Program : **B.Tech**

Subject Name: **Digital Communication**

Subject Code: **EC-502**

Semester: **5^{th}**

Unit 3

Digital Transmission Techniques: Phase shift Keying (PSK)- Binary PSK, differential PSK, differentially encoded PSK, Quadrature PSK, M-ary PSK. Frequency Shift Keying (FSK)- Binary FSK (orthogonal and non-orthogonal), M-ary FSK. Comparison of BPSK and BFSK, Quadrature Amplitude Shift Keying (QASK), Minimum Shift Keying (MSK)

3.1 Digital Modulation

Digital Modulation provides more information capacity, high data security, quicker system availability with great quality communication. Hence, digital modulation techniques have a greater demand, for their capacity to convey larger amounts of data than analog modulation techniques.
There are many types of digital modulation techniques and also their combinations, as listed below.

ASK – Amplitude Shift Keying

The amplitude of the resultant output depends upon the input data whether it should be a zero level or a variation of positive and negative, depending upon the carrier frequency.

FSK – Frequency Shift Keying

The frequency of the output signal will be either high or low, depending upon the input data applied.

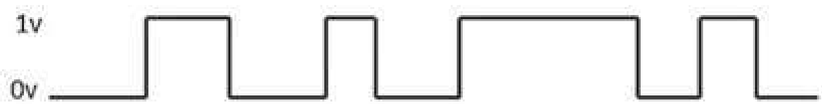
PSK – Phase Shift Keying

The phase of the output signal gets shifted depending upon the input. These are mainly of two types, namely Binary Phase Shift Keying (BPSK) and Quadrature Phase Shift Keying (QPSK), according to the number of phase shifts. The other one is Differential Phase Shift Keying (DPSK) which changes the phase according to the previous value.

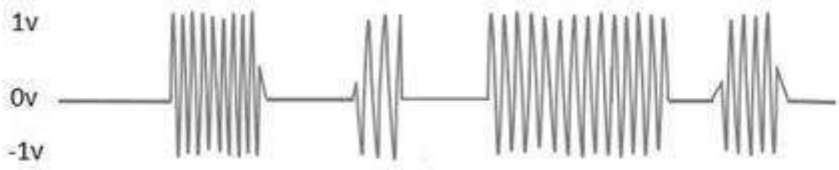
M-ary Encoding

M-ary Encoding techniques are the methods where more than two bits are made to transmit simultaneously on a single signal. This helps in the reduction of bandwidth.
The types of M-ary techniques are M-ary ASK, M-ary FSK & M-ary PSK.

Amplitude Shift Keying (ASK) is a type of Amplitude Modulation which represents the binary data in the form of variations in the amplitude of a signal.
Any modulated signal has a high frequency carrier. The binary signal when ASK modulated, gives a zero value for Low input while it gives the carrier output for High input.
The figure 3.1.1 represents ASK modulated waveform along with its input.



(a) ASK Modulation



(b) ASK Modulated Wave

Figure 3.1.1 ASK Modulation

To find the process of obtaining this ASK modulated wave, let us learn about the working of the ASK modulator.

3.2 ASK Modulator

The ASK modulator block diagram comprises of the carrier signal generator, the binary sequence from the message signal and the band-limited filter. Following is the block diagram of the ASK Modulator.

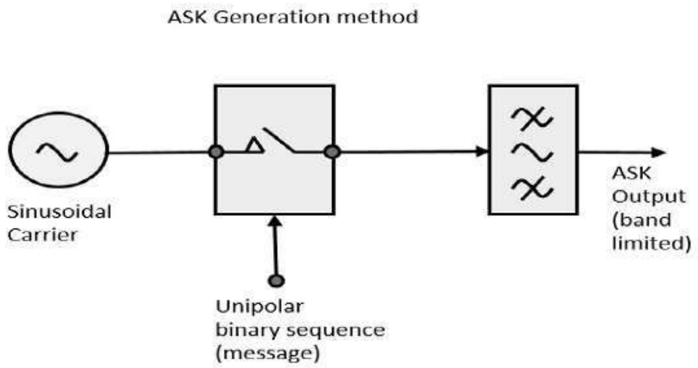


Figure 3.2.1 ASK Modulator

The carrier generator sends a continuous high-frequency carrier. The binary sequence from the message signal makes the unipolar input to be either High or Low. The high signal closes the switch, allowing a carrier wave. Hence, the output will be the carrier signal at high input. When there is low input, the switch opens, allowing no voltage to appear. Hence, the output will be low.
The band-limiting filter, shapes the pulse depending upon the amplitude and phase characteristics of the band-limiting filter or the pulse-shaping filter.

ASK Demodulator

There are two types of ASK Demodulation techniques. They are –

- Asynchronous ASK Demodulation/detection
- Synchronous ASK Demodulation/detection

The clock frequency at the transmitter when matches with the clock frequency at the receiver, it is known as a Synchronous method, as the frequency gets synchronized. Otherwise, it is known as Asynchronous.

Asynchronous ASK Demodulator

The Asynchronous ASK detector consists of a half-wave rectifier, a low pass filter, and a comparator. Following is the block diagram for the same.

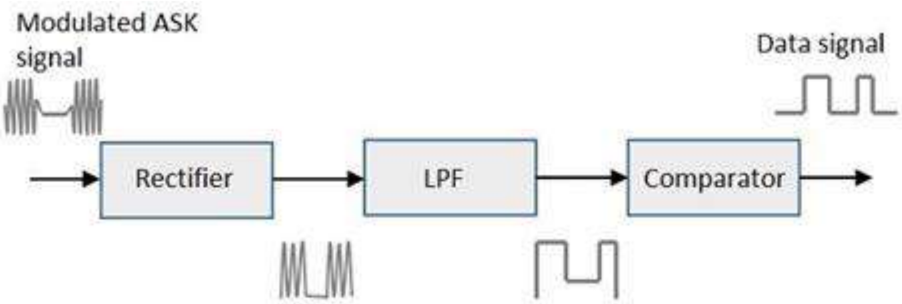


Figure 3.2.2 ASK Demodulator

The modulated ASK signal is given to the half-wave rectifier, which delivers a positive half output. The low pass filter suppresses the higher frequencies and gives an envelope detected output from which the comparator delivers a digital output.

Synchronous ASK Demodulator

Synchronous ASK detector consists of a Square law detector, low pass filter, a comparator, and a voltage limiter. Following is the block diagram for the same.

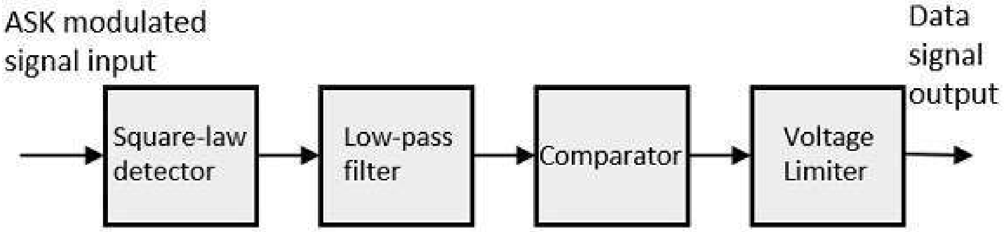


Figure 3.2.3 Synchronous ASK Demodulator

The ASK modulated input signal is given to the Square law detector. A square law detector is one whose output voltage is proportional to the square of the amplitude modulated input voltage. The low pass filter minimizes the higher frequencies. The comparator and the voltage limiter help to get a clean digital output.

3.3 Frequency Shift Keying

Frequency Shift Keying (FSK) is the digital modulation technique in which the frequency of the carrier signal varies according to the digital signal changes. FSK is a scheme of frequency modulation. The output of a FSK modulated wave is high in frequency for a binary High input and is low in frequency for a binary Low input. The binary 1s and 0s are called Mark and Space frequencies. The following image is the diagrammatic representation of FSK modulated waveform along with its input.

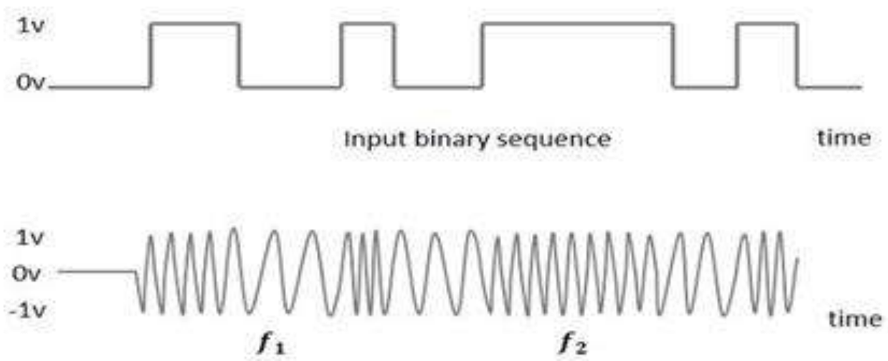


Figure 3.3.1 Frequency Shift Keying (FSK)

To find the process of obtaining this FSK modulated wave, let us know about the working of a FSK modulator.

FSK Modulator

The FSK modulator block diagram comprises of two oscillators with a clock and the input binary sequence. Following is its block diagram.

FSK Transmitter

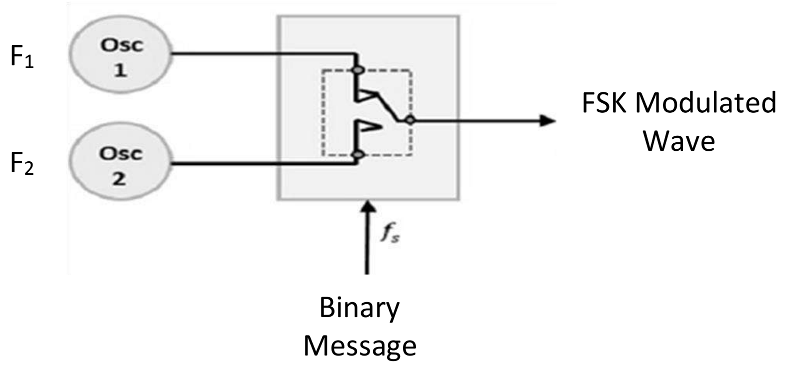


Figure 3.3.2 FSK Modulator

The two oscillators, producing a higher and a lower frequency signals, are connected to a switch along with an internal clock. To avoid the abrupt phase discontinuities of the output waveform during the transmission of the message, a clock is applied to both the oscillators, internally. The binary input sequence is applied to the transmitter so as to choose the frequencies according to the binary input.

FSK Demodulator

There are different methods for demodulating a FSK wave. The main methods of FSK detection are asynchronous detector and synchronous detector. The synchronous detector is a coherent one, while asynchronous detector is a non-coherent one.

Asynchronous FSK Detector

The block diagram of Asynchronous FSK detector consists of two band pass filters, two envelope detectors, and a decision circuit. Following is the diagrammatic representation.

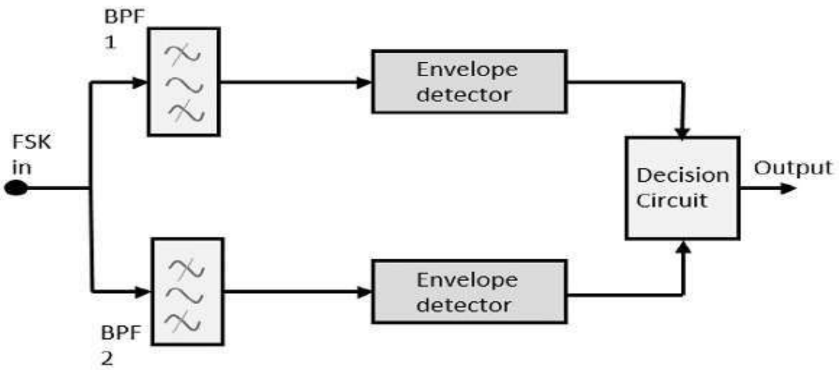


Figure 3.3.3 Asynchronous FSK Detector

The FSK signal is passed through the two Band Pass Filters (BPFs), tuned to Space and Mark frequencies. The output from these two Band Pass Filters look like ASK signal, which is then applied to the envelope detector. The signal in each envelope detector is modulated asynchronously.
The decision circuit chooses which output is more likely and selects it from any one of the envelope detectors. It also re-shapes the waveform to a rectangular one.

Synchronous FSK Detector

The block diagram of Synchronous FSK detector consists of two mixers with local oscillator circuits, two band pass filters and a decision circuit. Following is the diagrammatic representation.

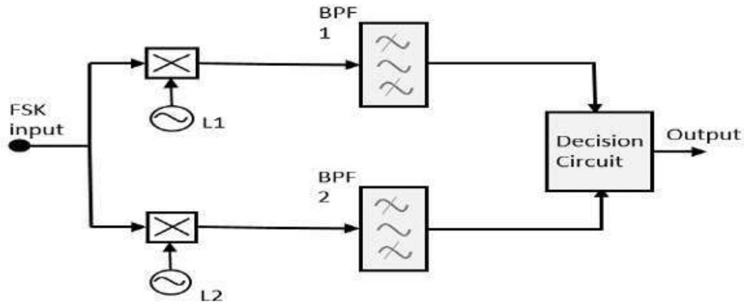


Figure 3.3.4 Synchronous FSK Detector

The FSK signal input is given to the two mixers with local oscillator circuits. These two are connected to two band pass filters. These combinations act as demodulators and the decision circuit chooses which output is more likely and selects it from any one of the detectors. The two signals have a minimum frequency separation.

For both of the demodulators, the bandwidth of each of them depends on their bit rate. This synchronous demodulator is a bit complex than asynchronous type demodulators.

3.4 Phase Shift Keying (PSK)

Phase Shift Keying (PSK) is the digital modulation technique in which the phase of the carrier signal is changed by varying the sine and cosine inputs at a particular time. PSK technique is widely used for wireless LANs, bio-metric, contactless operations, along with RFID and Bluetooth communications.

3.4.1 Binary Phase Shift Keying (BPSK)

In binary phase-shift keying (BPSK) the transmitted signal is a sinusoid of fixed amplitude. It has one fixed phase when the data is at one level and when the data is at the other level the phase is different by 180°. If the sinusoid is of amplitude A it has a power

$$P_s = \frac{1}{2}A^2 \; therefore \; A = \sqrt{2P_s}$$

Thus the transmitted signal is either

$$V_{BPSK}(t) = \sqrt{2P_s}.\cos(\omega_o t) \qquad ...3.4.1.1$$

or

$$V_{BPSK}(t) = -\sqrt{2P_s}.\cos(\omega_o t + \pi) \qquad ...3.4.1.2(a)$$

$$V_{BPSK}(t) = -\sqrt{2P_s}.\cos(\omega_o t) \qquad ...3.4.1.2(b)$$

In BPSK the data b(t) is a stream of binary digits with voltage levels which, as a matter of convenience, we take to be at + 1V and - 1V. When b(t) = 1V we say it is at logic level 1 and when b(t) = -l V we say it is at logic level 0.

Hence $V_{BPSK}(t)$ can be written, as

$$V_{BPSK}(t) = b(t)\sqrt{2P_s}.\cos(\omega_o t) \qquad ...3.4.1.3$$

In practice, a BPSK signal is generated by applying the waveform $\cos(\omega_o t)$, as a carrier, to a balanced modulator and applying the baseband signal b(t) as the modulating waveform. In this sense BPSK can be thought of as an AM signal.

BPSK Modulator:

The block diagram of Binary Phase Shift Keying consists of the balance modulator which has the carrier sine wave as one input and the binary sequence as the other input.

Following is the diagrammatic representation.

BPSK Modulator

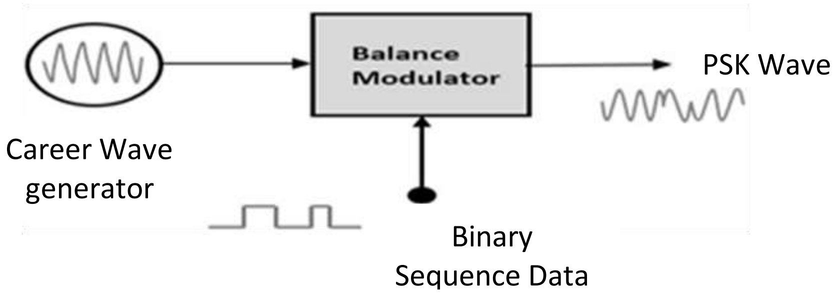


Figure 3.4.1 BPSK Modulator

The modulation of BPSK is done using a balance modulator, which multiplies the two signals applied at the input. For a zero binary input, the phase will be 0° and for a high input, the phase reversal is of 180°.

Following is the diagrammatic representation of BPSK Modulated output wave along with its given input.

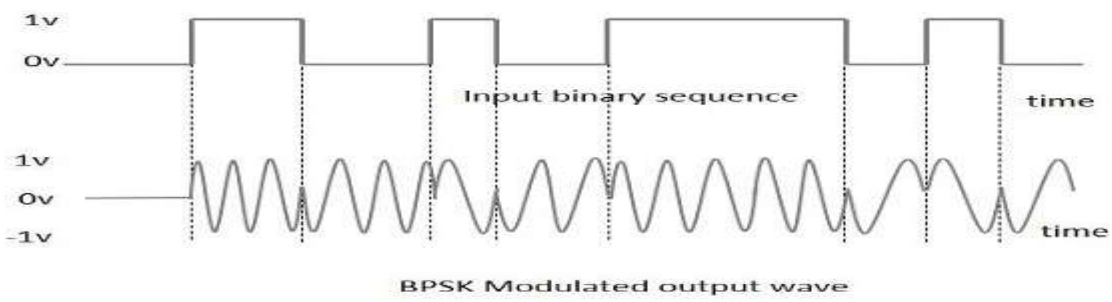


Figure 3.4.2 BPSK Modulated Waveform

The output sine wave of the modulator will be the direct input carrier or the inverted (180° phase shifted) input carrier, which is a function of the data signal.

