# Unit 1

## ASSIGNMENT SOLUTION

## CONTROL SYSTEM (BEC-302)

1. What are the basic components of a control system?
2. Compare open loop control system with close loop control system.
3. How many types of feedbacks are there? Explain the advantages and disadvantages of each.
4. Define feedback and its effect on control system.
5. Explain all rules for block diagram reduction techniques.
6. Find the transfer function for the given block diag. representations.

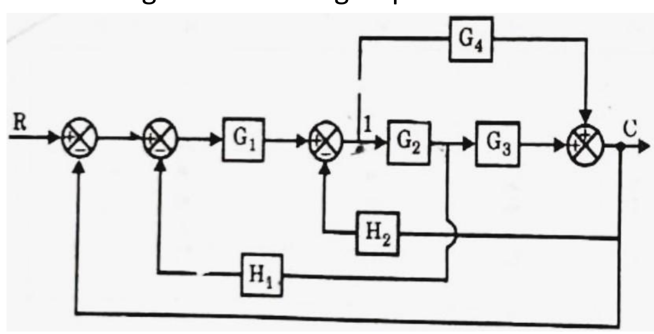
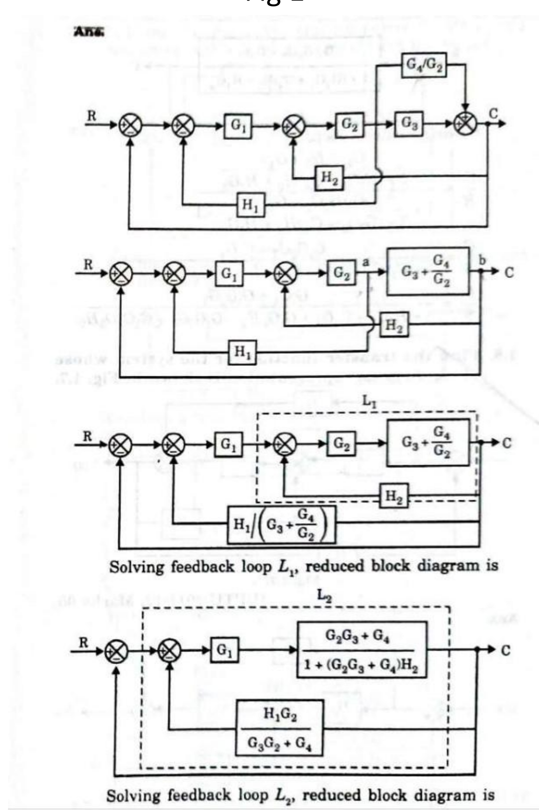
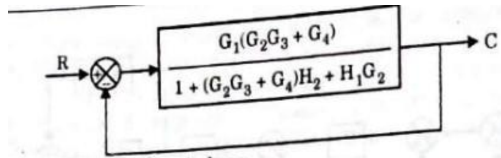



Fig 1



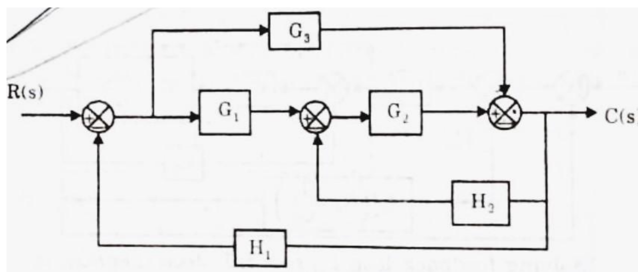


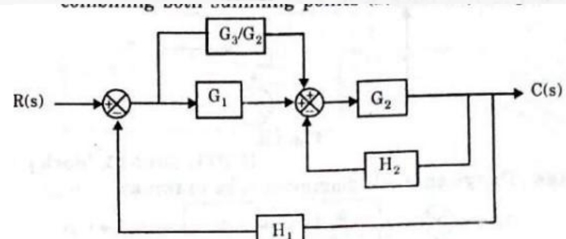

Solving feedback loop,

$$\frac{C}{R} = \frac{\dfrac{G_1(G_2G_3 + G_4)}{1 + (G_2G_3 + G_4)H_2 + H_1G_2}}{1 + \dfrac{G_1(G_2G_3 + G_4)}{1 + (G_2G_3 + G_4)H_2 + H_1G_2} \times 1}$$

