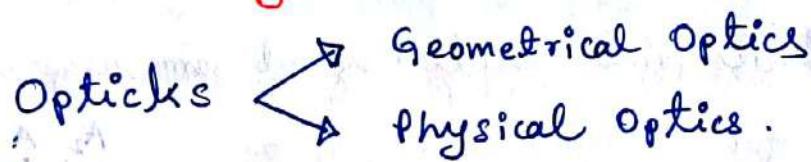


## PHYSICAL OPTICS

### (Diffraction and Holography)

- Books\*:
1. Opticks → Ghatak (6<sup>th</sup> Edition, Tata McGrawHill)  
⇒ Standard textbook, good for first time readers.
  2. Introduction to Geometrical and Physical Optics → B.K. Mathur (Old Book) ⇒ Good for concept building and theory learning.
  3. Fundamental of Optics (Tata McGrawHill) + Jenkins & White → Concise book, good for problem solving.
  4. Principles of Optics (Pergamon Press) → Born & Wolf  
⇒ Very good book for theory learning.
  5. Feynman lectures on physics Vol-1 → Feynman / Leighton / Sands (Narosa) ⇒ Short and concise for concept building.
  6. Optics → Hecht (Addison Wesley) ⇒ Good for problem solving and first time readers.
  7. Introduction to Holography → Toal (C&C Press) ⇒ New age book for basic holography principles.

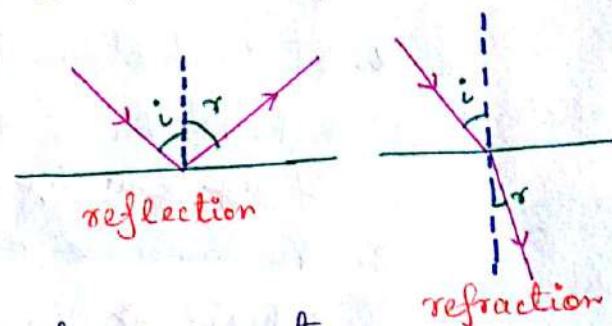


Geometrical Optics deals with refraction and reflection at surfaces, lenses, Matrix method, dispersion through prism, Aberrations and eyepieces and it terms on the particle (corpuscular) theory of light using Fermat's principle. Physical optics on the other hand deals with wave theory of light as Fresnel-Huygen's principle and discusses on Interference and Coherence, Diffraction, Polarisation (crystal optics), fiber optics and Holography.

## DIFFRACTION

Fermat's principle says that when a ray of light goes from one point to another through a set of media, it always follows a path along which the time taken is minimum.

$$\frac{dt}{dx} = 0 \text{ yields the "law of reflection"} \\ i = r. \text{ and the "Snell's}$$



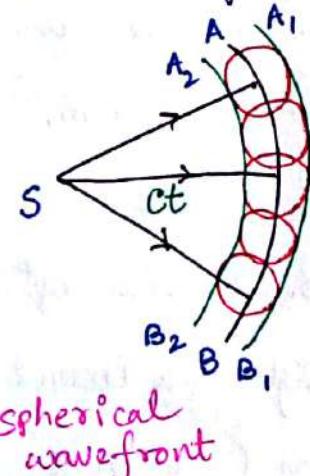
$$\text{law of refraction"} v_1 \sin i = v_2 \sin r$$

by conservation of the horizontal component of momentum.

The corpuscular model of light establish the rectilinear (straight line) propagation of light and propagation of light through vacuum.

## Wave theory and Huygens-fresnel principle

A source of light transmit wave that contain energy in all directions. A "wave front" is defined as the locus of all points which are in the same state of vibration (same phase). For example, circular ripples spreading out if a pebble is dropped, each circumferential point oscillating at same amplitude & same phase. Similarly for a light source, at a

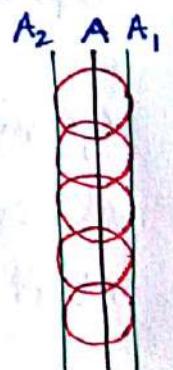


spherical wavefront

nearby location  $x=ct$  where AB is a spherical wavefront, while at large distance, AB is a plane wavefront.

Surface AB is called "primary wavefront".

The direction in which the wave is propagated is known as "ray" which is perpendicular to the wavefront.



plane wavefront

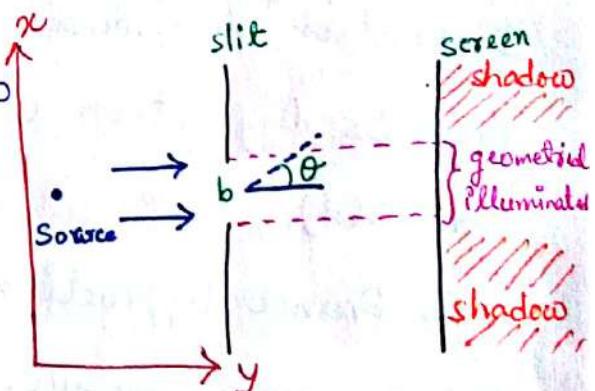
Huygen-fresnel principle tells that all points on the primary wavefront are considered to be the centres of disturbance and they

transmit secondary waves in all direction with the same velocity as the primary. So A, B, surface that touch the spheres after ct, distance is the "secondary wavefront"

Using Huygen-Fresnel principle, law of reflection ( $i = r$ ), law of refraction ( $v_1 \sin i = v_2 \sin r$ ), refraction of spherical wave at concave spherical surface ( $\frac{\mu}{v} - \frac{1}{u} = \frac{\mu-1}{R}$ ) and convex spherical surface ( $\frac{\mu-1}{R} = \frac{1}{v} - \frac{1}{u}$ ), lens formula for thin convex/concave lens ( $\frac{1}{f} = (\mu-1)(\frac{1}{R_1} - \frac{1}{R_2})$ ) can be obtained.

Why Diffraction? Wave-particle duality as in deBroglie's matter wave theory  $\lambda = \frac{h}{p}$  gives rise to Heisenberg's uncertainty principle  $\Delta x \Delta p_x \geq h$ .

If we illuminate a single slit (narrow opening) and if light propagation is rectilinear then there is no bending of light in the geometrical shadow.



But if a light quanta (photon) or electron pass through slit, then  $\Delta x \approx b$ , so  $\Delta p_x \approx \frac{h}{b}$ . As  $p_x = p \sin \theta$ , so  $\sin \theta \approx \frac{h}{pb} \approx \frac{\lambda}{b}$ .

When  $b \gg \lambda$ ,  $\sin \theta \rightarrow 0$  or almost no bending in geometrical shadow, while for  $b \approx \lambda$  then there will be significant bending. The bending of light round corners and spreading of light waves into the geometrical shadow of an object is called Diffraction.

## Difference between Interference & Diffraction

### Interference

- Result due to superposition of light from two different wavefront emanating from the same source.
- Fringes may/may-not be of same width.
- All bright bands are of uniform intensity.
- Points of minimum intensity are perfectly dark.

### Diffraction

- Result due to superposition of light from different parts of the same wavefront.
- Fringes are never of same width.
- All bright bands are of different intensity.
- Points of minimum intensity are not perfectly dark.

## Classification of Diffraction

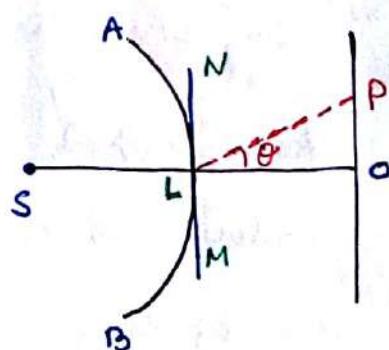
Diffraction phenomena are divided into two distinct classes, as Fresnel's diffraction (near field) and Fraunhofer diffraction (far field).

In Fresnel diffraction, source of light & screen are at finite distance from aperture. No concave/convex lenses are used so that incident wavefront is either spherical/cylindrical but not planar. So phase of secondary wavefront isn't same in the plane of aperture.

### Fresnel's assumptions

(a) A wavefront is divided into a large number of small areas (Fresnel's zone).

Secondary waves originating from various zones will interfere and the resultant effect can be noted at point P on the screen.

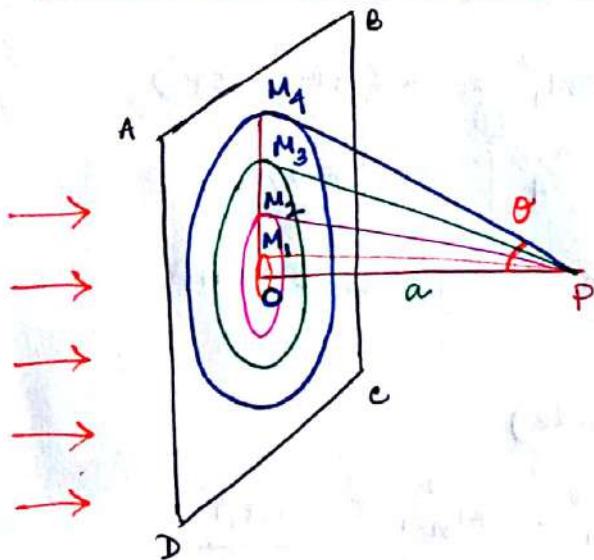


- (b) Resultant at P due to a particular zone will depend on the distance of the point from the zone.
- (c) Resultant at P will also depend on obliquity factor, which is proportional to  $(1 + \cos\theta)$ . So for a wavefront at L, maximum at O occurs for  $\theta = 0$ , while in LN or LM direction, intensity is half of O, as  $\theta = \frac{\pi}{2}$ . Along LS,  $\theta = \pi$ , so no intensity in reverse direction. (zone plate)

Fraunhofer diffraction occurs when source of light/screen are effectively infinite distances from aperture. Two convex lenses are used & incident wavefront is plain. Secondary wavelet from exposed portion of the wavefront at aperture are in the same phase at all points in plane of the aperture.

(plane transmission grating, concave reflection grating)

### Fresnel's half-period zone of a plain wave-front



- First half period zone  $a + \frac{\lambda}{2}$
- Second half period zone  $a + \lambda$
- Third half period zone  $a + \frac{3\lambda}{2}$
- Fourth half period zone  $a + 2\lambda$

Let us consider a plane wavefront of a monochromatic light at any particular instant. We want to find out the resultant amplitude at P due to all the wavelets coming from this wavefront.

According to Huygen's principle, every point on the plane wavefront may be regarded as the origin of the secondary wavelets & therefore the resultant effect at P due to the whole wavefront will be equal to the resultant of all these secondary wavelets.

The wavefront is divided into a number of Fresnel's half period zones - from P drop a perpendicular on ABCD at O (pole of the wave). Let  $OP = a$  and P as centre & radius  $(a + \frac{\lambda}{2})$ , draw a sphere cutting the wavefront in a circle at  $M_1$ ,

$PM_1 = a + \frac{\lambda}{2}$  so that the secondary wavelets from O & from the points on the circumference of  $M_1$  on reaching P will differ in phase by  $\frac{2\pi}{\lambda} (PM_1 - OP) = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi = \frac{T}{2}$  (half period)

Similarly other sphere of radii  $(a + \frac{3\lambda}{2}), (a + \frac{5\lambda}{2}), (a + \frac{7\lambda}{2}), \dots$  can be drawn that intersect at  $M_2, M_3, M_4, \dots$  so that the whole wavefront can be divided into several half period zones.

Amplitude due to wavelets produced by each zone is

- (i) Directly proportional to the area of the zone which is approximately equal.
- (ii) Varies inversely with the distance of zone from P.
- (iii) Varies with the obliquity factor  $(1 + \cos \theta)$ .

$$\text{Area of } 1^{\text{st}} \text{ half period zone} = \pi OM_1^2 = \pi (PM_1^2 - OP^2) \\ = \pi [(a + \frac{\lambda}{2})^2 - a^2] = \pi [a^2 + \frac{\lambda^2}{4}] \underset{\cancel{\lambda^2}}{\cancel{+ \frac{\lambda^2}{4}}} \simeq \underline{\pi a \lambda}.$$

$$\text{Similarly } OM_n^2 = PM_n^2 - OP^2 = (a + \frac{n\lambda}{2})^2 - a^2 = a n \lambda. \\ (\text{n}^{\text{th}} \text{ circle})$$

$$OM_{n-1}^2 = a(n-1)\lambda \quad ((n-1)^{\text{th}} \text{ circle}).$$

$$\text{So Area of } n^{\text{th}} \text{ zone} = \pi (OM_n^2 - OM_{n-1}^2) = \underline{\pi a \lambda}.$$

So radii of zone  $\propto \sqrt{n}$ .  
area of zone independent of n

## Schuster's Method :

For visible light,  $\lambda \approx$  small & so area of zone =  $\pi a^2$  but if  $\lambda$  is not very small then the area of half period zones of higher order decreases gradually. If the phase of the wavelets coming from  $O$  is zero then the phase of wavelets from intermediate points between  $O$  and  $M_1$  will vary from  $0$  to  $\pi$  (because  $\frac{2\pi}{\lambda} (PM_1 - OP) = \pi$ ).

$$\therefore \text{Average phase of all wavelets from } 1^{\text{st}} \text{ zone} = \frac{0+\pi}{2} = \frac{\pi}{2}.$$

Similarly phase difference of wavelets from  $M_1$  &  $M_2$  will be between  $\pi$  and  $2\pi$ , so that average phase of all wavelets from  $2^{\text{nd}}$  zone =  $\frac{\pi+2\pi}{2} = \frac{3\pi}{2}$ , from  $3^{\text{rd}}$  zone  $\frac{5\pi}{2}$ , from  $4^{\text{th}}$  zone  $\frac{7\pi}{2}$  & so on...

Resultant phase-difference between two consecutive zones =  $\pi$ .

Resultant phase-difference between two alternate zones =  $2\pi$ .

So if resultant from  $1^{\text{st}}$  half period zone is positive then  $2^{\text{nd}}$  half period zone is negative.

