Sem-III - Thermal Physics II

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Assignment II: $1^{st} - 2^{nd}$ law of Thermodynamics & Pure Substances

Submission due date: 28/01/2021

Q.1) If a gas is both ideal and paramagnetic obeying Curie's law, show that the entropy is given by

$$S = c_{V,M} lnT + nRlnV - \frac{M^2}{2C'_c} + constant,$$

where $c_{V,M}$ is the heat capacity at constant volume, magnetization assumed constant and C'_c is Curie's constant.

Q.2) The equation of state of a novel matter is $PV = AT^3$ with A a constant. The internal energy of the matter is $U = BT^n ln(V/V_0) + f(T)$. Using first law of thermodynamics, find B and n.

Q.3) Suppose an engine works between two reservoirs at T_1 and $T_2 (T_2 > T_1)$ until both reservoirs attain final temperature T_c. Show that $T_c > \sqrt{T_1 T_2}$. What is the maximum amount of work obtainable from this engine?

Q.4) A Carnot engine has an efficiency of 30% when the sink temperature is $27°C$. What must be the change in temperature of the source to make its efficiency 50%?

Q.5) An inventor claims to have developed an engine working between $600K$ and $300K$ to deliver an efficiency of 52%. Using Carnot's theorem, can you decipher whether this claim is valid?

Q.6) Two Carnot engines X and Y are operating in series. X receives heat at $1200K$ and rejects to a reservoir at temperature TK. The second engine Y receives the heat rejected by X and inturn rejects to a heat reservoir at $300K$. Calculate the temperature T for the situation when, (i) The work output of two engines are equal, (ii) The efficiency of two engines are equal.

Q.7) A Carnot's refrigerator takes heat from water at $0°C$ and discards it to a room temperature. $1Kg$ of water at $0°C$ is to be changed into ice at $0°C$. How many calories of heat are discarded to the room? What is the work done by the refrigerator in this process? What is the coefficient of performance $[P = Q_{cold}/(Q_{hot} - Q_{cold})]$ of the machine? Given, room temperature is $27°C$ and $1Cal = 4.2Joule$.

Q.8) A thermally conducting bar of length L, area A, density ρ is brought to a nonuniform temperature distribution by sandwiching between hot (temperature T_h) and cold reservoir (temperature T_c). The bar is removed from reservoirs, thermally insulated and kept at constant pressure. Show that the change in entropy of the bar is

$$\Delta S = c_p \rho AL \Big\{ 1 + ln\Big(\frac{T_h + T_c}{2}\Big) + \frac{T_c}{T_h - T_c} lnT_c - \frac{T_h}{T_h - T_c} lnT_h \Big\}.$$

Q.9) Consider a metal (say Copper) at $300K$ with the following values, $V = 7.06 cm^3/mol, K_T = 7.78 \times 10^{-12} N/m^2, \beta = 50.4 \times 10^{-6} K^{-1}, C_p = 24.5 J/molK$. Determine C_v.

Q.10) Prove that the ratio of adiabatic $\left[\alpha_S = \frac{1}{V}(\frac{\partial V}{\partial T})_S\right]$ to isobaric $\left[\alpha_P = \frac{1}{V}(\frac{\partial V}{\partial T})_P\right]$ coefficient of expansion is $\frac{1}{1-\gamma}$. Also, prove that the ratio of adiabatic $\left[E_S = -V(\frac{\partial P}{\partial V})_S\right]$ to isothermal $\left[E_T = -V(\frac{\partial P}{\partial V})_T\right]$ elasticities is equal to the ratio of specific heats.

Q.11) Prove that the ratio of adiabatic $\left[\beta_S = \frac{1}{P}(\frac{\partial P}{\partial T})_S\right]$ to isochoric $\left[\beta_V = \frac{1}{P}(\frac{\partial P}{\partial T})_V\right]$ pressure coefficient of expansion is $\frac{\gamma}{\gamma-1}$.

Q.12) (a) If equation of state of certain material satisfies $P = \frac{RT}{V}(1 + \frac{B''}{V})$ where $B'' = B''(T)$, show that

$$C_V = -\frac{RT}{V}\frac{d^2}{dT^2}(B''T) + C_V^{\infty},$$

where C_V^{∞} represents the value of C_V when V is very large. (b) In case $P = \frac{RT}{V}(1 + B'P)$ where $B' = B'(T)$, show that

$$C_P = RTP\frac{d^2}{dT^2}(B'T) + C_P^0,$$

where C_P^0 represents the value of C_P when pressure tends to zero.

Q.13) Using Berthelot's equation of state $P = \frac{RT}{V-b} - \frac{a}{TV^2}$, show that the critical constants are

$$P_c = \frac{1}{12b}\sqrt{\frac{2aR}{3b}},\ V_c = 3b,\ T_c = \sqrt{\frac{8a}{27bR}};\quad \frac{RT_c}{P_cV_c} = \frac{8}{3}.$$

Q.14) The boiling point of a liquid at pressure P_0 is T_0. Its molar latent heat of vaporisation is L and molar volume of the liquid phase is negligible as compared to vapour phase. The vapour phase obeys the ideal gas equation. Show that the boiling point T at pressure P is given by,

$$ln\Big(\frac{P}{P_0}\Big) = \frac{L}{RT_0}\Big(1 - \frac{T_0}{T}\Big).$$