Conduction

Transmission of Heat: CONDUCTION, CONVECTION, RADIATION
In conduction, head is transmitted from one point to other through the substance without actual motion of particles. Air or vacuum is poor conductor of heat, hence wooken fabric keeps us warm or thermos flook keeps thing isolated. In convection, head is transmitted by the actual motion of particles. Hot water circulation in heated kettle. Heat radiation is transmitted directly without any intervening medium.

Like sun radiation into earth by EM spectrum.

Coefficient of Thermal Conductivity

If we have a plane slab of area A, thickness & having temperature of & D2 at its two faces then if & amount of heat is transmitted in time t, then, & & A

 $\begin{array}{c|c}
0, & 0 \\
0, & 0
\end{array}$ $\begin{array}{c|c}
0, & 0 \\
0, & 0
\end{array}$

$$d(\theta_1 - \theta_2) \approx 8 = \frac{KA(\theta_1 - \theta_2)t}{\pi}$$

K = coeff." of thermal conductivity

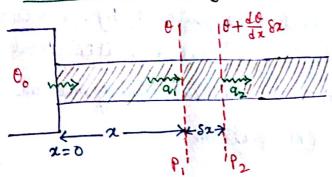
heat current
$$q = \frac{8}{t} = \frac{kA(\theta_1 - \theta_2)}{x} = \frac{\theta_1 - \theta_2}{x/kA} = \frac{\theta_1 - \theta_2}{\sqrt{kA}}$$
thermal resistance

This equation is similar to this law $I = \frac{V}{R}$, redefined in terms of thermal resistance of heat current. We know $R = \int_{A}^{L} \frac{L}{A} = \frac{1}{\sigma A}$ where $\sigma = \frac{1}{\sigma}$ is the electrical conductivity. Comparing with $R_{Th} = \frac{7}{KA}$ we can define the proportionality constant as coeff. of thermal conductivity. Dimension of $[Q] = [ML^{T} - \frac{1}{\sigma}]$, [X] = [L], [A] = [L], [O] = [O], [C] = [C].

(b) [C] = [C]

Mermal diffusivity is defined as the ratio of thermal conductivity to thermal capacity per unit volume. If Jo = dereity & S = specific $N = \frac{K}{mS} = \frac{K}{PS} = Thermometric conductivity$ heat then

Rectilinear Propagation of heat along a bar



Consider a bor of uniform onea of cross-section A contact with an oven at temperature 0, at 2=0. If O is the except appealure above the surroundings of the bar

at P1 at a distance x from the point of contact, then excent impureber at $\rho_2 = \theta + \frac{d\theta}{dx} \delta x$.

If heat flowing through P, in one second $q_1 = -KA \frac{d\theta}{dx} \Delta$ heat flowing through P_2 in one second $Q_2 = -KA \frac{d}{dx}(0 + \frac{d\theta}{dx} fx)$

so Heat gained per sound by the rod between P1 & P2

$$g = g_1 - g_2 = -kA \frac{d\theta}{dx} + kA \frac{d}{dx}(\theta + \frac{d\theta}{dx} \delta x)$$

$$= kA \frac{d^2\theta}{dx^2} \delta x$$

This amount of heat is used in two ways before sleady state is reached. 1 A part will increase the temperature, 2 Rest port is lost due to radiation from the exposed surface of the slab.

It rate of rise of temperature is do then heat used per second = (A 82) P x S x do 1 heat lost per second due to radiation muss specific .

= EPS20 where E = emissive power of surface, p= perimeter f 0 = average exceps of temperature within P. I Pz.

