UNIT: 2

Response of control system as a funct: of time $c(t)$

Introduction to time domain or time response analysis
(Variation of O/P w.r.t time)

The time response analysis is the analysis on time taken by the response of the system when subjected to an input.

Time response consists of two parts -

(i) Transient response → Temporary response → due to inertia or energy ~~change~~ storage

(ii) Steady state response → response of a system after a long time i.e. when the system settles down. (Final response)

$$c(t) = c_t + c_{ss}$$

c_t = Transient response
c_{ss} = Steady-state response
$c(t)$ = time response of a system

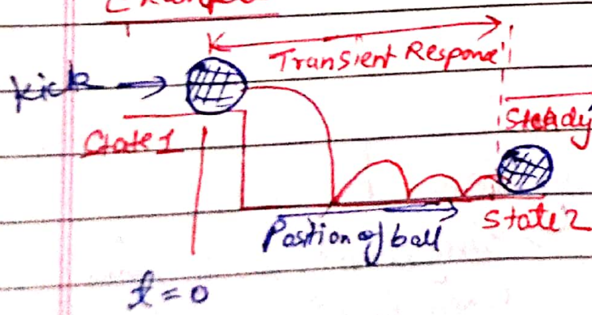
$$\lim_{t\to\infty} c_t \to 0 \qquad \lim_{t\to\infty} c(t) = c_{ss}$$

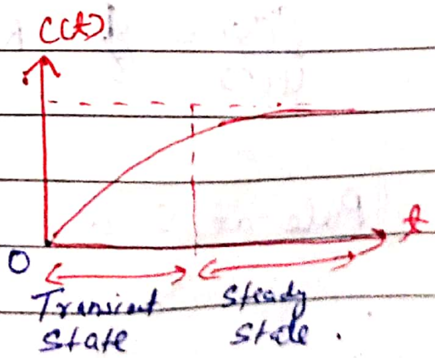
In transient response, there is a gradual change in system.

eg: fan, ~~bike~~

Time taken by a system to give an o/p for a given input. is ~~known~~ (Transient response)

Example:





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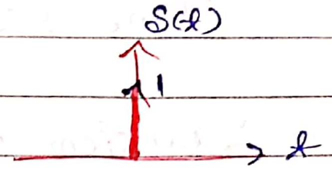
O/P of a system gradually ↑ from initial value towards final value

Standard Test Signals :

(i) Sudden shocks − Unit impulse signal.

$$r(t) = \delta(t) = 0 \quad t \neq 0$$
$$= 1 \quad t = 0$$



Laplace of $\delta(t) = 1$

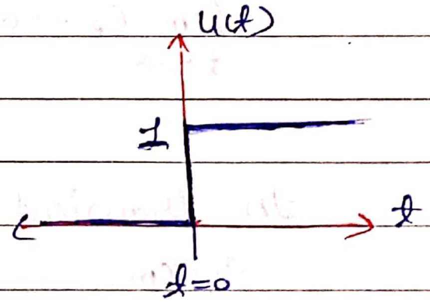
Pulse of zero duration and ∞ height.

1 represents the area under impulse fund^n.

$$\int_{-\infty}^{\infty} \delta(t)\, d\tau = 1$$

(ii) Sudden input (Unit step signal)

$$r(t) = u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$



$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

(running integration of impulse fund^n)

{ discontineous at t=0 because right hand limit & left hand limit about t=0 are unequal.

$0 \leftarrow 1 \rightarrow 1$

$$\delta(t) = \frac{d}{dt} u(t)$$

L.T

$$\frac{\delta(s)}{u(s)} \neq \frac{1}{s} \qquad R(s) = \frac{1}{s}$$

$\boxed{\text{Pole at } s = 0 \quad \text{Singularity function.}}$

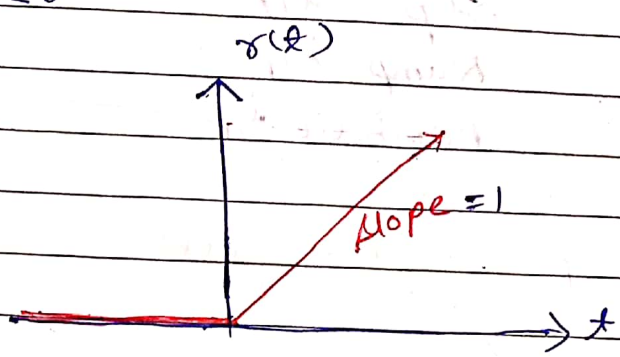
↳ Singularity is a point when fund^n is undefined

(3) Velocity time input = Unit Ramp Signal.

↳ velocity or derivative or slope = constant.

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$ Contineous at $t = 0$

$r(t) = t\,u(t)$



$$\boxed{\frac{dr(t)}{dt} = u(t)}$$

$$r(t) = \int_{-\infty}^{t} u(t)\,dt$$

$$\boxed{r(t) = \int_{-\infty}^{t} u(\tau)\,d\tau}$$

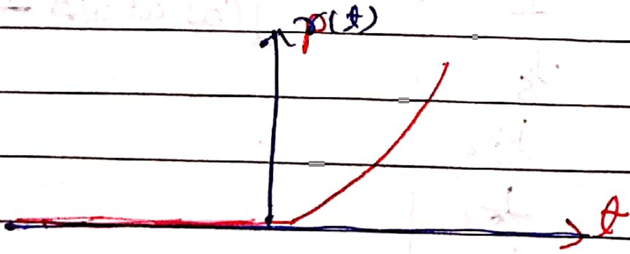
Laplace : $\boxed{R(s) = \frac{1}{s^2}}$

Two poles at origin.

4. Acceleration type input – Unit parabolic signal.

↳ acceleration or double derivative = constant.

$$p(t) = \begin{cases} t^2/2, & t \geq 0 \\ 0, & t < 0 \end{cases}$$ (Contineous at $t = 0$)



$$\boxed{\begin{aligned} &\frac{dp(t)}{dt} = r(t) = \text{ramp} \\ &p(t) = \int_{-\infty}^{t} r(\tau)\,d\tau \end{aligned}}$$

Laplace ⟶ $\boxed{R(s) = \frac{1}{s^3}}$ 3 poles at origin

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Response to standard i/p

Standard i/ps ⟶ diff. or Integrat[n]

Impulse i/p ⟶ Impulse response $h(t)$
Step i/p ⟶ ~~Impulse~~ step response $s(t)$
Ramp i/p ⟶ Ramp response $r(t)$
Parabolic i/p ⟶ Parabolic response $p(t)$

related by diff or int.

$h(t) = \frac{d}{dt} s(t)$ $\quad s(t) = \int_{-\infty}^{t} h(\tau)\, d\tau$.

$\frac{d}{dt} r(t) = u(t)$ or $\int u(t) = r(t)$

$\frac{d}{dt} u(t) = \delta(t)$

$\int \delta(t) = u(t)$

Transient Response Analyses

↳ Variation of response of a system with time.

