Sem-I - Newtonian Mechanics

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Assignment III: Gravitation

Submission due date: 05/02/2026

Q.1) **(a)** For a particle of mass m moving in a central potential $U(r)$, show the energy is given in polar coordinates by

$$E = \frac{1}{2}m\dot{r}^2 + U_{eff}(r),$$

where $U_{eff}(r) = U(r) + \frac{l^2}{2mr^2}$, l being the angular momentum. **(b)** Show that the effective potential of a particle of mass m in a central force is given by

$$U_{eff}(r) = U(r) + \frac{l^2}{2mr^2},$$

where l is the angular momentum. **(c)** A particle of mass m moving in a central force field describes a spiral $r = k\theta^2$, where k is a positive constant. (i) Find the force law. Given that the differential equation of the orbit is

$$\frac{l^2}{2mr^2}\left(\frac{d^2(1/r)}{d\theta^2} + \frac{1}{r}\right) = -f(r),$$

(ii) Compute the effective one-dimensional potential energy. (iii) Find the total energy of the system. **(d)** Consider a head-on elastic collision in one dimension between a heavy mass (m_1) and a light mass (m_2) ($m_1 >> m_2$), where m_2 is initially at rest. Show that after the collision, the light mass rebounds with a speed equal to about twice the initial speed of m_1. **(e)** State Kepler's laws of planetary motion. A particle of mass m moves under the action of a central force $f(r)\hat{\mathbf{r}}$. Show that the equation determining the orbit of the particle is

$$\frac{l^2}{2mr^2}\left(\frac{d^2(1/r)}{d\theta^2} + \frac{1}{r}\right) = -f(r),$$

where θ is the azimuthal angle and l is a constant of motion. Hence show that for an inverse square force, the trajectory is a conic section. **(f)** Prove that the total energy of a particle of mass m acted upon by a central force is given by,

$$E = \frac{l^2}{2m}\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right] + V(r),$$

where l is the angular momentum, $V(r)$ is the potential energy, $u = \frac{1}{r}$ and θ being the polar coordinates.

Q.2) **(a)** A particle is thrown from the Earth's surface with speed $v = \sqrt{\frac{3GM}{2R}}$, where M and R are mass and radius of the Earth respectively. What will be the nature of the orbit of the particle? **(b)** Find the central force for which the orbit is given by $r = ke^{a\theta}$, where a and k are constants. **(c)** A planet of mass m moves around the Sun of mass M. The nearest and the farthest distance of the planet from the Sun are a and b respectively. Find the magnitude of the angular momentum of the planet around the Sun in terms of m, M, a, b and G, where G is the gravitational constant. **(d)** A ball moving with speed of $9m/s$ strikes an identical stationary ball such that after the collision, the direction of each ball makes an angle 30° with the original line of motion. Find the speeds of two balls after collision. Is the kinetic energy conserved in this collision?

Q.3) **(a)** Find out the gravitational potential at a point inside a uniform solid sphere. **(b)** If the density of the material within a spherical body varies inversely as the distance from the centre, show that the gravitational field inside is the same everywhere. **(c)** Two particles of mass m_1 and m_2 move under mutual interacting force (central). Set up the equation of motion of the system and find an expression for the reduced mass. **(d)** A system of particles with masses m_i and position vectors $\bar{r}_i (i = 1, 2, \cdots, n)$ moves under its own mutual gravitational attraction alone. Write down the equation of motion $\bar{r}_i$. Show that a possible solution of the equation of motion is given by $\bar{r}_i = t^{2/3}\bar{a}_i$, where $\bar{a}_i$'s are constant vectors satisfying

$$\bar{a}_i = \frac{9G}{2} \sum_{j \neq i}^{n} \frac{m_j(\bar{a}_i - \bar{a}_j)}{\left[\bar{a}_i - \bar{a}_j\right]^3},$$

where G is the Gravitational constant. Show that for this system, the total angular momentum about the origin and the total linear momentum both vanish. What is the angular momentum about any other fixed point? **(e)** Consider a pair of stars of equal mass M rotating about their common centre of mass. The attraction between the stars is gravitational and the stars keep a separation l between them. Show that the time period of rotation of this double star system is given by $\pi l \sqrt{\frac{2l}{GM}}$, where G is the universal Gravitational constant.