

## Wave Motion

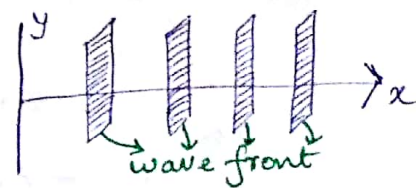
Equation of a progressive wave propagating through a continuous medium where the particles of the medium execute SHM is

$$y = A \sin(\omega t - kx) = A \sin \frac{2\pi}{\lambda} (vt - x)$$

for propagation along +ve x-direction.

Similarly  $y = A \sin(\omega t + kx) = A \sin \frac{2\pi}{\lambda} (vt + x)$

for propagation along -ve x-direction.

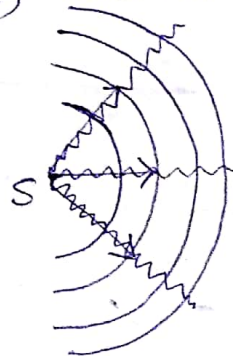


Note that if  $t$  increases by  $\Delta t$  and  $x$  by  $v\Delta t$ , then  $y$  will be restored to initial value. Thus a "disturbance" at one place is repeated at a position  $v\Delta t$  after time  $\Delta t$ , with propagating velocity  $v$ , that propagates without damping. In reality, its amplitude gradually diminishes due to resistive forces of the viscous medium.

Equation of such wave is  $y = A e^{-\gamma x} \sin \frac{2\pi}{\lambda} (vt - x)$  which is plane progressive wave.

If the wave is diverging then without any dissipation of energy (no damping), the amplitude falls off. In case of ~~spherical~~ spherical progressive wave,

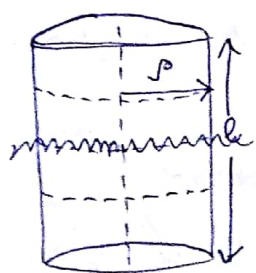
spherical progressive wave, Intensity  $I \propto \frac{W}{4\pi r^2}$



where  $W$  is the energy emitted from source.  $\therefore$  Amplitude<sup>2</sup>  $\propto \frac{1}{r^2}$

$\propto$  Amplitude  $\propto \frac{1}{r}$ . Such wave is represented as

$$y = \frac{A}{r} \sin \frac{2\pi}{\lambda} (vt - |r|) \quad |r| = \sqrt{x^2 + y^2 + z^2} \text{ (Expanding wave)}$$



For cylindrical progressive wave  $I = \frac{W}{2\pi r l} \propto I \propto \frac{1}{\sqrt{r}}$

$$\therefore A^2 \propto \frac{1}{\sqrt{r}} \propto A \propto \frac{1}{\sqrt[4]{r}}$$

Such wave is represented by  $y = \frac{A}{\sqrt{r}} \sin \frac{2\pi}{\lambda} (vt - r)$

The velocity  $v = \dot{x}$  is called the wave velocity as the disturbance propagates with this velocity. This is also called phase velocity as during motion of the wave, phase of the motion of the particles move with this velocity. But as the wave move

onward, particles vibrate about their mean position of rest which is called the particle velocity.  $v$

Now  $y = A \sin(\omega t - kx)$  gives  $v = \frac{dy}{dt} = A\omega \cos(\omega t - kx)$

and  $\frac{dy}{dx}$  = slope of the displacement of the particle  
 $= -Ak \cos(\omega t - kx) = -Ak \frac{v}{A\omega} = -\frac{k}{\omega} v$

$\therefore v = -\frac{\omega}{k} \frac{dy}{dx} = -v \frac{dy}{dx}$  → negative gradient of displacement.