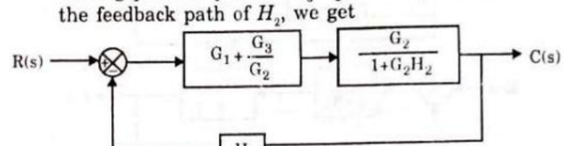
$$\frac{C}{R} = \frac{G_1G_2G_3 + G_1G_4}{1 + G_1(G_2G_3 + G_4)H_2 + H_2G_1 + G_1(G_2G_3 + G_4)}$$

$$\frac{C}{R} = \frac{G_1G_4 + G_1G_2G_3}{1 + H_1G_2 + G_1G_4 + G_1G_4H_2 + G_1G_2G_3 + G_1G_2G_3H_2}$$

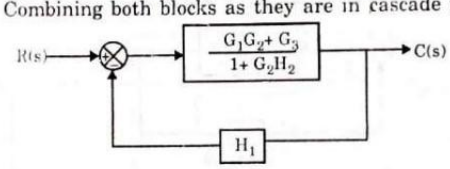



Adding parallel path of $G_3/G_2$ with $G_1$ and eliminating the feedback path of $H_2$, we get




Combining both blocks as they are in cascade form;




$\therefore$ $$\text{T.F} = \frac{G(s)}{1 + G(s)H(s)}$$

Here $G(s) = \dfrac{G_1G_2 + G_3}{1 + G_2H_2}$; $H(s) = H_1$

$\therefore$ $$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1G_2 + G_3}{1 + G_2H_2}}{1 + \dfrac{(G_1G_2 + G_3)}{1 + G_2H_2} \cdot H_1}$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2 + G_3}{1 + G_2H_2 + G_1G_2H_1 + G_3H_1}$$

