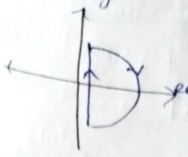
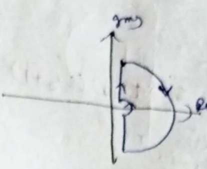
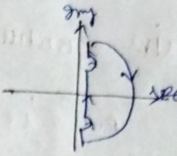
Types of Nyquist contours



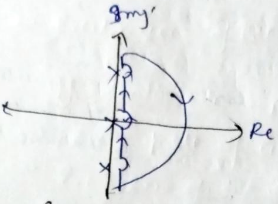
$G(s)H(s) = \frac{1}{(1+sT)}$



$G(s)H(s) = \frac{1}{s(sT+1)}$



$G(s)H(s) = \frac{1}{(s^2+a^2)}$



$G(s) \cdot H(s) = \frac{1}{s(s^2+a^2)}$

Note

If contour in s-plane is clockwise –

→ Every pole of GH inside contour → 1 A.C.W encirclement of origin in GH-plane.

→ Every zero of GH inside contour → 1 C.W encirclement of origin in GH-plane.

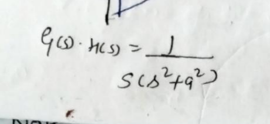
If contour in s-plane is anticlockwise –

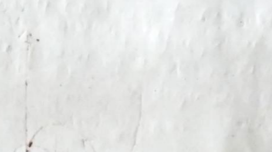
→ Every pole of GH inside contour → 1 C.W encirclement of origin in GH-plane

→ Every zero of GH inside contour → 1 A.C.W encirclement of origin in GH-plane.

Q

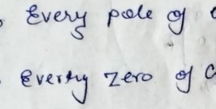
①

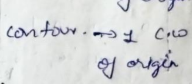




2 – poles inside contour.
2 – A.C.W encirclement.

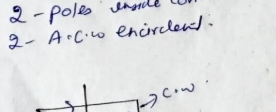
②

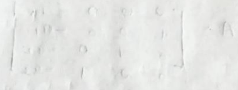




1 pole inside contour
1 A.C.W. encirclement

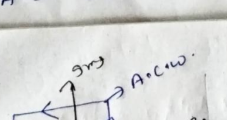
③

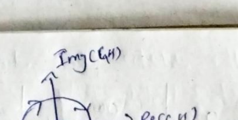




1 pole inside contour.
1 C.W encirclement

④





2 pole inside contour
2 – C.W encirclement.

Ex:- $G(s)H(s) = \dfrac{10}{s(s+2)}$

$F(s) = 1 + G(s)H(s) \Rightarrow 1 + \dfrac{10}{s(s+2)} = \dfrac{s^2+2s+10}{s(s+2)} = \dfrac{P(s)}{Q(s)}$

$\therefore$ $Q(s) = 0 \rightarrow$ poles of $1+G(s)H(s)$

i.e., $s = 0, -2 \Rightarrow$ open loop poles

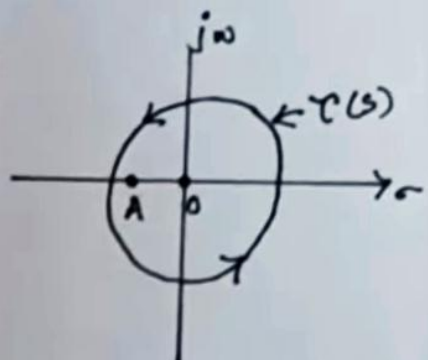
$P(s) = 0 \rightarrow$ Zeros of $1+G(s)H(s)$

i.e., $s = -1, -1 \Rightarrow$ closed loop poles

The s/m is absolutely stable if all zeros of $1+G(s)H(s)$ i.e., closed loop poles of the s/m are located in left half of s-plane.

2] Encirclement:

A point is said to be encircled by a closed Path if it is found to lie inside that closed path.



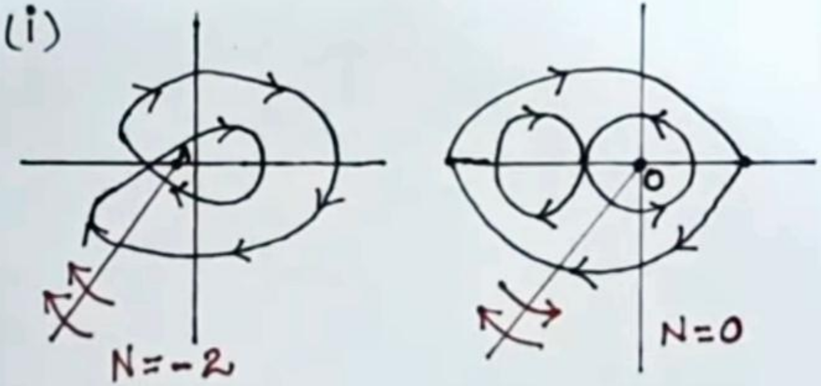
The points O, A → encircled by the colsed Path.

Counting Number of encirclements:

→ no of encirclement of point O & A is one in Anticlockwise direction.

For complicated cases.

(i)



3] Analytic Fun and Singularities:

A mathematical fun is said to be analytic at a point in a plane if its value & its derivative has finate existance at that point.