particle velocity

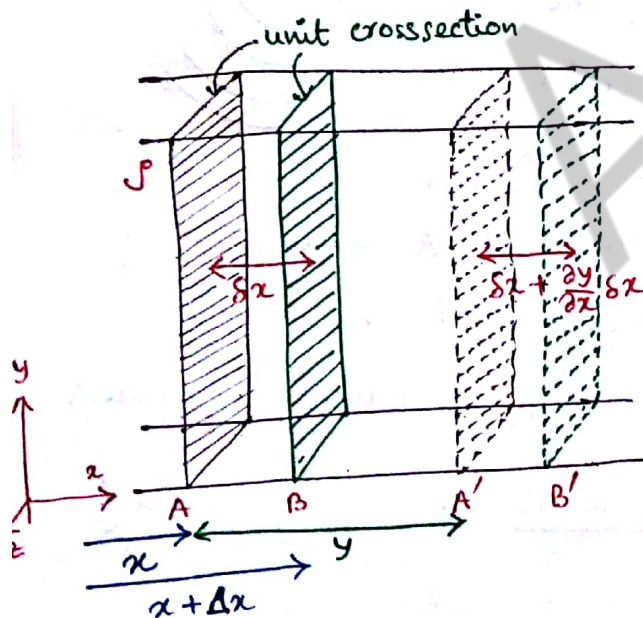
phase velocity

Also,  $\frac{\partial^2 y}{\partial t^2} = -A\omega^2 \sin(\omega t - kx)$ ,  $\frac{\partial^2 y}{\partial x^2} = -Ak^2 \sin(\omega t - kx)$

$\therefore \frac{\partial^2 y}{\partial x^2} = \left(\frac{k}{\omega}\right)^2 \frac{\partial^2 y}{\partial t^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$

$\therefore \frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$  is the differential equation for a progressive wave.

Plane longitudinal wave through an elastic fluid medium



Transverse wave cannot resist any shearing stress just like a fluid. So no transverse wave is formed in a fluid. Consider a unit crosssectional area in a fluid medium with its axis in the direction of propagation of the sound wave. Let A & B are two normal planar sections at  $x$  &  $x + \Delta x$  from some arbitrary origin.

In progression, let at an instant plane A is displaced by  $y$  to  $A'$  and at the same time the plane B is displaced to  $B'$  by an amount  $y + \frac{\partial y}{\partial x} \delta x$ .

$\therefore$  The actual position of  $A'$  plane is  $(x + y)$  &  $B'$  plane is  $(x + y + \delta x + \frac{\partial y}{\partial x} \delta x)$ .



Considering the tube has unit cross section, volume of fluid between A & B is  $\delta x$  & that between A' & B' is  $\delta x + \frac{\partial y}{\partial x} \delta x$ .  $\therefore$  Change in volume due to displacement  $\Delta V = \frac{\partial y}{\partial x} \delta x$  & hence the volume strain  $\frac{\Delta V}{V} = \frac{\frac{\partial y}{\partial x} \delta x}{\delta x} = \frac{\partial y}{\partial x}$ .

If  $\delta P$  be the excess pressure over the normal pressure on plane A that is transferred to A', then the Bulk modulus of the medium is

$$B = - \frac{\delta P}{\Delta V/V} = - \frac{\delta P}{\frac{\partial y}{\partial x}}, \quad \text{or} \quad \delta P = -B \frac{\partial y}{\partial x}. \quad \text{Negative sign}$$

indicating that the volume decreases with the increase in pressure.

Now the excess pressure over face B which is transferred to B' is

$$\begin{aligned} \delta P + \frac{\partial(\delta P)}{\partial x} \delta x &= -B \frac{\partial y}{\partial x} + \frac{\partial}{\partial x} \left( -B \frac{\partial y}{\partial x} \right) \delta x \\ &= -B \frac{\partial y}{\partial x} - B \frac{\partial^2 y}{\partial x^2} \delta x \quad [B \neq B(x)] \end{aligned}$$

$\therefore$  Excess pressure on the volume element is

$-B \frac{\partial y}{\partial x} - \left( -B \frac{\partial y}{\partial x} - B \frac{\partial^2 y}{\partial x^2} \delta x \right) = B \frac{\partial^2 y}{\partial x^2} \delta x$ . This pressure will create an acceleration along the direction of force within the volume element. If  $\rho$  is the density (medium) then using Newton's

2<sup>nd</sup> law  $\rho \delta x \frac{\partial^2 y}{\partial t^2} = B \frac{\partial^2 y}{\partial x^2} \delta x \quad \therefore \quad \boxed{\frac{\partial^2 y}{\partial t^2} = \frac{B}{\rho} \frac{\partial^2 y}{\partial x^2} = v^2 \frac{\partial^2 y}{\partial x^2}}$

Compressional wave due to longitudinal vibrations of a long thin rod:

Consider two normal cross section A & B at a distance  $x$  &  $x + \delta x$  from some arbitrary origin on a uniform thin rod whose length is large compared to its lateral dimension. Due to flow of compressional wave along the length, A is displaced to A' at a distance  $y$  & B is displaced to B' at a distance  $(y + \frac{\partial y}{\partial x} \delta x)$ . The actual position of A' and B' are  $(x+y)$  and  $(x+\delta x+y+\frac{\partial y}{\partial x} \delta x)$ . Thickness of the slice AB is  $\delta x$  and after time  $t$ , thickness of A'B' is  $\delta x + \frac{\partial y}{\partial x} \delta x$ .

