2023

PHYSICS — HONOURS — PRACTICAL

Paper : CC-8P

(Syllabus : 2019-2020 and 2018-2019)

Full Marks : 30

DAY - 1

[Experiment : 20, Laboratory Notebook : 5, Viva voce : 5]

1. (a) The heat equation in one dimension is given by $K\dfrac{\partial^2 u(x,t)}{\partial x^2} = \dfrac{\partial u(x,t)}{\partial t}$; K = Constant,

where $u(x, t)$ is the temperature function. Consider a wire of length $0 \le x \le \pi$. The boundary conditions at both ends are :

$$u(0, t) = 0 = u(\pi, t).$$

The initial temperature distribution is :

$$u(x, 0) = \sin x.$$

Write a Python program to solve the heat equation for the above system and plot the temperature distribution $u(x, t = 0.5\text{s})$. Take $K = 0.05$ unit.

[Code : 10 + Plot : 2, Total : 12]

(b) Bessel function of first kind of integer order with real value satisfy the relation :

$$J_{-n}(x) = (-1)^n J_n(x).$$

Using SciPy special module, write a Python program to plot $J_n(x)$ and $J_{-n}(x)$ in the same window for $n = 1$ and $n = 2$ in the range $0 \le x \le 10$. Comment how the above relation is satisfied from the plot.

[Code : 5 + Plot : 2 + Comment : 1, Total : 8]

2. (a) Compute the improper integral using quad function from SciPy package

$$I = \int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$$

Plot the integrand in the range $x \in [0, 0.98]$. Label the axes and write the integrand as a legend inside the graph.

[Program : 4 + Output : 1 + Graph : 1+½+½+1, Total : 8]

(b) Write a program to solve Laplace's equation on square grid

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0$$

Dirichlet Boundary conditions are :

Left B. C. $U(0, y) = \frac{y}{1+y^2}$, for $0 < y < 1$

Right B. C. $U(1, y) = \frac{y}{4+y^2}$, for $0 < y < 1$

Bottom B. C. $U(x, 0) = 0.\,0$, for $0 < x < 1$

Top B. C. $U(x, 1) = \frac{1}{(1+x)^2 + 1}$, for $0 < x < 1$

The exact analytical solution is $\frac{y}{(1+x)^2 + y^2}$. Plot the numerical solution (U_N) and exact analytical solution (U_A) with respect to x for a fixed y and with respect to y for a fixed x.

[Program : 8 + Graph : 2+2, Total : 12]

3. (a) (i) Plot Legendre's polynomial of orders $n = 1, 2, 3$ (ii) Verify the orthogonality relation :

$$\int_{-1}^{1} P_n(x) P_m(x)\,dx = \frac{2}{2n+1}\delta_{mn}, \text{ for } m, n = 1, 2,$$

where P_n is the nth order Legendre's polynomial. [Plot : 4 + Program : 6, Total : 10]

(b) Consider the following differential equation which describes damped harmonic motion :

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = 0,\ x(0) = 0, \frac{dx}{dt}(0) = 0$$

Consider $\gamma = 0.5$ unit and $k = 1$ unit. Write a Python program to solve the differential equation by using SciPy module. Plot x as a function of t for the range $0 \le t \le 10$.

[Code : 8 + Plot : 2, Total : 10]

4. (a) Solve using odeint package from SciPy :

$$\frac{d^2y}{dx^2} + y = 4x + 10\sin x, \quad y(\pi) = 0,\ y'(\pi) = 2\,.$$

Space domain : $x \in [\pi, 4\pi]$.

The exact solution is $y = 9\pi \cos x + 7\sin x + 4x - 5x\cos x$. Plot the numerical solution and exact solution in the same graph.

[Program : 8 + Graph : 2, Total : 10]

(b) The fourier expansion of a given function $F(x)$ periodic over interval $[-l, l]$ can be written as

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right)\right),$$

where $a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx,\ n = 0,1,2,3,\ldots\infty$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx,\ n = 1,2,3,\ldots\infty.$$

Analyse the Fourier series of a Sawtooth wave. Evaluate the Fourier coefficients and print first 10 coefficients. Plot the constructed series with the periodic function taken.

[Program : 7 + Graph : 3, Total : 10]

5. (a) The Fourier expansion of a given function $F(x)$ periodic over interval $[-l, l]$ can be written as

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right)\right),$$

where $$a_n = \frac{1}{l}\int_{-l}^{l} F(x)\cos\left(\frac{n\pi x}{l}\right)dx,\ n = 0,1,2,3,\ldots\infty$$

$$b_n = \frac{1}{l}\int_{-l}^{l} F(x)\sin\left(\frac{n\pi x}{l}\right)dx,\ n = 1,2,3,\ldots\infty.$$

Write a program to compute the first ten fourier's coefficients (a_n, b_n) of a square wave. Plot the constructed series and the square wave signal in the same graph using matplotlib. Set appropriate axis labelling and legend at suitable location in the graph.