Amplitude decreases due to obliquity factor  $(1 + \cos \theta)$ , so resultant amplitude

$$D = d_1 - d_2 + d_3 - d_4 + d_5 - \dots \pm d_n.$$

(i) If  $n = \text{odd}$ , to a first approximation  $d_2 = \frac{d_1 + d_3}{2}$ ,  $d_4 = \frac{d_3 + d_5}{2}$

$$\begin{aligned} \text{so that } D &= \frac{d_1}{2} + \left( \frac{d_1 + d_3}{2} - d_2 \right) + \left( \frac{d_3 + d_5}{2} - d_4 \right) + \dots + \frac{d_n}{2} \\ &= \frac{d_1}{2} + \frac{d_n}{2}. \end{aligned}$$

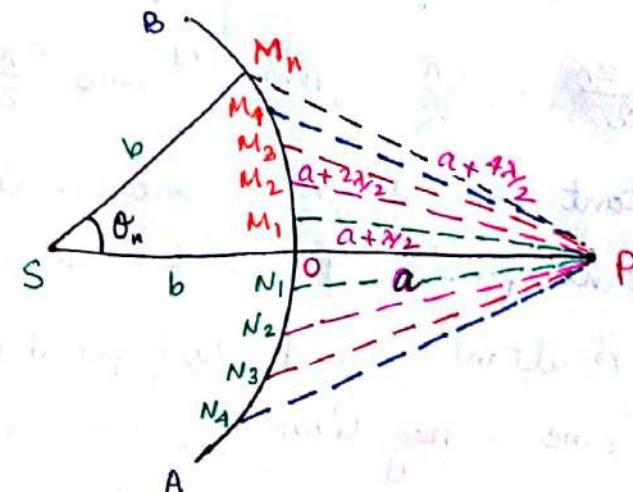
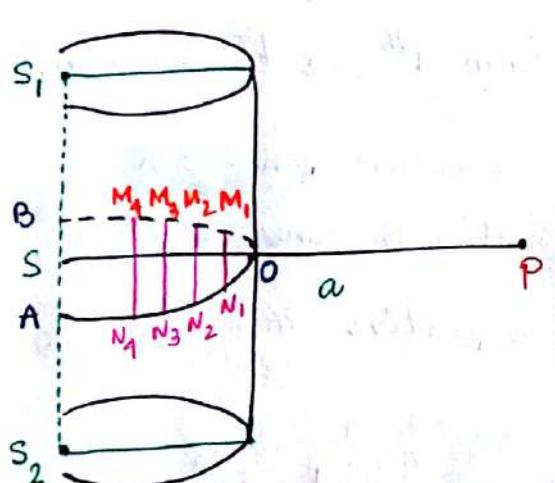
$$(ii) \text{If } n = \text{even}, D = \frac{d_1}{2} + \frac{d_{n-1}}{2} - d_n$$

If  $n$  is very large, then effect from  $n^{\text{th}}$  zone is negligible

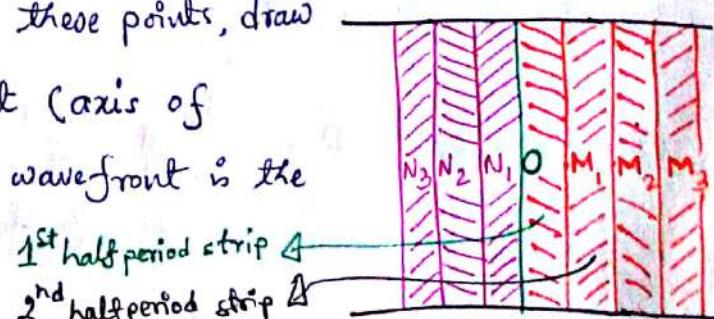


resultant amplitude due to whole wave is  $D = \frac{d_1}{2}$ . as 102  
intensity  $I = D^2 = \frac{d_1^2}{4}$ . If an obstacle is placed at O then  
the resultant disturbance at P is = half the disturbance due to  
wavelets from the 1st half-period zone with one-fourth the intensity.  
If obstacle at O blocks a considerable number of half-period  
zones, effect is negligible & no light is received at P - or light  
travels approximately in a straight line.

### Fresnel's half-period strip of a cylindrical wave-front



Consider a long and narrow slit  $S_1, S_2$ , when illuminated by monochromatic light of wavelength  $\lambda$ , produces cylindrical wavefront. To find the resultant amplitude, the wavefront can be divided into half period strips, with O as pole. Consider an equatorial section  $AOB$  through O in plane of paper. With P as centre & radius  $(a + \frac{\lambda}{2}), (a + \frac{2\lambda}{2}), \dots$  etc, draw arcs that cut  $AOB$  at point  $M_1, N_1, M_2, N_2, \dots$  etc. Through these points, draw lines parallel to length of slit (axis of wavefront) and the area of the wavefront is the half period strip.



Amplitude of the waves reaching P due to wavelets produced by each half-period strip is

- (i) Directly proportional to the area of the strip (not equal)
- (ii) Average distance of strip from P
- (iii) Varies with the obliquity factor  $(1 + \cos\theta)$

As length of strip is same, so areas are proportional to arcs

$$OM_1, M_1M_2, M_2M_3, \dots \text{ where } PM_n = a + \frac{n\lambda}{2}$$

from triangle  $PM_nS$ , we have  $PM_n^2 = SM_n^2 + PS^2 - 2SM_n PS \cos\theta_n$

$$\approx \left(a + \frac{n\lambda}{2}\right)^2 = b^2 + (a+b)^2 - 2b(a+b) \cos\theta_n \quad (1 - \theta_n/2)$$

$$\approx a^2 + an\lambda + \frac{n^2\lambda^2}{4} = 2b^2 + a^2 + 2ab - 2ab - 2b^2 + b(a+b)\theta_n^2$$

$$\text{or } an\lambda = b(a+b)\theta_n^2 \quad \text{or } \theta_n = \sqrt{\frac{an\lambda}{b(a+b)}} = k\sqrt{n}$$

$$\text{Now } OM_n = b\theta_n = bK\sqrt{n}$$

$$\text{So } OM_1 = bK, OM_2 = bK\sqrt{2}, OM_3 = bK\sqrt{3}$$

$$\text{So } M_1M_2 = bK(\sqrt{2}-1) = 0.414 bK$$

$$M_2M_3 = bK(\sqrt{3}-\sqrt{2}) = 0.318 bK$$

$$M_3M_4 = bK(\sqrt{4}-\sqrt{3}) = 0.268 bK, M_4M_5 = 0.236 bK, \dots$$

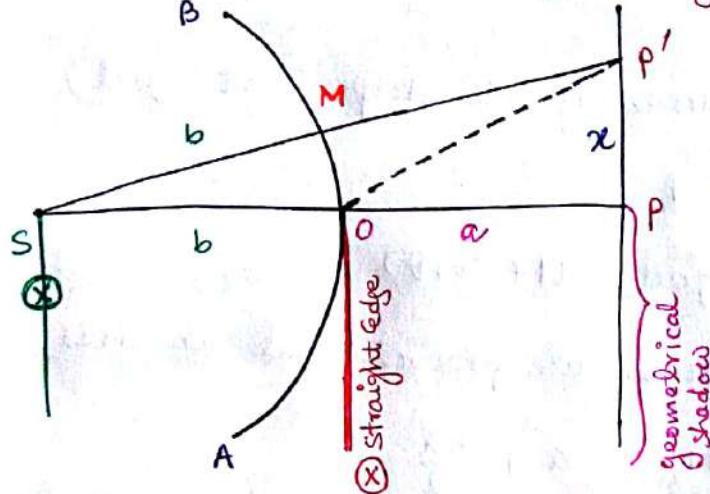
So area of strip initially decreases rapidly & then for increasing order more slowly. and because of opposite sign they cancel out each other. So the resultant at P is only due to first few half period strips.

$$D = d_1 - d_2 + d_3 - d_4 + \dots \approx \frac{d_1}{2} \quad (\text{from left side})$$

$$\approx \frac{d_1}{2} \quad (\text{from right side half wavefront})$$

$$\text{So resultant due to whole wavefront} = \frac{d_1}{2} \pm \frac{d_1}{2} = d_1 \quad (n \text{ odd}) \\ = 0 \quad (n \text{ even})$$

## Diffraction at a straight edge



⊗⊗ Out of plane of paper

Consider a straight edge at O and an illuminated narrow slit S parallel to each other. Dark & bright bands of unequal width of varying intensity is observed in geometrical shadow. We study intensity at P' with M as pole and construct Fresnel's half-period strip. The effect at P' depends upon the number of half-period strips contained in OM & BM.

Due to straight edge, the effect at P' is due to the upper half of the wavefront only, so displacement at P' is  $\frac{1}{2}$  of the displacement for whole wavefront or  $\frac{1}{4}$  of the full wavelet intensity.

# of half-period strips contained in OM depends on the path difference  $OP' - MP'$

$$\text{Now } OP' = \sqrt{a^2 + x^2} = a\left(1 + \frac{x^2}{a^2}\right)^{\frac{1}{2}}$$

$$\approx a\left(1 + \frac{x^2}{2a^2}\right) = a + \frac{x^2}{2a}$$

$$SP' = \sqrt{(a+b)^2 + x^2} = a+b + \frac{x^2}{2(a+b)}$$

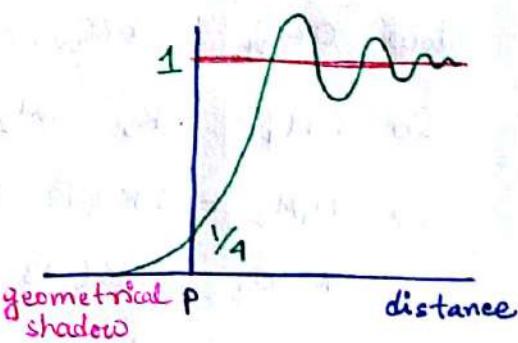
$$\therefore MP' = SP' - OP' = a + \frac{x^2}{2(a+b)}$$

$$\therefore \text{path difference } OP' - MP' = a + \frac{x^2}{2a} - a - \frac{x^2}{2(a+b)} = \frac{bx^2}{2a(a+b)}$$

for the displacement to be maximum,

$$\frac{bx^2}{2a(a+b)} = (2n+1)\frac{\lambda}{2} \quad \text{or} \quad x = \left[ \frac{a(a+b)(2n+1)\lambda}{b} \right]^{\frac{1}{2}}, n=0,1,2,\dots$$

$x \propto \sqrt{2n+1}$  (bright band)



for the displacement to be minimum,  $\frac{bx^2}{2a(a+b)} = n\lambda$

$$\therefore x = \left[ \frac{2a(a+b)n\lambda}{b} \right]^{\frac{1}{2}}, \quad n=1, 2, 3, \quad x \propto \sqrt{n} \text{ (dark band)}$$

Using these, wavelength of light can be found.

CW A narrow slit illuminated by light of  $\lambda = 5890\text{\AA}$  is located at a distance of 0.1 m from a straight edge. If the measurements are made at a distance of 0.5 m from the edge, calculate the distance between 1<sup>st</sup> & 2<sup>nd</sup> dark band.

$$\text{For } n\text{th dark band} \quad x = \sqrt{\frac{2a(a+b)n\lambda}{b}}$$

$$a = 0.5 \text{ m}$$

$$b = 0.1 \text{ m}$$

$$\lambda = 5890 \times 10^{-10} \text{ m}$$

$$\therefore x_2 - x_1 = \sqrt{\frac{2a(a+b)\lambda}{b}} (\sqrt{2} - 1)$$

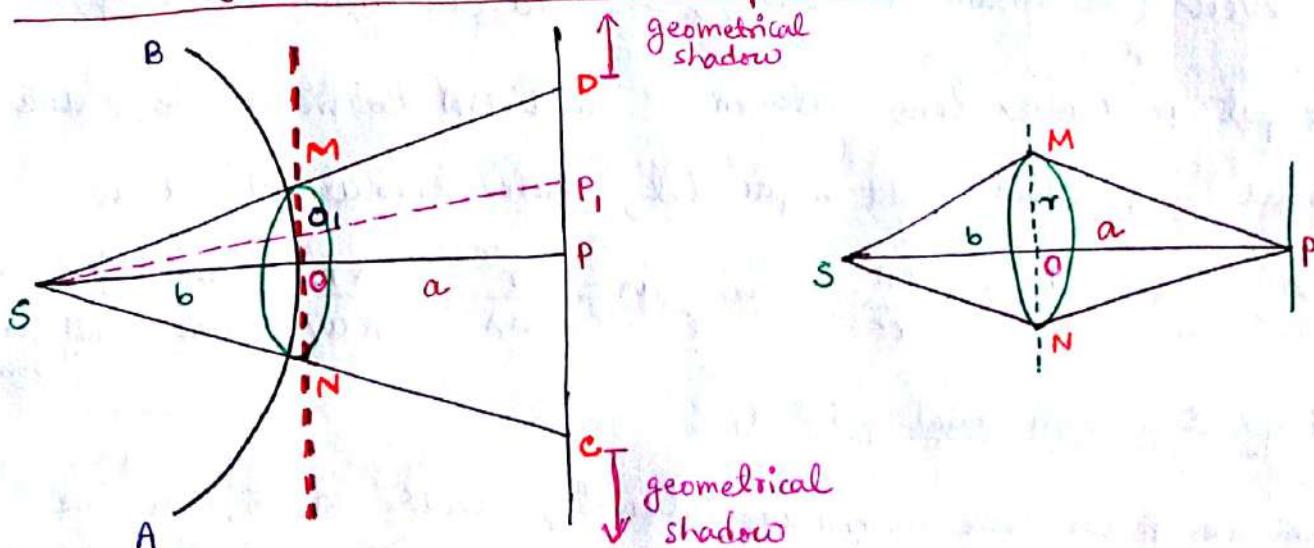
$$= 0.7786 \times 10^{-3} \text{ m.}$$

# Read about diffraction of light by a thin wire. fringe width

$$b = \frac{DA}{d}, \quad D = \text{distance between obstacle \& crosswire of Eyepiece},$$

$$\lambda = \text{wavelength of light.}$$

### Fresnel's diffraction at a circular aperture



from a point source S a wavefront (spherical) touches a circular aperture MN. To calculate the amplitude at screen P, we need to divide the wavefront MON into Fresnel's half-period zones about the pole O.

### Intensity at an axial point P :

If only the 1<sup>st</sup> half period zone is exposed then amplitude at P is twice the amplitude if the whole wavefront is exposed, or intensity will be four times. Let the amplitude is  $d_1$ .

If the screen is moved towards the aperture so that 1<sup>st</sup> & 2<sup>nd</sup> half-period zones are exposed then amplitude =  $d_1 - d_2 \approx 0$  as  $d_1 \approx d_2$  so dark & bright fringes will form as more half-period zones are exposed.

Path difference for waves reaching P along SMP & SOP is

$$= (SM + MP) - (SO + OP) = \sqrt{b^2 + r^2} + \sqrt{a^2 + r^2} - (b+a)$$

$$\approx b\left(1 + \frac{r^2}{2b^2}\right) + a\left(1 + \frac{r^2}{2a^2}\right) - (b+a) = \frac{r^2}{2}\left(\frac{1}{a} + \frac{1}{b}\right).$$

If the aperture contains n half-period zones then path difference is  $\frac{n\lambda}{2}$ . So  $\frac{r^2}{2}\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{n\lambda}{2}$  or  $\frac{1}{a} + \frac{1}{b} = \frac{n\lambda}{r^2}$ .

n = even (minimum intensity), n = odd (maximum intensity).