 $Q = A \delta x p S \frac{d\theta}{dt} + E p \delta x \theta = K A \frac{d^2\theta}{dx^2} \delta x$

fourier's differential $\frac{K}{J^{\circ}S} \frac{d^{\circ}O}{dx^{2}} = \frac{dO}{dt} + \frac{PE}{A_{J^{\circ}S}} O$ equation Special Cases 1: when heat lost by radiation is negligible: When rod is covered by insulating materials, heat lost EPSXO = 0 I total heat gained by rod is to raise the temperature, using $\frac{K}{\sqrt{s}} \frac{d^2 \sigma}{dx^2} = \frac{d\sigma}{dt} \qquad \sigma \sigma \qquad h \frac{d^2 \sigma}{dx^2} = \frac{d\sigma}{dt}$ Special Cases 2: after the steady state is reached: $\frac{d\theta}{dt} = 0$ and $\frac{d^2\theta}{dn^2} = \frac{PE}{KA}\theta = \mu \tilde{\theta}$ Mis is a Second order homogeneous linear differential equation If $0 = e^{mx}$ is the trial solution then $m^2 = \mu^2$ or $m = \pm \mu^2$ $O = A_1 e^{\mu x} + A_2 e^{-\mu x}$ If the bar is sufficiently long, we can assume that under steady state temperature of the surroundings.

no heat is lost from free end of the lar, as whole of the heat is lost from free and sides as radiation & free end will be at the

(a) when box is of infinite length: (Dirichlet B.C.) Boundary condition, n=0, 0=00

we see that $0 = A_1 e^{i\phi}$ can be true only if $A_1 = 0$, and $\theta_0 = A_2$.

mus after steady state is reached, temperature is exponentially distributed. This is useful in Ingen-Hausz experiment.

(b) When bar is of finite length: A=L, $\frac{d\theta}{dx}=0$ B.c.)

In this case
$$A_1 = \frac{\theta_0}{1 + e^{2\mu L}}$$
, $A_2 = \frac{\theta_0}{1 + e^{-2\mu L}}$
Solution $\theta = \theta_0 \left[\frac{e^{\mu x}}{1 + e^{2\mu L}} + \frac{e^{-\mu x}}{1 + e^{-2\mu L}} \right]$

Special Cases I at steady state ideal case when there is no loss of heat by radiation is. rod is thermally lagged & in steady state $h \frac{d^2 u}{dx^2} = \frac{d u}{dt} = 0 \quad \text{is} \quad \frac{d^2 u}{dx^2} = 0 \quad \text{(as } h \neq 0) \quad \text{[Laplace equation in Solving } \frac{d}{dx} \left(\frac{d u}{dx}\right) = 0 \quad \text{in} \quad \frac{d u}{dx} = \text{constant} = A \quad \text{electrostatico}$

∞ 0 = A2+B.

Find A 4 b using B.C. that n=0, $0=0_0$ x=1, $0=0_m$ (say)

at unknown distance l, the temperature is om.

 $\theta_0 = B$ and then $\theta_m = Al + \theta_0$ or $A = \frac{\theta_m - \theta_0}{l}$ $\theta_0 = \theta_0 - \frac{\theta_0 - \theta_m}{l}$

The decrement is linear, as solution of Laplace equation is always a straight line.

In steady state length up to which wax melts in wax coated bar from 0 = 0, $e^{-\mu x}$, $\ln \frac{0}{0} = -\mu x$ we see that if we have number of bars with conductivities K_1, K_2, K_3, \dots ele I wax melts up to length l_1, l_2, l_3, \dots etc. then at these length the temperature would be melting point of wax (say 0m).

s. In
$$\frac{\theta_m}{\theta_0} = -\mu_1 l_1 = -\mu_2 l_2 = -\mu_3 l_3 = \cdots$$

$$\frac{PE}{K_1A} M = \sqrt{\frac{PE}{K_2A}} l_2 = \sqrt{\frac{PE}{K_3A}} l_3 = ...$$

or l/sk = constant or ld SK

Hence in a steady state the length upto which the wax melts along a wax wated box is proportional to the square root of the coefficient of thermal conductivity of the material.

Periodic flow of heat: Propagation of heat wave in an insulated rod with one end heated sinusoidally.