[Program : 6 + Graph : (1+1)+(1+1), Total : 10]

(b) Using SciPy module, write a Python program to solve the differential equation describing the motion of a particle under gravity subjected to a resistive drag force :

$\frac{dv}{dt} = g - kv$, where $v = \frac{dx}{dt}$, 'x' being the displacement of the particle.

Consider $k = 0.01$ unit and $v(t = 0) = 0$. Plot $v(t)$ vs. t for $0 \le t \le 10$s.

[Code : 8 + Plot : 2, Total : 10]

6. (a) Write a Python program using SciPy to show the convolution of the following two Gaussian functions is also a Gaussian :

$$f(t)=\frac{1}{\sqrt{\pi}}e^{-(t-2)^2};\; g(t)=\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}$$

Plot the functions $f(t)$, $g(t)$ and the convoluted function on the same graph.

Note : $(f*g)(t) := \int_{-\infty}^{\infty} f(\tau)g(t-\tau)\,d\tau$ [Code : 7 + Plot : 3, Total : 10]

(b) Write a program to solve 1d wave equation :

$$\frac{\partial^2 U}{\partial t^2}=\frac{\partial^2 U}{\partial x^2}$$
$$U(0,t)=U(1,t)=0, \quad t>0$$
$$U(x,0)=\sin(\pi x), \quad 0\le x\le 1.$$
$$\left(\frac{\partial U}{\partial t}\right)_{x,t=0}=0, \quad 0\le x\le 1$$

Plot the numerical $U_N(x, t)$ solution in a graph with respect to x for different time instants $(0<t\le 1.5)$. Label the axes and put appropriate legends in the graph.

[Program : 7 + Graph : 1+(1+1), Total : 10]

7. (a) Verify the relation :

$$\int_{a-x_1}^{a+x_2} \delta(x-a) f(x) dx = f(a)$$

Limiting representation of Dirac-Delta function : $\delta_n(x-a) = \lim_{n\to\infty} \sqrt{\frac{n}{\pi}} e^{-n(x-a)^2}$.

Take, $f(x) = x^2$ and $a = 4$. Plot $\delta_n(x-a)$ vs. x for three values of n. Set appropriate axes labelling and put legend in the graph.

[Program : 6 + Graph : 2+2, Total : 10]

(b) Numerically verify the recursion relation of the given Legendre's polynomial :

$$\left(1-x^2\right) P_n'(x) = (n+1) x P_n(x) - (n+1) P_{n+1}(x)$$

$x \in [-1, 1]$. Take $n = 2, 4, 6$. Plot the right hand side and left hand side of the given relation in the same graph.

[Program : 6 + Graph : 4, Total : 10]

———————

2023

PHYSICS — HONOURS — PRACTICAL

Paper : CC-8P

(Syllabus : 2019-2020 and 2018-2019)

Full Marks : 30

DAY - 2

[Experiment : 20, Laboratory Notebook : 5, Viva voce : 5]

1. (a) Bessel function of first kind satisfy the recurrence relation :

$$J_{n+1}(x) = \frac{2n}{x} J_n(x) - J_{n-1}(x).$$

Use the above relation to write a Python program, by using only J_0 and J_1 from SciPy, to calculate and plot $J_2(x)$ and $J_3(x)$ for $0 \le x \le 20$.

[Code : 7 + Plot : 3, Total : 10]

(b) Write a program to solve the 1D Diffusion equation :

$$\frac{\partial U(x,t)}{\partial t} = D \frac{\partial^2 U(x,t)}{\partial x^2}$$

$D = 1.0$

$U(x,0) = 0, \quad 0 \le x \le 10$ Initial Condition

$U(0,t) = 0; \; U(10,t) = 100, \; \forall t$ Boundary Condition

Space Domain : $x \in [0, 10]$, Time Domain : $t \in [0, 5.0]$

Plot $U(x, t)$ with respect to x for different time instants.

[Program : 8 + Graph : 2, Total : 10]

2. (a) Differential equation for a swinging pendulum is $\frac{d^2\theta}{dt^2}+\frac{g}{L}\sin\theta=0$.

