Math Assignment

Name: Amit Yadav
Roll No. IEC2018081
Section: C2
Group: 7

Question: Examine the convergence of the following integral

$$\int_0^1 \left(\log\left(\frac{1}{x}\right)\right)^n dx.$$

Solution:

$$\int_0^1 \left(\log\left(\frac{1}{x}\right)\right)^n dx$$

put x=e^{-t} $\Rightarrow$ dx=-e^{-t}dt in above equ. and lower limit lnx=-t

$$t=\ln\left(\frac{1}{x}\right) \Rightarrow t=\ln\left(\frac{1}{0}\right)=\infty$$

and upper limit $t = \ln\left(\frac{1}{1}\right)=0$

Let

$$f(t)= \int_0^\infty \left(\log\left(e^t\right)\right)^n e^{-t}dt \Rightarrow \int_0^\infty t^n e^{-t}dt.$$

using Integration by Part

$$\int_0^\infty t^n e^{-t}dt= -t^n e^{-t}\Big|_0^\infty + \int_0^\infty n t^{n-1} e^{-t}dt \Rightarrow n\int_0^\infty t^{n-1} e^{-t}dt$$

Again by Integration by Part

$$n\int_0^\infty t^{n-1} e^{-t}dt \quad =n(n-1)\int_0^\infty t^{n-2} e^{-t}dt$$

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$$=n!\int_0^{\infty} e^{-t}dt \implies -n![e^{-t}]_0^{\infty} \implies -n!(0-1)=n!$$

Let $g(t)=\int_0^{\infty} e^{-t}dt$

$g(t)=\int_0^{\infty} e^{-t}dt = 1$ converges.

Using Limit Comparision Test:

$$\textit{Where } f(t) \geqslant 0 \textit{ and } g(t) > 0 \textit{ for all } x > a. \textit{ If } Lim_{t\to\infty}\frac{f(t)}{g(t)} = c \textit{ where } c \neq 0,$$

then both the integrals

$$\int_0^{\infty} f(t)dt \textit{ and } \int_0^{\infty} g(t)dt \textit{ both converge or both diverge.}$$

$$\textit{In case } c = 0, \textit{ then convergence of}$$

$$\int_0^{\infty} g(t)dt \;\textit{ implies convergence of } \int_0^{\infty} f(t)dt.$$

$$Lim_{t\to\infty}\frac{e^{-t}t^n}{e^{-t}} \;=\; t^n \textit{ as } t \to \infty \;\to t^n \to \infty.$$

$c \neq 0$ *Hence both converges or both diverges.*

since $g(t)$ *is convergence, so* $f(t)$ *will be also convergence.*

$\int_0^1 \left(\log\left(\frac{1}{x}\right)\right)^n dx=n!$ converges if n is finite else diverges.