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# CSC 250: Foundations of Computer Science I

## Fall 2023 - Lecture 10

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# Permutations

**Example** Alan, Cassie, Maggie, Seth and Roger want to take a photo in which three of the five friends are lined up in a row. How many different photos are possible?

| | | | | |
|---|---|---|---|---|
| $AMC$ | $AMS$ | $AMR$ | $ACS$ | $ACR$ |
| $ACM$ | $ASM$ | $ARM$ | $ASC$ | $ARC$ |
| $CAM$ | $MAS$ | $MAR$ | $CAS$ | $CAR$ |
| $CMA$ | $MSA$ | $MRA$ | $CSA$ | $CRA$ |
| $MAC$ | $SAM$ | $RAM$ | $SAC$ | $RCA$ |
| $MCA$ | $SMA$ | $RMA$ | $SCA$ | $RAC$ |
| $ASR$ | $MSR$ | $MCR$ | $MCS$ | $CRS$ |
| $ARS$ | $MRS$ | $MRC$ | $MSC$ | $CSR$ |
| $SAR$ | $SMR$ | $RMC$ | $CMS$ | $RCS$ |
| $SRA$ | $SRM$ | $RCM$ | $CSM$ | $RSC$ |
| $RSA$ | $MRS$ | $CRM$ | $SMC$ | $SCR$ |
| $RAS$ | $MSR$ | $CMR$ | $SCM$ | $SRC$ |

60, via an exhaustive (and exhausting!) list

Easier, using multiplication principle:

5 options for the person on the left, and

once we've chosen who should stand on the left, 4 options for the position in the middle

and once we've filled both those positions, 3 options for the person on the right.

This gives a total of $5 \times 4 \times 3 = 60$ possibilities.

We have listed all **Permutations** of the five friends taken 3 at a time.

$$\mathbf{P}(5,3) = 60$$

Image credit - https://www3.nd.edu

# Permutations

A **permutation** of **n** objects taken **k** at a time is an arrangement of k of the n objects in a specific <u>order</u>. The symbol for this number is $\mathbf{P}(\mathbf{n}, \mathbf{k})$.

Remember:

1. A permutation is an arrangement or sequence of selections of objects from a single set.
2. Repetitions are not allowed. Equivalently the same element may not appear more than once in an arrangement. (In the example above, the photo AAA is not possible).
3. the order in which the elements are selected or arranged is significant. (In the above example, the photographs AMC and CAM are different).

Image credit - https://www3.nd.edu

Permutation refers to the arrangement of objects in a definite order. That means permutation is the arrangement of objects in which order matters. The arrangement of r objects out of n objects can be calculated using the permutation formula.

## Permutation

Permutation is the arrangement of items in which **order matters**.

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

WallStreetMojo

Image credit - https://www.wallstreetmojo.com/

# Permutations

Here are some factorials.

$$\begin{aligned} 8! &= 8 - \text{factorial} \\ 11! &= 11 - \text{factorial} \\ 29! &= 29 - \text{factorial} \\ 2! &= 2 - \text{factorial, and so on} \end{aligned}$$

To compute factorials, rewrite the number before the exclamation point and all of the whole numbers that come before it.

$$\begin{aligned} 4! &= 4 \cdot 3 \cdot 2 \cdot 1 \\ 7! &= 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \\ 2! &= 2 \cdot 1 \\ 11! &= 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \end{aligned}$$

What are the values of these factorial numbers?

$$\begin{aligned} 4! &= 4 \cdot 3 \cdot 2 \cdot 1 = 24 \\ 5! &= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120 \\ 7! &= 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040 \end{aligned}$$

To compute ${}_nP_r$ you write:

$${}_nP_r = \frac{n!}{(n-r)!} = \frac{\Leftarrow \text{ total items!}}{\Leftarrow \text{ (total items-items taken at a time)!}}$$

To compute ${}_5P_3$ just fill in the numbers:

$${}_5P_3 = \frac{5!}{(5-3)!} = \frac{\Leftarrow \text{ total items!}}{\Leftarrow \text{ (total items-items taken at a time)!}}$$

$${}_5P_3 = \frac{5(4)(3)(2)(1)}{2(1)} = \frac{120}{2} = 60$$

There are 60 possible permutations.