Write a suitable Python program using odeint to solve the above equation. Plot θ as a function of t for $0 \le t \le 5$. The initial conditions are given as : $\theta(t=0)=\frac{\pi}{6}$, $\theta'(t=0)=0$. Take $L = 10$ cm, $g = 980$ cm/s^2.

[Code : 10 + Plot : 2, Total : 12]

(b) Write a Python program to numerically evaluate the following improper integral :

$$I(N)=\int_{-N}^{N} x^2 e^{-x^2}\,dx.$$

Increase 'N' gradually, so that $I(N)$ converges to $\sqrt{\pi}/2$. Plot $I(N)$ vs. N to show the convergence.

[Code : 6 + Plot : 2, Total : 8]

3. (a) Write a program to solve Laplace's equation on square grid :

$$\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}=0.$$

Dirichlet Boundary conditions are :

Left B. C. $U(0, y) = 0.0$ for $0 < y < 1$

Right B. C. $U(1, y) = y$ for $0 < y < 1$

Bottom B. C. $U(x, 0) = 0.0$ for $0 < x < 1$

Top B. C. $U(x, 1) = x$ for $0 < x < 1$

The exact analytical solution is $U_A(x, y) = xy$. Plot the numerical solution (U_N) and exact analytical solution (U_A) with respect to x for a fixed y and with respect to y for a fixed x.

[Program : 8 + Graph : 2+2, Total : 12]

(b) Compute the improper integral using quad function from SciPy package :

$$I=\int_{3}^{5}\frac{x^2}{\sqrt{(x-3)(5-x)}}\,dx.$$

Plot the integrand in the range $x \in [3.001, 4.999]$. Label the axes and write the integrand as a legend inside the graph.

[Program : 4 + Output : 1 + Graph : 1+½+½+1, Total : 8]

4. (a) Dirac-Delta function can be expressed as the limit of a Gaussian distribution :

$$\delta(x-x_0) = \lim_{\sigma \to 0} G_\sigma(x-x_0),$$

where $G_\sigma(x-x_0) = \dfrac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\dfrac{(x-x_0)^2}{2\sigma^2}\right\}$.

Take $x_0 = 1$. Plot $G_\sigma(x-x_0)$ for $\sigma = 0.1$, 0.05 and 0.001 to see that it really imitates the Dirac-Delta function. Hence, obtain the value of the integral and print the answer.

$$\int_0^\pi \left(x^3 - 4x + 2\right) G_\sigma(x-x_0)\, dx.$$

[Code : 10 + Plot : 2, Total : 12]

(b) Write a Python program using SciPy module to calculate the following Gaussian integral :

$$I = \int_0^\infty e^{-t^2/2}\, dt$$

Plot the integrand up to an upper limit of your choice. Print the value of the integral.

[Code : 6 + Plot : 2, Total : 8]

5. (a) Charging of a *CR* circuit is described by the differential equation :

$$R\frac{dq}{dt} = \frac{CV_0 - q}{C},$$

where R, C and V_0 are the resistance, capacitance and battery voltage. Write a Python program using SciPy module to solve this equation and plot $q(t)$ for $0 \le t \le 5$s. Take $R = 1\ \text{k}\Omega$, $C = 1\ \text{m}F$, $V_0 = 10$ V.

Given, at $t = 0$, $q(0) = 0$. [Code : 8 + Plot : 2, Total : 10]

(b) The Fourier expansion of a given function $F(x)$ periodic over interval $[-l, l]$ can be written as

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right)\right),$$

where $a_n = \frac{1}{l}\int_{-l}^{l} F(x)\cos\left(\frac{n\pi x}{l}\right)dx, \; n = 0,1,2,3,\ldots\infty$

$$b_n = \frac{1}{l}\int_{-l}^{l} F(x)\sin\left(\frac{n\pi x}{l}\right)dx, \; n = 1,2,3,\ldots\infty.$$

Write a program to compute the first ten Fourier's coefficients (a_n, b_n) of a square wave. Plot the constructed series and the square wave signal in the same graph using matplotlib. Set appropriate axis labelling and legend at suitable location in the graph.

[Program : 6 + Graph : (1+1)+(1+1), Total : 10]

6. (a) Numerically verify the recursion relation of the given Legendre's polynomial.

$$(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$$

$x \in [-1, 1]$. Take $n = 1, 2, 3, 4$. Plot the right hand side and left hand side of the given relation in the same graph.

[Program : 6 + Graph : 2, Total : 8]

(b) Write a program to solve the 1D Diffusion equation :

$$\frac{\partial U(x,t)}{\partial t} = D\frac{\partial^2 U(x,t)}{\partial x^2}$$

$D = 1.0$

$U(x,0) = \sin x, \quad 0 \leq x \leq \pi$ Initial Condition

$U(0,t) = U(\pi,t) = 0, \quad \forall t$ Boundary Condition

Space Domain : $x \in [0, \pi]$, Time Domain : $t \in [0, 2.0]$

Plot $U(x, t)$ with respect to x for different time instants.

