```
"""
Registration : xxxx
Description  : Raw-moments and cumulants of different distribution
Author       : AKB
"""

import numpy as np
from scipy.special import comb
from scipy.stats import moment, kstat, describe

def rawmoment(N,n=1):
    #Calculate raw moments by using recursion from central moments
    mu = np.mean(N)
    mun = 0
    for i in range(0,n+1):
        mun += comb(n,i)*moment(N,i)*np.power(mu,n-i)
    return mun

def cumulant(N,n=1):
    #Calculate the cumulant using the cumulant matrix, given in Eqn2.28 in Risken, Chap.2
    K = np.zeros((n,n))
    for i in range(n):
        K[i,0] = rawmoment(N,i+1)
        for j in range(1,i+1,1):
            K[i,j] = comb(i,j-1)*rawmoment(N,i-j+1)
        if (i != n-1): K[i,i+1] = 1.0
    Kn = np.power(-1,n-1)*np.linalg.det(K)
    return Kn

#Binomial Distribution
n,p = 20,0.5
N = np.random.binomial(20,0.5,100000)
print ('Binomial Distribution')
print ('Mean, Variance, Skewness and Kurtosis are: ', describe(N).mean, ',',
describe(N).variance,',', describe(N).skewness, 'and ', describe(N).kurtosis)
print ('The first four raw moments are: ',rawmoment(N,1),',',rawmoment(N,
2),',',rawmoment(N,3),'and ',rawmoment(N,4))
print ('The theoretical value of the first four raw moments are: ',n*p,',',n*p*(1-
p+n*p),',',n*p*(1-3*p + 3*n*p + 2*p**2 - 3*n*p**2 + n**2*p**2),'and ',n*p*(1-7*p + 7*n*p +
12*p**2 - 18*n*p**2 + 6*n**2*p**2-6*p**3+11*n*p**3-6*n**2*p**3+n**3*p**3))
print ('The first four central moments are: ',moment(N,1),',',moment(N,2),',',moment(N,
3),'and ',moment(N,4))
print ('The theoretical value of the first four central moments are: ',0.0,',',n*p*(1-
p),',',n*p*(1-p)*(1-2*p),'and ',n*p*(1-p)*(1+(3*n-6)*p*(1-p)))
print ('The first four cumulants are: ',kstat(N,1),',',kstat(N,2),',',kstat(N,3),'and
',kstat(N,4))
print ('The theoretical value of the first four cumulants are: ',n*p,',',n*p*(1-
p),',',n*p*(1-p)*(1-2*p),'and ', n*p*(1-p)*(1-6*p + 6*p**2))


#Poisson Distribution
lam = 4.0
N = np.random.poisson(lam,100000)
print ('\n\nPoisson Distribution')
print ('Mean, Variance, Skewness and Kurtosis are: ', describe(N).mean, ',',
describe(N).variance,',', describe(N).skewness, 'and ', describe(N).kurtosis)
print ('The first four moments are: ',rawmoment(N,1),',',rawmoment(N,2),',',rawmoment(N,
3),'and ',rawmoment(N,4))
print ('The theoretical value of the first four moments are: ',lam,',',lam*(1-
lam),',',lam*(1+3*lam+lam**2),',',lam*(1+7*lam+6*lam**2+lam**3))
print ('The first four central moments are: ',moment(N,1),',',moment(N,2),',',moment(N,
3),'and ',moment(N,4))
print ('The theoretical value of the first four central moments are:
',lam,',',lam,',',lam,',',lam*(1+3*lam))
print ('The first four cumulants are: ',kstat(N,1),',',kstat(N,2),',',kstat(N,3),'and
',kstat(N,4))
```

```
print ('The theoretical value of the first four cumulants is: ',lam)

#Normal Distribution
mu,sig = 2.0,0.1
N = np.random.normal(mu,sig,100000)
print ('\n\nNormal Distribution')
print ('Mean, Variance, Skewness and Kurtosis are: ', describe(N).mean, ',',
describe(N).variance,',', describe(N).skewness, 'and ', describe(N).kurtosis)
print ('The first four moments are: ',rawmoment(N,1),',',rawmoment(N,2),',',rawmoment(N,
3),'and ',rawmoment(N,4))
