PHYSICAL OPTICS

(Diffraction and Holography)

Books*:

1. Opticks → Ghatak (6th Edition, Tata Mc Graw Hill)
   ⇒ Standard textbook, Good for first time readers.

2. Introduction to Geometrical and Physical Optics → B.K. Mathur (Old Book) ⇒ Good for concept building and theory learning.

3. Fundamental of Optics (Tata McGraw Hill) → Jenkins & White ⇒ Concise book, good for Problem solving.

4. Principles of Optics (Pergamon Press) → Born & Wolf ⇒ Very good book for theory learning.

5. Feynman lectures on Physics Vol-1 → Feynman / Leighton / Sands (Narosa) ⇒ Short and concise for concept building.

6. Optics → Hecht (Addison Wesley) ⇒ Good for problem solving and first time readers.

7. Introduction to Holography → Toal (CRC Press) ⇒ New age book for basic holography principles.

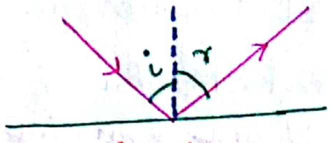
Opticks → Geometrical Optics
Opticks → Physical Optics.

Geometrical Optics deals with refraction and reflection at surfaces, lenses, Matrix method, dispersion through prism, Aberrations and eyepieces and it terms on the particle (corpuscular) theory of light using Fermat's principle. Physical optics on the other hand deals with wave theory of light as Fresnel-Hugen's principle and discusses on Interference and Coherence, Diffraction, Polarisation (crystal optics), Fiber optics and Holography.

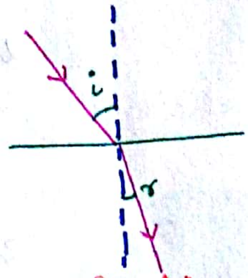
DIFFRACTION

Fermat's principle says that when a ray of light goes from one point to another through a set of media, it always follows a path along which the time taken is minimum.

$\frac{dt}{dx} = 0$ yields the "law of reflection" $i = r$. and the "Snell's law of refraction" $v_1 \sin i = v_2 \sin r$ by conservation of the horizontal component of momentum.



reflection

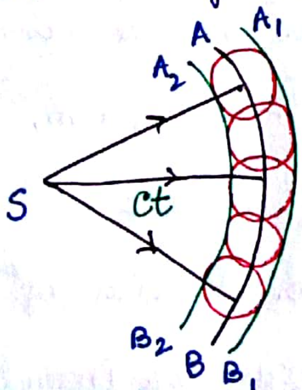


refraction

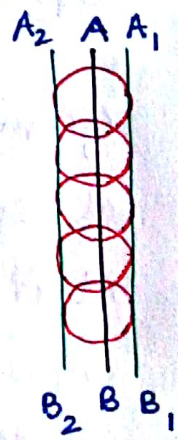
The corpuscular model of light establish the rectilinear (straight line) propagation of light and propagation of light through vacuum.

Wave theory and Huygens-Fresnel principle

A source of light transmit wave that contain energy in all directions. A "wave front" is defined as the locus of all points which are in the same state of vibration (same phase). For example, circular ripples spreading out in a pond as a pebble is dropped, each circumferential point oscillating at same amplitude & same phase. Similarly for a light source, at a nearby location $x = ct$ where AB is a spherical wavefront, while at large distance, AB is a plane wavefront. Surface AB is called "primary wavefront". The direction in which the wave is propagated is known as "ray" which is perpendicular to the wavefront.



