SPECIAL THEORY OF RELATIVITY

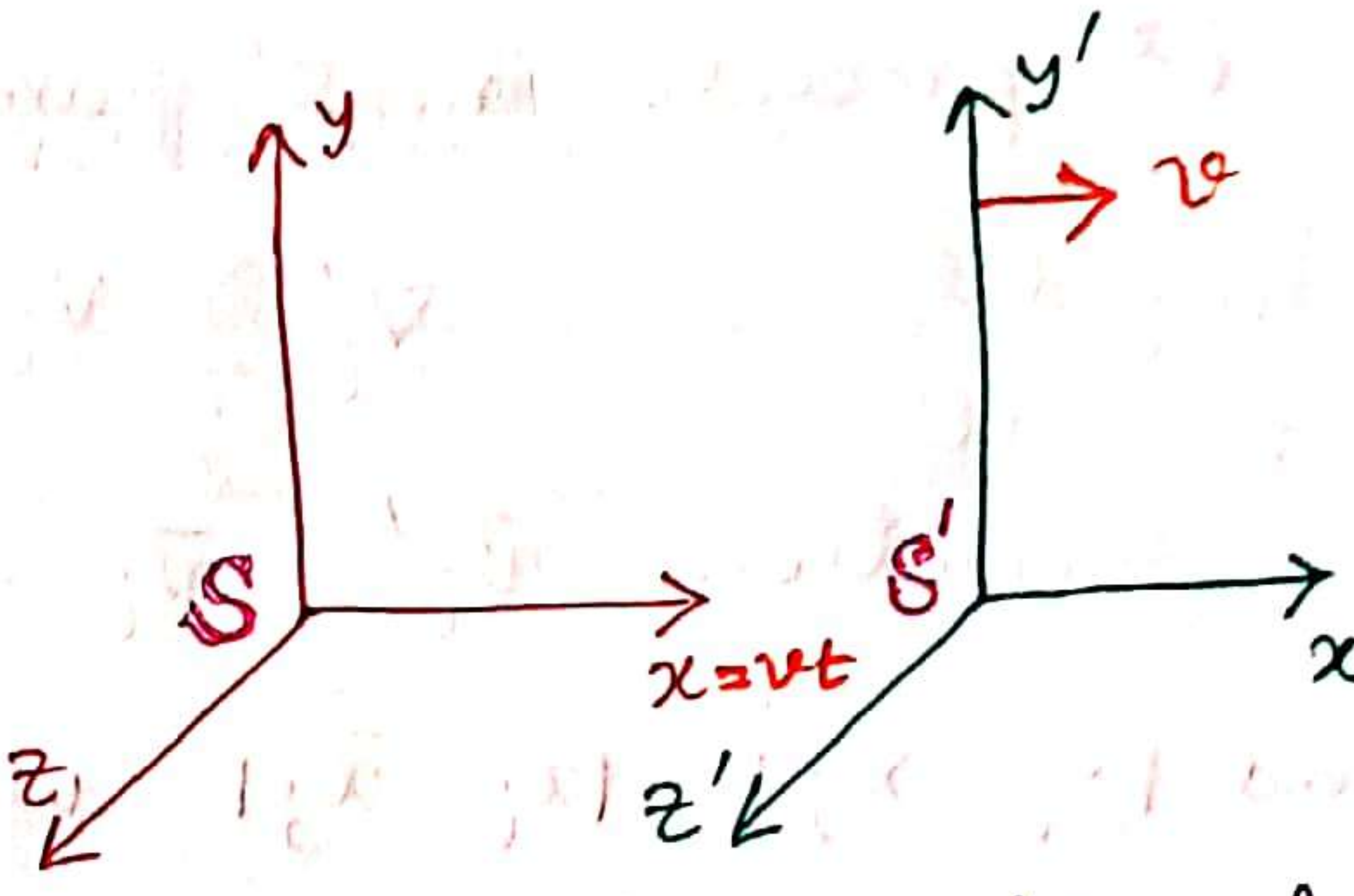
Books :

1. Introduction to Special Relativity → Resnick (Willey) ⇒ Good for first time readers to build concepts of SR (Einsteinian) without 4-vector (Poincare).
2. The Special Theory of Relativity → Bohm (Routledge)/ It's about time understanding Einstein's Relativity (Princeton University Press) → Mermin / Introduction to Special Relativity → Smith (Dover) / Relativity - The Special & General Theory → Einstein ⇒ Good for concept building.
3. Special Relativity → French. (CRC) ⇒ Berkeley Physics style book ; very good for minute details.
4. The Special Theory of Relativity → Banerji and Banerjee (PHI) ⇒ Student friendly nice book that consider both Einsteinian & Poincarian relativity without going too deep.
5. Classical Electrodynamics → Jackson (John Willey) ⇒ Chapter 11 ⇒ Tough but very good book to read about invariances and 4-vector formalism.
6. Classical Theory of Fields → Landau & Lifshitz (HB) ⇒ A book that any serious physics student can never afford to miss.
7. Modern Physics → Beiser / Mani-Mehta ⇒ Any standard modern physics book briefly touches Eeinsteinian Relativity which is usually easy to read for first time readers.
8. Tensor Analysis → B. Spain / Spiegel Schaum Series on Vector analysis ⇒ Read about tensors.