print ('The theoretical value of the first four moments are: ',mu,',',mu**2 +
sig**2,',',mu**3 + 3*mu*sig**2,'and ',mu**4 + 6*mu**2*sig**2 + 3*sig**4)
print ('The first four central moments are: ',moment(N,1),',',moment(N,2),',',moment(N,
3),'and ',moment(N,4))
print ('The theoretical value of the first four central moments are: ',0.0,',',sig**2,',',
0.0,'and ',3*sig**4)
print ('The first four cumulants are: ',kstat(N,1),',',kstat(N,2),',',kstat(N,3),'and
',kstat(N,4))
print ('The theoretical value of the first four cumulants are: ',mu,',',sig**2,',',
0.0,'and ',0.0)

#Uniform Distribution
a,b = -1,0
N = np.random.uniform(a,b,100000)
print ('\n\nUniform Distribution')
print ('Mean, Variance, Skewness and Kurtosis are: ', describe(N).mean, ',',
describe(N).variance,',', describe(N).skewness, 'and ', describe(N).kurtosis)
print ('The first four moments are: ',rawmoment(N,1),',',rawmoment(N,2),',',rawmoment(N,
3),'and ',rawmoment(N,4))
print ('The theoretical value of the first four moments are: ',0.5*(a+b),',',
(1/3.0)*(a**2+b**2+a*b),',',(1/4.0)*(a+b)*(a**2+b**2),'and ',(1/5.0)*(a**4 + b**4 + a**3*b
+ a*b**3 + a**2*b**2))
print ('The first four central moments are: ',moment(N,1),',',moment(N,2),',',moment(N,
3),'and ',moment(N,4))
print ('The theoretical value of the first four central moments are: ',0.0,',',
(1/12.0)*(b-a)**2,',',0.0,'and ',(1/80.0)*(b-a)**4)
print ('The first four cumulants are: ',kstat(N,1),',',kstat(N,2),',',kstat(N,3),'and
',kstat(N,4))
print ('The theoretical value of the first four cumulants are: ',-0.5,',',1/12.0,',',
0.0,'and ',-1/120.0)

#Exponential Distribution
lam = 1.5
N = np.random.exponential((1.0/lam),100000)
print ('\n\nExponential Distribution')
print ('Mean, Variance, Skewness and Kurtosis are: ', describe(N).mean, ',',
describe(N).variance,',', describe(N).skewness, 'and ', describe(N).kurtosis)
print ('The first four moments are: ',rawmoment(N,1),',',rawmoment(N,2),',',rawmoment(N,
3),'and ',rawmoment(N,4))
print ('The theoretical value of the first four moments are: ',(1.0/lam),',',(2.0/
(lam**2)),',',(6.0/(lam**3)),'and ',(24.0/(lam**4)))
print ('The first four central moments are: ',moment(N,1),',',moment(N,2),',',moment(N,
3),'and ',moment(N,4))
print ('The theoretical value of the first four central moments are: ',0,',',(1.0/
(lam**2)),',',(2.0/(lam**3)),'and ',(9.0/(lam**4)))
print ('The first four cumulants are: ',kstat(N,1),',',kstat(N,2),',',kstat(N,3),'and
',kstat(N,4))
print ('The theoretical value of the first four cumulants are: ',(1.0/lam),',',(1.0/
(lam**2)),',',(2.0/(lam**3)),'and ',(6.0/(lam**4)))


"""
Results:

Binomial Distribution
Mean, Variance, Skewness and Kurtosis are:  10.00857 , 5.021066765767658 ,
```

```
-0.012711951411309663 and  -0.09810797378410507
The first four raw moments are:  10.00857 , 105.19249000000002 , 1153.1897700000004 and
13119.532570000003
The theoretical value of the first four raw moments are:  10.0 , 105.0 , 1150.0 and
13072.5
The first four central moments are:  0.0 , 5.021016555100001 , -0.14302096505442452 and
73.15846014493519
The theoretical value of the first four central moments are:  0.0 , 5.0 , 0.0 and  72.5
The first four cumulants are:  10.00857 , 5.0210667657676575 , -0.14302525578327643 and
-2.4720220106092565
