CONTROL SYSTEM

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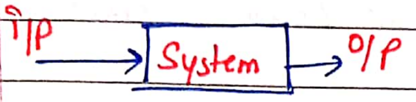
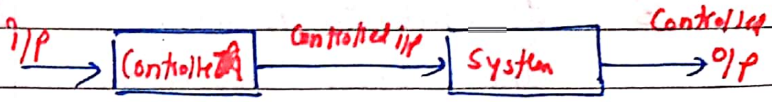
Basic concept of control System

System : System is a group of physical component which gives proper o/p for a given i/p.

Control System : Control sys. is a group of physical components which gives controlled o/p for a given i/p.





(a) System

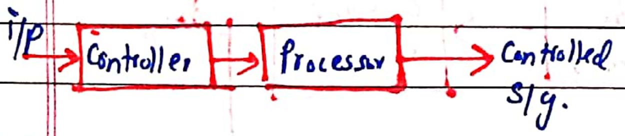
(b) Control System

⇒ Controlling Mechanism involve

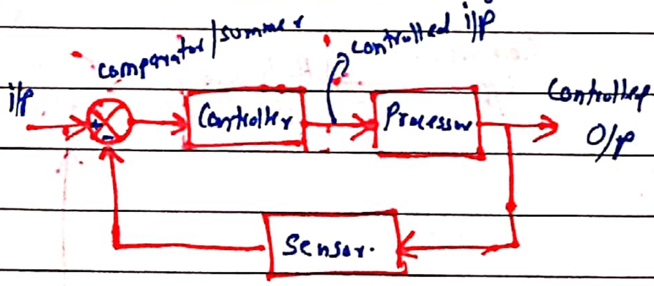
In general there are 2 types of control system

(a) Open loop Control System (Non-feedback C.S)

(b) Close loop control System. (Feedback control sy.)



(a)



(b)

→ Controlling mechanism is independent from the o/p

→ Controlled mechanism is depended on the o/p.

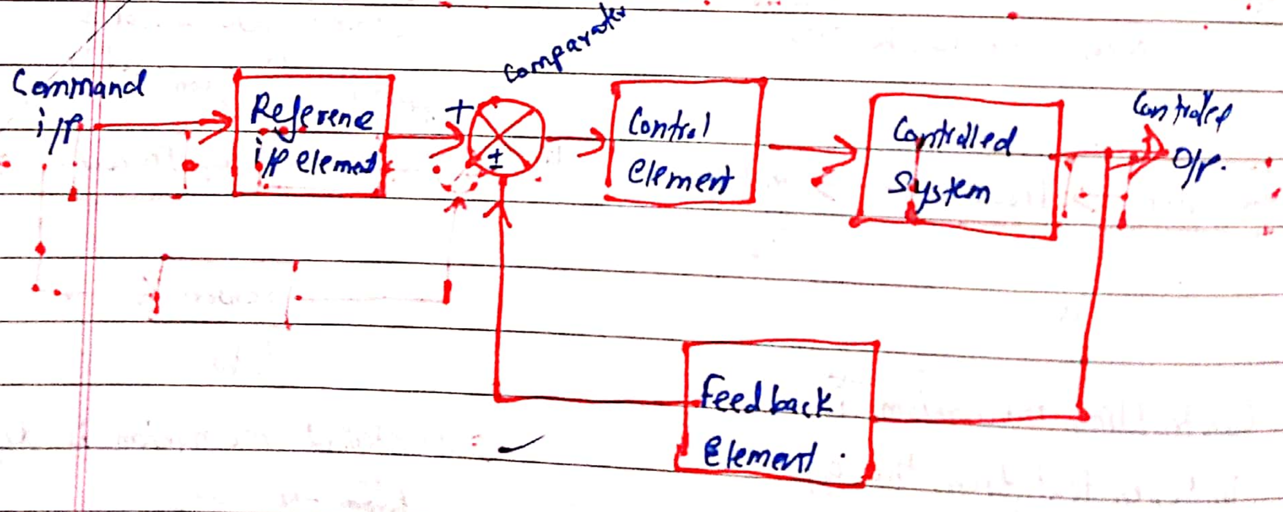
Example :→ Fan with regulator.
* Washing machine. (Normal)
* Normal iron box
* Traffic light system (in India.)

Example :→ A.C
→ Automatic Iron box

* Comparision b/w open loop & close loop:

# Open Loop System	Close loop System
(1) These are not reliable	(1) These are reliable
(2) It is easier to build	(2) It is difficult to build
(3) If calibration is good, they perform accurately.	(3) Accurate because of feedback
(4) Open Loop system are generally more stable. (because its location of pole is independent of any parameter)	(4) Less stable
(5) Optimization is not possible.	(5) Optimization is possible.

Basic elements or components of closed Loop system



(1) Command : The command is the externally produced i/p and independent of the feedback control system.

(2) Ref. i/p element: This produces the standard s/g proportional to the command.

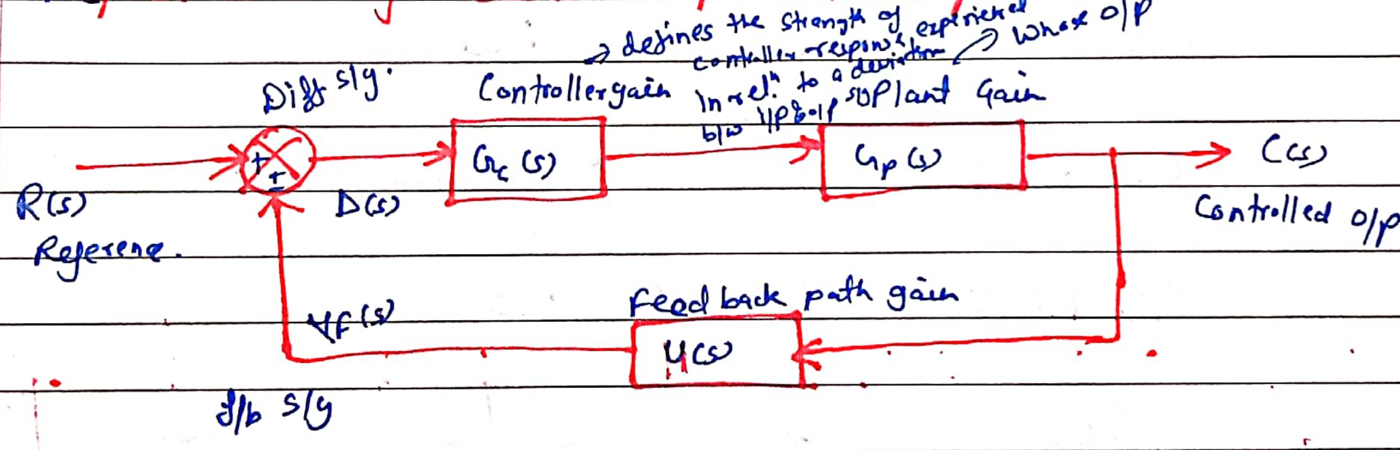
3. **Error detector / Comparator / Summer:** It receives the measured s/g & compare it with ref. i/p. The diff. of two s/gs produces the error s/g.

4. **Control element:** This regulate the o/p a/c to the s/g obtained from error detector.

5. **Controlled system:** This represents what we are controlling by feedback loop.

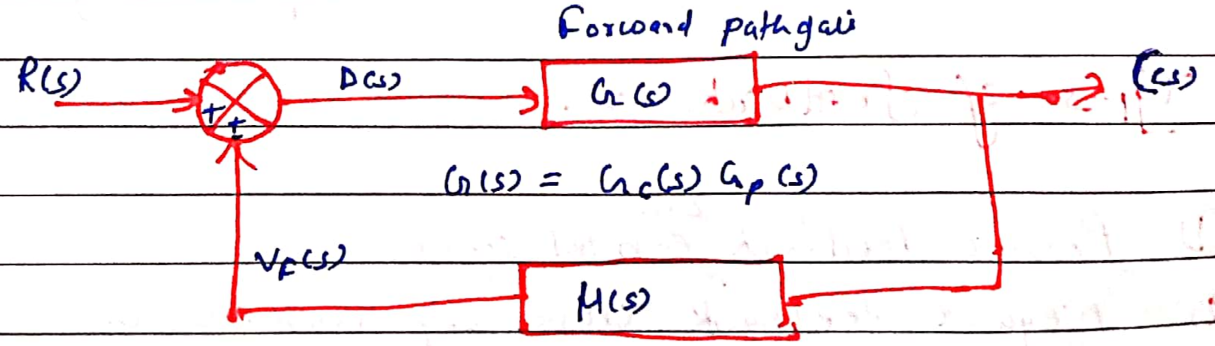
6. **Feed back element:** This element fed-back the o/p to the error detector for comparision with the ref. i/p.

Representation of closed loop control system:



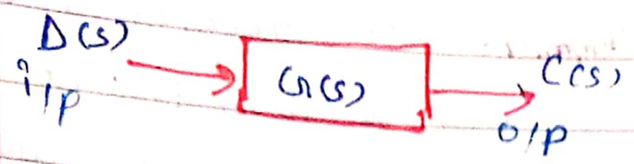
Controller gain → Tuning parameter

Process gain → Model parameter

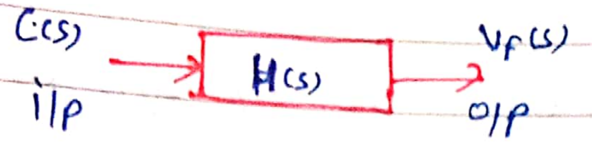


$$G(s) = G_c(s)\, G_p(s)$$

$$T(s) = \frac{C(s)}{R(s)} = \frac{o/p}{i/p} = \text{Transfer function} = \frac{G(s)}{[1 \mp H(s).G(s)]}$$



$$G(s) = \frac{C(s)}{D(s)} \quad -(1)$$



$$H(s) = \frac{V_f(s)}{C(s)} \quad -(2)$$

$$D(s) = R(s) \pm V_f(s) \quad -(5)$$

$$\left\{ \begin{array}{ll} C(s) = G(s).D(s) & -(3) \\ V_f(s) = H(s).C(s) & -(4) \end{array} \right.$$

So–

$$C(s) = G(s).D(s)$$

$$C(s) = G(s).[R(s) \pm V_f(s)] \quad -(6)$$

$$C(s) = G(s).[R(s) \pm H(s).C(s)]$$

$$C(s) = R(s)G.(s) \pm H(s).G(s).C(s)$$

$$C(s) \mp G(s).C(s) = R(s).G(s)$$

$$C(s)[1 \pm G(s)] = R(s)\,G(s)$$

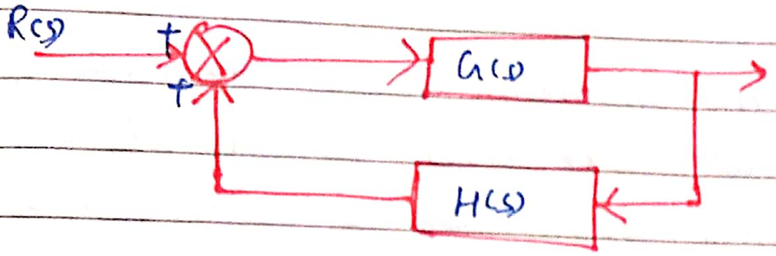
$$\frac{C(s)}{R(s)} = \frac{G(s)}{[1 \mp G(s)]} \Rightarrow \boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{(1 \mp G(s))}}$$

Close Loop control system.

Types of feedback c.s.