spherical wavefront



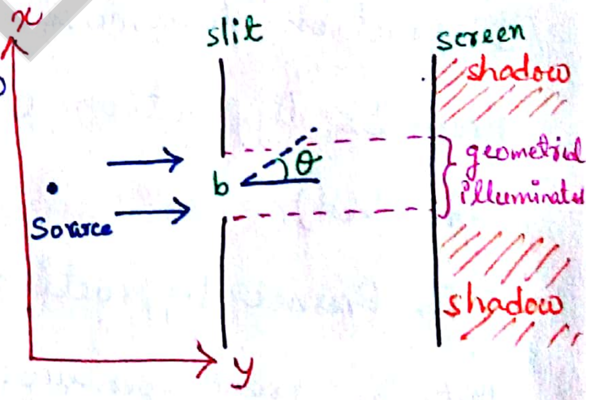
plane wavefront

Hygen-Fresnel principle tells that all points on the primary wavefront are considered to be the centres of disturbance and they transmit secondary waves in all direction with the same velocity as the primary. So A_1B_1 surface that touch the spheres after ct_1 distance is the "Secondary wavefront"

Using Huygen-Fresnel principle, law of reflection ($i = r$), law of refraction ($v_1 \sin i = v_2 \sin r$), refraction of spherical wave at concave spherical surface ($\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{R}$) and convex sphereical surface ($\frac{\mu-1}{R} = \frac{1}{v} - \frac{1}{u}$), lens formula for thin convex / concave lens ($\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$) can be obtained.

Why Diffraction? Wave-particle duality as in deBroglie's matter wave theory $\lambda = \frac{h}{p}$ gives rise to Heisenberg's uncertainty principle $\Delta x \, \Delta p_x \geq h$.

If we illuminate a single slit (narrow opening) and if light propagation is rectilinear then there is no bending of light in the geometrical shadow.



But if a light quanta (photon) or electron pass through slit, then $\Delta x \sim b$, so $\Delta p_x \sim \frac{h}{b}$. As $p_x = p \sin\theta$

So $\sin\theta \sim \frac{h}{pb} \sim \frac{\lambda}{b}$.

When $b \gg \lambda$, $\sin\theta \to 0$ or almost no bending in geometrical shadow, while for $b \sim \lambda$ then there will be significant bending.

The bending of light round corners and spreading of light waves into the geometrical shadow of an object is called Diffraction.

Difference between Interference & Diffraction

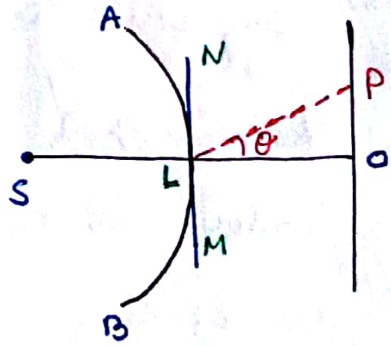
Interference	Diffraction
1. Result due to superposition of light from two different wavefront emanating from the same source.	1. Result due to superposition of light from different parts of the same wavefront.
2. Fringes may/may-not be of same width.	2. Fringes are never of same width
3. All bright bands are of uniform intensity	3. All bright bands are of different intensity
4. Points of minimum intensity are perfectly dark.	4. Points of minimum intensity are not perfectly dark.

Classification of Diffraction

Diffraction phenomena are divided into two distinct classes, as Fresnel's diffraction (near field) and Fraunhofer diffraction (far field).

In Fresnel diffraction, source of light & screen are at finite distance from aperture. No concave/convex lenses are used so that incident wavefront is either spherical/cylindrical but not planar. So phase of secondary wavefront isn't same in the plane of aperture.

Fresnel's assumptions



(a) A wavefront is divided into a large number of small area (Fresnel's zone). Secondary waves originating from various zones will interfere and the resultant effect can be noted at point P on the screen.