- rise time
- overshoot
- delaytime
- settling time
- peak time

↳ It tells about the speed of time i.e how fast a system goes to steadystate

Some Common L.T

$\delta(t)$	1
$u(t)$	$1/s$
$r(t)$	$1/s^2$
$p(t) = \frac{t^2}{2} u(t)$	$\frac{1}{s^3}$
$e^{-at} u(t)$	$\frac{1}{(s+a)}$
$\sin at\, u(t)$	$\frac{a}{s^2+a^2}$

$\boxed{\cos at\, u(t) \Rightarrow \frac{s}{s^2+a^2}}$

Standard test signal.

Name of the s/g	Time domain eq^n of signal, r(t)	Laplace transform of the signal, R(s)
Step	A	A/s
Unit step	1	$1/s$
Ramp	At	A/s^2
Unit Ramp	t	$1/s^2$
Parabolic	$At^2/2$	A/s^3
Unit parabolic	$t^2/2$	$1/s^3$
Impulse	$\delta(t)$	1

How to find poles of a function?

→ For a rational function we equate deno^n = 0 & find roots of the eq^n.

Que Calculate the step response of a system whose T.F is $H(s) = \frac{1}{(s+9)}$. Also find final value of step response.

$$T.F = \frac{L.T \text{ of } o/p}{L.T \text{ of } i/p}$$

Transfer funct^n = L.T of impulse res.

$$C(s) = R(s) \cdot H(s)$$

$$= \frac{1}{s} \cdot \frac{1}{(s+9)}$$

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$$C(s) = \frac{1}{s(s+a)} = \frac{1}{as} - \frac{1}{a(s+a)}$$

$$\boxed{c(t) = \frac{1}{a}\left[u(t) - e^{-at} \cdot u(t)\right]} \Rightarrow \frac{1}{a}\left[1-e^{-at}\right]u(t)$$

$$\lim_{t\to\infty} c(t) = c(\infty) = \frac{1}{a}$$

Pole – Zero plot

Zeroes are the value of 's' at which T.F become 0.
Equate the numr of T.F to 0 to compute zeroes.

$$T(s) = \frac{s+2}{s(s+1)(s+4)^2(s^2+2s+2)}$$

Zeroes $(s+2) = 0 \Rightarrow s = -2$

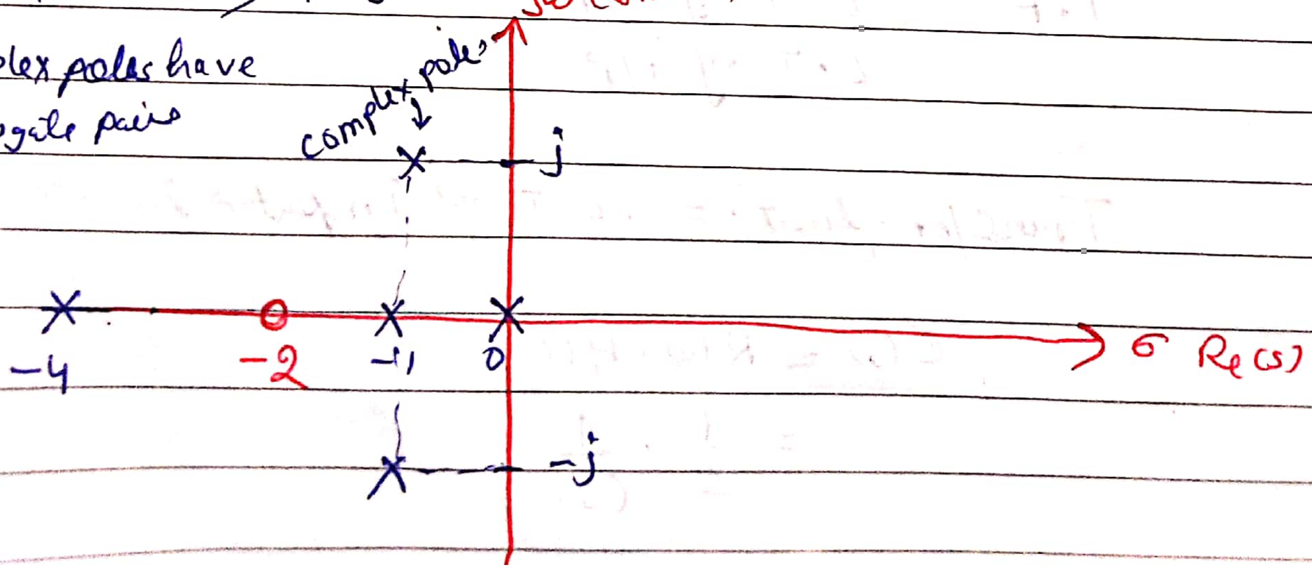
Poles $s(s+1)(s+4)^2(s^2+2s+2) = 0$

$s = 0$
$s = -1$
$s = -4 \longrightarrow$ order 2

denote Pole by X & zeroes by o

$s^2+2s+2=0 \Rightarrow -1 \pm j$

Complex poles have conjugate pairs



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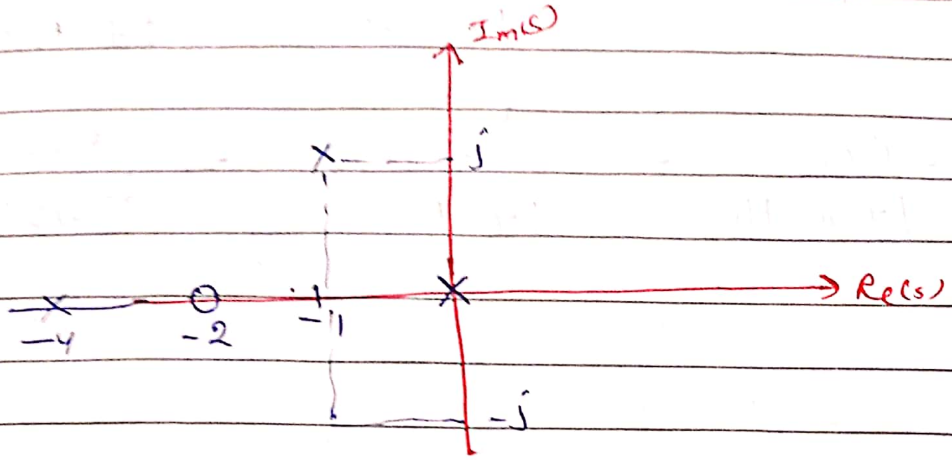
Q. Draw Pole-zero plot of $H(s) = \dfrac{K(s+2)}{s^2(s+4)(s^2+2s+2)}$



Types and order of the System:

Order : The order of differential eq^n governing the physical system.

→ Highest power of s in denominator of C.L.T.F

→ Used in transient analysis.