[Program : 10 + Graph : 2, Total : 12]

7. (a) Compute the integral

$$I = \int_{1}^{\infty} \frac{\ln x}{x^2}.$$

Plot the integrand up to certain upper limit of your choice.

[Program : 3 + Output : 1 + Graph : 2, Total : 6]

(b) Write a program to solve 1d wave equation :

$$\frac{\partial^2 U}{\partial t^2} = \frac{\partial^2 U}{\partial x^2}$$

$$U(0,t) = U(10,t) = 0, \qquad t > 0$$

$$U(x,0) = e^{-k(x-x_0)^2}, \qquad 0 \le x \le 10.$$

$$\left(\frac{\partial U}{\partial t}\right)_{x,t=0} = 0, \qquad 0 \le x \le 10$$

Take : $x_0 = 5.0$, $k = 10.0$. Plot the numerical $U_N(x, t)$ solution with respect to x for different time instants ($0 \le t \le 15$). Label the axes and put appropriate legends in the graph.

[Program : 10 + Graph : 2+(1+1), Total : 14]

2023

PHYSICS — HONOURS — PRACTICAL

Paper : CC-8P

(Syllabus : 2019-2020 and 2018-2019)

Full Marks : 30

DAY - 3

[Experiment : 20, Laboratory Notebook : 5, Viva voce : 5]

1. (a) Hermite Polynomials satisfy orthogonality relation :

$$\int_{-\infty}^{\infty} H_n(x)H_m(x)e^{-x^2}\,dx = 2^n.\,n!\sqrt{\pi}\,\delta_{nm}.$$

where, $\delta_{nm} = 1$ for $n = m$

$= 0$ for $n \neq m$.

Write a Python program using SciPy module to verify the above relation for $n = m = 1$ and $n = 1$, $m = 2$. [Code : 8, Total : 8]

(b) The heat equation in one dimension is given by :

$$K\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial u(x,t)}{\partial t};\ K \equiv \text{constant},$$

where $u(x, t)$ is the temperature function. Consider a wire of length $0 \leq x \leq \pi$.

The boundary conditions at both ends are :

$$u(0, t) = 0, \quad u(\pi, t) = 2\pi \text{ for } t > 0.$$

The initial temperature distribution is :

$$u(x, 0) = x + \sin x, \quad 0 \leq x \leq \pi.$$

Write a Python program to solve heat equation and plot the temperature distribution throughout the wire at $t = 10$s. Take $K = 1.0$ unit.

[Code : 10 + Plot : 2, Total : 12]

2. (a) Write a Python program to show the convolution of the following Gaussian functions is also a Gaussian :

$$f(x) = \frac{1}{\sqrt{\pi}} e^{-(x+1)^2}, \quad g(x) = \frac{1}{2\sqrt{\pi}} e^{-\frac{(x-1)^2}{4}}$$

Plot the functions $f(x)$, $g(x)$ and the convoluted function on the same graph.

Note: $(f * g)(t) := \int_{-\infty}^{\infty} f(\tau) g(t-\tau)\, d\tau$ [Code : 10 + Plot : 2, Total : 12]

(b) Solve

$$\frac{d^2x}{dt^2} + 4t = \sin^2 2t, \quad x(0) = 0, \quad \left.\frac{dx}{dt}\right|_0 = 0$$

by using odeint package from SciPy. Take $t \in [0, 5]$

Exact solution : $x(t) = -\frac{1}{96}\left(-3 + 24t^2 - 64t^3 + 3\cos 4t\right)$

(i) Plot $x(t)$ vs. t and compare it with exact solution.

(ii) Plot $\frac{dx}{dt}$ vs. t.

[Program : 5 + Graph : 1+1+1, Total : 8]

3. (a) The number of surviving nuclei in a radioactive sample is given by :

$$\frac{dN(t)}{dt} = -\lambda N(t)$$

Write a Python program to solve this equation to find the number of nuclei present after $t = 20$ years, if the sample had 10^4 nuclei at the beginning ($t = 0$). Take $\lambda = 6.93 \times 10^{-3}$ year^{-1}. Print the answer.

