Hydrostatics and Hydrodynamics

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Assignment III

Q.1) Water stands at a depth h in a large open drum whose side-walls are vertical. A hole is made in one of the walls at a depth x below the water surface. (i) Calculate the horizontal distance d from the wall at which the emerging stream of water strikes the ground. (ii) What should be the value of x which makes d maximum?

Q.2) Two drums A and B contain different liquids and both have a small hole on the vertical wall at a depth h below the liquid surface. The hole in drum A has half the cross-sectional area of the hole in drum B. (i) Using Toricelli's theorem, Calculate the ratio ρ_A/ρ_B of the densities of the liquids, if the mass flow rate is the same for the two holes. (ii) What is the ratio V_A/V_B of the volume flow rates from the two drums? (iii) At one instant, the liquid in drum A is $12.0\mathrm{cm}$ above the hole. If the drums are to have equal volume flow rates, what height above the hole must the liquid in drum B be just then?

Q.3) Two streams merge to form a river. One stream has a width of $8.2\mathrm{m}$, depth of $3.4\mathrm{m}$, and current speed of $2.3\mathrm{m/s}$. The other stream is $6.8\mathrm{m}$ wide and $3.2\mathrm{m}$ deep and flows at $2.6\mathrm{m/s}$. If the river has width of $10.5\mathrm{m}$ and speed at $2.9\mathrm{m/s}$, what is its depth?

Q.4) Dead Sea is about seven times saltier than any ocean due to the high evaporation rate and low rainfall. About one-third of the body of a swimmer floating in the Dead Sea is above the waterline. Assuming that the humanbody density is $0.98\mathrm{gm/cm^3}$, find the density of the water in the Dead Sea.

Q.5) Water flows along a horizontal tube of which the cross-section is not constant. Using Bernoulli's theorem, calculate the change in pressure when the velocity of flow changes from $10\mathrm{cm/s}$ to $20\mathrm{cm/s}$.

Q.6) A hypothetical material out of which an astronomical object is formed has an equation of state

$$p = \frac{1}{2}K\rho^2,$$

where p is the pressure and ρ is the mass density. Show that for this material under conditions of hydrostatic equilibrium, there is a linear relation between the density and the gravitational potential.

Q.7) The vorticity $\boldsymbol{\omega}$ of a flow field is defined as curl of the velocity vector $\boldsymbol{\omega} = \boldsymbol{\nabla} \times \mathbf{v}$. Show that for an incompressible fluid, the following relation holds between the velocity and vorticity vector fields,

$$\boldsymbol{\nabla} \cdot [(\mathbf{v} \cdot \boldsymbol{\nabla})\mathbf{v}] = \frac{1}{2}\nabla^2(\mathbf{v} \cdot \mathbf{v}) - \mathbf{v} \cdot \nabla^2\mathbf{v} - \boldsymbol{\omega} \cdot \boldsymbol{\omega}.$$

Q.8) An incompressible fluid of mass density ρ, viscosity η is pumped in steady-state laminar flow through a circular pipe of internal radius R and length L. The pressure at the inlet end is p_1 and the pressure at the exit is p_2 with $p_1 > p_2$. Let Q be the mass of fluid that flows through the pipe per unit time. Using steady state limit of the Euler's equation for viscous liquid (or Navier-Stokes equation for "wet water" as in Feynman Lectures)

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \boldsymbol{\nabla})\mathbf{v}\right] = -\boldsymbol{\nabla} p + \eta \nabla^2 \mathbf{v} + \mathbf{F}_{\mathtt{body}},$$

determine Q, which is the Poiseuille's equation. *[Hint: Switch to cylindrical coordinates.]*