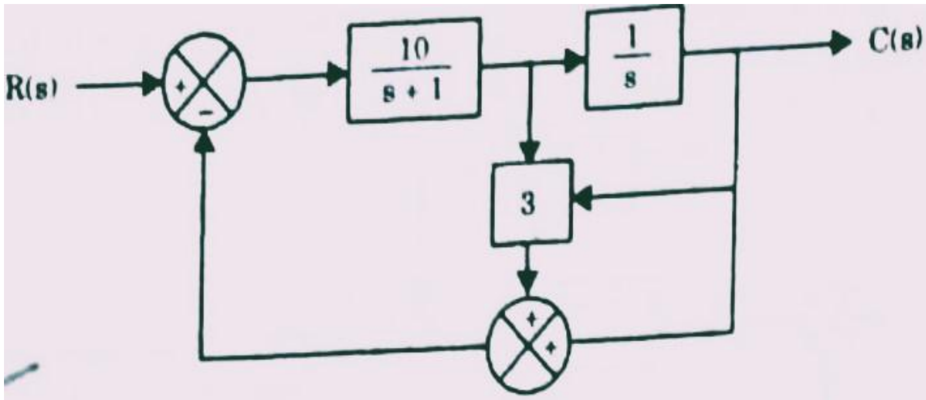



Fig 3

**Ans.** The given block diagram can be drawn as

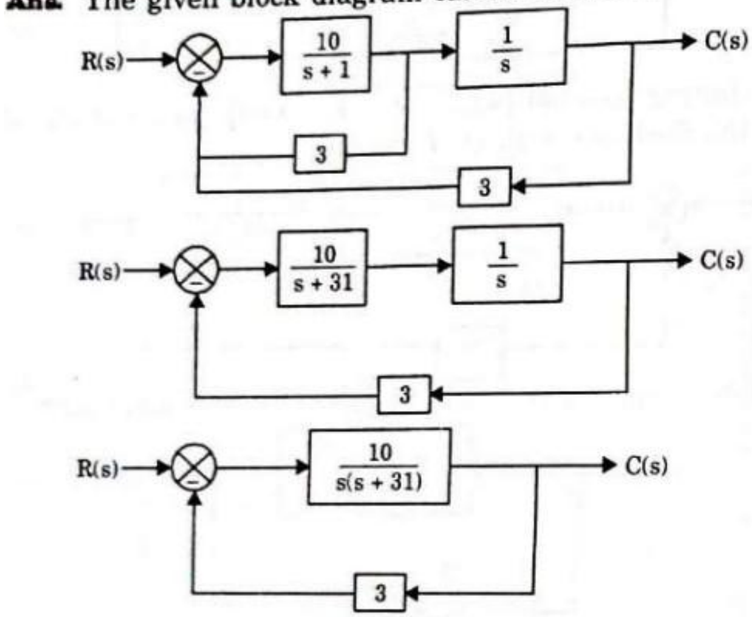



**Fig. 1.9.**

$$\frac{C(S)}{R(S)} = \frac{10}{s^2 + 31s + 30}$$

7. Determine the transfer function (C1/R1),(C1/R2),(C2/R1), and (C2/R2 ) for the given block diag. representation.

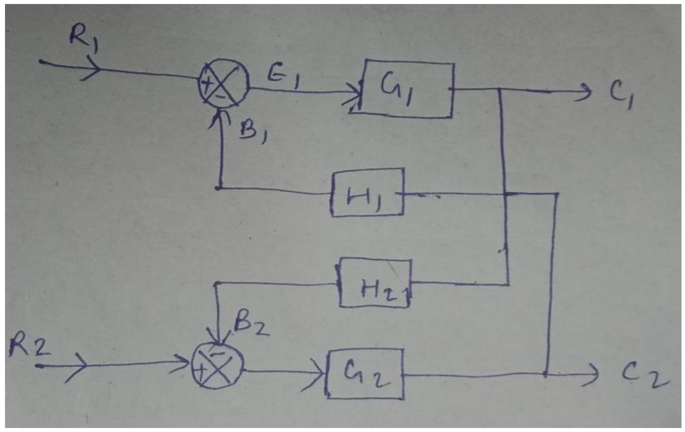



Fig 4



**Solution :**

$$E_2 = R_2 - B_2$$

$$B_2 = C_1 H_2 \qquad \text{...(1.119a)}$$

$$C_2 = E_2 G_2 \qquad \text{...(1.119b)}$$

$$\text{...(1.119c)}$$

put the value of $E_2$ from 1.119*a* in 1.119*c*

$$C_2 = G_2(R_2 - B_2) \qquad \text{...(1.119d)}$$

from (1.119*b*) put the value of $B_2$ in (1.119*d*)

$$C_2 = G_2(R_2 - C_1 H_2) \qquad \text{...(1.119e)}$$

$$B_1 = C_2 H_1 \qquad \text{...(1.119f)}$$

put the value of $C_2$ from (1.119*e*.) in (1.119*f*)

$$B_1 = G_2 H_1 (R_2 - C_1 H_2) \qquad \text{...(1.119g)}$$

$$E_1 = R_1 - B_1$$

$$E_1 = R_1 - G_2 H_1 (R_2 - C_1 H_2) \qquad \text{...(1.119h)}$$

Also, $$C_1 = E_1 G_1. \qquad \text{...(1.119i)}$$

from (1.119*h*) put the value of $E_1$ in (1.119*i*)

$$C_1 = G_1 [R_1 - G_2 H_1 (R_2 - C_1 H_2)] \qquad \text{...(1.119j)}$$

$$C_1(1 - G_1 G_2 H_1 H_2) = G_1 R_1 - G_1 G_2 H_1 R_2 \qquad \text{...(1.119K)}$$

When $R_1 = 0$ from (1.119*K*)

$$\boxed{\frac{C_1}{R_2} = \frac{-G_1 G_2 H_1}{1 - G_1 G_2 H_1 H_2}} \qquad \textbf{Ans.} \quad \text{...(1.119l)}$$

from (1.119*l*) put the value of $C_1$ in (1.119*e*)

$$C_2 = G_2\left[R_2 + \frac{G_1 G_2 H_1 H_2 R_2}{1 - G_1 G_2 H_1 H_2}\right]$$

$$\therefore \quad \boxed{\frac{C_2}{R_2} = \frac{G_2}{1 - G_1 G_2 H_1 H_2}} \qquad \textbf{Ans.} \quad \text{...(1.119m)}$$