∴ Change in length of the slice is  $\frac{\partial y}{\partial x} \delta x$  & hence the longitudinal strain =  $\frac{\frac{\partial y}{\partial x} \delta x}{\delta x} = \frac{\partial y}{\partial x}$ . If  $F$  is the stretching force per unit crosssectional area (or stress) of the plane A &  $Y$  is the Young's modulus of the material then  $Y = \frac{F}{\partial y / \partial x}$ , ∴  $F = Y \frac{\partial y}{\partial x}$ .

The stretching stress acting on B which is displaced to B' is  $Y \frac{\partial y}{\partial x} + \frac{\partial}{\partial x} \left( Y \frac{\partial y}{\partial x} \right) \delta x = Y \frac{\partial y}{\partial x} + Y \frac{\partial^2 y}{\partial x^2} \delta x$

∴ The total force on AB in the +x direction is  $Y \frac{\partial^2 y}{\partial x^2} \delta x$ .  
If  $\rho$  is material density, then  $\rho \delta x \frac{\partial^2 y}{\partial t^2} = Y \frac{\partial^2 y}{\partial x^2} \delta x$

$$\therefore \boxed{\frac{\partial^2 y}{\partial t^2} = \frac{Y}{\rho} \frac{\partial^2 y}{\partial x^2} = v^2 \frac{\partial^2 y}{\partial x^2}} \quad v = \sqrt{\frac{Y}{\rho}}$$

For iron,  $Y = 10^{12}$  dyne/cm<sup>2</sup>,  $\rho = 8.9$  gm/cc.  $v \approx 10^6$  cm/s.

If lateral strain is also accounted then,  $v = \sqrt{\frac{Y + \frac{4}{3}\eta}{\rho}}$ , where  $\eta$  = modulus of rigidity. This indicates that consideration of lateral strain enhances the velocity of longitudinal wave through the medium.

Longitudinal waves in a Gas  $\frac{\partial^2 y}{\partial t^2} = \frac{E}{\rho} \frac{\partial^2 y}{\partial x^2}$ ,  $v = \sqrt{\frac{E}{\rho}}$

where  $E$  is volume elasticity (bulk modulus) of gas,  $\rho$  = density.

Newton's Formula: Newton first calculated the velocity of sound wave in a gas, on the assumption that temperature variation is negligible, so in isothermal process, Boyle's law is applicable.

$$PV = \text{constant} \quad \therefore P\delta V + \delta P V = 0 \quad \therefore P = - \frac{\delta P}{\delta V/V} = E$$

$$\therefore v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{P}{\rho}} \quad \text{for air at standard pressure \& temperature (STP), } \rho = 1.29 \text{ kg/m}^3, P = 0.76 \text{ m of Hg}$$

$$= 0.76 \times 13.6 \times 10^3 \times 9.8 \text{ N/m}^2$$

$$v = 280 \text{ m/s}$$

But experimental value came as  $v = 332$  m/s.



Laplace's correction: Temperature correction, region of compression is heated & region of rarefaction is cooled. Since the thermal conductivity of a gas is small, that these thermal changes occur at much faster time scale that heat developed during compression & cooling due to rarefaction is not transferred out to thermalize. So in adiabatic condition,  $PV^\gamma = \text{constant}$ ,  $\gamma = C_p/C_v$

$$\therefore \Delta P V^\gamma + \gamma P V^{\gamma-1} \Delta V = 0 \quad \therefore \gamma P = - \frac{\Delta P V^\gamma}{V^{\gamma-1} \Delta V} = - \frac{\Delta P}{\Delta V/V} = E$$

$$\therefore v = \sqrt{\frac{\gamma P}{\rho}} \quad \gamma = 1.4 \text{ for air at STP, } v = 331.6 \text{ m/s.}$$

$$\text{Now } PV = \frac{m}{M} RT \text{ and } \rho = \frac{m}{V} \quad \therefore v = \sqrt{\frac{\gamma RT}{M}}$$

So velocity is independent of pressure or density,  $v \propto \sqrt{T}$ ,  
 $v \propto \frac{1}{\sqrt{M}}$ ,  $v \propto \sqrt{\gamma}$  i.e. whether monoatomic, diatomic gas etc.