Background: Special theory of relativity, as originally proposed by Einstein in 1905, resulted due to two basic inconsistencies that were posed from theory and experiment. The incompatibility of Newtonian mechanics (Galilean relativity) with Maxwell's equation of Electrodynamics, and the hard-to-throw "luminiferous aether" hypothesis that originated due to study of Optics were void after the null result obtained from Michelson-Morley Nobel prize winning experiment, lead Einstein to develop this theory that works for inertial frame of reference only. The noninertial effects were bundled into the General theory of Relativity. Special theory is accurate in producing results at relativistic speed, i.e close to the velocity of light. The theory is "special" because it's the special case of the "general" theory where the curvature of spacetime, designated by the energy-momentum tensor to cause gravity, is negligible - so it's flat!

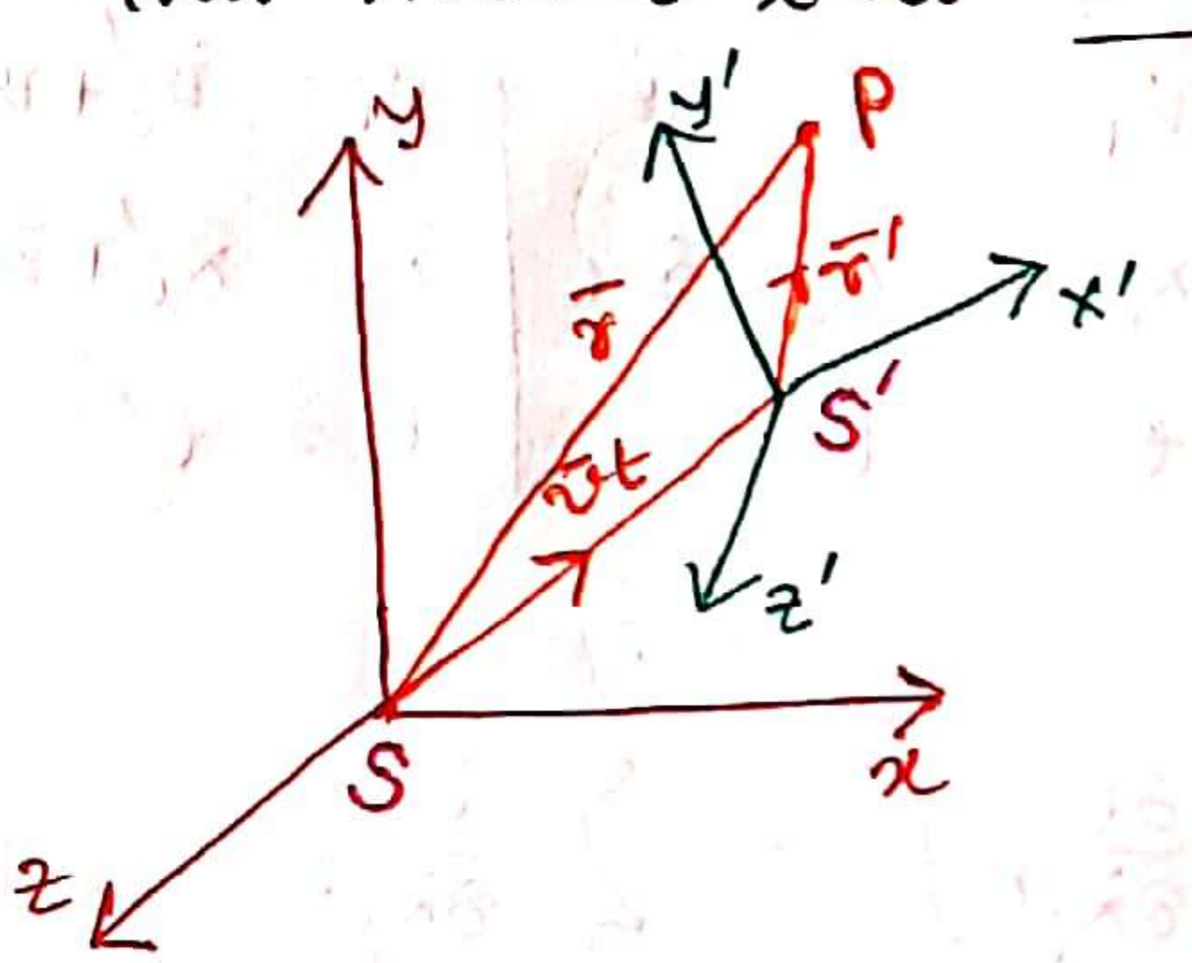
In Newtonian mechanics, speed of light has no special significance, so that according to $E = \frac{1}{2}mv^2$ in a particle accelerator, if energy of electron is increased 4 times, then velocity must be doubled, but experimentally it was found that a change from $0.9988c$ to $0.9999c$ happens for 10 MeV to 40 MeV increase of energy. So the connection between classical mechanics and electromagnetism was not understood. Here we look at the mathematical problem closely next.