The theoretical value of the first four cumulants are:  10.0 , 5.0 , 0.0 and  -2.5


Poisson Distribution
Mean, Variance, Skewness and Kurtosis are:  4.00587 , 4.0103956470564714 ,
0.48763605343158184 and  0.17112484870814226
The first four moments are:  4.00587 , 20.05735 , 116.39331 and  757.38391
The theoretical value of the first four moments are:  4.0 , -12.0 , 116.0 , 756.0
The first four central moments are:  0.0 , 4.010355543100001 , 3.9162474366240083 and
51.00104740248132
The theoretical value of the first four central moments are:  4.0 , 4.0 , 4.0 , 52.0
The first four cumulants are:  4.00587 , 4.010395647056471 , 3.9163649267885305 and
2.7533503532763346
The theoretical value of the first four cumulants is:  4.0


Normal Distribution
Mean, Variance, Skewness and Kurtosis are:  1.999999476669157 , 0.009992901815031431 ,
-0.011344365193204366 and  0.02783038627991674
The first four moments are:  1.999999476669157 , 4.009990708562915 , 8.059939183540289
and  16.240022063551255
The theoretical value of the first four moments are:  2.0 , 4.01 , 8.06 and
16.240299999999998
The first four central moments are:  0.0 , 0.009992801886013281 , -1.1332118692593105e-05
and  0.00030234730214344016
The theoretical value of the first four central moments are:  0.0 , 0.010000000000000002 ,
0.0 and  0.0003000000000000003
The first four cumulants are:  1.999999476669157 , 0.009992901815031945 ,
-1.1332458663493414e-05 and  2.7852198312781513e-06
The theoretical value of the first four cumulants are:  2.0 , 0.010000000000000002 , 0.0
and  0.0


Uniform Distribution
Mean, Variance, Skewness and Kurtosis are:  -0.49938590869560684 , 0.08366170542920796 ,
0.0005788927526283356 and  -1.2060191386134917
The first four moments are:  -0.49938590869560684 , 0.33304715461589063 ,
-0.2498631657558547 and  0.19990510258723604
The theoretical value of the first four moments are:  -0.5 , 0.3333333333333333 , -0.25
and  0.2
The first four central moments are:  0.0 , 0.08366086881215368 , 1.4008178530759192e-05
and  0.012556324947051665
The theoretical value of the first four central moments are:  0.0 , 0.08333333333333333 ,
0.0 and  0.0125
The first four cumulants are:  -0.49938590869560684 , 0.08366170542920792 ,
1.4008598785880607e-05 and  -0.008441268893531372
The theoretical value of the first four cumulants are:  -0.5 , 0.08333333333333333 , 0.0
and  -0.008333333333333333


Exponential Distribution
Mean, Variance, Skewness and Kurtosis are:  0.6635991711549315 , 0.4388044063364571 ,
1.9737326168332707 and  5.6299585274346775
The first four moments are:  0.6635991711549315 , 0.8791638782499058 , 1.7394916005265002
and  4.537809031450255
The theoretical value of the first four moments are:  0.6666666666666666 ,
```

```
0.8888888888888888 , 1.7777777777777777 and  4.7407407407407405
The first four central moments are:  0.0 , 0.4388000182923938 , 0.5737045227273051 and
1.6616593003868825
The theoretical value of the first four central moments are:  0 , 0.4444444444444444 ,
0.5925925925925926 and  1.7777777777777777
The first four cumulants are:  0.6635991711549315 , 0.43880440633645706 ,
0.5737217342645886 and  1.084110370613074
The theoretical value of the first four cumulants are:  0.6666666666666666 ,
0.4444444444444444 , 0.5925925925925926 and  1.1851851851851851
"""
```