Reception of BPSK:

The received signal has the form

$$V_{BPSK}(t) = b(t)\sqrt{2P_S}.\cos(\omega_o t + \theta) = b(t)\sqrt{2P_S}.\cos\omega_o(t + \theta/\omega_o) \quad \text{...3.4.1.4}$$

Here θ is a nominally fixed phase shift corresponding to the time delay θ/ω_o which depends on the length of the path from transmitter to receiver and the phase shift produced by the amplifiers in the" front-end" of the receiver preceding the demodulator. The original data b(t) is recovered in the demodulator. The demodulation technique usually employed is called synchronous demodulation and requires that there be available at the demodulator the waveform $\cos(\omega_o t + \theta)$. A scheme for generating the carrier at the demodulator and for recovering the baseband signal is shown in Fig. 3.4.1.1.

The received signal is squared to generate the signal

$$\cos^2(\omega_o t + \theta) = \frac{1}{2} + \frac{1}{2}\cos 2(\omega_o t + \theta) \quad \text{...3.4.1.5}$$

The DC component is removed by the band pass filter whose pass band is centered around 2fo and we then have the signal whose waveform is that of $\cos 2(\omega_o t + \theta)$. A frequency divider (composed of a flip-flop and narrow-band filter tuned to fo is used to regenerate the waveform $\cos(\omega_o t + \theta)$. Only the waveforms of the signals at the outputs of the squarer, filter and divider are relevant, not their amplitudes.

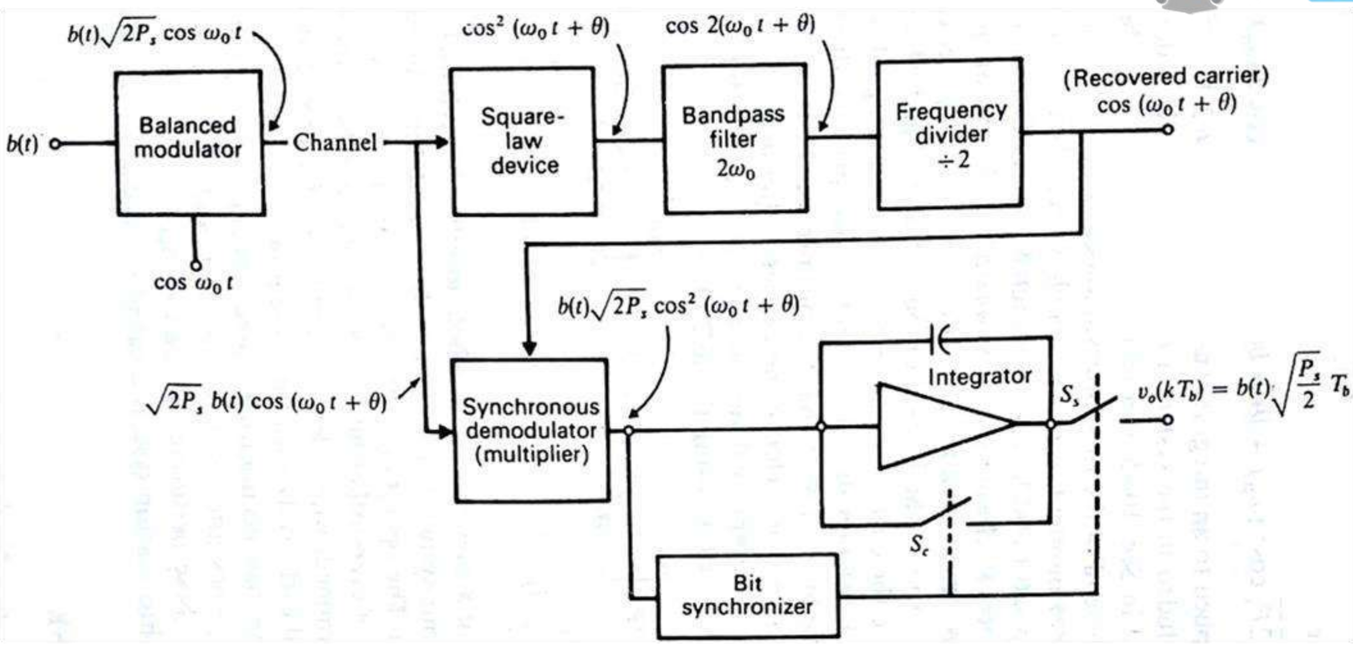


Figure 3.4.1.1 Reception of BPSK

Accordingly in Fig. 3.4.1.1 we have arbitrarily taken amplitudes to be unity. In practice, the amplitudes will be determined by features of these devices which are of no present concern. In any event, the carrier having been recovered, it is multiplied with the received signal to generate

$$b(t)\sqrt{2P_s}\cos^2(\omega_o t+\theta)=b(t)\sqrt{2P_s}\left[\frac{1}{2}+\frac{1}{2}\cos 2(\omega_o t+\theta)\right] \quad ...3.4.1.6$$

which is then applied to an integrator as shown.

We have included in the system a bit synchronizer. This device, whose operation is able to recognize precisely the moment which corresponds to the end of the time interval allocated to one bit and the beginning of the next. At that moment, it closes switch S, very briefly to discharge (dump) the integrator capacitor and leaves the switch S, open during the entire course of the ensuing bit interval, closing switch S, again very briefly at the end of the next bit time, etc. (This circuit is called an "integrate-and-dump" circuit.) The output signal of interest to us is the integrator output at the end of a bit interval but immediately before the closing of switch So. This output signal is made available by switch S; which samples the output voltage just prior to dumping the capacitor.

Let us assume for simplicity that the bit interval Tb is equal to the duration of an integral number n of cycles of the carrier of frequency f_o that is, $n\,.\,2\pi\ =\ \omega_0 T_b$ In this case the output voltage $v_0(kT_b)$ at the end of a bit interval extending from time $(k\ -\ 1)T_b$ to $(k)T_b$ is, using Eq. (3.4.1.6).

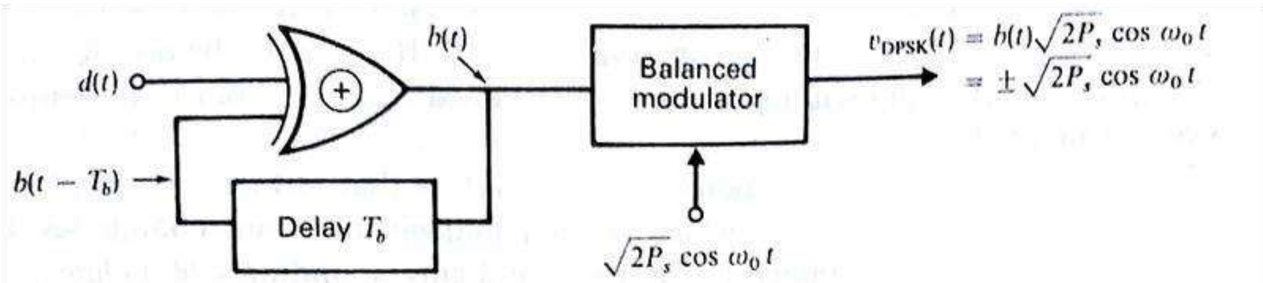
$$v_0(kT_b)=b(kT_b)\sqrt{2P_s}\int_{(k-1)T_b}^{kT_b}\frac{1}{2}\mathrm{dt}+b(kT_b)\sqrt{2P_s}\int_{(k-1)T_b}^{kT_b}\frac{1}{2}\cos 2(\omega_o t+\theta)\,\mathrm{dt} \quad ...3.4.1.7(a)$$

$$v_0(kT_b)=b(kT_b)\sqrt{\frac{P_s}{2}}T_b \quad ...3.4.1.7(b)$$

Since the integral of a sinusoid over a whole number of cycles has the value zero. Thus we see that our system reproduces at the demodulator output the transmitted bit stream b(t). The operation of the bit synchronizer allows us to sense each bit independently of every other bit. The brief closing of both switches, after each bit has been determined, wipes clean all influence of a preceding bit and allows the receiver to deal exclusively with the present bit.

3.4.2 Differential Phase Shift Keying (DPSK)

In BPSK, to regenerate the carrier we start by squaring $b(t)\sqrt{2P_s}.\cos(\omega_o t)$. Accordingly, if the received signal were instead $-b(t)\sqrt{2P_s}.\cos(\omega_o t)$ the recovered carrier would remain as before. Therefore we shall not be able to determine whether the received baseband signal is the transmitted signal $b(t)$ or it's negative $-b(t)$.
Differential phase-shift keying (DPSK) and differential encoded PSK (DEPSK) are modifications of BPSK which have the merit that they eliminate the ambiguity about whether the demodulated data is or is not inverted. In addition DPSK avoids the need to provide the synchronous carrier required at the demodulator for detecting a BPSK signal.
A means for generating a DPSK signal is shown in Fig. 3.4.2.1. The data stream to be transmitted, $d(t)$, is applied to one input of an exclusive-OR logic gate. To the other gate input is applied the output of the exclusive or gate $b(t)$ delayed by the time T_b, allocated to one bit. This second input is then $b(t-T_b)$. In Fig. 3.4.2.2 we have drawn logic waveforms to illustrate the response b(t) to an input d(t). The upper level of the waveforms corresponds to logic 1, the lower level to logic O. The truth table for the exclusive-OR gate is given in Fig 3.4.2.1, and with this table we can easily verify that the waveforms for (t), $b(t-T_b)$, and $b(t)$ are consistent with one another. We observe that, as required, $b(t-T_b)$ is indeed b(t) delayed by one bit time and that in any bit interval the bit b(t) is given $b(t) = d(t) \oplus b(t-T_b)$.



$d(t)$ logic level	$d(t)$ voltage	$b(t-T_b)$ logic level	$b(t-T_b)$ voltage	$b(t)$ logic level	$b(t)$ voltage
0	−1	0	−1	0	−1
0	−1	1	1	1	1
1	1	0	−1	1	1
1	1	1	1	0	−1

Figure 3.4.2.1 Means for generating DPSK

Because of the feedback involved in the system of Fig. 3.4.2.2 there is a difficulty in determining the logic levels in the interval in which we start to draw the waveforms (interval 1 in Fig. 3.4.2.2). We cannot determine b(t) in this first interval of our waveform unless we know b(k = 0). But we cannot determine b(0) unless we know both d(0) and b(-1), etc. Thus, to justify any set of logic levels in an initial bit interval we need to know the logic levels in the preceding interval. But such a determination requires information about the interval two bit times earlier and so on. In the waveforms of Fig. 3.4.2.2 we have circumvented the problem by arbitrarily assuming that in the first interval b(0) = 0. It is shown below that in the demodulator, the data will be correctly determined regardless of our assumption concerning b(0).

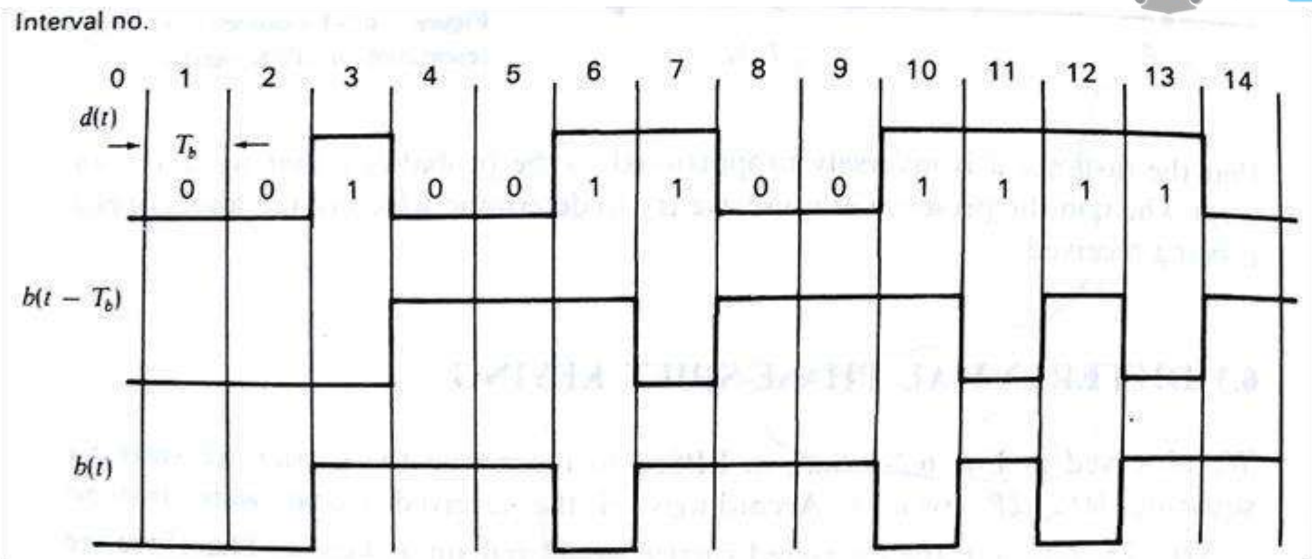


Figure 3.4.2.2 Logic waveforms to illustrate the response *b(t)* to an input *d(t)*.

We now observe that the response of b(t) to d(t) is that b(t) changes level at the beginning of each interval in which d(t) = 1 and b(t) does not change level when d(t) = 0. Thus during interval 3, d(3) = 1, and correspondingly b(3) changes at the beginning at that interval. During intervals 6 and 7, d(6) = d(7) = 1 and there are changes in b(t) at the beginnings of both intervals. During bits 10, 11, 12, and 13 d(t) = 1 and there are changes in b(t) at the beginnings of each of these intervals. This behavior is to be anticipated from the truth table of the exclusive-OR gate. For we note that when $d(t) = 0$, $b(t) = b(t - T_b)$ so that, whatever the initial value of $b(t - T_b)$,it reproduces itself. On the other hand when d(t) = 1, then $b(t) = b(t - T_b)$. Thus, in each successive bit interval b(t) changes from its value in the previous interval. Note that in some intervals where d(t) = 0 we have b(t) = 0 and in other intervals when d(t) = 0 we have b(t) = 1. Similarly, when d(t) = 1 sometimes b(t) = 1 and sometimes b(t) = 0. Thus there is no correspondence between the levels of d(t) and b(t), and the only invariant feature of the system is that a change (sometimes up and sometimes down) in b(t) occurs whenever d(t) = 1, and that no change in b(t) will occur whenever d(t) = 0.

Finally, we note that the waveforms of Fig. 3.4.2.2 are drawn on the assumption that, in interval 1, b(0) = 0. As is easily verified, if not intuitively apparent, if we had assumed b(0) = 1, the invariant feature by which we have characterized the system would continue to apply. Since b(0) must be either b(0) = 0 or b(0) = 1, there being no other possibilities, our result is valid quite generally. If, however, we had started with b(0) = 1 the levels b(l) and b(0) would have been inverted.

As is seen in Fig. 3.4.2.1 b(t) is applied to a balanced modulator to which is also applied the carrier $\sqrt{2P_s}.\cos(\omega_o t)$. The modulator output, which is the transmitted signal, is

$$\begin{aligned} V_{DPSK}(t) &= b(t)\sqrt{2P_s}.\cos(\omega_o t) \\ &= \pm\sqrt{2P_s}.\cos(\omega_o t) \end{aligned} \quad ...3.4.2.1$$

Thus altogether when d(t) = 0 the phase of the carrier does not change at the beginning of the bit interval, while when d(t) = 1 there is a phase change of magnitude π.

Reception:

A method of recovering the data bit stream from the DPSK signal is shown in Fig. 3.4.2.3. Here the received signal and the received signal delayed by the bit time 1'" are applied to a multiplier. The multiplier output is

$b(t).b(t-T_b)2P_s\cos(\omega_o t+\theta).\cos[\omega_o(t-T_b)+\theta]$
$$= b(t).b(t-T_b)P_s\left\{\cos\omega_o T_b + \cos\left[2\omega_o\left(t-\frac{T_b}{2}\right)+2\theta\right]\right\} \qquad ...3.4.2.2$$

and is applied to a bit synchronizer and integrator as shown in Fig. 6.2-1 for the BPSK demodulator. The first term on the right-hand side of Eq.(3.4.2.1) is, aside from a multiplicative constant, the waveform $b(t).b(t-T_b)$ which, as we shall see is precisely the signal we require. As noted previously in connection with BPSK, and so here, the output integrator will suppress the double frequency term. We should select $\omega_o T_b$ so that $\omega_o T_b = 2n\pi$ with n an integer. For, in this case we shall have $\cos\omega_o T_b = +1$ and the signal output will be as large as possible.