When $R_2 = 0$, (1.119*K*) becomes

$$C_1 = G_1 [R_1 + C_1 G_2 H_1 H_2]$$

$$C_1 = G_1 R_1 + C_1 G_1 G_2 H_1 H_2$$

$$\therefore \quad \boxed{\frac{C_1}{R_1} = \frac{G_1}{1 - G_1 G_2 H_1 H_2}} \qquad \textbf{Ans.} \quad \text{...(1.119n)}$$

When $R_2 = 0$, (1.119*e*) becomes

$$C_2 = -G_2 C_1 H_2$$

from (1.119*n*) put the value of $C_1$

$$\therefore \quad C_2 = \frac{-G_1 G_2 R_1 H_2}{1 - G_1 G_2 H_1 H_2}$$

$$\boxed{\frac{C_2}{R_1} = \frac{-G_1 G_2 H_2}{1 - G_1 G_2 H_1 H_2}} \qquad \textbf{Ans.} \quad \text{...(1.119n')}$$

8. Find the transfer function for the signal flow graph.

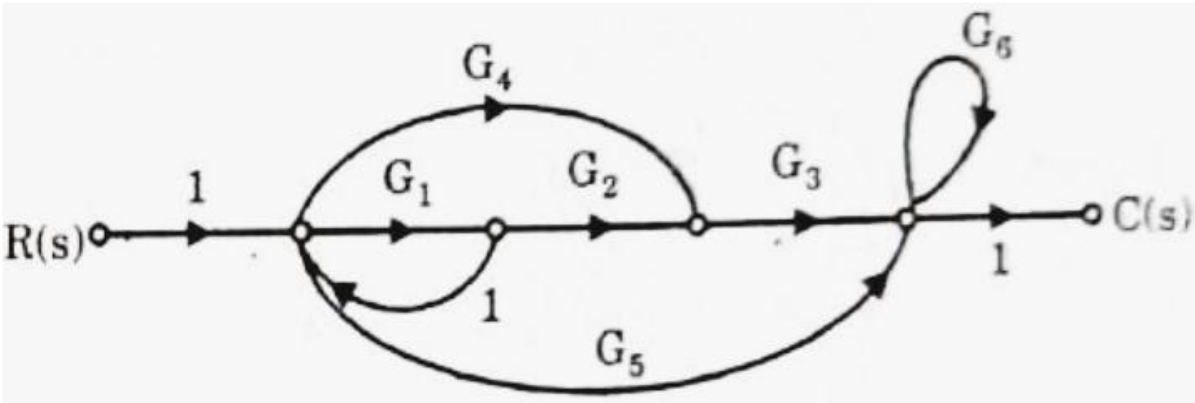



Fig 5

**Ans:** Number of forward paths

$$T_1 = G_1G_2G_3,\quad T_2 = G_4G_3,\quad T_3 = G_5$$

Number of individual loops

$$L_1 = G_1,\quad L_2 = G_6$$

Number of non touching loops, $L_1L_2 = G_1G_6$

$$\Delta_1 = 1,\ \Delta_2 = 1,\ \Delta_3 = 1$$

$$\text{T.F.} = \frac{T_1\,\Delta_1 + T_2\Delta_2 + T_2\Delta_3}{1-(L_1+L_2)+(L_1\,L_2)}$$

$$\text{T.F.} = \frac{G_1\,G_2\,G_3 + G_4\,G_3 + G_5}{1-G_1-G_6+G_1\,G_6}$$

9. What is signal flow graph? How it is constructed?
10. What are the two different methods to obtain SFG? Explain with example.

