2\times 110+220=440$

Hence, the required cost of producing 110 units is Rs. 440.

15. Here $p=5$ and $\alpha=60°$

$\therefore$ The required equation of the line is

$$x\cos\alpha+y\sin\alpha=p$$

$$x\cos 60° + y\sin 60° = 5$$

or $\Rightarrow$ $$x + \sqrt{3}y = 10 \quad \left(\because \sin 60° = \frac{\sqrt{3}}{2} \text{ and } \cos 60° = \frac{1}{2}\right)$$

16. Here $(x_1, y_1) \equiv (3, -4)$ and $\theta = 60°$

The required equation of the line in the symmetric form is $\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta}$

$$\Rightarrow \quad \frac{x-3}{\cos 60°} = \frac{y-(-4)}{\sin 60°}$$

$$\Rightarrow \quad \sqrt{3}x - y = 4 + 3\sqrt{3}$$

17. We have $2x + y = 4$ ...(1)

and $x - y + 1 = 0$ ...(2)

Solving equations (1) and (2), we get

$$x = 1 \quad \text{and} \quad y = 2$$

$\therefore$ The point of intersection of (1) and (2) is (1, 2).

Again $2x - y - 1 = 0$ ...(3)

$x + y - 8 = 0$ ...(4)

Solving equations (3) and (4), we get $(x, y) \equiv (3, 5)$

$\therefore$ The point of intersection of (3) and (4) is (3, 5)

$\therefore$ The required equation of the straight line joining the points of intersection (1, 2), (3, 5) is

$$y - 2 = \frac{(5-2)}{(3-1)} \cdot (x - 1)$$

$$\Rightarrow \quad 3x - 2y + 1 = 0$$

18. The equation of the line through the point (4, 5) is

$$y - 5 = m(x - 4) \quad ...(1)$$

where m is the slope of the line.

Now the given line is $2x - y + 7 = 0$

$$\Rightarrow \quad y = 2x + 7 \quad ...(2)$$

If m_1 be the slope of the line (2), then $m_1 = 2$

If equation (1) makes an angle of 45° with equation (2), then we have

$$\tan 45° = \frac{m_1 \sim m}{1 + m \cdot m_1} = \frac{2 - m}{1 + 2m} \quad i.e., \quad \frac{2 \sim m}{1 + 2m}$$

$\therefore$ either $1 = \frac{m-2}{1+2m}$ or $1 = \frac{2-m}{1+2m}$

If $\frac{m-2}{1+2m} = 1$ then $m = -3$

If $\frac{2-m}{1+2m} = 1$ then $m = \frac{1}{3}$

Hence, from (1) the required equation of the two lines is $y - 5 = -3(x-4)$ and $y - 5 = \frac{1}{3}(x - 4)$

$$\Rightarrow \quad 3x - y - 17 = 0 \quad \text{and} \quad x - 3y + 11 = 0$$

19. The equation of any straight line parallel to the line $8x + 7y + 5 = 0$ is $8x + 7y + c = 0$ ...(1)

where c is an arbitrary constant.

If the line (1) passes through the point $(5, -6)$, then

$$8 \times 5 + 7 \times (-6) + c = 0 \Rightarrow c = 2$$

Hence from (1), the required equation of the straight line is $8x + 7y + 2 = 0$

20. Solving $x + y = 8$ and $3x - 2y + 1 = 0$, we get the point of intersection.

$\therefore$ The point of intersection is (3, 5).

Now, the equation of the line joining the points

$$(3, 4) \text{ and } (5, 6) \text{ is } (y - 4) = \frac{(6-4)}{(5-3)}(x - 3)$$

$$\Rightarrow \quad x - y + 1 = 0 \quad ...(1)$$

$\therefore$ The equation of the line parallel to the line $x - y + 1 = 0$ is

$$x - y + c = 0 \quad ...(2)$$

Where c is an arbitrary constant. If the line passes through the point (3, 5) then

$$3 - 5 + c = 0 \quad \text{or} \quad c = 2$$

Hence from (2), the required equation of the line is $x - y + 2 = 0$.