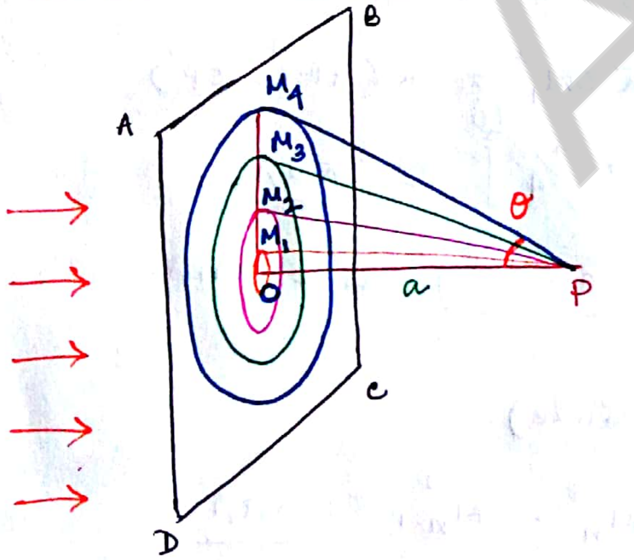
(b) Resultant at P due to a particular zone will depend on the distance of the point from the zone.

(c) Resultant at P will also depend on obliquity factor, which is proportional to $(1+\cos\theta)$. So for a wavefront at L, maximum at O occurs for $\theta = 0$, while in LN or LM direction, intensity is half of O, as $\theta = \pi/2$. Along LS, $\theta = \pi$, so no intensity in reverse direction. (zone plate)

Fraunhofer diffraction occur when source of light/screen are effectively infinite distances from aperture. Two convex lenses are used & incident wavefront is plain. Secondary wavelet from exposed portion of the wavefront at aperture are in the same phase at all points in plane of the aperture.

(plane transmission grating, concave reflection grating)

Fresnel's half-period zone of a plain wave-front



- First half period zone $a + \lambda/2$
- Second half period zone $a + \lambda$
- Third half period zone $a + 3\lambda/2$
- fourth half period zone $a + 2\lambda$

Let us consider a plane wavefront of a monochromatic light at any particular instant. We want to find out the resultant amplitude at P due to all the wavelets coming from this wavefront.

According to Huygen's principle, every point on the plane wavefront may be regarded as the origin of the secondary wavelets & therefore the resultant effect at P due to the whole wavefront will be equal to the resultant of all these secondary wavelets.

The wavefront is divided into a number of Fresnel's half period zones - from P drop a perpendicular on ABCD at O (pole of the wave). Let OP = a and P as centre & radius $(a+\lambda/2)$, draw a sphere cutting the wavefront in a circle at M_1,

$PM_1 = a + \lambda/2$ so that the secondary wavelets from O & from the points on the circumference of M_1 on reaching P will differ in phase by $\frac{2\pi}{\lambda}(PM_1 - OP) = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi = \frac{T}{2}$ (half period)

Similarly other sphere of radii $(a+\frac{2\lambda}{2}), (a+\frac{3\lambda}{2}), (a+\frac{4\lambda}{2}), \ldots$ can be drawn that intersect at $M_2, M_3, M_4, \ldots$ so that the whole wavefront can be divided into several half period zones.

Amplitude due to wavelets produced by each zone is

(i) Directly proportional to the area of the zone which is approximately equal.

(ii) Varies inversely with the distance of zone from P.

(iii) Varies with the obliquity factor $(1+\cos\theta)$.

Area of 1st half period zone $= \pi\, OM_1^2 = \pi(PM_1^2 - OP^2)$

$= \pi[(a+\lambda/2)^2 - a^2] = \pi[a\lambda + \cancel{\frac{\lambda^2}{4}}] \simeq \pi a\lambda.$

Similarly $OM_n^2 = PM_n^2 - OP^2 = (a+\frac{n\lambda}{2})^2 - a^2 = an\lambda$ (n^{th} circle)

$OM_{n-1}^2 = a(n-1)\lambda$ ($(n-1)^{th}$ circle).