Consider a system of infinite length, well insulated (no loss due to roidiation) whose one end is connected to an heat source from where heat is supplied not continuously but periodically. with Oo amplifude and w being the angular frequency.

ro= o, e wt

Using fourier's equation $h \frac{d^2o}{dx^2} = \frac{do}{dt} + \frac{PE}{Aps} o'$ without radiation loss, the unidirectional heat equation is $h \frac{d^2\sigma}{dx^2} = \frac{d\sigma}{dt}$, $h = \frac{K}{\sqrt{r}S}$

is the thermal diffusivily of the rod.

Let 0 = u(x) + v(x,t) is a trial solution. then separating the variables, $\frac{d^2u}{dx^2} = 0$, $h\frac{d^2v}{dx^2} = \frac{dv}{dt}$

me solution of re-equation can be $v = F(x)e^{i\beta t}$

 $\delta = \int_{0}^{\infty} f(x) =$

Taking the trial solution as F(x) = Aemx we obtain,

 $m^2 = \frac{1}{h}$ or $m = \pm \sqrt{\frac{1}{h}}$ & $f(x) = A_1 e^{-\frac{1}{h}} + A_2 e^{-\frac{1}{h}}$

As 2 - 00 yields F(x) - 00 (uphysical), so A, = 0

 $F(x) = A_2 e^{-\int \frac{iP}{h} x}$

Now $(1+i)^{7} = 2i$ $= \frac{1}{2}(1+i)^{7} \propto \sqrt{i} = \pm \frac{1}{\sqrt{2}}(1+i)$ $c_{0} f(x) = A_{2} e^{-(1+i)\sqrt{\frac{15}{2h}}x} + A_{3} e^{(1+i)\sqrt{\frac{15}{2h}}x}$

is
$$v = f(x)$$
 eight = $\begin{bmatrix} A_2 e^{-\int \frac{\pi}{2h}x} e^{\frac{i}{2h}x} - \int \frac{\pi}{2h}x \end{bmatrix}$
Here also, as $x \to \infty$, $v \to \infty$ (unphysical), hence $A_3 = 0$.

8. $v(x,t) = A_2 e^{-\int \frac{\pi}{2h}x} e^{\frac{i}{2h}x} - \int \frac{\pi}{2h}x \end{bmatrix}$
Putting the boundary condition for $0 = 0$ eight at $x = 0$, were $0 = 0$ eight $0 = 0$.

Hence $v(x,t) = A_2 e^{\frac{\pi}{2h}x} e^{\frac{i}{2h}x} = \frac{\pi}{2h} e^{\frac{\pi}{2h}x} e^{\frac{\pi}{2h}x} = 0$.

Therefore $v(x,t) = 0$ e $v(x,t) = 0$ e $v(x,t) = 0$ eight $v(x,t) = 0$ eight

In a periodic flow of heat along an iron bor, the periodic time is 4 minute. If the temperature bravels maximum 6 cm in 1 minute, calculate the thermal conductivity of iron. Density of iron = 7.8 gm/cm³, specific heat of iron = 0.11 Cal/gm c.

$$n\theta = \sqrt{2\omega h} = \sqrt{\frac{4\pi k}{T_0 s}} \quad cr \quad n\theta^2 = \frac{4\pi k}{T_0 s}$$

As $\frac{\partial}{\partial t} = v = \sqrt{\frac{4\pi k}{T_0 S}}$ or $k = \frac{\partial J S}{4\pi T}$

Here v = 6 cm/min = 0.1 cm/sec, $T = 4 \text{min} = 4 \times 60 = 240 \text{ see}$ S = 0.11 cal/gm°c, $J^0 = 9.8 \text{ gm/cm}^3$ $K = \frac{v^2 T J^0 S}{4 \pi} = \frac{0.1^2 \times 240 \times 9.8 \times 0.11}{4 \times 3.14} = 0.1639 \text{ cal cm/see}^{-1} \text{ e}^{-1}$