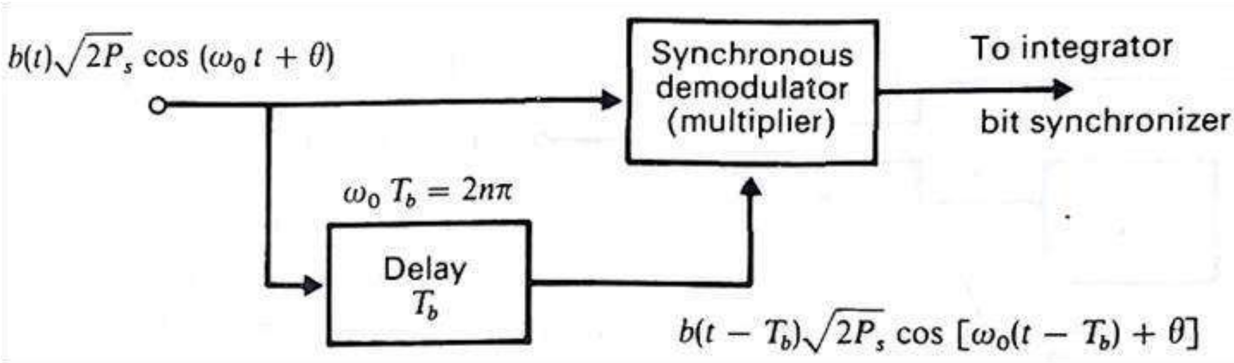


Figure 3.4.2.3 Methods of recovering data from DPSK

Further, with this selection, the bit duration encompasses an integral number of clock cycles and the integral of the double-frequency term is exactly zero.
The transmitted data bit d(t) can readily be determined from the product $b(t).b(t-T_b)$. If d(t) = 0 then there was no phase change and $b(t) = b(t-T_b)$ both being + 1V or both being - 1V. In this case $b(t).b(t-T_b)$= 1. If however, d(t) = 1 then there was a phase change and either b(t) = 1V with $b(t-T_b)$ = - I V or vice versa. In either case $b(t).b(t-T_b) = -1$.

OR
Differential Phase Shift Keying (DPSK) the phase of the modulated signal is shifted relative to the previous signal element. No reference signal is considered here. The signal phase follows the high or low state of the previous element. This DPSK technique doesn't need a reference oscillator.
The following figure represents the model waveform of DPSK.

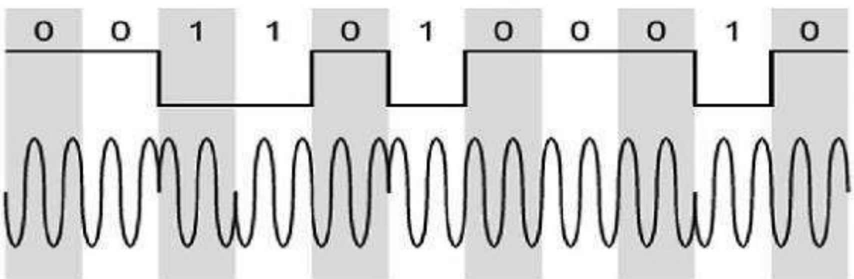


Figure 3.4.9 Differential Phase Shift Keying (DPSK)

It is seen from the above figure that, if the data bit is Low i.e., 0, then the phase of the signal is not reversed, but continued as it was. If the data is a High i.e., 1, then the phase of the signal is reversed, as with NRZI, invert on 1 (a form of differential encoding).
If we observe the above waveform, we can say that the High state represents an M in the modulating signal and the Low state represents a W in the modulating signal.

DPSK Modulator

DPSK is a technique of BPSK, in which there is no reference phase signal. Here, the transmitted signal itself can be used as a reference signal. Following is the diagram of DPSK Modulator.

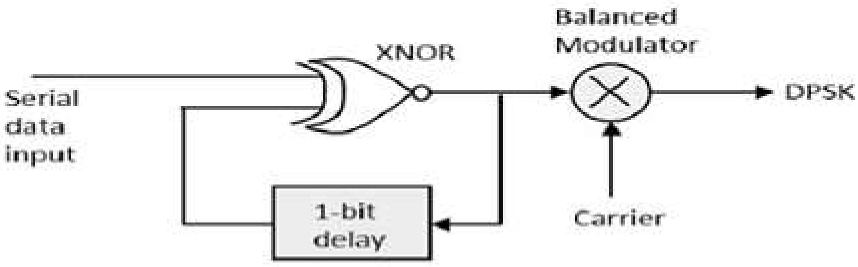


Figure 3.4.10 DPSK Modulator

DPSK encodes two distinct signals, i.e., the carrier and the modulating signal with 180° phase shift each. The serial data input is given to the XNOR gate and the output is again fed back to the other input through 1-bit delay. The output of the XNOR gate along with the carrier signal is given to the balance modulator, to produce the DPSK modulated signal.

DPSK Demodulator

In DPSK demodulator, the phase of the reversed bit is compared with the phase of the previous bit. Following is the block diagram of DPSK demodulator.

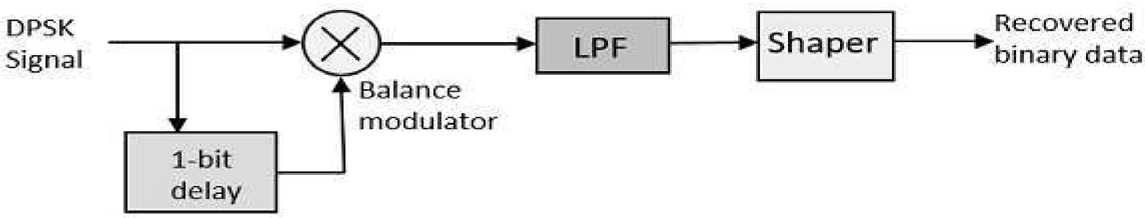


Figure 3.4.11 DPSK Demodulator

From the above figure, it is evident that the balance modulator is given the DPSK signal along with 1-bit delay input. That signal is made to confine to lower frequencies with the help of LPF. Then it is passed to a shaper circuit, which is a comparator or a Schmitt trigger circuit, to recover the original binary data as the output.

The word binary represents two bits. M represents a digit that corresponds to the number of conditions, levels, or combinations possible for a given number of binary variables.

This is the type of digital modulation technique used for data transmission in which instead of one bit, two or more bits are transmitted at a time. As a single signal is used for multiple bit transmission, the channel bandwidth is reduced.

3.4.3 Differentially Encoded Phase Shift Keying (DEPSK)

The DPSK demodulator requires a device which operates at the carrier frequency and provides a delay of T_b. Differentially-encoded PSK eliminates the need for such a piece of hardware. In this system, synchronous demodulation recovers the signal b(t), and the decoding of b(t) to generate d(t) is done at baseband.

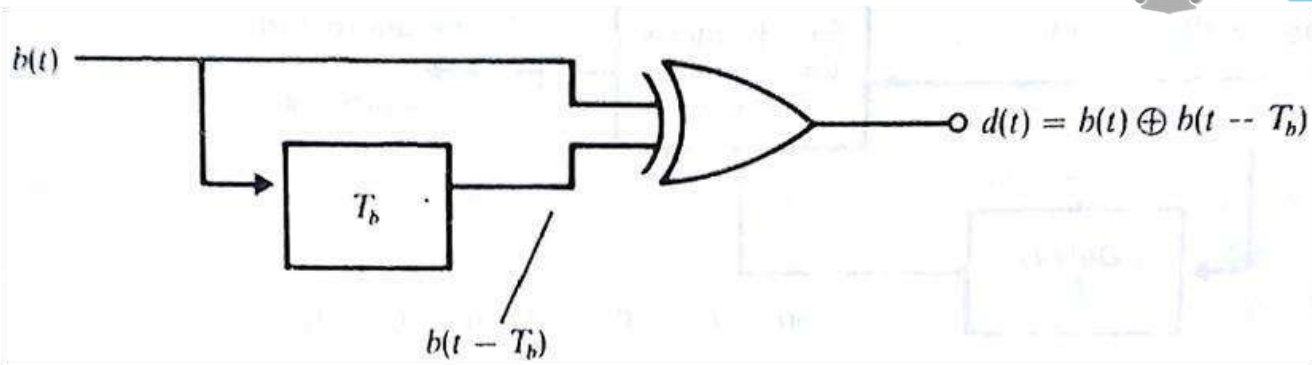


Figure 3.4.3.1 Baseband Decoder to obtain d(t) from b(t)

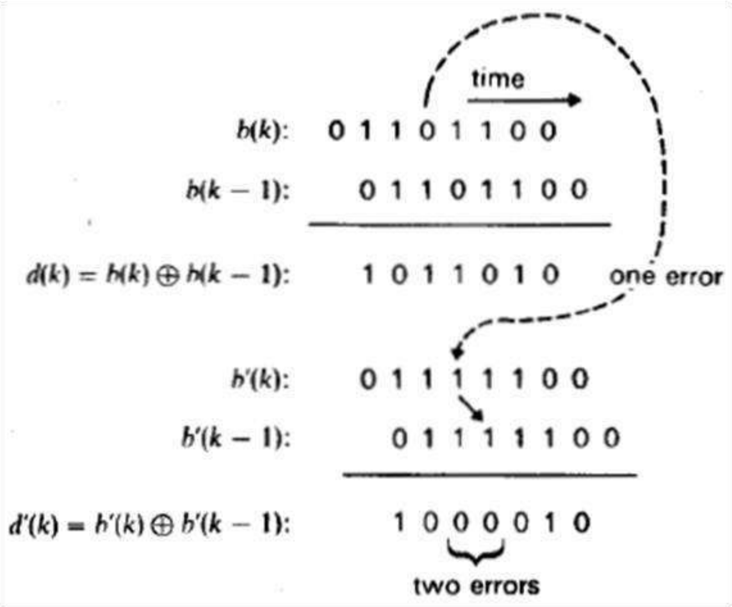


Figure 3.4.3.2 Errors in DEPSK occurs in pairs

The transmitter of the DEPSK system is identical to the transmitter of the DPSK system shown in Fig. 3.4.2.1, The signal b(t) is recovered in exactly the manner shown in Fig. 3.4.2.1 for a BPSK system. The recovered signal is then applied directly to one input of an exclusive-OR logic gate and to the other input is applied $b(t - T_b)$ (see Fig. 3.4.3.1). The gate output will be at one or the other of its levels depending on whether $b(t) = b(t - T_b)$ or $b(t) = \overline{b(t - T_b)}$. In the first case b(t) did not change level and therefore the transmitted bit is d(t) = 0. In the second case d(t) = 1.

We have seen that in DPSK there is a tendency for bit errors to occur in pairs but that single bit errors are possible. In DEPSK errors always occur in pairs. The reason for the difference is that in DPSK we do not make a hard decision, in each bit interval about the phase of the received signal. We simply allow the received signal in one interval to compare itself with the signal in an adjoining interval and, as we have seen, a single error is not precluded. In DEPSK, a firm definite hard decision is made in each interval about the value of b(t). If we make a mistake, then errors must result from a comparison with the preceding and succeeding bit. This result is illustrated in Fig.3.4.3.2. It is shown the error-free signals $b(k), b(k - 1)$ and $d(k) = b(k) \oplus b(k - 1)$. We have assumed that b'(k) has a single error. Then b'(k - 1) must also have a single error. We note that the reconstructed waveform d'(k) now has two errors.

3.4.4 Quadrature Phase Shift Keying (QPSK)

This is the phase shift keying technique, in which the sine wave carrier takes four phase reversals such as 0°, 90°, 180°, and 270°.

If these kinds of techniques are further extended, PSK can be done by eight or sixteen values also, depending upon the requirement.

QPSK Modulator

The mechanism by which a bit stream b(t) generates a QPSK signal for transmission is shown in Fig. 3.4.4.1 and relevant waveforms are shown in Fig. 3.4.4.2. In these waveforms we have arbitrarily assumed that in every case the active edge of the clock waveforms is the downward edge. The toggle flip-flop is driven by a clock waveform whose period is the bit time Tb. The toggle flip-flop generates an odd clock waveform and an even waveform. These clocks have periods 2Tb. The active edge of one of the clocks and the active edge of the other are separated by the bit time Tb. The bit stream $b(t)$ is applied as the data input to both type-D flip-flops, one driven by the odd and one driven by the even clock waveform. The flip-flops register alternate bits in the stream b(t) and hold each such registered bit for two bit intervals, that is for a time Tb. In Fig. 3.4.4.2 we have numbered the bits in $b(t)$. Note that the bit stream $b(t)$ (which is the output of the flip-flop driven by the odd clock) registers bit 1 and holds that bit .for time 2Tb, then registers bit 3 for time 2Tb, then bit 5 for 2Tb, etc. The even bit stream $b_e(t)$ holds, for times 2Tb each the alternate bits numbered 2.4, 6, etc.
The bit stream $b(t)$ (which. as usual, we take to be $b_e(t)$= ± 1 volt) is superimposed on a carrier $\sqrt{P_s}.\sin(\omega_o t)$ by the use of two multipliers (i.e., balanced modulators) as shown, to generate two signals $S_e(t)$ and $S_o(t)$. These signals are then added to generate the transmitted output signal Vm(t) which is

$$v_m(t) = \sqrt{P_s}.b_o(t)\sin(\omega_o t) + \sqrt{P_s}.b_e(t)\cos(\omega_o t)$$

As may be verified, the total normalized power of $v_m(t)$ is P_s.

The QPSK Modulator uses a bit-splitter, two multipliers with local oscillator, a 2-bit serial to parallel converter, and a summer circuit. Following is the block diagram for the same.

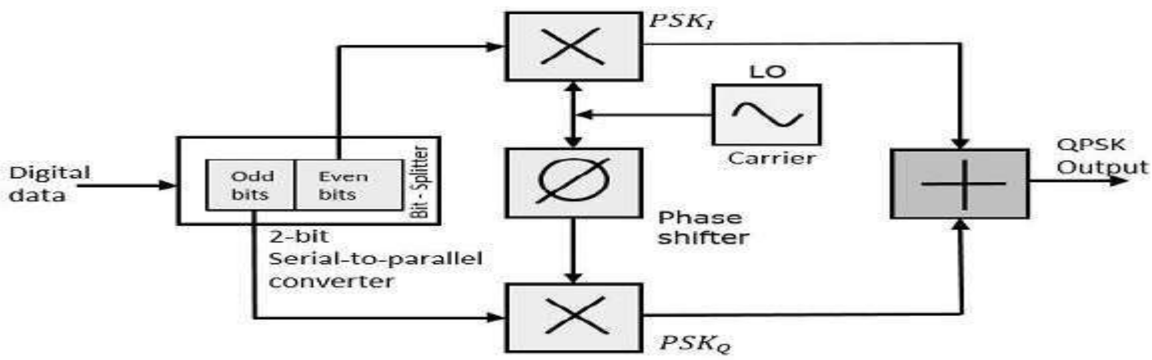


Figure 3.4.6 QPSK Modulator

At the modulator's input, the message signal's even bits (i.e., 2nd bit, 4th bit, 6th bit, etc.) and odd bits (i.e., 1st bit, 3rd bit, 5th bit, etc.) are separated by the bits splitter and are multiplied with the same carrier to generate odd BPSK (called as PSK_I) and even BPSK (called as PSK_Q). The PSK_Q signal is anyhow phase shifted by 90° before being modulated.

The QPSK waveform for two-bits input is as follows, which shows the modulated result for different instances of binary inputs.

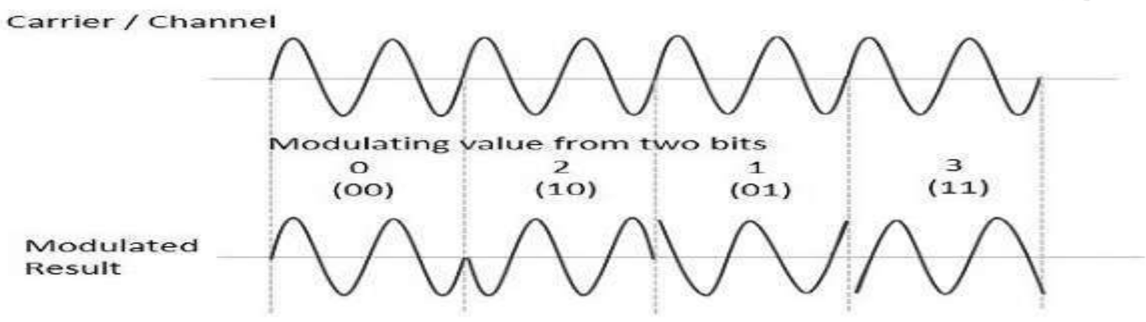


Figure 3.4.7 QPSK Waveforms

QPSK Demodulator

The QPSK Demodulator uses two product demodulator circuits with local oscillator, two band pass filters, two integrator circuits, and a 2-bit parallel to serial converter. Following is the diagram for the same.

