

Amazon Forest Dieback Tipping Points Thresholds

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I. Introduction

Nearly every Earth system is influenced by global temperature. Further, many of these systems have a “tipping point”, defined as a point where after the conditions are met, the system undergoes irreversible change. Some examples include the Indian monsoon seasons, ice sheet loss, and Atlantic Meridional Overturning Circulation. As temperatures rise due to global warming, the tipping points of certain systems can occur at a lower threshold, making determining such tipping points crucial. However, since global warming levels are unlikely to slow before the end of the century [1], if we can briefly overshoot a temperature threshold it may be possible for a system to return to its original state before irreversible change. [2]

The focus of this study is Amazon rainforest dieback. How global warming will affect the Amazon has been studied in detail. It is generally accepted that the Amazon will experience dieback due to rising temperatures, but at what timescale and severity is debated. There are many different approaches to modeling Amazon dieback. [3] uses 24 general circulation models to construct probability density functions of biomass change. This study is relatively optimistic, stating dieback can be mitigated if the benefits of higher CO₂ levels for plants outweigh other negative impacts of climate change like drought. Conversely, [4] predicts rapid dieback by the middle of the century if current levels of emissions continue. The study uses the Hadley Centre general circulation model (HadCM3) and a multilayer canopy light interception model to show that when more dynamics are considered, dieback is confirmed.

II. Modeling Approach

This study uses a simplified TRIFFID model first proposed by [5]. It models vegetation fraction v , with a single species Lotka-Volterra model given by

$$\frac{dv}{dt} = gv(1 - v) - \gamma v \quad 1$$

The term γ is the disturbance rate. The term g represents the growth term given by

$$g = g_0 \left(1 - \left(\frac{T_l - T_{opt}}{\beta} \right)^2 \right) \quad 2$$

Where g_0 is the maximum growth rate, T_{opt} is the optimal temperature for plant growth, and β is the half-width of the growth versus temperature curve. The local temperature T_l is given by

$$T_l = T_f + (1 - v)a \quad 3$$

Where T_f is the temperature with total forest cover, taken from pre-industrial conditions simulated in [5] and a is the difference between surface temperature of bare soil and forest. Note that there exists a natural feedback loop in this system. As T_l increases and

moves farther from T_{opt} the growth term decreases which causes v to decrease, which then increases T_l .

The values for each parameter are summarized in Table 1

Parameter	Value	Units
a	5	°C
β	10	°C
g_0	2	yr ⁻¹
γ	0.2	yr ⁻¹
T_{opt}	28	°C
T_f	25	°C

Table 1 Parameter Values

To further simulate global warming, we modify eq (3) to include a warming rate parameter. We have,

$$T_l = T_f + (1 - v)a + r(t) \quad 4$$

Where $r(t)$ is defined with the following piecewise function:

$$r(t) = \begin{cases} r_{warming}t & 0 < t \leq t_{end\ warming} \\ T_{overshoot} & t_{end\ warming} < t \leq t_{end\ overshoot} \\ r_{cooling}t & t_{end\ overshoot} < t \leq t_{end\ cooling} \\ T_{stable} & t_{end\ cooling} < t \leq t_{end} \end{cases}$$

The parameters of $r(t)$ are summarized in Table 2

Parameter	Description
$r_{warming}$	Rate of global warming
$r_{cooling}$	Rate of global cooling
$T_{overshoot}$	Degrees above pre-industrial conditions
T_{stable}	Stabilized temperature above pre-industrial conditions

Table 2 Parameter Values for $r(t)$

To implement the model, a combination of manual Runge Kutta 4 and Python's pre-built implementation was used. RK4 was chosen because of the relative accuracy compared to computation time.

III. Simulations and Results

a. Identifying tipping points

The first simulation ran used a manual implementation of RK4 to determine the tipping point of the system when warming increases linearly with time. The initial condition used for each simulation was $v = 0.82$ which comes from [5] and represents the vegetation fraction in the year 2000 which will serve as time 0. The results for a warming parameter of 4 °C per 100 years are given in Figure. 1.