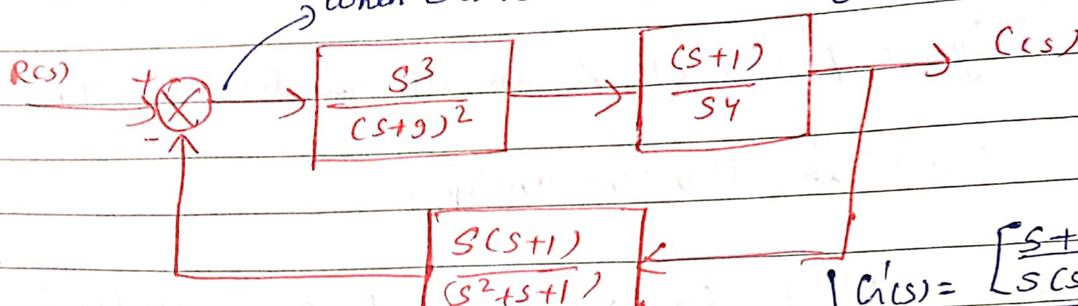
Type : → Used in steady-state error analysis.

→ No. of poles at origin for O.L.T.F ie G(s).H(s)

→ Defination varies with type of error.

Q. The type of the system given above is —.

When E(s) is mentioned: use G(s) H(s) for type. else convert to unity feedback



$$T(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{G'(s)}{1+G'(s)}$$

Equivalent unity f/b system —

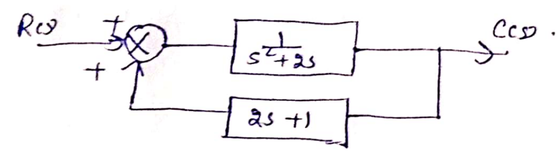
$$G'(s) = \frac{G(s)}{\left[1+G(s)H(s) - G(s)\right]}$$

$$G(s) = \frac{(s+1)}{s\{s+9\}^2} \qquad H(s) = \frac{s(s+1)}{\left[s^2+s+1\right]}$$

$$G'(s) = \frac{\left[\frac{s+1}{s(s+9)^2}\right]}{\left[1 + \frac{(s+1)}{s(s+9)^2} \cdot \frac{s(s+1)}{(s^2+s+1)} - \frac{(s+1)}{s(s+9)^2}\right]}$$

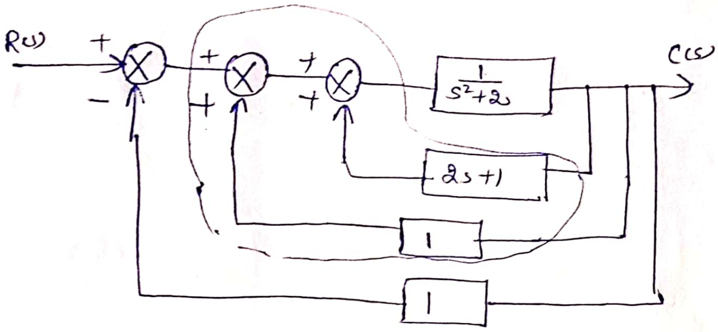
$$G'(s) = \frac{(s+1)}{s(s+9)^2 + (s+1)\left[\frac{s^2+s-s^2-s-1}{s^2+s+1}\right]}$$

Q



Method 1

①



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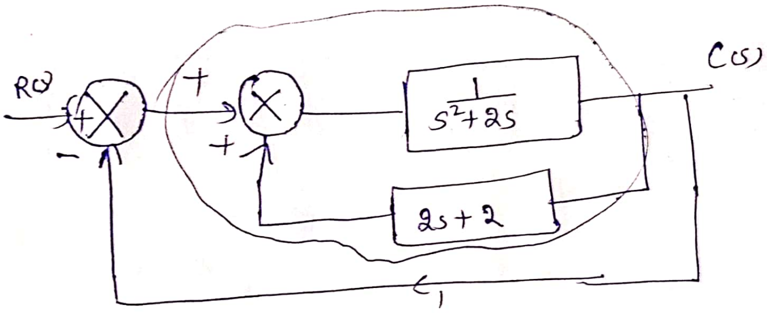
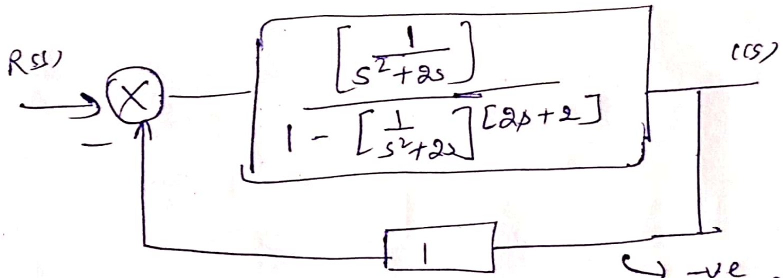
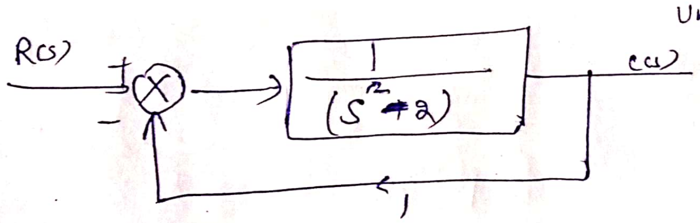




→ −ve unity f/b.



$$\text{Type} = \frac{1}{(s^2+2)\,s^0} \Rightarrow \boxed{\text{Type} = 0}$$

Method 2 $\quad G_1'(s) = \dfrac{G_1(s)}{1 - G_1(s)\cdot H(s) - G_1(s)}$

$$G_1'(s) = \frac{\frac{1}{(s^2+2s)}}{1 - \frac{1}{(s^2+2s)}\cdot[2s+1] - \frac{1}{(s^2+2s)}}$$

$$G_1'(s) = \frac{\frac{1}{(s^2+2s)}}{\frac{[s^2+\cancel{2s}-\cancel{2s}-1-1]}{(s^2+2s)}}$$

$$G_1'(s) = \frac{1}{(s^2-2)} = \frac{1}{s^0[s^2-2]}$$

$$\boxed{\text{Type} = 0}$$

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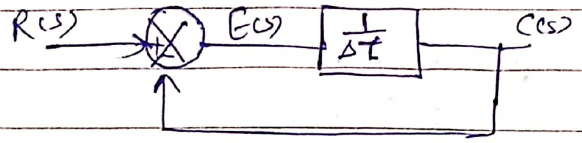
Time response of first order control system to unit step signal.

First order system.



$$T(s) = \frac{\left(\frac{1}{s\tau}\right)}{\left[1+\left(\frac{1}{s\tau}\right)\right]}$$

$$T(s) = \frac{1}{(1+s\tau)}$$

{ Max. power of 's' is 1 in den. so order is 1 & only 1 pole at origin so type is also 1.