Newtonian & Galilean Relativity



At non-relativistic speed, laws of physics are invariant in all inertial (Galilean) frames of reference. This is the principle of Galilean relativity which was established by taking Galilean transformation between two inertial frames S and S'. $x' = x - vt,\ y = y',\ z = z'$ so that $u_{x'} = u_x - v,\ u_{y'} = u_y,\ u_{z'} = u_z$ & $a_{x'} = a_x,\ a_{y'} = a_y,\ a_{z'} = a_z$.

This means that "true force" measured in different inertial frames are equal and there is no way to distinguish among the infinitely many frames in which frame the true force is measured. This also applies for frames that are not parallel or relative velocities are also not parallel.



$$\bar{r}' = \bar{r} - \bar{v}t,\quad t' = t \quad \text{so that } \bar{u}' = \bar{u} - \bar{v} \text{ and } \bar{a}' = \bar{a}.$$

In Galilean relativity, the length of a rod is invariant for both observers in S and S'.

If (x_1, y_1, z_1) and (x_2, y_2, z_2) are coordinates of two end points of the rod in S frame and (x_1', y_1', z_1') and (x_2', y_2', z_2') are that in S' frame which are in relative motion with each other having velocity v along x-direction, then

$$x_2' - x_1' = x_2 - vt - (x_1 - vt) = x_2 - x_1,\quad y_2' - y_1' = y_2 - y_1$$
$$z_2' - z_1' = z_2 - z_1$$

According to Pythagoras theorem, $l = l'$ where $l = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

In general case, $(\bar{r}_2' - \bar{r}_1')^2 = (\bar{r}_2 - \bar{r}_1)^2$, where $\bar{x}' = \bar{x} - \bar{v}t$.

Consider a group of Newtonian particles interacting via 2-body central field potential $V_{ij}(|\bar{x}_i - \bar{x}_j|)$. Equation of motion of the i^{th} particle in s' frame is

$$\boxed{m_i \frac{d\bar{v}_i'}{dt'} = -\nabla_i' \sum_j V_{ij}(|\bar{x}_i' - \bar{x}_j'|)}$$

with the Galilean transformation $\bar{v}_i' = \bar{v}_i - \bar{v}$, $\bar{\nabla}_i' = \bar{\nabla}_i$, $\frac{d\bar{v}_i'}{dt'} = \frac{d\bar{v}_i}{dt}$ as $\bar{v} \neq \bar{v}(t)$

and $|\bar{x}_i' - \bar{x}_j'| = |\bar{x}_i - \bar{x}_j|$, we get back Newton's law in S-frame.

$$\boxed{m_i \frac{d\bar{v}_i}{dt} = -\bar{\nabla}_i \sum_j V_{ij}(|\bar{x}_i - \bar{x}_j|)}$$

(Galilean invariance)

If however the same transformation is applied to wave equation then a field $\phi(\bar{x}', t')$ satisfying wave equation in s' frame

$$\boxed{\underbrace{\left(\sum_i \frac{\partial^2}{\partial \bar{x}_i'^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t'^2}\right)}_{\Box'^2} \phi(\bar{x}', t') = 0} \quad (1)$$

$$\left\{\begin{array}{l|l} \bar{v}_i' = \bar{v}_i - \bar{v} & \bar{v}_i = \bar{v}_i' + \bar{v} \\ \bar{x}_i' = \bar{x}_i - \bar{v}t & \bar{x}_i = \bar{x}_i' + \bar{v}t \\ t' = t & t = t' \end{array}\right.$$

