Sem-I - Newtonian Mechanics

(Instructor: AKB, Department of Physics, Asutosh College)

Assignment II: Non-inertial Frame of Reference

Submission due date: 30/12/2021

Q.1) Show that for a single particle with constant mass, the equation of motion implies the following differential equation for the kinetic energy:

$$\frac{dE}{dt} = \mathbf{F} \cdot \mathbf{v},$$

while if the mass varies with time, the corresponding equation is

$$\frac{d(mE)}{dt} = \mathbf{F} \cdot \mathbf{p}.$$

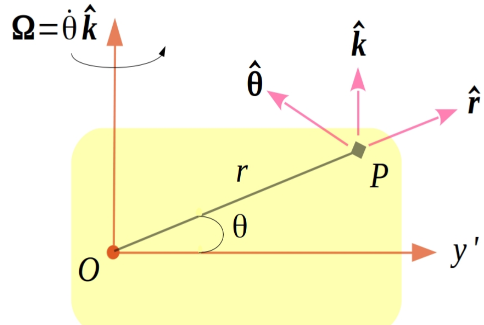
Q.2) **(a)** Assuming that Earth is spherical, calculate the difference in apparent acceleration of gravity at the equator and the poles. **(b)** Calculate the rate of rotation of the plane of oscillation of a Foucault pendulum at latitude 30 degrees and hence determine the time it will take to turn through a complete right angle.

Q.3)

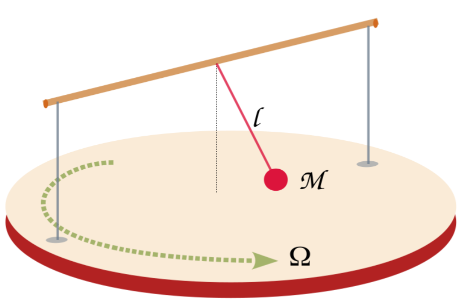
In figure beside, the polar coordinates (r, θ) are measured relative to an inertial frame $\mathcal{F}$, while $\mathcal{F}'$ is a frame rotating about the z-axis passing through $\mathcal{O}$ with angular velocity $\boldsymbol{\Omega}$. By using the velocity and acceleration transformation formula between inertial and rotating coordinate system, show that

$$\mathbf{v} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}}, \quad \mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\boldsymbol{\theta}}.$$

[Note here as $\dot{\boldsymbol{\Omega}} \neq 0$, therefore unlike our derivation in class, you also have to consider $\dot{\boldsymbol{\Omega}} \times \mathbf{r}'$ term along with fictitious acceleration terms in the rotating coordinate system.]



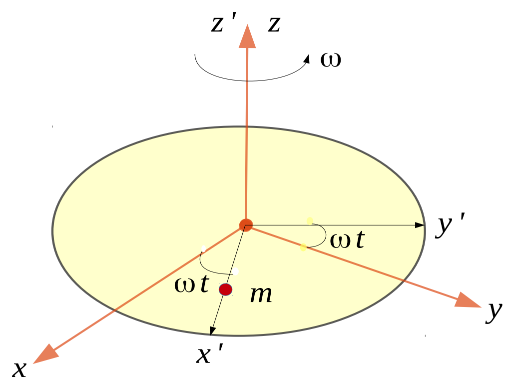
Q.4)



As shown beside, a pendulum is rigidly fixed to an axle held by two supports so that it can swing only in a plane perpendicular to the axle. The supports are mounted on a platform which rotates with constant angular velocity $\boldsymbol{\Omega}$. The pendulum consists of a mass $\mathcal{M}$ attached to a massless rod of length l. Find the pendulum's oscillation frequency, assuming that the amplitude is small.

Q.5) An ocean current circulating counter-clockwise when viewed from directly overhead was discovered in a well-isolated layer beneath the surface. The period of rotation was 14 hours ($\omega = \frac{2\pi}{14}$ hr^{-1}). In Earth's surface fixed coordinate frame, the ocean current is x-axis pointing south, y-axis pointing east, z-axis pointing vertically upward and $\mathbf{\Omega} = -\Omega sin\lambda\hat{\mathbf{i}} + \Omega cos\lambda\hat{\mathbf{k}}$ where λ is colatitude ($\Omega = \frac{2\pi}{24}$ hr^{-1}). At what latitude and in which hemisphere was the current detected?

Q.6)



A turn-table marked with three orthogonal axes (x', y', z') is rotating on the Earth, assumed to be an inertial frame (x, y, z), with constant angular velocity $\boldsymbol{\omega}$ about z-axis. At $t = 0$, both frames coincide with each other. A ball of mass m is rolling outward without slipping along the x'-axis with a constant velocity v. What is the total force exerted by the turn-table on the ball? Find out the total force as measured in the Earth frame coordinates (x, y, z).