11. Draw the SFG of the given block diag. and find its transfer function.

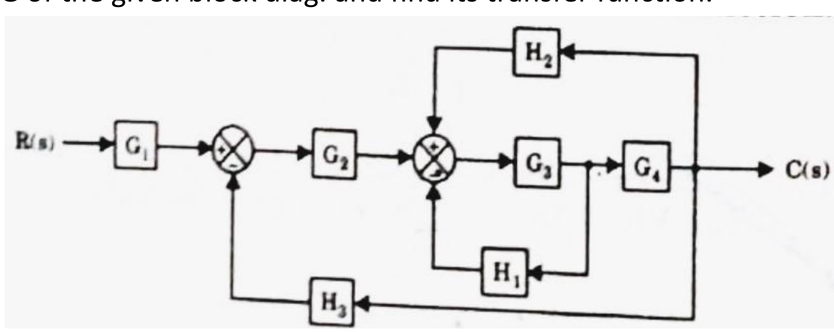



Fig 6

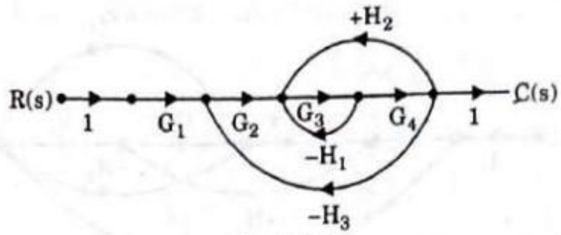



**Fig. 1.15.**

The forward path is $P_1 = G_1\,G_2\,G_3\,G_4$

Loops are :

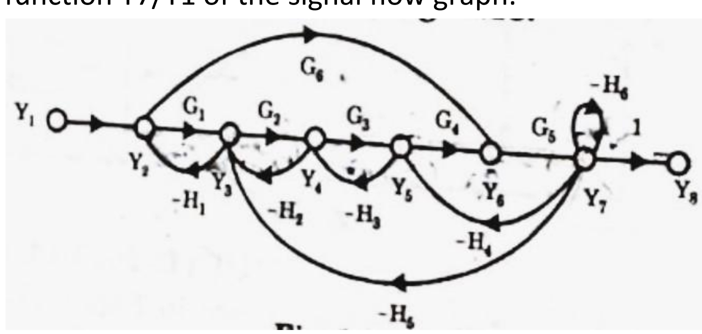
$$L_1 = -\,G_3\,H_1$$
$$L_2 = -\,G_2\,G_3\,G_4\,H_3$$
$$L_3 = G_3\,G_4\,H_2$$

$\Delta_1 = 1 -$

Here, non-touching loops are not present

$$\therefore \quad \Delta = 1 - [L_1 + L_2 + L_3]$$
$$= 1 - [-\,G_3 H_1 - G_2 G_3 G_4 H_3 + G_3 G_4 H_2]$$
$$= 1 + G_3 H_1 + G_2 G_3 G_4 H_3 - G_3 G_4 H_2$$
$$\Delta_1 = 1$$
$$\therefore \quad \text{T.F.} = \frac{P_1\,\Delta_1}{\Delta}$$
$$\text{T.F.} = \frac{G_1\,G_2\,G_3\,G_4}{1 + G_3\,H_1 + G_2\,G_3\,G_4\,H_3 - G_3\,G_4\,H_2}$$

12. Find the transfer function Y7/Y1 of the signal flow graph.



Ans. Using Mason's Gain formula,

$$T(s) = \frac{C(s)}{R(s)} = \left[\frac{\sum_{i=1}^{K} P_i \Delta_i}{\Delta}\right]$$

$$\frac{Y_7}{Y_1} = \left(\frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}\right)$$

($\because$ Two forward paths exist here)

where, $\Delta$ = 1 − (Sum of all loops) + (Sum of all two non-touching loops) − (Sum of all three non-touching loops) + ...

