STABILITY

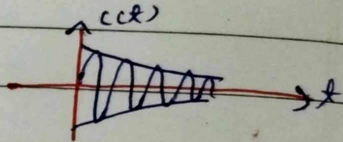
Stability is very important to analize & design the system. The system can be classified as –
→ (i) Absolute stability → True or false (ii) Relative stability → Relation b/w 2 stable system & comparing

① **Stable System:**

If the o/p of system approaches to a finite value or bounded value, it is called stable system.

Consider a system 1, if bounded i/p r(t) is applied to system 1 & its o/p is finite or a bounded value, then this system is stable.

r(t) Bounded i/p → System 1 → Bounded o/p c(t)



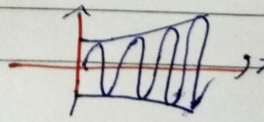
If due to i/p, oscillations setup in o/p & if these oscillations disappear with time, system is called stable.

② **Unstable system:**

If o/p of a system approaches to infinite value, it is calle unstable system.

Consider a system 2, whose o/p ↑sed indefinitely with bounded i/p r(t), then this system is unstable.

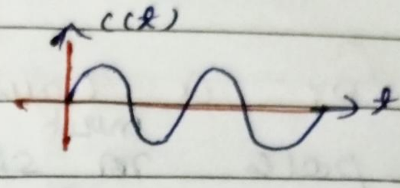
Bounded i/p (r(t)) → System 2 → Unbounded o/p



Marginally stable system :

If due to i/p , oscillations set-up in o/p & if these oscillations continue with constant amplitude , then it is marginally stable or limitedly stable system.

Bounded i/p $r(t)$ → System3 → Bounded o/p $c(t)$



Conclusions :

(A) In both cases (Stable - unstable), provided that the i/p of system is bounded (in limits)

(B) If unbounded o/p obtained by applying unbounded i/p , then nothing can be said about stability

(C) A LTI system is stable if —

(i) The system is excited by Bounded i/p & o/p obtained is also bounded (BIBO)

(ii) In absence of i/p , o/p becomes zero.

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Absolute Stablity by –
R-H criteria (Routh Hurwitz)

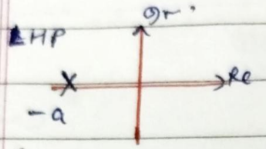
Relative Stability by –
Root locus, Nyquist plot, Bode plot.

For a causal system to be stable all the poles must ~~should~~ lie on LHS of the pole

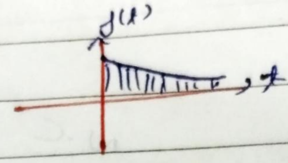
EFFECT OF LOCATION OF POLES ON STABILITY.

① Poles on negative real axis.

$F(s) = \dfrac{1}{(s+a)}$



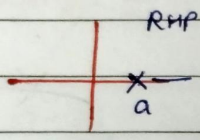
$f(t) = e^{-at}\,u(t)$



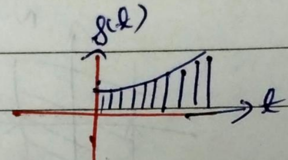
STABLE

② Poles on +ve real axis.

$F(s) = \dfrac{1}{(s-a)}$



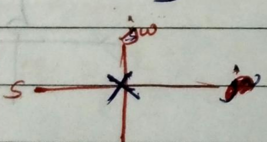
$f(t) = e^{at}\,u(t)$



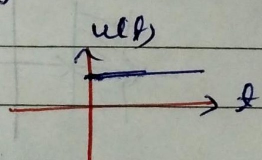
UNSTABLE.

③ Poles at origin.

$F(s) = \dfrac{1}{s}$



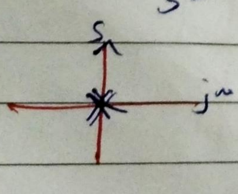
$f(t) = u(t)$



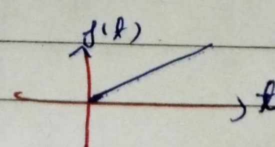
MARGINALLY STABLE.

④ Dual poles at origin

$F(s) = \dfrac{1}{s^2}$



$f(t) = t\,u(t)$



UNSTABLE.

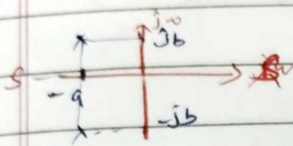
(4) Complex poles in Left hand side of the S-plane.

$$F(s) = \frac{1}{(s+a)^2 + b^2}$$

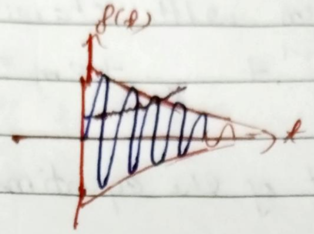
$s = -a \pm jb$

If $b = 1$ then,

$s = -a \pm j$



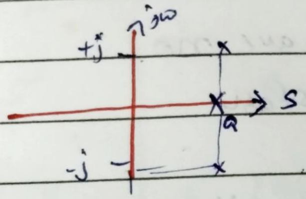
$f(t) = e^{-at} \sin t$



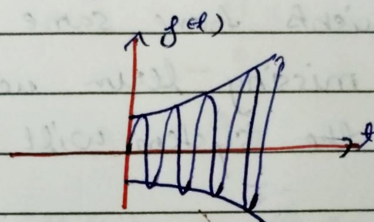
STABLE.

(5) Complex poles on Right hand side of the S-plane

$$F(s) = \frac{1}{(s-a)^2 + 1^2}$$



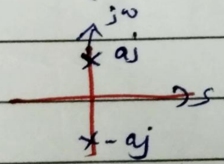
$f(t) = e^{at} \sin t$



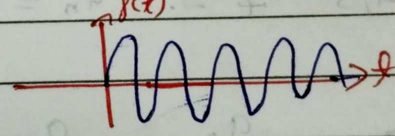
UNSTABLE

(6) Pole on jω-axis.

$$F(s) = \frac{1}{(s^2 + a^2)}$$



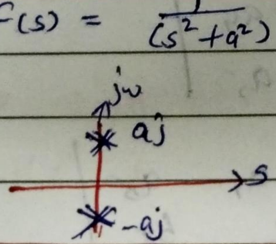
$f(t) = \sin(t)$



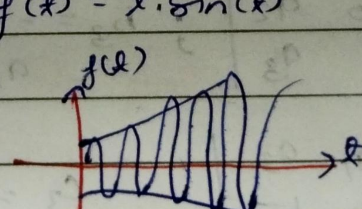
MARGINALLY STABLE

(7) Dual poles on jω-axis.

$$F(s) = \frac{1}{(s^2 + a^2)^2}$$



$f(t) = t \cdot \sin(t)$



UNSTABLE.

CONDITIONS FOR STABILITY

Consider a system with characteristics eq^n.

$$a_0 s^m + a_1 s^{m-1} + \dots + a_m = 0$$

(i) All the cofficient of the equations should have same sign.

(ii) There should be no missing terms.

If above two conditions are not satisfied the system will be unstable. But if all the cofficients have same sign and there is no missing term we have no gurantee that the system will be stable.

For stability we use Routh-Horwitz criterian

The R-H criterion -

$$1+G(s).H(s) \rightarrow a_0 s^n + a_1 s^{n-1} + \dots + a_n = 0$$

$$\begin{array}{c|cccc} s^n & a_0 & a_2 & a_4 & \dots \\ s^{n-1} & a_1 & a_3 & a_5 & \dots \\ s^{n-2} & b_1 & b_3 & b_5 & \dots \\ s^{n-3} & c_1 & c_3 & c_5 & \dots \\ \vdots & & & & \end{array}$$

$$b_1 = \frac{-1}{a_1}\begin{vmatrix} a_0 & a_2 \\ a_1 & a_3 \end{vmatrix} ; \quad b_3 = \frac{-1}{a_1}\begin{vmatrix} a_0 & a_4 \\ a_1 & a_5 \end{vmatrix}$$

$$c_1 = -\frac{1}{b_1}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \quad c_3 = \frac{-1}{b_1}\begin{vmatrix} a_1 & a_5 \\ b_1 & b_5 \end{vmatrix}$$

⇒ Example :

$$a_1 s^4 + a_2 s^3 + a_3 s^2 + a_4 s^1 + a_5 s^0 = 0$$

s^4	a_1	a_3	a_5
s^3	a_2	a_4	0
s^2	$b_1 = \left(\frac{a_2 a_3 - a_1 a_4}{a_2}\right)$	$b_2 = \left(\frac{a_2 a_5 - 0}{a_2}\right)$	0
s^1	$c_1 = \frac{b_1 a_4 - b_2 a_2}{b_1}$	$c_2 = 0$	0
s^0	$d_1 = \left(\frac{c_1 b_2 - 0}{c_1}\right)$	0	0

If all the coff. in first column are of the same sign (+ve), then given eq. has no roots with +ve real parts. Hence the system is stable.

No sign change = No poles on RHS.
LHP p = 4

Q_1 $s^4 + 2s^3 + 6s^2 + 4s + 1 = 0$

Row 1	s^4	1	6	1
Row 2	s^3	2	4	0
Row 3	s^2	$4 \Rightarrow \left[\frac{2\times6-4\times1}{2}\right]$	$1 \Rightarrow \left[\frac{2\times1-1\times0}{2}\right]$	$0 \Rightarrow [0]$
Row 4	s^1	$3.5 \Rightarrow \left[\frac{4\times4-2\times1}{4}\right]$	$0 \Rightarrow \left[\frac{4\times0-0}{4}\right]$	$0 \Rightarrow 0$
Row 5	s^0	$1 \Rightarrow \left[\frac{3.5\times1-0}{3.5}\right]$	0	0

All coff. in the first column are of the same sign (+ve), the given eqn has no roots with +ve real axis.

STABLE

Q_2 $s^4 + 2s^3 + 3s^2 + 4s + 5 = 0$

$s^4 + 2s^3 + 3s^2 + 4s^1 + 5s^0 = 0$

s^4	1	3	5
s^3	2	4	0
s^2	$\left[\frac{2\times3-4\times1}{2}\right]=1$	$\left[\frac{2\times5-0\times1}{2}\right]=5$	0
s^1	$\left[\frac{1\times4-2\times5}{1}\right]=-6$	$\left[\frac{1\times0-0}{1}\right]=0$	0
s^0	$\left[\frac{-6\times5-0}{-6}\right]=5$		

$\Downarrow$

s^4	1	3	5
s^3	2	4	0
s^2	1	5	0
s^1	-6	0	0
s^0	5	0	0

UNSTABLE

No. of sign change = 2 = No. of poles on RHS

Poles on LHP = 4 − 2 = 2

Special case of R-H criterion

Case-1 If the first element in any one row of routh array is zero, but the other elements are not zero.

Then—

→ We replace zero element by ϵ where $\epsilon \to 0$ & has the same ~~size~~ sign as 1st element of previous row.

→ Continue routh-array and judge sign of elements by putting $\epsilon \to 0$

Q $1s^5 + 2s^4 + 1s^3 + 2s^2 + 3s^1 + 10 = 0$

s^5	1	1	3	0
s^4	2	2	10	0
s^3	$\left[\frac{2-2}{2}\right] = 0 \to \epsilon$	$\left(\frac{6-10}{2}\right) = -2$	0	0
s^2	$\left(\frac{2\epsilon+4}{\epsilon}\right)$	10		
s^1	$\left[\frac{\frac{-4\epsilon-8}{\epsilon} - 10\epsilon}{\left(\frac{2\epsilon+4}{\epsilon}\right)}\right] < 0$			
s^0	10			

2 sign changes in 1st column = 2 poles on RHP

Total ⇒ 5.

So 3 in LHP

Q

$s^3 + s + 2 = 0 \Rightarrow s^3 + 0s^2 + 1s^1 + 2s^0 = 0$

s^3	1	1
s^2	$0 \to \epsilon$	2
s^1	$\left(\frac{\epsilon - 2}{\epsilon}\right) \to -ve$	0
s^0	2	

$\boxed{\epsilon \geq 0}$

2 sign change = 2 roots in RHS plan

CASE 2 When all the elements in any one row of R-Array are zero. then –

→ Two row preceding row of zeroes is called auxiliary equations.

s^2	a	b
s^1	0	0

$as^2 + b = 0 \to$ auxillary eq^n

Roots of auxillary eq^n are also the roots of C.E.