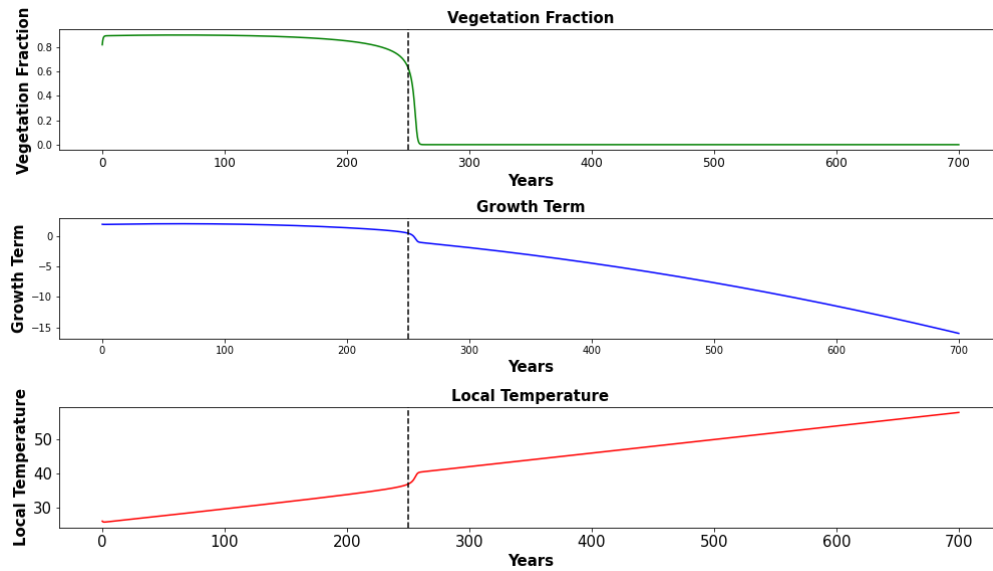


Figure 1 Vegetation fraction, growth term and local temperature

The simulation was run for 500 years so there was a total temperature increase of 20 °C due to global warming. This illustrates the fast-onset nature of the tipping points for this system. At $t = 250$ years we observe the vegetation fraction sharply decrease. The figure also demonstrates the feedback nature of the system. After the additional warming is halted, the local temperature immediately and sharply decreases.

Using Python's implementation of RK4, the tipping points for various levels of warming were simulated. Warming rates range from 1.5 °C to 5 °C per 100 years. The results are shown in Figure 2.

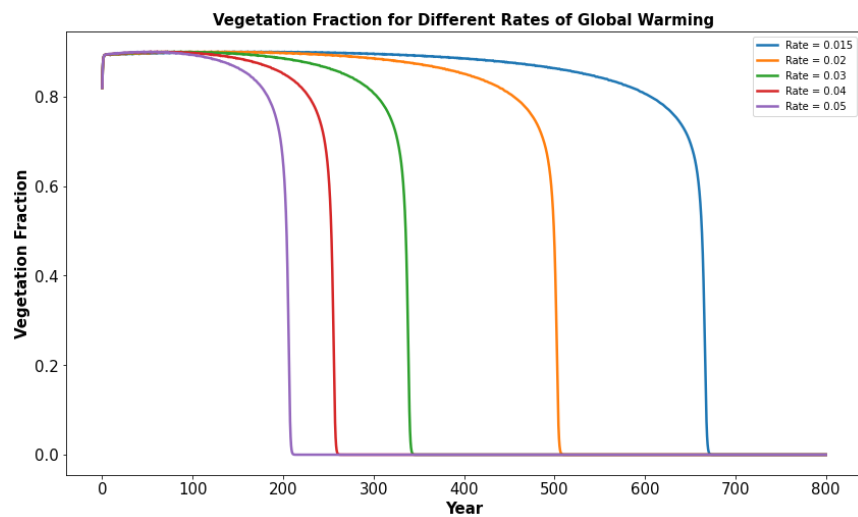


Figure 2 Tipping points corresponding to different rates of linear warming

We observe that as the level of warming increases, the tipping point, marked by the sharp decrease in vegetation, occurs more rapidly. In these scenarios, there is no possibility for the system to return to the original equilibrium.

b. Identifying overshoot thresholds

We will focus on the case where warming increases at a rate of 4 degrees per 100 years. From Figure 2 we can determine the tipping point for this rate of warming occurs after 200 years. Thus, we want to know how many years the threshold temperature can be surpassed and yet the system does not transition into the total dieback state.