So Area of n^{th} zone $= \pi(OM_n^2 - OM_{n-1}^2) = \pi a\lambda.$

So $\boxed{\begin{array}{l}\text{radii of zone} \propto \sqrt{n} \\ \text{area of zone independent of } n\end{array}}$

Schuster's Method :

For visible light, $\lambda \simeq$ small & so area of zone $= \pi a \lambda$ but if λ is not very small then the area of half period zones of higher order decreases gradually. If the phase of the wavelets coming from O is zero then the phase of wavelets from intermediate points between O and M_1 will vary from 0 to π (because $\frac{2\pi}{\lambda}(PM_1 - OP) = \pi$).

$\therefore$ Average phase of all wavelets from 1^{st} zone $= \frac{0+\pi}{2} = \frac{\pi}{2}$.

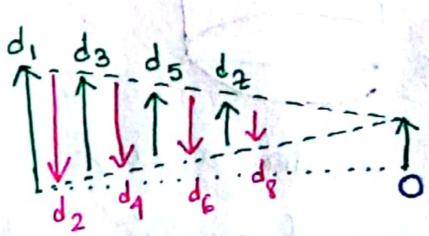
Similarly phase difference of wavelets from M_1 & M_2 will be between π and 2π, so that average phase of all wavelets from 2^{nd} zone

$= \frac{\pi + 2\pi}{2} = \frac{3\pi}{2}$, from 3^{rd} zone $\frac{5\pi}{2}$, from 4^{th} zone $\frac{7\pi}{2}$ & so on...

Resultant phase-difference between two consecutive zones $= \pi$.

Resultant phase-difference between two alternate zones $= 2\pi$.

So if resultant from 1^{st} half period zone is positive then 2^{nd} half period zone is negative.



Amplitude decreases due to obliquity factor $(1 + \cos\theta)$, so resultant amplitude

$$D = d_1 - d_2 + d_3 - d_4 + d_5 - \cdots \pm d_n.$$

(i) If $n =$ odd, to a first approximation $d_2 = \frac{d_1 + d_3}{2}$, $d_4 = \frac{d_3 + d_5}{2}$

so that $$D = \frac{d_1}{2} + \left(\frac{d_1 + d_3}{2} - d_2\right) + \left(\frac{d_3 + d_5}{2} - d_4\right) + \cdots + \frac{d_n}{2}$$

$$= \frac{d_1}{2} + \frac{d_n}{2}.$$

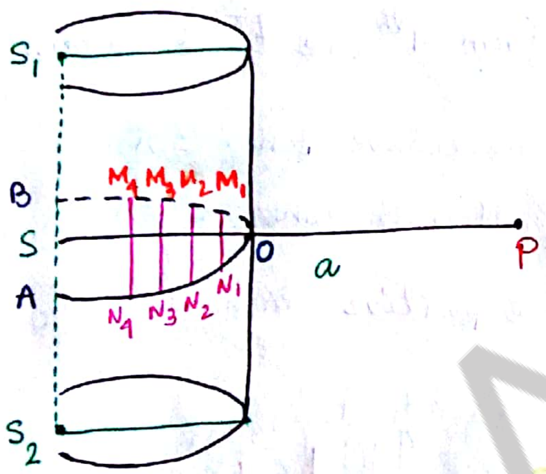
(ii) If $n =$ even, $D = \frac{d_1}{2} + \frac{d_{n-1}}{2} - d_n$

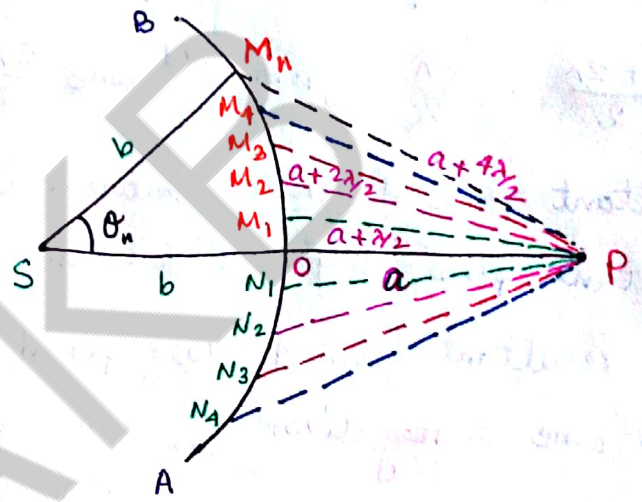
If n is very large, then effect from n^{th} zone is negligible &