$P_1 = G_1G_2G_3G_4G_5$

$P_2 = G_6G_5$

Number of loops present are :

$L_1 = -H_1G_1,$

$L_2 = -H_2G_2,$

$L_3 = -H_3G_3,$

$L_4 = -H_4G_4G_5,$

$L_5 = -H_6$

$L_6 = G_5G_6H_5H_1,$

$L_7 = -G_3G_2G_4G_5H_5,$

$L_3L_5 = H_3G_3H_6$

Number of two non-touching loop :

$L_1L_3 = H_1G_1H_3G_3$

$L_1L_4 = H_1G_1H_4G_4G_5$

$L_1L_5 = H_1G_1H_6$

$L_2L_4 = H_2G_2H_4G_4G_5$

$L_2L_5 = H_2G_2H_6$

Number of three non-touching loops :

$L_1L_3L_5 = -H_1G_1H_3G_3H_6$

$$\frac{Y_7}{Y_1} = \frac{G_1G_2G_3G_4G_5\Delta_1 + G_6G_5\Delta_2}{\begin{array}{l}1 + H_1G_1 + H_2G_2 + H_3G_3 + H_4G_4G_5 + H_6 - \\ G_5G_6H_5H_1 + G_3G_2G_4G_5H_5 - G_6H_4H_3H_2H_1 + \\ H_1G_1H_3G_3 + H_1G_1H_4G_4G_5 + \\ H_1G_1H_6 + H_2G_2H_4G_4G_5 + H_2G_2H_6 + H_3G_3H_6 + \\ H_1G_1H_3G_3H_6\end{array}}$$

where, $\Delta_1 = 1 - [0] = 1$

$\Delta_2 = 1 + G_2H_2 + G_3H_3$

13. Find the transfer function C/R for the signal flow graph.

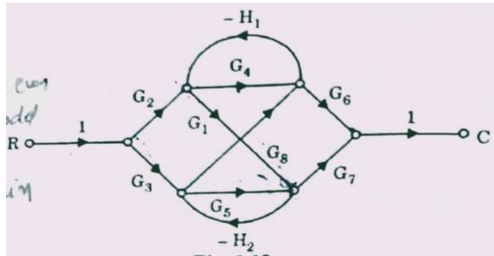



Fig 8

: The Mason's gain formula of given by, $\frac{C}{R} = \frac{\Sigma P_k \Delta_k}{\Delta}$

where,

$\Delta = 1$ – (some of all loops) + (some of non-touching loops) .......

$P_k$ = forward paths

$\Delta_k = 1$ – (loop not touching $P_k$)

**Forward paths of SFG :**

$$P_1 = G_2 G_4 G_6$$
$$P_2 = G_3 G_5 G_7$$
$$P_3 = G_2 G_1 G_7$$
$$P_4 = G_3 G_8 G_6$$
$$P_5 = -G_2 G_1 H_2 G_8 G_6$$
$$P_6 = -G_3 G_8 H_1 G_1 G_7$$

**Loops of SFG :**

$$L_1 = -G_4 H_1, \quad L_2 = -G_5 H_2, \quad L_3 = G_1 H_2 G_8 H_1$$

**Non-touching loops of SFG :** There is one pair having gain product

$$L_1 L_2 = G_4 H_1 G_5 H_2.$$
$$\Delta = 1 + G_4 H_1 + G_5 H_2 - G_1 H_2 G_8 H_1 + G_4 H_1 G_5 H_2$$
$$\Delta_1 = 1 + G_5 H_2$$
$$\Delta_2 = 1 + G_4 H_1$$
$$\Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1$$

$$\therefore \quad \frac{C}{R} = T = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6}{\Delta}$$

$$= \frac{G_2 G_4 G_6 (1 + G_5 H_2) + G_3 G_5 G_7 (1 + G_4 H_1) + G_2 G_1 G_7}{1 + G_4 H_1 + G_5 H_2 - G_1 H_2 G_8 H_1 + G_4 G_5 H_1 H_2} + \frac{G_3 G_8 G_6 - G_2 G_6 G_8 G_1 H_2 - G_3 G_7 G_8 G_1 H_1}{1 + G_4 H_1 + G_5 H_2 - G_1 H_2 G_8 H_1 + G_4 G_5 H_1 H_2}$$