Plot $N(t)$ vs. t for t in $\left[0, \frac{5}{\lambda}\right]$. [Code : 8 + Plot : 2, Total : 10]

(b) The Fourier expansion of a given function $F(x)$ periodic over interval $[-l, l]$ can be written as

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right)\right),$$

where $a_n = \frac{1}{l}\int_{-l}^{l} F(x)\cos\left(\frac{n\pi x}{l}\right)dx,\ n = 0, 1, 2, 3, ..., \infty$

$$b_n = \frac{1}{l}\int_{-l}^{l} F(x)\sin\left(\frac{n\pi x}{l}\right)dx,\ n = 1, 2, 3, ..., \infty.$$

Write a program to compute the first five Fourier's coefficients (a_n, b_n) of a square wave. Plot the constructed series and the square wave signal in the same graph using matplotlib. Set appropriate axes labelling and legend at suitable location in the graph.

[Program : 6 + Graph : (1+1)+(1+1), Total : 10]

4. (a) Write a program to solve 1d wave equation :

$$\frac{\partial^2 U}{\partial t^2} = \frac{\partial^2 U}{\partial x^2}$$
$$U(0,t) = U(10,t) = 0, \qquad t > 0$$
$$U(x,0) = e^{-k(x-x_0)^2}, \qquad 0 \le x \le 10.$$
$$\left(\frac{\partial U}{\partial t}\right)_{x,t=0} = 0, \qquad 0 \le x \le 10.$$

Take : $x_0 = 5.0$, $k = 10.0$. Plot the numerical $U_N(x, t)$ solution with respect to x for different time instant $t(0 \le t \le 20)$. Label the axes and put appropriate legends in the graph.

[Program : 10 + Graph : 2+(1+1), Total : 14]

(b) Compute the integral

$$\int_1^2 \frac{x\,dx}{\sqrt{x-1}}$$

Plot the integrand in the range $x \in [1.001, 2.0]$. Label the axes and write the integrand as a legend inside the graph.

[Program : 3 + Output : 1 + Graph : 1+1, Total : 6]

5. (a) Write a program to solve Laplace's equation on square grid

$$\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}=0$$

Dirichlet Boundary conditions are :

Left B. C. $U(0, y)=\dfrac{y}{1+y^2}$ for $0<y<1$

Right B. C. $U(1, y)=\dfrac{y}{4+y^2}$ for $0<y<1$

Bottom B. C. $U(x, 0)=0.0$ for $0<x<1$

Top B. C. $U(x, 1)=\dfrac{1}{(1+x)^2+1}$ for $0<x<1$

The exact analytical solution is $\dfrac{y}{(1+x)^2+y^2}$. Plot the numerical solution (U_N) and exact analytical solution (U_A) with respect to x and y in the same graph.

[Program : 8 + Graph : 2+2, Total : 12]

(b) Verify the orthogonality relation :

$$\int_{-1}^{1} P_n(x)P_m(x)dx=\frac{2}{2n+1}\delta_{mn}, \text{ for } m, n = 1, 2, 3, 4,$$

where P_n is the nth order Legendre's polynomial.

[Program : 8, Total : 8]

6. (a) Dirac-Delta function can be expressed as the limit of Gaussian distribution :

$$\delta(x-x_0)=\lim_{\sigma\to 0} G_\sigma(x-x_0),$$

where, $$G_\sigma(x-x_0)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{(x-x_0)^2}{2\sigma^2}\right\}.$$

Take $x_0=0$. Plot $G_\sigma(x-x_0)$ for $\sigma=1.0$, 0.5 and 0.005 to see that it really imitates the Dirac-Delta function. Hence, obtain the value of the following integral :

$$\int_{-1}^{1} e^{x+2}\, G_\sigma(x-x_0)\,dx\,.$$

Print the answer. [Code : 9 + Plot : 2, Total : 11]

(b) Suppose $f(t)=e^{-at^2}$ and $g(t)=e^{-bt^2}$, are two Gaussians, where $a, b>0$. Taking suitable values of a and b, show that the convolution of these two Gaussians is also a Gaussian. Plot them.

Note : $(f*g)(t):=\int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau\,.$ [Program : 7 + Graph : 2, Total : 9]

7. (a) Consider the following forced harmonic motion :

$$\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}+kx=F\cos\omega t,$$

where the exact solution of the amplitude of the steady state, $A=\dfrac{F}{\sqrt{\left(k-\omega^2\right)^2+\gamma^2\omega^2}}$. Take $\gamma=0.1$, $k=2$ and $F=1$. Solve the differential equation by using odeint from SciPy module. Plot the amplitude resonance curve. Compare with the exact formula.