$F(s) = \dfrac{25}{s(s+1)}$

F(s) is analytic at all points in s-plan except $s=0$ & $s=-1$ $\because F(s) = \infty$

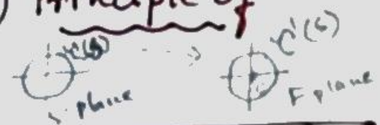
Poles of the fun are Singularities, if it is having only one value of S

Ex:- $F(s) = \sqrt{s}$ & $s = 9$

$\therefore s = +3$ & -3 → not Single valued.

We will assume the transfer fun of s/m are Single valued.

4] Mapping theorem (or) Principle of Argument



Mapping Theorem satates that the mapped locus $\Upsilon'(s)$ encircles the new origin of F-plane as many times as the difference between the number of Zeros & Poles of F-plane which are encircled by $\Upsilon(s)$ Path in S-plane.

$$\boxed{N = Z - P}$$

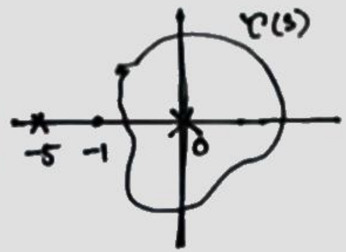
N → Encirclements of origin of F-plane.

P → no of poles of F(s) encircled by $\Upsilon(s)$ Path in splane.

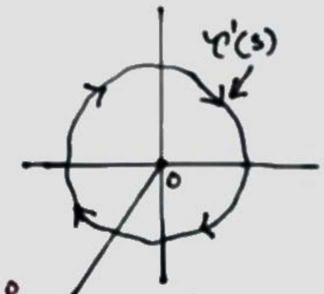
Z → no of Zeros of F(s) encircled by $\Upsilon(s)$ Path in S-plane.

The statement is also called as Principle of Argument.

(i) P>Z: 1 Pole & no zeros.

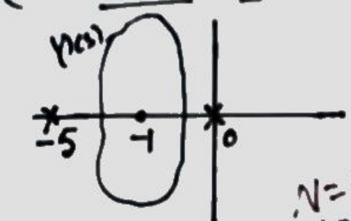


S-plane

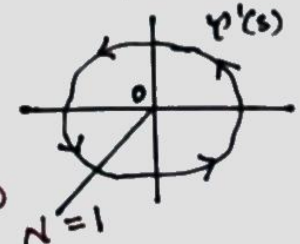


$N = Z - P$
$N = 0 - 1$ $N = -1$ F-plane

(ii) P<Z: 1 zero & zero poles

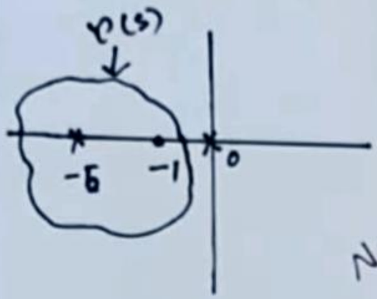


$N = Z - P$
$N = 1 - 0$

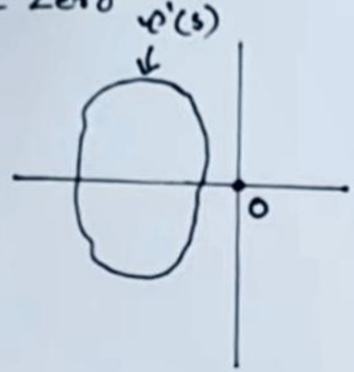


$N = 1$

iii) P = Z : 1 pole & 1 zero



$N = Z - P$
$= 1 - 1$
$N = 0$



6] Nyquist Stability Criterion :

Select a single valued fun F(s) as 1+G(s)H(s) ; G(s) H(s) → open loop T.F.

$\therefore F(s) = 1 + G(s) H(s)$

Now,

$$\boxed{\begin{aligned} \text{Poles of } 1+G(s)H(s) &= \text{Poles of } G(s)H(s) \\ &= \text{open loop poles} \end{aligned}}$$

$$\boxed{\text{Zeros of } 1+G(s)H(s) = \text{Closed loop poles}}$$

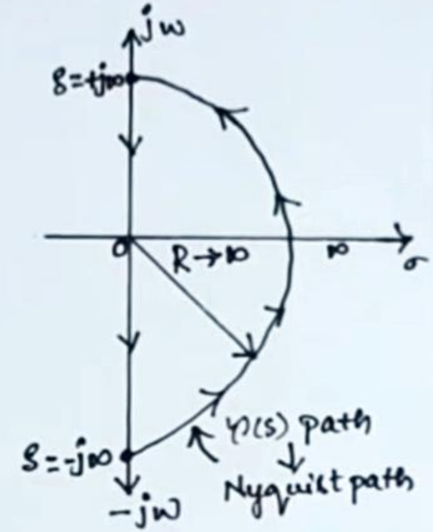
For stability, all Zeros of 1+G(s)H(s) must be in left half of S-plane.

location of zeros ⇒ "Unknown".

→ Nyquist suggested,

→ rather analizing presence of zeros in left half,

→ better to examine the presence of any one zero in right half.



→ We know the poles of G(s) H(s), which are encircled by the Nyquist path.

→ Now map all the points of Nyquist path into F-plane with the help of mapping fun to get γ'(s)

This mapping obtained in F-plane by mapping all the points on Nyquist path is called Nyquist plot.

→ We can determine the no of encirclements of origin by Nyquist plot in F-plance (N)

→ $\therefore N = Z - P$

as N & P are known ⇒ we can find Z