Replace row of zeroes by coffiicient of derivative of auxillary eq^n

$$\boxed{\frac{d}{ds}(as^2 + b) = 2s}$$

Properties of auxiliary equations.

① Auxiliary equation is always of even order.

s^3	a	b
s^2	0	0

$\boxed{s^3 \quad a \quad b} \rightarrow$ A.E.

$$as^3 + bs = 0$$
$$s[as^2 + b] = 0$$
$$s = 0, \quad s = \pm\sqrt{\frac{-b}{a}}$$

If A.E has odd order then everything remains same except the fact that there is a pole at origin.

② Root of A.E are symmetrically placed wrt imaginary axis.

↳ No. of roots of AE in RHP = No. of roots of A.E in LHP

* No. of sign changes below AE in R-array = no. of roots of A.E in RHP = no. of roots of A.E in LHP

* No. of poles on Im axis = order of A.E − 2[no. of sign change below A.E]

* No. of poles of C.E in RHP = total no. of sign change in 1^{st} column of R-array.

* No. of poles of C.E in LHP = order of C.E − Img. − RHP.

Q $s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$

s^6	1	8	20	16
s^5	2	12	16	0
A.E → s^4	$\left(\frac{16-12}{2}\right) = 2$	$\left(\frac{40-16}{2}\right) = 12$	16	0
→ s^3	0	0	0	0
s^2				
s^1				
s^0				

A.E

$$2s^4 + 12s^2 + 16s^0 = 0$$

$$s^4 + 6s^2 + 8s^0 = 0$$

A.E ⇒ $s^4 + 6s^2 + 8 = 0$

$s^4 + 6s^2 + 8 = 0$
$(s^2+4)(s^2+2) = 0$
$s = \pm j2, \pm j\sqrt{2}$
Simple poles on Img. axis
↳ marginally stable

$$\frac{d}{ds}[s^4 + 6s^2 + 8] \Rightarrow 4s^3 + 12s$$

$$\Rightarrow 4[s^3 + 3s]$$

Now

s^6	1	8	20	16
s^5	2	12	16	0
E → s^4	2	12	16	0
s^3	4	12	0	0
s^2	$\left(\frac{48-24}{4}\right) = 6$	$\frac{64}{4} = 16$	0	0
s^1	$\left(\frac{72-64}{6}\right) = 8/6$	0	0	0

Total sign change = 0 = 0 poles in RHP

Img. = 4 − 0 = 0

$\boxed{LHP = 6 - 4 = 2}$

Q $s^5 + 2s^4 + 3s^3 - 6s^2 + 2s' - 4 = 0$

	s^5	1	3.	2	0
A.E →	$\boxed{s^4}$	-2	-6	-4	A.E.
→	s^3	$\left[-\frac{2\times3+6}{-2}\right] = 0$	0	0	
	s^2				
	s^1				
	s^0				

A.E ⇒ $-2s^4 - 6s^2 - 4s^0 = 0$

A.E $\boxed{-2s^4 - 6s^2 - 4 = 0}$

or

$\frac{d}{ds}(A.E) = \frac{d}{ds}[-2s^4 - 6s^2 - 4] \Rightarrow -8s^3 - 12s = 0$

Now –

	s^5	1	3	2	0
A.E →	s^4	-2	-6	-4	0
*	s^3	-8	-12	0	0
	s^2	$\frac{48-24}{-8} = -3.$	-4	0	0
	s^1	$\frac{+36-32}{-3} = -\frac{4}{3}$	0	0	0
	s^0	-4	0	0	0

No. of sign change below A.E. = 0 LHP = 0 RHP.

Poles of img. ax = 4 - 0 - 0 = 4

Total sign change = 1 = RHP

LHP = 5 - 4 - 1 = 0   Unstable

Q $1 + G(s).H(s) = s^4 + s^3 - 3s^2 - s + 2 = 0$

Calculate all poles –

s^4	1	-3	2
s^3	1	-1	0
A.E → s^2	$\frac{-3+1}{1} = -2$	2	0
s^1	0	0	0
s^0			

$$A.E \Rightarrow -2s^2 + 2s^0 = 0$$

$$\boxed{A.E \; -2s^2 + 2 = 0}$$

$$\frac{d}{ds}(A.E) = \frac{d}{ds}[-2s^2 + 2] \Rightarrow -4s$$

Now →

s^4	1	-3	2
s^3	1	-1	0
A.E → s^2	-2	2	0
s^1	-4	0	0
s^0	2	0	0

No. of sign change below A.E = 1

1 LHP = 1 RHP.

Img = Order of A.E − 2[Sign change below A.E]

$= 2 - 2[1] = 0$

Total RHP = 2 , LHP = 4 − 2 = 2

Root Locus

{ Locus is a path traced by a point or a particle that move under certain condition

Root locus is the locus of ~~...~~ roots of characteristic eq^n as certain parameter in system is varied.

* $G(s) = \dfrac{K}{s(s+2)(s+4)}$

Step 1 : To find poles & zeros -

P → S = 0, -2, -4   [No. of poles = 3]

Z → No zeroes.

Step 2 Plot the poles & zeroes on the graph & find the existence of root locus.

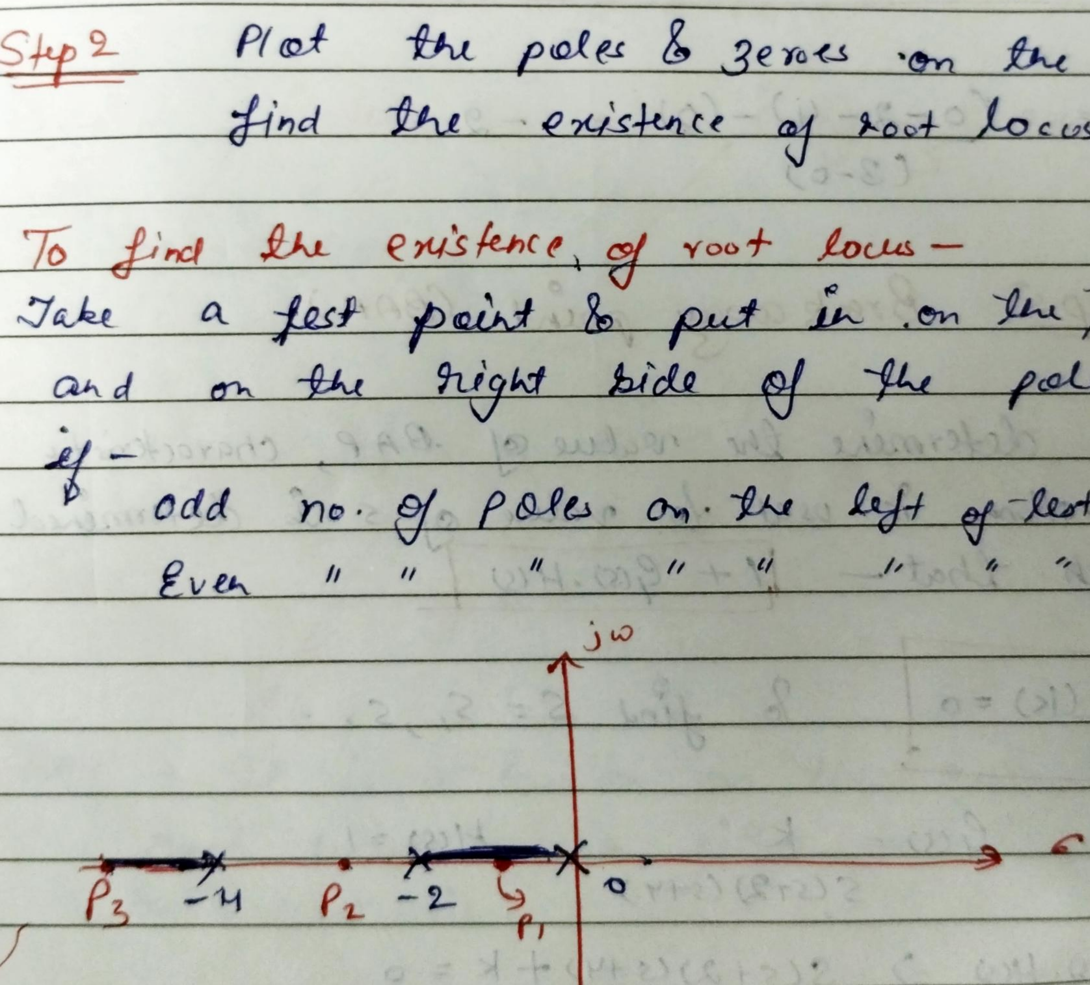
To find the existence of root locus -

Take a test point & put in on the just left and on the right side of the poles one by one

if -

odd no. of poles on the left of test point = ✓ exist

Even " " " " " " " " = X Not exist.



{ P_1, P_2, P_3 are the test point

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Step 4 Find the asymptodes & centroid.

$$\boxed{\text{Asymptode} = \frac{180[2q+1]}{P-Z}} \qquad q = 0, 1, \ldots (P-Z-1)$$

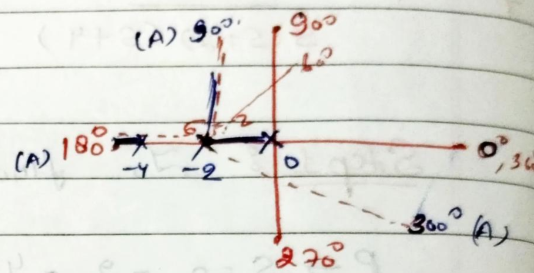
So -

$$A = \frac{180(2q+1]}{(3-0)} = 60(2q+1) \quad \left\{\begin{array}{l}\text{where} - \\ q = 0, 1, 2.\end{array}\right.$$

$q = 0 \rightarrow 60(2\times0+1) = 60^\circ$

$q = 1 \rightarrow 60[2\times1+1] = 180^\circ$

$q = 2 \rightarrow 60[2\times2+1] = 300^\circ$



$$\boxed{\text{Centroid} = \frac{\text{Sum of poles} - \text{Sum of zeroes}}{P-Z}}$$

$$\sigma = C = \frac{(0-2-4)-(0)}{(3-0)} = -2.$$

Step 5 Break away point (BAP)

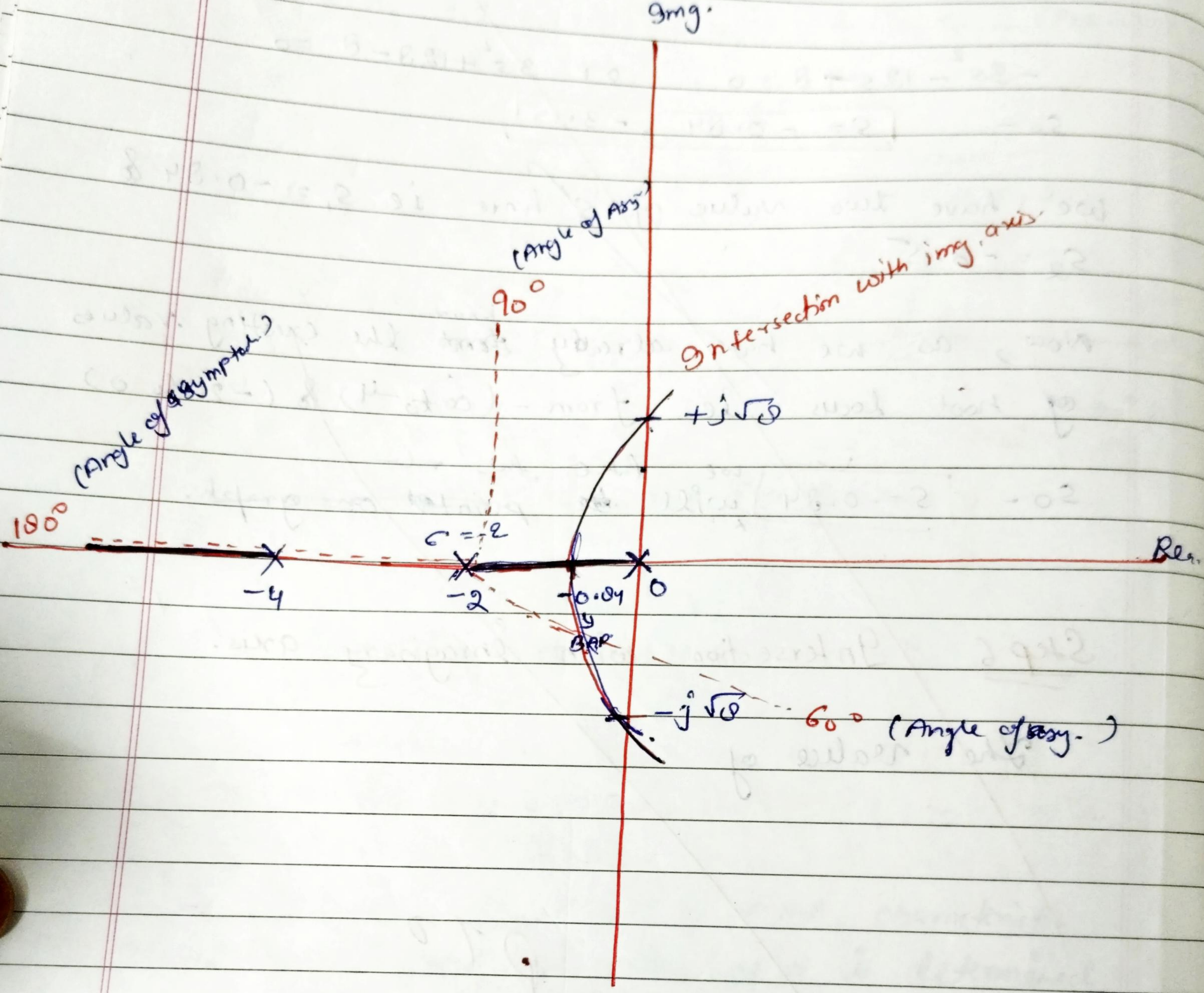
To determine the value of BAP, characteristic equations is used & value of s is determined such that — $\boxed{1 + G(s).H(s)}$