### Energy Transport in Travelling Waves

When a plane progressive harmonic wave passes through a medium, medium particles contain extra energy due to SHM in terms of KE & PE. Total energy is conserved.

$$y = a \sin \frac{2\pi}{\lambda} (vt - x) \text{ is displacement}$$

$$\text{velocity of each particle } v = \frac{dy}{dt} = \frac{2\pi va}{\lambda} \cos \frac{2\pi}{\lambda} (vt - x)$$

$$\text{For } \rho = \text{density, KE per unit volume} = \frac{1}{2} \rho v^2 = \frac{1}{2} \rho \left( \frac{2\pi va}{\lambda} \right)^2 \cos^2 \frac{2\pi}{\lambda} (vt - x)$$

$$\text{Since this varies over time, average KE density of the medium is } \langle KE \rangle = \frac{\rho}{2} \frac{4\pi^2 v^2 a^2}{\lambda^2} \underbrace{\langle \cos^2 \frac{2\pi}{\lambda} (vt - x) \rangle}_{= 1/2} = \frac{\rho \pi^2 v^2 a^2}{\lambda^2}$$

To evaluate the average P.E. we calculate work done for the decrease of volume  $\delta V$  against an average pressure  $\frac{P + (P + \delta P)}{2}$  is

$$dW = \left( P + \frac{\delta P}{2} \right) \delta V. \quad \text{But } B = - \frac{\delta P}{\delta V/V} \text{ and } \delta V = - \frac{\partial y}{\partial x} \delta x$$

$$= \left( P - B/2 \frac{\partial y}{\partial x} \right) \left( - \frac{\partial y}{\partial x} \delta x \right)$$

$$\therefore \delta P = - \frac{B}{V} \delta V = - B \frac{\partial y}{\partial x}$$

$$\therefore \delta V/V = \frac{\partial y}{\partial x}$$

$$= -p \frac{\partial y}{\partial x} \delta x + \frac{B}{2} \left( \frac{\partial y}{\partial x} \right)^2 \delta x$$

∴ The work done per unit volume  $W = -p \frac{\partial y}{\partial x} + \frac{B}{2} \left( \frac{\partial y}{\partial x} \right)^2$

$$= p \frac{2\pi a}{\lambda} \cos \frac{2\pi}{\lambda} (vt - x) + \frac{B}{2} \left( \frac{2\pi a}{\lambda} \right)^2 \cos^2 \frac{2\pi}{\lambda} (vt - x)$$

$$\therefore \text{Time averaged P.E.} \propto \langle \text{P.E.} \rangle = \frac{2\pi a p}{\lambda} \langle \cos \frac{2\pi}{\lambda} (vt - x) \rangle + \frac{4\pi^2 a^2 B}{2\lambda^2} \langle \cos^2 \frac{2\pi}{\lambda} (vt - x) \rangle$$

$$= \frac{\pi a^2 B}{\lambda^2}$$

$$\langle \text{PE} \rangle = \frac{\pi^2 \rho v^2 a^2}{\lambda^2} = \langle \text{KE} \rangle$$

$$\text{as } v = \sqrt{B/\rho}$$

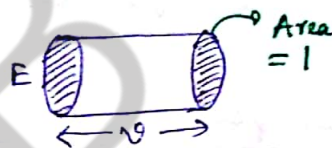
So average value of PE & KE are same. Therefore total energy of the medium is

$$E = \frac{2\pi^2 \rho v^2 a^2}{\lambda^2}$$

which is the energy crossing unit

area per unit time or intensity.  $I \propto a^2, \propto \rho, \propto 1/\lambda^2, \propto v^3$

$$I = vE = \frac{2\pi^2 \rho v^3 a^2}{\lambda^2}$$



### Unit of Intensity: Bel & Decibel

The ratio of sound intensity from low to high in the detectable range is  $1:10^{14}$ . So logarithmic ratio is 14:1 for high to low. For all practical purposes, absolute intensity is unnecessary & relative values have more practical significance.