Consider i/p of s/g $R(s)$ = Unit step s/g

$$r(t) = u(t) = 1 \quad t \geq 0$$

$$R(s) = LT[r(t)] = LT[1] = \frac{1}{s} \quad , t \geq 0$$

we know -

$$T(s) = \frac{C(s)}{R(s)} = \frac{1}{1+s\tau} \Rightarrow C(s) = R(s)\frac{1}{1+s\tau}$$

$$C(s) = \frac{1}{s}\cdot\frac{1}{(1+s\tau)}$$

$$C(s) = \frac{1}{s}\cdot\frac{1}{(1+s\tau)} \Rightarrow \left(\frac{1}{s}\right)\times\frac{1}{\tau\left[\frac{1}{\tau}+s\right]} \Rightarrow \frac{1}{s}\frac{1/\tau}{\left[s+1/\tau\right]}$$

By partial fraction -

$$C(s) = \frac{1}{s} - \frac{1}{\left(s+\frac{1}{\tau}\right)}$$

To find $c(t)$ we have to take I.L.T of $C(s)$

$$c(t) = L^{-1}\left[\frac{1}{s} - \frac{1}{(s+\tau^{-1})}\right] = L^{-1}\left[\frac{1}{s}\right] - L^{-1}\left[\frac{1}{s+\tau^{-1}}\right]$$

$$c(t) = \left[1 - e^{-\frac{1}{\tau}t}\right]u(t) \quad \cancel{t \geq 0}$$

$$c(t) = \left[1 - e^{-\frac{1}{\tau}t}\right] \cdots \quad t \geq 0$$

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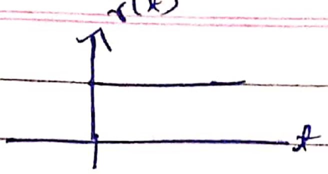
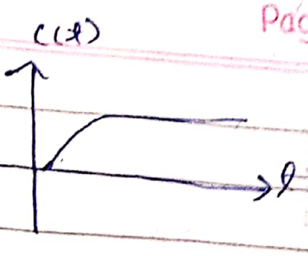




Error in 1^{st} order system :-

$$E(s) = R(s) - C(s)$$

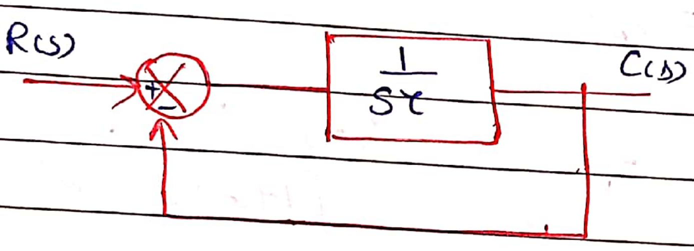
$$\downarrow$$

$$e(t) = r(t) - c(t)$$

$$e(t) = 1 - [1 - e^{-1/\tau\, t}]$$

$$\boxed{e(t) = e^{-1/\tau\, t}}$$

Time response of first order system for unit impulse



$$T(s) = \frac{C(s)}{R(s)} = \frac{\left(\frac{1}{s\tau}\right)}{\left[1 + \frac{1}{s\tau}\cdot 1\right]}$$

$$\boxed{T(s) = \frac{1}{1 + s\tau}}$$

Here, consider input sig $R(s) =$ ~~δ(t)~~ Impulse (Unit impulse sig)

$$r(t) = \delta(t)$$

$$R(s) = LT[r(t)] = LT[\delta(t)] = 1$$

$$\frac{C(s)}{R(s)} = \frac{1}{[1 + s\tau]}$$

$$C(s) = R(s) \cdot \frac{1}{[1 + s\tau]} = 1 \cdot \frac{1}{[1 + s\tau]} = \frac{1/\tau}{(s + 1/\tau)}$$

To find $c(t)$, take $L.T$ of $C(s) \Rightarrow L.T^{-1}\left[\frac{1/\tau}{s + 1/\tau}\right]$

$$c(t) = \frac{1}{\tau} e^{-1/\tau\, t} u(t) \quad \text{or} \quad \frac{1}{\tau} e^{-1/\tau\, t}, \; t \geq 0$$

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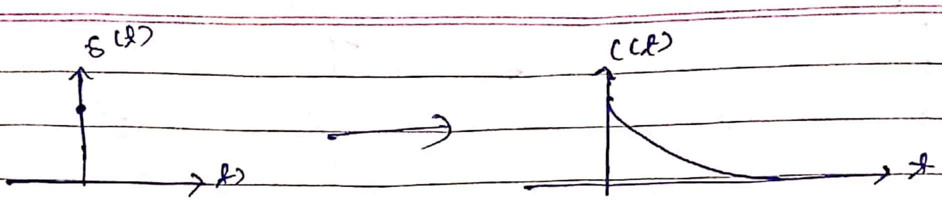
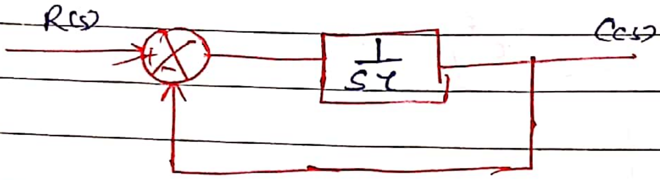


(a) i/p (b) o/p

Time response of first order system for unit ramp s/g.



$$T(s) = \frac{C(s)}{R(s)} = \frac{1}{(1+s\tau)};$$

For a unit ramp s/g as input –

$r(t) = \text{ramp s/g} = t \quad ; \quad t \geq 0$

$R(s) = L \cdot T[r(t)] = 1/s^2$

$$\frac{C(s)}{R(s)} = \frac{1}{[1+s\tau]} \Rightarrow C(s) = R(s)\frac{1}{[1+s\tau]} = \frac{1}{s^2} \cdot \frac{1}{[1+s\tau]}$$

$$C(s) = \frac{1}{s^2} \cdot \frac{1}{[1+s\tau]}$$

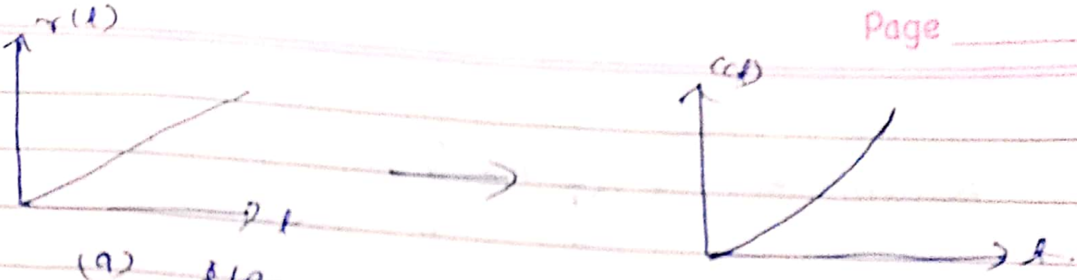
By partial fraction $\Rightarrow C(s) = \frac{1-s\tau}{s^2} + \frac{\tau^2}{(1+s\tau)}$

$$C(s) = \frac{1}{s^2} - \frac{1\tau}{s} + \frac{\tau^2}{(1+s\tau)} \Rightarrow \frac{1}{s^2} - \frac{1}{s}\tau + \frac{\tau}{(s+1/\tau)}$$