If we put a convex lens between S and MN to make the plane wavefront (incident light is parallel, source lies at  $\infty$ ) then

$$b = \infty, \text{ so } \frac{1}{a} = \frac{n\lambda}{r^2} \text{ or } n = \frac{r^2}{a\lambda} = \frac{\pi r^2}{\pi a \lambda} = \frac{\text{area of aperture}}{\text{area of half-period zone}}.$$

### Intensity at a non-axial point P<sub>1</sub> :

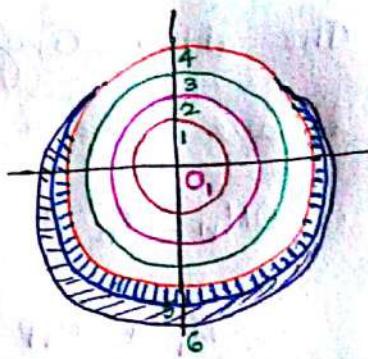
Suppose at P we have maximum intensity with n = 5. As we move up to P<sub>1</sub>, the pole shifts to O<sub>1</sub>. Here suppose only 4 zones are completely exposed while  $\frac{1}{2}$  of 5<sup>th</sup> and 6<sup>th</sup> zone are exposed, so that resultant displacement at P<sub>1</sub>

$$= d_1 - d_2 + d_3 - d_4 + \frac{d_5}{2} - \frac{d_6}{2}$$

$$= \frac{d_1}{2} + \left( \frac{d_1 + d_3}{2} - d_2 \right) + \left( \frac{d_3 + d_5}{2} - d_1 \right) - \frac{d_6}{2}$$

$$= \frac{d_1}{2} - \frac{d_6}{2}. \text{ So the intensity will be minimum.}$$

If we move up to  $P_2$  then intensity will be maximum as first 3 zones are completely exposed and  $\frac{1}{2}$  of 4<sup>th</sup>, 5<sup>th</sup>, 6<sup>th</sup>, 7<sup>th</sup> zones are exposed. So there will be concentric alternating bright & dark rings.



### Diffraction at a circular obstacle

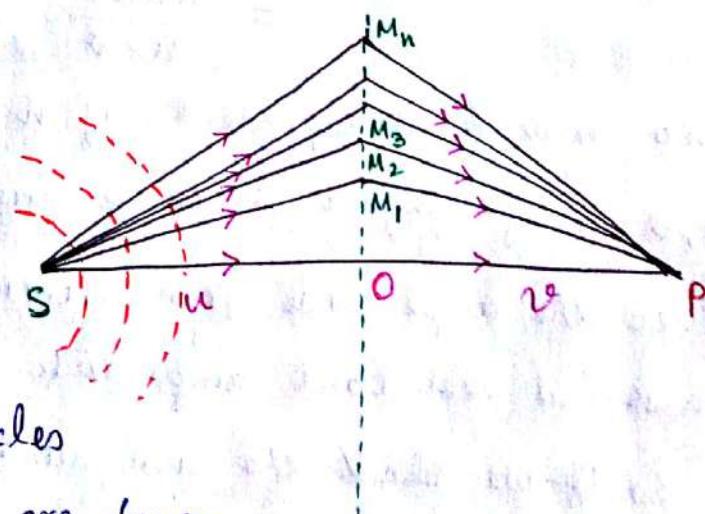
Here if the obstacle obstructs only the 1<sup>st</sup> half-period zone then at  $P_1$ , displacement =  $-d_2 + d_3 - d_4 + d_5 \dots = -\frac{d_2}{2}$ .  $\therefore I \propto \frac{d_2^2}{4}$ .

If size of obstacle is increased or point  $P$  is brought near so that 2<sup>nd</sup>, 3<sup>rd</sup>, ... etc zones are obstructed then displacement is  $\frac{d_3}{2}, -\frac{d_4}{2}$ , ... or  $I \propto \frac{d_3^2}{4}, \frac{d_4^2}{4}, \dots$  So  $P$  remain always bright, which is actually the geometrical shadow.

for any other point  $P_1, P_2$ , etc within the geometrical shadow, diffracted waves interfere due to phase difference & produce interference band (circular). Outside the geometrical shadow, we get diffraction band of unequal width.

### Zone Plate

The idea of Fresnel's half period zone can be used to construct a transparent plate on which circles with radii proportional to  $\sqrt{n}$  are drawn.



The alternating annular zones are blocked, so that the plate behaves like a convex lens. So by construction  $OM_1 = r_1$ ,  $OM_2 = r_2$ ,  $\dots OM_n = r_n$  are the radius of the circles and

$$SM_1 + M_1 P = SO + OP + \lambda/2$$

$$SM_2 + M_2 P = SO + OP + \lambda$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots$$

$$SM_n + M_n P = SO + OP + n\lambda/2. \quad \text{--- (1)}$$

So annular rings are half-period zones, consecutive zone differs by  $\lambda/2$ .

$$\text{If } SO = u, OP = v \text{ then } SM_n = \sqrt{SO^2 + OM_n^2} = \sqrt{u^2 + r_n^2} \approx u \left( 1 + \frac{r_n^2}{2u^2} \right) \\ = u + \frac{r_n^2}{2u}, (u \gg r_n).$$

$$\text{Similarly } M_n P = \sqrt{v^2 + r_n^2} \approx v + \frac{r_n^2}{2v}.$$

$$\text{So from equation (1), } u + \frac{r_n^2}{2u} + v + \frac{r_n^2}{2v} = u + v + n\lambda/2$$

$$\text{or } r_n^2 \left( \frac{1}{u} + \frac{1}{v} \right) = n\lambda$$

$$\text{Applying the sign convention } u \rightarrow -u, r_n^2 = \frac{n\lambda uv}{u-v}$$

$$\text{So } r_n \propto \sqrt{n} \quad \text{for } \lambda, u, v = \text{constant.}$$

$$\text{Area of } n^{\text{th}} \text{ zone} = \pi(r_n^2 - r_{n-1}^2) = \pi \left[ \frac{n\lambda uv}{u-v} - \frac{(n-1)\lambda uv}{u-v} \right] \\ = \frac{\pi \lambda uv}{u-v} \cdot \neq f(n)$$

So area is independent of  $n$  & decreases for decreasing  $u, v$  or if object or image are brought near to the zoneplate.

Since the amplitude from alternate zones will have opposite ~~faces~~ phases so resultant amplitude at  $P$  will be  $d = d_1 - d_2 + d_3 - d_4 + \dots$

So if we block the even number or the odd number of half period zones then the resultant amplitude at  $P$  will be either

$$d = d_1 + d_3 + d_5 + \dots \quad \text{or} \quad d = d_2 + d_4 + d_6 + \dots$$

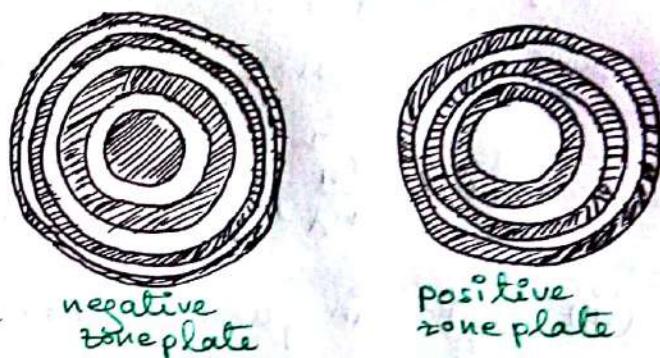
So intensity at P is many times brighter than that due to all exposed zones, so that light from S can be focussed at P. So the result is similar to a convex lens. When  $u=\infty$ ,  $v=f$  so that  $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$  where  $f = \frac{n^2}{n\lambda}$  is the focal length of the zoneplate. So zoneplate acts like a convex lens.

### Construction of zoneplate:

Zoneplate is a system of areas corresponding to half-period zones.

Concentric circles with radii  $\propto$

Natural numbers are drawn on a white paper/glass. Alternate zones are painted black - if odd zones are transparent & even zones are opaque then it's a positive zone plate, otherwise a negative zone plate.



### Phase reversal in a zoneplate: R.W. Wood coated the even no.

zones with a thin film of transparent substance made of Gelatin mixed with  $K_2Cr_2O_7$ , instead of painting them black. As a result, the phase of the waves traversing the even numbered zones change phase  $\pi$ , producing intensity at P 4-fold as

$$d = d_1 + d_2 + d_3 + d_4 + \dots$$

Such type of zoneplates are used as objectives of telescope, photographic camera etc.

## Difference between a zoneplate & convex lens

(i) For a particular wavelength, a convex lens has a single focal length, but for a zone plate, there are a number of focal lengths between the plate and brightest focus (multiple foci)

$f = \frac{n^2}{n\lambda}$ . So for a fixed distance of object, lens produces one image whereas zoneplate produces a number of images. Depending on the position of screen, it may contain 3 or 5 or 7 half-period zones & intensity of image decreases with decreasing focal length.

$$f_1 = \frac{r_n^2}{n\lambda}, \quad f_2 = \frac{r_n^2}{3n\lambda}, \quad f_3 = \frac{r_n^2}{5n\lambda}, \dots$$

(ii) Light in passing through the lens takes equal time to go from S to P through any part of lens whereas in a zoneplate disturbances from any transparent zone reach P one-period later than the disturbances from the next inner zone.

(iii) Focal length of a lens is  $\frac{1}{f} = (\mu - 1)(\frac{1}{R_1} - \frac{1}{R_2})$  whereas for zoneplate is  $\frac{1}{f} = \frac{n\lambda}{r_n^2}$ .

(iv) Focal length of lens is proportional to  $\lambda$ , so is greater for red rays than violet. Focal length is inversely proportional to  $\lambda$  for zoneplate so is greater for violet than red rays.

Q What is the radius of 1<sup>st</sup> zone of a zoneplate of focal length 0.2 m for a light of wavelength 5000 Å?

$$\lambda = 5000 \text{ Å} = 5 \times 10^{-7} \text{ m}, \quad f = 0.2 \text{ m}, \quad n=1, \quad \text{so } f = \frac{r_1^2}{\lambda}$$

$$\therefore r_1 = \sqrt{f\lambda} = 3.16 \times 10^{-4} \text{ m.}$$

CW Calculate the radii of the first 3 clear elements of a zone plate which is designed to bring a parallel beam of light of wavelength  $6000\text{\AA}$  to the first focus at a distance of 2 metres.

parallel light mean  $u = \infty$ ,  $v = f$  (first focus)

$$v = f = 2 \text{ m.}, \lambda = 6000 \text{\AA} = 6 \times 10^{-7} \text{ m.}, r_n = \sqrt{n\lambda f}.$$

$$\text{for 1st clear zone } n=1, r_1 = \sqrt{\lambda f} = 1.095 \times 10^{-3} \text{ m.}$$

$$\text{for 2nd clear zone } n=3, r_2 = \sqrt{3\lambda f} = 1.897 \times 10^{-3} \text{ m.}$$

$$\text{for 3rd clear zone } n=5, r_3 = \sqrt{5\lambda f} = 2.449 \times 10^{-3} \text{ m.}$$

CW A plane wavefront ( $\lambda = 6000\text{\AA}$ ) advancing towards a point is divided into a number of half-period zones. Amplitude contribution of these half-period zones is 1, 0.98, 0.96, ... 0. Compare the intensities at the point when first 31, 4, 36 half period zone are only exposed.

$$d_1 = 1, d_2 = 0.98, d_3 = 0.96, \dots d_n = 0.$$

$$\text{So } \frac{d_1 + d_3}{2} = d_2, d_1 - d_2 = 1 - 0.98 = 0.02$$

$$d_{31} = 1 - 30 \times 0.02 = 0.40$$

$$d_{35} = 1 - 34 \times 0.02 = 0.32$$

$$d_{36} = 1 - 35 \times 0.02 = 0.30$$

$$\text{So resultant amplitude of 31 zones} = \frac{d_1}{2} + \frac{d_{31}}{2} = \frac{1}{2} + \frac{0.4}{2} = 0.7$$

$$\begin{aligned} \text{resultant amplitude of 36 zones} &= \frac{d_1}{2} + \frac{d_{35}}{2} - d_{36} \\ &= \frac{1}{2} + \frac{0.32}{2} - 0.3 = 0.36. \end{aligned}$$

$$\therefore \frac{I_{31}}{I_{36}} = \frac{0.7^2}{0.36^2} = 3.78.$$

CW A zoneplate is found to give series of images of a point source on the axis. If the strongest and the 2nd strongest images are at distances of 0.3 m and 0.06 m respectively from the zoneplate (both on same side) calculate the distance of the source from the zoneplate, principle focal length

and radius of 1st zone for  $\lambda = 5 \times 10^{-7} \text{ m}$ .

$$\lambda = 5 \times 10^{-7} \text{ m}, v_1 = 0.3 \text{ m}, v_2 = 0.06 \text{ m}.$$

$$f_1 = \frac{r_n^2}{\lambda}, f_2 = \frac{r_n^2}{3\lambda}. \therefore f_2 = f_1/3.$$

If  $u$  is the distance of the object from zone plate then

$$\frac{1}{u} = \frac{1}{v_1} - \frac{1}{f_1} \text{ and } \frac{1}{u} = \frac{1}{v_2} - \frac{1}{f_2}$$

$$\therefore \frac{1}{v_1} - \frac{1}{f_1} = \frac{1}{v_2} - \frac{3}{f_1} \Rightarrow \frac{2}{f_1} = \frac{1}{v_2} - \frac{1}{v_1} = \frac{1}{0.06} - \frac{1}{0.3}$$

$$\therefore f_1 = 0.15 \text{ m.} \rightarrow \text{principal focal length}$$

$$\text{Now } r_1 = \sqrt{f_1 \lambda} = 0.274 \times 10^{-4} \text{ m.} \rightarrow \text{radius of 1st zone}$$

$$\frac{1}{u} = \frac{1}{v_1} - \frac{1}{f_1} = \frac{1}{0.3} - \frac{1}{0.15}, u = -0.3 \text{ m.}$$

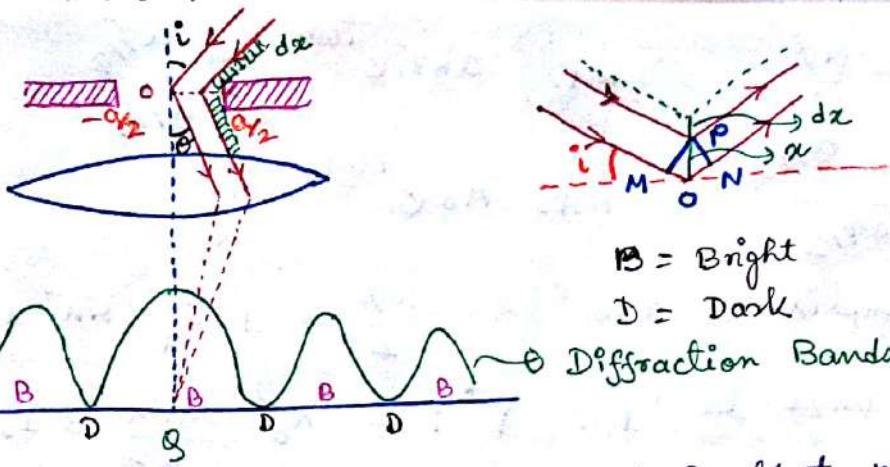
$\rightarrow$  distance from source to zone plate

### Fraunhofer Diffraction (far-field)

Single slit diffraction : A single slit is a vacant space which is obstructed by two sharp opaque regions. When a monochromatic light is incident on such a slit, it is found that the intensity on the opposite side has a variation and maximum & minimum brightness are observed, which are called diffraction bands. This phenomena occurs due to diffraction of light.