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Heat flow in three dimensions

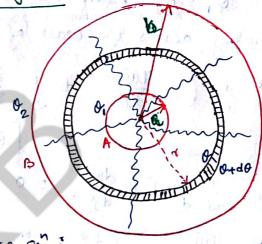
We have learned fouriers law in one dimension,

 $h \frac{d^2 \sigma}{dx^2} = \frac{d\sigma}{dt} + \mu^2 h \sigma$. In three dimensions, we have

 $h \nabla \theta = \frac{d\theta}{dt} + \mu h \theta$. In steady state, $\frac{d\theta}{dt} = 0$ and without radiation loss, $\mu^2 = 0$ yields $\nabla^2 \theta = 0$ This is called Laplace equation of heat flow. Compare with Electrostatics, Laplace equation

@ Spherical shell Method (Radial flow)

consider a sphrerical shell of inner radius a and outer radius b. Let of by are the temperature at inside inside inside the sphere. We want to find out temperature at a <r<b.



In some spherical polar coordinates, Laplace en is

$$\alpha \frac{45}{7} \frac{34}{9} \left(45 \frac{34}{9} \right) = 0$$

or
$$d\theta = \frac{c_1}{r^2} dr$$
 or $\theta = -\frac{c_1}{r} + c_2$

Now we use Dirichlet Boundary condition H = 0, at r = a. $H = 0_2$ at r = b.

$$O_1 = -\frac{C_1}{a} + C_2$$

$$O_2 = -\frac{C_1}{b} + C_2$$

$$O_3 = -\frac{C_1}{b} + C_2$$

$$O_4 = \frac{ab(O_1 - O_2)}{a - b}$$

$$\circ \circ C_2 = o_1 + \frac{c_1}{a} = o_1 + \frac{(o_1 - o_2)b}{a - b} = \frac{a - b}{a - b}.$$

co The temperature at any distance or s $(H) = \left[\frac{ab(\theta_1 - \theta_2)}{b - a}\right] \frac{1}{7} + \frac{a\theta_1 - b\theta_2}{a - b}$ 6) Cylindrical flow of heat consider a cylindrical tube of length l, inner radius a 4 outer radius b with temperature of inner surface 0, 2 outer surface 02 with 0,>02 where heat is conducted radially across the wall of the tube. Het Laplace eq. becomes. 1 do (sam) + 1 2 do 2 + 302 = 0 $[\mathbb{H} \neq \mathbb{H}(0, \emptyset)]$ or $[\mathbb{H} \neq \mathbb{H}(0, \emptyset)]$ dA = Gds or A = glus + c2 We use Dirichlet boundary condition, A = 0, at s=a .. 0, = c, lna + c2 00 0,-02 = Gln %. 02 = 9 ln b + c2 € G = 0,-02 en 96 $\begin{array}{ll} \text{c. } c_2 = \theta_1 - c_1 \ln \alpha = \theta_1 - \frac{(\theta_1 - \theta_2)}{\ln \theta_b} \ln \alpha = \frac{\theta_1 (\ln \alpha - \ln b)}{\ln \theta_b} - \frac{(\theta_1 - \theta_2) \ln \alpha}{\ln \theta_b} \end{array}$ = 02 lna - 0, lnb So the temperature at any distance $r\ddot{s}$, $(\vec{H}) = \frac{\theta_1 + \theta_2}{m g_b} ln s^{o} + \frac{\theta_2 ln a - \theta_1 ln b}{ln g_b}$

Using Fourier's law at unit time, Q = K 2772 do

 $\alpha = \alpha \int_{\alpha}^{b} \frac{dr}{r} = 2\pi k l \int_{\alpha}^{\theta_{2}} d\theta = 2\pi k l \left(\theta_{2} - \theta_{1}\right).$

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or Orlin
$$\frac{b}{a} = 2\pi k l (\theta_2 - \theta_1)$$
 or $\theta_1 = \frac{2\pi k l (\theta_1 - \theta_2)}{\ln \theta_b}$

$$\sigma_1 = \frac{\theta_1 \ln \theta_2}{2\pi l (\theta_1 - \theta_2)}$$

Wiedemann-franz law The law states that ratio of thermal and electrical conductivities for all metals is directly proportion to the absolute temperature of the body.