[Program : 7 + Graph : 2+1, Total : 10]

(b) Compute the improper integral

$$\int_a^b dx\frac{1}{\sqrt{(b-x)(x-a)}}$$

Take the following combinations of a and b :

$a=1,\ b=2$

$a=2,\ b=4$

Plot the integrand vs. x for any set of (a, b).

[Program : 6 + Output : 1+1 + Plot : 2, Total : 10]

2023

PHYSICS — HONOURS — PRACTICAL

Paper : CC-8P

(Syllabus : 2019-2020 and 2018-2019)

Full Marks : 30

DAY - 4

[Experiment : 20, Laboratory Notebook : 5, Viva voce : 5]

1. (a) Consider a rod (length L) of uniform cross-section and made of homogeneous material which is perfectly insulated so that heat flows only along the length. Two ends of the rod are inserted into two ice baths to maintain fixed temperature at both the ends and $U\left(\frac{L}{2}, 0\right) = 100$. Consider the following 1D diffusion equation :

$$\frac{\partial U(x,t)}{\partial t} = \frac{\partial^2 U(x,t)}{\partial x^2}$$

Using one initial condition and two boundary conditions, solve this partial differential equation and plot the temperature profile along the length of the rod with time.

[Program : 10 + Graph : 2, Total : 12]

(b) Evaluate the Integral for $n = -3/2, -1/2, 1/2, 1, 3/2, 5/2$.

$$I(n) = \int_0^\infty e^{-x} x^{n-1} dx.$$

[Program : 4 + Output : 4, Total : 8]

2. (a) Suppose $f(t) = e^{-at}$ and $g(t) = \sin(bt)$. Taking suitable values of a and b, compute the convolution of the given functions. Plot $f(t)$, $g(t)$ and convoluted function.

Note : Convolution over finite range : $(f * g)(t) := \int_0^t f(\tau)g(t-\tau)d\tau$.

[Program : 8 + Graph : 2, Total : 10]

(b) Solve

$$\frac{d^2x}{dt^2} + Ax + Bx^3 = C\cos\omega t$$

by using odeint package from SciPy. Take : $A = 1.2$, $B = 0.2$, $C = 1.8$ and $\omega = 1.5$, $t \in [0, 15]$

(i) Plot x vs. t. (ii) Plot $\frac{dx}{dt}$ vs. t.

Label the axes and put appropriate legends in the graph.

[Program : 7 + Graph : 1+1+1, Total : 10]

3. (a) The Fourier expansion of a given function $F(x)$ periodic over interval $[-l, l]$ can be written as

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right)\right),$$

where $$a_n = \frac{1}{l}\int_{-l}^{l} F(x)\cos\left(\frac{n\pi x}{l}\right)dx,\ n = 0, 1, 2, 3, ..., \infty$$

$$b_n = \frac{1}{l}\int_{-l}^{l} F(x)\sin\left(\frac{n\pi x}{l}\right)dx,\ n = 1, 2, 3, ..., \infty.$$

Write a program to compute the first five Fourier's coefficients (a_n, b_n) of a square wave. Plot the constructed series and the square wave signal in the same graph using matplotlib. Set appropriate axes labelling and legend at suitable location in the graph.

[Program : 6 + Graph : (1+1)+(1+1), Total : 10]

(b) An useful improper integral is :

$$\int_0^{\infty} x^n e^{-\alpha x} dx = \frac{n!}{\alpha^{n+1}}$$

(i) Using SciPy module, show the result for $n = 3$, $\alpha = 1$:

$$\int_0^{\infty} x^3 e^{-x} dx = 6$$

(ii) Such integrals are usually performed numerically by gradually increasing the upper limit to a large value (say, 10^3). Plot the value of the integral : $I(a) = \int_0^{a} x^3 e^{-x} dx$ against the upper limit 'a'. The plot must converge to the desired answer.

[Code : 3+5+ Plot : 2, Total : 10]

4. (a) Write a program to solve Laplace's equation on square grid

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0$$

Dirichlet Boundary conditions are :

Left B. C. $U(0, y) = 0.0$ for $0 < y < 1$

Right B. C. $U(1, y) = y$ for $0 < y < 1$

Bottom B. C. $U(x, 0) = 0.0$ for $0 < x < 1$

Top B. C. $U(x, 1) = x$ for $0 < x < 1$

The exact analytical solution is $U_A(x, y) = xy$. Plot the numerical solution (U_N) and exact analytical solution (U_A) with respect to x and y in the same graph.

[Program : 10 + Graph : 1+1, Total : 12]

(b) Verify the relation :

$$\int_{-\infty}^{\infty} \delta(x-a) f(x) dx = f(a)$$

Limiting representation of Dirac-Delta function : $\delta_n(x-a) = \lim_{n \to \infty} \sqrt{\frac{n}{\pi}} e^{-n(x-a)^2}$.