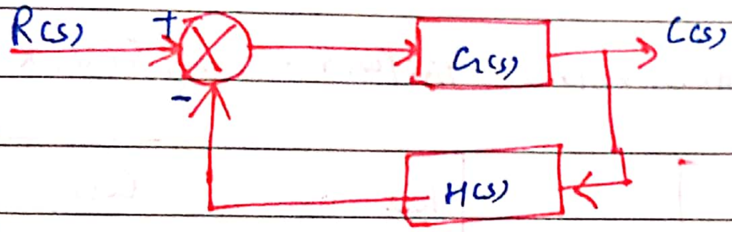
1. Positive feedback C.L control system
2. Negative feedback close loop control system.

① **Positive feedback** : The positive feedback add the reference i/p, (R(s)) and feedback output.



$$\boxed{T(s) = \frac{G(s)}{1 - G(s).H(s)}}$$

② **Negative Feedback** The negative feedback reduces the error b/w the reference i/p R(s) and the system output.



$$\boxed{T(s) = \frac{G(s)}{1 + G(s).H(s)}}$$

Keypoints :

① Positive feedback is used for designing of oscillator & multivibrator

② Negative f/b is used for designing of amplifiers.

③ Positive f/b decreases the stability of system.

④ Negative f/b ↑ the stability of system.

⑤ Positive f/b is also referred as regenerative f/b system

⑥ Neg. f/b is also referral as degenerative f/b system

7. If there is no information is given about feedback then by defaud we will considered it as a -ve f/b.

8. If there is no information about feedback element then by default wee will consider it as a unity f/b system ie. $H(s) = 1$. (gain is 1)

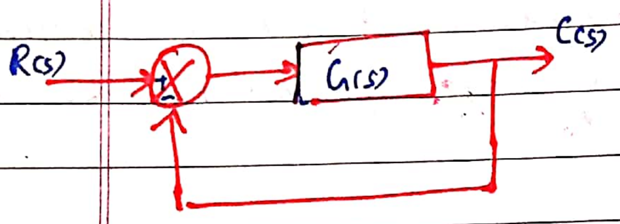


Fig: Negative Unity Feedback system

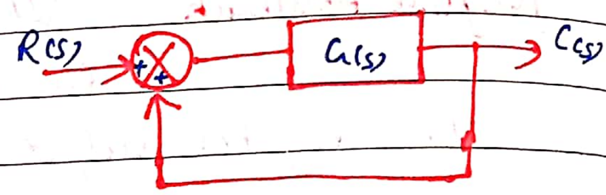


Fig: Positive unity feedback system

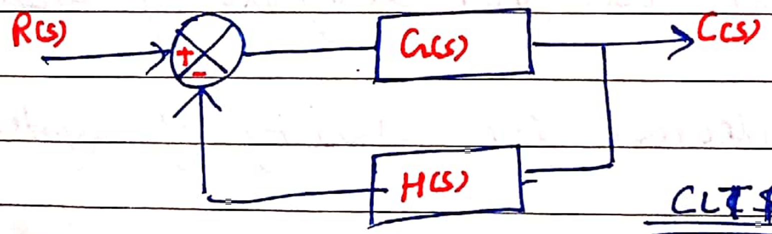
$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1-G(s)}$$

* Mostly we use -ve f/b unity f/b system for designing and analysis of control system.

Concept of open loop transfer funct^n: (OLTF)

↳

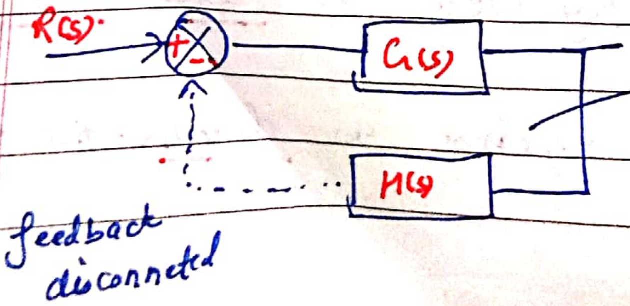


$$T(s) = \frac{G(s)}{1 + G(s)H(s)}$$

CLTF. (Close loop Transfer system)

$\boxed{\text{Product of } G(s) \ \& \ H(s) = \text{Open loop transfer system}}$

↳



→ $G(s) \cdot H(s)$ = O-loop transfer function

$$T(s) = \frac{G(s)}{[1 + G(s).H(s)]} = \frac{G(s)}{[1 + O.L.T.F]} = \frac{Num.(s)}{Den.(s)}$$

Roots of Num.(s) → Zeros.
Roots of Den.(s) → Poles

$\boxed{Den.(s) = 1 + O.L.T.F}$ ⇒ Characteristic Eq^n / Poles of T.F
or Roots of Den.(s)

Stability of Close loop transfer fun^n (CLTF) = f(poles)

= f(poles)
= f [roots of D(s)]
= f [fun^n of OLTF]

1. If we change OLTF
2. 1 + OLTF → change → Den.(s)
3. Roots of Den.(s) = Change
4. Poles of T(s) → Change
5. Stability of T(s) → Change
6. Performance of T(s) → Change.

In unity feedback system → H(s) = 1.

$$OLTF = G(s).H(s) = G(s)$$

$$T(s) = \frac{G(s)}{1 + G(s).1} = \frac{OLTF}{1 + OLTF}.$$

$$\boxed{C.L.T.F = \frac{OLTF}{1 + OLTF}}$$

Effect of Feedback:

$$T(s) = \frac{G(s)}{[1+G(s).H(s)]}$$

① Effect of feedback on overall Gain
② Effect of feedback on sensitivity
③ Effect of feedback on Stability.

On — Overall gain

① Overall gain of negative feedback close loop control sys is the ratio of '$G(s)$' and $(1+G(s)H(s))$.

- So the overall gain may ↑ or ↓ depending on the value of $[1+G(s).H(s)]$

② If the value of $[1+G(s).H(s)]$ is less than 1, than overall gain ↑. In this case $G(s).H(s)$ value is −ve because the gain of the feedback path is negative.

③ If the value of $[1+G(s)H(s)]$ is greater than 1 then the overall gain decreases. In this case, '$G(s)H(s)$' value is positive because of the gain of feedback path is positive.

On Sensitivity

① Sensitivity of the the overall gain of −ve f/b C.L.C.S.(T) to the variation in open loop gain (G) is defined as −

$$\text{Sensitivity} \Rightarrow S = \frac{\%\text{ change in } T\ (T.F)}{\%\text{ change in } G\ (\text{gain})} = \frac{\frac{\partial T}{T}}{\frac{\partial G}{G}} = \frac{\partial T}{\partial G} \times \frac{G}{T} \quad —①$$

By partial diffⁿ w.r.t G both side —

$$\frac{\partial T}{\partial G} = \frac{\partial}{\partial G}\left[\frac{G}{1+GH}\right] = \frac{[1+GH] - G \cdot H}{(1+GH)^2}$$

Put ① in

$$\frac{\partial T}{\partial G} = \frac{1}{(1+G.H)^2} \quad —②$$

$$\left\{ T = \frac{G}{1+GH} \quad \text{or} \quad \frac{G}{T} = 1+GH \quad —③ \right.$$

Put eqⁿ ③ in ①

$$S = \frac{\partial T}{\partial G} \cdot \frac{G}{T} = \frac{\partial T}{\partial G}\left[1+G.H\right]$$

$$S = \frac{1}{(1+G.H)^2} \cdot [1+G.H] = \frac{1}{(1+G.H)}$$

$$\boxed{S = \frac{1}{(1+G.H)}} \quad \text{Sensitivity}$$

* So, we got the ~~overall gain~~ sensitivity of close loop C.S as the reciprocal of (1+G.H). So, sensitivity may ↑ or ↓ depending on the value of (1+G.H)

→ If $(1+G.H)\downarrow$ $\qquad S\uparrow$

→ If $(1+G.H)\uparrow$ $\qquad S\downarrow$

Effect on Stability

A system is said to be stable, if its o/p is under control. Otherwise it is said to be unstable.

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$$T(s) = \frac{G(s)}{(1+G(s)\cdot H(s))} = \frac{Num.(s)}{Den.(s)}$$

If Den. value $= 0$ ie $1 + G(s)\cdot H(s) = 0$

$\rightarrow G(s).H(s) = -1$

then the o/p of the control system will be ∞.

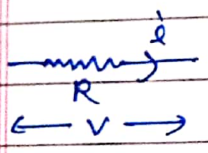
So control system become unstable.

↳ Modeling of Physical Systems (Electrical N/w & Mechanical System).

Mathematical modeling of electrical systems.

Basic elements of an electrical system are –

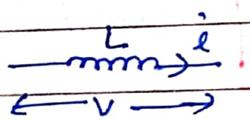
① Resistor



$\boxed{V = i.R}$

or $\boxed{i = (V/R)}$

② Inductor

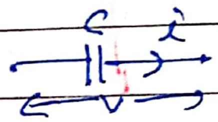


$[L = d\phi/dt]$

$\boxed{V = L\, di/dt}$

$\boxed{i = \frac{1}{L}\int V.dt}$

③ Capacitor.

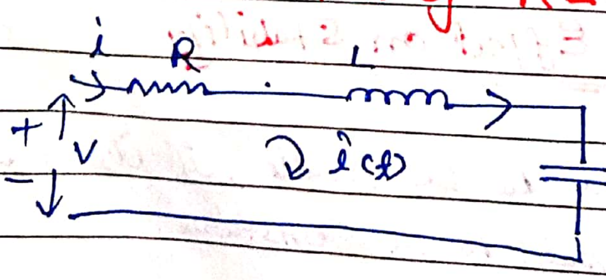


$[q = C.V$ or $C = q/V]$

$\boxed{i = C\, dv/dt}$

$\boxed{V = \frac{1}{C}\int i\, dt}$

Series combination of RLC.



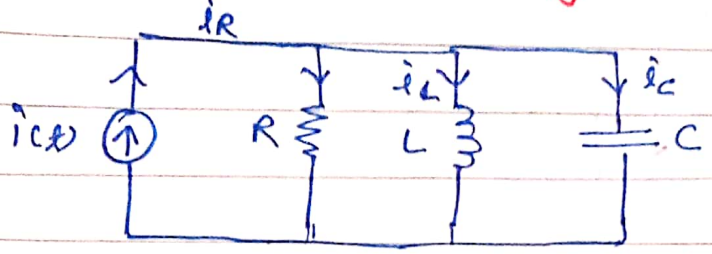
KVL

$$V = R\,i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i\,dt$$

$i = dq/dt$

$$\boxed{V = R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{1}{C}q}$$

Parallel combination of RLC



By Krichoff current law -

$$i(t) = i_R + i_L + i_c$$

$$\left\{ \begin{aligned} i_R &= v/R \\ i_L &= 1/L \int v\,dt \\ i_c &= C\frac{dv}{dt} \end{aligned} \right.$$

$$i(t) = \frac{v}{R} + \frac{1}{L}\int v\,dt + C\frac{dv}{dt}$$

$$\left\{ v = \frac{d\phi}{dt} \right.$$

$$\boxed{i(t) = \frac{1}{R}\frac{d\phi}{dt} + \frac{1}{L}\phi + C\frac{d^2\phi}{dt^2}}$$ → Differential eqn model

Transfer function model.

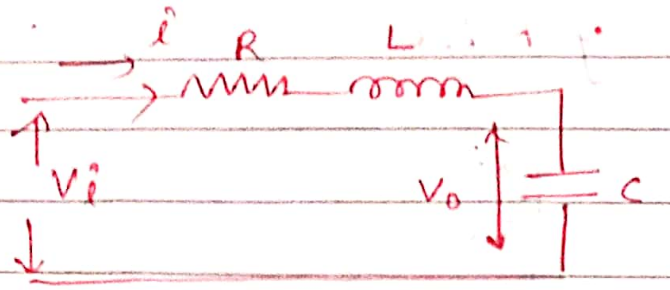
For

$$V = R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{q}{C}$$ { Apply L.T on this.

$$\Rightarrow V(s) = R\,s\,q(s) + L\left[s^2 q(s)\right] + \frac{q(s)}{C}$$

$$\Rightarrow V(s) = q(s)\left[Rs + Ls^2 + \frac{1}{C}\right]$$

Mathematical modeling of a control system is done to develop a set of mathematical equation, so that the system can be simulated & analysed.



$$V_i = Ri + L\frac{di}{dt} + V_o \quad -①$$

$$\left\{ i = C\frac{dV_o}{dt} \right\} - ②$$ current across capacitor.