To find $c(t)$, take ILT of $C(s)$

$$c(t) = [t - \tau \cdot 1 + \tau \cdot e^{-1/\tau\, t}]u(t)$$

$$c(t) = [t - \tau + \tau e^{-1/\tau\, t}] \quad t > 0$$



(a) i/p (b) o/p

Error $= r(t) - c(t)$

$e(t) = t - (t - \tau + \tau e^{-t/\tau})$

$\boxed{e(t) = \tau - \tau e^{-t/\tau}}$

When we give ramp s/g as an i/p then due to error we get c(t) as in fig (b)

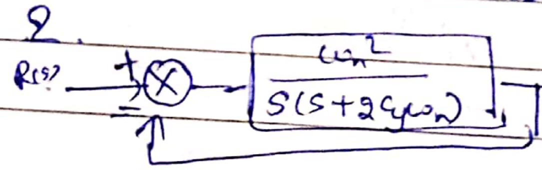
Steady state error — e_{ss}

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{t \to \infty} [\tau - \tau e^{-t/\tau}]$$

$$\boxed{e_{ss} = \tau}$$

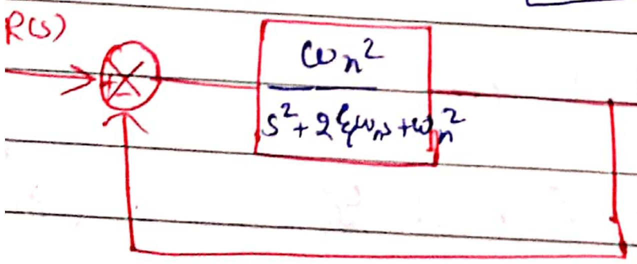
SECOND ORDER SYSTEM

For the system to be in second order, its denominator the max. value of 's' in den^r. should be 2.



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$\boxed{H(s) = 1}$



Where —

ω_n = Undamped freq

ζ = damping ratio

$$\boxed{\zeta = \frac{\text{Actual damping}}{\text{Critical damping}}}$$

if —

$\zeta = 0$ (Undamped system)

$0 < \zeta < 1$ (underdamped system)

$\zeta = 1$ (Critically damped system)

$\zeta > 1$ (over damped system)

$$\boxed{T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}}$$

$\omega_d \rightarrow$ damped freq.

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Denominator $\rightarrow \underset{a}{1}s^2 + \underset{b}{2\xi\omega_n}s + \underset{c}{\omega_n^2} = 0$

Roots $\Rightarrow$ $$S_1, S_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$S_1, S_2 = \frac{-2\xi\omega_n \pm \sqrt{(2\xi\omega_n)^2 - 4\omega_n^2}}{2}$$

$$S_1, S_2 = \frac{-2\xi\omega_n \pm \sqrt{2^2[(\xi\omega_n)^2 - \omega_n^2]}}{2}$$

$$= \frac{-\xi\omega_n \pm \sqrt{\xi^2\omega_n^2 - \omega_n^2}}{1}$$

$$\boxed{S_1, S_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}}$$

So- If $\xi = 0$ (undamped system)

$$\boxed{S_1, S_2 = 0 \pm \omega_n\sqrt{-1} \Rightarrow \pm j\omega_n}$$ {Purely imaginary roots

If $\xi = 1$ (critically damped system)

$$\boxed{S_1, S_2 = -\omega_n \pm \omega_n\sqrt{0} = -\omega_n}$$ {Purely real roots

If $0 < \xi < 1$ (Under damped system)

$$S_1, S_2 = -\xi\omega_n \pm j\omega_n\sqrt{1 - \xi^2}$$ {Take -ve common from root.

$$\boxed{S_1, S_2 = -\xi\omega_n \pm j\omega_n\omega_d}$$ {$\omega_d = \sqrt{1 - \xi^2}$

Roots are complex conjugate.

If $\xi > 1$ (Over damped system)

$$\boxed{S_1, S_2 = -\xi n \pm \omega_n\sqrt{\xi^2 - 1}}$$ (Complex root)

Response of undamped 2^{nd} order System for Unit step i/p.

$$T(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$\xi = 0$ for (Undamped syst.)

$$T(s) = \frac{\omega_n^2}{s^2 + \omega_n^2}$$

controlled o/p

$$T(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + \omega_n^2} \quad \text{— (1)}$$

Ref. i/p

For unit step sig as an i/p -

$r(t) = u(t) = 1$

$R(s) = LT[u(t)] = 1/s$ — (2)

Put (2) in (1)

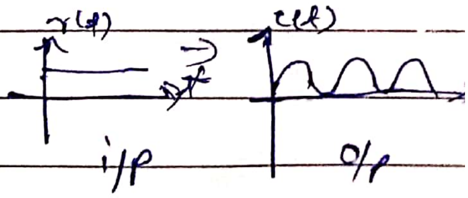
$$T(s) = \frac{1}{s} \cdot \frac{\omega_n^2}{(s^2 + \omega_n^2)} = \frac{A}{s} + \frac{Bs + C}{(s^2 + \omega_n^2)}$$

{ By partial fraction

$$C(s) = \frac{A(s^2 + \omega_n^2) + s(Bs + C)}{s(s^2 + \omega_n^2)}$$

{ $A = 1$, $B = -1$, $C = 0$

$$C(s) = \frac{1}{s} - \frac{s}{(\omega_n^2 + s^2)}$$



To find $c(t)$ take I.L.T of $C(s)$

$$c(t) = [u(t) - \cos\omega_n t \, u(t)]$$

or

$$c(t) = [1 - \cos\omega_n t] \; ; \text{ for } t > 0$$

TIME DOMAIN SPECIFICATIONS :

The desired performance characteristics of control system are specified in terms of time domain specifications.

① Delay time (t_d) ③ Peak time (t_p) ⑤ Settling time (t_s)
② Rise time (t_r) ④ Maximum overshoot (M_p) ⑥ Steady-state error (e_{ss})

① Delay time (t_d) :

It is the time required for the response to reach 50% of the final value in first attempt.

$$\boxed{t_d = \frac{1 + 0.7\xi}{\omega_n}}$$

② Rise time (t_r)

It is the time required for the response to rise from 10 to 90% of the final value for overdamped system & 0 to 100% of the final value for undamped system.