 $\frac{K}{\sigma} \times T$ or $\frac{K}{\sigma T} = \text{constant} = 2\sqrt{\frac{6}{K}} \frac{K_B^2}{e^2} = L \left(\frac{\text{Lorest2}}{\text{number}} \right)$

Physically this means that substances which are good conductor of heat are also good conductor of electricity.

Doude's theory of electrical conduction

Drude in 1900 introduced the concept of free electron gas model of metals, I obtained the electric conductivity of the metal. All metals (conductors) contain a huge number of nearly free electrons that behave as gas atoms in Kinetic theory. If m is mass of electron l v is velocity at temperature T,

 $\frac{1}{2}mv_{rms}^2 = \frac{3}{3}k_BT \quad oo \quad v_{rms} = \sqrt{\frac{3k_BT}{m}}$

If we apply an electric field E & then electron will experience a force eE and accelarate with eE. Now as the

electron moves to hit an atom or ion, if a is the mean interatomic distance that is gone in time t

then average drift relocity of the electron

So
$$N_d = \frac{eE}{2m}t = \frac{eE}{2m}\frac{\lambda}{v}$$

So The current density $J = nev_d = \frac{ne\lambda}{2mv_{rm}}E = \sigma E$

and Thermal conductivity $K = \frac{1}{3} n \bar{c} \lambda \frac{dE}{d\bar{t}} + \beta r$ only translational engage, $E = \frac{3}{2} K_B T$, $K = \frac{1}{3} N \bar{c} \lambda \frac{3}{2} K_B = \frac{1}{2} N \bar{c} \lambda K_B$

$$\frac{k}{\sigma} = \frac{1}{2} \frac{n C A k_B}{n e^2 A} \frac{2m \sqrt{\frac{3k_B T}{m}}}{m}$$

$$= 2 \sqrt{\frac{8k_B T}{m \Lambda}} \frac{m \chi k_B}{2m e^2 \chi} m \sqrt{\frac{3k_B T}{m}} = 2 \sqrt{\frac{6}{\Lambda}} \frac{k_B^2}{e^2} T$$

 $\frac{1}{100} = \frac{1}{100} = \frac{1}$

Heat conduction through a slab of varying thickness

To form ice, 80 cals of heat are given out at 0°C when some thick ice layer has formed, heat given out how to conduct through this thickness. Let us find out the time required to increase the icelayer from x_1 to x_2 . If at t, ice formed is a then within time dt, dx thickness of ice is formed, then the heat liberated is $g = Adx_0L$. This heat flows in dt from o'c to outside temperature -0°C.

i. $g = \frac{KA[0-(-10)]dt}{x} = \frac{KA0}{x}dt$

 $\partial_{x} A dx \int L = \frac{k A \theta}{x} dt \quad \text{or} \quad x dx = \frac{k \theta}{\sqrt{\rho} L} dt$

Integrating, $\frac{1}{2}\chi^2 = \frac{KQ}{\sqrt{\rho_L}}t + C$

Now at t=0, $x=x_1$, $t=t_2$, $x=x_2$

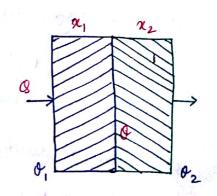
$$\frac{1}{2}x_1^2 = c \qquad \frac{1}{2}x_2^2 = \frac{k\theta}{\nu L}t_2 + \frac{1}{2}x_1^2$$