Put ② in ①

$$V_i = RC\frac{dV_o}{dt} + LC\frac{d^2V_o}{dt} + V_o$$

Divide the above equation by LC –
we get –

$$\frac{1}{LC} \cdot V_i = \frac{R}{L}\frac{dV_o}{dt} + \frac{d^2V_o}{dt} + \frac{1V_o}{LC}$$

$$\frac{1}{LC} V_i(s) = \frac{R}{L} s V_o(s) + s^2 V_o(s) + \frac{1}{LC} V_o(s)$$

$$\frac{1}{LC} V_i(s) = V_o(s)\left[\frac{R}{L}s + s^2 + \frac{1}{LC}\right]$$

$$\boxed{T(s) = \frac{V_o(s)}{V_i(s)} = \frac{(1/LC)}{\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}}$$

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→ All physical systems are Non-linear.

→ Linearisation is necessary to design & analyse the C.S.

→ Linear system can be represented in s-domain as Transfer funct and in time domain as linear differential equation.

→ Finally, all the tools such as Bode plot, Root locus, Nyquist plot etc can be used.

Mechanical Systems.

Based on Type of motion, mechanical systems are classified into two types.

① Translational Mechanical sys. ② Rotational Mechanical system.

Translational: ↓ moves along St. line

Rotational: ↓ moves along an axis.

Elements: Mass, Spring, Damper.

List of symbols used-

x = displacement, m

$v = dx/dt$ = velocity = m/sec

$a = dv/dt = d^2x/dt^2$ = accleration; m/sec^2

f = applied force, N

f_m = opposing force by mass.

f_b = opposing force by elasticity

Elements: moment of inertia of mass, Dashpot, ~~torsional~~ torsional spring

List of symbols used.

θ = angular displacement, rad.

$\frac{d\theta}{dt}$ = angular velocity, rad/sec

T = applied Torque, NM

$\frac{d^2\theta}{dt^2}$ = angular acc^n, rad/sec^2

J = moment of inertia kg-m^2/rad

B = rotational friction coeff. N-M/(rad/sec)

K = stiffness of spring N-m/rad

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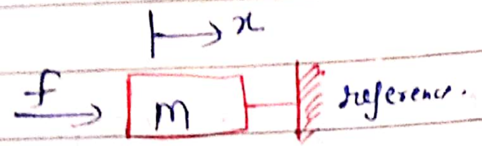
① **Mass:** If a force is applied on Mass M, then it is opposed by an opposing force due to mass.

$$f_m \propto a$$

$$f_m = Ma = M\frac{d^2x}{dt^2}$$

{ direction of F & x = same }

$$\left\{ a = \frac{dv}{dt} = \frac{d^2x}{dt^2} \right.$$



A/c to Newton's law —

applied force = opposing force by mass.

f = applied force.
f_m = opposing force.
a = acc^n.

$$f = f_m$$

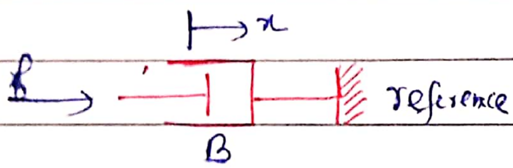
$$f = f_m = M\frac{d^2x}{dt^2}$$

② **Dashpot / Damper:** If a force is applied on damper B, it is opposed by an opposing force due to friction.

$$f_b \propto v$$

{ $v \to$ velocity; B = frict^n coffiid }

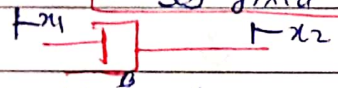
$$f_b = Bv$$



A/c to Newton 2nd law —

When one side is fixed to ref.

$$\boxed{f = f_b = B\frac{dx}{dt}}$$

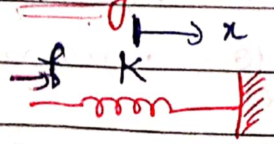


When both side is not fixed.

$$f_b \propto \frac{d}{dt}(x_1 - x_2)$$

$$\boxed{f_b = B\frac{d}{dt}(x_1 - x_2)}$$

③ **Spring** If a force is applied on spring K, it is opposed by opposing force due to elasticity.

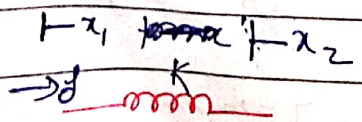


When 1 end is fixed to ref.

$$f_K \propto x$$

{ For spring Restoring force = opposing force }

$$f_K = Kx = f$$



{ K = elasticity constt or spring constant }

When 2 ends are free.

$$f_K \propto (x_1 - x_2)$$

$$f_K = K(x_1 - x_2)$$

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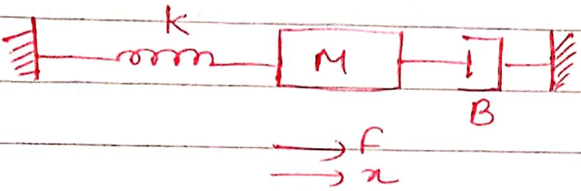
Mathematical model of spring-Mass-Damper arrangement



$$F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx \longrightarrow \text{Differential equation}$$

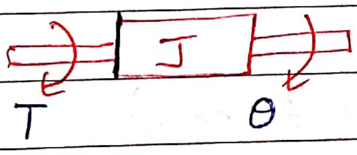
Taking L·T both side -

$$F(s) = Ms^2X(s) + BsX(s) + KX(s)$$

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{Ms^2+Bs+K}} \longrightarrow \text{Transfer function}$$

Rotational mechanical System :

① Mass



J = ideal mass element

Elasticity & frictional coff are negligible.

T_J = opposing torque

$$T_J \propto \frac{d^2\theta}{dt^2}$$

$$T = T_J = J\frac{d^2\theta}{dt^2}$$

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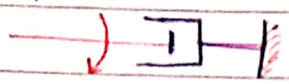
② Dashpot

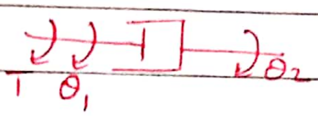
→ If dashpot is fixed it to a reference then the opposing torque due to its friction of dashpot $T_b \propto \frac{d\theta}{dt}$



$$T_b \propto \frac{d\theta}{dt}$$

$$\boxed{T_b = B\frac{d\theta}{dt}}$$

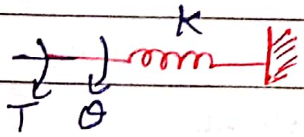
→ If 2 ends are not fixe.



$$T_b \propto \frac{d}{dt}(\theta_1 - \theta_2)$$

$$\boxed{T = T_b = B\frac{d(\theta_1 - \theta_2)}{dt}}$$

③ Spring

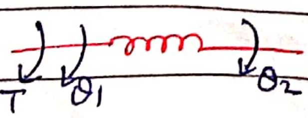
→ 

(1 end is fixed.)

$$T_K \propto \theta$$

$$T_K = K\theta$$

$$\boxed{T = T_K = K\theta}$$

→ 

(If both ends are free)

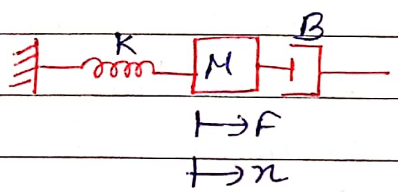
$$\boxed{T = T_K = K(\theta_1 - \theta_2)}$$

Analogous System.

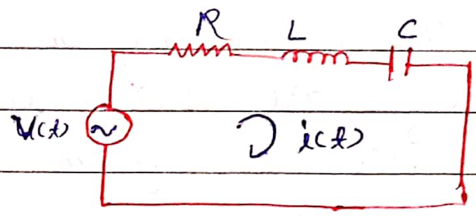
① Force – Voltage Analogy
② Force – Current analogy.

Two systems are said to be analogous to each other, if following conditions are satisfied.

↳ The two systems are physically different.
↳ Differential eqⁿ modelling of these two systems are same.



System 1



System 2.

$$F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx \quad -①$$

[Second order diff. eqⁿ in x]

Compare

By KVL –

$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

Put $i = dq/dt$

$$V = R\,dq/dt + L\,d^2q/dt^2 + q/C \quad -②$$

[Second order diff. eqⁿ in q]

$$V = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} \quad -③$$

Date
Page

Translational Mech system	Electrical system.
Force (F)	Voltage (V)
Mass (M)	Inductance (L)
Friction coeff (B)	Resistance (R)
Spring constant (K)	Reciprocal of C ($\frac{1}{C}$)
Displacement (x)	Charge (q)
Velocity (v)	Current (i)

Keypoints:

1. In mechanical system, the elements having same velocity are said to be in series in electrical system the elements in series will have same current.

2. Each node (mass) in the mechanical system corresponds to a closed loop in electrical system

3. Number of ~~masses~~ meshes in electrical system is equal to the no. of masses in mass in mech. system

4. The elements connected b/w two masses in mechanical system is represented as a common element b/w two meshes in electrical system.

Scanned with CamScanner

Example :

$F \rightarrow V$

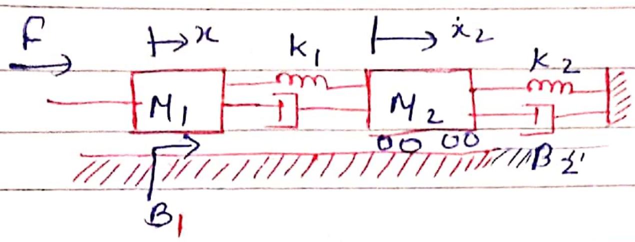
$M \rightarrow L$

$B \rightarrow R$

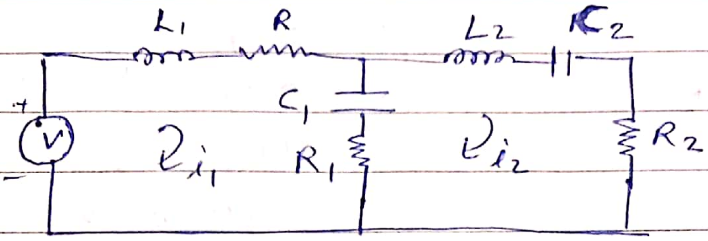
$K \rightarrow 1/C$

$x \rightarrow q$

(1)

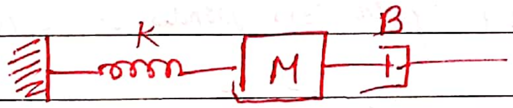


Sol^n



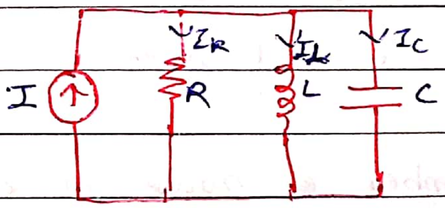
(2) FORCE - CURRENT ANALOGY

In this analogy, the mathematical equations of the translational mechanical system are compared with the nodal equations of the electrical system.