$$\boxed{\frac{d(K)}{ds} = 0} \quad \text{\& find } S = S_1, S_2 \ldots$$

So — $G(s) = \dfrac{K}{S(S+2)(S+4)}$ $\qquad H(s) = 1$

$1 + G(s).H(s) \Rightarrow S(S+2)(S+4) + K = 0$

$$\boxed{K = -s^3 - 6s^2 + 8s}$$



Root Locus of $\dfrac{K}{S(S+2)(S+4)}$

$$\frac{dk}{ds} = \frac{d}{ds}[-s^3 - 6s^2 + 8s] = 0 \Rightarrow -3s^2 - 12s + 8 = 0$$

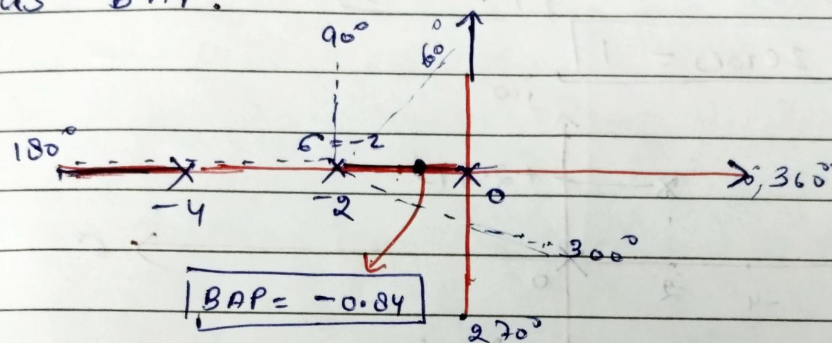
or $3s^2 + 12s + 8 = 0$; so $\rightarrow$ $S = -0.84$ & -3.15

Here we have 2 values of 's' i.e. $S_1 = -0.84$ & $S_2 = -3.15$

But, we know that the existing value of root locus is from $(-\infty$ to $-4)$ & $(-2$ to $0)$.

$\therefore$ we will take $S_1 = -0.84$ in the graph.

as BAP.



Step 6 Intersection with imaginary axis.

The value of K & the point at which root locus branches crosses the imaginary axis is determined by Routh-criterion to the characteristic equation.

Characteristics equations – $S^3 + 6s^2 + 8s + K = 0$

R-H criteria –

s^3	1	8	0
s^2	6	K	0
s^1	$\left(\frac{48-K}{6}\right)$	0	0
s^0	K	0	0

$\frac{48-K}{6} > 0$

$\boxed{K \Rightarrow 48}$

A.E $= 6s^2 + K = 0$; $6s^2 + 48 = 0$ so – $s^2 = -8$

$\boxed{s = \pm j\sqrt{8}}$

Root Locus for complex poles.

$$G(s) = \frac{K(s+9)}{s(s^2+4s+11)}$$

Step 1 Find poles & zeroes —

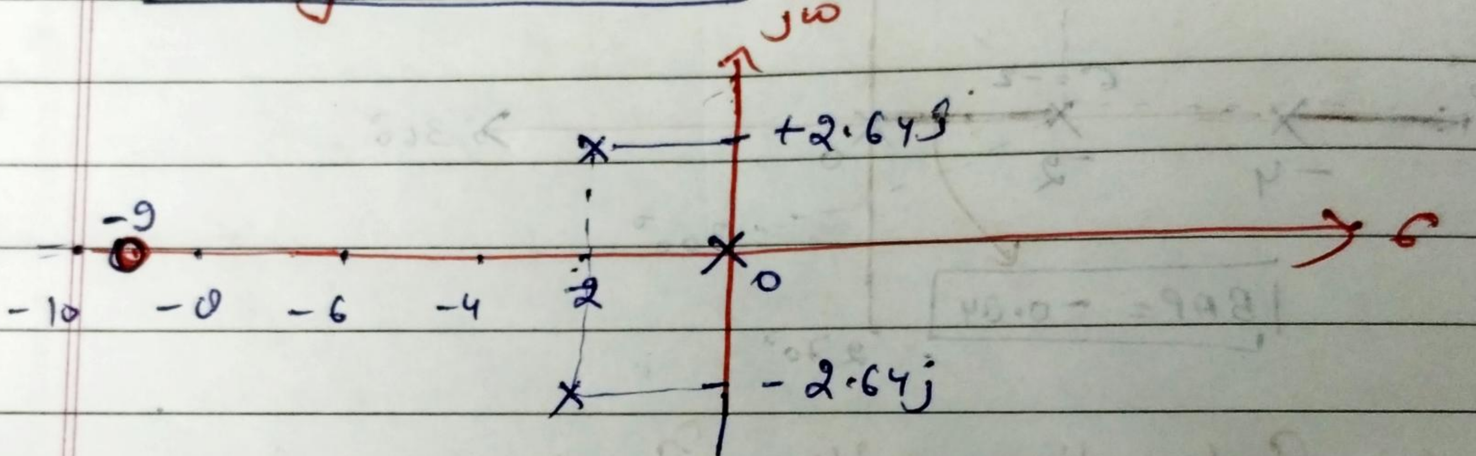
Poles $\Rightarrow$ $s = 0$, $(s^2+4s+11) = 0$

$\Rightarrow s = 0$, $(-2+2.64j)$, $(-2-2.64j)$

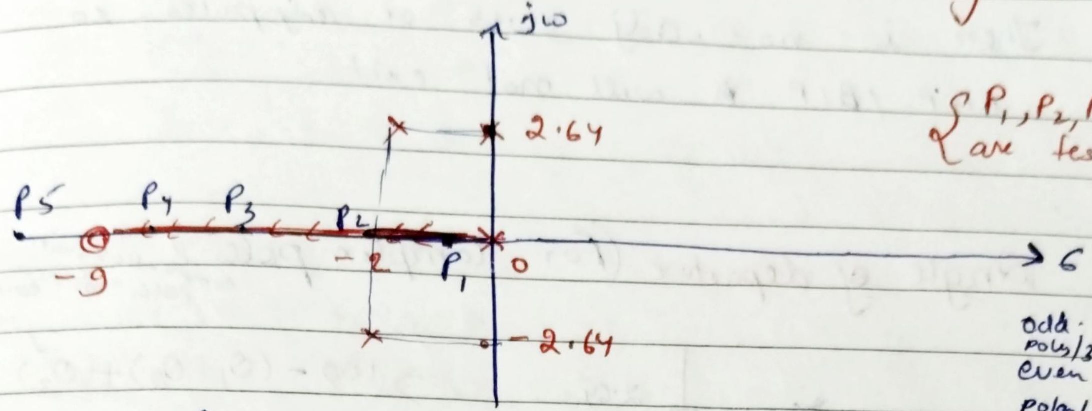
$\boxed{\text{No. of poles} \Rightarrow 3}$

Zeroes $= (s+9) = 0$, $\boxed{s = -9}$

$\boxed{\text{No. of zeroes} = 1}$



Step 2 To find the root locus on real axis (i.e to find the existence of root locus)



{ P1, P2, P3, P4 & P5 are test points

odd poles/zero ✓
even poles/zero ✗

starts from poles & ends at zeroes / ∞.

Step 3 Find the angle of asymptotes & centroid.

(Asy. starts from 6)

$$\boxed{A = \pm \frac{180(2q+1)}{(P-Z)}} \Rightarrow \pm \frac{180(2q+1)}{(3-1)} = \frac{180(2q+1)}{2}$$

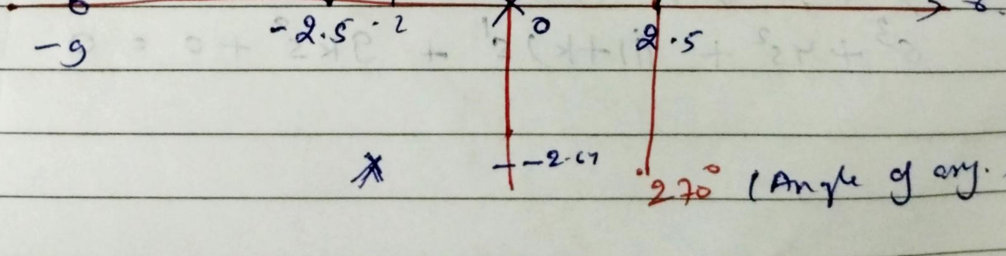
$A = \pm 90(2q+1)$ $\quad$ { $q = 0, 1, \ldots (P-Z-1)$

$q = 0 \rightarrow A = \pm 90[0+1] = 90^\circ$ $\quad$ $= 0, 1, (3-1-1)$

$q = 1 \rightarrow A = \pm 90[3] = 270^\circ$ $\quad$ $= 0, 1.$

$$\text{Centroids} = \frac{\Sigma P - \Sigma Z}{(P-Z)} \Rightarrow \frac{0 - 2 + 2.64j - 2 - 2.64j - 9}{(3-1)}$$

$$6 = \text{Cen}' = \frac{-4+9}{2} = \frac{5}{2} = 2.5 \qquad \boxed{6 = 2.5}$$



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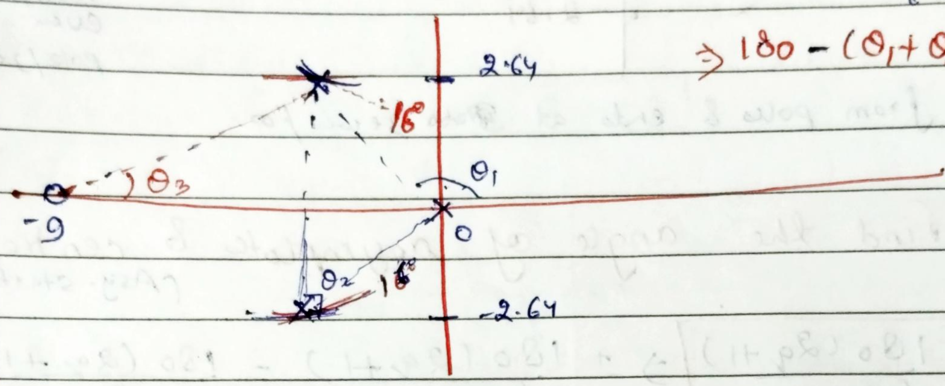
Step 4 To find the break in point / Breakaway point.
(break in point → for adj zeros; Breakaway point → for adj poles)

There is no adj zeros or adj poles. so BAP / BIP will not exist.

Step 5 Angle of departure (For complex poles)

$\Rightarrow 180 - (\theta_1 + \theta_2) + (\theta_3)$
(complex poles with poles and; complex poles with zero)



Using protector. measure, $\theta_1, \theta_2, \theta_3$
so-

$\theta_1 = 127°$, $\theta_2 = 90°$, $\theta_3 = 21°$

Angle of ~~of~~ dep. $= 180 - (127 + 90°) + 21$
$= -16°$

Step 6 Find crossing point on img. axis.