You can use this formula anytime that you are looking to figure out permutations!

Image credit - https://flexbooks.ck12.org/

# Permutations
## Formula Derivation

Here We are making group of n different objects, selected r at a time equivalent to filling r places from n things.

| | | | | | |
|---|---|---|---|---|---|
| r-places: | 1 | 2 | 3 | 4 | r |
| The number of choices: | $n$ | $(n-1)$ | $(n-2)$ | $(n-3)$ | $n-(r-1)$ |

Permutations and Combinations

The number of ways of arranging = The number of ways of filling r places.

${}^nP_r = n.\ (n\text{-}1).\ (n\text{-}2)\ldots(n\text{-}r\text{+}1) = n/(n\text{-}r)!$

$$\frac{n.(n-1).(n-2)\cdots(n-r+1).(n-r)!}{(n-r)!} = \frac{n!}{(n-r)!} = {}^nP_r$$

so **nPr formula** we have to use is

${}^nP_r = n!/(n\text{-}r)!$

Image credit - https://lambdageeks.com/

Let us assume that there are r boxes and each of them can hold one thing. There will be as many permutations as there are ways of filling in $r$ vacant boxes by $n$ objects.

No. of ways the first box can be filled: $n$

No. of ways the second box can be filled: $(n - 1)$

No. of ways the third box can be filled: $(n - 2)$

No. of ways the fourth box can be filled: $(n - 3)$

No. of ways $r$th box can be filled: $(n - (r - 1))$

Therefore, no. of ways of filling in $r$ boxes in succession can be given by:

$n\,(n - 1)\,(n - 2)\,(n\text{-}3)\ldots(n - (r - 1))$

This can be written as:

$n\,(n - 1)\,(n - 2)\ldots(n - r + 1)$

The no. of permutations of $n$ different objects taken $r$ at a time, where $0 < r \le n$ and the objects do not repeat is:

$n\,(n - 1)\,(n - 2)\,(n - 3)\ldots(n - r + 1)$.

$\Rightarrow$ nPr = n ( n – 1) ( n – 2)( n – 3). . .( n – r + 1)

Multiplying and divided by $(n - r)\,(n - r - 1)\ldots 3 \times 2 \times 1$, we get

$$nP_r = \frac{[n(n-1)(n-2)(n-3)\ldots(n-r+1)(n-r)(n-r-1)..3\times2\times1]}{(n-r)(n-r-1)..3\times2\times1} = \frac{n!}{(n-r)!}$$

Hence,

$$nP_r = \frac{n!}{(n-r)!}$$

Where $0 < r \le n$

Image credit -https://byjus.com/

# Permutations
## Examples

Eight students should be accomodated in two 3-bed and one 2-bed rooms. In how many ways can they be accomodated?

HIDE THE SOLUTION   SHOW ALL SOLUTIONS

Solution:

Room #1: n1 = 3
Room #2: n2 = 3
Room #3: n3 = 2
n = 3 + 3 + 2 = 8

$$P'_{3,3,2}(8) = \frac{8!}{3!3!2!}$$

$$P'_{3,3,2}(8) = \frac{40320}{72}$$

$$P'_{3,3,2}(8) = 560$$

There are 560 ways of accomodating the students.

6. There are 4 czech and 3 slovak books on the bookshelf. Czech books should be placed on the left side of the bookshelf and slovak books on the right side of the bookshelf. How many ways are there to arrange the books?

HIDE THE SOLUTION   SHOW ALL SOLUTIONS

Solution:

Arranging czech books: P(4) = 4! =24
Arranging slovak books: P(3) = 3! = 6

N = P(4).P(3) = 24.6 = 144

The books may be arranged in 144 ways.

Image credit - https://www.priklady.eu/

Eight cars enter a race. The three fastest cars will be given first, second, and third places. How many arrangements of first, second, and third places are possible with eight cars?

**Solution**

Here, the order does matter since they are not just picking any 3 cars regardless of how fast they drive. They are picking the three fastest cars to give them first, second, third places.

Evaluate $_nP_r$ with n = 8 and r = 3

$_8P_3 = 8(8 - 1)(8 - 2)$

$_8P_3 = 8(7)(6) = 336$

There are 336 possible arrangements of first, second, and third places.