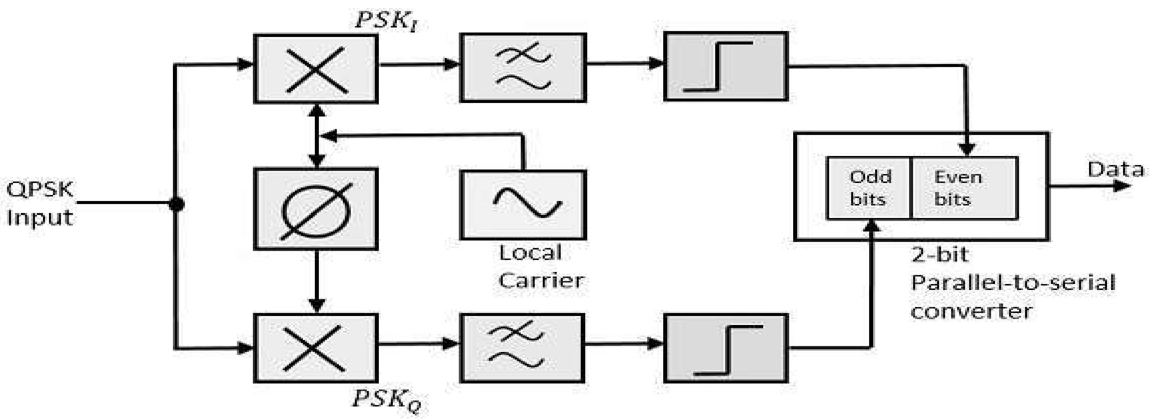


Figure 3.4.8 QPSK Demodulator

The two product detectors at the input of demodulator simultaneously demodulate the two BPSK signals. The pair of bits are recovered here from the original data. These signals after processing, are passed to the parallel to serial converter.

3.5 M-ary Equation

If a digital signal is given under four conditions, such as voltage levels, frequencies, phases, and amplitude, then M = 4. The number of bits necessary to produce a given number of conditions is expressed mathematically as $N=\log 2M$. Where N is the number of bits necessary M is the number of conditions, levels, or combinations possible with N bits.

The above equation can be re-arranged as

$$2N=M$$

For example, with two bits, $2^2 = 4$ conditions are possible.

Types of M-ary Techniques

In general, Multi-level (M-ary) modulation techniques are used in digital communications as the digital inputs with more than two modulation levels are allowed on the transmitter's input. Hence, these techniques are bandwidth efficient.

There are many M-ary modulation techniques. Some of these techniques, modulate one parameter of the carrier signal, such as amplitude, phase, and frequency.

M-ary ASK

This is called M-ary Amplitude Shift Keying (M-ASK) or M-ary Pulse Amplitude Modulation (PAM).

The amplitude of the carrier signal, takes on M different levels.

Representation of M-ary ASK

$S_m(t) = A_m \cos(2\pi f_c t) \qquad A_m \in (2m-1-M)\Delta, m = 1,2, \dots \dots M$ and $0 \le t \le T_s$

Some prominent features of M-ary ASK are −

- This method is also used in PAM.
- Its implementation is simple.
- M-ary ASK is susceptible to noise and distortion.

M-ary FSK

This is called as M-ary Frequency Shift Keying (M-ary FSK).

The frequency of the carrier signal, takes on M different levels.

Representation of M-ary FSK

$$S_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left(\frac{\pi}{T_s}(n_c + i)t\right) \qquad 0 \le t \le T_s \quad i = 1,2, \dots \dots M$$

Where $fc = nc/\,2Ts$

for some fixed integer n.

Some prominent features of M-ary FSK are −

- Not susceptible to noise as much as ASK.
- The transmitted M number of signals are equal in energy and duration.
- The signals are separated by 12Ts
- Hz making the signals orthogonal to each other.
- Since M signals are orthogonal, there is no crowding in the signal space.
- The bandwidth efficiency of M-ary FSK decreases and the power efficiency increases with the increase in M.

M-ary PSK

This is called as M-ary Phase Shift Keying (M-ary PSK).

The phase of the carrier signal, takes on M different levels.

Representation of M-ary PSK

$$S_i(t) = \sqrt{\frac{2E}{T}} \cos(\omega_{0t} + \phi_i t) \qquad 0 \le t \le T \; and \quad i = 1,2, \dots \dots M$$

$\phi_i(t) = \frac{2\pi i}{M}$ where $i = 1,2, \dots \dots M$

Some prominent features of M-ary PSK are −

- The envelope is constant with more phase possibilities.
- This method was used during the early days of space communication.
- Better performance than ASK and FSK.
- Minimal phase estimation error at the receiver.
- The bandwidth efficiency of M-ary PSK decreases and the power efficiency increases with the increase in M.

So far, we have discussed different modulation techniques. The output of all these techniques is a binary sequence, represented as 1s and 0s. This binary or digital information has many types and forms, which are discussed further.

3.6 Comparison of BPSK and BFSK

The two modulation techniques can be compared in the following manners:

1. The bandwidth required for transmitting the BFSK signal is 4f (f is the frequency of the data signal), whereas the bandwidth requirement for the BPSK signal is only 2f. Therefore BPSK is a better option.

2. In BPSK the information of the message is stored in the phase variations of the career wave whereas in case of the BFSK scheme, the information is available as the frequency variations of the career wave. Now we know that the noise can affect the frequency of the career wave but cannot affect the phase of the career signal. Therefore the BPSK Scheme is again a better option.

3. The noise will also occupy some frequency it may damage the signal flatly or frequency selectively. But in PSK there is very less chance in the changing of phase f the signal. Hence for noisy channel PSK is better than FSK.

3.7 Quadrature Phase Shift Keying (QPSK)

We have seen that when a data stream whose bit duration is T_b is to be transmitted by BPSK the channel bandwidth must be nominally $2f_b$ where $f_b = I/T_b$.
Quadrature phase-shift keying allows bits to be transmitted using half the bandwidth. In QPSK system we use the type-D flip-flop as a one bit storage device.

D Flip Flop

The type-D flip-flop represented in Fig3.7.4.1 has a single data input terminal (D) to which a data stream d(t) is applied. The operation of the flip-flop is such that at the" active" edge of the clock waveform the logic level at D is transferred to the output Q. Representative waveforms are shown in Fig. 3.7.4.2 We assume arbitrarily that the negative-going edge of the clock waveform is the active edge.

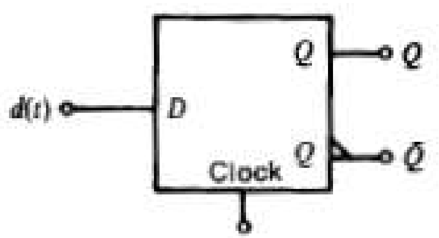


Figure 3.7.4.1 D Flip Flop

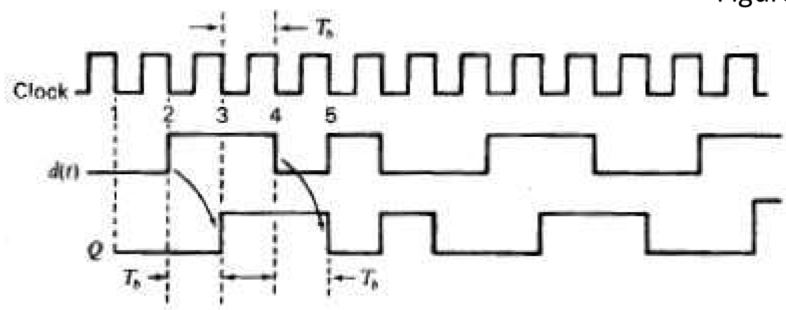


Figure 3.7.4.2 Waveforms showing D Flip Flop Characteristics

QPSK Transmitter (Modulator):
The mechanism by which a bit stream b(t) generates a QPSK signal for transmission is shown in Fig. 3.7.4.3 and relevant waveforms are shown in Fig. 3.7.4.4. In these waveforms we have arbitrarily assumed that in every case the active edge of the clock waveforms is the downward edge. The toggle flip-flop is driven by a clock waveform whose period is the bit time Tb. The toggle flip-flop generates an odd clock waveform and an even waveform. These clocks have periods 2Tb. The active edge of one of the clocks and the active edge of the other are separated by the bit time Tb. The bit stream $b(t)$ is applied as the data input to both type-D flip-flops, one driven by the odd and one driven by the even clock waveform. The flip-flops register alternate bits in the stream b(t) and hold each such registered bit for two bit intervals, that is for a time Tb. In Fig. 3.4.4.4 we have numbered the bits in $b(t)$. Note that the bit stream $b(t)$ (which is the output of the flip-flop driven by the odd clock) registers bit 1 and holds that bit .for time 2Tb, then registers bit 3 for time 2Tb, then bit 5 for 2Tb, etc. The even bit stream $b_e(t)$ holds, for times 2Tb each the alternate bits numbered 2.4, 6, etc.

The bit stream $b(t)$ (which. as usual, we take to be $b_e(t)$= ± 1 volt) is superimposed on a carrier $\sqrt{P_s}.\sin(\omega_o t)$ by the use of two multipliers (i.e., balanced modulators) as shown, to generate two signals $S_e(t)$ and $S_o(t)$. These signals are then added to generate the transmitted output signal Vm(t) which is

$$v_m(t) = \sqrt{P_s}.b_o(t)\sin(\omega_o t) + \sqrt{P_s}.b_e(t)\cos(\omega_o t)$$

As may be verified, the total normalized power of $v_m(t)$ is P_s.

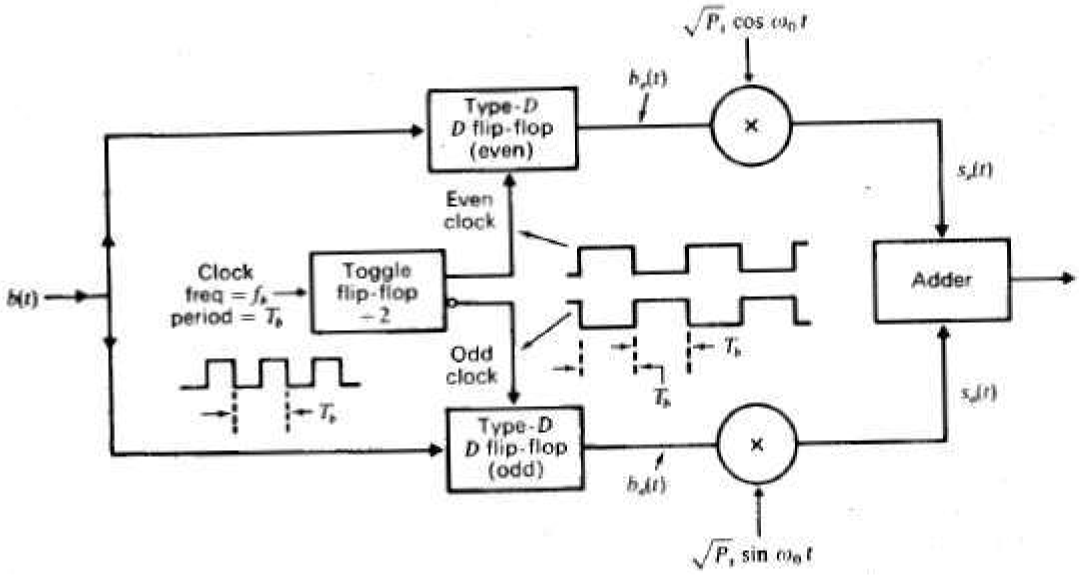


Figure 3.7.4.3 QPSK Transmitter

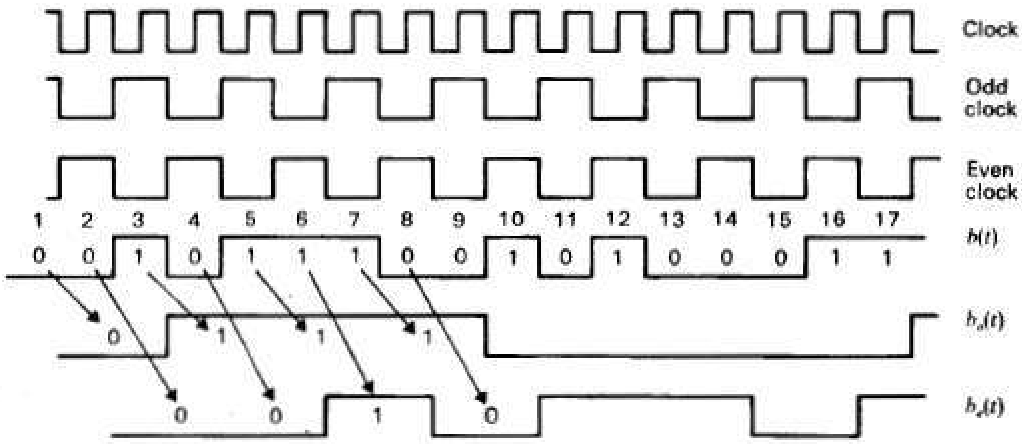


Figure 3.7.4.3 Waveforms for the QPSK Transmitter

In BPSK, the bit duration is T_b, and the generated signal has a nominal bandwidth of $2x1/T_b$. In the waveforms of $b_o(t)$ and $b_e(t)$, the bit times are each $1/2T_b$, hence both $b_o(t)$ and $b_e(t)$ have nominal bandwidth which are half of the bandwidth in BPSK.

Phasor Diagram:

When b_0 = 1 the signal $s_o(t)$ = $\sqrt{P_s}.\sin(\omega_o t)$, and $s_o(t)$ = $-\sqrt{P_s}.\sin(\omega_o t)$ when b_o = -1. Correspondingly, for $b_e(t)$ = ± 1, $s_e(t)$ = ±.$\sqrt{P_s}.(t)\cos(\omega_o t)$ These four signals have been represented as phasors in Fig. 3.7.4.4. They are in mutual phase quadrature. Also drawn are the phasors representing the four possible output signals $v_m(t) = so(t) + Se(t)$. These four possible output signals have equal amplitude $\sqrt{2P_s}$ and are in phase quadrature; they have been identified by their corresponding values of b_o and b_e. At the end of each bit interval (i.e., after each time T_b) either b_o, or b_e can change, but both cannot change at the same time. Consequently, the QPSK system shown in Fig. 3.7.4.3 is called offset or staggered QPSK and abbreviated OQPSK. After each time T_b, the transmitted signal, if it changes, changes phase by 90° rather than by 180° as in BPSK.

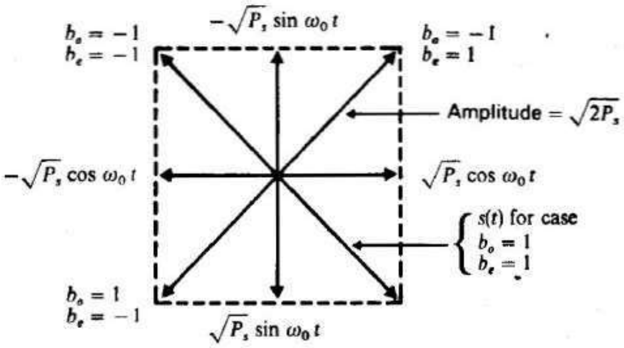


Figure 3.7.4.4 Phasor diagrams for the sinusoids of Fig 3.7.4.2

Non-offset QPSK

Suppose that in Fig. 3.7.4.3 we introduce an additional flip-flop before either the odd or even flip-flop. Let this added flip-flop be driven by the clock which runs at the rate f_b. Then one or the other bit streams, odd or even, will be delayed by one bit interval. As a result, we shall find that two bits which occur in time sequence (i.e., serially) in the input bit stream b(t) will appear at the same time (i.e., in parallel) at the outputs of the odd and even flip-flops. In this case $b_e(t)$ and $b_o(t)$ can change at the same time, after each time $2T_b$, and there can be a phase change of 180° in the output signal. There is no difference, in principle, between a staggered and non-staggered system.

In practice, there is often a significant difference between QPSK and OQPSK. At each transition time, T" for OQPSK and $2T_b$ for QPSK, one bit for OQPSK and perhaps two bits for QPSK change from 1V to -1V or -1V to 1V. Now the bits be(t) and bo(t) can, not change instantaneously and, in changing, must pass through zero and dwell in that neighborhood at least briefly. Hence there will be brief variations in the amplitude of the transmitted waveform. These variations will be more pronounced in QPSK than in OQPSK since in the first case both be(t) and bo(t) may be zero simultaneously so that the signal amplitude may actually be reduced to zero temporarily.

Symbol versus Bit Transmission

In BPSK we deal individually with each bit of duration T_b. In QPSK we lump two bits together to form what is termed a symbol. The symbol can have anyone of four possible values corresponding to the two-bit sequences 00, 01, 10, and 11. We therefore arrange to make available for transmission four distinct signals. At the receiver each signal represents one symbol and, correspondingly, two bits. When bits are transmitted, as in BPSK, the signal changes occur at the bit rate. When symbols are transmitted the changes occur at the symbol rate which is one-half the bit rate. Thus the symbol time is $T_s=2T_b$.

The QPSK Receiver

A receiver for the QPSK signal is shown in Fig. 3.7.4.5. Synchronous detection is required and hence it is necessary to locally regenerate the carriers $\cos\omega_o t$ and $\sin\omega_o t$. The scheme for carrier regeneration is similar to that employed in BPSK. In that earlier case we squared the incoming signal, extracted a waveform at twice the carrier frequency by filtering, and recovered the carrier by frequency dividing by two. In the present case, it is required that the incoming signal be raised to the fourth power after which filtering recovers a waveform at four times the carrier frequency and finally frequency division by four regenerates the carrier. In the present case, also, we require both $\cos\omega_o t$ and $\sin\omega_o t$.