resultant amplitude due to whole wave is $D = \frac{d_1}{2}$

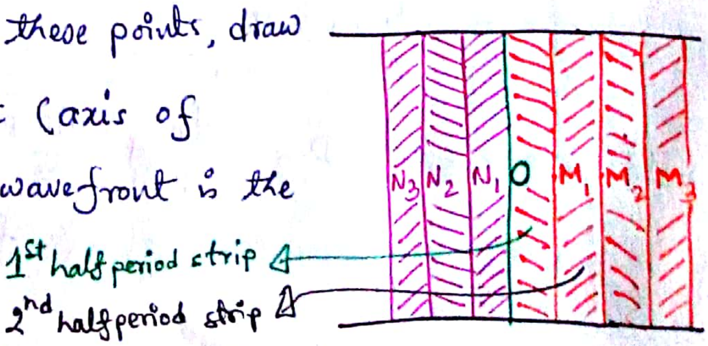
intensity $I = D^2 = \frac{d_1^2}{4}$. If an obstacle is placed at O then the resultant disturbance at P is = half the disturbance due to wavelets from the 1st half-period zone with one-fourth the intensity. If obstackle at O blocks a considerable number of half-period zones, effect is negligible & no light is received at P - or <u>light travels approximately in a straight line.</u>

Fresnel's half-period strip of a cylindrical wavefront





Consider a long and narrow slit S_1S_2, when illuminated by monochromatic light of wavelength λ, produces cylindrical wavefront. To find the resultant amplitude, the wavefront can be divided into half period strips, with O as pole. Consider an equatorial section AOB through O in plane of paper. With P as centre & radius $(a + \frac{\lambda}{2})$, $(a + \frac{2\lambda}{2})$, ... etc, draw arcs that cut AOB at point M_1N_1, M_2N_2, ... etc. Through these points, draw lines parallel to length of slit (axis of wavefront) and the area of the wavefront is the half period strip.



Amplitude of the waves reaching P due to wavelets produced by each half-period strip is

(i) Directly proportional to the area of the strip (not equal)

(ii) Average distance of strip from P

(iii) Varies with the obliquity factor $(1+\cos\theta)$

As length of strip is same, so areas are proportional to arcs $OM_1, M_1M_2, M_2M_3, \dots$ where $PM_n = a + \frac{n\lambda}{2}$

from triangle PM_nS, we have $PM_n^2 = SM_n^2 + PS^2 - 2SM_n PS \cos\theta_n$

$$\text{or } \left(a+\frac{n\lambda}{2}\right)^2 = b^2 + (a+b)^2 - 2b(a+b)\underbrace{\cos\theta_n}_{(1-\theta_n^2/2!)}$$

$$\text{or } \cancel{a^2} + an\lambda + \cancel{\frac{n^2\lambda^2}{4}}^{\to 0} = \cancel{2b^2} + \cancel{a^2} + \cancel{2ab} - \cancel{2ab} - \cancel{2b^2} + b(a+b)\theta_n^2$$

$$\text{or } an\lambda = b(a+b)\theta_n^2 \quad \text{or} \quad \theta_n = \sqrt{\frac{an\lambda}{b(a+b)}} = K\sqrt{n}$$

Now $OM_n = b\theta_n = bK\sqrt{n}$.

So $OM_1 = bK$, $OM_2 = bK\sqrt{2}$, $OM_3 = bK\sqrt{3}$.