or
$$t_2 = \sqrt{\frac{\alpha_L}{2\kappa_0}} (\chi_2^2 - \chi_1^2)$$

If at t=0, x=0 then time required to form a layer of thickness x

$$\dot{b} = \frac{\int L}{2KO} \chi^2$$

Heat conduction through a composite slab consider a slab made of two materials of thickness of and ond or and conductivities K, and K2. At steady state, heat enters at 0,



$$S = \frac{K_1 A (\theta_1 - \theta)}{\chi_1} = \frac{K_2 A (\theta - \theta_2)}{\chi_2}$$

$$V Q_1 = \frac{A (\theta_1 - \theta)}{\frac{\chi_1}{K_1}} = \frac{A (\theta - \theta_2)}{\frac{\chi_2}{K_2}} = \frac{A (\theta_1 - \theta_2)}{\frac{\chi_1}{K_1} + \frac{\chi_2}{K_2}}$$

If the composite slab can be replaced by a single slab of thickness $\alpha_1 + \alpha_2$ such that it will conduct in unit time heat G under temperature difference $\theta_1 - \theta_2$, then the equivalent conductivity be

K., then
$$8 = \frac{KA(\theta_1 - \theta_2)}{\alpha_1 + \alpha_2} = \frac{A(\theta_1 - \theta_2)}{\frac{\alpha_1}{K_1} + \frac{\alpha_2}{K_2}}$$

$$\frac{\alpha_1 + \alpha_2}{K_1 + \frac{\alpha_2}{K_2}}$$

$$\frac{\alpha_1 + \alpha_2}{K_1 + \frac{\alpha_2}{K_2}}$$

If we have
$$n > 2$$
 slabs then $\frac{\chi_1 + \chi_2 + \chi_3 + \cdots}{K} = \frac{\chi_1 + \chi_2 + \chi_3 + \cdots}{K}$

HW O One end of a metal rod is in contact with a source of heat at 100°C. In the steady state the temperature at a point 10 cm from the source is 60°C. find the temperature at a point 20 cm from the cource.

2) Suppose 10 cm of ice has already formed on a pond so that the air outside is at -5°C. How long will it take from the next milimeter outside is at -5°C. How long will it take from the next milimeter outside is at -5°C. How long will it take from the next milimeter outside is at -5°C. How long will it take from the next milimeter outside is at -5°C. How long will it take from the next milimeter outside is at -5°C. How long will it take from the next milimeter outside is at -5°C. How long will it take from the next milimeter outside is at -5°C. How long will it take from the next milimeter outside is at -5°C. How long will it take from the next milimeter outside is at -5°C. How long will it take from the next milimeter outside is at -5°C. How long will it take from the next milimeter outside is at -5°C. How long will it take from the next milimeter outside is at -5°C. How long will it take from the next milimeter outside is at -5°C. How long will it take from the next milimeter outside is a second of the next milimeter outside is at -5°C. How long will it take from the next milimeter outside it is a second of the next milimeter outside it is a second of the next milimeter outside it is a second of the next milimeter outside it is a second of the next milimeter outside it is a second of the next milimeter outside it is a second of the next milimeter outside it is a second of the next milimeter outside it is a second of the next milimeter outside it is a second of the next milimeter outside it is a second of the next milimeter outside it is a second of the next milimeter outside it is a second of the next milimeter outside it is a second of the next milimeter outside it is a second of the next milimeter outside it is a second of the next milimeter outside it is a second of the next milimeter outside it is a second of the next milimeter outside it is a second of the next milimeter outside it is a second of the next milimeter outside it is a second of the next milimeter outside it is a second of the next milimeter

(3) A lake is covered with ice 2 cm thick. Temperature of air is -15°C. Find the rate of thickening of ice in cm/hour. For ice given K=0.004 cgs unit, p = 0.9 gm/ce, L = 80 cal/gm.

A Two equal boars of copper & aluminium are welded end to end and lagged. If the free ends of the copper & aluminium are maintained at 100°C and 0°C respectively. Find the temperature of welded surface K of Cu & Al are 0.92 and 0.5 cgs unit respectively.