The incoming signal is also applied to two synchronous demodulators consisting, as usual, of a multiplier (balanced modulator) followed by an integrator. The integrator integrates over a two-bit interval of duration $T_s=2T_b$ and then dumps its accumulation. As noted previously, ideally the interval 2Tb = T_s should encompass an integral number of carrier cycles. One demodulator uses the carrier $\cos\omega_o t$ and the other the carrier $\sin\omega_o t$. We recall that when sinusoids in phase quadrature are multiplied, and the product is integrated over an integral number of cycles, the result is zero. Hence the demodulators will selectively respond to the parts of the incoming signal involving respectively $b_e(t)$ or $b_o(t)$.

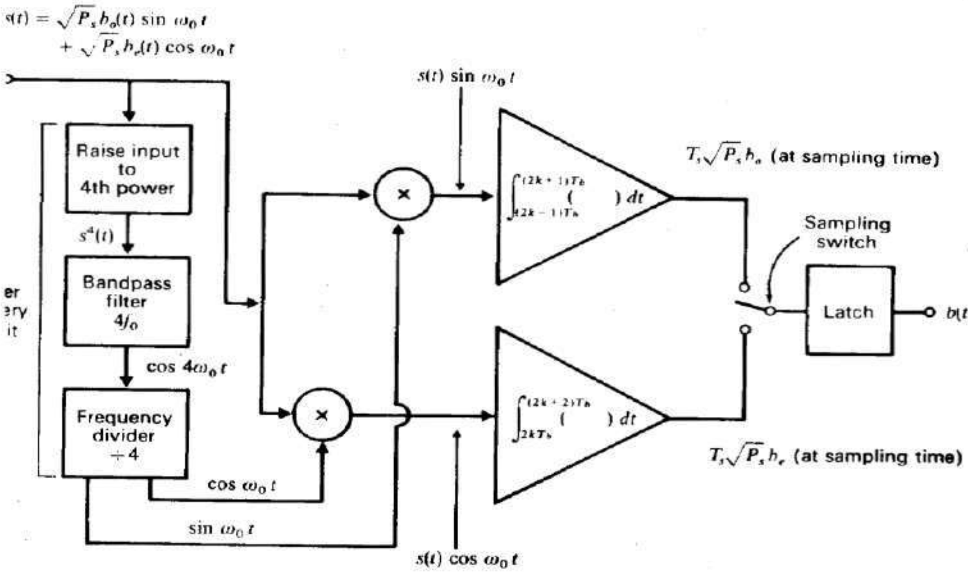


Figure 3.7.4.5 A QPSK Receiver

Of course, as usual, a bit synchronizer is required to establish the beginnings and ends of the bit intervals of each bit stream so that the times of integration can be established. The bit synchronizer is needed as well to operate the sampling switch. At the end of each integration time for each individual integrator, and just before the accumulation is dumped, the integrator output is sampled. Samples are taken alternately from one and the other integrator output at the end of each bit time T_b and these samples are held in the latch for the bit time T_b Each individual integrator output is sampled at intervals 2 T_b. The latch output is the recovered bit stream b(t).

The voltages marked on Fig. 3.7.4.5 are intended to represent the waveforms of the signals only and not their amplitudes. Thus the actual value of the sample voltages at the integrator outputs depends on the amplitude of the local carrier, the gain, if any, in the modulators and the gain in the integrators. We have however indicated that the sample values depend on the normalized power P_s of the received signal and on the duration T_s of the symbol.

3.8 M ARY PSK

In BPSK we transmit each bit individually. Depending on whether b(t) is logic 0 or logic 1, we transmit one or another of a sinusoid for the bit time T_b, the sinusoids differing in phase by $2\pi/2 = 180^0$. In QPSK we lump together two bits. Depending on which of the four two-bit words develops, we transmit one or another of four sinusoids of duration $2T_b$ the sinusoids differing in phase by amount $2\pi/4 = 90°$. The scheme can be extended. Let us lump together N bits so that in this N-bit symbol, extending over the time NT_b, there are 2N = M possible symbols. Now let us represent the symbols by sinusoids of duration $NT_b = T_s$ which differ from one another by the phase $2\pi / M$. Hardware to accomplish such M-ary communication is available.

Thus in M-ary PSK the waveforms used to identify the symbols are

$$v_m(t) = \sqrt{2P_s}\cos(\omega_o t + \emptyset_m) \text{ (m=0, 1, ..., M-1)} \qquad ...3.8.1$$

Where phase angle is given by

$$\emptyset_m = (2m+1)\frac{\pi}{M} \qquad ...3.8.2$$

The waveforms of Eq. (3.8.1) are represented by the dots in Fig. 3.8.1 in a signal space in which the coordinate axes are the orthonormal waveforms $u1(t) = \sqrt{2/T_s}\cos(\omega_o t)$ and $u2(t) = \sqrt{2/T_s}\sin(\omega_o t)$. The distance of each dot from the origin is $\sqrt{E_s} = \sqrt{P_s T_s}$

From Eq. (3.5.1) we have

$$v_m(t) = \left(\sqrt{2P_s}\cos\emptyset_m\right)\cos(\omega_o t) - \left(\sqrt{2P_s}\sin\emptyset_m\right)\sin(\omega_o t) \qquad ...3.8.3$$

Defining p_e and p_o by

$$p_e = \sqrt{2P_s}\cos\emptyset_m \qquad ...3.8.4a$$
$$p_o = \sqrt{2P_s}\sin\emptyset_m \qquad ...3.8.4b$$

Equation 3.5.3 becomes

$$v_m(t) = p_e\cos(\omega_o t) - p_o\sin(\omega_o t) \qquad ...3.8.5$$

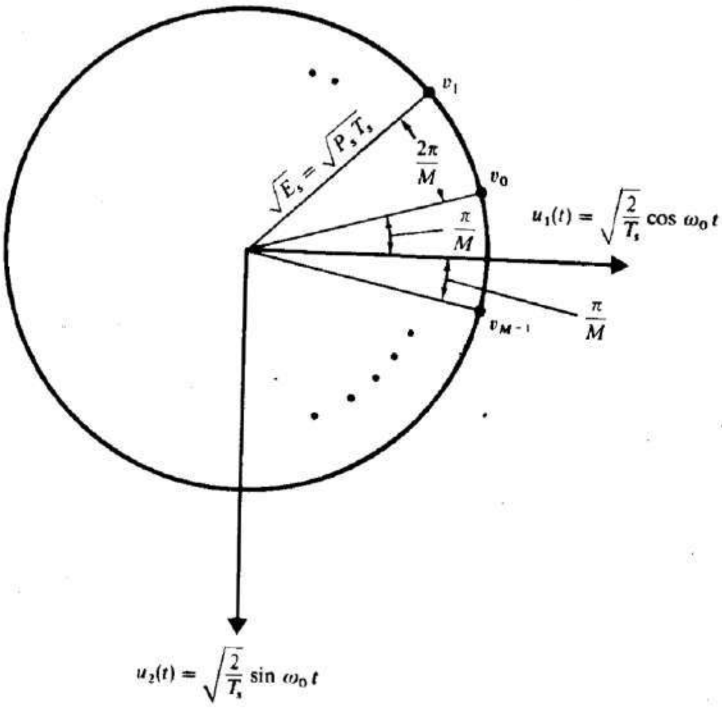


Figure 3.8.1 Graphical representation of M-ary PSK Signals

M-ary Transmitter and Receiver

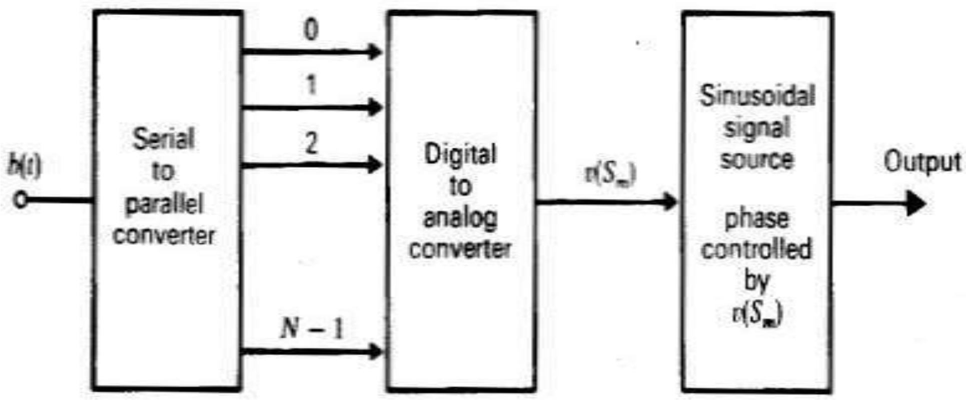


Figure 3.8.2 M Ary Transmitter

The transmitter, the bit stream b(t) is applied to a serial-to-parallel converter. This converter has facility for storing the N bits of a symbol. The N bits have been presented serially, that is, in time sequence, one after another. These N bits, having been assembled, are then presented all at once on N output lines of the converter, that is they are presented in parallel. The converter output remains unchanging for the duration NTb of a symbol during which time the converter is assembling a new group of N bits. Each symbol time the converter output is updated.

The converter output is applied to a D/A converter. This D/A converter generates an output voltage which assumes one of 2N = M different values in a one to-one correspondence to the M possible symbols applied to its input. That is, the D/A output is a voltage v(Sm) which depends on the symbol Sm (m = 0, 1,... ,M - 1). Finally v(Sm) is applied as a control input to a special type of constant amplitude sinusoidal signal source whose phase 4>m is determined by v(Sm). Altogether, then, the output is a fixed amplitude, sinusoidal waveform, whose phase has a one-to-one correspondence to the assembled N-bit symbol. The phase can change once per symbol time.

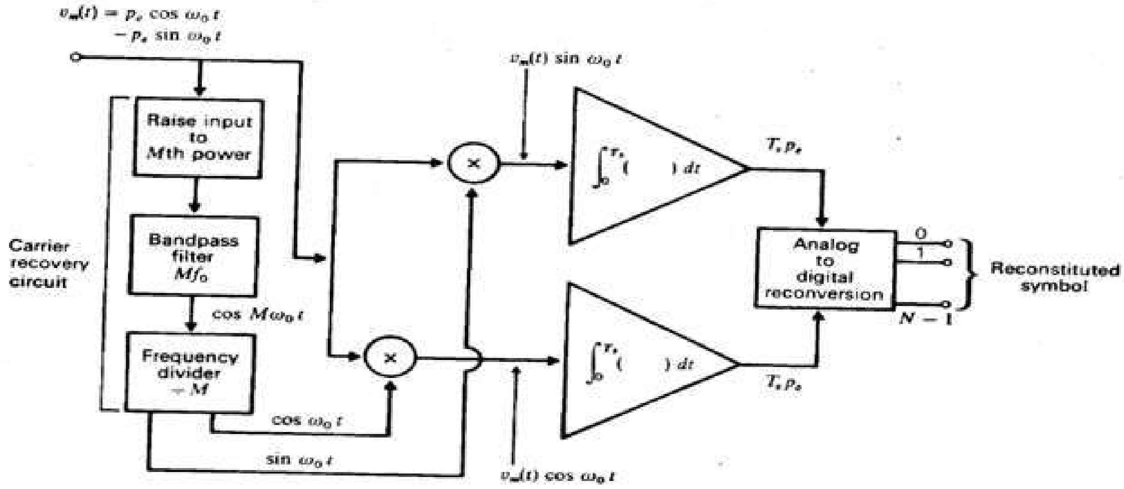


Figure 3.8.3 M Ary Transmitter

The carrier recovery system requires, in the present case a device to raise the received signal to the Mth power, filter to extract the Mfo component and then divide by M.

Since there is no staggering of parts of the symbol, the integrators extend their integration over the same time interval. Of course, again, a bit synchronizer is needed.

The integrator outputs are voltages whose amplitudes are proportional to TsPe and TsPo respectively and change at the symbol rate. These voltages measure the components of the received signal in the directions of the quadrature phasors $\sin w_0 t$ and $\cos w_0 t$. Finally the signals TsPe and TsPo are applied to a device which reconstructs the digital N-bit signal which constitutes the transmitted signal.

Current operating systems are common in which M = 16. In this case the bandwidth is B = 2fh/4 =fb/2 in comparison to B =fb for QPSK. PSK systems transmit information through signal phase and not through signal amplitude. Hence such systems have great merit in situations where, on account of the vagaries of the transmission medium, the received signal varies in amplitude.

3.9 Quadrature Amplitude Shift Keying (QASK)

In BPSK, QPSK, and M-ary PSK we transmit, in any symbol interval, one signal or another which are distinguished from one another in phase but are all of the same amplitude. In each of these individual systems the end points of the signal vectors in signal space falls on the circumference of a circle. Now we have noted that our ability to distinguish one signal vector from another in the presence of noise will depend on the distance between the vector end points. It is hence rather apparent that we shall be able to improve the noise immunity of a system by allowing the signal vectors to differ, not only in their phase but also in amplitude. We now describe such an amplitude and phase shift keying system. Like QPSK it involves direct (balanced) modulation of carriers in quadrature (i.e., $\cos \omega_o t$ and $\sin \omega_o t$) in quadrature and hence abbreviated as QAPSK or simply QASK.

For Example consider to transmit a symbol for every 4 bits. There are then 2^4= 16 different possible symbols and we shall have to be able to generate 16 distinguishable signals. One possible geometrical representation is shown in figure 3.9.1. In this configuration each signal point is equally distant from its neighbors, the distance being d=2a.

Let us assume that all 16 signals are equally likely. Because of the symmetrical placement around the origin, we can determine the average energy associated with a signal, from the four signals in the first quadrant. The average normalized energy of a signal is

$$E_s = \frac{1}{4}[(a^2 + a^2) + (9a^2 + a^2) + (a^2 + 9a^2) + (9a^2 + 9a^2)] \qquad ...3.9.1$$

$$E_s = 10a^2$$

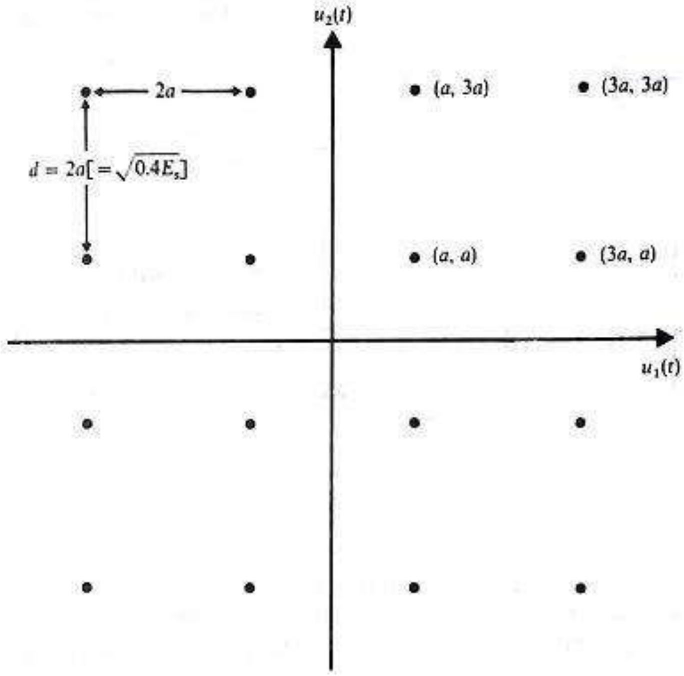


Figure 3.9.1 Geometrical Representation of 16 signals QASK

$$a = \sqrt{0.1E_s} \qquad ...3.9.2$$

$$d = 2\sqrt{0.1E_s} \qquad ...3.9.3$$

In present case each symbol represent 4 bits, the normalized symbol energy is $E_s = 4E_b$ where E_b is the normalized bit energy. Therefore

$$a = \sqrt{0.1E_s} = \sqrt{0.4E_b}$$

$$\text{and } d = 2\sqrt{0.4E_b} \qquad ...3.9.4$$

This distance is significantly less than the distance between adjacent QPSK signals where, $d = 2\sqrt{E_b}$; however the distance is greater than 16 MPSK where

$$d = \sqrt{16E_b sin^2\frac{\pi}{16}} = 2\sqrt{0.15E_b} \qquad ...3.9.5$$

Thus 16 QASK will have a lower error rate than 16 MPSK, but higher than QPSK.

