CSC 250: Foundations of Computer Science I

Fall 2023 - Lecture 11

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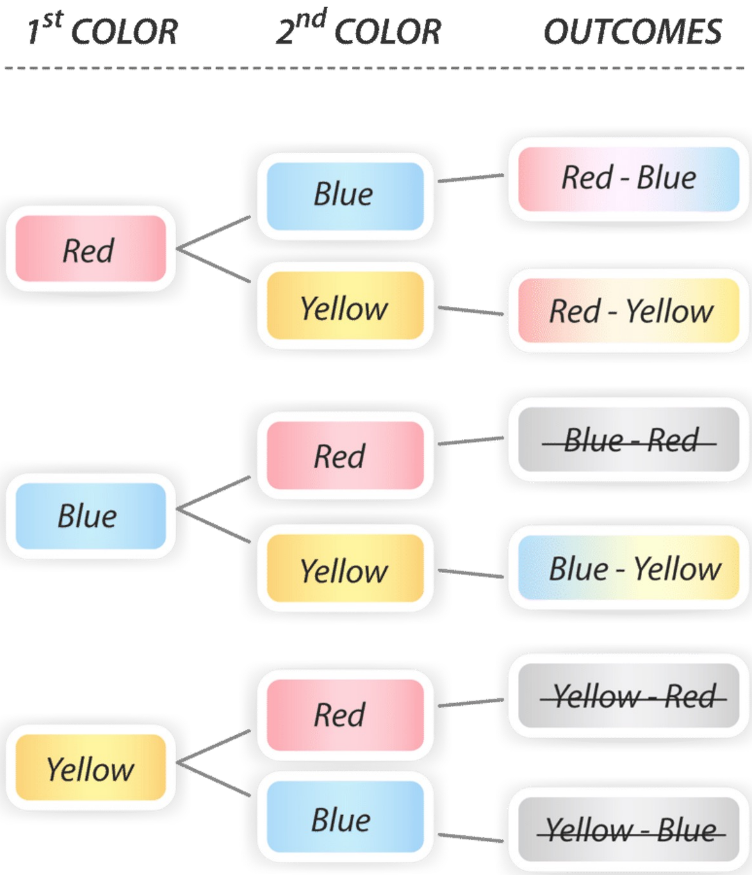
Combination
Formula Derivation



The diagram shows all 6 permutations of the 3 colors. But we are counting combinations here, so order doesn't matter. Therefore, in this tree diagram, we will cross out all outcomes that are repeats. For example, the first red-blue is no different from blue-red, so we'll cross out blue-red. In all, there are 3 combinations that are not repeats.

5 people: A B C D E

Choose 2 of them: 5 x 4 = 20 ways

AB	BA	CA	DA	EA
AC	BC	CB	DB	EB
AD	BD	CD	DC	EC
AE	BE	CE	DE	ED

AD and DA is counted as separated possibilities.

If we only cared about what 2 people end up with and not the order in which they were chosen, then every possible pair, we have twice in this table, because there is two possible orderings that group could have been chosen in.

So we have divide 20 by (the number of ways we could have chosen the group = 2)

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4 people: Choose 3 of them in a group

A B C D

4 x 3 x 2 = 24

ABC	**BAC**	**CAB**	DAB
ABD	BAD	CAD	DAC
ACB	**BCA**	**CBA**	DBA
ACD	BCD	CBD	DBC
ADB	BDA	CDA	DCA
ADC	BDC	CDB	DCB

There are 6 different ways of choosing the same 3 people. In other words, there are 6 ways (3!) of ordering 3 things.

nCr = nPr / r! (order doesn't matter)

Combination

A combination is a selection of items from a set that has distinct members, such that the **order of selection does not matter**

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

$$P(n,r) = \frac{n!}{(n-r)!}$$

Image credit - https://www.thoughtco.com/

$_5C_3$ of these

COMBINATIONS	PERMUTATIONS					
{A, B, C} $\longrightarrow$	ABC	BCA	CAB	CBA	BAC	ACB
{A, B, D} $\longrightarrow$	ABD	BDA	DAB	DBA	BAD	ADB
{A, B, E} $\longrightarrow$	ABE	BEA	EAB	EBA	BAE	AEB
{A, C, D} $\longrightarrow$	ACD	CDA	DAC	DCA	CAD	ADC
{A, C, E} $\longrightarrow$	ACE	CEA	EAC	ECA	CAE	AEC
{A, D, E} $\longrightarrow$	ADE	DEA	EAD	EDA	DAE	AED
{B, C, D} $\longrightarrow$	BCD	CDB	DBC	DCB	CBD	BDC
{B, C, E} $\longrightarrow$	BCE	CEB	EBC	ECB	CBE	BEC
{B, D, E} $\longrightarrow$	BDE	DEB	EBD	EDB	DBE	BED
{C, D, E} $\longrightarrow$	CDE	DEC	ECD	EDC	DCE	CED