So that $\sum_i \frac{\partial^2}{\partial \bar{x}_i'^2} = \sum_i \frac{\partial}{\partial \bar{x}_i'}\frac{\partial}{\partial \bar{x}_i'}$

$$= \sum_i \Big(\frac{\partial}{\partial \bar{x}_i}\underbrace{\frac{\partial \bar{x}_i}{\partial \bar{x}_i'}}_{=1} + \frac{\partial}{\partial t}\underbrace{\frac{\partial t}{\partial \bar{x}_i'}}_{=0}\Big)\Big(\frac{\partial}{\partial \bar{x}_i}\underbrace{\frac{\partial \bar{x}_i}{\partial \bar{x}_i'}}_{=1} + \frac{\partial}{\partial t}\underbrace{\frac{\partial t}{\partial \bar{x}_i'}}_{=0}\Big) = \sum_i \frac{\partial^2}{\partial \bar{x}_i^2}$$

but $$\frac{\partial^2}{\partial t'^2} = \frac{\partial}{\partial t'}\frac{\partial}{\partial t'} = \Big(\frac{\partial}{\partial t}\underbrace{\frac{\partial t}{\partial t'}}_{=1} + \frac{\partial}{\partial \bar{x}_i}\underbrace{\frac{\partial \bar{x}_i}{\partial t'}}_{=\bar{v}}\Big)\Big(\frac{\partial}{\partial t}\underbrace{\frac{\partial t}{\partial t'}}_{=1} + \frac{\partial}{\partial \bar{x}_i}\underbrace{\frac{\partial \bar{x}_i}{\partial t'}}_{=\bar{v}}\Big)$$

$$= \Big(\frac{\partial}{\partial t} + \bar{v}\cdot\frac{\partial}{\partial \bar{x}_i}\Big)\Big(\frac{\partial}{\partial t} + \bar{v}\cdot\frac{\partial}{\partial \bar{x}_i}\Big) = \frac{\partial^2}{\partial t^2} + 2\bar{v}\cdot\frac{\partial}{\partial \bar{x}_i}\frac{\partial}{\partial t} + \Big(\bar{v}\cdot\frac{\partial}{\partial \bar{x}_i}\Big)\Big(\bar{v}\cdot\frac{\partial}{\partial \bar{x}_i}\Big)$$

So (1) becomes

$$\boxed{\left(\sum_i \frac{\partial^2}{\partial x_i^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{2}{c^2}\Big(\bar{v}\cdot\frac{\partial}{\partial \bar{x}_i}\Big)\frac{\partial}{\partial t} - \frac{1}{c^2}\Big(\bar{v}\cdot\frac{\partial}{\partial \bar{x}_i}\Big)\Big(\bar{v}\cdot\frac{\partial}{\partial \bar{x}_i}\Big)\right)\phi(\bar{x}, t) = 0}$$

So wave equation is not invariant under Galilean transformation. This is reasonable for sound consisting of compression & rarefaction that changes with the choice of reference frame, but not EM wave. Schrödinger equation however is invariant under Galilean transformation, $-\frac{\hbar^2}{2m}\nabla'^2\psi' + V'\psi' = i\hbar\frac{\partial \psi'}{\partial t'}$ becomes