Generation of QPSK

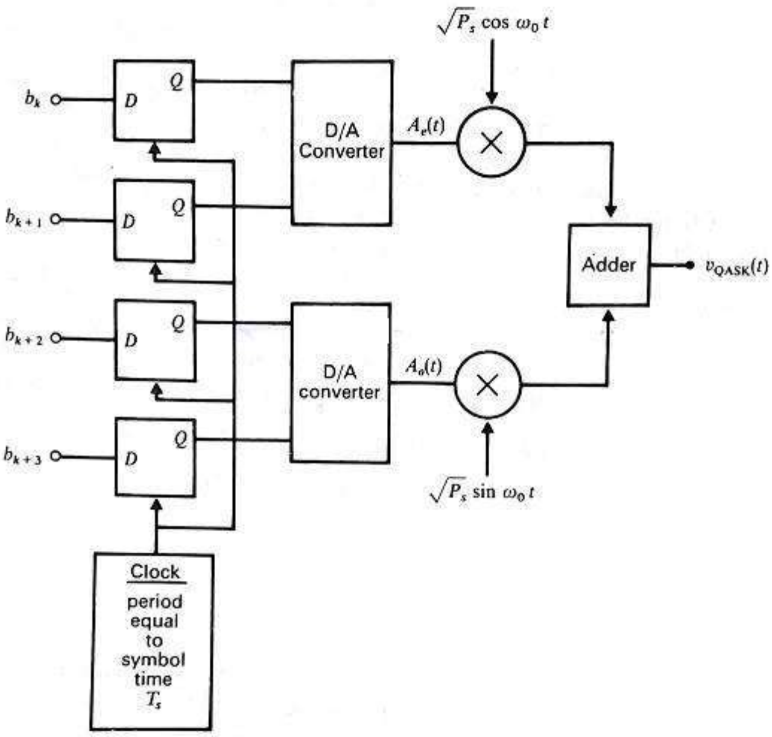


Figure 3.9.2 Generation of QPSK Signal

QASK generator for 4 bit symbol is shown. The 4 bit symbol $b_{k+3}\ b_{k+2}\ b_{k+1}\ b_k$ is stored in the 4 bit register made up of four flip flops. A new symbol is presented once per interval $T_s= 4T_b$ and the content of the register is correspondingly updated at each active edge of the clock which also have a period T_s. Two bits are presented to one D/A converter and two to second converter. The converter output $A_e(t)$ modulates the balanced modulator whose input career is the even function $\sqrt{P_s}\cos(\omega_o t)$ and $A_o(t)$ modulates the modulator whose input career is the odd function career. Then the transmitted signal is

$$v_{QASK}(t) = A_e(t)\sqrt{P_s}.\cos(\omega_o t) + A_o(t)\sqrt{P_s}.\sin(\omega_o t) \qquad \text{...3.9.6}$$

Bandwidth of QASK

The Bandwidth of the QASK signal is

$$B=2f_b/N$$

Which is the same as in the case of M ary PSK. With N=4 corresponding to 16 possible distinguishable signals we have $B_{QASK(16)}=f_b/2$ which is one fourth of the bandwidth required for binary PSK.

QASK Receiver

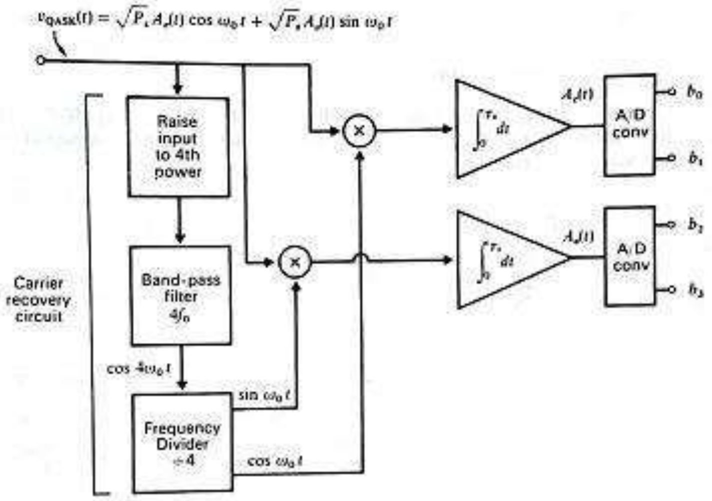


Figure 3.9.3 The QASK Receiver

It is similar to QPSK receiver, where a set of quadrature careers for synchronous demodulation is generated by raising the received signal to the power 4, extracting the components at frequency $4f_o$ and then dividing the frequency by 4.

In present case since the coefficients A_e and A_o are not of fixed value we have to enquire whether the career is still recoverable. We have

$$v^4_{QASK}(t) = P_s^2[A_e(t).\cos(\omega_o t) + A_o(t).\sin(\omega_o t)]^4 \qquad \text{...3.9.7}$$

Neglecting all the terms not at the frequency $4f_o$, we have

$$\frac{v^4_{QASK}(t)}{P_s} = \left[\frac{A_e^4(t) + A_o^4(t) - 6A_e^2(t)A_o^2(t)}{8}\right]\cos(4\omega_o t) + \left[\frac{A_e(t)A_o(t)[A_e^2(t) - A_o^2(t)]}{2}\right]\sin(4\omega_o t) \qquad \text{...3.9.8}$$

The average value of the coefficient of $\cos 4\omega_o t$ is not zero whereas the average value of the coefficient of $\sin 4\omega_o t$ is zero. Thus a narrow filter centered at $4f_o$ will recover the signal at $4f_o$.

After getting the careers, two balanced modulators together with two integrators recover the signals $A_e(t)$ and $A_o(t)$ as shown in figure. The integrators have an integration time equal to the symbol time T_s. Finally the original input bits are recovered by using A/D Converter.

3.10 Binary Frequency Shift Keying (BFSK)

In binary frequency-shift keying (BFSK) the binary data waveform d(t) generates a binary signal

$$v_{BFSK}(t) = \sqrt{2P_s}\cos[\omega_o t + d(t)\Omega t] \quad \text{...3.10.1}$$

Here d(t) = + 1 or -1 corresponding to the logic levels 1and 0 of the data waveform. The transmitted signal is of amplitude $\sqrt{2P_s}$ and is either

$$v_{BFSK}(t) = S_H(t) = \sqrt{2P_s}\cos(\omega_o + \Omega)\,t \quad \text{...3.10.2}$$

$$v_{BFSK}(t) = S_L(t) = \sqrt{2P_s}\cos(\omega_o - \Omega)\,t \quad \text{...3.10.3}$$

and thus has an angular frequency $\omega_o + \Omega$ or $\omega_o - \Omega$ with Ω a constant offset from the nominal carrier frequency ω_o. We shall call the higher frequency ω_H(= $\omega_o + \Omega$) and the lower frequency ω_L.(= $\omega_o - \Omega$). We may conceive that the BFSK signal is generated in the manner indicated in Fig. 3.7.1. Two balanced modulators are used, one with carrier ω_H and one with carrier ω_L. The voltage values of $P_H(t)$ and of $P_L(t)$ are related to the voltage values of d(t) in the following manner

d(t)	P_H(t)	P_L(t)
+1V	+1V	0V
-1V	0V	+1V

Thus when d(t) changes from +1 to -1 P_H changes from 1 to 0 and P_L from 0 to 1. At any time either P_H or P_L is 1 but not both so that the generated signal is either at angular frequency ω_H or at ω_L.

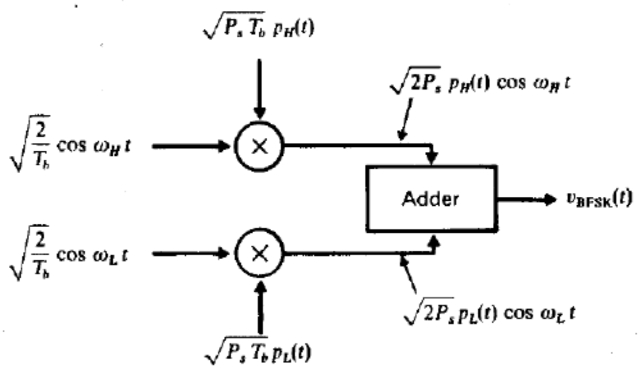


Figure 3.10.1 A representation of a manner in which a BFSK signal can be generated.

Spectrum of BPSK

In terms of variables P_H and P_L the BFSK signal is given by

$$v_{BFSK}(t) = \sqrt{2P_s}P_H\cos[\omega_H t + \theta_H] + \sqrt{2P_s}P_L\cos[\omega_L t + \theta_L] \quad \text{...3.10.4}$$

where we have assumed that each of the two signals are of independent and random, uniformly distributed phase. Each of the terms in Eq. (3.10.4) looks like the signal $\sqrt{2P_s}b(t)\cos\omega_o t$ which we encountered in BPSK and for which we have already deduced the spectrum, but there is an important difference. In the BPSK case, b(t) is bipolar, i.e., it alternates between + 1 and – 1 while in the present case P_H and P_L are unipolar, alternating between + 1 and 0. We may, however, rewrite P_H and P_L as the sums of a constant and a bipolar variable, that is

$$P_H(t) = \frac{1}{2} + \frac{1}{2}P'_H(t) \quad \text{...3.10.5a}$$

$$P_L(t) = \frac{1}{2} + \frac{1}{2}P'_L(t) \quad \text{...3.10.5b}$$

In above equations $P'_H(t)$ and $P'_L(t)$ are bipolar alternating between +1 and -1 and are complementary i.e. when $P'_H(t) = +1$, $P'_L(t) = -1$ and vice versa. Then from equation 4,

$$v_{BFSK}(t) = \sqrt{\frac{P_s}{2}}\cos[\omega_H t + \theta_H] + \sqrt{\frac{P_s}{2}}\cos[\omega_L t + \theta_L] + \sqrt{\frac{P_s}{2}}P'_H\cos[\omega_H t + \theta_H] + \sqrt{\frac{P_s}{2}}P'_L\cos[\omega_L t + \theta_L] \quad ...3.10.6$$

The first two terms in Eq. (3.10.6) produce a power spectral density which consists of two impulses, one at f_H and one at f_L. The last two terms produce the spectrum of two binary PSK signals one centered about f_H and one about f_L. The individual power spectral density patterns of the last two terms are for the case f_H - f_L = $2f_b$. For this separation between f_H and f_L we observe that the overlapping between the two parts of the spectra is not large and we may expect to be able, to distinguish the levels of the binary waveform d(t). In any event, with this separation the bandwidth of BFSK is

$$BW_{BFSK} = 4f_b \quad ...3.7.7$$

which is twice the bandwidth of BPSK.

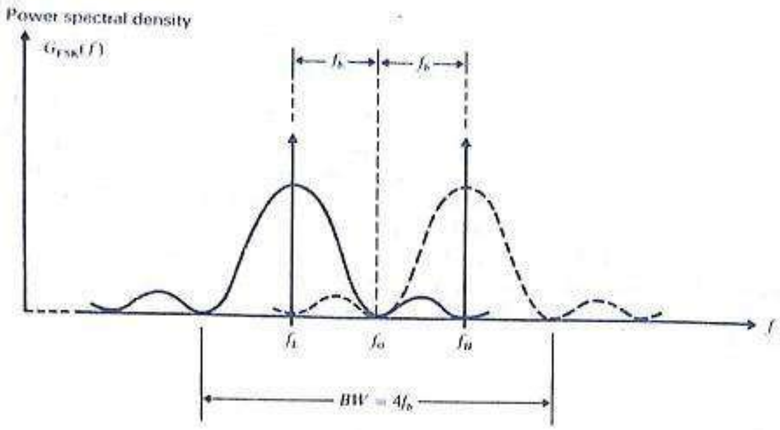


Figure 3.10.2 Power Spectral Densities of Equation 3.7.6

BFSK Receiver:

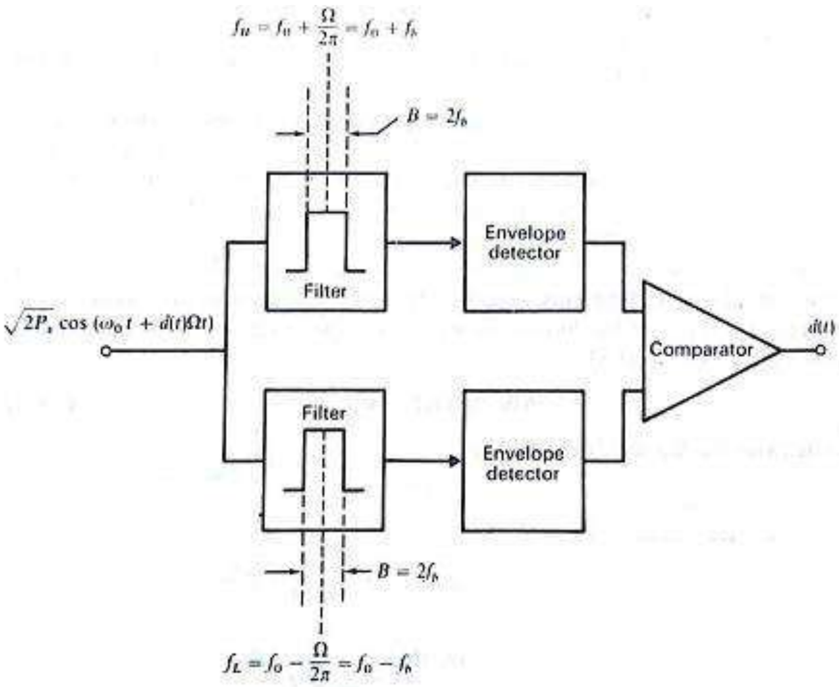


Figure 3.10.3 A Receiver for BFSK Signal

A BFSK signal is typically demodulated by a receiver system as in Fig.3.10.3. The signal is applied to two bandpass filters one with center frequency at f_H the other at f_L. Here we have assumed, that f_H - f_L = $2(\Omega/2\pi)$ = $2f_b$. The filter frequency ranges selected do not overlap and each filter has a passband wide enough to encompass a main lobe in the spectrum of Fig. 3.10.2. Hence one filter will pass nearly all the energy in the transmission at f_H the other will perform similarly for the transmission at f_L. The filter outputs are applied to envelope detectors and finally the envelope detector outputs are compared by a comparator. A comparator is a circuit that accepts two input signals. It generates a binary output which is at one level or the other depending on which input is larger. Thus at the comparator output the data d(t) will be reproduced.

When noise is present, the output of the comparator may vary due to the systems response to the signal and noise. Thus, practical systems use a bit synchronizer and an integrator and sample the comparator output only once at the end of each time interval T_b.

Geometrical Representation of Orthogonal BFSK

In M-ary phase-shift keying and in quadrature-amplitude shift keying, any signal could be represented as $C_1u_1(t) + C_2u_2(t)$. There $u_1(t)$ and $u_2(t)$ are the orthonormal vectors in signal space, that is, $u1(t) = \sqrt{\frac{2}{T_s}}.\cos(\omega_o t)$and $u2(t) = \sqrt{\frac{2}{T_s}}.\sin(\omega_o t)$. The functions u1 and u2 are orthonormal over the symbol interval T_S. And, if the symbol is a single bit, $T_S = T_b$. The coefficients C1 and C2 are constants. The normalized energies associated with $C_1u_1(t)$ and with $C_2u_2(t)$are respectively C_1^2 and C_2^2 and the total signal energy is $C_1^2 + C_2^2$. In M-ary PSK and QASK the orthogonality of the vectors u_1 and u_2 results from their phase quadrature. In the present case of BFSK it is appropriate that the orthogonality should result from a special selection of the frequencies of the unit vectors. Accordingly, with m and n integers, let us establish unit vectors

$$u_1(t) = \sqrt{\frac{2}{T_b}}\cos 2\pi m f_b t \qquad ...3.10.8$$

and

$$u_2(t) = \sqrt{\frac{2}{T_b}}\cos 2\pi n f_b t \qquad ...3.10.9$$

Where $f_b=1/T_b$. The vectors U1 and U2 are the m^{th} and n^{th} harmonics of the (fundamental) frequency f_b. As we are aware, from the principles of Fourier analysis, different harmonics (m ± n) are orthogonal over the interval of the fundamental period $T_b = 1/f_b$.

If now the frequencies f_H and f_L in a BFSK system are selected to be (assuming m > n)

$$f_H = mf_b \qquad ...3.\ 10.10a$$

and

$$f_L = nf_b \qquad ...3.\ 10.10b$$

Then corresponding signal vectors are

$$S_H(t) = \sqrt{E_b}u_1(t) \qquad ...3.\ 10.11a$$

and

$$S_L(t) = \sqrt{E_b}u_2(t) \qquad ...3.\ 10.11b$$

The signal space representation of these signals is shown in Fig. 3. 10.4. The signals, like the unit vectors are orthogonal. The distance between signal end points is therefore

$$d = \sqrt{2E_b}$$

Note that this distance is considerably smaller than the distance separating end points of BPSK signals, which are antipodal.

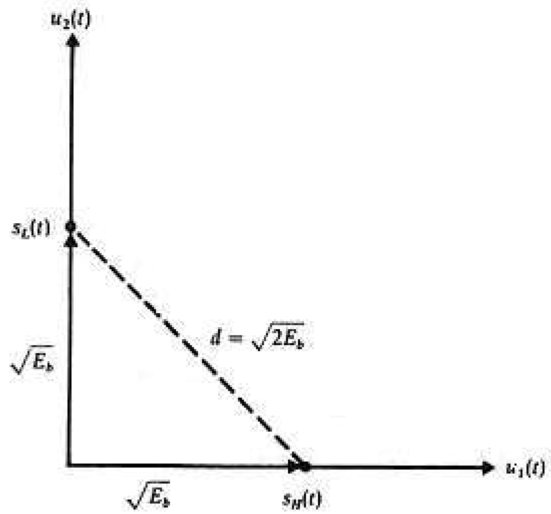


Figure 3.10.4 Signal Space representation of BFSK

Geometrical Representation of Non-Orthogonal BFSK

When the two FSK signals $S_H(t)$ and $S_L(t)$ are not orthogonal, the Gram-Schmidt procedure can still be used to represent the signals of Eqs. 3.10.2 and 3.10.3.

Let us represent the higher frequency signal SH(t) as:

$$S_H(t) = \sqrt{2P_S}\cos\omega_H t = S_{11}u_1(t)\ 0 \le t \le T_b \qquad \text{...3. 10.12a}$$

and

$$S_L(t) = \sqrt{2P_S}\cos\omega_L t = S_{12}u_1(t) + S_{22}u_2(t)\ 0 \le t \le T_b \qquad \text{...3. 10.12b}$$

The representation of these two signals in signal space is shown in Fig. 3. 10.5.