5% → 95% for Critically damped sys

$$\boxed{t_r = \frac{\pi - \phi}{\omega_d}}$$

Where —

$$\omega_d = \omega_n \sqrt{1-\xi^2}$$

$$\phi = \tan^{-1} \frac{\sqrt{1-\xi^2}}{\xi}$$

③ Peak time (t_p) :

Peak time is the time required for the response to reach the peak value of the time response.

$$\boxed{t_p = \frac{n\pi}{\omega_d}}$$

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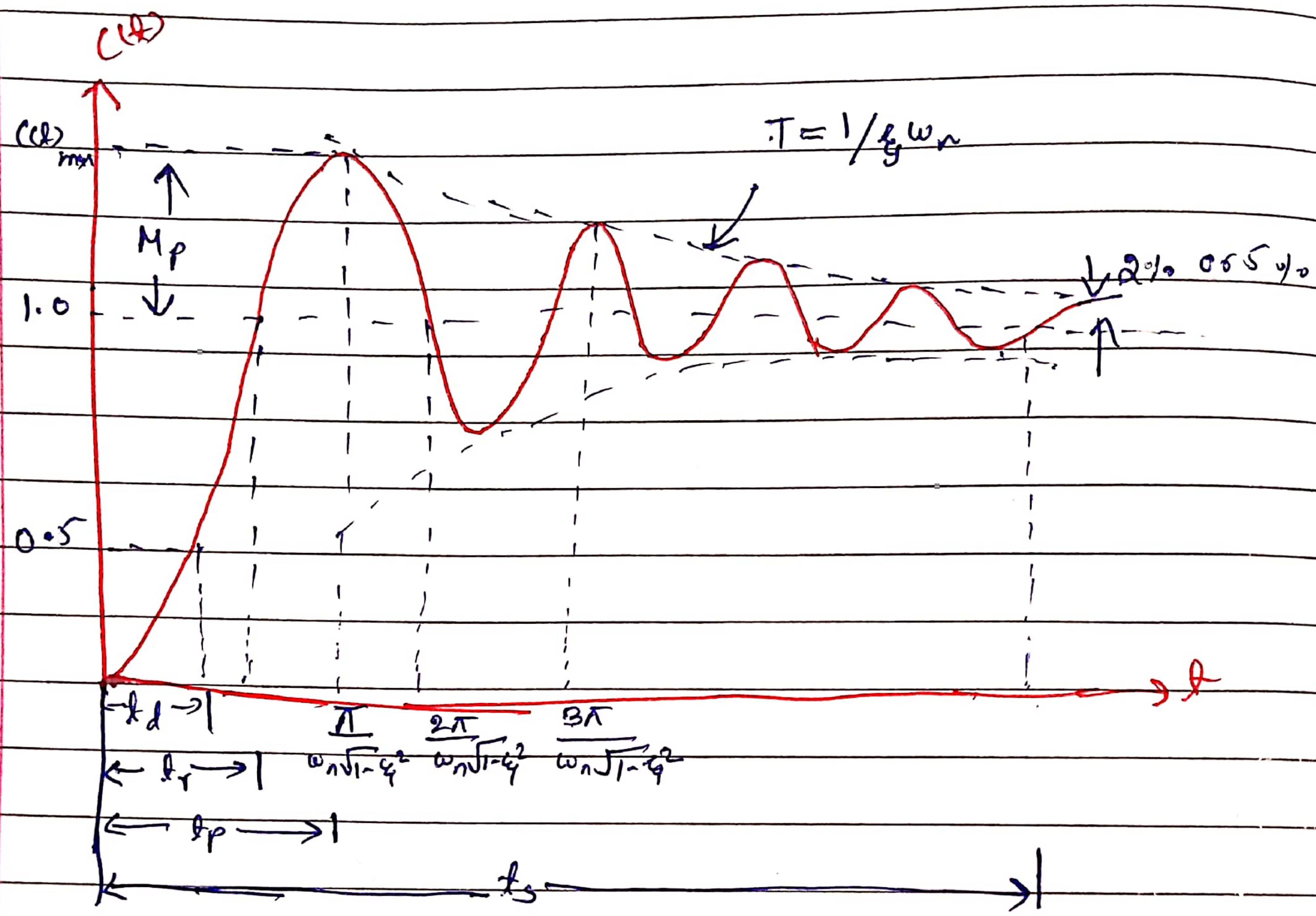

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Maximum overshoot / Peak overshoot :

It is the difference b/w the time response peak and steady state output & it is defined as-

$$M_p = C(t_p) - 1$$

$$M_p = C(t_p) - C(\infty)$$

% maximum overshoot.

$$\% M_p = \frac{C(t_p) - C(\infty)}{C(\infty)} \times 100$$

$$\boxed{\% M_p = e^{-\frac{\pi \xi}{\sqrt{1-\xi^2}}} \times 100}$$

Setting time (t_s)

It is the time required for the response to reach and stay within a specified tolerance band (usually 2 to 5 %) of the final value.

For 2 % tolerance

$$\boxed{t_s = \frac{4}{\xi \omega_n}}$$

For 5 % tolerance

$$\boxed{t_s = \frac{3}{\xi \omega_n}}$$

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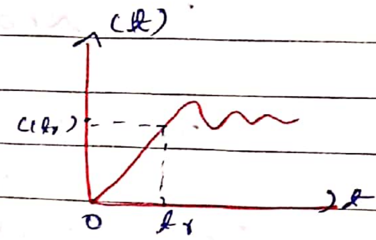
Derivation :

(i) Rise time -

Response of underdamped system —



$$C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \phi)$$

$$C(t) = C(t_r) = 1.$$

$$1 = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \phi)$$

$$0 = -\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \phi)$$

So—

$$\sin(\omega_d t_r + \phi) = 0 \quad -(1)$$

$$\sin\phi = 0, \quad \{\phi = 0, \pi, 2\pi \ldots$$

$$\sin\pi = 0$$

So -

$$\sin(\omega_d t_r + \phi) = \sin\pi.$$

$$\{\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\omega_d t_r + \phi = \pi$$

$$\omega_d t_r = \pi - \phi$$

$$t_r = \frac{(\pi - \phi)}{\omega_d} = \frac{(\pi - \phi)}{\omega_n\sqrt{1-\xi^2}}$$

So—

$$\boxed{t_r = \frac{(\pi - \phi)}{\omega_n\sqrt{1-\xi^2}}}$$

$$\left\{\tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right) = \phi\right.$$

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(ii) Peak time :

We know that -

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)$$

To calculate peak point -

$$\frac{d}{dt}(c(t))\Big|_{t=t_p} = 0$$

$$0 = \frac{d}{dt_p}(1) - \frac{d}{dt_p}\left[\frac{e^{-\xi\omega_n t_p}\sin(\omega_d t_p+\phi)}{\sqrt{1-\xi^2}}\right]$$

$$0 = 0 - \frac{1}{\sqrt{1-\xi^2}}\left[e^{-\xi\omega_n t}(-\xi\omega_n)\sin(\omega_d t_p+\phi) + e^{-\xi\omega_n t}\omega_d\cos(\omega_d t_p+\phi)\right]$$