Image credit - https://www.basic-mathematics.com/

# Permutations
## Examples

The total number of batting orders is the number of ways to arrange 8 players in order from a squad of 16.

Evaluate ${}_nP_r$ with n = 16 and r = 8

${}_{16}P_8 = 16(16 - 1)(16 - 2)(16 - 3)(16 - 4)(16 - 5)(16 - 6)(16 - 7)$

${}_{16}P_8 = 16(15)(14)(13)(12)(11)(10)(9)$

${}_{16}P_8 = 518{,}918{,}400$

Next time a baseball coach says that he had looked at all possible batting orders and picked the best ones, just say, "sure."

6 boys and 8 girls will have a presentation in class today. If the teacher is going to allow the girls to go first, how many different arrangement are there for the presentation?

**Solution**

If the girls present first, then the number of arrangement is ${}_8P_8$

Then, when the boys present, the number of arrangements is ${}_6P_6$

Using the multiplication principle, the total number of arrangement is ${}_8P_8 \times {}_6P_6$

${}_8P_8 \times {}_6P_6 = (8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1)(6 \times 5 \times 4 \times 3 \times 2 \times 1)$

${}_8P_8 \times {}_6P_6 = (40320)(720) = 29{,}030{,}400$

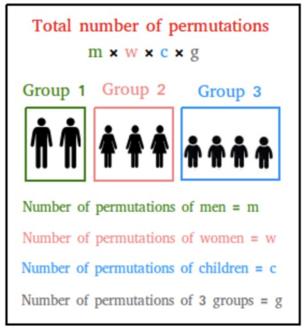
Image credit - https://www.basic-mathematics.com/

# Permutations
## Examples

A photographer is trying to take a picture of two men, three women, and four children. If the men, the women, and the children are always together, how many ways can the photographer arrange them?

**Solution**

Since the men, the women, and the children will stay together, we will have three groups. Below, see a picture of this situation.




Then, the problem has the following **four tasks**:

**Task 1**: Find the number of ways the 2 men can be arranged (${}_2P_2$)

**Task 2**: Find the number of ways the 3 women can be arranged (${}_3P_3$)

**Task 3**: Find the number of ways the 4 children can be arranged (${}_4P_4$)

**Task 4**: Find the number of ways the 3 groups can be arranged (${}_3P_3$)

Then, use the fundamental counting principle shown below to find the total number of permutations of all 4 tasks.

Therefore, evaluate ${}_2P_2$ , ${}_3P_3$ , ${}_4P_4$ , and ${}_3P_3$ and then multiply ${}_2P_2$ , ${}_3P_3$ , ${}_4P_4$ , and ${}_3P_3$ together.

$${}_2P_2 = 2(2 - 1) = 2(1) = 2$$

$${}_3P_3 = 3(3 - 1)(3 - 2) = 3(2)(1) = 6$$

$${}_4P_4 = 4(4 - 1)(4 - 2)(4 - 3) = 4(3)(2)(1) = 24$$

$${}_3P_3 = 6$$

$${}_2P_2 \times {}_3P_3 \times {}_4P_4 \times {}_3P_3 = 2 \times 6 \times 24 \times 6 = 1728$$

The photographer has 1728 ways to arrange these people. **He better not make a big fuss about it**!

Image credit - https://www.basic-mathematics.com/

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# Permutations

## Examples

**1.** A pizza parlor offers 10 toppings.

a. How many 3-topping pizzas could they put on their menu? Assume double toppings are not allowed.

b. How many total pizzas are possible, with between zero and ten toppings (but not double toppings) allowed?

c. The pizza parlor will list the 10 toppings in two equal-sized columns on their menu. How many ways can they arrange the toppings in the left column?

▼ Solution.

a. $\binom{10}{3} = 120$ pizzas. We must choose (in no particular order) 3 out of the 10 toppings.

b. $2^{10} = 1024$ pizzas. Say yes or no to each topping.

c. $P(10, 5) = 30240$ ways. Assign each of the 5 spots in the left column to a unique pizza topping.