System1

$$\boxed{F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx} \quad -(1)$$



System 2

Applying nodal analysis (KCL)

$$I = I_R + I_L + I_C$$

$$I = \frac{V}{R} + \frac{1}{L}\int V \cdot dt + C\frac{dV}{dt}$$

$$I = \frac{1}{R}\frac{d\phi}{dt} + \frac{1}{L}\phi + C\frac{d^2\phi}{dt^2} \qquad \left\{ V = \frac{d\phi}{dt} \right.$$

$$\boxed{I = C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + \frac{1}{L}\phi} \quad -(2)$$

Translational Mechanical System	Electrical System.
Force (F)	Current (I)
Mass (M)	Capacitance (C)
Friction coff. (B)	Reciprocal of R. ($1/R$)
Spring constant (K)	Reciprocal of L ($1/L$)
Displacement (x)	Magnetic flux (ϕ)
Velocity (v)	Voltage (v)

Keypoints

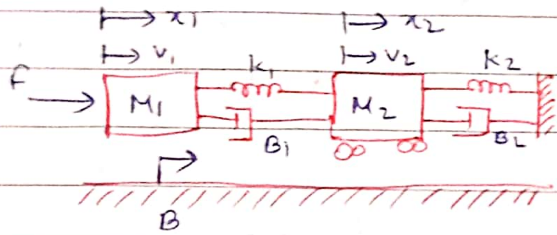
→ In mechanical system the elements in parallel will have same force. & similarly ~~parallels~~ in electrical system parallel elements will have same voltage

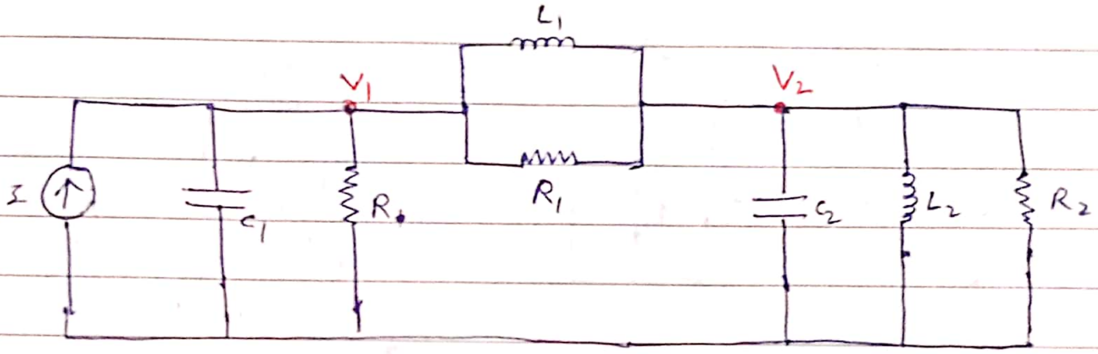
→ Each node (mass) in mechanical system corresponds to a node in electrical system.

→ Number of nodes in electrical system is equal to number of nodes in mechanical system.

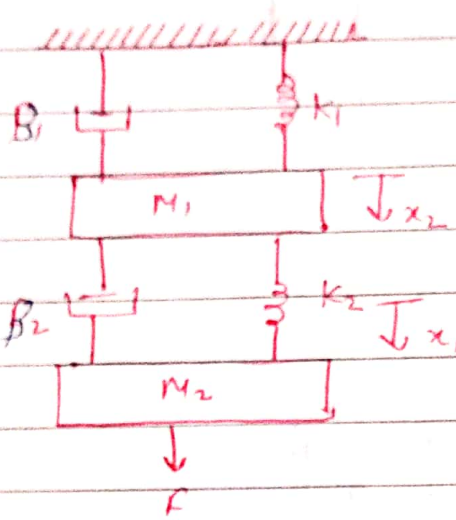
→ The element connected b/w two nodes in mechanical system is represented as common element b/w nodes in electrical system.

Example :

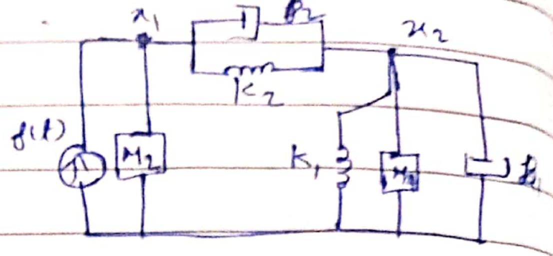




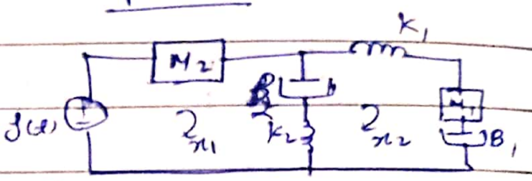
Q Write down the system equations for the given mechanical systems.



F-C —

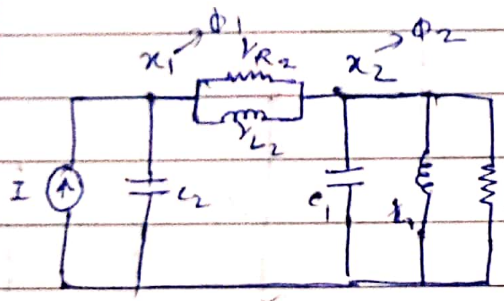


F.V —

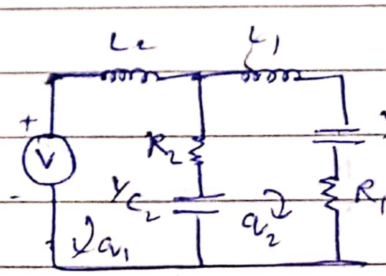


Sol^n

Force-current



Force-voltage

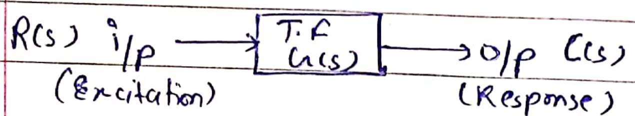


BLOCK DIAGRAM.

Pictorial representation of the components integrated in a control system and the flow of signals.

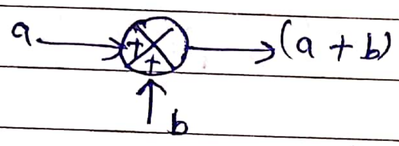
Block diagram elements are —

① Block —

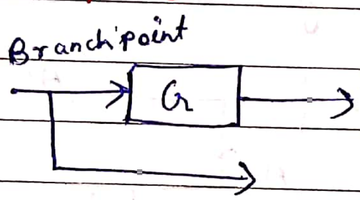


$$G(s) = \frac{O/P}{I/P} = \frac{C(s)}{R(s)}$$

② Summing points —



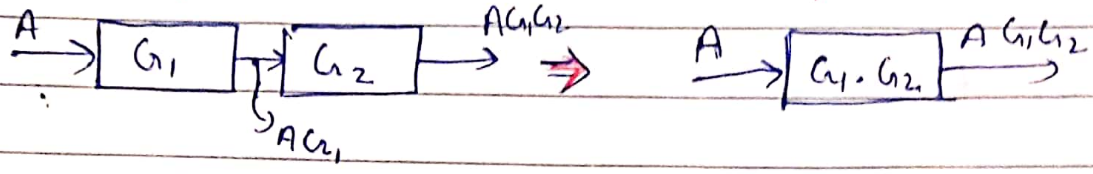
③ Branch point —



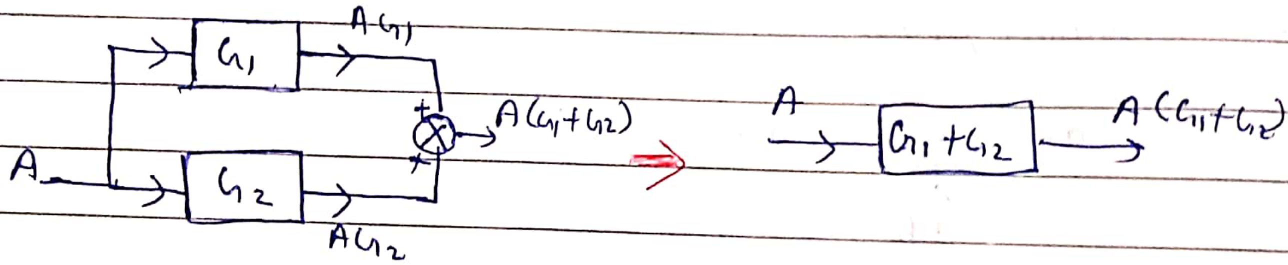
BLOCK DIAGRAM REDUCTION RULES

Transfer function : $H(s) = \dfrac{C(s) \rightarrow o/p \ (L.T)}{R(s) \rightarrow i/p \ (L.T)}$

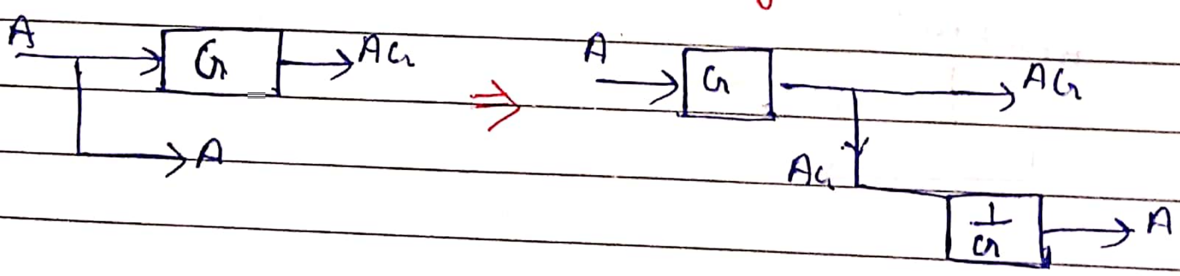
Rule 1 Combining the blocks in cascade / series.



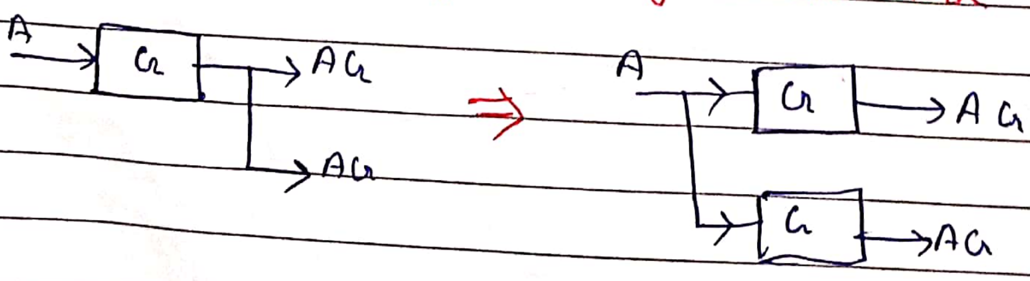
Rule 2 Combining the parallel blocks.



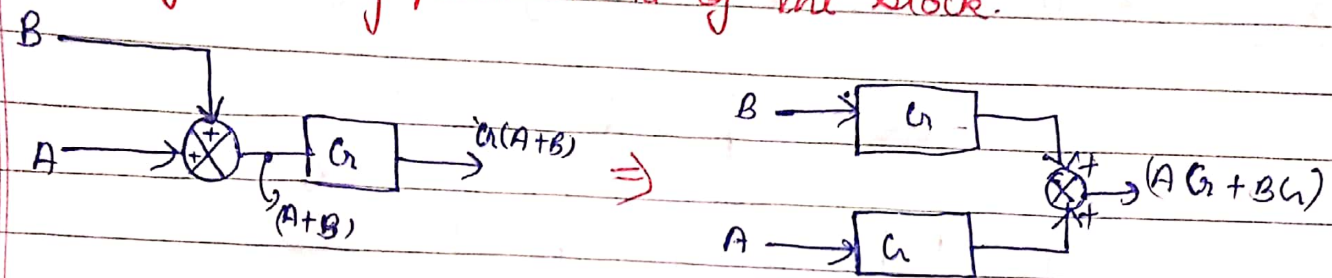
Rule 3 Moving the branch point ahead of the block.