Bel & Decibel are logarithmic units of relative intensity. If ratio of sound intensity is 10:1 then difference of intensity is "1 Bel".  $N = \log_{10} I_1/I_2$  where  $N$  = number of bels and

$I_1$  &  $I_2$  are intensity of two sounds. A "decibel (db)" is  $\frac{1}{10}$ th of

Bel.  $n = 10 \log_{10} I_1/I_2$  where  $n$  = no. of decibels.

$$1 \text{ bel} = 10 \text{ dB} = 10^1 : 1$$

$$\therefore 1 \text{ dB} = 10^{0.1} : 1$$

$$\therefore N \text{ bels} = 10N \text{ dB} = 10^N : 1$$

$$= 1.26 : 1$$

$$0.1N \text{ bels} = n \text{ dB} = 10^{0.1n} : 1$$



### Intensity level (IL)

IL is ratio of intensity to standard intensity  $I_0$  where threshold is  $10^{-12} \text{ watt/m}^2$  or  $10^{-16} \text{ watt/cm}^2$  which corresponds to lower limit of intensity for audability.

$$\therefore (IL)_{\text{bel}} = \log_{10} \left( \frac{I}{I_0} \right), \quad (IL)_{\text{db}} = 10 \log_{10} \left( \frac{I}{I_0} \right)$$

This means  $I = 10^2 I_0$ ,  $(IL)_{\text{bel}} = 2 \text{ bels} = 20 \text{ decibels}$

$$I_{\text{max}} = 10^{14} I_0, \quad (IL)_{\text{bel}} = 14 \text{ bels} = 140 \text{ decibels.}$$

Conversely Intensity 140 db means  $140 = 10 \log_{10} \frac{I}{I_0}$

$$\therefore I = I_0 \times 10^{14} = 10^{-16} \times 10^{14} = 10^{-2} \text{ watt/cm}^2$$

### Relation between wave intensity & mean square of excess pressure

The excess pressure due to propagation of sound wave is

$$p = -B \frac{\partial y}{\partial x} \quad \text{where } B = \rho v^2 = \text{Bulk modulus of the medium}$$

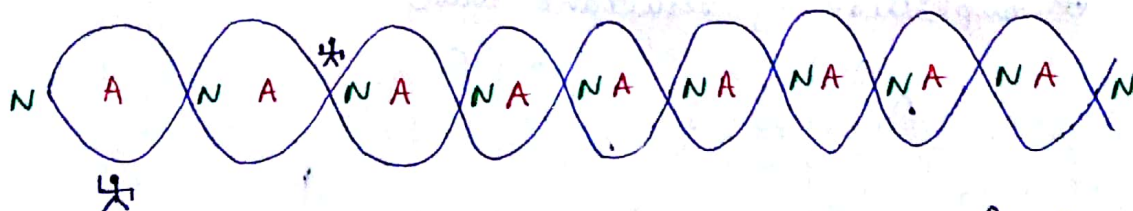
$$\text{displacement } y = a \sin \frac{2\pi}{\lambda} (vt - x)$$

$$\therefore p = \rho v^2 a \frac{2\pi}{\lambda} \cos \frac{2\pi}{\lambda} (vt - x) \quad \text{or } p^2 = \frac{4\pi^2 v^4 \rho^2 a^2}{\lambda^2} \cos^2 \frac{2\pi}{\lambda} (vt - x)$$

$$\therefore \langle p^2 \rangle = \frac{4\pi^2 v^4 \rho^2 a^2}{\lambda^2} \langle \cos^2 \frac{2\pi}{\lambda} (vt - x) \rangle = \frac{2\pi^2 v^4 \rho^2 a^2}{\lambda^2}$$

$$\text{But we know } I = \frac{2\pi^2 v^3 \rho a^2}{\lambda^2} = \frac{\langle p^2 \rangle}{\rho v}$$

So whenever pressure is maximum,  $I$  is maximum  $\rightarrow$  loud sound (Node)  
pressure is minimum,  $I$  is minimal  $\rightarrow$  very low sound (Antinode)



$y \propto \sin()$ ,  $p \propto \cos()$ .  $p$  and  $y$  are  $\frac{\pi}{2}$  phase different  
so whenever  $y$  is minimum,  $p$  is maximum and vice versa.