$3! \cdot {}_5C_3$ of these

Image credit - https://www.math10.com/

Combination

Examples

In how many ways you can choose 8 of 32 playing cards not considering their order?

$$C(8,32) = \binom{32}{8}$$

$$C(8,32) = \frac{32!}{24!8!}$$

$$C(8,32) = \frac{32.31.30.29.28.27.26.25}{8.7.6.5.4.3.2.1}$$

$$C(8,32) = 10518300$$

The playing cards can be chosen in 10 518 300 ways.

Image credit - https://www.priklady.eu/

Marcos is handing out Frisbees to his teammates
He is handing them out 2 at a time and there are three colors: yellow, red, and blue. How many color combinations are possible?

$${}_3C_2 = \frac{(3)(2)(1)}{(2 \cdot 1)(1)} = \frac{6}{2} = 3$$

There are 18 students in a classroom. How many different eleven-person students can be chosen to play in a soccer team?

The order in which students are listed once the students are chosen does not distinguish one student from another. You need the number of combinations of 18 potential students chosen 11 at a time.

Evaluate ${}_nC_r$ with n = 18 and r = 11

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

$${}_{18}C_{11} = \frac{18!}{11!(18-11)!}$$

$${}_{18}C_{11} = \frac{18!}{11!(7)!}$$

$${}_{18}C_{11} = \frac{18\times17\times16\times15\times14\times13\times12\times11!}{11!(7\times6\times5\times4\times3\times2)}$$

$${}_{18}C_{11} = \frac{18\times17\times16\times15\times14\times13\times12}{(7\times6\times5\times4\times3\times2)}$$

$${}_{18}C_{11} = \frac{160392960}{5040}$$

$${}_{18}C_{11} = 31824$$

There are 31824 different eleven-person students that can be chosen from a group of 18 students.

Image credit - https://www.basic-mathematics.com/

Combination

Examples

A company has 20 male employees and 30 female employees. A grievance committee is to be established. If the committee will have 3 male employees and 2 female employees, how many ways can the committee be chosen?

This problem has the following two tasks:

Task 1: choose 3 males from 20 male employees

Task 2: choose 2 females from 30 female employees

${}_{20}C_3 \times {}_{30}C_2 = 1140 \times 435 = 495900$

The number of ways the committee can be chosen is 495900

Multiplication Principle

Eight candidates are competing to get a job at a prestigious company. The company has the freedom to choose as many as two candidates. In how many ways can the company choose two or fewer candidates.

The company can choose 2 people, 1 person, or none.

Notice that this time we need to use the addition principle as opposed to using the multiplication principle.

What is the difference? The key difference here is that the company will choose either 2, 1, or none. The company will not choose 2 people and 1 person at the same time. This does not make sense!

${}_8C_2 + {}_8C_1 + {}_8C_0 = 28 + 10 + 1 = 39$

The company has 39 ways to choose two or fewer candidates.

Addition Principle

Image credit - https://www.basic-mathematics.com/

Combination

Examples

It is compulsory to answer 10 questions in an examination choosing at least 4 questions from each part A and part B. If there are 6 questions in part A and 7 questions in part B, in how many ways can 10 questions be attempted?

Case I: 4 questions from part A and 6 questions from part B

Number of ways of choosing 4 questions from part A = $^{6}C_{4}$ = 15

Number of ways of choosing 6 questions from part B = $^{7}C_{6}$ = 7

Total number of ways = 15 × 7 = 105

Case II: 5 questions from part A and 5 questions from part B

Number of ways of choosing 5 questions from part A = $^{6}C_{5}$ = 6

Number of ways of choosing 5 questions from part B = $^{7}C_{5}$ = 21

Total number of ways = 6 × 21 = 126

Case II: 6 questions from part A and 4 questions from part B

Number of ways of choosing 6 questions from part A = $^{6}C_{6}$ = 1

Number of ways of choosing 4 questions from part B = $^{7}C_{4}$ = 35

Total number of ways = 1 × 35 = 35

Required number of ways = 105 + 126 + 35 = 266.