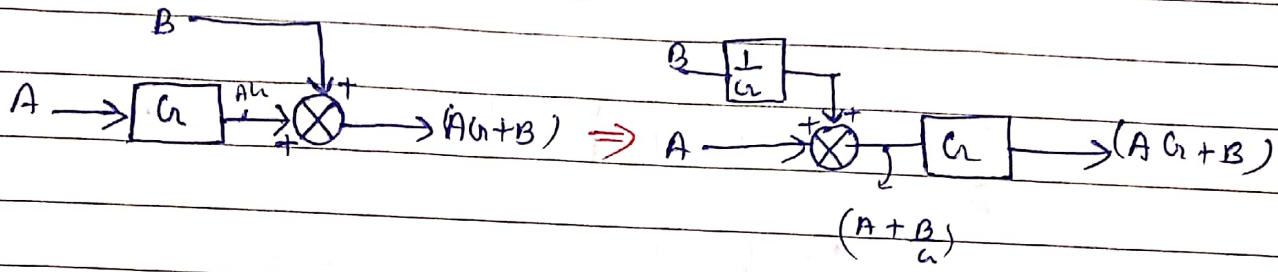
Rule 4 Moving the branch point before the block.



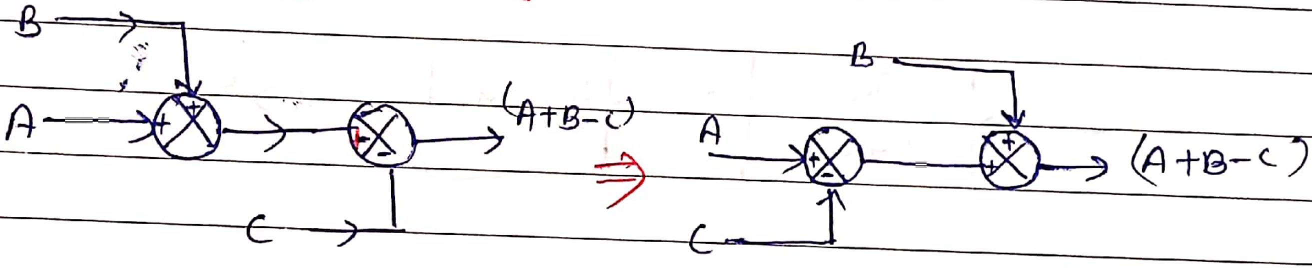
Rule 5 — Moving summing point ahead of the block.



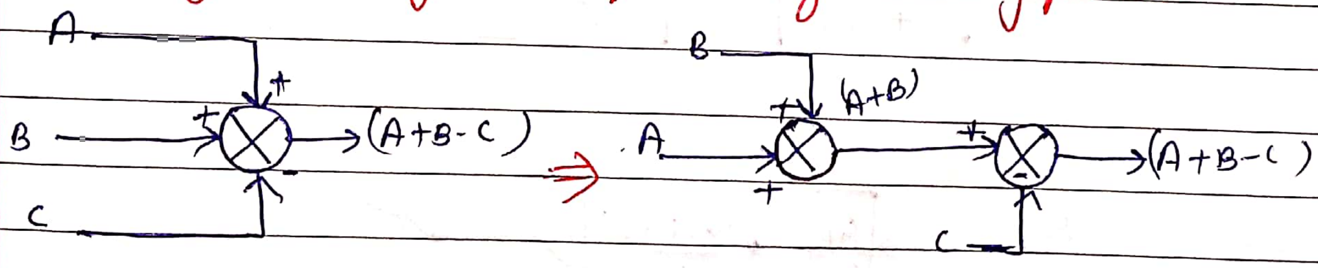
Rule 6 — Moving summing point before the block



Rule 7 — Interchanging summing points.

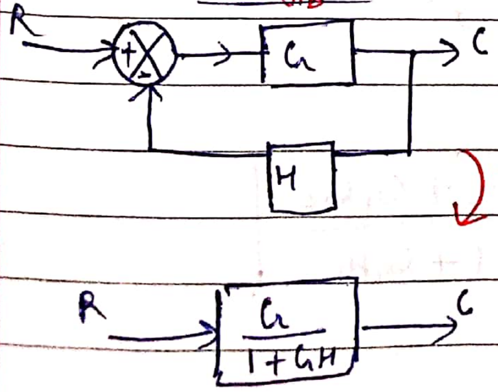


Rule 8 — Splitting summing points / combining summing points.

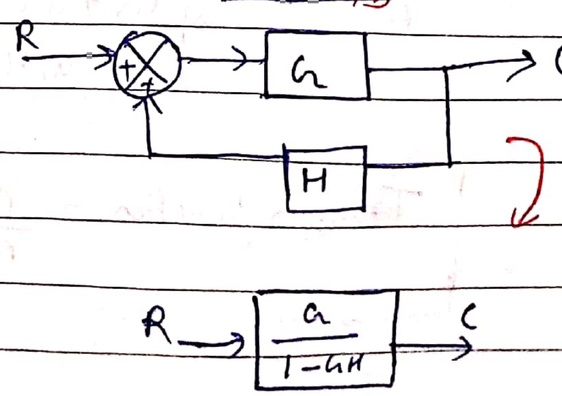


Rule 9 — Elimination of feedback.

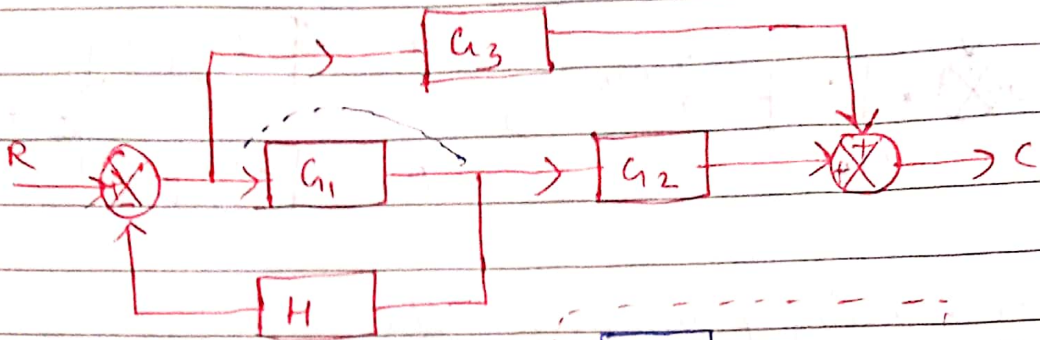
−ve f/b



+ve f/b

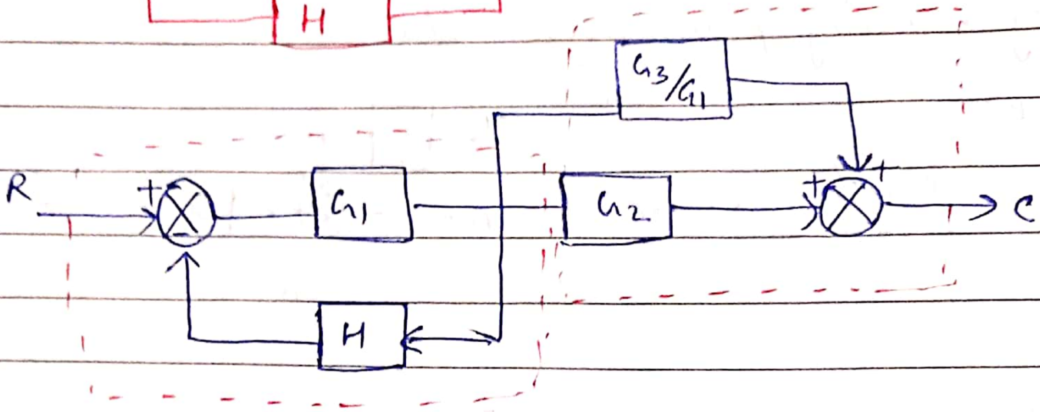


Q1 Reduce the block diagram & find the T.F

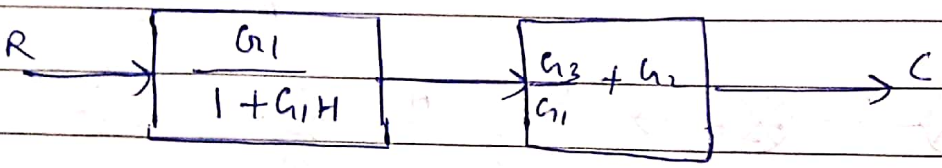


Sol^n

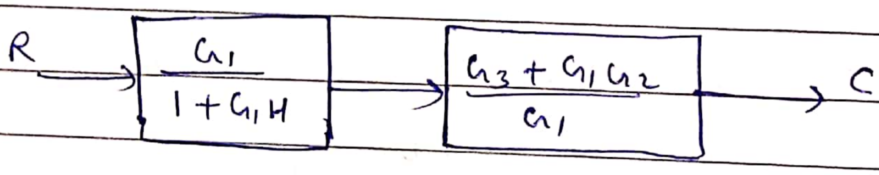
$\Rightarrow$



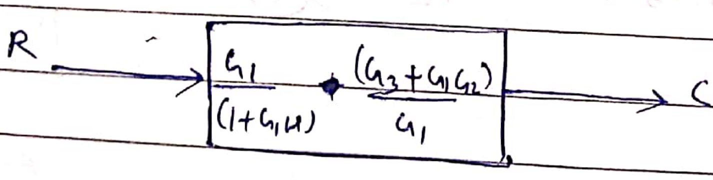
$\Rightarrow$



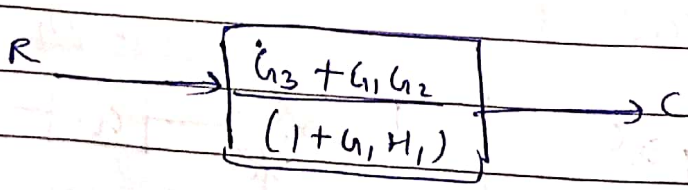
$\Rightarrow$



$\Rightarrow$



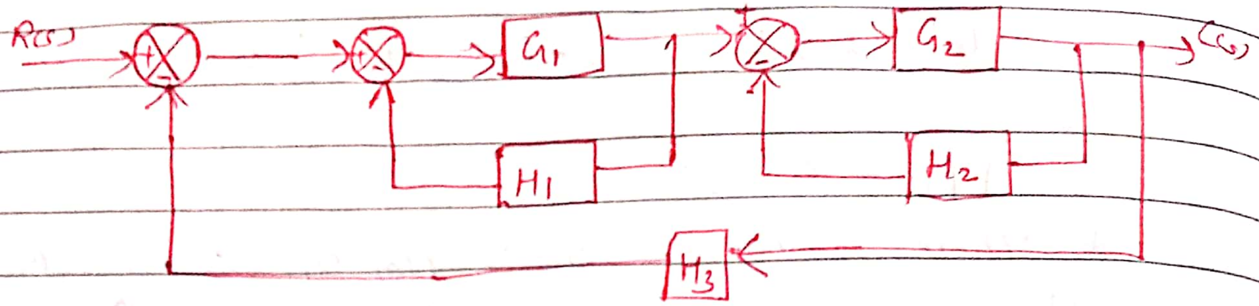
$\Rightarrow$



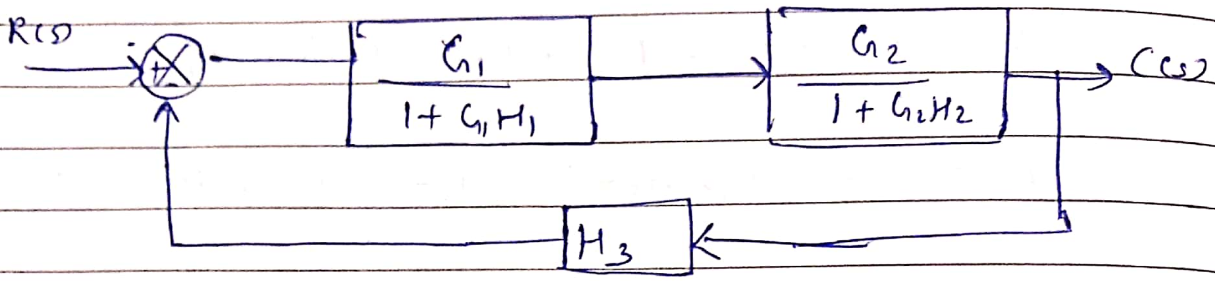
$$\boxed{\text{Transfer function} = \frac{C}{R} = \frac{G_3+G_1G_2}{1+G_1H_1}}$$

Block dig: Que 2.