Activate

**2.** A combination lock consists of a dial with 40 numbers on it. To open the lock, you turn the dial to the right until you reach a first number, then to the left until you get to second number, then to the right again to the third number. The numbers must be distinct. How many different combinations are possible?

▼ Solution.

Despite its name, we are not looking for a combination here. The order in which the three numbers appears matters. There are $P(40, 3) = 40 \cdot 39 \cdot 38$ different possibilities for the “combination”. This is assuming you cannot repeat any of the numbers (if you could, the answer would be $40^3$).

Activate

Image credit - https://discrete.openmathbooks.org/

# Permutations
## Examples

Example 1): There is a train whose 7 seats are kept empty, then how many ways can three passengers sit.

solution: Here n=7, r=3

so Required number of ways=

${}^{n}P_{r} = n!/(n-r)!$

${}^{7}P_{3}$ = 7!/(7-3)! = 4!.5.6.7/4! = 210

In 210 ways 3 passengers can sit.

**Example** 2) How many ways can 4 people out of 10 women be chosen as team leaders?

solution: Here n=10, r=4

so Required number of ways=

${}^{n}P_{r} = n!/(n-r)!$

${}^{10}P_{4}$ = 10!/(10-4)! = 6!7.8.9.10/6! = 5040

In 5040 ways 4 women can be chosen as team leaders.

**Example** 3) How many permutations are possible from 4 different letter, selected from the twenty-six letters of the alphabet?

solution: Here n=26, r=4

so Required number of ways=

${}^{n}P_{r} = n!/(n-r)!$

${}^{26}P_{4}$ = 26!/(26-4)! = 22!.23.24.25.26/22! = 358800

In 358800 ways, 4 different letter permutations are available.

**Example** 5) Find out the number of ways a judge can award a first, second, and third place in a contest with 18 competitors.

solution: Here n=18, r=3

so Required number of ways=

${}^{n}P_{r} = n!/(n-r)!$

${}^{18}P_{3}$ = 18!/(18-3)! = 15!.16.17.18/15! = 4896

Among the 18 contestants, in 4896 number of ways, a judge can award a 1st, 2nd and 3rd place in a contest.

Image credit - https://lambdageeks.com/

## Permutations

### Examples

**Example** In how many ways can you choose a President, secretary and treasurer for a club from 12 candidates, if each candidate is eligible for each position, but no candidate can hold 2 positions? Why are conditions 1, 2 and 3 satisfied here?

$\mathbf{P}(12, 3) = 12 \times 11 \times 10 = 1,320.$

Condition 1 is satisfied because we have a single set of 12 candidates for all 3 positions.

Condition 2 is satisfied because no one can hold more than one position.

Condition 3 is satisfied because being president is different than being treasurer or secretary.

**Example** You have been asked to judge an art contest with 15 entries. In how many ways can you assign $1^{\text{st}}$ , $2^{\text{nd}}$ and $3^{\text{rd}}$ place? (Express your answer as $\mathbf{P}(n, k)$ for some $n$ and $k$ and evaluate.)

$\mathbf{P}(15, 3) = 15 \cdot 14 \cdot 13 = 2,730.$

**Example** Ten students are to be chosen from a class of 30 and lined up for a photograph. How many such photographs can be taken? (Express your answer as $\mathbf{P}(n, k)$ for some $n$ and $k$ and evaluate.)

$\mathbf{P}(30, 10) = 30 \cdot 29 \cdot 28 \cdot 27 \cdot 26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21.$ Note $30 - 10 = 20$ and we stopped at 21.

$\mathbf{P}(30, 10) = 109,027,350,432,000$

Image credit - https://www3.nd.edu

# Permutations
## With and Without Repetition

## Repetition of an Event

If one event with **n** outcomes occurs **r** times with repetition allowed, then the number of ordered arrangements is $n^r$

**Example 1** What is the number of arrangements if a die is rolled

(a) 2 times ? $6 \times 6 = 6^2$

(b) 3 times ? $6 \times 6 \times 6 = 6^3$

(b) r times ? $6 \times 6 \times 6 \times \ldots\ldots = 6^r$

Image credit - https://documents.uow.edu.au

**Example 2**

(a) How many different car number plates are possible with 3 letters followed by 3 digits?