Image credit - https://byjus.com/

A box contains two white balls, three black balls and four red balls. In how many ways can three balls be drawn from the box, if at least one black ball is to be included in the draw?

Number of white balls = 2

Number of black balls = 3

Number of red balls = 4

Number of non black balls = 2 + 4 = 6

Number of black balls to be drawn	Number of red balls to be drawn	Total balls to be drawn
$^{3}C_{1}$	$^{6}C_{2}$	3
$^{3}C_{2}$	$^{6}C_{1}$	3
$^{3}C_{3}$	$^{6}C_{0}$	3

Number of ways

$$= (^{3}C_{1} \cdot {}^{6}C_{2}) + (^{3}C_{2} \cdot {}^{6}C_{1}) + (^{3}C_{3} \cdot {}^{6}C_{0})$$

$$= (3 \cdot 15) + (3 \cdot 6) + (1 \cdot 1)$$

$$= 45 + 18 + 1$$

$$= 64$$

Image credit - https://www.onlinemath4all.com/

Combination
Examples

Eg 2. A committee of 5 people is to be chosen from a group of 6 men and 4 women. How many committees are possible if

a) there are no restrictions?

Solution : $^{10}C_5$

b) one particular person must be chosen on the committee?

Solution : $\underline{1} \times {}^{9}C_4$

c) one particular woman must be excluded from the committee?

Solution : $^{9}C_5$

Eg 2. A committee of 5 people is to be chosen from a group of 6 men and 4 women. How many committees are possible if:

d) there are to be 3 men and 2 women?

Solution : **Men & Women =** $^{6}C_3 \times {}^{4}C_2$

e) there are to be men only?

Solution : $^{6}C_5$

f) there is to be a majority of women?

Solution :

3 Women & 2 men Or 4 Women & 1 man

$= {}^{4}C_3 \times {}^{6}C_2 + {}^{4}C_4 \times {}^{6}C_1$

Image credit - University of Wollongong Australia

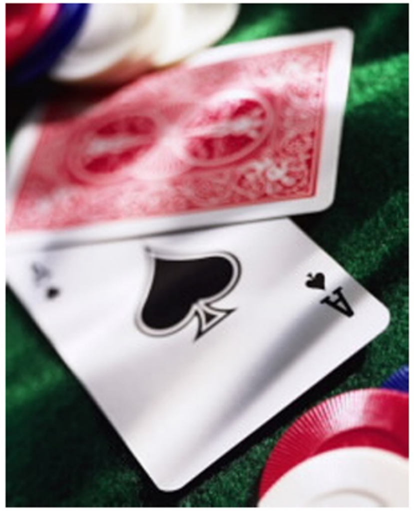
Combination
Examples

Eg 3. In a hand of poker, 5 cards are dealt from a regular pack of 52 cards.

(i) What is the total possible number of hands if there are no restrictions?

Solution :

$^{52}C_5$



Eg 3. In a hand of poker, 5 cards are dealt from a regular pack of 52 cards.

ii) In how many of these hands are there:

a) 4 Kings?

Solution : $^{4}C_4 \times {}^{48}C_1$ or 1×48

b) 2 Clubs and 3 Hearts?

Solution : $^{13}C_2 \times {}^{13}C_3$

Image credit - University of Wollongong Australia

Combination
Examples

Eg 3. In a hand of poker, 5 cards are dealt from a regular pack of 52 cards.

ii) In how many of these hands are there:

c) all Hearts?

Solution : $^{13}C_5$

d) all the same colour?

Solution : Red or Black = $^{26}C_5 + {}^{26}C_5 = 2 \times {}^{26}C_5$

Eg 3. In a hand of poker, 5 cards are dealt from a regular pack of 52 cards.

ii) In how many of these hands are there:

e) four of the same kind?

Solution :

$^{4}C_4 \times {}^{48}C_1 \times 13 = 1 \times 48 \times 13$

f) 3 Aces and two Kings?

Solution : $^{4}C_3 \times {}^{4}C_2$

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