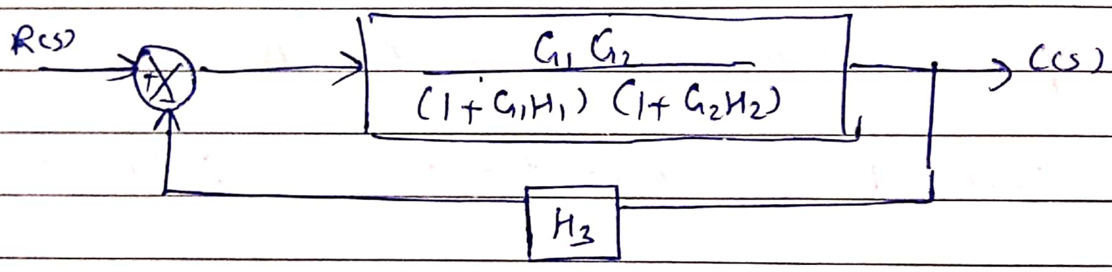
Que2



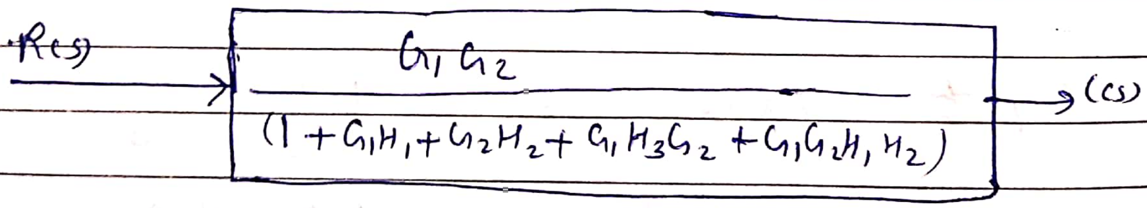
⇒ I



II



III



$$T(s) = \frac{C(s)}{R(s)} = \frac{G_1 G_2}{(1+G_1H_1+G_2H_2+G_1G_2H_3+G_1G_2H_1H_2)}$$

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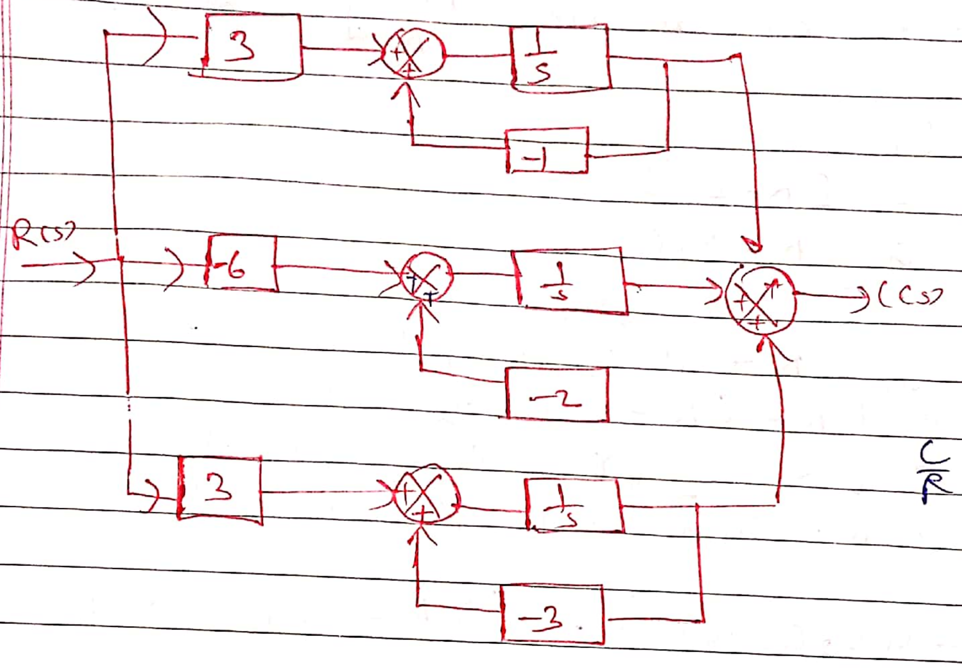
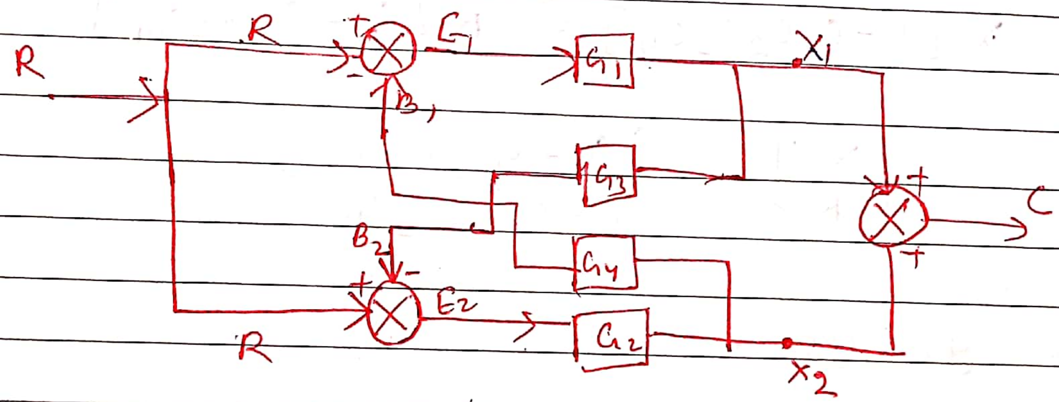
Ques



Find $\frac{C(s)}{R(s)}$

$$\frac{C}{R} = \frac{2}{s+1} + \frac{6+3}{(s+2)(s+3)}$$

Q



$$E_1 = R - B_1 \quad -(1)$$
$$E_2 = R - B_2 \quad -(2)$$
$$X_1 = E_1 G_1 \quad -(3)$$
$$X_2 = E_2 G_2 \quad -(4)$$
$$B_1 = X_2 G_4 \quad -(5)$$
$$B_2 = X_1 G_3 \quad -(6)$$

$$C = X_1 + X_2 \quad -(7)$$

Divide eqn (7) by (R) both side to get T.F (c)

$$\frac{C}{R} = \frac{X_1}{R} + \frac{X_2}{R}$$

$$T.F = \frac{X_1}{R} + \frac{X_2}{R}$$

First we have to find $\left[\frac{X_1}{R}\right]$ So—

$$X_1 = E_1 G_1$$
$$X_1 = G_1 [R - B_1]$$
$$X_1 = G_1 [R - X_2 G_4]$$
$$X_1 = G_1 [R - E_2 G_2 G_4]$$
$$X_1 = G_1 [R - (R - B_2) G_2 G_4]$$
$$X_1 = G_1 [R - (R - X_1 G_3) G_2 G_4]$$
$$X_1 = G_1 [R - G_2 G_4 R + G_2 G_3 G_4 X_1]$$
$$X_1 = [G_1 R - G_1 G_2 G_4 R + G_1 G_2 G_3 G_4 X_1]$$

$$[X_1 - G_1 G_2 G_3 G_4 X_1] = [G_1 R - G_1 G_2 G_4 R]$$

$$X_1 [1 - G_1 G_2 G_3 G_4] = R [G_1 - G_1 G_2 G_4]$$

$$\left[\frac{X_1}{R}\right] = \frac{[G_1 - G_1 G_2 G_4]}{1 - G_1 G_2 G_3 G_4}$$

To find $\left(\frac{X_2}{R}\right)$ —

$$X_2 = E_2 G_2$$
$$X_2 = G_2 [R - B_2]$$
$$X_2 = G_2 [R - X_1 G_3]$$
$$X_2 = G_2 [R - E_1 G_1 G_3]$$
$$X_2 = G_2 [R - G_1 G_3 (R - B_1)]$$
$$X_2 = G_2 [R - G_1 G_3 (R - X_2 G_4)]$$
$$X_2 = G_2 [R - G_1 G_3 R + G_1 G_3 G_4 X_2]$$
$$X_2 = [G_2 R - G_1 G_2 G_3 R + G_1 G_2 G_3 G_4 X_2]$$
$$X_2 [1 - G_1 G_2 G_3 G_4] = R [G_2 - G_1 G_2 G_3]$$

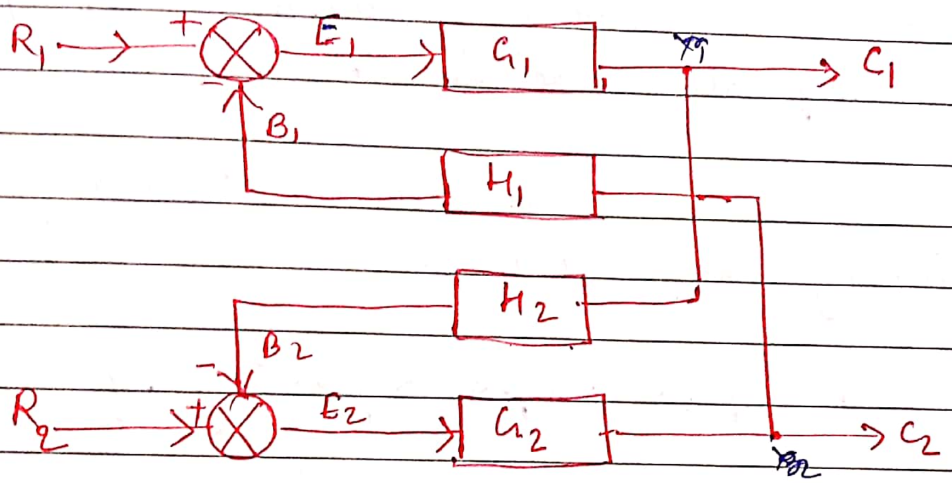
$$\frac{X_2}{R} = \frac{G_2 - G_1 G_2 G_3}{[1 - G_1 G_2 G_3 G_4]}$$

$$T.F = \frac{C}{R} = \frac{X_1}{R} + \frac{X_2}{R}$$

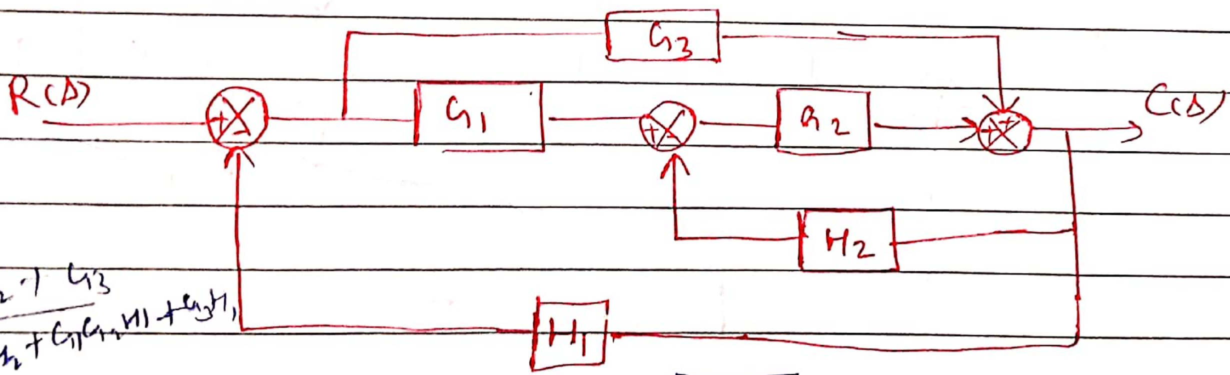
$$\boxed{T.F = \frac{C}{R} = \frac{G_1 + G_2 - G_1 G_2 G_4 - G_1 G_2 G_3}{[1 - G_1 G_2 G_3 G_4]}}$$

Ques Determine the transfer function $\left(\frac{C_1}{R_1}\right), \left(\frac{C_2}{R_2}\right), \left(\frac{C_1}{R_2}\right)$ & $\left(\frac{C_2}{R_1}\right)$ from the block diagram given below.