14. What is modelling in electrical system?

15. Find the transfer function of the circuit.

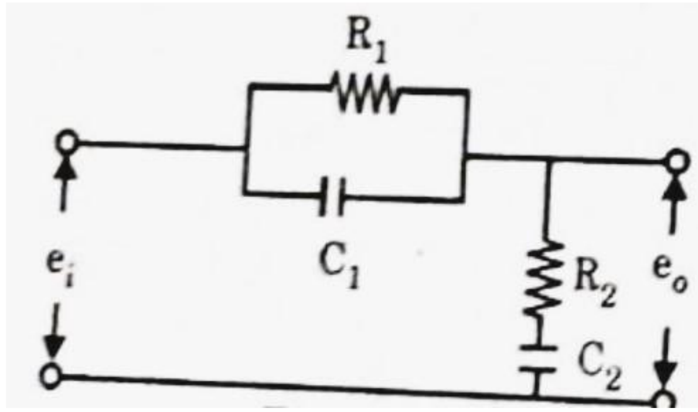



Fig 9

$L$ = inductance
$i$ = current
In alternating current circuit, the stored energy keeps on varying as the current changes.

Voltage $\boxed{e = -L\frac{di}{dt}}$

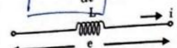



Fig. 1.21.

(*iii*) **Capacitance :** It stores energy and energy is $\frac{1}{2}CV^2$

where, $C$ = capacitance.
$V$ = voltage across it.

Stored energy varies with variation in the voltage. Energy is maximum when current is maximum.

$$V = \frac{q}{C}$$

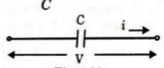



Fig. 1.22.

$q$ = stored charge

$$\boxed{q = \int i\,dt}$$

**1.19. Find the transfer function of the circuit.**

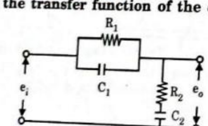



Fig. 1.23.

**Ans.**

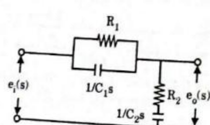



Fig. 1.24.

$i(s)$ = Laplace transform of current

$$e_i(s) = \left[\left\{\frac{R_1\left(\frac{1}{C_1 s}\right)}{R_1 + \left(\frac{1}{C_1 s}\right)}\right\} + R_2 + \frac{1}{C_2 s}\right] i(s)$$

$$= \left[\frac{R_1}{(R_1 C_1 s + 1)} + R_2 + \frac{1}{C_2 s}\right] i(s)$$

$$= \left[\frac{R_1}{(R_1 C_1 s + 1)} + \frac{(R_2 C_2 s + 1)}{C_2 s}\right] i(s)$$

$$i(s) = \frac{[(R_1 C_1 s + 1) C_2 s] e_i(s)}{R_1 C_2 s + (R_1 C_1 s + 1)(R_2 C_2 s + 1)}$$

$$e_o(s) = \left(R_2 + \frac{1}{C_2 s}\right) i(s)$$

$$= \left[\frac{(R_2 C_2 s + 1)}{C_2 s}\right] i(s)$$

$$e_o(s) = \frac{\left[\frac{(R_2 C_2 s + 1)}{C_2 s}\right][(R_1 C_1 s + 1) C_2 s] e_i(s)}{R_1 C_2 s + (R_1 C_1 s + 1)(R_2 C_2 s + 1)}$$

$$\therefore \quad \frac{e_o(s)}{e_i(s)} = \frac{(R_2 C_2 s + 1)(R_1 C_1 s + 1)}{R_1 C_2 s + (R_1 C_1 s + 1)(R_2 C_2 s + 1)}$$

**1.20. How modeling of mechanical system done ?**

**OR**

**What are the elements of mechanical system ?**

**Ans.** (*i*) **Translation Motion :** It takes place along a straight line and variables are displacement, velocity and acceleration.

Newton's law of linear motion is applied *i.e.*, product of mass and acceleration is equal to the algebraic sum of forces acting to it.