So $M_1M_2 = bK(\sqrt{2}-1) = 0.414\ bK$

$M_2M_3 = bK(\sqrt{3}-\sqrt{2}) = 0.318\ bK$

$M_3M_4 = bK(\sqrt{4}-\sqrt{3}) = 0.268\ bK$, $M_4M_5 = 0.236\ bK, \dots$

So area of strip initially decreases rapidly & then for increasing order more slowly. and because of opposite sign they cancel out each other. So the resultant at P is only due to first few half period strips.

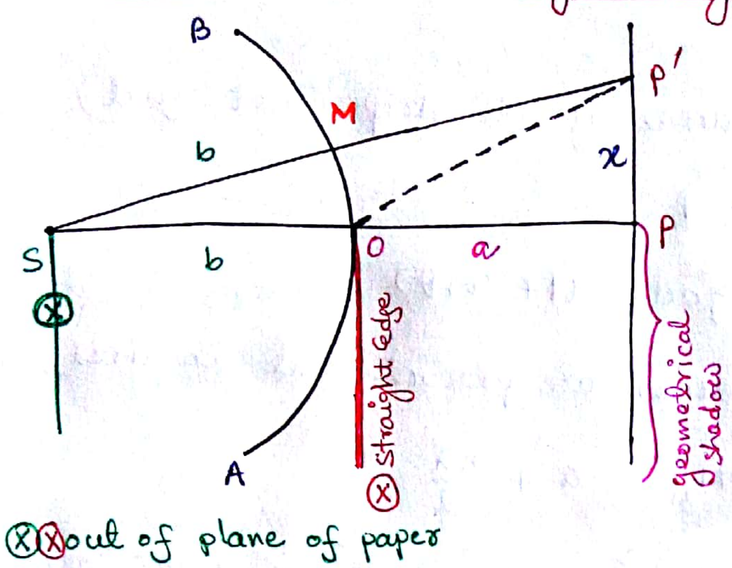
$$D = d_1 - d_2 + d_3 - d_4 + \dots \approx \frac{d_1}{2} \text{ (from half wavefront) (left side)}$$

$$\approx \frac{d_1}{2} \text{ (from right side half wavefront)}$$

So resultant due to whole wavefront $= \frac{d_1}{2} \pm \frac{d_1}{2} = d_1$ (n odd)

$= 0$ (n even)

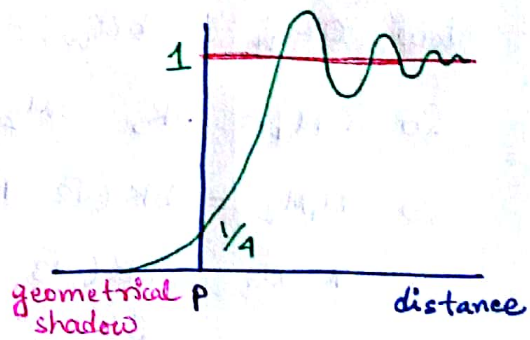
Diffraction at a straight Edge



Consider a straight edge at O and an illuminated narrow slit S parallel to each other. Dark & bright bands of unequal width of varying intensity is observed in geometrical shadow. We study intensity at P' with M as pole and construct Fresnel's half-period strip. The effect at P' depends upon the number of half-period strips contained in OM & BM.

Due to straight edge, the effect at P is due to the upper half of the wavefront only, so displacement at P is $\frac{1}{2}$ of the displacement for whole wavefront or $\frac{1}{4}$ of the full wavelet intensity.

\# of half-period strips contained in OM depends on the path difference $OP' - MP'$



$$\text{Now } OP' = \sqrt{a^2 + x^2} = a\left(1 + \frac{x^2}{a^2}\right)^{1/2}$$

$$\simeq a\left(1 + \frac{x^2}{2a^2}\right) = a + \frac{x^2}{2a}$$

$$SP' = \sqrt{(a+b)^2 + x^2} = a + b + \frac{x^2}{2(a+b)}$$

$$\therefore MP' = SP' - SM = a + \frac{x^2}{2(a+b)}$$

$$\therefore \text{Path difference } OP' - MP' = a + \frac{x^2}{2a} - a - \frac{x^2}{2(a+b)} = \frac{bx^2}{2a(a+b)}$$