21. Length of the perpendicular from the point $(3, -2)$ to the straight line $12x - 5y + 6 = 0$ is

$$\frac{12 \times 3 - 5 \times -2 + 6}{\sqrt{(12)^2 + (-5)^2}} = \frac{36 + 10 + 6}{\sqrt{169}} = 4 \text{ units.}$$

22. Putting $y = 0$ in $5x + 12y - 30 = 0$, we get

$$5x - 30 = 0 \quad \text{or} \quad x = 6$$

$\therefore$ (6, 0) is a point on the first line $5x + 12y - 30 = 0$

Required distance between the parallel lines = Perpendicular distance of the point (6, 0) from the second line $5x + 12y - 4 = 0$.

$$= \frac{5.6 + 12.0 - 4}{\sqrt{5^2 + 12^2}} = \frac{30 - 4}{13} = 2 \text{ units}$$

23. The equation of the line through the point of intersection of $2x - 3y + 1 = 0$ and $x + y - 2 = 0$, is

$$(2x - 3y + 1) + k(x + y - 2) = 0$$

i.e., $$(2 + k)x + (k - 3)y + (1 - 2k) = 0 \quad ...(1)$$

If this line is parallel to the y-axis, then its equation must be of the form $x = h$, i.e, the coefficient of y in (1) must be zero.

$$\therefore \quad k - 3 = 0 \quad \text{or} \quad k = 3$$

Hence, from (1) the required equation of the line is $(2 + 3)x + 0 \cdot y + (1 - 2 \times 3) = 0$ [Putting $k = 3$]

$$\Rightarrow \quad x = 1$$

24. The equation of any line passing through the point of intersection of the lines $x + 2y - 3 = 0$ and $4x - y + 7 = 0$ is

$$x + 2y - 3 + k(4x - y + 7) = 0 \quad ...(1)$$

or $$(1 + 4k)x(2 - k)y + (7k - 3) = 0 \quad ...(2)$$

m_1 = Slope of the line (2) $= \frac{4k + 1}{k - 2}$

and m_2 = (Slope of the line $y - x + 10 = 0$) = 1

If the line (1) be parallel to the line $y - x + 10 = 0$

then $$\frac{4k+1}{k-2} = 1 \Rightarrow k = -1$$

Hence from (1), the required equation of the line is

$$(x + 2y - 3) - 1.(4x - y + 7) = 0$$

$$\Rightarrow \quad 3x - 3y + 10 = 0$$

25. Solving $2x - y + 5 = 0$ and $5x + 3y - 4 = 0$, we get $x = -1$ and $y = 3$ i.e., the point of intersection of the given lines is $(-1, 3)$.

$\therefore$ The equation of any line perpendicular to the line

$x - 3y + 21 = 0$ is $3x + y + k = 0$ ...(1)

If this line (1) passes through the point $(-1, 3)$ then

$$3 \times -1 + 3 + k = 0 \Rightarrow k = 0$$

$\therefore$ From (1), the required equation of the line is $3x + y = 0$.

26. The equation of any line passing through the intersection of the lines $3x + 4y - 7 = 0$ and $x - y + 2 = 0$ is

$$(3x + 4y - 7) + k(x - y + 2) = 0 \quad ...(1)$$

Slope of the line $= -\frac{3 + k}{4 - k} = 3$

$$\Rightarrow \quad k = \frac{15}{2}$$

Hence, from (1) the required equation of the line is

$$(3x + 4y - 7) + \frac{15}{2}(x - y + 2) = 0$$

$$\Rightarrow \quad 21x - 7y + 16 = 0$$

27. The equation of any line passing through the point of intersection of the lines $3x - 4y + 1 = 0$ and $5x + y - 1 = 0$ is

$$(3x - 4y + 1) + k(5x + y - 1) = 0 \quad ...(1)$$

for intercept of this line with the x-axis, $y = 0$

$$\therefore \quad 3x + 1 + k(5x - 1) = 0$$

$$\Rightarrow \quad x = \frac{k - 1}{5k + 3}$$

For intercept of the line (1) on the y-axis, $x = 0$

$$\therefore \quad -4y + 1 + k(y - 1) = 0$$

$$\Rightarrow \quad y = \frac{k - 1}{k - 4}$$

Since the intercepts on the axes are equal.

$$\therefore \quad \frac{k - 1}{5k + 3} = \frac{k - 1}{k - 4} \Rightarrow k = 1, \text{ or } x = -\frac{7}{4}$$

But $k \neq 1$, because if $k = 1$, the line (1) becomes $8x - 3y = 0$ which passes through the origin and therefore cannot make non-zero intercepts on the axis.