By formulating the model such that $r(t)$ is piecewise defined, we can test many different levels of warming, how many years that level of warming occurs, years of overshoot, and rate of cooling. In the figures 3-6, warming was introduced for 200 years at a rate of 4 °C per 100 years and cooling to 1.5 °C for 150 years. Overshoot levels were 5 – 20 years respectively.

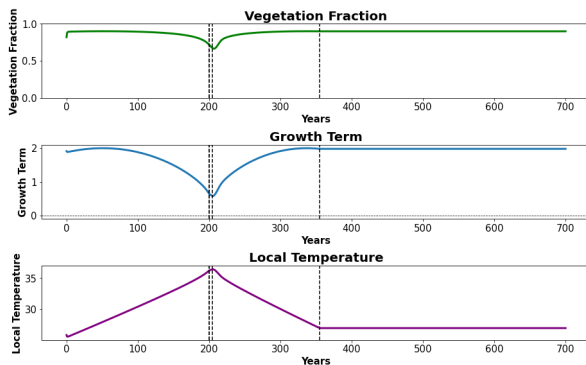


Figure 3 Overshoot by 5 years

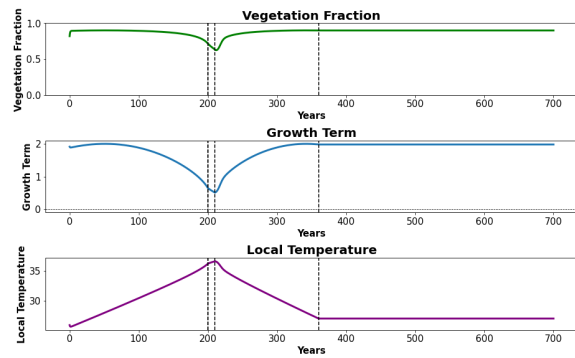


Figure 4 Overshoot by 10 years

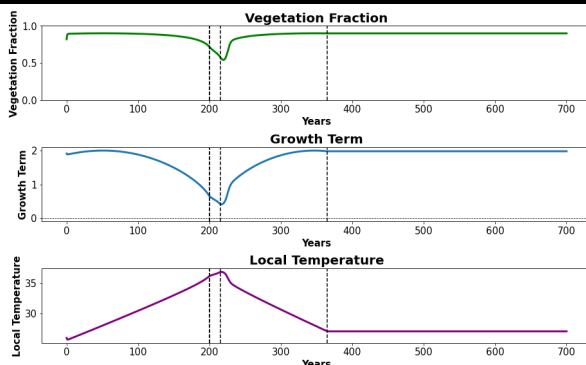


Figure 5 Overshoot by 15 years

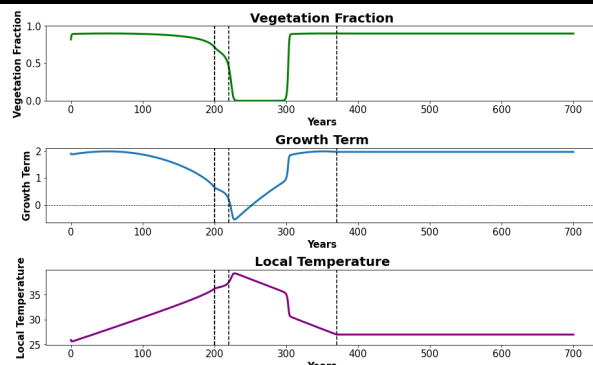


Figure 6 Overshoot by 20 years

Due to the simplicity of the model, it is possible for the vegetation fraction to appear to reach 0 but then eventually increase in the 20-year overshoot case. In actuality, the values of vegetation fraction are asymptotically close to 0 but do not actually reach 0 which allows the system to eventually return to high levels of vegetation. Further, if the Amazon rainforest reached the point of being bare soil, there would be other significant changes to the climate which would make it impossible to return to high levels of vegetation. Thus, while the model suggests the 20-year overshoot returns to ~0.87 vegetation fraction, this still would indicate an unsafe overshoot.

c. Comparison of manual and benchmark solutions

We will compare the manual solutions with different timestep sizes to a benchmark solution using Python's implementation of RK4 with a step size of 0.001. The metric used is mean squared error. The comparison is summarized in figures 7 and 8.