For the displacement to be maximum,

$$\frac{bx^2}{2a(a+b)} = (2n+1)\frac{\lambda}{2} \quad \text{or} \quad x = \left[\frac{a(a+b)(2n+1)\lambda}{b}\right]^{1/2}, \; n = 0, 1, 2, \ldots$$

$x \propto \sqrt{2n+1}$ (bright band)

For the displacement to be minimum, $\frac{bx^2}{2a(a+b)} = n\lambda$

or $x = \left[\frac{2a(a+b)\,n\lambda}{b}\right]^{1/2}$, $n = 1, 2, 3$, $\quad x \propto \sqrt{n}$ (dark band)

Using these, wavelength of light can be found.

CW A narrow slit illuminated by light of $\lambda = 5890\,Å$ is located at a distance of 0.1 m from a straight edge. If the measurements are made at a distance of 0.5 m from the edge, calculate the distance between 1st & 2nd dark band.

For nth dark band $\quad x = \sqrt{\frac{2a(a+b)n\lambda}{b}}$

$a = 0.5\,m$
$b = 0.1\,m$
$\lambda = 5890 \times 10^{-10}\,m$

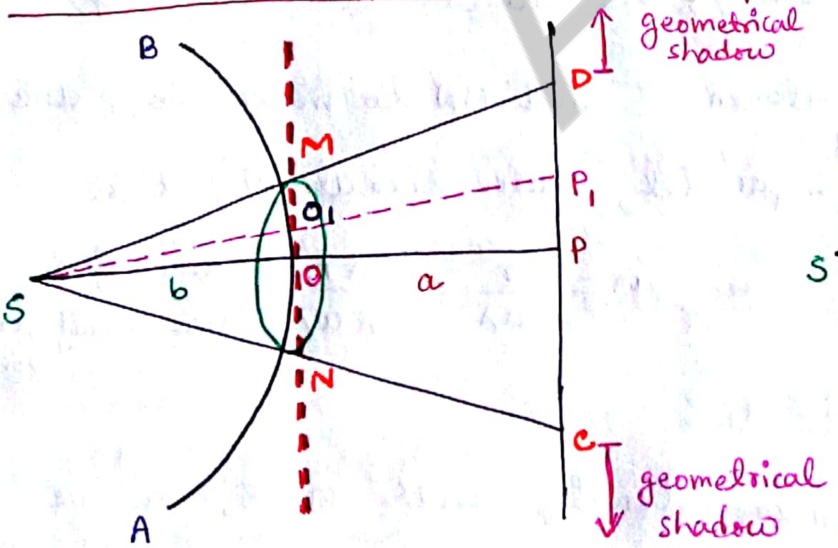
$\therefore x_2 - x_1 = \sqrt{\frac{2a(a+b)\lambda}{b}}\,(\sqrt{2} - 1)$

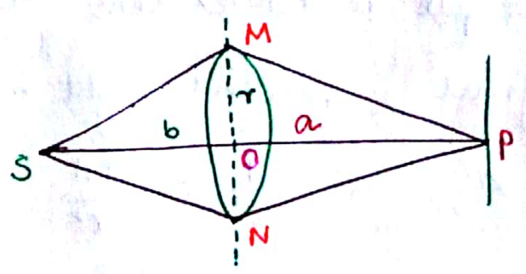
$= 0.7786 \times 10^{-3}\,m.$

Read about diffraction of light by a thin wire. Fringe width

$\beta = \frac{D\lambda}{d}$, D = distance between obstacle & crosswire of Eyepiece,
λ = wavelength of light.

Fresnel's diffraction at a circular aperture





From a point source S a wavefront (spherical) touches a circular aperture MN. To calculate the amplitude at screen P, we need to divide the wavefront MON into Fresnel's half-period zones about the pole O.