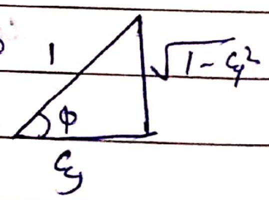
$$\xi\omega_n \sin(\omega_d t_p+\phi) = \omega_d\cos(\omega_d t_p+\phi).$$

$$\frac{\sin(\omega_d t_p+\phi)}{\cos(\omega_d t_p+\phi)} = \frac{\omega_d}{\xi\omega_n} = \frac{\omega_n\sqrt{1-\xi^2}}{\xi\omega_n} = \frac{\sqrt{1-\xi^2}}{\xi}$$

$$\tan(\omega_d t_p+\phi) = \frac{\sqrt{1-\xi^2}}{\xi}$$

Sin Cos tan
$\left(\frac{P}{H}\right)$ $\left(\frac{B}{H}\right)$ $\left(\frac{P}{B}\right)$

$$\sin(\omega_d t_p+\phi)\cdot\cos\phi - \cos(\omega_d t_p+\phi)\cdot\sin\phi = 0$$



$$\sin A \cos B - \cos A \sin B = \sin(A-B)$$

$\sin\phi = \sqrt{1-\xi^2}$

$\cos\phi = \xi$

$$\sin(\omega_d t_p + \phi - \phi) = 0$$

$$\sin(\omega_d t_p) = 0$$

$\{\sin\pi = 0$

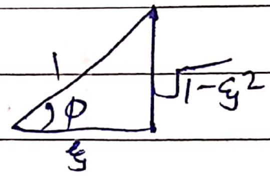
$$\sin(\omega_d t_p) + \phi = \sin\pi$$

$$\boxed{t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}$$

3) Peak overshoot (Maximum overshoot)

We know that -

$$C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t + \phi) \quad -(1)$$



We know by definition -

$$M_p = \frac{C(t_p) - C(\infty)}{C(\infty)}$$

Ratio of {Time taken from max. value to final value / to the final value

$$C(t_p) = 1 - \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\sin(\omega_d t_p + \phi)$$

$\{ t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$

Putting value of t_p -

$$C(t_p) = 1 - \frac{e^{-\xi\omega_n \times \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}}\sin(\omega_d \pi/\omega_d + \phi)$$

$$= 1 - \frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}}\sin(\pi + \phi)$$

$\{ \sin(\pi + \phi) = \sin(180 + \phi) = -\sin\phi$

$$= 1 + \frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}}\sin(\phi)$$

$\{ \sin\phi = \sqrt{1-\xi^2}$ from $\triangle$

$$C(t_p) = 1 + \frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}} \times \sqrt{1-\xi^2}$$

$$\boxed{C(t_p) = 1 + e^{-\xi\pi/\sqrt{1-\xi^2}}}$$

For $c(\infty) \rightarrow$ Put $t = \infty$ in eq? ①

$$c(\infty) = 1 - \frac{e^{-\infty(\xi\omega_n)}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)$$

$$c(\infty) = 1 - 0 = 1.$$

$$M.P = \frac{1 + e^{-\xi\pi/\sqrt{1-\xi^2}} - 1}{1}$$

$$\boxed{M.P = e^{-\xi\pi/\sqrt{1-\xi^2}} \times 100}$$

Settling time (t_s)

We know that –

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \sin(\omega_d t + \phi)$$

For 2% tolerance band –

$$c(t) \approx$$

$$\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} = 0.02$$

If 'ξ' value is very less — then denominator = 1.

$$\boxed{e^{-\xi\omega_n t_s} = 0.02}$$

$$-\xi\omega_n t_s = \ln(0.02)$$

$$-\xi\omega_n t_s = -4$$

$$\boxed{t_s = \frac{4}{\xi\omega_n}}$$

For 5 % —

$$\frac{e^{-\xi\omega_n t_s}}{\sqrt{1-\xi^2}} = 0.05$$

For low value of ξ —

$$e^{-\xi\omega_n t_s} = 0.05$$

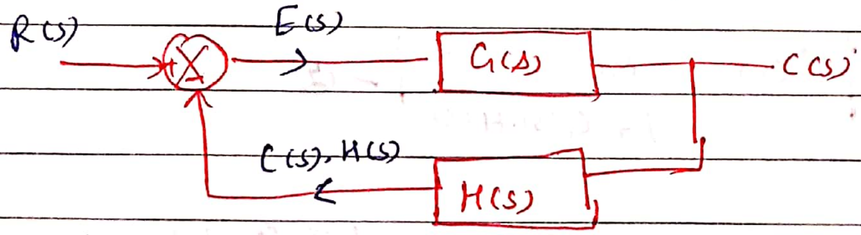
$$-\xi\omega_n t_s = \ln(0.05)$$

$$-\xi\omega_n t_s = -3$$

$$\boxed{t_s = \frac{3}{\xi\omega_n}}$$

Steady State Error —

The steady state error is the value of error when $t \to \infty$. The steady state error is the measure of system accuracy. These error arise from the nature of input, type of system & for non-linearity of system component.



$$E(s) = R(s) - C(s).H(s) \quad -(1)$$

$$C(s) = E(s)\,G(s) \quad -(2)$$

Put (2) in (1)

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$$E(s) = R(s) - E(s) \cdot G(s) \cdot H(s)$$

$$E(s)\left[1 + G(s) \cdot H(s)\right] = R(s)$$

$$\boxed{E(s) = \frac{R(s)}{1 + G(s) \cdot H(s)}}$$

To find $e(t)$ take I-L-T of $E(s)$.

$$e(t) = \mathcal{L}^{-1}\left[E(s)\right] = \mathcal{L}^{-1}\left[\frac{R(s)}{1 + G(s) \cdot H(s)}\right]$$

For steady state error:

$$\boxed{e_{ss} = \lim_{t \to \infty} e(t)}$$

$$e(t) \Rightarrow e(\infty) = \lim_{s \to 0} s\,E(s)$$ {By final value theorem}

or

$$\boxed{e_{ss} = \lim_{s \to 0} s\,E(s)}$$

Steady state error when the i/p is unit step s/g.

we know —

$$\boxed{e_{ss} = \lim_{s \to 0} s\,E(s)} \quad — ①$$

where — $$\boxed{E(s) = \frac{R(s)}{1 + G(s) \cdot H(s)}} \quad — ②$$

$$r(t) = u(t)$$ ∵ $R(s) = L.T.\,[r(t)]$

$$\boxed{R(s) = 1/s} \quad — ③$$

$$e_{ss} = \lim_{s \to 0} s\,E(s)$$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{1/s}{[1 + G(s) \cdot H(s)]}$$

$$e_{ss} = \lim_{s \to 0} \frac{1}{1 + G(s) \cdot H(s)}$$

$$\boxed{e_{ss} = \frac{1}{1 + \lim_{s \to 0} G(s) \cdot H(s)}}$$

$$\boxed{e_{ss} = \frac{1}{1 + K_p}}$$

$K_p \to$ static position error const.