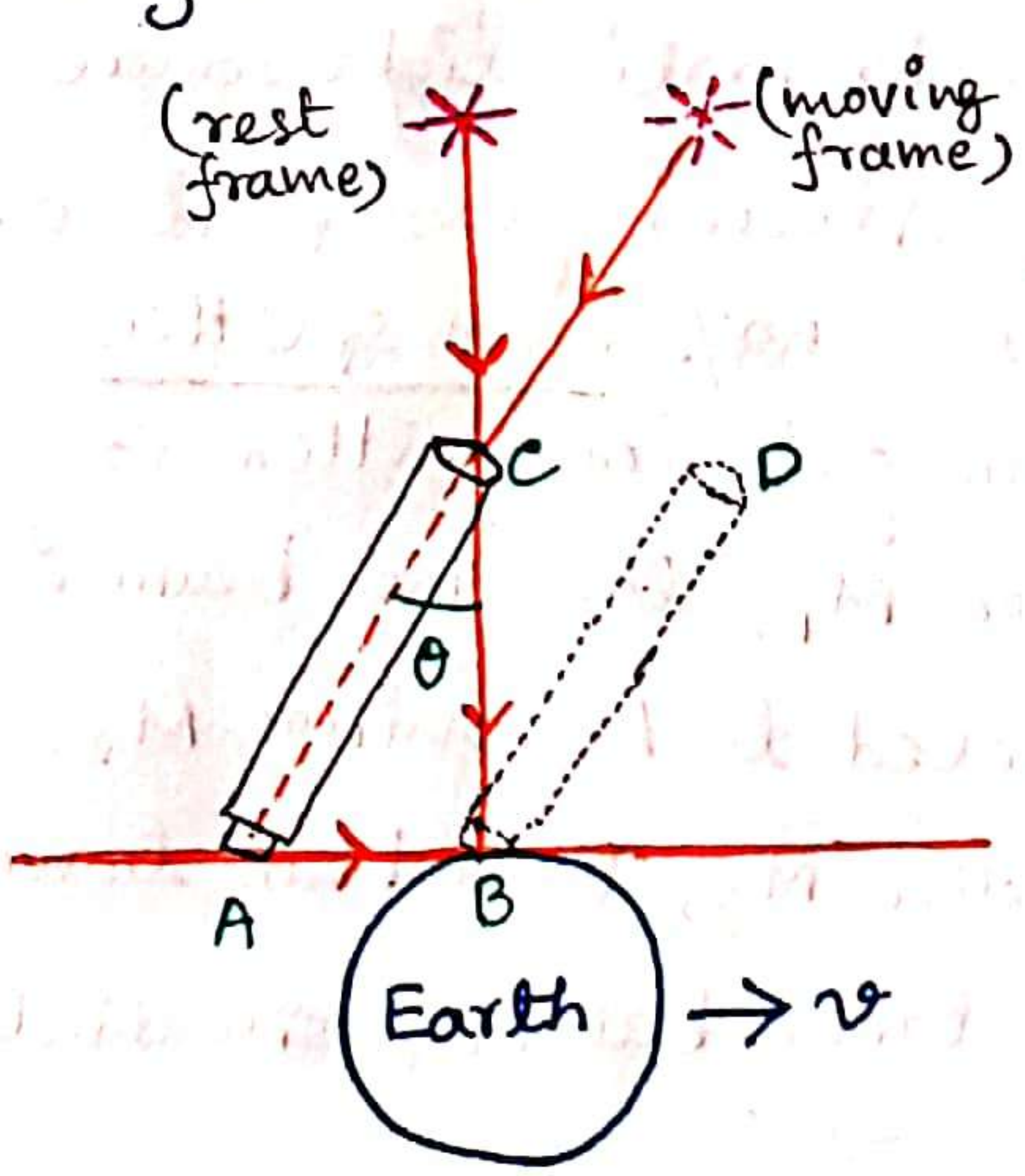
$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial \psi}{\partial t}$ with $V' = V$ and $\psi = \psi' e^{\left[i\frac{m}{\hbar}\bar{v}\cdot\bar{x} - i\frac{mv^2}{2\hbar}t\right]}$

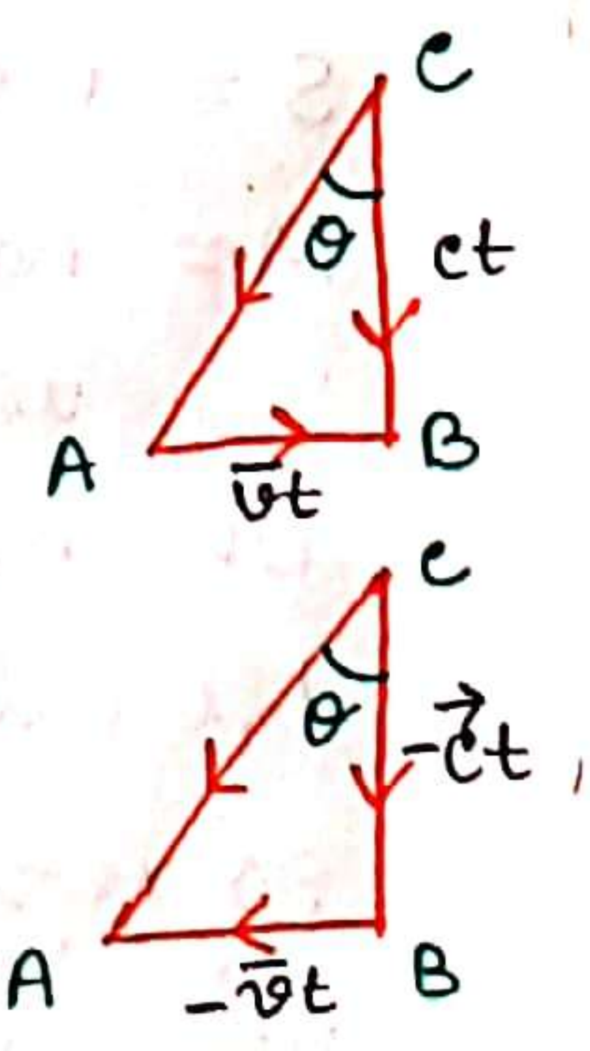
The luminiferous Ether & Search for Special Privileged frame

"Luminiferous aether", meaning light-bearing ether, was came into theory since Newton to accomodate the idea of light propagation through an invisible medium ether, similar to sound propagation. Einstein remarked in 1895 that the velocity of a wave is proportional to square root of elastic forces & inversely proportional to the mass of ether moved (dragged) by these forces. $v = \sqrt{E/\rho}$ 19th century believed that this velocity is the absolute velocity of Earth (with respect to ether frame) and tried to find this special frame using a series of optics experiments. The contradictory and negative result with prediction framed the theoretical ground of special relativity.