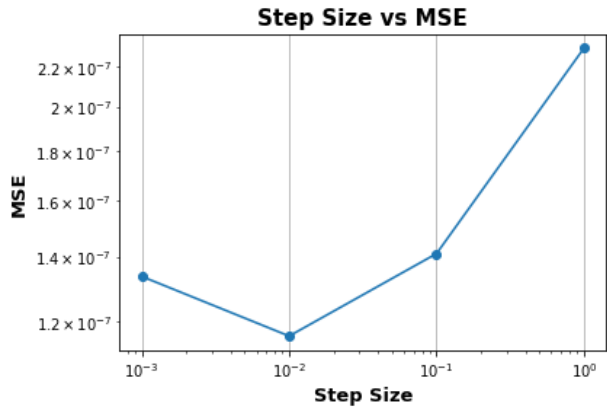


Figure 7 Step size vs MSE with benchmark solution

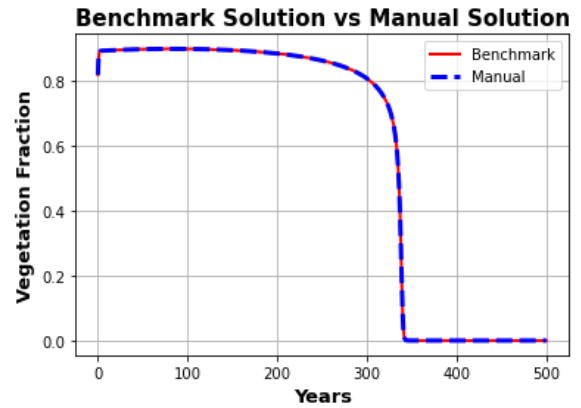


Figure 8 Benchmark and manual solution

We see that as the step size increases the error increases which corresponds with the Lax-Richtmyer Equivalence Theorem condition for consistency. The benchmark and manual solutions very close and even a step size of 1 has MSE on the order of 10^{-7} .

d. Comparison of results to literature

In Ritchie et al. the tipping point thresholds are generally lower than the results shown in this report. This could be due to several reasons. First, T_f , the temperature with total vegetation cover, could be different than what was used in the paper. Additionally, the paper does not indicate what function was used to model warming rates which would heavily impact when the tipping points occur. Due to time limitations, global warming was modeled by a linear piecewise function but using a polynomial or exponential informed by current warming rates would yield more accurate results.

IV. Conclusions

The model confirms that tipping points of Earth systems are directly influenced by global temperatures. We have observed the rate of warming is directly correlated with how fast tipping points occur. It is also interesting to note that due to the feedback nature of the model, the equilibrium point does not occur at $T_l = 28^\circ\text{C}$ which is the optimal growth rate. This is because a maximum growth rate would cause high levels of tree cover which would lower temperatures. So, rising global temperatures would not move the system to equilibrium.

There are many improvements that can be made to the model. As mentioned before, the warming function could be made more accurate with polynomial or exponential components instead of linear. Additionally, the growth rate parameter could include effects like deforestation, CO2 levels, and rainfall. Finally, the model considers only one type of vegetation but could be expanded to include more.

References

1. Raftery, A. E., Zimmer, A., Frierson, D. M., Startz, R. & Liu, P. Less than 2 °C warming by 2100 unlikely. *Nat. Clim. Chang.* **7**, 637–641 (2017).
2. Ritchie, Paul DL, et al. "Overshooting tipping point thresholds in a changing climate." *Nature* 592.7855 (2021): 517-523.
3. Rammig, Anja, et al. "Estimating the risk of Amazonian forest dieback." *New Phytologist* (2010): 694-706.
4. Huntingford, Chris, et al. "Using a GCM analogue model to investigate the potential for Amazonian forest dieback." *Theoretical and Applied Climatology* 78 (2004): 177-185.
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