#### Theory :

Let a parallel beam of monochromatic light of wavelength  $\lambda$  is incident on a narrow slit of width  $a$  in a direction making an angle  $i$  with the normal. After diffraction, they are scattered in various directions. We will calculate the intensity at screen due to the rays diffracted at an angle  $\theta$  with the normal.



$$\angle OPM = i$$

$$\angle OPN = \theta.$$

B = Bright

D = Dark

Diffraction Bands

The rays coming from O and P that meet at Q will have path difference = OM + ON =  $x(\sin i \pm \sin \theta)$ . So the phase difference between these two waves is  $\frac{2\pi}{\lambda} x(\sin i \pm \sin \theta)$ .

Let the displacement at any point Q due to secondary waves from the origin O (midpoint of the slit) is proportional to

$$y = R.P. \text{ of } r e^{iwt} \quad \left[ \begin{array}{l} \text{for derivation, we consider the nature of wavefronts} \\ \text{spatial part outside the calculation} \end{array} \right]$$

Then the amplitude of the wave coming from the point P that meets at point Q is  $y = R.P. kr e^{i[w t \pm \frac{2\pi}{\lambda} x(\sin i \pm \sin \theta)]}$

$$= R.P. kr e^{i(wt \mp \delta)} \quad , \quad k = \text{constant.}$$

If we consider the change of phase over a small distance  $dx$  which is negligible compared to  $x$ , then the displacement at Q due to waves from a region  $dx$  after  $x$  will be given by

$$y = R.P. kr e^{i(wt \pm \delta)} dx.$$

∴ The total displacement at Q due to the whole slit is

$$Y = R.P. kr e^{iwt} \int_{-\alpha/2}^{\alpha/2} e^{\pm i\delta} dx, \quad \text{let } \delta = \phi x \text{ where}$$

$$\phi = \pm \frac{2\pi}{\lambda} (\sin i \pm \sin \theta)$$

$$= R.P. kr e^{iwt} \int_{-\alpha/2}^{\alpha/2} e^{i\phi x} dx = R.P. kr e^{iwt} \left[ \frac{e^{i\phi x}}{i\phi} \right]_{-\alpha/2}^{\alpha/2}$$

$$= R.P. \frac{kr e^{iwt}}{\phi} 2 \left[ \frac{e^{i\phi \alpha/2} - e^{-i\phi \alpha/2}}{2i} \right]$$

$$= R.P. \frac{2k\pi e^{iwt}}{\phi} \sin \frac{\alpha\phi}{2} = R.P. akr e^{iwt} \frac{\sin \frac{\alpha\phi}{2}}{\alpha\phi/2}$$

$$= R.P. A_0 e^{iwt} \frac{\sin \frac{\alpha\phi}{2}}{\alpha\phi/2} = R.P. A_0 e^{iwt} \frac{\sin \alpha}{\alpha}$$

where  $A_0$  is total amplitude and

$$\alpha = \frac{\alpha\phi}{2} = \pm \frac{\alpha\pi}{\lambda} (\sin i \pm \sin \theta)$$

$$\therefore \text{Intensity at } \phi \text{ will be } I = Y^* Y = A_0^2 \frac{\sin^2 \alpha}{\alpha^2} = I_0 \frac{\sin^2 \alpha}{\alpha^2}$$

which will be extremum (minimum or maximum) when

$$\frac{dI}{d\alpha} = 0 \Rightarrow \frac{d}{d\alpha} \left( I_0 \frac{\sin^2 \alpha}{\alpha^2} \right) = 0.$$

$$\Leftrightarrow 2I_0 \frac{\sin \alpha}{\alpha} \frac{\alpha \cos \alpha - \sin \alpha}{\alpha^2} = 0 \Leftrightarrow 2I_0 \frac{\sin \alpha}{\alpha^2} \left( \cot \alpha - \frac{1}{\alpha} \right) = 0$$

$$\Leftrightarrow 2I \left( \cot \alpha - \frac{1}{\alpha} \right) = 0.$$

$$\begin{aligned} \text{Also, } \frac{d^2 I}{d\alpha^2} &= 2 \frac{dI}{d\alpha} \left( \cot \alpha - \frac{1}{\alpha} \right) + 2I \left( -\operatorname{cosec}^2 \alpha + \frac{1}{\alpha^2} \right) \\ &= 4I \left( \cot \alpha - \frac{1}{\alpha} \right)^2 + 2I \left[ -\left( 1 + \cot^2 \alpha \right) + \frac{1}{\alpha^2} \right] \\ &= 4I \left( \cot \alpha - \frac{1}{\alpha} \right)^2 - 2I \left( 1 + \cot^2 \alpha - \frac{1}{\alpha^2} \right) \end{aligned}$$

$\therefore \frac{dI}{d\alpha} = 0$  only when (i)  $I = 0$  or (ii)  $\cot \alpha = \frac{1}{\alpha}$ .

when  $\cot \alpha = \frac{1}{\alpha} \Rightarrow \frac{d^2 I}{d\alpha^2} = -2I < 0$  (maximum)

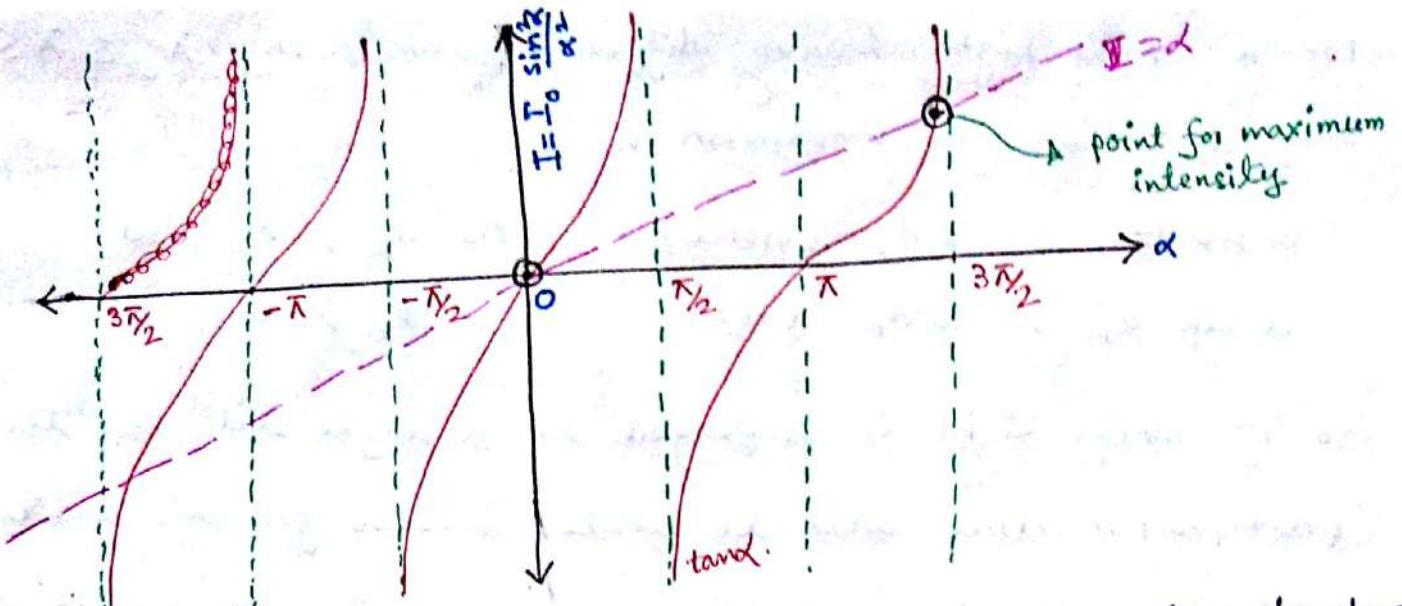
So other condition will give us the condition for minimum,  $I = 0$

or  $\sin \alpha = 0$ . But  $\alpha \neq 0$ , so  $\alpha = \delta\pi, \delta = \pm 1, \pm 2, \pm 3, \dots$

$$\Leftrightarrow \alpha = \frac{\alpha\phi}{2} = \frac{\alpha}{2} \frac{2\pi}{\lambda} (\sin i \pm \sin \theta) = \delta\pi.$$

$$\Leftrightarrow \alpha(\sin i \pm \sin \theta) = \delta\pi \quad (\text{minimum}) \quad (\text{oblique incidence})$$

For normal incidence  $i=0$ ,  $\therefore \alpha \sin \theta = \delta\pi$  (normal incidence)



The values of  $\alpha$  that satisfy the relation  $\cot \alpha = \frac{1}{d}$  or  $\tan \alpha = d$  can be obtained graphically by plotting  $I = \alpha$  and  $I = \tan \alpha$  on the same graph. The intensity is maximum when  $\alpha = 0$  is called the principal maximum. The other value of  $\alpha$  which will give maximum will be obtained from intersection of this points which will occur at  $\alpha = \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}$ , etc.

Principal Maxima : For the principle maxima,  $\alpha=0$  and taking

$$\lim_{d \rightarrow 0} \frac{\sin d}{d} = 1, \text{ we get } I_{\text{principal}} = I_0$$

$$\text{Hence when } \alpha \rightarrow 0, \text{ we obtain } \frac{\alpha d}{2} \rightarrow 0$$

$$\Rightarrow \varphi = \frac{2\pi}{\lambda} (\sin i \pm \sin \theta) \rightarrow 0$$

i.e.  $i \rightarrow 0$  &  $\theta \rightarrow 0$ . Thus, principal maxima is obtained at the middle when angle of incidence & diffraction is zero. As the value of  $\alpha$  increases, the value of  $\frac{\sin \alpha}{\alpha}$  decreases, so the intensity of secondary maxima reduces.

White light :

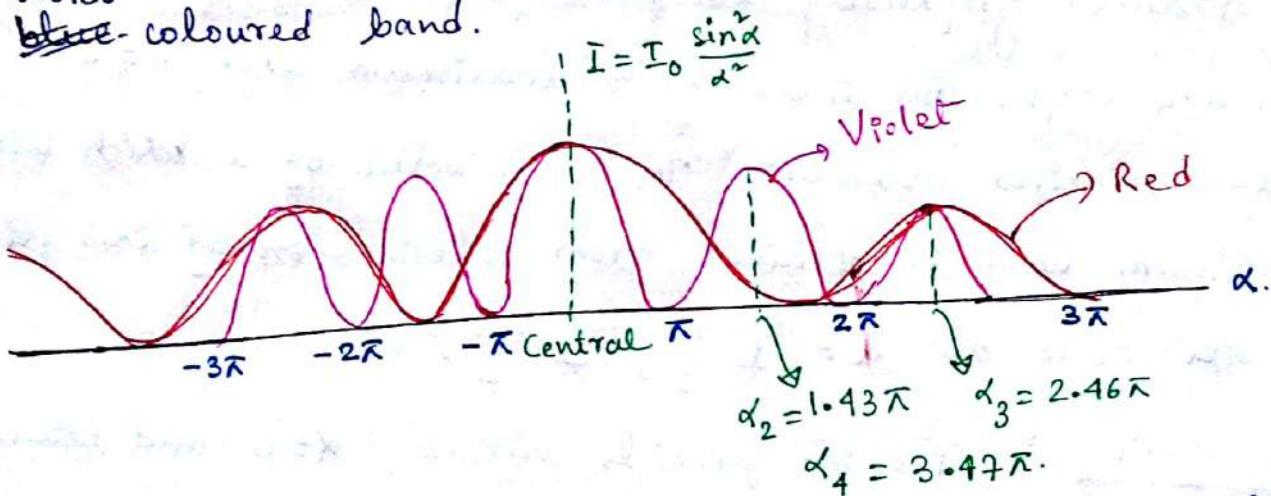
for normal incidence ( $i=0$ ), the condition of minimum for diffraction in single slit is  $a \sin \theta = \lambda$ .

for a white light having different wavelength  $\lambda_v \leq \lambda \leq \lambda_R$   
 then condition for minimum is

$$a \sin \theta_v = s \lambda_v \text{ (violet)} \quad \text{As } \lambda_R > \lambda_v, \text{ so}$$

$$a \sin \theta_R = s \lambda_R \text{ (red)} \quad \boxed{\theta_R > \theta_v}$$

So if white light is employed to a single slit, then the central (principle) maxima will be white, because for this condition  $\theta \rightarrow 0$ . But for other secondary maxima, the band will be coloured with the red-coloured band further apart from the violet  
~~blue~~-coloured band.



So secondary maxima do not fall half-way between two minima.

Secondary Minima: The direction of secondary minima is

$$a \sin \theta = s \lambda \quad (\text{normal incidence})$$

$$\text{So } \alpha = \frac{a\pi}{\lambda} \sin \theta = s\pi, \quad s = \pm 1, \pm 2, \dots, \pm n.$$

If  $\alpha = \pm \pi$ ,  $\sin \theta = 0$ , Intensity  $I = 0$

So various diffraction minima occur at  $\alpha = \pm \pi, \pm 2\pi, \dots, \pm n\pi$ .  
 $(n = \text{integer})$

$$\text{Again, } \frac{a\pi}{\lambda} \sin \theta = s\pi \text{ gives } \sin \theta = s \frac{\lambda}{a} = \pm \frac{n\lambda}{a}$$

So position of 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup> etc minima are given by  $\pm \frac{\lambda}{a}, \pm \frac{2\lambda}{a}, \pm \frac{3\lambda}{a}, \dots$  for various values of  $\sin \theta$ .