## Phase velocity and Group velocity

A single progressive wave along +ve x axis is represented by  $y = a \sin(\omega t - kx) = a \sin \frac{2\pi}{\lambda} (vt - x)$  where  $v$  is the "phase velocity" of the wave, as with this velocity phase of a wave moves from one point to another point.

But when two or more such harmonic waves of slightly different frequencies are superposed, the anharmonic phenomena of beat occurs. These beats are generally known as modulations and such anharmonic motion has modulated amplitude that repeats at a frequency called beat frequency or modulation frequency. It carries energy from one point to another with a velocity different from those of the harmonic waves. These travelling modulations that consist of group of harmonic waves are called wave packets or wave groups. The velocity with which this modulated amplitude moves is called "group velocity".

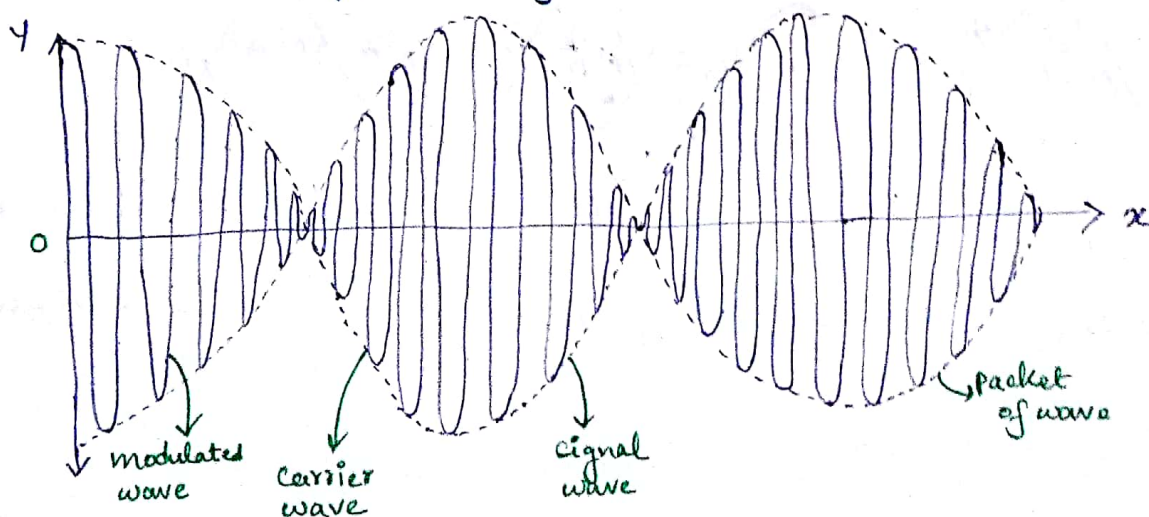
If we superpose two different waves that are slightly different in frequency/wavelength, then resultant amplitude is

$$y = y_1 + y_2 = a \sin(\omega t - kx) + a \sin[(\omega + \Delta\omega)t - (k + \Delta k)x]$$

$$\therefore y = 2a \cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right) \sin\left[(\omega + \frac{\Delta\omega}{2})t - (k + \frac{\Delta k}{2})x\right]$$

$$\therefore y = A \sin\left[(\omega + \frac{\Delta\omega}{2})t - (k + \frac{\Delta k}{2})x\right]$$

⇓ amplitude of resultant wave





This represents a travelling wave at a frequency  $\omega + \frac{\Delta\omega}{2}$ , wave vector  $k + \frac{\Delta k}{2}$  and amplitude  $2a \cos(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x)$  which is modulated by an envelope of frequency  $\Delta\omega$ . Thus we can say that velocity of this amplitude is  $\frac{\Delta\omega}{\Delta k}$  and in  $\lim_{\Delta k \rightarrow 0}$ , we get  $V_g = \frac{d\omega}{dk}$  is the group velocity. Note that the phase velocity is  $V_p = \frac{\omega}{k}$ .