(a) Aberration of light (Bradley, 1727)

"Aberration" means propagation in moving bodies. If Earth is considered an ~~non~~ inertial frame, then on a windless rainy day a man standstill on ground will see raindrops atop (zenith) coming vertically downwards, which isn't the case if the man starts to move. Similarly if light is seen coming from a star to a man with telescope (astronomer) from zenith, the apparent direction of light from star will not be vertical because of Earth velocity v. The angle θ between actual & apparent direction is called aberration.





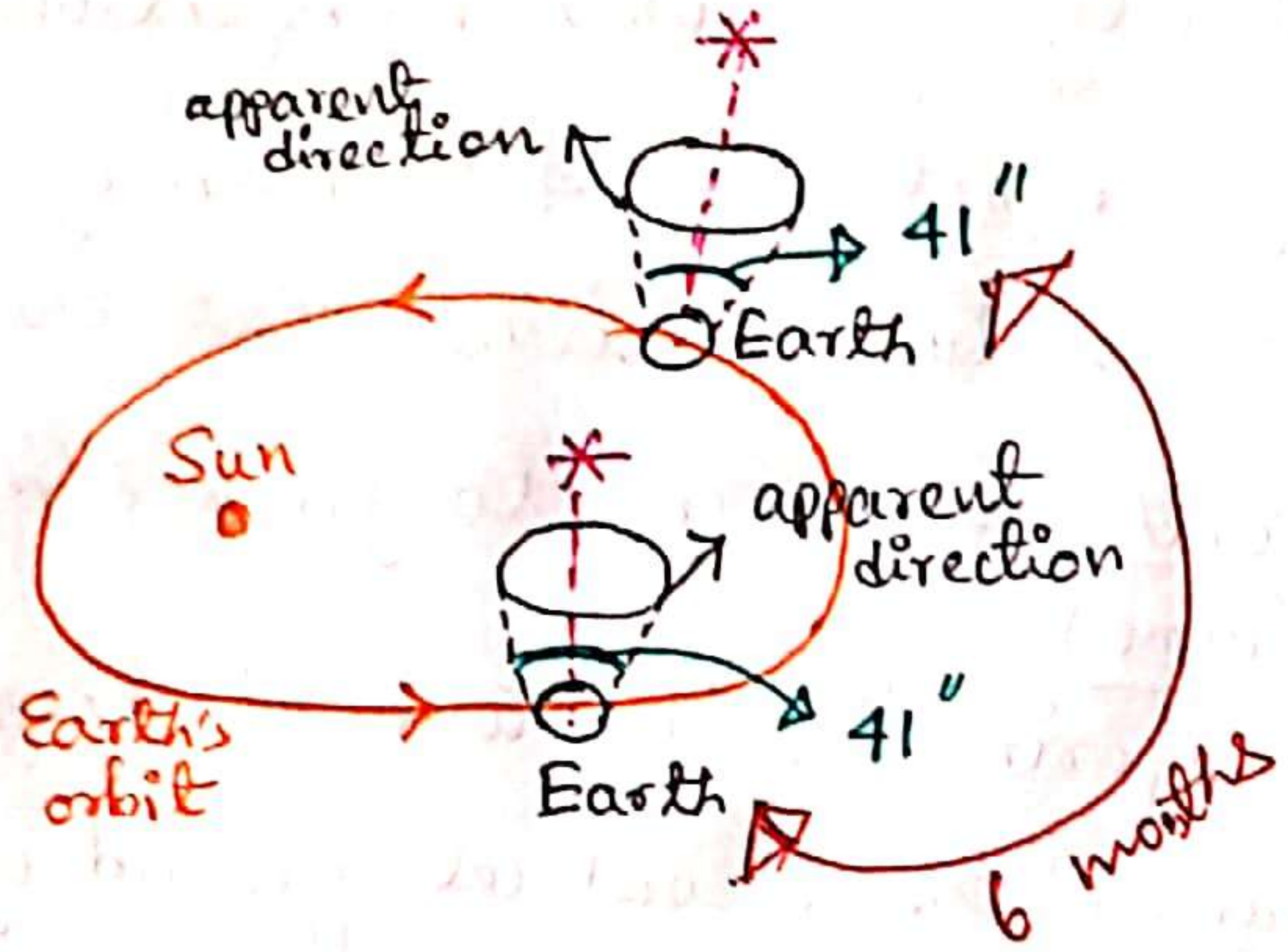
By the time light enters through C to come out from B of the telescope, due to Earth's velocity A has moved to B.