Referring to this figure we see that the distance separating S_H and S_L is:

$$d^2_{BFSK} = (S_{11} - S_{12})^2 + S_{22}^2 = S_{11}^2 - 2S_{11}S_{12} + S_{12}^2 + S_{22}^2 \qquad \text{...3. 10.13}$$

In order to determine d^2_{BFSK} when the two signals are not orthogonal we must evaluate S11, S12, and S22 using Eqs. (3. 10.12). From Eq. 3. 10.12a we have:

$$S_{11}^2 = 2P_S\int_0^{T_b}\cos^2\omega_H t\,dt = E_b\left[1 + \frac{\sin 2\omega_H T_b}{2\omega_H T_b}\right] \qquad \text{...3. 10.14}$$

Using Eq. (3. 10.12b) we first determine S_{12} by multiplying both sides of the equation by $u_1(t)$ and integrating from $0 \le t \le T_b$. The result is:

$$S_{12} = \sqrt{2P_S}u_1(t)\ \cos\omega_L t\,dt$$

$$S_{12} = \frac{E_b}{S_{11}}\left[\frac{\mathrm{Sin}(\omega_H-\omega_L)T_b}{(\omega_H-\omega_L)T_b} - \frac{\mathrm{Sin}(\omega_H+\omega_L)T_b}{(\omega_H+\omega_L)T_b}\right] \qquad \text{...3.7.15a}$$

By using equation 3.10.12a where,

$$u_1(t) = (\sqrt{2P_S}/S_{11})\ \cos\omega_H t \qquad \text{...3.7.15b}$$

Finally, S22 is found from Eq. 3.10.12b by squaring both sides of the equation and then integrating from 0 to T_b. Since u_1 and u_2 are orthogonal, the result is:

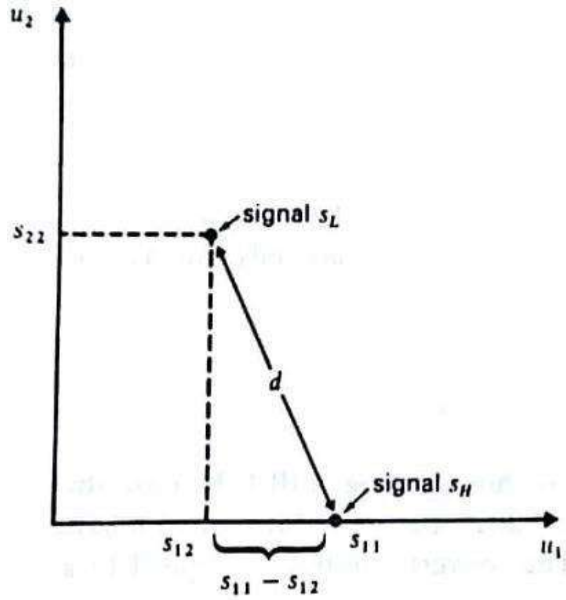


Figure 3.10.5 Signal Space representation of BFSK when $S_H(t)$ and $S_L(t)$ are not orthogonal

$$\int_0^{T_b} S_L^2(t)\,dt = 2P_S\int_0^{T_b} cos^2\,\omega_H t\,dt = S_{12}^2 + S_{22}^2 \qquad \text{...3. 10.16a}$$

Therefore

$$S_{12}^2 + S_{22}^2 = E_b\left[1 + \frac{\sin 2\omega_L T_b}{2\omega_L T_b}\right] \qquad \text{...3. 10.16b}$$

The distance d between SH and SL given in Eq. (6.8-14) can now be determined by substituting Eqs. 3. 10.14, 3. 10.15a, and 3. 10.16b into Eq. 3.10.13 The result is:

$$d^2 = E_b\left[1 + \frac{\sin 2\omega_H T_b}{2\omega_H T_b}\right] - 2E_b\left[\frac{\mathrm{Sin}(\omega_H-\omega_L)T_b}{(\omega_H-\omega_L)T_b} + \frac{\mathrm{Sin}(\omega_H+\omega_L)T_b}{(\omega_H+\omega_L)T_b}\right] + E_b\left[1 + \frac{\sin 2\omega_L T_b}{2\omega_L T_b}\right] \qquad \text{...3. 10.18}$$

In above equation,

$$\left|\frac{\sin 2\omega_H T_b}{2\omega_H T_b}\right| \ll 1$$

$$\left|\frac{\sin 2\omega_L T_b}{2\omega_L T_b}\right| \ll 1$$

And

$$\left|\frac{\mathrm{Sin}(\omega_H+\omega_L)T_b}{(\omega_H+\omega_L)T_b}\right| \ll \left|\frac{\mathrm{Sin}(\omega_H-\omega_L)T_b}{(\omega_H-\omega_L)T_b}\right|$$

Then the final result is then

$$d^2 \cong 2E_b\left[1-\frac{\mathrm{Sin}(\omega_H-\omega_L)T_b}{(\omega_H-\omega_L)T_b}\right] \qquad ...3.10.19$$

Here $S_H(t)$ and $S_L(t)$ are orthogonal $(\omega_H\text{-}\omega_L)T_b=2\pi(m\text{-}n)f_bT_b=2\pi(m\text{-}n)$ and the above equation given $d = \sqrt{2E_b}$.

Note that if $(\omega_H\text{-}\omega_L)T_b=3\pi/2$, the distance d increases and becomes,

$$d_{opt} = \left[2E_b\left(1+\frac{2}{3\pi}\right)\right]^{1/2} = \sqrt{2.4E_b} \qquad ...3.\ 10.20$$

d^2 is increased by 20%.

3.11 Comparison of BFSK and BPSK

Let us start with the BFSK signal

$$v_{BFSK}(t) = \sqrt{2P_s}\cos[\omega_o t + d(t)\Omega t]$$

Using the trigonometric identity for the cosine of the sum of two angles and recalling that cos θ = cos (- θ) while sin θ = - sin (-θ) we are led to the alternate equivalent expression

$$v_{BFSK}(t) = \sqrt{2P_s}\cos\Omega t\cos\omega_o t - \sqrt{2P_s}d(t)\sin\Omega t\sin\omega_o t \qquad ...3.11.1$$

Note that the second term in above equation looks like the signal encountered in BPSK i.e., a carrier $\sin\omega_0 t$ multiplied by a data bit d(t) which changes the carrier phase. In the present case however, the carrier is not of fixed amplitude but rather the amplitude is shaped by the factor sin Ωt. We note further the presence of a quadrature reference term $\cos\Omega t\cos\omega_0 t$ which contains no information. Since this quadrature term carries energy, the energy in the information bearing term is thereby diminished. Hence we may expect that BFSK will not be as effective as BPSK in the presence of noise. For orthogonal BFSK, each term has the same energy, hence the information bearing term contains only one-half of the total transmitted energy.

The Generation of BFSK is easier but it has many disadvantages…

- Bandwidth is greater in comparison with BPSK, almost double. (Because we are using two career signals.)
- Error rate of BFSK is higher

3.12 M-ARY FSK

An M-ary FSK communications system is shown in Fig. 3.12.1. It is an obvious extension of a binary FSK system. At the transmitter an N-bit symbol is presented each T_S, to an N-bit D/A converter. The converter output is applied to a frequency modulator, i.e., a piece of hardware which generates a carrier waveform whose frequency is determined by the modulating waveform. The transmitted signal, for the duration of the symbol interval, is of frequency f_0 or f_1 ...or f_{M-1} with $M = 2^N$. At the receiver, the incoming signal is applied to M paralleled bandpass filters each followed by an envelope detector. The bandpass filters have center frequencies $f_0, f_1, \ldots, f_{M-1}$. The envelope detectors apply their outputs to a device which determines which of the detector indications is the largest and transmits that envelope output to an N-bit AID converter.

The probability of error is minimized by selecting frequencies $f_0, f_1, \ldots, f_{M-1}$ so that the M signals are mutually orthogonal. One commonly employed arrangement simply provides that the carrier frequency be successive even harmonics of the symbol frequency $f_S=1/T_S$. Thus the lowest frequency, say f_0, is $f_0 = k f_S$, while $f_1 = (k + 1) f_S$, $f_2 = (k + 2) f_S$ etc. In this case, the spectral density patterns of the individual possible transmitted signals overlap in the manner shown in Fig. 3.12.1, which is an extension to M-ary FSK of the pattern of Fig. 3.9.2, which applies to binary FSK. We observe that to pass M-ary FSK the required spectral range is

$$B = 2Mf_S \qquad ...3.12.1$$

Since $f_S = f_b/N$ and $M=2^N$, we have

$$B = 2^{N+1}f_b/N \qquad ...3.12.2$$

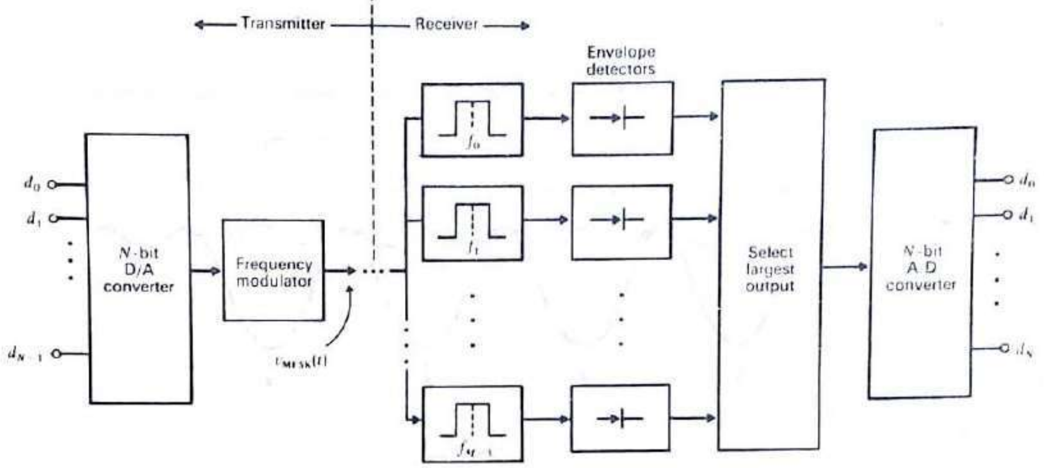


Figure 3.12.1 An M-ARY Communication System

Note that M-ary FSK requires a considerably increased bandwidth in comparison with M-ary PSK. However, as we shall see, the probability of error for M-ary FSK decreases as M increases, while for M-ary PSK, the probability of error increases with M.

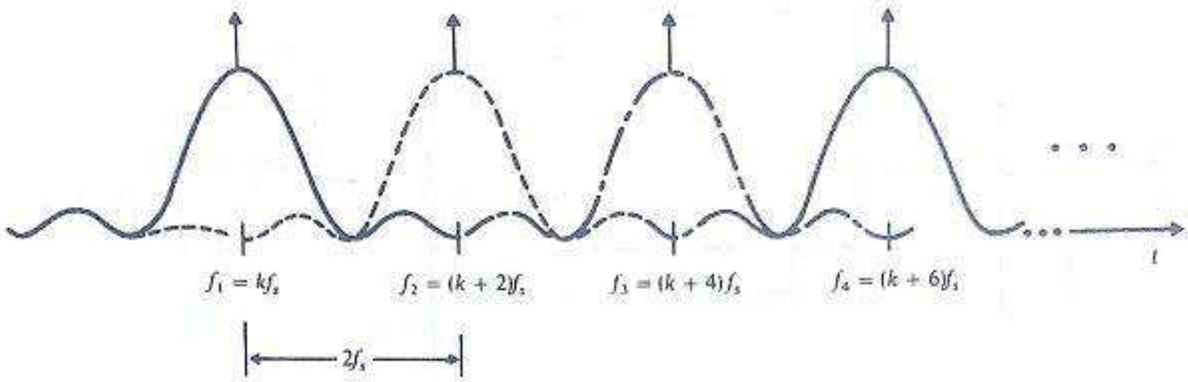


Figure 3.12.2 Power Spectral Density of an M-ARY FSK (Four Frequencies are shown)

Geometrical Representation of an M-ARY FSK

In Fig 3.7.4, we provided a signal space representation for the case of orthogonal binary FSK.

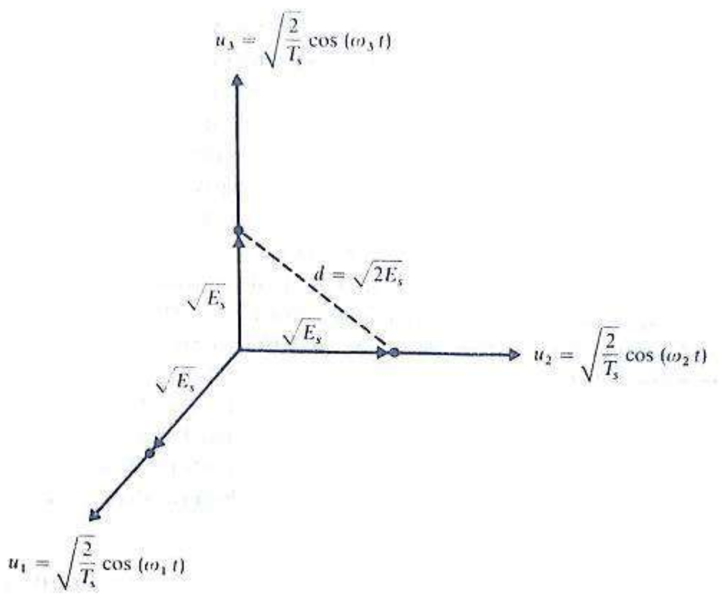


Figure 3.12.3 Geometrical representation of orthogonal M-ary FSK (M = 3) when the frequencies are selected to generate orthogonal signals.

The case of M-ary orthogonal FSK signals is clearly an extension of this figure. We simply conceive of a coordinate system with M mutually orthogonal coordinate axes. The square of the length of the signal vector is the normalized signal energy. Note that, as in Fig. 3.7.4, the distance between signal points is 3.9.3.

Note that this value of d is greater than the values of d calculated for M-ary PSK with the exception of the cases M = 2 and M = 4. It is also greater than d in the case of 16-QASK.

$$d = \sqrt{2E_s} = \sqrt{2NE_b} \quad \text{...3.12.3}$$

3.13 Minimum Shift Keying (MSK)

There are two important differences between QPSK and MSK:

1. In MSK the baseband waveform, that multiplies the quadrature carrier, is much "smoother" than the abrupt rectangular waveform of QPSK. While the spectrum of MSK has a main center lobe which is 1.5 times as wide as the main lobe of QPSK; the side lobes -in MSK are relatively much smaller in comparison to the main lobe, making filtering much easier.
2. The waveform of MSK exhibits phase continuity, that is, there are not abrupt. phase changes as in QPSK. As a result we avoid the inter-symbol interference caused by nonlinear amplifiers.

The waveforms of MSK are shown in figure 3.13.1. In (a) we start with a typical data bit stream b(t). This bit stream is divided into an odd an even bit stream in (b) and (c), as in the manner of OQPSK. The odd stream $b_o(t)$ consists of the alternate bits b_1, b_3, etc., and the even stream $b_e(t)$ consists of b_2, b_4; etc. Each bit in both streams is held for two bit intervals 2 $T_b = T_s$, the symbol time. The staggering, which is optional in QPSK, is essential in MSK. The staggering is that the changes in the odd and even stream do not occur at the same time.

Also generated at the MSK transmitter are the waveforms $\sin 2\pi(t/4T_b)$ and $\cos 2\pi(t/4T_b)$ as in (d). These waveforms, and their phases with respect to the bit streams $b_o(t)$ and $b_e(t)$, meet the essential requirements that $\sin 2\pi(t/4T_b)$ passes through zero precisely at the end of the symbol time in $b_e(t)$ and $\cos 2\pi(t/4T_b)$ passes through zero at the end of the symbol time in $b_o(t)$. We now .generate the products $b_e(t) \sin 2\pi(t/4T_b)$ and $b_o(t) \cos 2\pi(t/4T_b)$ which are shown in (e) and (f).

In MSK the transmitted signal is

$$v_{MSK}(t) = \sqrt{2P_s}\left[b_e(t)\sin 2\pi\left(\frac{t}{4T_b}\right)\right]\cos\omega_o t + \sqrt{2P_s}\left[b_0(t)\cos 2\pi\left(\frac{t}{4T_b}\right)\right]\sin\omega_o t \quad \text{...3.13.1}$$

In MSK, the carriers are multiplied by the" smoother" waveforms shown in Fig. 3.13.1(e) and (f). As we may expect, the side lobes generated by these smoother waveforms will be smaller than those associated with the rectangular waveforms and hence easier to suppress as is required to avoid interchannel interference.

In Eq. (3.13.1) MSK appears as a modified form of OQPSK, which we can call" shaped Q'PSK". We can, however, rewrite the equation to make it apparent that MSK is an FSK system. Applying the trigonometric identities for the products of sinusoids we find that Eq. (3.10.1) can be written:

$$v_{MSK}(t) = \sqrt{2P_s}\left[\frac{b_o(t)+b_e(t)}{2}\right]\sin(\omega_o+\Omega)t + \sqrt{2P_s}\left[\frac{b_o(t)-b_e(t)}{2}\right]\cos(\omega_o+\Omega)t \quad \text{...3.13.2a}$$

$$\text{Where } \Omega = 2\pi/4T_b = 2\pi(fb/4) \quad \text{...3.13.2b}$$

If we define $C_H = (b_0+b_e)/2$, $C_L==(b_0-b_e)/2$, $\omega_H = \omega_0 + \Omega$, $\omega_H = \omega_0 - \Omega$ then equations 3.10.2 becomes

$$v_{MSK}(t) = \sqrt{2P_s}C_H(t)\sin\omega_H t + \sqrt{2P_s}C_L(t)\omega_L t \quad \text{...3.13.3}$$

Now b; = + 1 and be = + 1, so that as is easily verified, if b_0, = b_e then CL = 0 while $C_H = b_0 = \pm 1$. Further, if $b_0 = -b_e$ then $C_H = 0$ and $C_L = b_0 = + 1$.