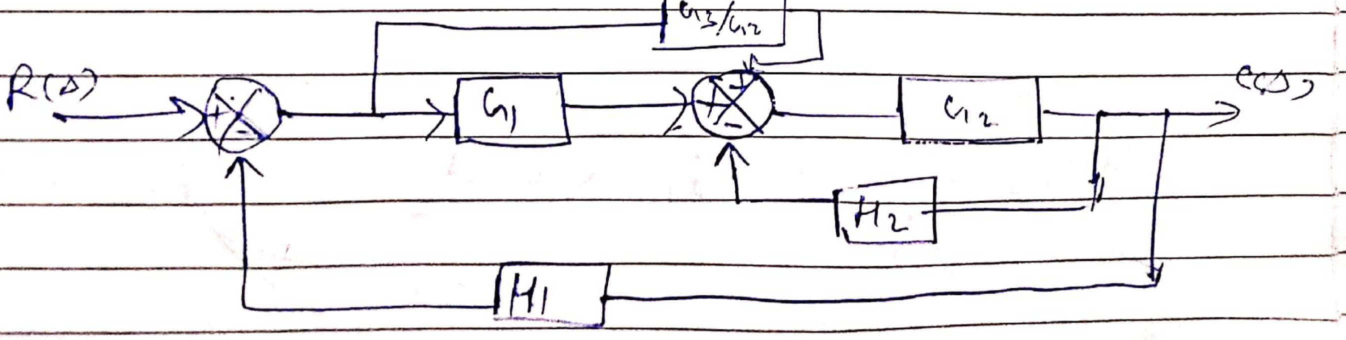
$\Rightarrow$



Ques)



$$\frac{C}{R} = \frac{G_1 G_2 + G_3}{1 + G_2 H_2 + G_1 G_2 H_1 + G_3 H_1}$$



SIGNAL FLOW GRAPH (SFG)

→ SFG is a diagram which represents a set of simultaneous equations.

→ This method was developed by S.J Mason. This method doesn't require any reduction technique.

→ It consist of nodes and these nodes are connected by a directed line called branches.

→ Every branch has an arrow which represents the flow of signal.

MASON'S GAIN FORMULA -

The overall Transfer function (transmittance) b/w i/p node & o/p node of a SFG is given by Mason's gain formula.

$$\boxed{T = \frac{\sum^{1} g_K \Delta_K}{\Delta}} \quad \text{or} \quad \boxed{T = \frac{\sum^{2} P_K \Delta_K}{\Delta}}$$

where -

T = Transfer function

Δ = 1 - [sum of all individual loop gain] + [sum of all possible gain product of two non-touching loops] - [sum of all possible gains products of 3 non-touching loops] + - - - - -.

P_K / g_K = gain of the K^{th} forward path

Δ_K = the parts of Δ not touching the k^{th} forward path.

Method to obtain Signal flow Graph.

① From the system equations
② From the Block diagram.

① From Block diagram:

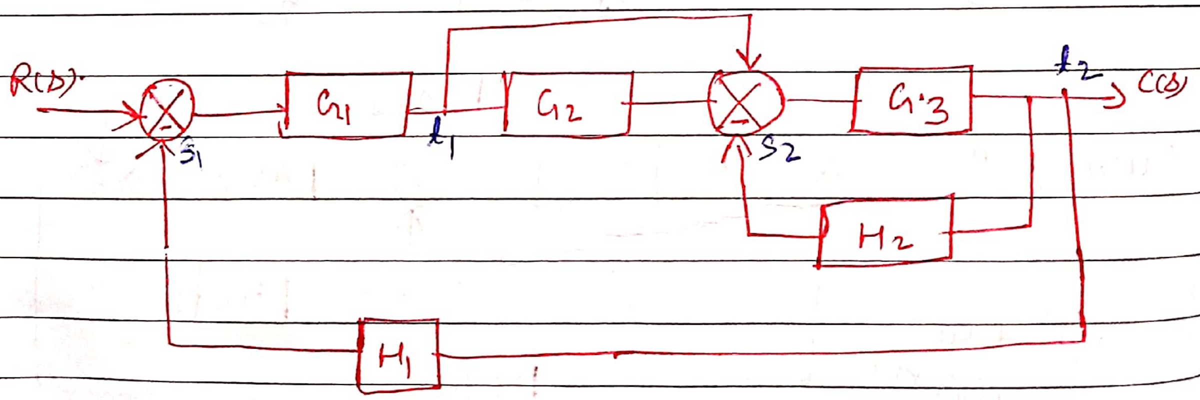
Process:

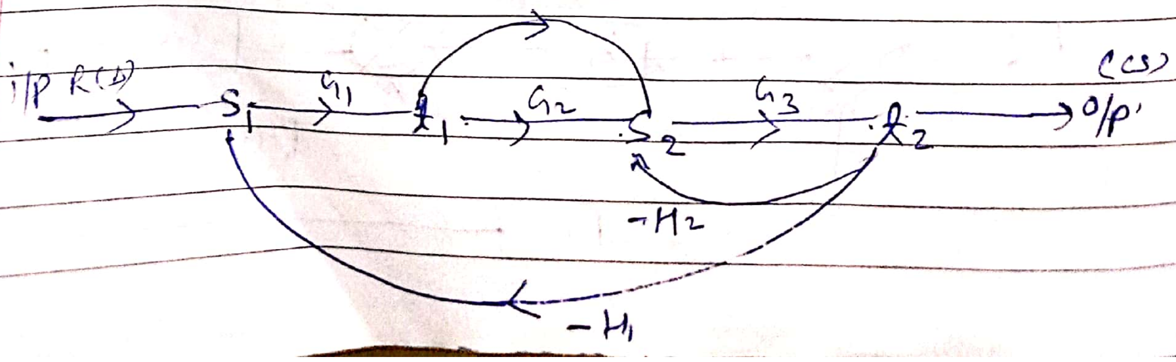
→ Name all the summing points & take off points in the block diagram.

→ Represent each summing point & take-off point by a separate node in SFG.

→ Convert them by the branches instead of blocks indicating block T.F as the branch gain.

→ Show the i/p & output separately if required.





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From the system equations:

→ Represent each variable by a separate node.

→ Use the property that value of the variable represented by a node is an algebric sum of all the s/g entering at that node, to simulate the equations.

→ Coefficients of the variable in the eqn are to be represented as the branch gain, joining the nodes in sfg

→ Show the i/p & o/p separately to complete sfg.

Example: $V_1 = 2V_i + 3V_2$ —(1)

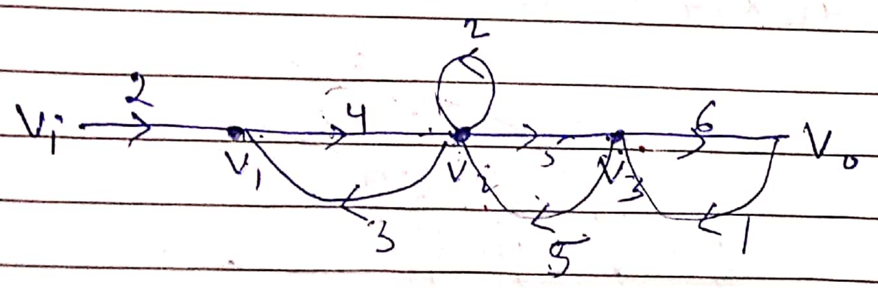
$V_2 = 4V_1 + 5V_3 + 2V_2$ —(2)

$V_3 = 5V_2 + V_0$ —(3)

$V_0 = 6V_3$ —(4)

Where V_i = i/p & V_0 = o/p

System variables → V_1, V_2, V_3



Eqn (1)
Starting point V_i
Ending point V_0.

$V_i \xrightarrow{2} V_1$

$V_2 \xrightarrow{3} V_1$

$V_1 \xrightarrow{4} V_2$

$V_3 \xrightarrow{5} V_2$

$V_2 \xrightarrow{2} V_2$

$V_2 \xrightarrow{5} V_3$

$V_0 \xrightarrow{1} V_3$

$V_3 \xrightarrow{6} V_0$

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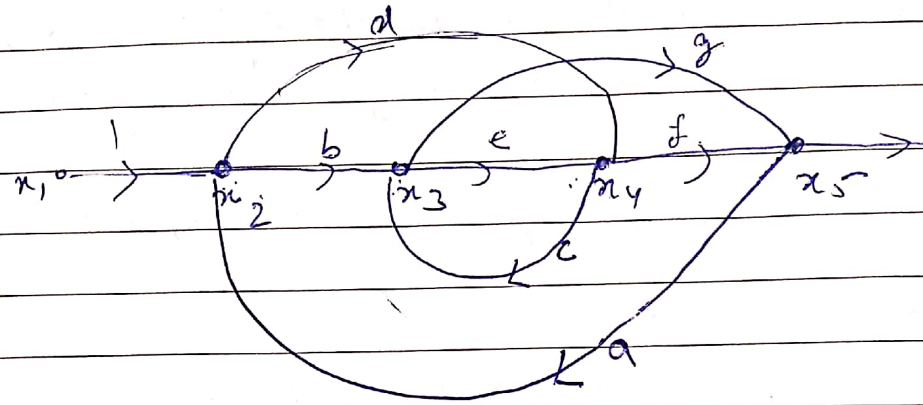
Q Draw the s/g flow graph for the following set of eq^n.

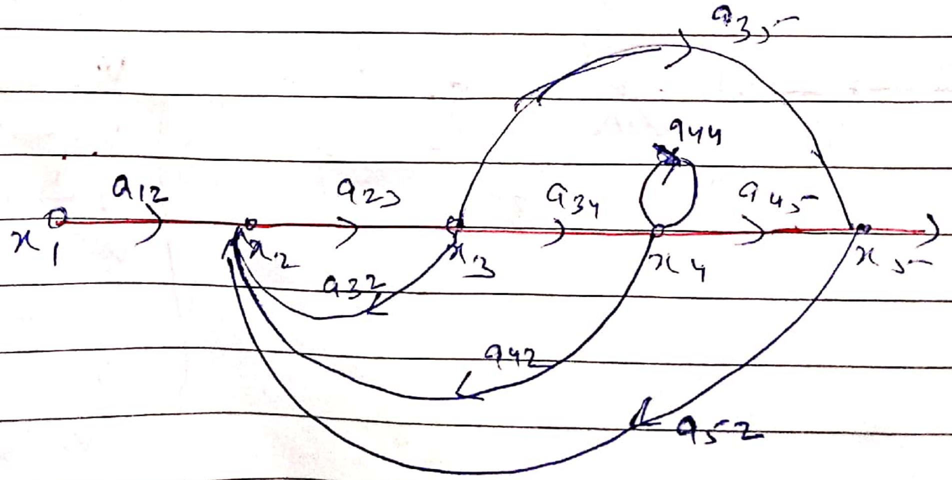
$$x_2 = x_1 + a x_5$$
$$x_3 = b x_2 + c x_4$$
$$x_4 = d x_2 + e x_3$$
$$x_5 = f x_4 + g x_3$$
$$x_6 = x_5$$