$\therefore \quad k = -\frac{7}{4}$ and from (1), we get

$$3x - 4y + 1 - \frac{7}{4}(5x + y - 1) = 0$$

$\Rightarrow 23x + 23y = 11$, which is the required equation of the line.

28. We have $\frac{x}{a} - \frac{y}{b} = 1$ ...(1)

Since (1) passes through the point (8, 6)

$$\therefore \quad \frac{8}{a} - \frac{6}{b} = 1 \quad ...(2)$$

The line (1) meets the x-axis at the point given by $y = 0$ and from (1) $x = a$ i.e, the line (1) meets the x-axis at the point $A(a, 0)$.

Similarly, the line meets the y-axis $(x = 0)$ at the point $B(0, -b)$.

By the given condition, area of $\Delta = 12$

$$\Rightarrow \quad \frac{1}{2}ab = 12$$

$$\Rightarrow \quad ab = 24 \Rightarrow b = \frac{24}{a}$$

Substituting $b = \frac{24}{a}$ in (2), we get

$$\frac{8}{a} - \frac{6}{\frac{24}{a}} = 1 \Rightarrow a = 4 \text{ or } -8 \text{ and } b = 6 \text{ or } -3$$

Hence, from (1) the equation of the straight line are

$$\frac{x}{4} - \frac{y}{6} = 1 \text{ and } \frac{x}{-8} - \frac{-y}{-3} = 1$$

$$\Rightarrow \quad 3x - 2y = 12 \text{ and } 3x - 8y + 24 = 0$$

29. The equation of the lines may be written as $3x + 4y + 2 = 0$ and $-5x + 12y + 6 = 0$ in which the constant terms 2 and 6 are both positive.

The equation of the bisector of the angle in which the origin lies is $\frac{3x + 4y + 2}{\sqrt{3^3 + 4^2}} = \frac{-5x + 12y + 6}{\sqrt{(-5)^2 + (12)^2}}$

$$\Rightarrow \quad 16x - 12y - 1 = 0$$

The equation of the other bisector is

$$\frac{3x + 4y + 2}{\sqrt{3^2 + 4^2}} = \frac{-(-5x + 12y + 6)}{\sqrt{(-5)^2 + (12)^2}}$$

$$\Rightarrow \quad x + 8y + 4 = 0$$

30. Let the equations of the sides BC, CA and AB of the triangle ABC be represented by

$$2y - x = 5 \quad ...(1)$$

$$y + 2x = 7 \quad ...(2)$$

$$y - x = 1 \quad ...(3)$$

Solving the above 3 equations (1), (2) and (3), we get $A(2, 3)$, $B(3, 4)$ and $C\left(\frac{9}{5}, \frac{17}{5}\right)$

$\therefore$ The area of ΔABC

$$= \frac{1}{2}\left\{2 \times 4 - 3 \times 3 + 3 \times \frac{17}{5} - 4 \times \frac{9}{5} + \frac{9}{5} \times 3 - \frac{17}{5} \times 2\right\}$$

$$= \frac{1}{2}\left(8 - 9 + \frac{51}{5} - \frac{36}{5} + \frac{27}{5} - \frac{34}{5}\right) = \frac{3}{10} \text{ units.}$$

31. Let $A(1, 2)$, $B(2, 3)$ and $C(4, 3)$ be the vertices of ΔABC.

$m_1 =$ Slope of $BC = \frac{3 - 3}{4 - 2} = 0$ i.e., BC is parallel to the x-axis.