Thus, depending on the value of the bits b_o and b_e in each bit interval, the transmitted signal is at angular frequency ω_H or at ω_L precisely as in FSK and the magnitude of the amplitude is always equal to $\sqrt{2P_s}$.

In MSK, the two frequencies f_H and f_L, are chosen to insure that the two possible signals are orthogonal over the bit interval T_b. That is, we impose the constraint that

$$\int_0^{T_b} \sin \omega_H t \sin \omega_L t \, dt = 0 \qquad ...3.13.4$$

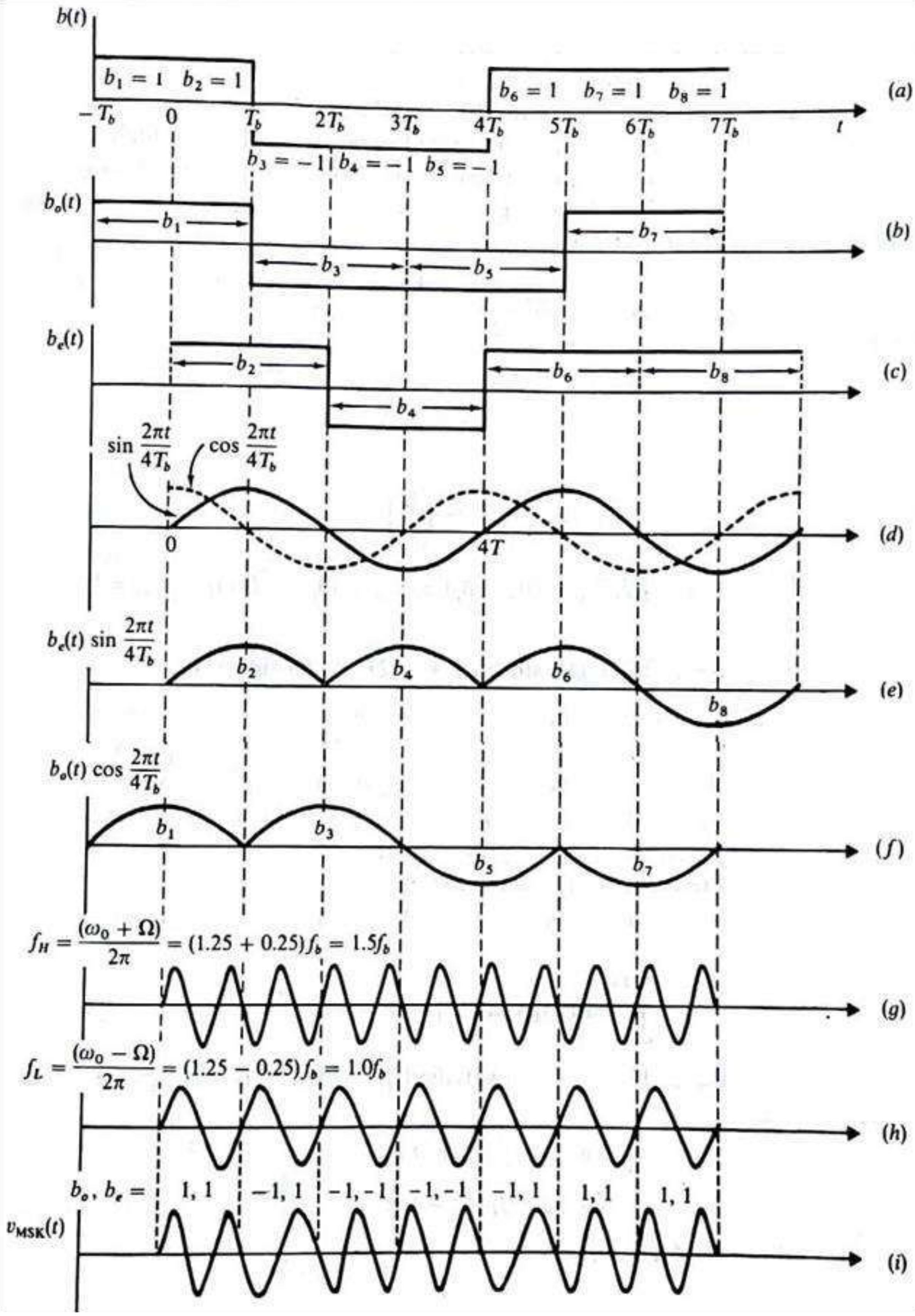


Figure 3.13.1 MSK Waveforms

The equation 3.13.4 will be satisfied provided that it is arranged, with m and n integers, that

$$2\pi(f_H - f_L)T_b = n\pi \qquad ...3.13.5a$$

and

$$2\pi(f_H + f_L)T_b = m\pi \qquad ...3.13.5b$$

Also

$$f_H = f_0 + \frac{f_b}{4} \qquad \text{...3.13.06a}$$

and

$$f_L = f_0 - \frac{f_b}{4} \qquad \text{...3.13.6b}$$

From equations 3.13.5 and 3.13.6

$$f_b T_b = f_b \frac{1}{f_b} = 1 = n \qquad \text{...3.13.7a}$$

and

$$f_0 = \frac{m}{4} f_b \qquad \text{...3.13.7b}$$

Equation (3.13.7a) shows that since n = 1, f_H and f_L are as close together as possible for orthogonality to prevail. It is for this reason that the present system is called" minimum shift keying." Equation (3.13.7b) shows that the carrier frequency f_o is an integral multiple of $f_b/4$. Thus

$$f_H = (m+1)\frac{f_b}{4} \qquad \text{...3.13.8a}$$

and

$$f_L = (m-1)\frac{f_b}{4} \qquad \text{...3.13.8b}$$

Signal Space Representation of MSK

The signal space representation of MSK is shown in Fig. 3.13.2 The orthonormal unit vectors of the coordinate system are given by $u_H(t) = \sqrt{2/T_s}\,\sin\omega_H t$ and $u_L(t) = \sqrt{2/T_s}\,\sin\omega_L t$. The end points of the four possible signal vectors are indicated by dots. The smallest distance between signal points is

$$d = \sqrt{2E_s} = \sqrt{4E_b} \qquad \text{...3.13.9}$$

just as for the case of QPSK.

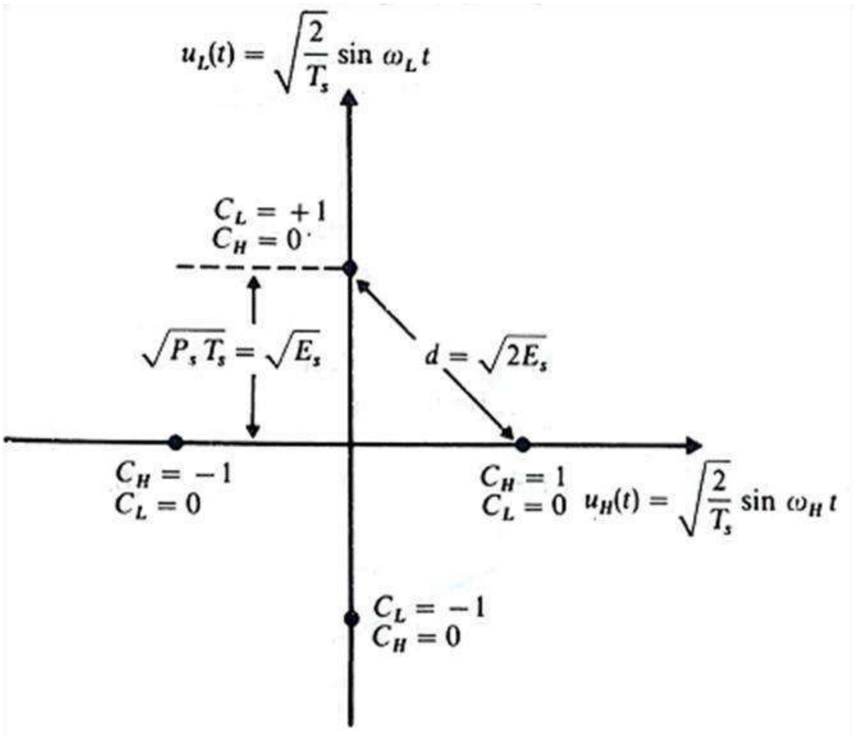


Figure 3.13.2 Signal Space Representation of MSK

We recall that QPSK generates two BPSK signals which are orthogonal to one another by virtue of the fact that the respective carriers are in phase quadrature. Such phase quadrature can also be characterized as time quadrature since, at a carrier frequency f_0 a phase shift of $\pi/2$ is accomplished by a time shift in amount $1/4f_0$, that is $\sin 2\pi f_0(t+1/4f_0) = \sin(2\pi f_0 t + \pi/2) = \cos(2\pi f_0 t)$. It may be noted that in MSK we have again two BPSK signals. Here, however, the respective carriers are orthogonal to one another by virtue of the fact that they are in frequency quadrature.

Generation and Reception of MSK

One way to generate a MSK signal is the following: We start with sin Ωt and sin ω_0t, and use 90° phase shifters to generate sin (Ωt + π/2) = cosΩt and sin (ω_0t + π/2) = cosω_0t. We then use multipliers to form the products sinΩt cosω_0t and cosΩt sin ω_0t. Additional multipliers generate $\sqrt{2P_s}b_e(t)\sin\Omega t\cos\omega_0 t$ and $\sqrt{2P_s}b_0(t)\cos\Omega t\sin\omega_0 t$. Finally an adder is used to form the sum. An alternative and more favored scheme is shown in Fig. 3.13.3a. This technique has the merit that it avoids the need for precise 90° phase shifters at angular frequencies ω_o and Ω.

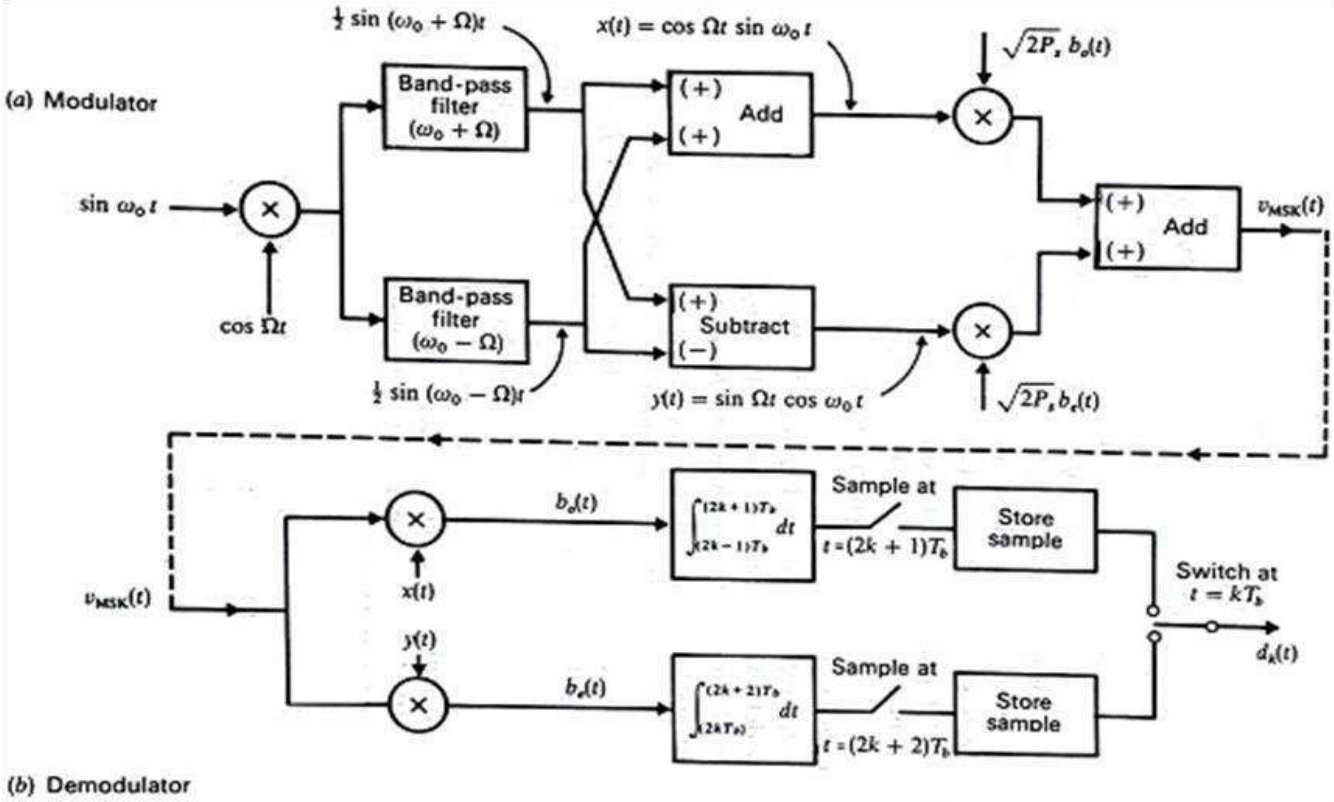


Figure 3.13.3 MSK Modulation and demodulation

The MSK receiver is shown in Fig. 3.13.3b. Detection is performed synchronously, i.e., by determining the correlation of the received signal with the waveform x(t) = cosΩt sin ω_0t to determine bit b_0(t), and with y(t) =sinΩt cosω_0t to determine bit b_e(t). The integration is performed over the symbol interval. The integrators integrate over staggered overlapping intervals of symbol time T_s = $2T_b$.

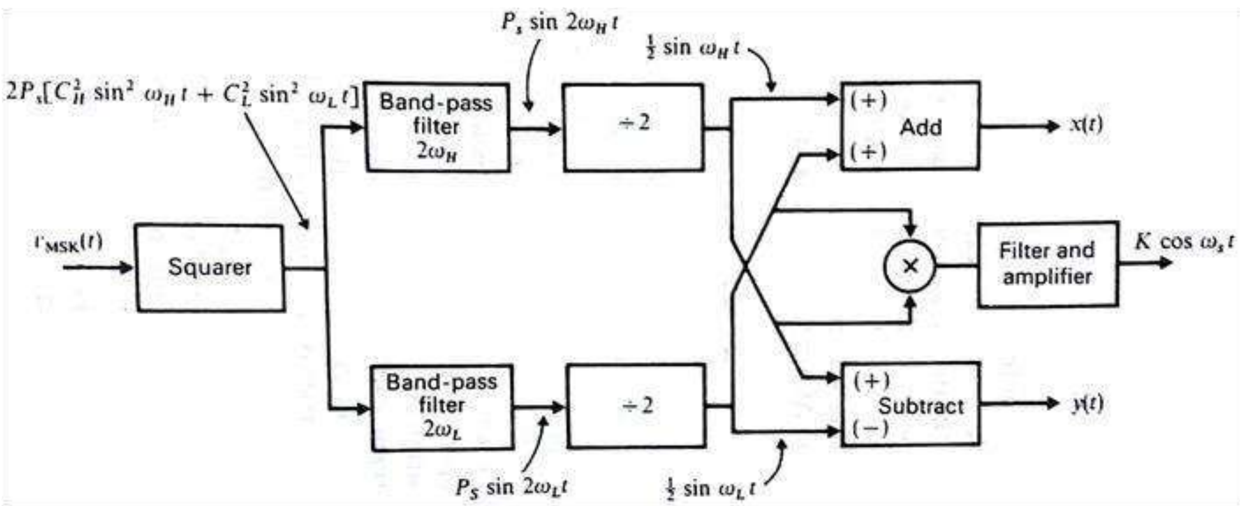


Figure 3.13.4 Regeneration of x(t) and y(t)

At the end of each integration time the integrator output is stored and then the integrator output is dumped. The switch at the output swings back and forth at the bit rate so that finally the output waveform is the original data bit stream $d_k(t)$.

At the receiver we need to reconstruct the waveforms x(t) and y(t). A method for locally regenerating x(t) and y(t) is shown in Fig. 3.10.4. From Eq3.13.3 we see that MSK consists of transmitting one of two possible HPSK signals, the first at frequency $\omega_0 - \Omega$ and the second at frequency $\omega_0 + \Omega$. Thus as in BPSK detection, we first square and filter the incoming signal. The output of the squarer has spectral components at the frequency $2\omega_H = 2(\omega_0 + \Omega)$ and at $2\omega_L = 2(\omega_0 - \Omega)$. These are separated out by band pass filters. Division by 2 yields waveforms $(1/2)\sin \omega_H t$ and $(1/2)\sin \omega_L t$ from which, as indicated, x(t) and y(t) are regenerated by addition and subtraction respectively. Further, the multiplier and low-pass filter shown regenerate a waveform at the symbol rate Is $=f_b/2$ which can 'be used to operate the sampling switches in Fig. 3.13.3b.