$$\therefore \frac{d\omega}{dk} = \frac{d}{dk}(kV_p) \quad \text{or} \quad V_g = V_p + k \frac{dV_p}{dk}$$

$$\text{Using } k = \frac{2\pi}{\lambda}, \quad dk = -\frac{2\pi}{\lambda^2} d\lambda, \quad V_g = V_p - \frac{2\pi}{\lambda} \frac{\lambda^2}{2\pi} \frac{dV_p}{d\lambda}$$

$$\text{or } \boxed{V_g = V_p - \lambda \frac{dV_p}{d\lambda}}$$

Dispersive Medium A medium where the phase velocity of any wave varies with frequency/wavelength is called a dispersive medium. Glass, water & all transparent substances are dispersive for electromagnetic waves. In dispersive medium  $\lambda_1 \neq \lambda_2 \neq \lambda_3 \neq \dots \neq \lambda_n$  and  $V_g$  is always greater than  $V_p$ . However in nondispersive medium  $\lambda_1 = \lambda_2 = \dots = \lambda_n$  and  $V_g = V_p$ . Vacuum is nondispersive to EM waves but dispersive for de Broglie wave (matter wave). Note that all mediums are nondispersive for sound waves, therefore all waves of different wavelength move with a constant velocity in a medium.

$$\text{Now } \frac{1}{V_g} = \frac{dk}{d\omega} = \frac{d}{d\omega}\left(\frac{\omega}{V_p}\right) = \frac{1}{V_p} - \frac{\omega}{V_p^2} \frac{dV_p}{d\omega}$$

$$\text{As } \omega = 2\pi\nu, \quad d\omega = 2\pi d\nu. \quad \therefore \frac{1}{V_g} = \frac{1}{V_p} - \frac{\nu}{V_p^2} \frac{dV_p}{d\nu}$$

But  $V_p = \frac{c}{n}$  where  $n$  is the refractive index of the medium.

$$\therefore \frac{1}{V_g} = \frac{1}{V_p} - \frac{\nu}{(c/n)^2} \frac{d}{d\nu}\left(\frac{c}{n}\right) = \frac{1}{V_p} + \frac{\nu^2}{c^2} \frac{dn}{d\nu} = \frac{1}{V_p} + \frac{1}{\lambda} \frac{dn}{d\nu}$$

$$\left[ \text{as } c = \nu\lambda \quad \text{or} \quad \nu = \frac{c}{\lambda} \quad \text{or} \quad d\nu = -\frac{c}{\lambda^2} d\lambda \right] \quad = \frac{1}{V_p} - \frac{\lambda}{c} \frac{dn}{d\lambda}$$

$$\therefore V_g = \left( \frac{1}{V_p} - \frac{\lambda}{c} \frac{dn}{d\lambda} \right)^{-1} = \left( \frac{1}{V_p} \right)^{-1} \left( 1 - \frac{\lambda V_p}{c} \frac{dn}{d\lambda} \right)^{-1} \approx V_p \left( 1 + \frac{\lambda}{n} \frac{dn}{d\lambda} \right)$$

$$\therefore \boxed{V_g = V_p \left( 1 + \frac{\lambda}{n} \frac{dn}{d\lambda} \right)}$$