Take, $f(x) = x^2$ and $a = 2$. Plot $\delta_n(x-a)$ vs. x for four values of n. Set appropriate axes labelling and put legend in the graph.

[Program : 6 + Graph : 2, Total : 8]

5. (a) Wave equation in 1D is given by

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2},\ c = \text{velocity.}$$

Consider $c = 1$, $0 \le x \le \pi$ and $0 \le t \le 10$s. The initial conditions are :

$$y(x,0) = \sin(x),\quad \frac{\partial y\,(x,0)}{\partial t} = 0$$

The boundary conditions are :

$$y(0, t) = 0 = y(\pi, t).$$

Write a suitable Python program to solve this differential equation. Print $y\left(\frac{\pi}{2}, 5\right)$.

Plot $y(x, t)$ vs. x for a given t. [Code : 10 + Plot : 2, Total : 12]

(b) Hermite Polynomials satisfy orthogonality relation :

$$\int_{-\infty}^{\infty} H_n(x) H_m(x) e^{-x^2} dx = 2^n . n! \sqrt{\pi}\, \delta_{nm}$$

where, $\delta_{nm} = 1$ for $n = m$

$= 0$ for $n \neq m$

Write a Python program using SciPy module to verify the above relation for $n = m = 1$ and $n = 1$, $m = 2$. [Code : 8, Total : 8]

6. (a) Solve $\frac{d^2x}{dt^2} - \epsilon\left(1-x^2\right)\frac{dx}{dt} + x = 0, x(0) = 0.75, \frac{dx}{dt}(0) = 0$ by odeint package from SciPy.

Take $\epsilon = 3.5$.

(i) Plot $x(t)$ vs. t and $\frac{dx}{dt}$ vs. t in the same graph using matplotlib. Label the x-axis as 'Time' and y-axis as 'Displacement or Velocity'. Set legends at the appropriate position for two different graphs.

(ii) Plot $\frac{dx}{dt}$ vs. $x(t)$. Label the x-axis as 'Position' and y-axis as 'Velocity'. Set title as 'Phase plot'.

[Program : 6 + Graph : (2+2)+(1+1), Total : 12]

(b) Write a python program to numerically evaluate the Bose-Einstein integral :

$$\int_0^\infty \frac{x}{e^x - 1} dx = \frac{\pi^2}{6}$$

Plot the integrand for $0 \le x \le 10$. Start with a small non-zero value as the lower limit.

[Code : 6 + Plot : 2, Total : 8]

7. (a) Using SciPy module, write a Python program to solve the differential equation describing the motion of a particle under gravity subjected to a resistive drag force :

$$\frac{dv}{dt} = g - kv, \text{ where } v = \frac{dx}{dt},$$

'x' being the displacement of the particle.

Consider $k = 0.01$ unit and $v(t = 0) = 0$. Plot $v(t)$ vs. t for $0 \le t \le 10$s.

[Code : 10 + Plot : 2, Total : 12]

(b) Numerically verify the recursion relation of the given Legendre's polynomial.

$$\left(1-x^2\right)P_n'(x) = (n+1)xP_n(x) - (n+1)P_{n+1}(x)$$

$x \in [-1, 1]$. Take $n = 2, 4$. Plot the right hand side and left hand side of the given relation in the same graph.

[Program : 5 + Graph : 3, Total : 8]

2022

PHYSICS — HONOURS — PRACTICAL

Paper : CC-8P

[Mathematical Physics-II]

(Syllabus : 2019-2020)

Full Marks : 30

The figures in the margin indicate full marks.

[Distribution of Marks : LNB - 5, Viva - 5, Experiment - 20, Total - 30]

1. (a) Consider the following Gaussian integral :

$$\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-(x-\mu)^2/2\sigma^2}\,dx$$

Take suitable values of σ and μ and plot the function. Find the value of the integral.

(b) Suppose $F(t) = -at^2$ and $G(t) = -bt^2$, are two Gaussians, where $a, b > 0$. Taking suitable values of a and b, show that the convolution of these two Gaussians is also a Gaussian. Plot them.

8+12

2. (a) Consider the following damped harmonic motion :

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = 0$$

Take $\gamma = 0.4$ and $k = 1$. Solve the differential equation by using a suitable function from scipy module and plot the decay of x versus t for a definite range.