$CE = 1 + G(s) \cdot H(s) \Rightarrow 1 + \frac{K(s+9)}{s(s^2+4s+11)} = 0$

$s[s^2+4s+11] + K(s+9) = 0$

$s^3 + 4s^2 + 11s + sK + 9K = 0$

$s^3 + 4s^2 + 11s + (sK)$

$s^3 + 4s^2 + (11+K)s^1 + 9Ks^0 + 0 = 0$

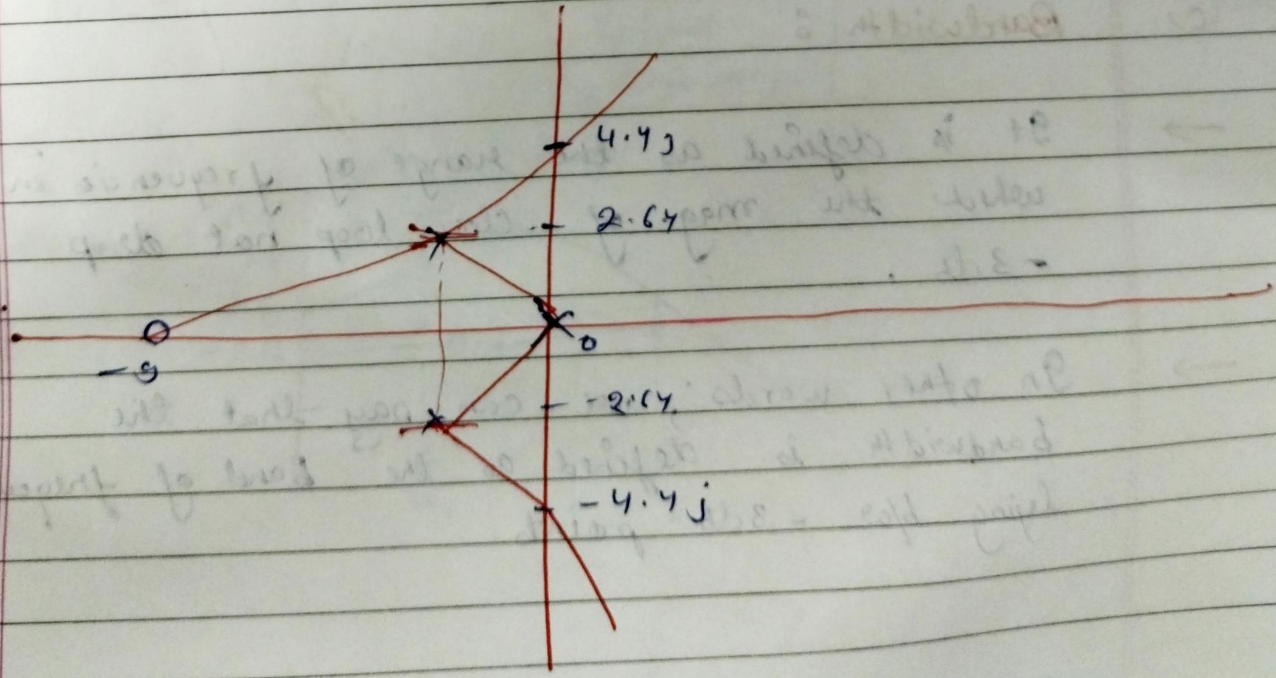
s^3	1	$(11+K)$	0
s^2	4	$9K$	0
s^1	$\frac{4(11+K)-9K}{4}$	0	0
s^0	$9K$	0	0

$$4(11+K)-9K>0$$
$$44+4K-9K>0$$
$$44-5K>0$$
$$\frac{44}{5} \geq K$$
$$\boxed{8.8 = K}$$

$$9K = 0$$
$$9 \times \frac{44}{5} \Rightarrow$$
$$\boxed{79.2}$$

$$A.E = 4s^2 + 9K = 0$$
$$4s^2 + 9K = 0$$
$$s^2 = \frac{-79.2}{4} \Rightarrow -19.8$$

$$s = \pm j\sqrt{19.8} \Rightarrow \pm j\, 4.4$$
$$\boxed{s = \pm 4.4 j}$$



FREQUENCY DOMAIN SPECIFICATIONS.

(a) **Resonant Peak: M_r** $\boxed{M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}}$

→ The max. value of magnitude is known as resonant peak.

→ The magnitude of resonant peak gives the inf: about the relative stability of the system.

→ A large value of resonant peak implies undesirable transient response.

$\boxed{\omega_d = \omega_n\sqrt{1-\xi^2}}$

(b) **Resonant frequency (ω_r)** $\boxed{\omega_r = \omega_n\sqrt{1-2\xi^2}} = \frac{\sqrt{1-2\xi^2}}{\sqrt{1-\xi^2}}$

→ The freq. at which magnitude has max. value is known as resonant freq.

→ If ω_r is large, the time response is fast.

(c) **Bandwidth:**

→ It is defined as the range of frequencies in which the mag. of close loop not drop -3 db.

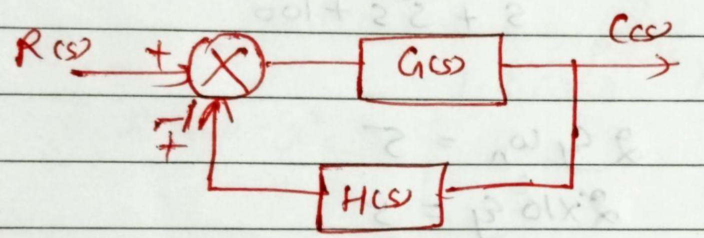
→ In other words, we can say that the bandwidth is defined as the band of frequencies lying b/w -3 db points.

(d) Cut-off frequency (ω_b) $\boxed{\omega_b = \omega_n \sqrt{(1-2\zeta^2)+\sqrt{2-4\zeta^2+4\zeta^4}}}$

The freq. at which the mag. is 3db below its zero frequency value is called as cut-off. freq. (ω_b)

(e) Cut-off rate.

The cut-off rate is the slope of the log mag. curve near the ~~cd~~ cut-off freq.



$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)\,H(s)}}$$

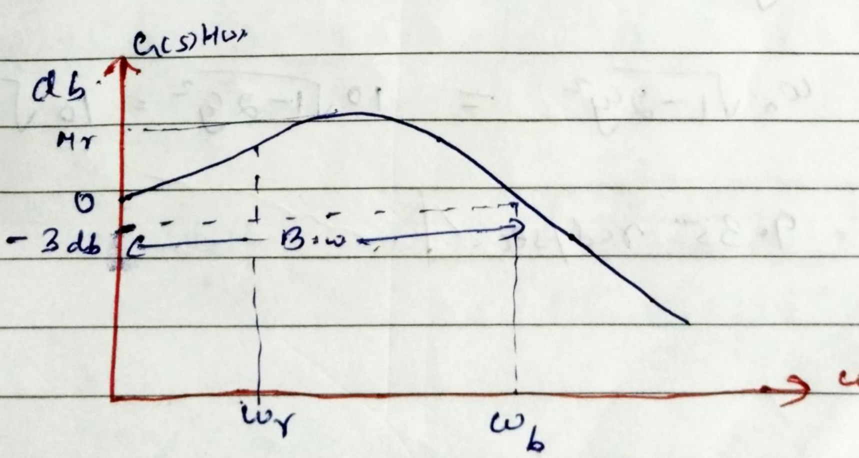
In freq. domain $\rightarrow$ s is replace by $j\omega$ $\{s \rightarrow j\omega\}$

so –

$$T(j\omega) = \frac{C(j\omega)}{R(j\omega)} = \frac{G(j\omega)}{1 \pm G(j\omega)H(j\omega)}$$

or –

$$\boxed{M(j\omega) = \frac{G(j\omega)}{1 \pm G(j\omega)H(j\omega)}}$$



Q. For unity feedback sys. $G(s) = \frac{100}{s(s+5)}$

Determine — (i) Resonance Peak (M_r)

(ii) Resonance freq. (ω_r)

$\{H(s)=1\}$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{100}{s(s+5)}}{1+\left[\frac{100}{s(s+5)}\right]} = \frac{100}{s^2+5s+100} \quad -(1)$$

compare eq (1) with the transfer funct. of 2nd order system so —

$$T.f = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} = \frac{100}{s^2+5s+100}$$

$\omega_n^2 = 100$; $2\zeta\omega_n = 5$

$\boxed{\omega_n = 10}$

$2\times 10\,\zeta = 5$

$\zeta = 1/4 = 0.25$

$\boxed{\zeta = 0.25}$

(i) Resonance Peak :

$$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = \frac{1}{2\zeta\sqrt{1-(0.25)^2}} = 2.0655$$

(ii) Resonant freq.

$$\omega_r = \omega_n\sqrt{1-2\zeta^2} = 10\sqrt{1-2\zeta^2} = 10\sqrt{1-2(0.5)^2}$$

$\boxed{\omega_r = 9.35 \text{ rad/sec.}}$

Q For a closed loop control system $G(s) = \frac{100}{s(s+8)}$ and $H(s) = 1$. Determine the resonant peak & resonant frequency.

$\Rightarrow$ $$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s).H(s)} = \frac{\frac{100}{s(s+8)}}{1+\left[\frac{100}{s(s+8)}\right].1} = \frac{100}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\frac{C(s)}{R(s)} = \frac{100}{s^2+8s+100} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$\boxed{\omega_n = 10 \text{ rad/sec}}$; $\boxed{\xi = 0.4}$

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = 1.3638 \quad \& \quad \omega_r = \omega_n\sqrt{1-2\xi^2} = 8.2462 \text{ rad/sec}$$

Effect of adding a zero to the forward path.

$$G(s) = \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$

Let a zero $(1+sT)$ is added to the forward path transfer function. then –

$$G(s) = \frac{\omega_n^2(1+sT)}{s(s+2\xi\omega_n)}$$

So –

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s).1} \Rightarrow \frac{\left[\frac{\omega_n^2(1+sT)}{s(s+2\xi\omega_n)}\right]}{\left\{1+\frac{\omega_n^2(1+sT)}{s(s+2\xi\omega_n)}\right\}}$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2(1+Ts)}{s^2+(2\xi\omega_n+T\omega_n^2)s+\omega_n^2}$$

$$\text{Bandwidth} = \left[-x + \frac{1}{2}\sqrt{x^2+4\omega_n^2} \right]^2$$

Where, $x = 4\zeta\omega_n^2 + 4\zeta\omega_n^3 T - 2\omega_n^2 - \omega_n^4 \tau^2$

So, when a zero is added to the forward path transfer function, the B.W of the closed loop system will increase. Rise tym ↓, Stability ↑.

Effect of adding a pole to the forward path

Let pole $(1+s\tau)$ is added to the forward path function then ~~equation~~ —

$$G(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)(1+s\tau)}$$

→ Calculation of B.W is very complicated.
So B.W of close loop control system decrease

→ According to Kuo*, for small value of T, the B.W of the system ↑ but resonant peak M_r also ↑.

→ But when T becomes large, the B.W ↓ but resonant peak M_r ↑

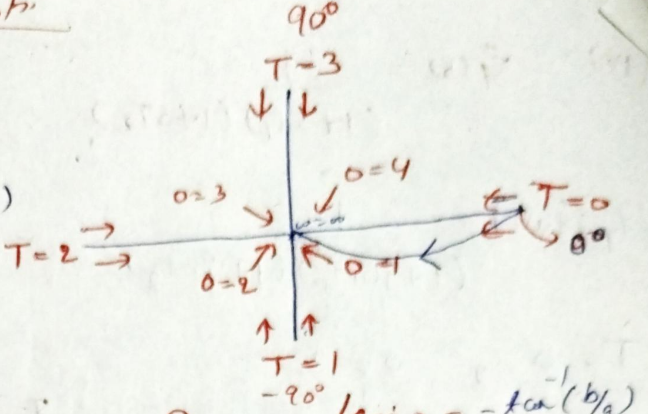
→ The rise time ↑ with ↓ in B.W.

→ Stability of the system ↓ with ↓ B.W.

Polar-Plot

Types of systems

(i) $G(s) = \dfrac{1}{(1+sT)} = \dfrac{1}{(1+j\omega T)}$



T → Type , O = Order

→ T = 0

→ O = 1.