Secondary Maxima The direction of  $n^{\text{th}}$  secondary maximum is

$$\sin\theta = \pm \frac{(2n+1)\lambda}{2a}.$$

$$\text{So } d = \pm \frac{\pi}{\lambda} a \left( \frac{(2n+1)\lambda}{2a} \right) = \pm (2n+1) \frac{\pi}{2} = \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, \pm \frac{7\pi}{2}, \dots$$

$$1^{\text{st}} \text{ secondary maximum } d = \frac{3\pi}{2}, \text{ so } I = I_0 \frac{\sin^2 d}{d^2} = \frac{1}{9\pi^2} I_0 \approx \frac{I_0}{22}$$

$$2^{\text{nd}} \text{ secondary maximum } d = \frac{5\pi}{2}, \text{ so } I = I_0 \frac{\sin^2 d}{d^2} = \left(\frac{2}{5\pi}\right)^2 I_0 \approx \frac{I_0}{61}$$

CW A parallel beam of light of wavelength  $5 \times 10^{-7} \text{ m}$  is incident normally on a narrow slit of width 0.2 mm. The Fraunhofer diffraction pattern is observed on a screen which is placed at the focal plane of a convex lens of focal length 50 cm. Calculate the distance between the first two minima and the first two maxima on the screen. Assume that the lens is placed very close to the slit.

$$a = 0.2 \text{ mm} = 2 \times 10^{-4} \text{ m}, \lambda = 5 \times 10^{-7} \text{ m}, f = 0.5 \text{ m}.$$

$$\text{for secondary minima, } a \sin\theta = \pm 8\lambda.$$

and  $\theta \ll 1$ ,  $\sin\theta \approx \theta$ .  $\therefore$  Angular diffraction for 1<sup>st</sup> minima

$$\theta_1 = \frac{\lambda}{a} = \frac{5 \times 10^{-7}}{2 \times 10^{-4}} = 2.5 \times 10^{-3} \text{ radian}$$

$$\text{Angular diffraction for 2<sup>nd</sup> minima } \theta_2 = \frac{2\lambda}{a} = \frac{10 \times 10^{-7}}{2 \times 10^{-4}} = 5 \times 10^{-3} \text{ rad}$$

$\therefore$  Separation between 1<sup>st</sup> & 2<sup>nd</sup> minimum  $x = f(\theta_2 - \theta_1)$

$$= (5 - 2.5) \times 10^{-3} \times 0.5 = 0.125 \times 10^{-3} \text{ m.}$$

1<sup>st</sup> and 2<sup>nd</sup> maxima will occur at  $d = 1.43\pi$  &  $2.46\pi$ .

$\therefore$  Angular diffraction for 1<sup>st</sup> maxima  $\theta'_1 = 1.43 \times 2.5 \times 10^{-3} \text{ rad}$

Angular diffraction for 2<sup>nd</sup> maxima  $\theta'_2 = 2.46 \times 2.5 \times 10^{-3} \text{ rad}$

$\therefore$  Separation between 2<sup>nd</sup> & 1<sup>st</sup> maxima  $x' = f(\theta'_2 - \theta'_1)$

$$= 0.1288 \times 10^{-2} \text{ m.}$$

## Double slit diffraction:

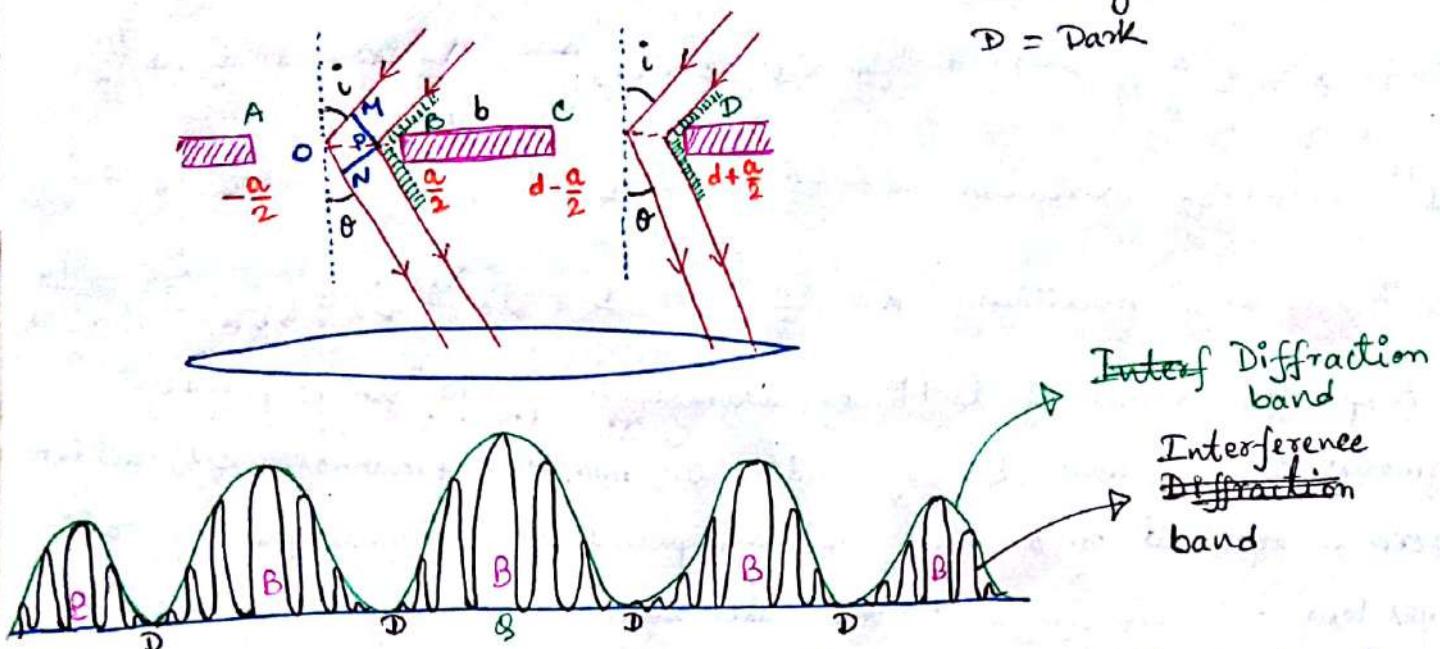
$$a+b = d$$

$$\angle OPM = i$$

$$\angle OPN = \theta$$

B = Bright

D = Dark



When a parallel beam of wavelength  $\lambda$  is made incident normally on a surface containing two narrow and close slits AB and CD, each of width  $a$  and kept separated by an opaque space BC of width  $b$ , we get a series of dark & bright bands on the screen. The two points on two slits (A, C) or (B, D) which are separated by a distance  $a+b=d$  are called "Corresponding points".

If  $\delta$  is the phase difference between the rays diffracted from origin O (which is the mid point of the first slit) and from P (which is at a distance  $x$  from O), then

$$\delta = \frac{2\pi}{\lambda} x (\sin i^o \pm \sin \theta) = \frac{2\pi x}{\lambda} \phi \quad \text{where } \phi = \sin i^o \pm \sin \theta.$$

If  $y = R.P.$  of  $re^{i\omega t}$  is the displacement of the wave at any point Q due to diffraction from the origin O, then the displacement at Q due to the rays diffracted from P at a distance  $x$  from O is

$$y = R.P. \text{ of } k r e^{i(\omega t \pm \delta)}, \quad k = \text{constant}.$$

Hence the displacement at Q due to rays diffracted from a small region  $dx$  of the first slit is

$$y = R.P. K r e^{i(\omega t \pm \delta)} dx = R.P. R e^{\frac{i 2\pi}{\lambda} (ct \pm x\phi)} [ \omega = \frac{2\pi c}{\lambda} = 2\pi v ]$$

$$= R.P. R e^{i\sigma} e^{i\phi} dx [ R = Kr ] [ \delta = \frac{2\pi x\phi}{\lambda} ] [ \sigma = \frac{2\pi ct}{\lambda} ] [ \phi = \frac{2\pi d}{\lambda} ]$$

So resultant displacement at Q due to the two slits together would be

$$y = R.P. R e^{i\sigma} \left[ \int_{-\alpha/2}^{\alpha/2} e^{i\phi} dx + \int_{d-\alpha/2}^{d+\alpha/2} e^{i\phi} dx \right]$$

$$= R.P. \alpha R e^{i\sigma} \left[ \frac{\sin(\alpha/2)}{(\alpha/2)} + e^{id} \frac{\sin(\alpha/2)}{(\alpha/2)} \right] [ \alpha = \frac{\alpha l}{2} = \frac{\pi a \phi}{\lambda} ]$$

$$= R.P. \alpha R \frac{\sin d}{\alpha} (1 + e^{id}) e^{i\sigma} [ A_0 = \alpha R, A = A_0 \frac{\sin d}{\alpha} ]$$

$$= R.P. A (1 + \cos(id) + i \sin(id)) (\cos \sigma + i \sin \sigma) [ C = 1 + \cos(id), D = \sin(id) ]$$

$$= R.P. A (C + iD)(\cos \sigma + i \sin \sigma)$$

$$= \cancel{R.P.} A (C \cos \sigma - D \sin \sigma) [ C = E \cos \gamma, D = E \sin \gamma ]$$

$$= A E \cos(\sigma + \gamma)$$

$$\text{so that } E = \sqrt{C^2 + D^2}$$

$$= \sqrt{1 + \cos^2(id) + \sin^2(id)}$$

$$= \sqrt{2(1 + \cos id)}$$

$$= \sqrt{4 \cos^2 id / 2}$$

So the amplitude of resultant displacement is  $A \sqrt{C^2 + D^2}$  or the resultant intensity  $I = A^2(C^2 + D^2)$

$$= 4A^2 \cos^2 \left( \frac{id}{2} \right) = 4A^2 \cos^2 \beta [ \beta = \frac{id}{2} = \frac{\pi d}{\lambda} (\sin \gamma + \cos \gamma) ]$$

$$= 4A_0^2 \frac{\sin^2 d}{\alpha^2} \cos^2 \beta$$

$$= I_1 \times I_2$$

$$I_2 = \cos^2 \beta ?$$

This indicate that two different system of fringes will appear. Intensity  $I_1$  is due to diffraction produced by each individual slit while intensity  $I_2$  is due to the interference of the diffracted

says at an angle  $\theta$  from the corresponding points of each slit. (interference).

Condition for Minima: The intensity  $I$  would be zero when either  $I_1 = 0$  (Diffraction minima) or  $I_2 = 0$  (Interference minima).

(i)  $I_1 = 4A_0^2 \frac{\sin^2 d}{d^2} = 0$  gives  $d = n\pi$ ,  $n = \pm 1, \pm 2, \dots, {}^{th}$

or  $\frac{\pi a}{\lambda} \sin \theta = n\pi$  (for normal incidence)

or  $a \sin \theta = n\lambda$

and  $a(\sin i \pm \sin r) = n\lambda$  (for oblique incidence)

[Diffraction minima]

(ii)  $I_2 = \cos^2 \beta = 0$  gives  $\beta = (2n+1) \frac{\pi}{2}$

or  $\frac{\pi d}{\lambda} \sin r = (2n+1) \frac{\pi}{2}$  (for normal incidence)

or  $d \sin r = (2n+1) \frac{\lambda}{2}$

and  $d(\sin i \pm \sin r) = (2n+1) \frac{\lambda}{2}$  (for oblique incidence)

Angular separation between two consecutive minima

$$\sin \theta_1 = \pm \frac{3\lambda}{2d}, \sin \theta_2 = \pm \frac{5\lambda}{2d}, \text{ so } \sin \theta_2 - \sin \theta_1 = \pm \frac{\lambda}{d}$$

Condition for Maxima:

for central maxima  $d = 0$  so that  $\lim_{d \rightarrow 0} \frac{\sin d}{d} = 1$ .

So for diffraction maxima (secondary),  $d = \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, \dots$

$$\text{so } \sin \theta_1 = \pm \frac{3\lambda}{a}, \pm \frac{5\lambda}{a}, \pm \frac{7\lambda}{a}, \dots$$

[Diffraction maxima]

when width  $a$  of each slit is very small, then  $d$  is small, so  $I_1 \rightarrow 4A_0^2$  becomes a constant. Under this condition, the maxima of the resultant  $I$  will be solely controlled by  $I_2$ . Hence for normal

incidence, interference maximum is given by  $\cos^2 \beta = 1$  or

$$\beta = \frac{\pi}{\lambda} d \sin \theta = n\pi \Rightarrow d \sin \theta = n\lambda, \quad n=0, \pm 1, \pm 2, \dots$$

(normal incidence)

and  $d(\sin i \pm \sin \theta) = n\lambda$  (oblique incidence) [Interference maxima]

Angular separation between two consecutive maxima

$$\sin \theta_1 = \pm \frac{\lambda}{d}, \quad \sin \theta_2 = \pm \frac{2\lambda}{d}, \quad \text{so } \sin \theta_2 - \sin \theta_1 = \pm \frac{\lambda}{d}$$

So the angular separation between two consecutive maxima & minima are equal.  $= \pm \frac{\lambda}{d}$ .

### Missing order in double slit diffraction pattern

For normal incidence when the condition for maxima of interference pattern and minima of diffraction pattern for a given value of  $\theta$  are simultaneously satisfied, then the interference maxima will be missing.

