HW 1. $x = a\sin(\omega t + \phi)$ At $t = 0$, $x = x_0$

$\therefore x_0 = a\sin\phi$, $\dot{x} = a\omega\cos(\omega t + \phi)$, At $t = 0$, $\dot{x} = v_0$

$v_0 = a\omega\cos\phi$.

$\therefore \tan\phi = \dfrac{\sin\phi}{\cos\phi} = \dfrac{x_0/a}{v_0/a\omega} = \dfrac{x_0\omega}{v_0}$

also, $a^2\sin^2\phi + a^2\cos^2\phi = x_0^2 + \dfrac{v_0^2}{\omega^2}$

or $a^2(\sin^2\phi + \cos^2\phi) = x_0^2 + \dfrac{v_0^2}{\omega^2}$. $\therefore a = \sqrt{x_0^2 + \dfrac{v_0^2}{\omega^2}}$.

2. Loss contact = upward force $\geq$ weight mg.

$x = a\sin(\omega t + \phi)$

$\therefore$ Upward force $= m\ddot{x} = ma\omega^2\sin(\omega t + \phi) = m\omega^2 x$.

So loss contact condition, $m\omega^2 x \geq mg$

or $x \geq \dfrac{g}{\omega^2} \geq \dfrac{9.81}{4\pi^2 5^2} \geq 10^{-2}$ metre.

3. $v = \omega\sqrt{a^2 - x^2}$

$\therefore 80 = \omega\sqrt{a^2 - 3^2}$ & $60 = \omega\sqrt{a^2 - 4^2}$

or $\dfrac{80}{60} = \dfrac{4}{3} = \dfrac{\sqrt{a^2 - 3^2}}{\sqrt{a^2 - 4^2}}$ or $\dfrac{16}{9} = \dfrac{a^2 - 9}{a^2 - 16}$

or $a = 5$ cm.