$\boxed{s \to j\omega}$

Magnitude $= |G(j\omega)| = \dfrac{1}{\sqrt{1+\omega^2T^2}}$; Phase angle $\angle G(j\omega) = -\tan^{-1}(b/a)$

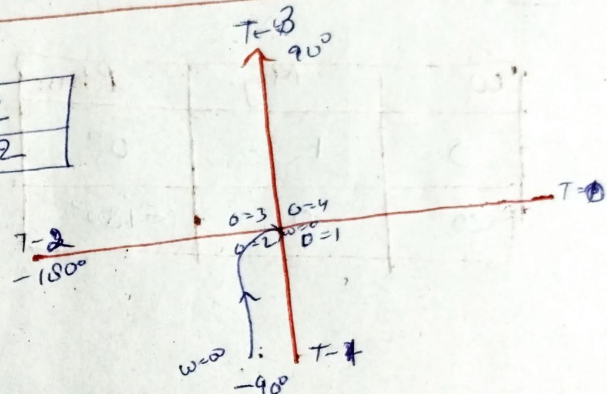
$= -\tan^{-1}(\omega T)$

{ Starts from Type & ends at ~~pole~~ order

ω	M	P
0	1	0°
∞	0	-90°

(ii) $G(s) = \dfrac{1}{s(1+sT)}$

T = 1
O = 2



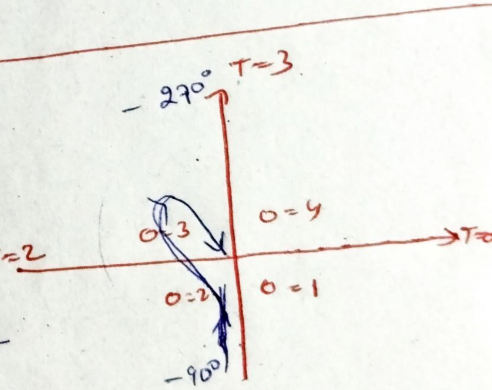
$G(j\omega) = \dfrac{1}{j\omega(1+j\omega T)}$

Mag $= |G(j\omega)| = \dfrac{1}{\omega\sqrt{1+\omega^2T^2}}$

Phase $= \angle G(j\omega) = -90 - \tan^{-1}(\omega T)$

ω	M	P
0	∞	-90°
∞	0	-180°

(iii) $G(s) = \dfrac{1}{s(sT_1+1)(sT_2+1)}$ T = 1, O = 3



Mag $= |G(j\omega)| = \dfrac{1}{\omega\sqrt{1+\omega^2T_1^2}\sqrt{1+\omega^2T_2^2}}$

Phase $= \angle G(j\omega) = -90° - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2$

ω	M	P
0	∞	-90°
∞	0	-270°

Polar plot

$1+G(s)$ $\quad 1+K\left[\frac{N(s)}{D(s)}\right]$

(IV) $G(s) = \dfrac{1}{(1+sT_1)(1+sT_2)}$

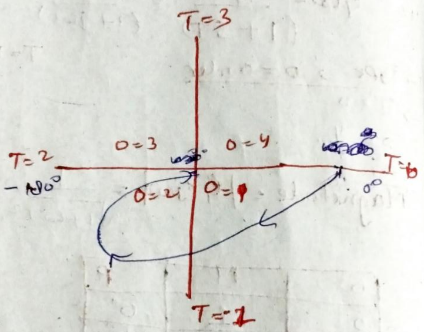
$G(j\omega) = \dfrac{1}{(1+j\omega T_1)(1+j\omega T_2)}$

$T = 0$, $\quad 0 = 2$

$|G(j\omega)| = \dfrac{1}{\sqrt{1+\omega^2 T_1^2}\sqrt{1+\omega^2 T_2^2}}$

$\angle G(j\omega) = -\tan^{-1}\omega T_1 - \tan^{-1}\omega T_2$

ω	Mag.	Phase
0	1	0°
∞	0	-180°



Type of Nyquist Contour.

→ To judge stability, we need to determine n of poles in RHP, so nyquist contour in s-pl must encircle entire right half-plane.

→ Must not pass through any pole.

$G(s) \cdot H(s) = \dfrac{1}{(1+sT)}$

(IV) $G(s) = \dfrac{1}{(1+sT_1)(1+sT_2)}$

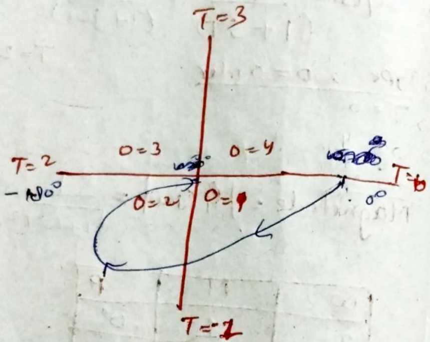
$G(j\omega) = \dfrac{1}{(1+j\omega T_1)(1+j\omega T_2)}$

$T = 0$, $O = 2$

$|G(j\omega)| = \dfrac{1}{\sqrt{1+\omega^2 T_1^2}\sqrt{1+\omega^2 T_2^2}}$

$\angle G(j\omega) = -\tan^{-1}\omega T_1 - \tan^{-1}\omega T_2$

ω	Mag.	Phase
0	1	0°
∞	0	-180°



Type of Nyquist Contour.

→ To judge stability, we need to determine no. of poles in RHP, so nyquist contour in s-plane must encircle entire right half-plane.

→ Must not pass through any pole.

Stability using Nyqcuist plot

$$\boxed{N = P - Z}$$

N is positive for anticlockwise encirclement around (-1, 0).

N is negative for clockwise encirclement around (-1, 0) → critical point

P is open loop poles of system on RHP.

Z is close loop poles of system on RHP

Nyquist contour:

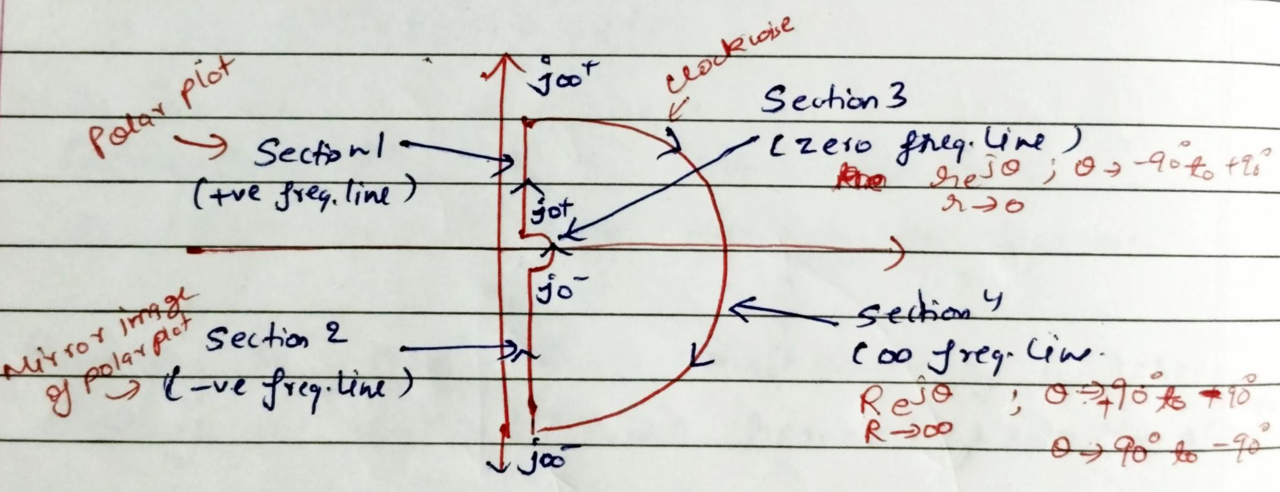


fig: S- plane of C.E.

↳ Open-loop transfer function (OLTF)

$G(s) = \frac{K\ N(s)}{D(s)}$ → Roots of zeros of OLTF
→ Roots of poles of OLTF.

↳ Close-loop transfer function (CLTF)

$CLTF = \frac{G(s)}{1+G(s).1} = \frac{K\ N(s)}{1+ G(s)}$ → ~~Roots of zeros of CLTF~~
→ ~~Roots of poles of CLTF~~.

$CLTF = \frac{K\ \cancel{N(s)}}{1+ [\cancel{K N}}$

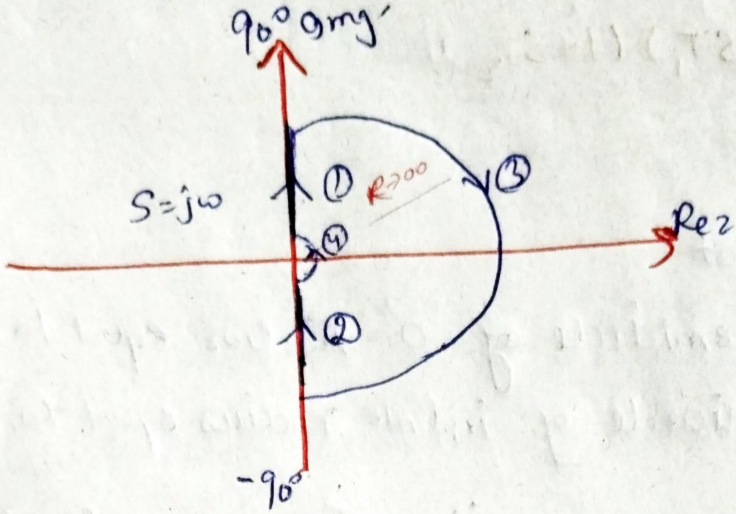
$CLTF = \frac{G(s)}{1+G(s)} = \frac{K\frac{N(s)}{D(s)}}{1+K\left[\frac{N(s)}{D(s)}\right]} = \frac{K\ N(s)}{D(s)+K\ N(s)}$ → Roots of zeros of CLTF
→ Roots of poles of CLTF.

↳ Characteristics equation (C.E)

$C.E = 1+G(s) = 1+K\frac{N(s)}{D(s)} = \frac{D(s)+KN(s)}{D(s)}$ → Roots of zeros of C.E
→ Roots of poles of C.E.

$$\boxed{\begin{array}{l} \text{Poles of C.E = Pole of OLTF} \\ \text{Zeros of C.E = Poles of CLTF} \end{array}}$$

MAPPING OF NYQUIST CONTOUS



Curve 1 : $\boxed{S = j\omega}$ (Polar plot)

ω varies from $0 \rightarrow \infty$

Curve 2 : $\boxed{S = -j\omega}$ (Inverse-polar-plot)

we get a mirror image of polar plot about real axis.