Type 0 $(N = 0)$

$$\boxed{e_{ss} = \frac{1}{1 + K_p}}$$

$$K_p = \lim_{s \to 0} G(s) \cdot H(s) \Rightarrow \lim_{s \to 0} \frac{K\,(S + Z_1)(S + Z_2) + \dots}{s^0 (S + P_1)(S + P_2) - \dots}$$

$$K_p = \frac{K\,(Z_1, Z_2 \dots}{P_1, P_2 \dots} \to \text{Constant}$$

$\left\{ \begin{array}{l} Z \to \text{zeros} \\ P \to \text{Poles.} \end{array} \right.$

$K_p =$ Constant.

So –

$$\boxed{e_{ss} = \frac{1}{1 + \text{Constant}}} = \text{Constant.}$$

Loop T.f of $G(s)\cdot H(s)$

Type 1 (N=1)

$$= \frac{K(s+z_1)(s+z_2)+\ldots}{s^N(s+P_1)(s+P_2)\ldots}$$

We know -

$$e_{ss} = \frac{1}{1+k_p}$$

$$K_p = \lim_{s\to 0} G(s)H(s) = \frac{s(s+z_1)(s+z_2)\ldots}{s^1(s+P_1)(s+P_2)\ldots}$$

$$\boxed{K_p = \infty}$$

So -

$$e_{ss} = \frac{1}{1+\infty} = 0$$

$$\boxed{e_{ss} = 0}$$

Steady state error when the i/p is unit ramp s/g.

we know that -

$$e_{ss} = \lim_{s\to 0} sE(s) \quad -(1)$$

$$E(s) = \frac{R(s)}{1+G(s)\cdot H(s)} \quad -(2)$$

i/p ramp s/g -

$$r(t) = t, \quad t > 0$$

So for $R(s) \rightarrow L.T[r(t)]$

$$\boxed{R(s) = \frac{1}{s^2}}$$

$$E(s) = \frac{1/s^2}{[1+G(s)\cdot H(s)]}$$

So -

$$e_{ss} = \lim_{s\to 0} s\cdot \frac{1/s^2}{[1+G(s)\cdot H(s)]}$$

$$e_{ss} = \lim_{s \to 0} s\,E(s)$$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{1/s}{[1 + G(s) \cdot H(s)]}$$

$$e_{ss} = \lim_{s \to 0} \frac{1}{1 + G(s) \cdot H(s)}$$

$$\boxed{e_{ss} = \frac{1}{1 + \lim_{s \to 0} G(s) \cdot H(s)}}$$

$$\boxed{e_{ss} = \frac{1}{1 + k_p}}$$

$k_p \rightarrow$ static position error const

Type 0 $(N = 0)$

$$\boxed{e_{ss} = \frac{1}{1 + k_p}}$$

$$k_p = \lim_{s \to 0} G(s) \cdot H(s) \Rightarrow \lim_{s \to 0} \frac{K\,(s + z_1)(s + z_2) + \ldots}{s^0\,(s + P_1)(s + P_2) - \ldots}$$

$$k_p = \frac{K\,(z_1, z_2 \ldots)}{P_1, P_2 \ldots} \rightarrow \text{constant}$$

$\begin{cases} z \rightarrow \text{zeros} \\ P \rightarrow \text{Poles} \end{cases}$

k_p = constant.

So –

$$\boxed{e_{ss} = \frac{1}{1 + \text{constant}}} = \text{constant.}$$

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Type 1 (N=1)

Loop T.F of $G(s)\cdot H(s)$

$$= \frac{K(s+z_1)(s+z_2)+..}{s^N(s+P_1)(s+P_2)..}$$

We know -

$$e_{ss} = \frac{1}{1+k_p}$$

$$K_p = \lim_{s\to 0} G(s)H(s) = \frac{s(s+z_1)(s+z_2)...}{s^1(s+P_1)(s+P_2)....}$$

$$\boxed{K_p = \infty}$$

So -

$$e_{ss} = \frac{1}{1+\infty} = 0$$

$$\boxed{e_{ss} = 0}$$

Steady state error when the i/p is unit ramp s/g.

we know that -

$$e_{ss} = \lim_{s\to 0} s\,E(s) \quad -(1)$$

$$E(s) = \frac{R(s)}{1+G(s).H(s)} \quad -(2)$$

i/p ramp s/g -

$$r(t) = t, \quad t>0$$

So - for $R(s) \to L.T[r(t)]$

$$\boxed{R(s) = \frac{1}{s^2}}$$

$$E(s) = \frac{1/s^2}{[1+G(s).H(s)]}$$

So -

$$e_{ss} = \lim_{s\to 0} s\cdot\frac{1/s^2}{[1+G(s).H(s)]}$$

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$$e_{ss} = \lim_{s\to 0} \frac{1}{s(\cancel{1} + \cancel{G}(s)H(s))}$$

$$e_{ss} = \lim_{s\to 0} \frac{1}{s + s \cdot G(s)H(s)}$$

$$e_{ss} = \lim_{s\to 0} \frac{1}{s + s \cdot G(s)H(s)} \Rightarrow \frac{1}{0 + \lim_{s\to 0} s\,G(s)H(s)}$$

$$e_{ss} = \frac{1}{\left[\lim_{s\to 0} s\,G(s)\cdot H(s)\right]} = \frac{1}{K_V} \Rightarrow \boxed{e_{ss} = \frac{1}{K_v}}$$

$$\boxed{K_V = \lim_{K\to 0} s\,G(s)\cdot H(s)}$$

$K_V \rightarrow$ static velocity error coff.

Type 0 $(N=0)$

$$K_V = \lim_{s\to 0} s\,G(s)\cdot H(s)$$

$$K_V = \lim_{s\to 0} s \cdot \frac{K(s+z_1)(s+z_2)\ldots}{s^N(s+P_1)(s+P_2)}$$

$$K_V = \lim_{s\to 0} s \cdot \frac{K(s+z_1)(s+z_2)\ldots}{s^0(s+P_1)(s+P_2)}$$

$$K_V = \lim_{s\to 0} \frac{0\,K(s+z_1)(s+z_2)\ldots}{(s+P_1)(s+P_2)}$$

$$\boxed{K_V = 0}$$

$K_V = 0$

$$\boxed{e_{ss} = \frac{1}{K_V} = \frac{1}{0} = \infty}$$

Type 1 $(N=1)$

$$K_V = \lim_{s\to 0} s\,G(s)\cdot H(s)$$

$$K_V = \lim_{s\to 0} \cancel{s} \frac{K(s+z_1)(s+z_2)\ldots}{\cancel{s^1}(s+P_1)(s+P_2)\ldots}$$

$$K_V = \frac{K z_1 z_2}{P_1 P_2 \ldots} = \text{constant}$$

$K_V = \text{Constant}$

$$e_{ss} = \frac{1}{K_V} = \frac{1}{\text{constant}} = \text{Constant}$$

$$\boxed{e_{ss} = \text{constant}}$$

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