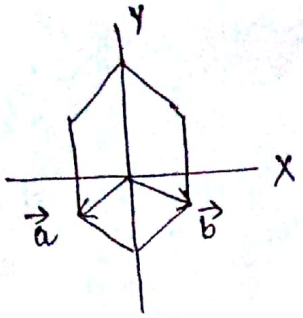
1. 

$\vec{a} = \frac{\sqrt{3}a}{2}\hat{i} + \frac{a}{2}\hat{j}$, $\vec{b} = -\frac{\sqrt{3}a}{2}\hat{i} + \frac{a}{2}\hat{j}$,

$\vec{c} = c\hat{k}$

$V = \vec{a}\cdot\vec{b}\times\vec{c} = \left(\frac{\sqrt{3}a}{2}\hat{i} + \frac{a}{2}\hat{j}\right)\cdot\left[\left(-\frac{\sqrt{3}a}{2}\hat{i} + \frac{a}{2}\hat{j}\right)\times c\hat{k}\right]$

$= \left(\frac{\sqrt{3}a}{2}\hat{i} + \frac{a}{2}\hat{j}\right)\cdot\left(\frac{\sqrt{3}ac}{2}\hat{j} + \frac{ac}{2}\hat{i}\right)$

$= \frac{\sqrt{3}a^2c}{4} + \frac{\sqrt{3}a^2c}{4} = \frac{\sqrt{3}}{2}a^2c.$

2. Mass per molecule $= \frac{M}{N}$

If n number of molecules in a unit cell, mass of unit cell $= \frac{nM}{N}$. Crystal density ρ

$\therefore$ Volume of unit cell $V = \frac{nM}{\rho N}$

or $a^3 = \frac{nM}{\rho N}$. $\Rightarrow a = \left(\frac{nM}{\rho N}\right)^{1/3}$

For fcc crystal, number of molecules per unit cell $n = 4$.

$\therefore a = \left(\frac{4M}{\rho N}\right)^{1/3}$.

3. $\vec{a}^{**} = 2\pi\frac{\vec{b}^*\times\vec{c}^*}{\vec{a}^*\cdot\vec{b}^*\times\vec{c}^*}$, $\vec{b}^{**} = 2\pi\frac{\vec{c}^*\times\vec{a}^*}{\vec{a}^*\cdot\vec{b}^*\times\vec{c}^*}$, $\vec{c}^{**} = 2\pi\frac{\vec{a}^*\times\vec{b}^*}{\vec{a}^*\cdot\vec{b}^*\times\vec{c}^*}$

$\vec{b}^* = 2\pi\frac{\vec{c}\times\vec{a}}{\vec{a}\cdot\vec{b}\times\vec{c}}$, $\vec{c}^* = 2\pi\frac{\vec{a}\times\vec{b}}{\vec{a}\cdot\vec{b}\times\vec{c}}$, $\vec{a}^* = 2\pi\frac{\vec{b}\times\vec{c}}{\vec{a}\cdot\vec{b}\times\vec{c}}$

$\vec{b}^*\times\vec{c}^* = (2\pi)^2\frac{(\vec{c}\times\vec{a})\times(\vec{a}\times\vec{b})}{(\vec{a}\cdot\vec{b}\times\vec{c})^2} = (2\pi)^2\left[\frac{\vec{a}(\vec{c}\times\vec{a}\cdot\vec{b}) - b(\vec{c}\times\vec{a}\cdot\vec{a})}{(\vec{a}\cdot\vec{b}\times\vec{c})^2}\right]$

$= (2\pi)^2\frac{\vec{a}}{\vec{a}\cdot\vec{b}\times\vec{c}}$

$\vec{a}^*\cdot\vec{b}^*\times\vec{c}^* = (2\pi)^3\frac{\vec{a}\cdot\vec{b}\times\vec{c}}{(\vec{a}\cdot\vec{b}\times\vec{c})^2} = \frac{(2\pi)^3}{\vec{a}\cdot\vec{b}\times\vec{c}}$

$\therefore a^{**} = 2\pi\frac{(2\pi)^2\frac{\vec{a}}{\vec{a}\cdot\vec{b}\times\vec{c}}}{(2\pi)^3\frac{1}{\vec{a}\cdot\vec{b}\times\vec{c}}} = \vec{a}$, $\vec{b}^{**} = 2\pi\frac{(2\pi)^2\frac{\vec{b}}{\vec{a}\cdot\vec{b}\times\vec{c}}}{(2\pi)^3\frac{1}{\vec{a}\cdot\vec{b}\times\vec{c}}} = \vec{b}$

$c^{**} = 2\pi\frac{(2\pi)^2\frac{\vec{c}}{\vec{a}\cdot\vec{b}\times\vec{c}}}{(2\pi)^3\frac{1}{\vec{a}\cdot\vec{b}\times\vec{c}}} = \vec{c}$.

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