Curve 3 : Clockwise Infinite semicircle

$s = Re^{j\theta}$; $R \rightarrow \infty$, $\theta \rightarrow \pi/2$ to $-\pi/2$

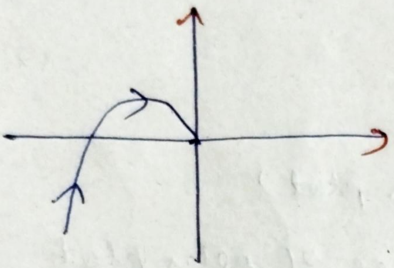
Curve 4 : Anticlockwise semicircle

$s = \lim_{R \to 0} re^{j\theta}$; $\theta \rightarrow -\pi/2$ to $\pi/2$

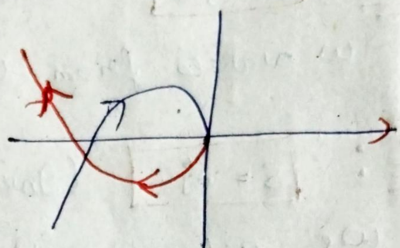
Q $G(s).H(s) = \frac{1}{s(1+sT_1)(1+sT_2)}$

⇒ (i) Polar plot
(ii) Inverse polar plot
(iii) Anticlockwise semicircle of '0' radius equal to (P−Z)
(iv) Clock-wise semicircle of infinite radius equal to type of syst

(i) polar plot



(ii) Inverse polar plot



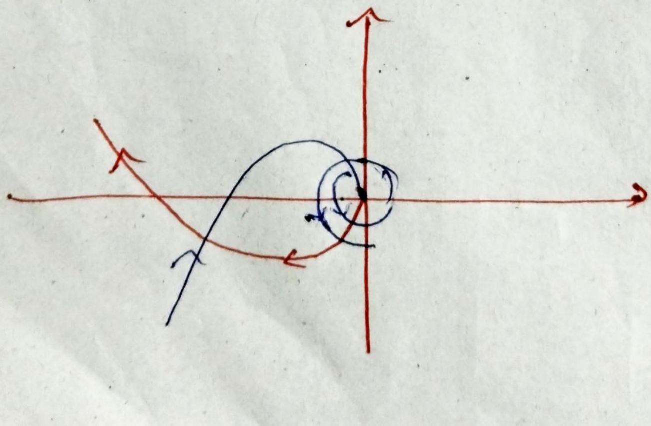
(iii) $G(s)H(s) = \lim_{R\to\infty} Re^{j\theta} = \lim_{R\to\infty} \frac{1}{Re^{j\theta}(1+RT_1e^{j\theta})(1+RT_2e^{j\theta})}$

$= \lim_{R\to\infty} \frac{1}{R^3 T_1 T_2}$ At $R\to\infty$ $(1+RT_1e^{j\theta}) \to RT_1e^{j\theta}$
$(1+RT_2e^{j\theta}) \to RT_2e^{j\theta}$

$= \lim_{R\to\infty} \frac{1}{R^3 T_1 T_2} \cdot e^{-j3\theta}$

$\frac{1}{R^3} \to 0$: $\theta \Rightarrow 90^\circ$ to -90°

$-3\theta \Rightarrow -270^\circ$ to 270°. 3 semicircle



(iv) $G(s) \cdot H(s) = \dfrac{1}{s(sT_1+1)(sT_2+1)}$

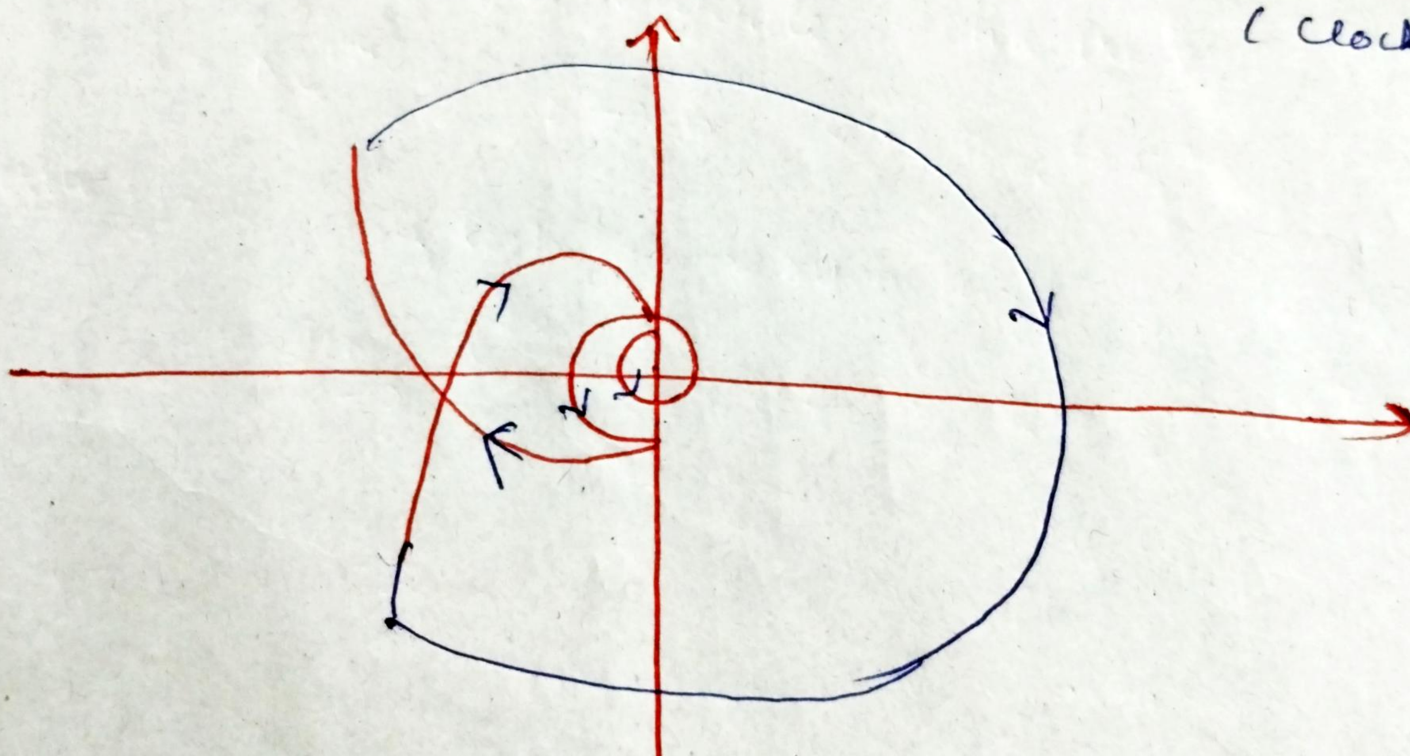
Put $s = \lim\limits_{r \to 0} re^{j\theta}$

$$G(s) \cdot H(s) = \frac{1}{re^{j\theta}(1+rT_1 e^{j\theta})(1+rT_2 e^{j\theta})}$$

$$= \frac{1}{re^{j\theta}} \Rightarrow \frac{1}{r} \cdot e^{-j\theta}$$

$\frac{1}{r} \to \infty$, $-\theta$: 90° to -90°

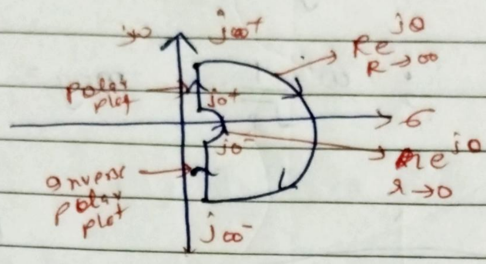
1 semicircle of ∞ radius (clockwise)



Alternate method

Q $G(s) \cdot H(s) = \dfrac{10}{s(s+1)}$

Step 1 Nyquist contour



Step 2 Analysis of section 1 (polar plot)

$G(s) \cdot H(s) = \dfrac{10}{s(s+1)}$; $G(s)H$.

Put $s \to j\omega$ in above eqⁿ.

$$\left(G(j\omega) \cdot H(j\omega)\right) = \frac{10}{j\omega(j\omega+1)}$$

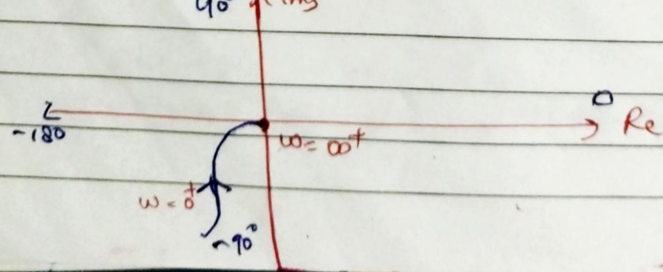
Magnitude:

$$|G(j\omega)\, H(j\omega)| = \frac{10}{\omega\sqrt{\omega^2+1}}$$

Angle

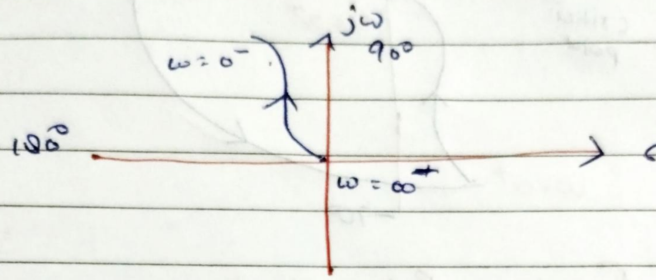
$$\angle G(j\omega) \cdot H(j\omega) = -90^\circ - \tan^{-1}\omega$$

ω	$\lvert G(j\omega)H(j\omega) \rvert$	$\angle G(j\omega) \cdot H(j\omega)$
0^+	∞	-90
∞^+	0	-180°



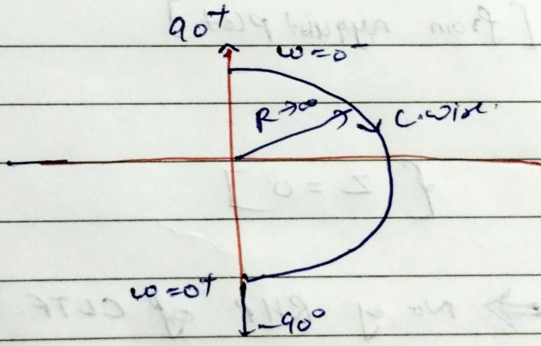
Step 3 Analysis of section 2 (Mirror image of polar plot)

ω	$\lvert G(j\omega) H(j\omega) \rvert$	$\angle G(j\omega) H(j\omega)$
∞^-	0	$+180^\circ$
0^-	∞	$+90^\circ$



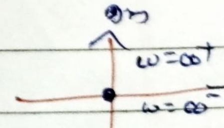
Step 4 Analysis of zero freq.

ω	$\lvert G(j\omega) \cdot H(j\omega) \rvert$	$\angle G(j\omega) \cdot H(j\omega)$
0^-	∞	$+90^\circ$
0^+	∞	-90°



No. of ∞ mag. half circle = No. of poles at origin = 1

Direct' of ∞ mag. half circle is always clock wise.



Step 5: Analysis of ∞ freq. line.

ω	$\lvert G(j\omega) H(j\omega) \rvert$	$\angle G(j\omega) \cdot H(j\omega)$
∞^+	0	-180°
∞^-	0	$+180^\circ$

Date
Page

Step 6 Nyquist pole.

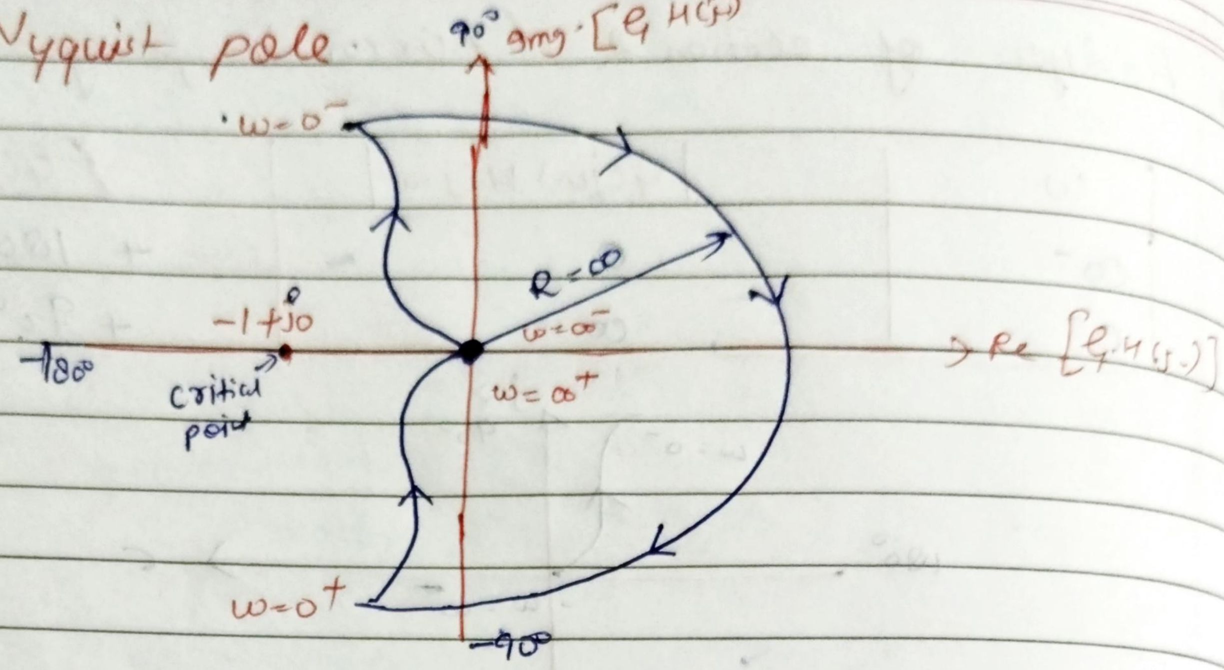


Fig: $G(j\omega) H(j\omega)$

Step 7 Nyquist Stability criteria.