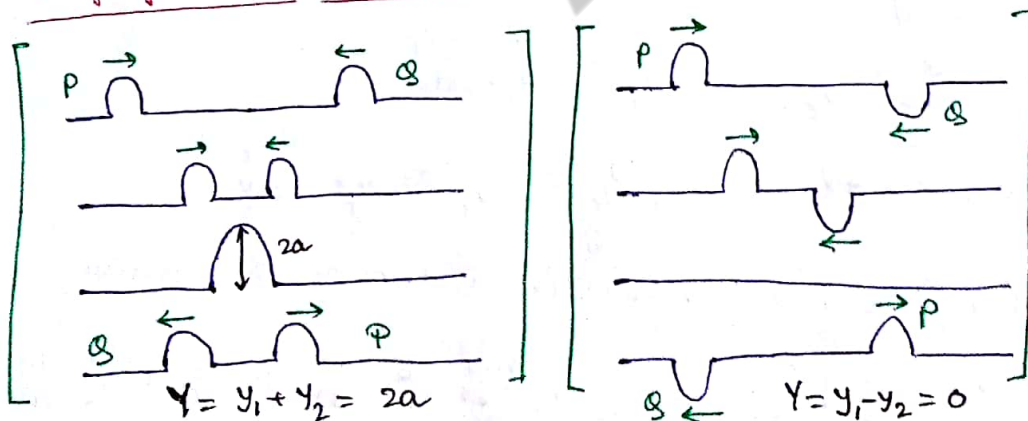
## Loudness of Sound : Phon & Sone

Intensity of sound refers to a purely physical aspect of sound which is the amount of energy reaching unit area in unit time. But loudness of sound is a subjective phenomena & it refers to the sensation produced to ear. Sensation of loudness depends on the intensity of sound, but loudness is not proportional to intensity. According to Weber & Fechner,  $\Delta L \propto \frac{\Delta I}{I}$  where  $\Delta L$  is increment in loudness &  $\Delta I$  is that of intensity.

$$\therefore \Delta L \propto K \frac{\Delta I}{I} \quad \text{or} \quad L = K \ln I \quad \text{where } K \text{ is some constant.}$$

The absolute unit of intensity is  $\text{Watt}/\text{cm}^2$  but it is expressed in decibel. The unit of loudness is "phon". To measure loudness of sound in phons, another pure tone of frequency 1000 cycles per second (C.P.S.) is taken whose intensity is gradually altered till its loudness becomes exactly equal to that of the given sound. If then the intensity level of the pure tone is  $n$  decibel, then the loudness of that sound is  $n$  phon. Phon is a small unit, another larger unit is "sone" which is equal to 10 times of phon.

## Superposition of Waves



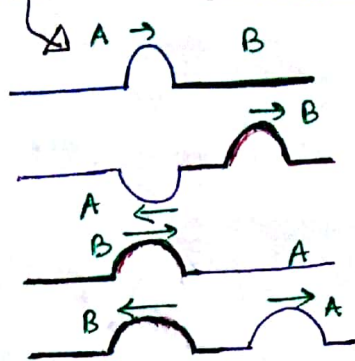
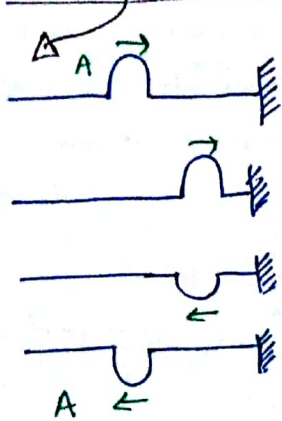
Different waves can pass through each other simultaneously through the same medium. We can hear distinctly the conversation in room, or when dropped a pebble in

pond, vice ripples passing (solitonic waves satisfying KdV equation).

"Processes by which different trains of waves travelling through a medium simultaneously overlap into one another without losing their individual nature/shape is called superposition of waves."



## Reflection & refraction of waves



} rarer to denser

} denser to rarer

$$y_p = a \sin \frac{2\pi}{\lambda} (vt - x)$$

$$y_r = -a \sin \frac{2\pi}{\lambda} (vt + x) \\ = a \sin \left[ \frac{2\pi}{\lambda} (vt + x) \pm \pi \right]$$

When reflected from denser to rarer medium, no phase change occurs. But from rarer to denser reflection  $180^\circ$  phase change occurs.  $\lambda$  and  $v$  of refracted wave different than incident wave. But  $v$  will remain unchanged & according to

Snell's law,  $n = \frac{v_i}{v_r} = \frac{\lambda_i}{\lambda_r}$

## Comparison between Interference & Beats / Progressive & Stationary wave

### Interference

1. Two waves must have same frequency.
2. Phase difference of two waves is constant.
3. Position of maximum & minimum intensity remains unchanged.

### Beats

1. Two waves must have slightly different frequencies.
2. Phase difference varies from 0 to  $\pi$  with time.
3. Positions of maximum & minimum intensity changes continuously.

### Progressive wave

1. Produced due to continuous periodic vibrations of medium particles.
2. Particles execute identical periodic motion about mean position.
3. Wave advances with definite velocity.
4. Wave retains its shape.

### Stationary wave

1. Produced due to superposition of two identical progressive waves (collinear) in opposite directions.
2. Except particles at nodes, execute motions of varying amplitudes.
3. Wave does not advance in medium.
4. Wave change by shrinking to straight line twice in each period.