**Solution:** $26 \times 26 \times 26 \times 10 \times 10 \times 10 = 26^3 \times 10^3$

(b) How many of these number plates begin with ABC ?

**Solution:** $1 \times 1 \times 1 \times 10 \times 10 \times 10 = 10^3$

(c) If a plate is chosen at random, what is the probability that it begins with ABC?

**Solution:** $\dfrac{10^3}{26^3 \times 10^3} = \dfrac{1}{26^3}$

## Factorial Representation

$n! = n(n-1)(n-2)\ldots\ldots\ldots.3 \times 2 \times 1$

For example $5! = 5.4.3.2.1$ **Note** $0! = 1$

**Example**

a) In how many ways can 6 people be arranged in a row?

**Solution** : 6.5.4.3.2.1 = 6!

b) How many arrangements are possible if only 3 of them are chosen?

**Solution:** 6.5.4 = 120

# Permutations
## With and Without Repetition

### Permutation word problems with repetitions

**Word problem #6**

How many four- letter passwords can be made using the six letters a, b, c, d, e, and f?

With no repetitions, you can use the formula $_nP_r = n(n - 1)(n - 2)(n - 3) \ldots$ and evaluate $_6P_4$.

$_6P_4 = 6(6 - 1)(6 - 2)(6 - 3) = 6 \times 5 \times 4 \times 3 = 360$

> Notice that there are 6 choices for the first letter, 5 choices for the second letter, 4 choices for the third letter, and 3 choices for the fourth letter.

However, with repetitions, notice that there are 6 choices for the first letter, 6 choices for the second letter, 6 choices for the third letter, and 6 choices for the fourth letter.

$_6P_4$ with repetitions $= 6 \times 6 \times 6 \times 6 = 1296$

Image credit - https://www.basic-mathematics.com/

These are the easiest to calculate.

When a thing has ***n*** different types ... we have ***n*** choices each time!

For example: choosing ***3*** of those things, the permutations are:

$$\mathbf{n \times n \times n}$$

*(n multiplied 3 times)*

More generally: choosing ***r*** of something that has ***n*** different types, the permutations are:

$$\mathbf{n \times n \times \ldots} \textit{ (r times)}$$

(In other words, there are **n** possibilities for the first choice, THEN there are **n** possibilites for the second choice, and so on, multplying each time.)

Which is easier to write down using an exponent of **r**:

$$\mathbf{n \times n \times \ldots (r \text{ times}) = n^r}$$

> Example: in the lock above, there are 10 numbers to choose from (0,1,2,3,4,5,6,7,8,9) and we choose 3 of them:
>
> $$\mathbf{10 \times 10 \times \ldots (3 \text{ times}) = 10^3 = 1{,}000 \text{ permutations}}$$

So, the formula is simply:

$$\mathbf{n^r}$$

where ***n*** is the number of things to choose from,
and we choose ***r*** of them,
repetition is allowed,
and order matters.

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# Permutations
## Involving Conditions

**Math 3201**

## 2.3D1 Permutation Problems Involving Conditions

Permutation problems sometimes involve **conditions**. For example, in certain situations, objects may be arranged in a line where two or more objects must be placed together, or certain objects must be placed in certain positions.

We will look specifically at three types of conditions:

**Case 1:** When an element must be placed in a specific position. For example, Morgan must be the first student in the lineup.

**Case 2:** When two or more elements must be grouped together. For example Michelle and Alicia must sit together in class.

**Case 3:** When two or more elements must **not** be grouped together. For example, the red book and the blue book must not be placed next to each other on the shelf.

Image credit - http://mrpaynemath.weebly.com

**Case 1: When An Element Must Be Placed in a Specific Position**

We have already did problems like this. We solved by drawing a diagram and using the **Fundamental Counting Principle**. While there are other ways that we can work these out, the method we already used is probably the easiest, so we will continue to go with that.

**Example 1:**
At a used car lot, seven different car models are to be parked close to the road for easy viewing. The three red cars must be parked so there is a red car at each end, and one exactly in the middle. How many ways could the seven cars be parked?

$$P = \underset{R}{\underline{3}} \times \underline{4} \times \underline{3} \times \underset{R}{\underline{2}} \times \underline{2} \times \underline{1} \times \underset{R}{\underline{1}} = 144$$

* take care of conditions first!!!