(b) Consider the Wave functions for 1D quantum harmonic oscillator :

$$\psi_n(x) = \frac{1}{\sqrt{2^n n!}}\left(\frac{1}{\pi}\right)^{\frac{1}{4}} \exp\left(-\frac{x^2}{2}\right) H_n(x)$$

where H_n is the Hermite polynomial and we consider $\frac{m\omega}{h} = 1$. Plot the wave functions for $n = 0$, 1 and 2 in a single graph.

12+8

3. (a) Consider the following forced harmonic motion :

$$\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}+kx=F\cos\omega t$$

where the exact solution of the amplitude of the steady state $= F/\sqrt{(k-\omega)^2+\gamma^2\omega^2}$. Take $\gamma=0.5$, $k=5$ and $F=1$. Solve the differential equation by using a suitable function from scipy module and plot the amplitude resonance curve. Compare with the exact formula.

(b) Prove the orthogonality relation :

$$\int_{-1}^{1}P_m(x)P_n(x)=\frac{2}{2n+1}\delta_{mn}$$

where P_m is the m-th order Legendre polynomial. 12+8

4. (a) Analyse in a Fourier series of a Sawtooth wave whose window size is 6 and frequency is 2. Evaluate the Fourier coefficients and plot the constructed series with the given periodic function.

(b) Consider a rod (length L) of uniform cross-section and made of homogeneous material which is perfectly insulated so that heat flows only along the length. Two ends of the rod are inserted in two ice baths to maintain fixed temperature at both the ends and heat at the middle. Consider the following diffusion equation :

$$\frac{\partial u}{\partial t}=D^2\frac{\partial^2 u}{\partial x^2}$$

where D is the diffusion coefficient and may be set to unity. Forming one initial condition and two boundary conditions, solve this partial differential equation and plot the temperature profile (temperature along the length of the rod). 8+12

5. (a) Evaluate the Fresnel Integral :

$$\int_0^\infty \sin x^2 dx$$

Plot the solution up to a certain value of the upper limit. Comment on the plot.

(b) Write a python program to analyse in a Fourier series of a full-wave rectifier signal :

$$f(x)=|\sin(x)|$$

Evaluate the Fourier coefficients and plot the constructed series with the given periodic function. 10+10

3. (a) Consider the following radioactive decay equation :

$$\frac{dx}{dt} = -kx$$

Solve the differential equation by using a suitable function from scipy module and plot decay of x versus t.

(b) Prove the orthogonality relation :

$$\int_{-1}^{1} P_m(x) P_n(x) = \frac{2}{2n+1} \delta_{mn}$$

where P_m is the m-th order Legendre polynomial. 12+8

4. (a) Analyse in a Fourier series of a square wave whose window size is 6 and frequency is 2. Evaluate the Fourier coefficients and plot the constructed series with the given periodic function.

(b) Consider a rod (length L) of uniform cross-section and made of homogeneous material which is perfectly insulated so that heat flows only along the length. The right end of the rod is inserted in an ice bath to maintain fixed temperature at that end and heated at the left end. Consider the following diffusion equation :

$$\frac{\partial u}{\partial t} = D^2 \frac{\partial^2 u}{\partial x^2}$$

where D is the diffusion coefficient and may be set to unity. Forming one initial condition and two boundary conditions, solve this partial differential equation and plot the temperature profile (temperature along the length of the rod). 8+12

5. (a) Evaluate the Fresnel Integral :

$$\int_0^{\infty} \sin x^2 dx$$

Plot the solution up to a certain value of the upper limit. Comment on the plot.

(b) Write a python program to anlyse in a Fourier series of a symmetric triangular signal. Evaluate the Fourier coefficients and plot the constructed series with the given periodic function. 10+10

6. (a) Write a suitable python program to calculate the integral : $\int_0^\infty e^{-x^2} dx$. Set the accuracy level to 0.0001.

(b) Write a suitable program to solve the following equation :

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \text{ where } 0 < x < \pi,\ t > 0$$ with the initial and boundary conditions :

$$u(x,0) = 1,\ \frac{\partial u(x,0)}{\partial t} = 0$$

$$u(0, t) = 0,\ u(\pi, t) = 0.$$

8+12

7. (a) Verify the following identity with a python program :

$$\frac{\sin(n+1)\theta}{\sin\theta} = \sum_{l=0}^{n} P_l(\cos\theta) P_{n-l}(\cos\theta),$$ where the symbols have their usual meanings.

(b) Consider the Cauchy problem :

$$\frac{\partial u}{\partial t} = x\frac{\partial u}{\partial x} - u + 1,\ -\infty < x < \infty, t \geq 0.$$

$u(x, 0) = \sin x,\ -\infty < x < \infty$

Solve this problem and discuss the behaviour of the solution for large time.

10+10