$\therefore$ The perpendicular from $A(1, 2)$ to BC is parallel to the y-axis and its equation is $x = h$, which passes through $A(1, 2)$

$\therefore h = 1$ i.e, the equation of the perpendicular from $A(1, 2)$ on BC is $x = 1$ ...(1)

$$m_2 = \text{Slope of } AC = \frac{3 - 2}{4 - 1} = \frac{1}{3}$$

If m_2' be the slope of a line perpendicular to AC then $m_2 m_2' = -1$ or $\frac{1}{3} \cdot m_2' = -1$ or $m_2' = -3$

$\therefore$ The equation of the perpendicular from $B(2, 3)$ on AC whose slope is -3 is

$$y - 3 = -3(x - 2)$$
$$3x + y = 9 \quad \ldots(2)$$

$\Rightarrow$ The orthocentre is the point of intersection of two lines (1) and (2)

$\therefore$ From (1) and (3), we get $3 \times 1 + y = 9$

$\Rightarrow$ $y = 6$

$\therefore$ The required coordinates of the orthocentre are (1, 6)

32. Let $A(x_1, y_1)$ be the third vertex. Let AD, BE, CF be the perpendiculars from the vertices on the opposite side BC, CA, AB respectively. Then the orthocentre $O(0, 0)$ is the point of intersection of AD, BE and CF.

Since AD i.e. OA is perpendicular to BC.

$\therefore$ Slope of $BA \times$ slope of $BC = -1$

or $$\frac{y_1 - 0}{x_1 - 0} \times \frac{3-(-1)}{-2-5} = -1$$

$\Rightarrow$ $$y_1 = \frac{7x_1}{4} \quad \ldots(1)$$

Again, since OB is perpendicular to CA

$\therefore$ $$\frac{-1-0}{5-0} \times \frac{y_1 - 3}{x_1 + 2} = -1$$

$\Rightarrow$ $$5x_1 + 10 = y_1 - 3$$

$\Rightarrow$ $$x_1 = -4 \quad \left(\because y_1 = \frac{7x_1}{4}\right)$$

From (1), $y_1 = \frac{7x_1}{4} = \frac{7 \times (-4)}{4} = -7$

Hence, the required coordinates of the third vertex A are $(x_1, y_1) = (-4, -7)$

33. Let (x_1, y_1) be the third vertex then $y_1 = x_1 + 3 \ldots(1)$

The area of the triangle formed by the points (2, 1), (3, − 2) and (x_1, y_1)

$$= \frac{1}{2}\{-4 - 3 + 3y_1 + 2x_1 + x_1 - 2y_1\}$$
$$= \frac{1}{2}(3x_1 + y_1 - 7)$$

By the given condition,

$$\pm \frac{1}{2}(3x_1 + y_1 - 7) = 5$$

$\Rightarrow$ $$3x_1 + y_1 - 7 = \pm 10$$

$\Rightarrow$ $$3x_1 + y_1 = 17 \quad \ldots(2)$$

and $$3x_1 + y_1 = -3 \quad \ldots(3)$$

Solving (1) and (2) we get $x_1 = \frac{7}{2}, y_1 = \frac{13}{2}$

Solving (1) and (3) we get $x_1 = -\frac{3}{2}$ and $y_1 = \frac{3}{2}$

Hence, the coordinates of the third vertex is either $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(-\frac{3}{2}, \frac{3}{2}\right)$

34. Equation of any line L perpendicular to

$$5x - y = 1 \text{ is } x + 5y = k \quad \ldots(1)$$

Where k is an arbitrary constant.

If this line cuts x-axis at A and y-axis at B, then for A, $y = 0$ and from (1) $x = k$ i.e., A is the point $(k, 0)$ for B, $x = 0$ and from (1) $y = \frac{k}{5}$ i.e., B is the point $\left(0, \frac{k}{5}\right)$

$\therefore$ Area of the given $\Delta OAB = \frac{1}{2}(x_1 y_2 - x_2, y_1)$

$$= \frac{1}{2}\left(\frac{k^2}{5} - 0\right) = \frac{k^2}{10}$$

But the given condition $\frac{k^2}{10} = 5$

or $$k^2 = 50 \quad \therefore k = \pm 5\sqrt{2}$$

Hence, from (1), the required equation of the line is

$$x + 5y = 5\sqrt{2} \quad \text{or} \quad x + 5y = -5\sqrt{2}$$

35. Let $ABCD$ be the square and let (1, 2) and (3, 8) be the coordinates of opposite vertices A and C respectively.

The equation of the diagonal AC is $y - 2 = \frac{8-2}{3-1}(x - 1)$

$\Rightarrow$ $$3x - y = 1$$