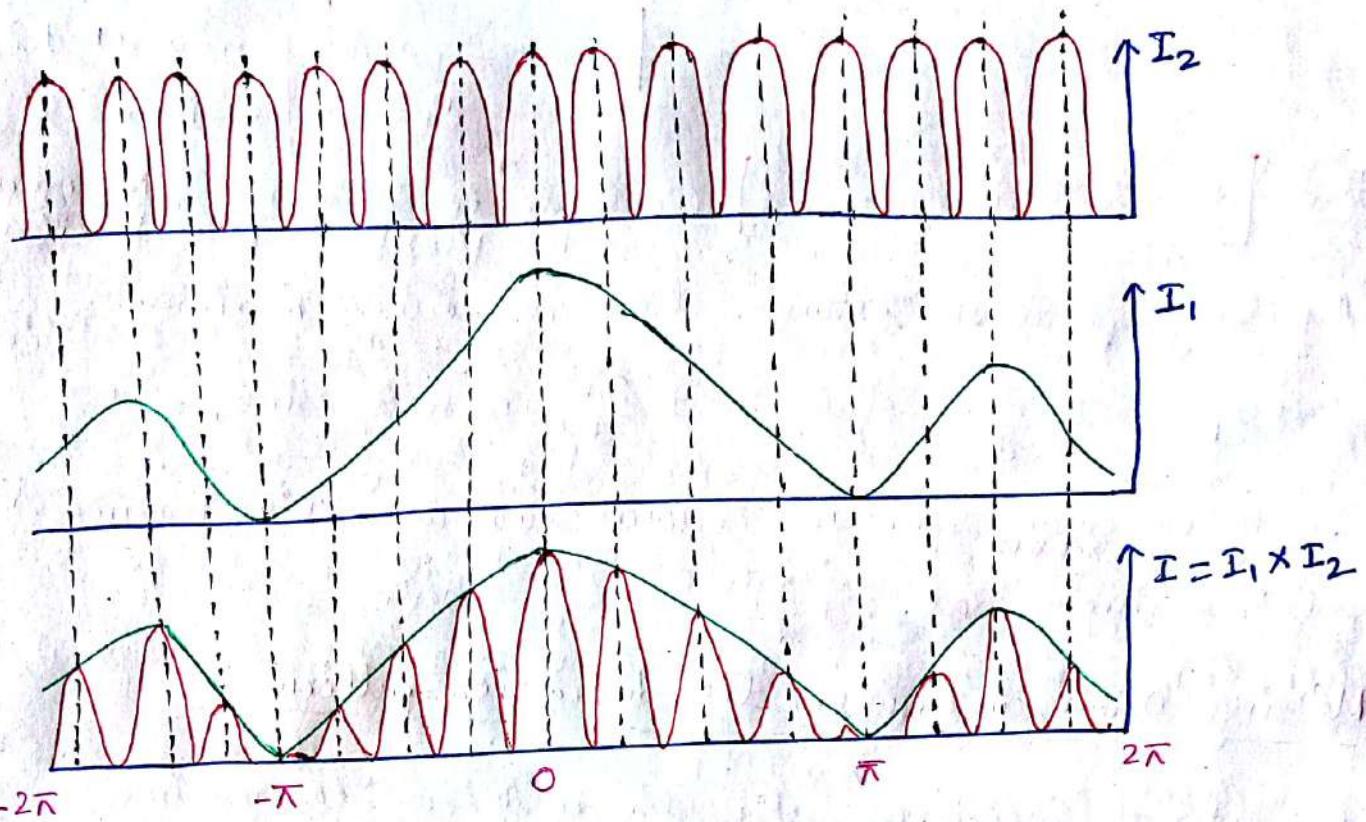
$$\text{Thus } d = \frac{\pi}{\lambda} a \sin \theta = 1\pi \quad (\text{diffraction minima})$$

$$\beta = \frac{\pi}{\lambda} d \sin \theta = n\pi \quad (\text{interference maxima})$$

$$\therefore \frac{\beta}{\alpha} = \frac{d}{a} = \frac{n}{\lambda}. \quad \text{When } d = 2a, n = 2, 4, \dots$$

Hence 2, 4, 6, ... etc orders of interference maxima are absent which corresponds to 1, 2, 3, ... etc orders of diffraction dark bands. Similarly when  $d = 3a$ ,  $n = 3, 6, 9, \dots$  etc orders of interference maxima are absent, which corresponds to 1, 2, 3, ... etc orders of diffraction dark bands.

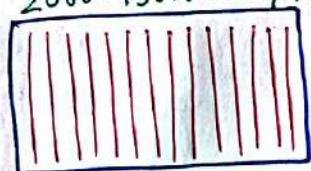
## Complete double slit pattern



## Plane Transmission Diffraction Grating

A plane transmission grating consists of a number of parallel and equidistant lines ruled over optically plane & parallel glass plate by means of a fine diamond point worked with a ruling engine. Number of such ruled lines per inch ranges between 2000 - 15000. Each ruled line behaves as an opaque space, and the transparent portion between two consecutive ruled lines behaves as a slit. If "a" be the width of the clear space and "b" be the width of the ruled lines (opaque space) then the distance  $d = a+b$  is called "Grating constant". The two points on the consecutive clear space whose distance is  $d$  are called "corresponding points".

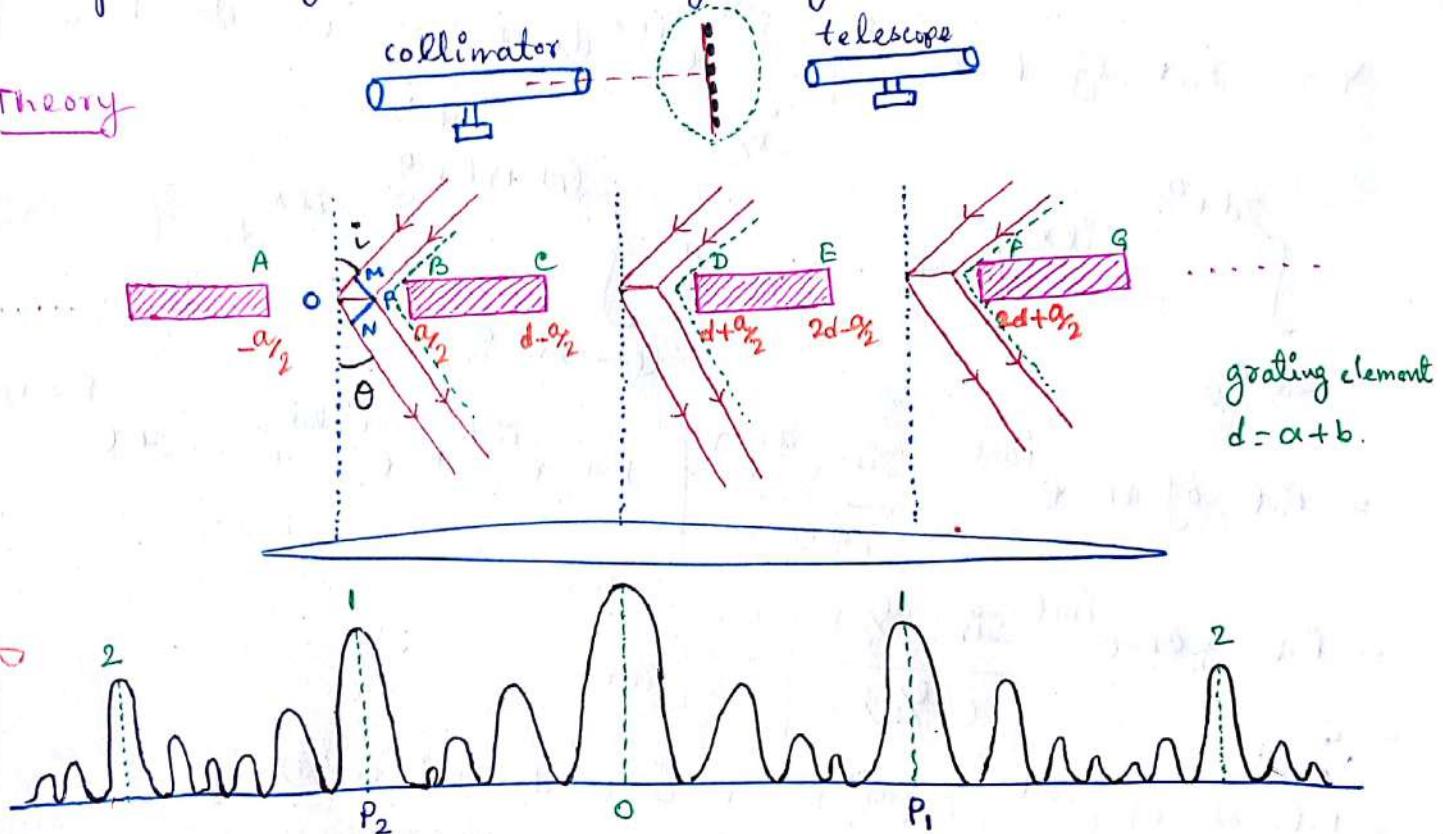
2000-15000 lines/inch



Master / Replica  
Grating.

Drawing exactly parallel & equidistant lines on a glass plate by a diamond point is an extremely difficult task & so the grating becomes very costly. To cut the cost, a cast of these ruled surface is made with some transparent material & such cast is called "replica grating". Cellulose Acetate is properly diluted is put on the surface of master grating and then dried to a thin film. This film is mounted on a transparent glass plate to form the replica grating.

### Theory



A parallel beam of monochromatic light of wavelength  $\lambda$  is incident at an angle  $i$  with the normal to the grating surface, in which there are  $N$  number of parallel equidistant slits of width  $a$  and opaque space of width  $b$ , and are diffracted at an angle  $\theta$ .

Let  $y = R.P. \propto e^{i\omega t}$  is the displacement of the wave reaching a point on screen which is diffracted from the middle point  $O$  of the

first slit. Then the displacement of the wave from a point P at a distance  $x$  from O will be

$$y = \text{R.P. of } Kr e^{i(\omega t \pm \delta)}, \quad k = \text{constant} \quad [R = KR]$$

$$= \text{R.P. of } R e^{i(\omega t + lx)} \quad \delta = lx$$

$$l = \frac{2\pi}{\lambda} \left[ \frac{\sin i}{\sin \theta} \pm \frac{\sin \theta}{\sin i} \right]$$

and the displacement from a further small distance

$dx$  after  $x$ , will be  $y = \text{R.P. of } R e^{i(\omega t + lx) dx}$ .

∴ Amplitude due to all the points in slit will be

$$y = \text{R.P. of } R e^{i\omega t} \left[ \int_{-\alpha/2}^{\alpha/2} e^{ilx} dx + \int_{d-\alpha/2}^{d+\alpha/2} e^{ilx} dx + \int_{2d-\alpha/2}^{2d+\alpha/2} e^{ilx} dx + \dots + \int_{(N-1)d-\alpha/2}^{(N-1)d+\alpha/2} e^{ilx} dx \right]$$

$$= \text{R.P. of } aR e^{i\omega t} \frac{\sin(\alpha l/2)}{(al/2)} \left[ 1 + e^{ild} + e^{i2ld} + \dots + e^{i(N-1)ld} \right]$$

$$= \text{R.P. of } aR e^{i\omega t} \frac{\sin(\alpha l/2)}{(al/2)} \left[ \frac{1 - e^{iNld}}{1 - e^{ild}} \right]$$

$$= \text{R.P. of } aR e^{i\omega t} \frac{\sin(\alpha l/2)}{al/2} \left[ \frac{1 - \cos(Nld) - i \sin(Nld)}{1 - \cos(lid) - i \sin(lid)} \right].$$

So the intensity at that particular point will be

$$I = Y^* Y = a^2 R^2 \frac{\sin^2(\alpha l/2)}{(al/2)^2} \frac{2(1 - \cos Nld)}{2(1 - \cos lid)}$$

$$= A_0^2 \frac{\sin^2 d}{d^2} \frac{\sin^2 N\beta}{\sin^2 \beta}$$

$$= I_1 \times I_2 \quad \begin{array}{l} \text{Diffraction} \\ \text{Interference} \end{array}$$

$$A_0 = aR$$

$$\alpha = \frac{al}{2}$$

$$\beta = \frac{ld}{2}$$

Here, the first factor  $I_1 = A_0^2 \frac{\sin^2 \alpha}{d^2}$  gives the intensity distribution in the diffraction pattern due to single slit and  $I_2 = \frac{\sin^2 N\beta}{\sin^2 \beta}$  gives the interference pattern produced by the beams coming from  $N$  number of slits.

### Condition for Principal Maxima:

When  $\beta = \pm n\pi$  with  $n=1, 2, 3, \dots$  we see that  $I_2 = \frac{\sin^2 N\beta}{\sin^2 \beta}$ ,

gives indeterminate form, but

$$\lim_{\beta \rightarrow n\pi} \frac{\sin N\beta}{\sin \beta} = \lim_{\beta \rightarrow n\pi} \frac{\frac{d}{d\beta} (\sin N\beta)}{\frac{d}{d\beta} (\sin \beta)} = \lim_{\beta \rightarrow n\pi} \frac{\cos N\beta \times N}{\cos \beta} = \pm N.$$

So resultant intensity is  $I = A_0^2 \frac{\sin^2 \alpha}{d^2} N^2$  which is very large as  $N$  is large. So the condition for principal maxima is

$$\beta = n\pi \quad \text{or} \quad \frac{ld}{2} = n\pi \quad \text{or} \quad \frac{ld}{\lambda} (\sin i + \sin \theta) \times \frac{d}{2} = n\pi$$

$$\therefore d(\sin i + \sin \theta) = n\lambda, \quad n=0, \pm 1, \pm 2, \dots$$

These are the position of maxima and the  $\pm$  sign indicates that there are two principal maxima of same order that lie on either side of  $0^{\text{th}}$  order maxima.

### Condition for Secondary Maxima & Minima:

$I_2 = \frac{\sin^2 N\beta}{\sin^2 \beta}$  will be extremum (minimum or maximum) when

$$\frac{dI_2}{d\beta} = 0 \quad \text{and} \quad \frac{d^2 I_2}{d\beta^2} = \begin{cases} \text{positive (minimum)} \\ \text{negative (maximum)} \end{cases}$$

$$\text{So } \ln I_2 = 2 \ln \frac{\sin N\beta}{\sin \beta} = 2(\ln \sin N\beta - \ln \sin \beta).$$

Differentiating,  $\frac{1}{I_2} \frac{dI_2}{d\beta} = 2(N \cot N\beta - \omega t \beta)$

$$\therefore \frac{dI_2}{d\beta} = 2I_2(N \cot N\beta - \cot \beta).$$

$$\text{and } \frac{d^2 I_2}{d\beta^2} = 2 \frac{dI_2}{d\beta} (N \cot N\beta - \cot \beta) + 2I_2(-N^2 \operatorname{cosec}^2 N\beta + \operatorname{cosec}^2 \beta)$$

$$= 2 \frac{dI_2}{d\beta} (N \cot N\beta - \cot \beta) + 2I_2(1 - N^2 + \cot^2 \beta - N^2 \cot^2 N\beta)$$

$\frac{dI_2}{d\beta} = 0$  when (a)  $I_2 = 0$  that means when  $\sin N\beta = 0$   
 but  $\sin \beta \neq 0$  so that the factor  $\frac{\sin N\beta}{\sin \beta}$  becomes zero. So for  
 minima,  $N\beta = m\pi$ ,  $m = \pm 1, \pm 2, \pm 3, \dots$

$$\therefore N \frac{2\pi}{\lambda} \frac{1}{2} d(\sin \beta \pm \sin \theta) = m\pi$$

$$\therefore d(\sin \beta \pm \sin \theta) = \frac{m\lambda}{N} \quad (\text{Condition for secondary minima})$$

Point to realize is for  $m = 0, N, 2N, \dots$   $\sin \beta$  is also zero so that we obtain principal maxima. There are  $(N-1)$  minima between two consecutive principal maxima.