$N = P - Z$ [Anticlkwise]

$P = 0$ [from OLTF]

$N = 0$ [from nyquist plot]

$N = P - Z$

$0 = 0 - Z$ $\{Z = 0\}$

So-

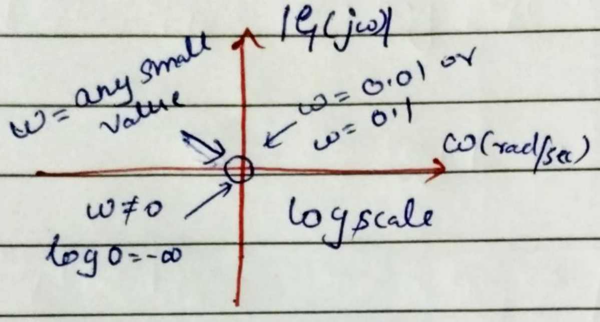
$\boxed{Z = 0}$ $\Rightarrow$ No of RHP of CLTF

(STABLE CLTF)

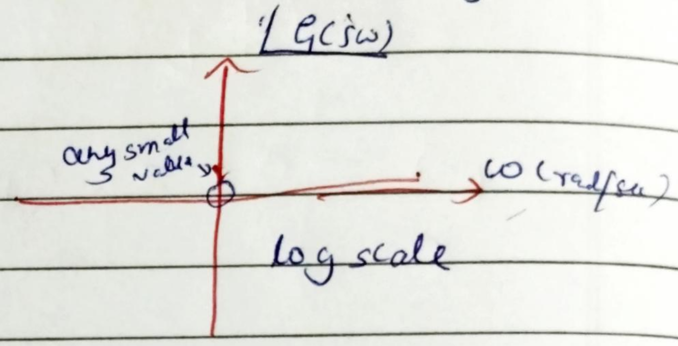
Bode plot : Bode plot is a graphical representation of the transfer function for determining the stability of the control system.

Bode plot consist of 2 separate plot -

(i) $20 \log_{10} |G(j\omega)|$ vs $\log \omega$ (ii) Phase shift (ϕ) vs $\log \omega$



Bode Magnitude plot



Bode phase plot.

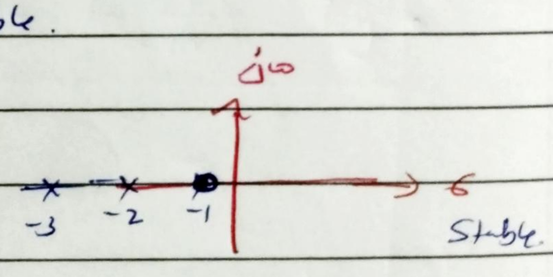
MPS and NMPS

MPS — Min. phase system [Consist only L-H poles & zeros]

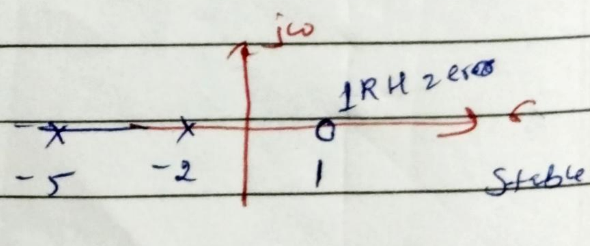
NMPS — Non-min phase system [Consist atleast 1 pole or zero in R-H of the s-plane]

* MPS with L-H pole & zeros → Stable
* Preferred ✓ NMPS with 1 R-H zero → Stable
* NMPS with 1 RH pole → Unstable.

$$G_1(s) = \frac{(1+s)}{(2+s)(3+s)} \rightarrow MPS$$



$$G_2(s) = \frac{(1-s)}{(s+5)(s+2)} \rightarrow NMPS$$



(i) $\boxed{G(s) = \frac{1}{s}}$

Put $\boxed{s = j\omega}$

$G(j\omega) = \frac{1}{(j\omega)}$ | Magnitude $|G(j\omega)| = \frac{1}{\omega}$ | Angle $\angle G(j\omega) = -90^\circ$

In dB →

$|G(j\omega)|_{dB} = 20 \log |G(j\omega)| = 20 \log\left(\frac{1}{\omega}\right)$

$= 20 \log 1 - 20 \log \omega = 0 - 20 \log \omega$

$\boxed{|G(j\omega)|_{dB} = -20 \log \omega}$

ω	$\lvert G(j\omega)\rvert_{dB}$
0.1	$-20 \log(0.1) = 20 \, dB$
1.0	$-20 \log(1) = 0 \, dB$
10.0	$-20 \log 10 = -20 \, dB$

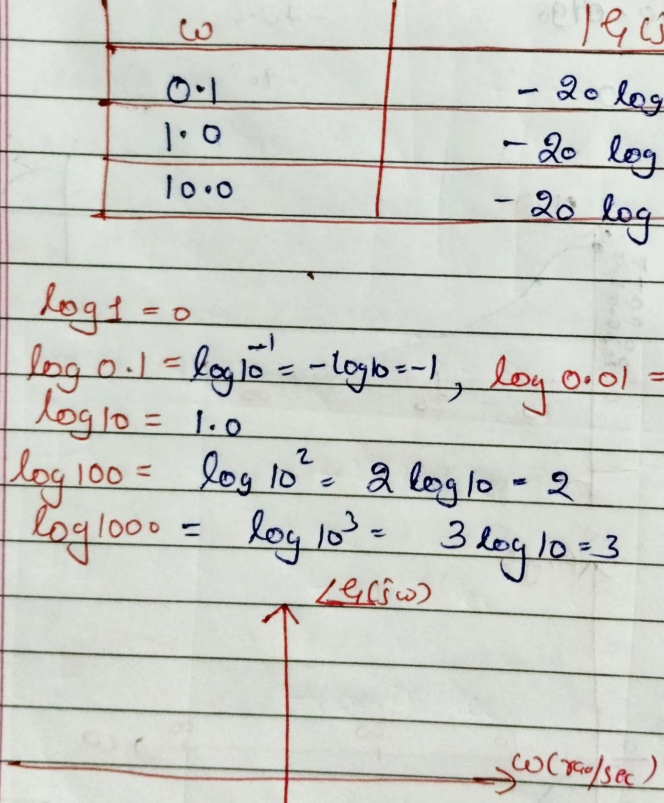
$\log 1 = 0$

$\log 0.1 = \log 10^{-1} = -\log 10 = -1$, $\log 0.01 = \log\left(\frac{1}{100}\right) = \log 10^{-2} = -2 \log 10 = -2$

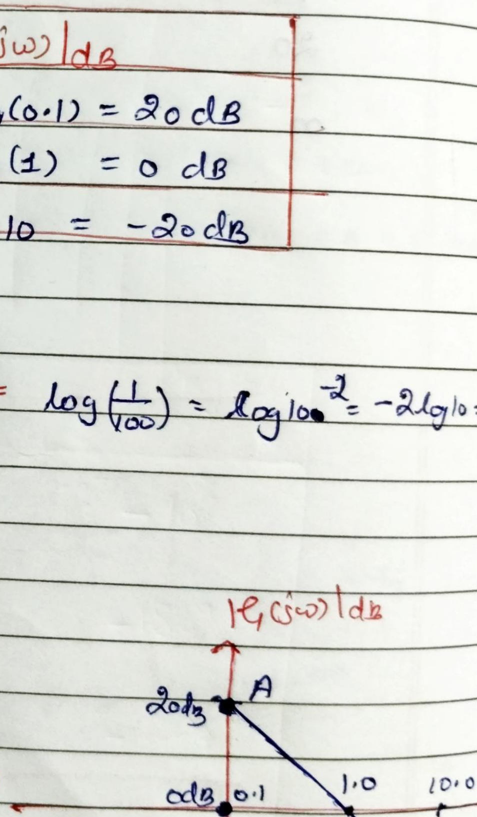
$\log 10 = 1.0$

$\log 100 = \log 10^2 = 2 \log 10 = 2$

$\log 1000 = \log 10^3 = 3 \log 10 = 3$



(i) Bode phase plot



log scale

(ii) Bode Mag. plot

$$Slope_{AB} = \frac{Y_B - Y_A}{\log\omega_B - \log\omega_A} = \frac{Y_B - Y_A}{\log\left(\frac{\omega_B}{\omega_A}\right)} = \frac{0-20}{\log\left(\frac{1.0}{0.1}\right)}$$

$$Slope_{AB} = \frac{-20}{\log 10} = -20\ dB/dec.$$

$$Slope_{BC} = \frac{Y_C - Y_B}{\log(\omega_C/\omega_B)} = \frac{-20-0}{\log(10/1)} = -20\ dB/dec$$

$$Slope_{AC} = \frac{Y_C - Y_A}{\log(\omega_C/\omega_A)} = \frac{-20-20}{\log(10/0.1)} = \frac{-40}{\log 100} = -20\ dB/dec$$

(ii) $G(s) = \frac{1}{s^2}$

Put $s = j\omega$

	Magnitude	Phase
$G(j\omega) = \frac{1}{(j\omega)^2}$	$\lvert G(j\omega)\rvert = \frac{1}{\omega^2}$	$\angle G(j\omega) = -180^\circ$

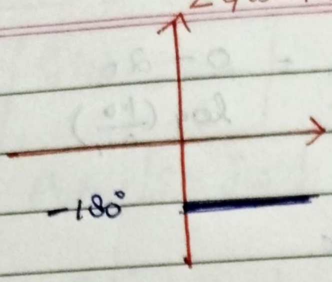
In dB-

$$|G(j\omega)|_{dB} = 20\log|G(j\omega)| = 20\log\frac{1}{\omega^2} = 20\log 1 - 20\log\omega^2$$

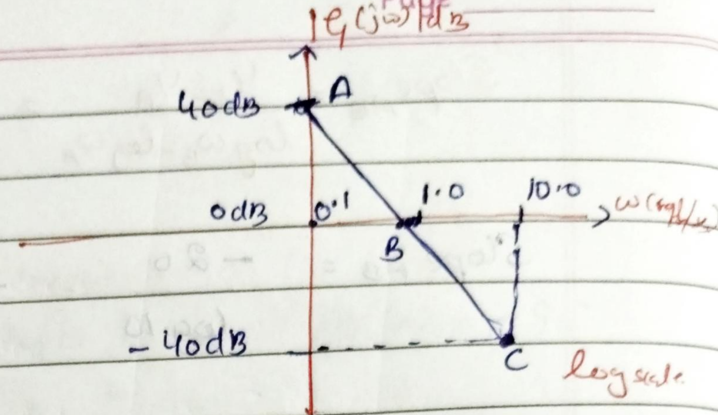
$$\boxed{|G(j\omega)|_{dB} = -40\log\omega}$$

ω	$\lvert G(j\omega)\rvert_{dB}$
0.1	$-40\log 0.1 = 40\ dB$
1.0	$-40\log 1 = 0$
10.0	$-40\log 10 = -40\ dB$

ω (r/sec)



(i) Bode phase plot



(ii) Bode Mag. plot

$$\text{Slope}_{AB} = \frac{Y_B - Y_A}{\log(\omega_B/\omega_A)} = \frac{0-40}{\log(1/0.1)} = -40\ dB/dec$$

$$\text{slope}_{BC} = \frac{Y_C - Y_B}{\log(\omega_C/\omega_B)} = \frac{-40-0}{\log(10/1)} = -40\ dB/dec.$$

$$\text{Slope}_{AC} = \frac{Y_C - Y_A}{\log(\omega_C/\omega_A)} = \frac{-40-40}{\log(10/0.1)} = \frac{-80}{2} = -40\ dB/dec$$

Conclusion -

$G(s)$	Slope	Angle.
$\frac{1}{s}$	$-20dB/dec$	$-90°$
$\frac{1}{s^2}$	$-40dB/dec$	$-180°$
$\frac{1}{s^3}$	$-60dB/dec$	$-270°$
$\frac{1}{s^N}$	$-20N\ dB/dec.$	$-90°N$