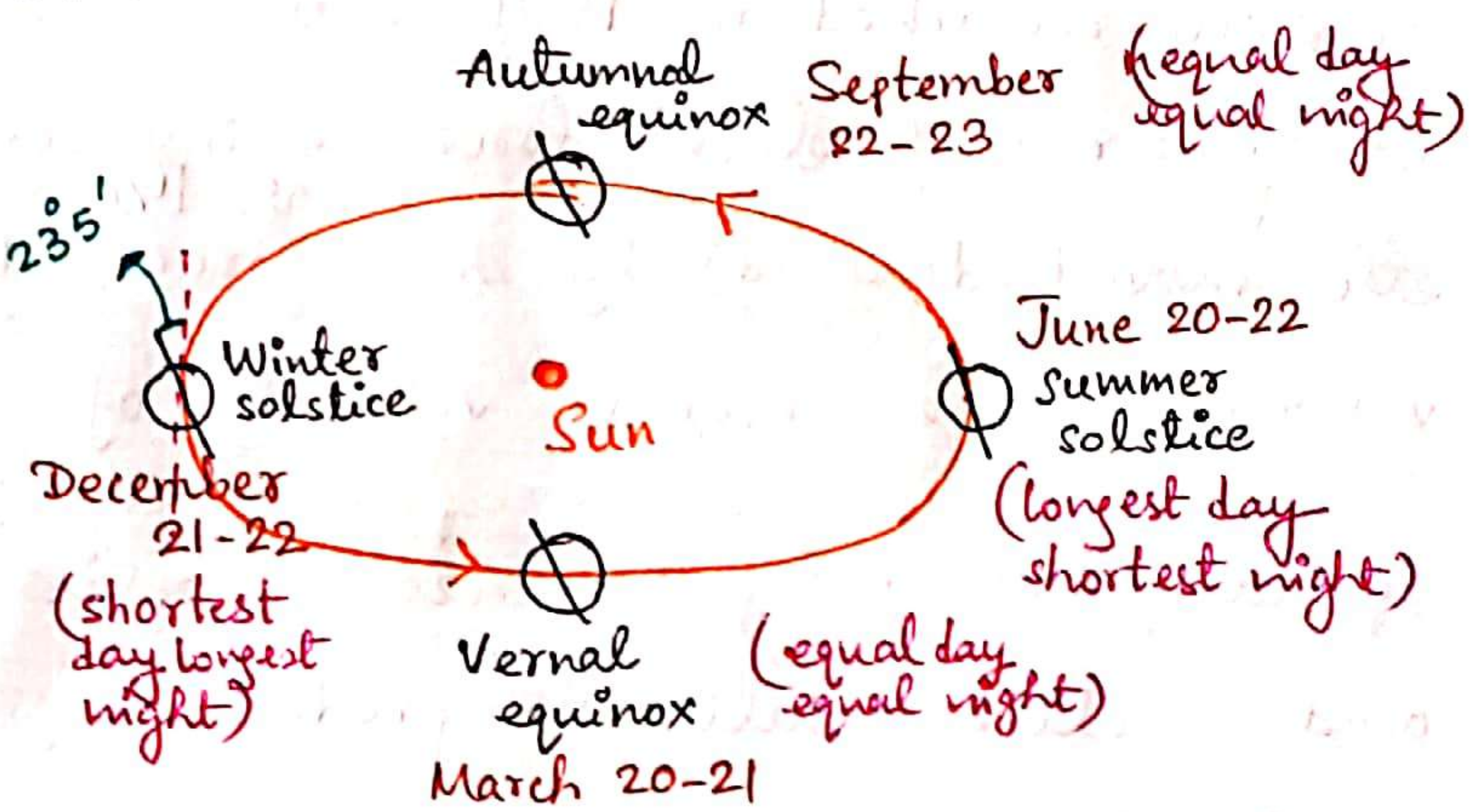
So $CB = ct$ and $AB = vt$

$\therefore \tan\theta = 1/\frac{CB}{BA} = \frac{v}{c}$ or $\theta = \tan^{-1}\left(\frac{v}{c}\right)$

We know that Earth has daily and annual rotation, so for an equitorial observer, Earth's daily velocity is approximately $\frac{1}{2}$ Km/s while that of annual is 30 Km/s. Taking this into account

$$\theta = \tan^{-1}\frac{v}{c} = \tan^{-1}\frac{3\times10^4}{3\times10^8} = 20.5 \text{ arc seconds.}$$