(*ii*) **Mass :** The function of mass in linear motion is to store kinetic energy. Mass cannot store potential energy.

$$M\frac{d^2x}{dt^2} = f(t)$$

16. How modelling of mechanical system is done?
17. Draw the electrical analogous circuit of the system given below

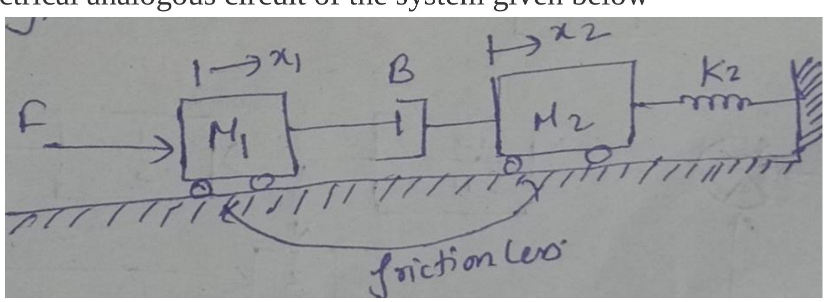
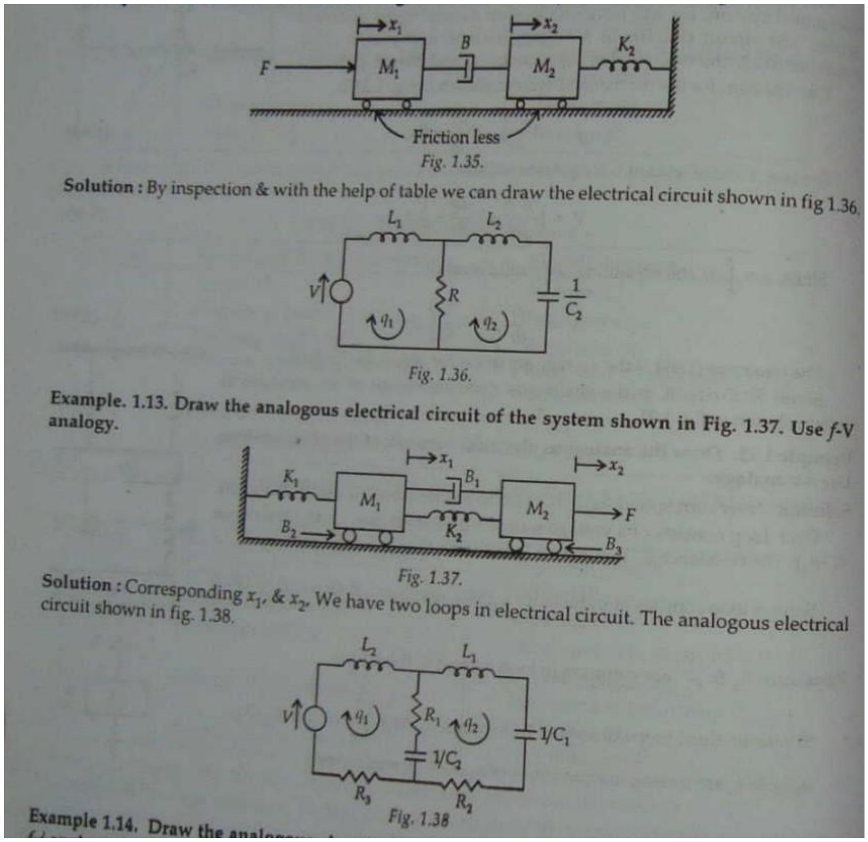



Fig 10



Friction less

*Fig. 1.35.*

**Solution :** By inspection & with the help of table we can draw the electrical circuit shown in fig 1.36.

*Fig. 1.36.*

**Example. 1.13. Draw the analogous electrical circuit of the system shown in Fig. 1.37. Use *f*-V analogy.**

*Fig. 1.37.*

**Solution :** Corresponding $x_1$, & $x_2$. We have two loops in electrical circuit. The analogous electrical circuit shown in fig. 1.38.

*Fig. 1.38*

**Example 1.14. Draw the anal...**

18. What are the physical quantities (i) force (ii) mass (iii) damper (iv) displacement and (v) velocity analogous to in the force current analogy and force voltage analogy?
19. Explain : (i) force-voltage analogy (ii) force-current analogy