(B) $G(s) = s$

Put $s = (j\omega)$

$G(j\omega) = j\omega$

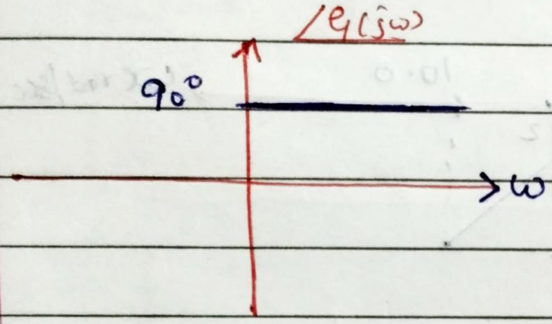
Mag. $|G(j\omega)| = \omega$

Phase. $\angle G(j\omega) = 90^\circ$

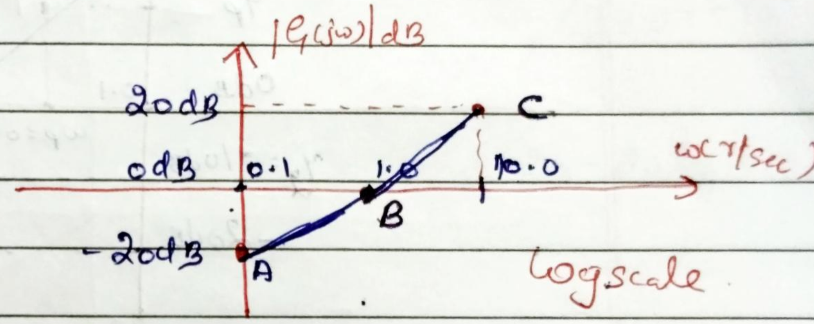
In dB–

$|G(j\omega)|_{dB} = 20 \log \omega$

ω	$\lvert G(j\omega)\rvert_{dB}$
0.1	$20 \log(0.1) = -20\,dB$
1.0	$20 \log(1) = 0$
10	$20 \log(10) = 20\,dB$



(i) Bode Phase Plot



(ii) Bode Mag. plot

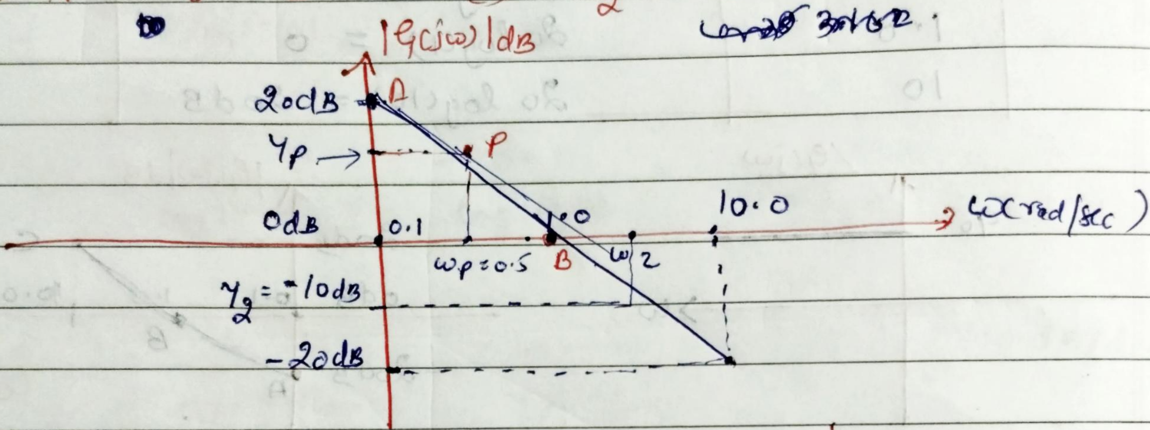
$$\text{Slope}_{AB} = \frac{Y_B - Y_A}{\log(\omega_B/\omega_A)} = \frac{0-(-20)}{\log(1/0.1)} = 20\,dB/dec$$

$$\text{Slope}\,BC = \frac{Y_B - Y_C}{\log(\omega_B/\omega_C)} = \frac{0-20}{\log(1/10)} = 20\,dB/dec.$$

$$\text{Slope}\,AC = \frac{Y_C - Y_A}{\log(\omega_C/\omega_A)} = \frac{20-(-20)}{\log(10/0.1)} = 20\,dB/dec.$$

$G(s)$	Slope	Angle.
S^1	20 dB/dec	90°
S^2	40dB/dec	180°
S^3	60 dB/dec	270°
S^N	20N dB/dec.	$90^\circ N$

Q (i) $Y_P = ?$ dB. (ii) $\omega_2 = ?$ rad/sec.



$$\text{Slope}_{AB} = \frac{0-20}{\log\left[\frac{1}{0.1}\right]} = -20\,dB/dec$$

$$\text{Slope } AP = \frac{Y_P - Y_A}{\log(\omega_P/\omega_A)} = \frac{Y_P - 20}{\log\left(\frac{0.5}{0.1}\right)} = \frac{Y_P - 20}{\log(5)}$$

$$-20 = \frac{Y_P - 20}{0.698} \Rightarrow Y_P = -20 \times 0.698 + 20$$

$$\boxed{Y_P = 6.02\ dB}$$

$$\text{Slope } B2 = \frac{Y_2 - Y_B}{\log(\omega_2/\omega_B)} =$$

$$-20 = \frac{-10-0}{\log(\omega_2/1)} = \frac{-10}{\log \omega_2}$$

$$\log \omega_2 = \frac{10}{20} = 0.5$$

$$\boxed{\omega_2 = 10^{0.5} = 3.162\ r/sec}$$

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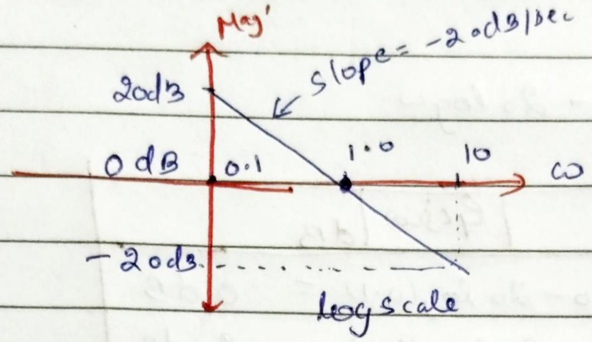
Q. Draw a Bode-plot for a given T.f $-$ $G(s) = K/s$

Case I $K = 1.0$ $G(s) = \frac{1}{s}$

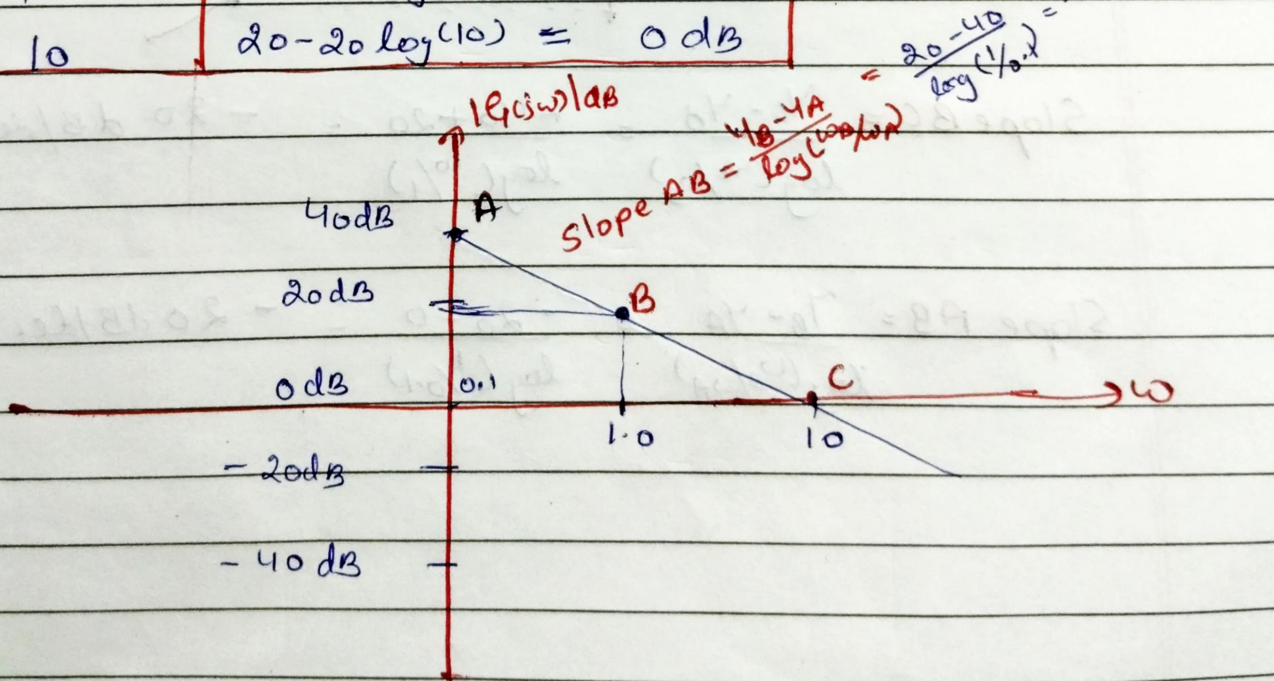


Case II $K > 1.0$

Assume $\boxed{K = 10}$ $G(s) = 10/s$ $|G(j\omega)| = 10/\omega$, $\angle G(j\omega) = -90^\circ$

$$|G(j\omega)|_{dB} = 20\log 10/\omega = 20\log 10 - 20\log\omega = 20 - 20\log\omega$$

ω	$\lvert G(j\omega)\rvert_{dB}$	
0.1	$20 - 20\log(0.1) =$	40dB
1.0	$20 - 20\log(1.0) =$	20dB
10	$20 - 20\log(10) =$	0dB



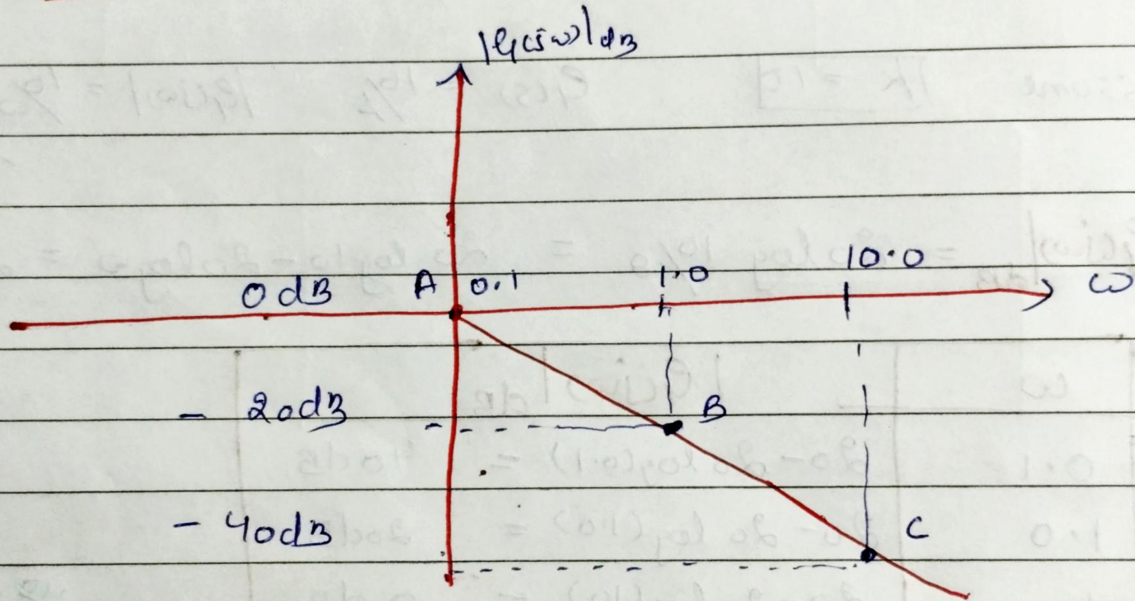
Date
Page

Case 3 : $K < 1.0$ Assume $K = 0.1$

$G(s) = 0.1/s$, $|G(j\omega)| = \frac{0.1}{\omega}$, $|G(j\omega)|_{dB} = 20\log 0.1 - 20\log\omega$

$|G(j\omega)|_{dB} = -20 - 20\log\omega$

ω	$\lvert G(j\omega)\rvert_{dB}$
0.1	$-20 - 20\log(0.1) = 0\,dB$
1.0	$-20 - 20\log(1.0) = -20\,dB$
10.0	$-20 - 20\log(10.0) = -40\,dB$



Slope BC $= \frac{y_C - y_B}{\log(\omega_C/\omega_B)} = \frac{-40+20}{\log(10/1)} = -20\ dB/dec.$

Slope AB $= \frac{y_B - y_A}{\log(\omega_B/\omega_A)} = \frac{-20-0}{\log(1/0.1)} = -20\ dB/dec.$