(b) If however  $N \cot N\beta - \omega t \beta = 0$  then we obtain

$$\frac{d^2 I_2}{d\beta^2} < 0. \text{ So this condition yields the secondary maxima}$$

$$\text{so } N \cot N\beta - \omega t \beta = 0 \Rightarrow N \tan \beta = \tan N\beta.$$

$$\therefore N^2 \frac{\sin^2 \beta}{\cos^2 \beta} = \frac{N^2 \sin^2 N\beta}{N^2 \cos^2 N\beta} \quad \text{or} \quad \frac{\sin^2 N\beta}{\sin^2 \beta} = N^2 \frac{\cos^2 N\beta}{\cos^2 \beta}$$

$$\therefore \frac{\sin^2 N\beta}{\sin^2 \beta} = N^2 \frac{1}{\sec^2 N\beta \cos^2 \beta} = N^2 \frac{1}{(1 + \tan^2 N\beta) \cos^2 \beta} = \frac{N^2}{(1 + N^2 \tan^2 \beta) \cos^2 \beta}$$

$$= \frac{N^2}{\cos^2 \beta + N^2 \sin^2 \beta} = \frac{N^2}{1 + (N^2 - 1) \sin^2 \beta}. \quad (\text{as } N \tan \beta = \tan N\beta)$$

∴ Intensity of secondary maxima  $I_{sm} = I_1 \frac{N^2}{1 + (N^2 - 1) \sin^2 \beta}$

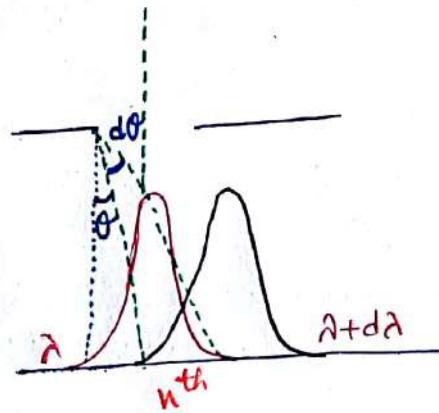
$$= \frac{I_{PM}}{1 + (N^2 - 1) \sin^2 \beta} \quad \text{where intensity of principal maxima} = N^2.$$

So as  $N$  increases,  $I_{sm} \ll I_{PM}$ , so secondary maxima will not be visible with a grating with 15000 lines/inch.

### Width of the Principal Maxima:

To calculate the width of the  $n^{th}$  principal maxima, let  $\theta$  and  $\theta + d\theta$  be the angles of diffraction for the peak of the  $n^{th}$  principal maxima & the first minima adjacent to it. So

the angular width of the principal maxima is  $2d\theta$ .



Now, the intensity of the ray diffracted at an angle  $\theta$  is

$$I = I_1 \frac{\sin^2 N\beta}{\sin^2 \beta} \quad \text{where } \beta = \frac{\pi d \sin \theta}{\lambda} \text{ for normal incidence.}$$

$$\text{so that the amplitude is } A = \sqrt{I_1} \frac{\sin(\frac{N\pi d \sin \theta}{\lambda})}{\sin(\frac{\pi d \sin \theta}{\lambda})}$$

The amplitude of the secondary minimum will be

$$\begin{aligned} A' &= \sqrt{I_1} \frac{\sin(\frac{N\pi d}{\lambda} \sin(\theta + d\theta))}{\sin(\frac{\pi d}{\lambda} \sin(\theta + d\theta))} \\ &= \sqrt{I_1} \frac{\sin[\frac{N\pi d}{\lambda} \sin \theta + \frac{N\pi d}{\lambda} \cos \theta d\theta]}{\sin[\frac{\pi d}{\lambda} \sin \theta + \frac{\pi d}{\lambda} \cos \theta d\theta]} \end{aligned}$$

$$\begin{aligned} [\sin d\theta &\approx d\theta] \\ \cos d\theta &\approx 1 \\ \text{to first approximation} \end{aligned}$$

Substituting the condition for the principal maxima  $d \sin \theta = n\lambda$ ,

$$A' = \sqrt{I_1} \frac{\sin[Nn\pi + Nn\pi \cot \theta d\theta]}{\sin[n\pi + n\pi \cot \theta d\theta]} = \sqrt{I_1} \frac{\sin(Nn\pi \cot \theta d\theta)}{\sin(n\pi \cot \theta d\theta)}$$

first secondary minima for a given  $\lambda$  could be obtained when  $A' = 0$ , i.e.  $Nn\pi \cot \theta d\theta = \pi$ .  $\therefore d\theta = \frac{1}{Nn \cot \theta}$

$\therefore$  The width of the principal maxima  $= 2d\theta = \frac{2}{Nn \cot \theta}$ .

Here, as  $\theta$  increases,  $\cot \theta$  decreases. Therefore the width of the principal maxima increases for higher order number.

CW A parallel beam of Sodium light is allowed to be incident normally on a plane grating having 4250 lines/cm and a 2<sup>nd</sup> order spectral line is observed to be deviated through 30°. Calculate the wavelength of the spectral line.

$$n = 2, \theta = 30^\circ, N = 4250, \text{ so } d = a+b = \frac{1}{4250} \text{ cm}$$

So from the condition of secondary maxima  $d \sin \theta = n\lambda$  we have

$$\frac{1}{4250} \sin 30^\circ = 2\lambda \quad \therefore \lambda = 5882 \times 10^{-8} \text{ cm} = 5882 \text{ Å}.$$

CW In a plane transmission grating the angle of diffraction for second order maxima for wavelength  $5 \times 10^{-5}$  cm is 30°. Calculate the number of lines in 1 cm of the grating surface.

If  $N$  lines/cm exists then Grating element  $d = a+b = \frac{1}{N}$  cm.

$$n = 2, \theta = 30^\circ, \lambda = 5 \times 10^{-5} \text{ cm}. \text{ From } d \sin \theta = n\lambda \text{ we have}$$

$$\frac{1}{N} \sin 30^\circ = 2 \times 5 \times 10^{-5} \quad \therefore N = 5000/\text{cm}.$$

CW A wire grating is made of 200 wires/cm placed at equal distance apart. The diameter of each wire is 0.025 mm. Calculate the angle of diffraction for the 3<sup>rd</sup> order spectrum and also find the absent spectra if any.  $\lambda = 6000 \text{ Å}$ .

Grating element  $d = a+b = \frac{1}{200} \text{ cm} = 0.005 \text{ cm}, n = 3$ .

$$\lambda = 6000 \text{ Å} = 6000 \times 10^{-8} \text{ cm}.$$

$$0.005 \sin\theta = 3 \times 6000 \times 10^{-8} \quad \text{or} \quad \theta = 2^\circ 4'$$

Now  $b$  = width of opacity = diameter of wire = 0.0025 cm.

So width of transparency  $a = d - b = 0.005 - 0.0025 = 0.0025$  cm.

Order of absent spectrum  $n = \frac{d}{a} = \frac{0.005}{0.0025} = 2$ . So the second order spectrum will be missing.

Q Show that in a diffraction grating with grating element  $1.5 \times 10^{-6}$  m and light of wavelength 550 nm, third and higher order principal maxima are not visible.

Grating element  $d = a+b = 1.5 \times 10^{-6}$  m,  $\lambda = 5500 \times 10^{-10}$  m.

As  $\sin 90^\circ = 1$  so the maximum angle of diffraction is  $90^\circ$  and let  $n$  be the maximum number of order of spectrum that can be observed.

So from  $d \sin\theta = n\lambda$ ,  $n = \frac{d}{\lambda} = \frac{1.5 \times 10^{-6}}{5500 \times 10^{-10}} = 2.72 < 3$ .

Thus, only 2<sup>nd</sup> order will be visible. No 3<sup>rd</sup> or higher order is possible.

Q If we use white light source with a diffraction grating, with 15000 lines/inch, show that the 2<sup>nd</sup> and 3<sup>rd</sup> order spectra overlap.

Given  $\lambda_{\text{violet}} = 4000 \text{\AA}$  and  $\lambda_{\text{red}} = 7000 \text{\AA}$  in the white light beam.

Grating element  $d = a+b = \frac{1}{15000}$  inch =  $\frac{2.54}{15000}$  cm =  $1.69 \times 10^{-6}$  m.

$\lambda_{\text{violet}} = 4000 \times 10^{-8}$  cm =  $4 \times 10^{-7}$  m.

$\lambda_{\text{red}} = 7000 \times 10^{-8}$  cm =  $7 \times 10^{-7}$  m.

In the 2<sup>nd</sup> order spectrum,  $\sin\theta_{\text{violet}}^{(2)} = \frac{2 \times \lambda_{\text{violet}}}{d} = \frac{2 \times 4 \times 10^{-7}}{1.69 \times 10^{-6}} = 0.1731$

So  $\theta_{\text{violet}}^{(2)} = 28.2^\circ$ .

and  $\sin\theta_{\text{red}}^{(2)} = \frac{2 \times \lambda_{\text{red}}}{d} = \frac{2 \times 7 \times 10^{-7}}{1.69 \times 10^{-6}} = 0.8284$ .  $\therefore \theta_{\text{red}}^{(2)} = 55.9^\circ$

In the 3<sup>rd</sup> order spectrum,  $\sin\theta_{\text{violet}}^{(3)} = \frac{3 \times \lambda_{\text{violet}}}{d} = \frac{3 \times 4 \times 10^{-7}}{1.69 \times 10^{-6}} = 0.71$

$$\therefore \theta_{\text{violet}}^{(3)} = 45.23^\circ$$

and  $\sin \theta_{\text{red}}^{(3)} = \frac{3 \times 7 \times 10^{-7}}{1.69 \times 10^{-6}} = 1.2426 > 1$  (impossible as  $\sin 90^\circ = 1$ )

As  $\theta_{\text{red}}^{(2)} > \theta_{\text{violet}}^{(3)}$ , so the 2<sup>nd</sup> & 3<sup>rd</sup> order spectra will overlap. And as  $\theta_{\text{red}}^{(3)} > 90^\circ$ , so 3<sup>rd</sup> order spectrum of red light will not be observed.

Q A diffraction grating used at normal incidence gives a line ( $5400\text{\AA}$ ) in certain order superimposed on another line ( $4050\text{\AA}$ ) of ( $5400\text{\AA}$ ) in the next higher order. If the angle of diffraction is  $30^\circ$ , how many lines/cm are there on the grating?

$$\lambda_1 = 5400\text{\AA} = 5400 \times 10^{-10} \text{ m}, \quad \lambda_2 = 4050\text{\AA} = 4050 \times 10^{-10} \text{ m.}$$

$\theta_1 = \theta_2 = 30^\circ, n = ?$ . Two lines of  $n^{\text{th}}$  &  $(n+1)^{\text{th}}$  order superpose, so  $d \sin \theta_1 = n \lambda_1, d \sin \theta_2 = (n+1) \lambda_2$

$$\therefore \frac{n+1}{n} = \frac{\lambda_1}{\lambda_2} = \frac{540}{405}. \quad \therefore n = 3.$$

Then from  $d \sin 30^\circ = 3 \times 5400 \times 10^{-10}$  we obtain  $d = 324 \times 10^{-8} \text{ m}^{-8}$

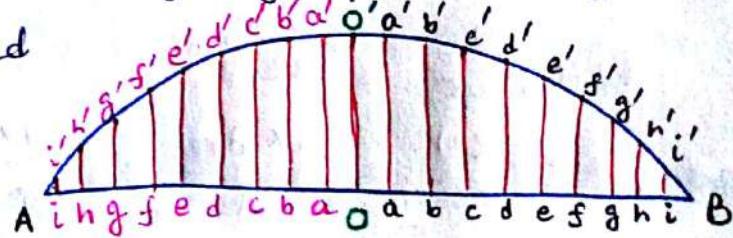
$$\therefore \text{Number of lines/cm} = \frac{1}{d} = \frac{1}{324 \times 10^{-8} \times 10^{-2}} / \text{cm} = 3086 / \text{cm.}$$

### Rowland's Concave Reflection Grating

In 1882, Henry Rowland employed a grating ruled on a concave polished spherical mirror usually of speculum metal (alloy of tin & copper), with lines equally spaced along the chord AB but unequally spaced along the arc.

$$oa = ab = bc = cd = \dots$$

$$o'a' \neq a'b' \neq b'c' \neq c'd' \neq \dots$$



It produces diffraction & focuses the spectrum without using an additional lens. The spectrum is thus free from chromatic & other aberrations. Diffraction at UV range can also be achieved in this grating which is usually absorbed by glass. The polystrip between the rulings because of their small width offer limitations to the incident beam & thus causing it diffract in all directions after reflection.

### Advantages of Concave reflecting grating over Plane transmission grating

- (i) In performing an experiment with a plane transmission grating we require two convergent lenses to form the spectra. But in case of concave grating, the concave surface itself serves the purpose of the convergent lens. So chromatic aberrations can be eliminated.
- (ii) Glass lenses are unsuitable for the investigation of UV and IR rays. But in a concave grating there is no such restriction.
- (iii) In plane transmission grating, part of light energy is absorbed by lenses and the grating plate itself, so the received intensity of spectrum is low. Because of absence of such absorption in concave grating, the spectrum is more intense.
- (iv) In a "normal spectrum" the distance between two lines is proportional to the wavelength difference. Plane transmission grating spectrum is not normal but concave reflection grating spectrum is normal.
- (v) It is difficult to rule more than 2500 lines/inch in case of a plane transmission grating as with very small wavelengths, the spectra is very close to the central maximum. This is mostly solved in concave grating.

## Resolving Power and Dispersive Power

Magnifying Power: It is defined as the ratio of the angle subtended by the image at the eye, as seen through the instrument to the angle subtended by the object at the centre of the unaided eye, when the image and the object both lie at the same distance from the eye. This distance for a microscope is the distance of distinct vision and for a telescope it is infinity.

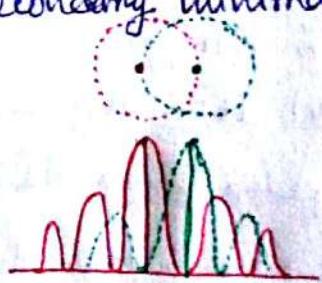
Angular Dispersive Power: It is defined as the change in the angle of diffraction corresponding to a unit change in the wavelength.  $\frac{d\theta}{d\lambda}$ . For the principal maximum in the  $n^{\text{th}}$  order

for a diffraction grating  $d \sin \theta = n\lambda$ .