**Example 2:**
Vanessa has a blue book, a red book, a yellow book and a green book that she wants to put in line on a shelf. If the red book must go at either end, how many possible arrangements are there?

$$P = \underline{3} \times \underline{2} \times \underline{1} \times \underset{R}{\underline{1}} = 6$$

or +

$$P = \underset{R}{\underline{1}} \times \underline{3} \times \underline{2} \times \underline{1} = \frac{6}{12}$$

Find your way here

# Permutations
## Involving Conditions

**Case 2: When Two or More Elements Must Be Grouped Together**

We will also look at cases in which two elements are to be grouped together. For example, how many arrangements of the word MATH exist if T and H must always be kept together.

Homework

**Steps:**

1. Treat any elements that must be grouped together as a single element.
2. Determine the total number of groups you have, $n$, and determine the number of ways they can be arranged, $n!$, using the **Fundamental Counting Principle**.
3. Look at the group that contains more than one element. Call the number of elements in that group $e$. Figure out how many ways the letters can be arranged in that group $e!$.
4. Since there are ***two conditions*** that must both be met this is an "AND" situation. Thus we must multiply $n!$ and $e!$. For example, there will be a certain number of ways the groups themselves can be arranged, and a certain number of ways the elements within the large group can be arranged.
5. Multiply the number of arrangements for the two conditions:

$$\# \text{ of possible arrangements} = n! \times e!$$

**Example 3:**
How many arrangements of the word FAMILY exist if A and L must always be together?

$$P = \overset{5}{\underset{e}{\boxed{2 \times 1}}} \times \underset{n}{4 \times 3 \times 2 \times 1}$$

$$P = e! \cdot n! = 2! \cdot 5! = 240$$

**Example 4:**
At a used car lot, seven different car models are to be parked close to the road for easy viewing. The three red cars must be parked side by side. How many ways can the seven cars be parked?

$$P = \overset{5}{\boxed{\underset{R}{3} \times \underset{R}{2} \times \underset{R}{1}}} \times 4 \times 3 \times 2 \times 1 = 3! \cdot 5! = 720$$

Image credit - http://mrpaynemath.weebly.com

Image credit - http://mrpaynemath.weebly.com

# Permutations
## Involving Conditions

**Case 3: When Two or More Elements Must Not Be Grouped Together**

We will not calculate the number of arrangements in these problems directly, but rather we will use the following:

| Number of arrangements when two elements **cannot be grouped together** | = | Total number of possible arrangements | − | Number of possible arrangements when the two arrangements **are** grouped together |
|---|---|---|---|---|

This is called **indirect reasoning.**

**Example 5:**
Michael, Bradley, Jarod and Andrew are to be arranged in a line from left to right. How many ways can they be arranged if Michael and Bradley are **not** to stand next to each other?

$$P(\text{all arrangements}) = \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 24$$

$$P(\text{together}) = \boxed{\underset{M}{2} \times \underset{B}{1}} \times 2 \times \underline{1} = 2! \cdot 3! = 12$$

$$P(\text{not together}) = P(\text{all}) - P(\text{together})$$
$$= 24 - 12$$
$$= 12$$

**Example 6:**
Todd, Jean, Kyle, Colin and Lori are to be arranged in a line from left to right.

(A) How many ways can they be arranged?

$$P = \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 120$$

(B) How many ways can they be arranged if Jean and Lori cannot be side by side?

$$P(S \times S) = \boxed{\underset{J}{2} \times \underset{L}{1}} \times \underline{3} \times \underline{2} \times \underline{1} = 2! \cdot 4! = 48$$

$$P(S \times S)' = 120 - 48 = 72$$

(C) How many ways can they be arranged if Kyle and Colin must be side by side?

$$P = \boxed{\underset{K}{2} \times \underset{C}{1}} \times \underline{3} \times \underline{2} \times \underline{1} = 48$$

(D) How many ways can they be arranged if Jean must be at one end of the line? HINT: Jean must be in the first OR the last location.

$$P(\text{Beginning}) \text{ or } P(\text{end})$$

$$P = \underset{J}{1} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} + \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} \times \underset{J}{1} = 24 + 24 = 48$$