$x_1 \xrightarrow{1} x_2$
$x_5 \xrightarrow{a} x_2$
$x_2 \xrightarrow{b} x_3$
$x_4 \xrightarrow{c} x_3$
$x_2 \xrightarrow{d} x_4$
$x_3 \xrightarrow{e} x_4$
$x_4 \xrightarrow{f} x_5$
$x_3 \xrightarrow{g} x_5$
$x_5 \xrightarrow{1} x_6$

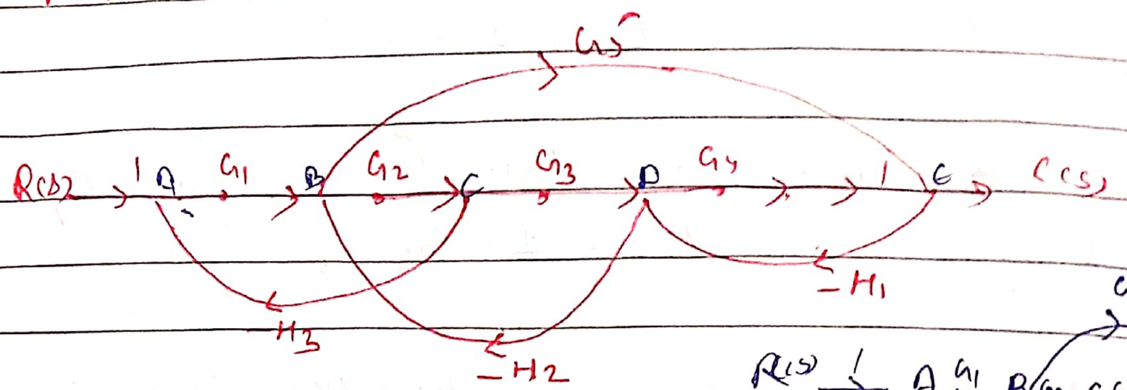


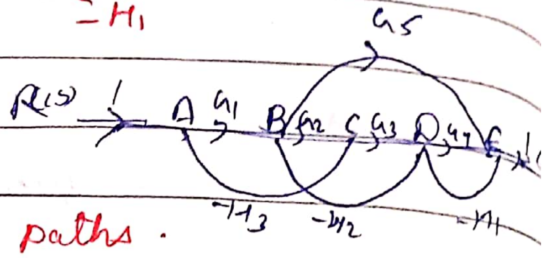
Q

$$x_2 = a_{12} x_1 + a_{32} x_3 + a_{42} x_4 + a_{52} x_5$$
$$x_3 = a_{23} x_2$$
$$x_4 = a_{34} x_3 + a_{44} x_4$$
$$x_5 = a_{35} x_3 + a_{45} x_4$$



Q For the SFG in figure below. Obtain the T.F

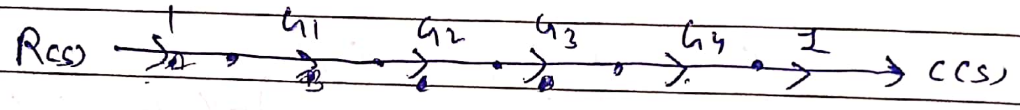




Sol^n Step 1 Total no. of forward paths.

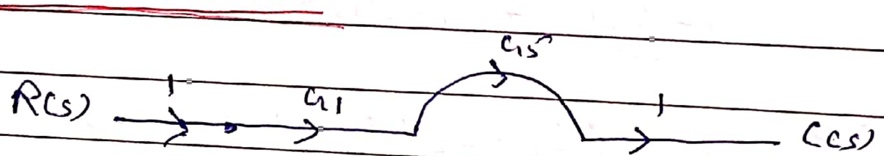
There are two forward paths.

Forward Path 1



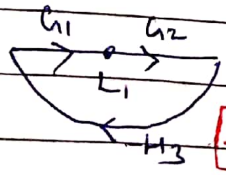
$P_1 = G_1 G_2 G_3 G_4$

Forward Path 2

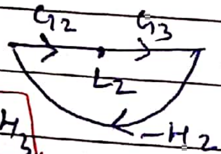


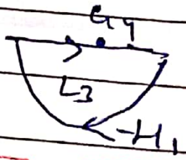
$P_2 = G_1 \cdot G_5$.

Step 2 Obtain total no. of single loops.

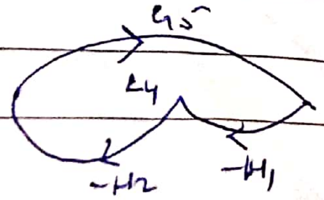


$\boxed{L_1 = P_{11} = -G_1 G_2 H_3}$





$\boxed{L_2 = P_{21} = -G_1 G_3 H_2}$ $\boxed{L_3 = P_{31} = -G_4 H_1}$



$\boxed{L_4 = P_{41} = G_5 H_1 H_2}$

Step 3 : Obtain total number of two non-touching loops & their gain product.

$$P_{12} = L_1 \times L_3 = (-G_1 G_2 H_3)(-G_4 H_1)$$

$$\boxed{P_{12} = G_1 G_2 G_4 H_1 H_3}$$

* There is no 3 non-touching loops so its value = 0

Step 4 Value of system determinant 'Δ'

$$\Delta = 1 - (P_{11} + P_{21} + P_{31} + P_{41}) + P_{12}$$

$$\Delta = 1 - (-G_1 G_2 H_3 - G_2 G_3 H_2 - G_4 H_1 + G_5 H_1 H_2) + G_1 G_2 G_4 H_1 H_3$$

$$\boxed{\Delta = 1 + G_1 G_2 H_3 + G_2 G_3 H_2 + G_4 H_1 - G_5 H_1 H_2 + G_1 G_2 G_4 H_1 H_3}$$

Step 5 Value of Δ_K ie Δ_1, Δ_2

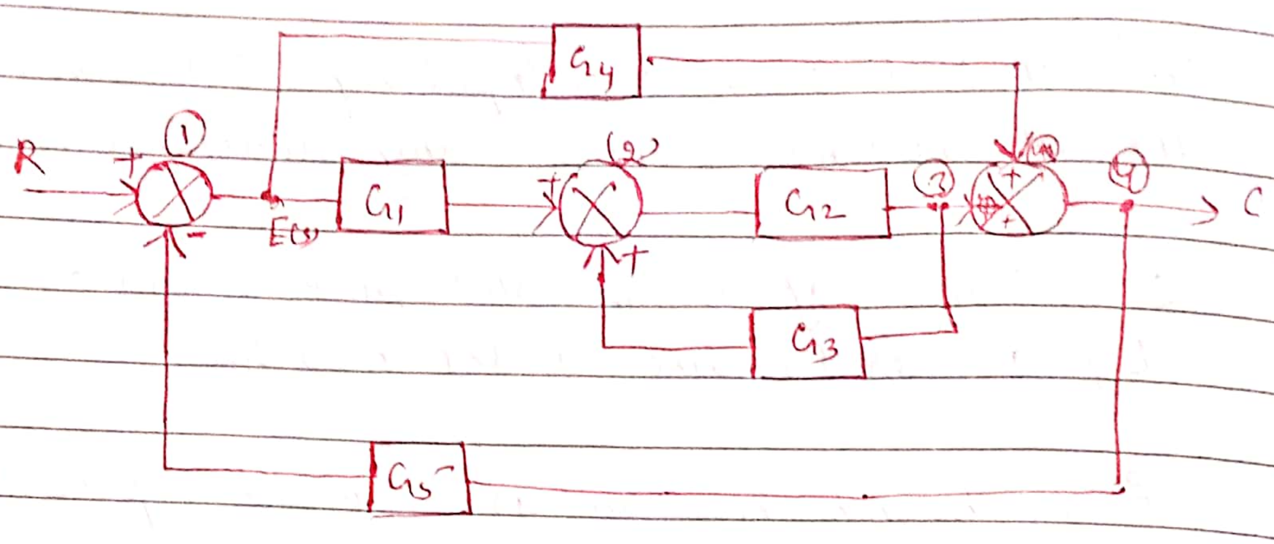
Take P_1, $\Delta_1 = 1 - 0 = 1$ $\therefore P_1 \Delta_1 = G_1 G_2 G_3 G_4$

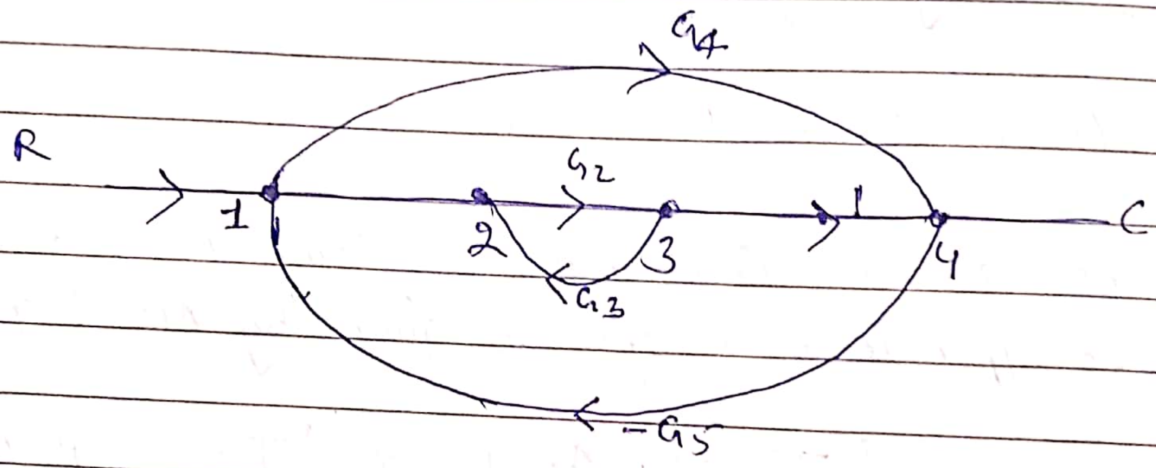
Take P_2, $\Delta_2 = 1 - 0 = 1$ $\therefore P_2 \Delta_2 = G_1 G_5$

Step 6 Value of system T.F using Mason's gain formula

$$\frac{C(s)}{R(s)} = \frac{1}{\Delta} \sum_{k=1,2} P_k \Delta_K = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4 + G_1 G_5}{1 + G_1 G_2 H_3 + G_2 G_3 H_2 + G_4 H_1 - G_5 H_1 + G_1 G_2 G_4 H_1 H_3}}$$





(A) Forward paths —

(i) $g_1 = G_1 G_2$ (ii) $g_2 = G_4$

(B) There are three individual loops —

(i) $L_1 = G_2 G_3$ (2 - 3 - 2)

(ii) $L_2 = -G_1 G_2 G_5$ (1 - 2 - 3 - 4 - 1)

(iii) $L_3 = -G_4 G_5$ (1 - 4 - 1)

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Since all three loops touching the forward path g_1, therefore — $\Delta_1 = 1 - 0 = 1$

The first loop L_1 do not touch the forward path g_2 therefore — $\Delta_2 = 1 - G_2 G_3$

There are two non-touching loops L_1 & L_3

$\therefore$ There ~~gr~~ products —

$$L_1 \cdot L_3 = -G_2 G_3 G_4 G_5$$

$$T = \frac{C}{R} = \frac{\sum g_K \Delta_K}{\Delta} = \frac{g_1 \Delta_1 + g_2 \Delta_2}{\Delta}$$

$$T = \frac{G_1 G_2 + G_4 (1 - G_2 G_3)}{1 - (G_2 G_3 - G_1 G_2 G_5 - G_4 G_5) - (-G_2 G_3 G_4 G_5)}$$

$$\boxed{T = \frac{C}{R} = \frac{G_1 G_2 + G_4 - G_2 G_3 G_4}{1 - G_2 G_3 + G_1 G_2 G_5 + G_4 G_5 - G_2 G_3 G_4 G_5}}$$

Where $\Delta = 1 - (L_1 + L_2 + L_3) + (L_1 L_3)$