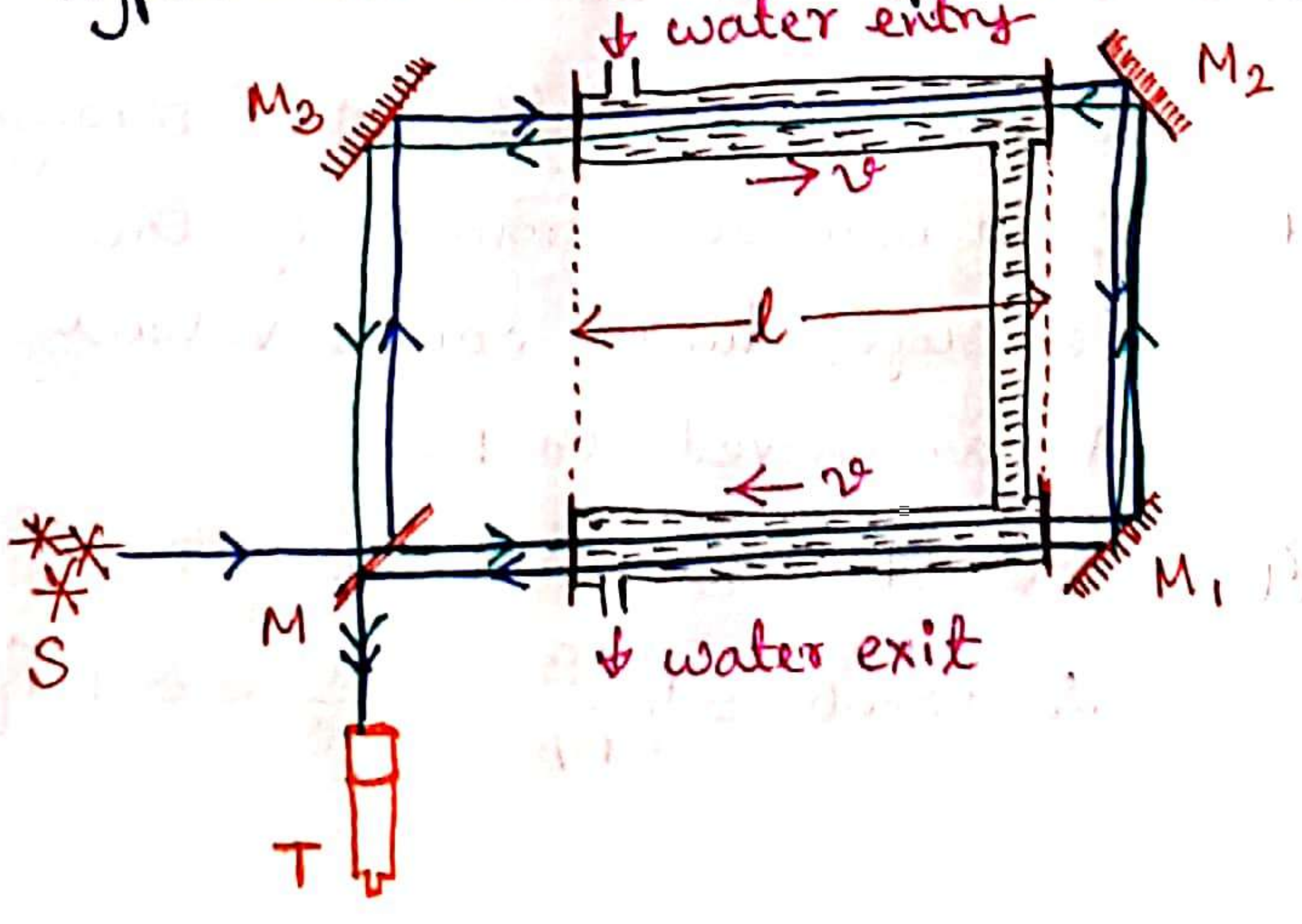




So if an astronomer observes a star overhead for a year, it will create an ellipse of angular diameter of 41" with the zenith which Bradley tested with γ-Draconis star to experimentally confirm Earth's absolute velocity to be 30 km/s, and ether is not dragged around with Earth.

(b) Fizeau's Experiment (1851) after Fresnel's hypothesis (1817)

In 1817, Fresnel theoretically predicted that light will be partially dragged along flowing water to contradict the ether-drag hypothesis that was experimentally verified by Fizeau.



S = monochromatic light source

M = half-silvered glass plate as used as 50% beam splitter.

One beam gets transmitted to hit mirror M_1, the other beam is 90° reflected to hit mirror M_3. Using another M_2, both beam traverse equal distance but in opposite direction $(=2l)$