Differentiating,  $d \cos \theta d\theta = n d\lambda \Rightarrow \frac{d\theta}{d\lambda} = \frac{n}{d \cos \theta}$ . So the dispersive power of a grating increases with (i) order of spectrum, (ii) deereement of grating element or with increase in number of lines/cm.

Resolving Power: It is defined as the ratio of the wavelength of a line in the spectrum to the least difference in the wavelength of the next line that can just be seen as separate  $\frac{\lambda}{d\lambda}$ . It is also defined as the reciprocal of the smallest angle subtended at the objective by two point objects which can just be distinguished as separate. Two lines of wavelength  $\lambda$  &  $\lambda + d\lambda$  are resolved if primary maxima of  $\lambda$  falls on the first secondary minima of the other. → Rayleigh's Criteria.

So the least angular separation between the principal maxima of the diffraction pattern of



The two spectral lines in a given diffraction order should be equal to half the angular width of either principal maximum.

$$\text{Resolving power} = \frac{\lambda}{d\lambda}$$

### Resolving Power of Plane Transmission Grating

for normal incidence,  $n^{\text{th}}$  order principal maxima for a plane diffraction grating is  $d \sin \theta = n\lambda$  — ① If the incident light contains two wavelength  $\lambda$  &  $\lambda + d\lambda$  then according to Rayleigh's criteria, the two lines will be separated when the peak of  $n^{\text{th}}$  principal maxima of  $\lambda + d\lambda$  falls on the first minima of  $\lambda$  after  $n^{\text{th}}$  principal maxima of  $\lambda$ . The increase in the angle of diffraction for the  $n^{\text{th}}$  principal maxima of  $\lambda + d\lambda$  can be obtained by differentiating the condition of principal maxima,

$$d \cos \theta d\theta = n d\lambda \quad \text{— ②} \quad \text{But } d\theta = \frac{1}{Nn \cot \theta} \quad \text{— ③}$$

$$\text{Dividing ② by ③, } \cot \theta d\theta = \frac{d\lambda}{\lambda}$$

$$\frac{d\theta}{d\lambda} = \frac{n}{d \cos \theta}$$

$$\text{So resolving power} = \frac{\lambda}{d\lambda} = \frac{1}{\cot \theta d\theta} = Nn$$

$$\frac{\lambda}{d\lambda} = Nn$$

$N$  = Total number of rulings in the grating

$n$  = Order number of the spectrum.

If  $N' = \text{number of lines/cm of grating surface}$  then  $d = \frac{1}{N'}$ ,

$$\text{So } \frac{\lambda}{d\lambda} = NN = \frac{ds \sin \theta}{\lambda} N = \frac{N}{N'} \frac{\sin \theta}{\lambda}$$

Thus the change in number of lines on the grating will not increase the resolving power. But angular dispersive power

$\frac{d\theta}{d\lambda} = \frac{n}{\cos\theta} = \frac{NN'}{\cos\theta}$ . So dispersive power varies proportionately with number of lines in a grating. Therefore a grating having higher dispersive power than another does not necessarily has a higher resolving power.

$$\text{Resolving power of telescope} = \frac{1}{d\theta} = \frac{d}{1.22\lambda}$$

$d$  = Diameter of the objective of telescope.

CW A telescope has an objective of diameter 2.54 m. Assuming the mean wavelength of light to be  $5.5 \times 10^{-5}$  cm, estimate the smallest angular separation of two stars which can be resolved.

$$d = 2.54 \text{ m}, \lambda = 5.5 \times 10^{-5} \text{ cm} = 5.5 \times 10^{-7} \text{ m.}$$

$$\text{So angular separation } d\theta = \frac{1.22 \times 5.5 \times 10^{-7}}{2.54} = 2.642 \times 10^{-7} \text{ rad.}$$

CW and R.P. =  $\frac{1}{d\theta} = 3.785 \times 10^6$

A spectrometer having a very large circular scale reading upto 1 second is fitted with a telescope of 1 inch objective. Is it useful to resolve a Sodium source?

$$d = 1 \text{ inch} = 2.54 \text{ cm}, \lambda (\text{sodium D1-D2 lines})$$

$$= 5893 \text{ Å} = 5893 \times 10^{-8} \text{ cm.}$$

$$\therefore d\theta = \frac{1.22\lambda}{d} = \frac{1.22 \times 5893 \times 10^{-8}}{2.54} \text{ rad} = \frac{1.22 \times 5893 \times 10^{-8} \times 180 \times 3600}{2.54 \pi} \text{ sec}$$

= 5.836 sec. Since the angular resolution  $\sim 6$  sec so having a scale reading upto 1 sec is of no use. If we have to use this spectroscope then the aperture of telescope objective should be increased so that  $d\theta \sim 1$  sec. So  $d = 6 \text{ inch} = 0.15 \text{ m.}$

CQ Calculate the angular dispersion for a diffraction grating having 14438 lines/inch when used in the 3<sup>rd</sup> order at 4200 Å.

$$n = 3, \lambda = 4200 \times 10^{-10} \text{ m.} \quad \text{and} \quad d = a + b = \frac{2.54 \times 10^{-2}}{14438} = 0.1759 \times 10^{-5} \text{ m}$$

Angular dispersion power  $\frac{d\theta}{d\lambda} = \frac{n}{d \cos \theta}$ . from the condition of principal maxima,  $d \sin \theta = n\lambda$ ,  $\cos \theta = \sqrt{1 - \frac{n^2 \lambda^2}{d^2}} = \sqrt{1 - \left(\frac{3 \times 4200 \times 10^{-10}}{0.1759 \times 10^{-5}}\right)^2} = 0.6978$ .

$$\text{So angular dispersion } d\theta = \frac{n d\lambda}{d \cos \theta} = \frac{3 \times 10^{-10}}{0.1759 \times 10^{-5} \times 0.6978} \text{ rad} \\ = 0.014^\circ \quad (\text{converted as } 180^\circ = \pi \text{ rad})$$

Q2 In a 2<sup>nd</sup> order spectrum of a plane diffraction grating, a certain spectral line appears at an angle of  $10^\circ$ , while another of wavelength  $0.5 \times 10^{-10} \text{ m}$  greater appears at an angle 3" greater. Find the wavelength of the lines & the minimum grating width required to resolve them.

Given  $\sin 10^\circ = 0.1736$  &  $\cos 10^\circ = 0.9848$ .

$$n = 2, d\lambda = 0.5 \times 10^{-10} \text{ m}, d\theta = 3'' = \frac{3\pi}{180 \times 3600} = 0.1455 \times 10^{-4} \text{ rad}$$

$$\text{Now } \frac{d\theta}{d\lambda} = \frac{0.1455 \times 10^{-4}}{0.5 \times 10^{-10}} = \frac{n}{d \cos \theta} = \frac{2}{d \cos 10^\circ} = \frac{2}{d \times 0.9848}$$

$$\text{or } d = 0.6979 \times 10^{-5} \text{ m}$$

If  $\lambda_1$  is the wavelength of the first line, then  $d \sin \theta = n\lambda$ ,

$$\lambda_1 = \frac{0.6979 \times 10^{-5} \times 0.1736}{2} = \underline{\underline{6057.77 \text{ \AA}}}$$

$$\text{and } \lambda_2 = \lambda_1 + d\lambda = 6057.77 + 0.5 = \underline{\underline{6058.27 \text{ \AA}}}$$

$$\text{As resolving power } \frac{\lambda}{d\lambda} = nN, \text{ so } N = \frac{\lambda}{n d\lambda} = \frac{6057.77}{2 \times 0.5} = 6058$$

$$\text{Grating width} = Nd = 6058 \times 0.6979 \times 10^{-5} = 0.04228 \text{ m} \\ = \underline{\underline{4.228 \text{ cm}}}$$

CW A diffraction grating which has 4000 lines to a cm is used at normal incidence. Calculate the dispersive power of the grating in the 3<sup>rd</sup> order spectrum in the wavelength region 5000 Å.

$$n = 3, \quad N = 4000 \text{ lines/cm}, \quad \lambda = 5000 \text{ Å}$$

$$\therefore d = a + b = \frac{1}{4000} = 2.5 \times 10^{-4} \text{ cm}$$

$$\text{So from } ds \sin \theta = n\lambda, \text{ we get } \sin \theta = \frac{3 \times 5000 \times 10^{-8}}{2.5 \times 10^{-4}} = 0.6.$$

$$\text{So dispersive power } \frac{d\theta}{d\lambda} = \frac{nN}{\cos \theta} = \frac{3 \times 4000}{\sqrt{1 - (0.6)^2}} = 15000.$$

CW find the number of lines a grating should have to resolve the 1<sup>st</sup> order doublet having a wavelength difference 0.6 nm at 589.3 nm.

$$n=1, \quad d\lambda = 0.6 \text{ nm} = 0.6 \times 10^{-9} \text{ m}, \quad \lambda = 589.3 \times 10^{-9} \text{ m.}$$

If  $N$  is the no. of lines on grating to resolve the doublet then

$$\text{from } \frac{\lambda}{d\lambda} = N \quad \Rightarrow \quad N = \frac{\lambda}{d\lambda} = \frac{589.3 \times 10^{-9}}{0.6 \times 10^{-9}} = 982.$$

### Difference between Grating and Prism Spectrum

#### Prism Spectrum

1. There is only one spectrum.

2. Deviation of ray depends on angle of prism, refractive index of material &  $\lambda$ . It is more for violet than for red light.

3. Dispersive power of prism is more for violet than red, so violet is less intense than red.

#### Grating spectrum

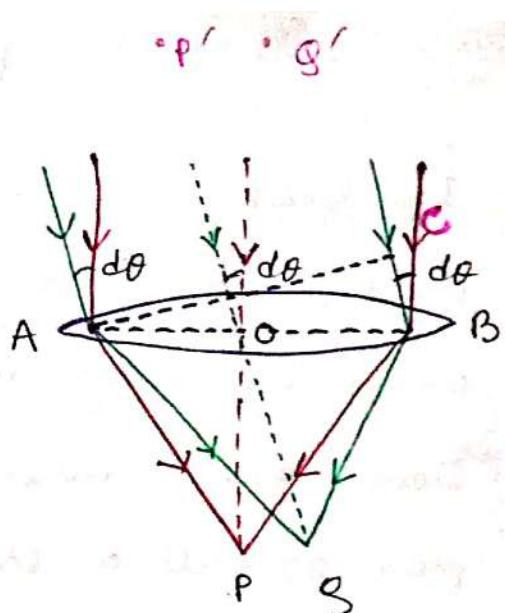
1. There are a number of spectra on both side of central maxima.

2. Deviation of ray is directly proportional to  $\lambda$  & inversely proportional to grating element & independent of r.i. of grating material.

3. Dispersive power of grating is  $\frac{n}{d \cos \theta}$ . As  $\theta \ll$ , so angular separation is small between two lines in a particular order of spectrum, but  $\propto N$  and decreases with  $d \gg$ .

## Resolving Power of a Telescope

Consider two points  $p'$  and  $q'$  on the object plane of a telescope with aperture  $d$  with an angular separation  $d\theta$ . Light from  $p'$  comes normally in parallel rays to form image at focal plane so that  $AP = BP$ . If point



$q'$  has to be resolved, then according to "Rayleigh's criterion", the first maximum of image  $q$  should fall on the first secondary minimum of image  $p$  - meaning, path difference between  $q$  &  $p$  is  $\frac{\lambda}{2}$ .

$$\text{Now } BC = AB d\theta \quad \Rightarrow \quad \frac{\lambda}{2} = d d\theta \quad \Rightarrow \quad d\theta = \frac{\lambda}{d}$$

Using Bessel's functions, Airy showed that for a circular aperture  $\lambda$  should be replaced by  $1.22\lambda$ . so that resolving power of a telescope  $= \left(\frac{1.22\lambda}{d}\right)^{-1} = \frac{d}{1.22\lambda}$  as resolving power of telescope  $= \frac{1}{d\theta}$ .

## HOLOGRAPHY

In Greek, holography means whole writing. A regular photograph is a two dimensional snapshot of a 3D object whose intensity distribution projected on a plane is recorded.

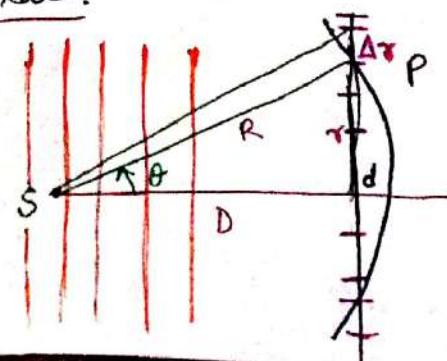
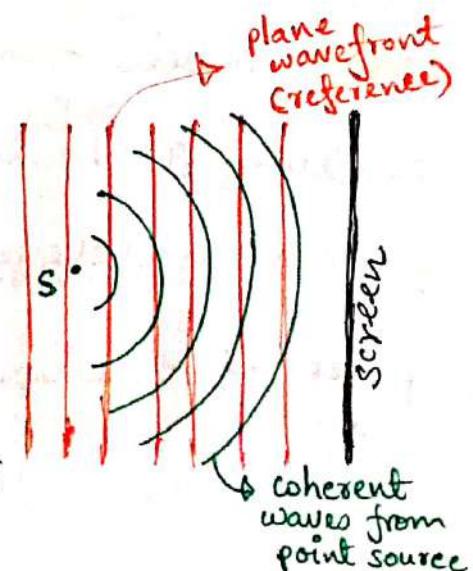
Denis Gabor invented in 1947 a technique in which the lost phase as well as the 3D nature of intensity is recorded. It is a two step process by which (1) an object illuminated by coherent light makes interference fringes in a photographic emulsion & (2) reillumination of the developed interference pattern by light of same wavelength to produce 3D image.

Coherent source (Laser) was developed in 1960 & Gabor received Nobel prize in Physics in 1971 for 3D lensless method of photography.

### Basic Principles of Holography

Consider the interference pattern caused by coherent monochromatic plane waves incident on a point scatterer S. Concentric bright & dark circles will be formed by superposition of scattered light & reference beam. The developed screen (plate) contain light & dark partially absorbing fringe pattern, called "Gabor zone plate".

Since the reference beam is assumed to be in constant phase across the hologram plate fringes at any point P will be separated by  $\Delta r$  that correspond to path difference of  $\lambda$ .



$$\Delta r \sin\theta = \lambda.$$

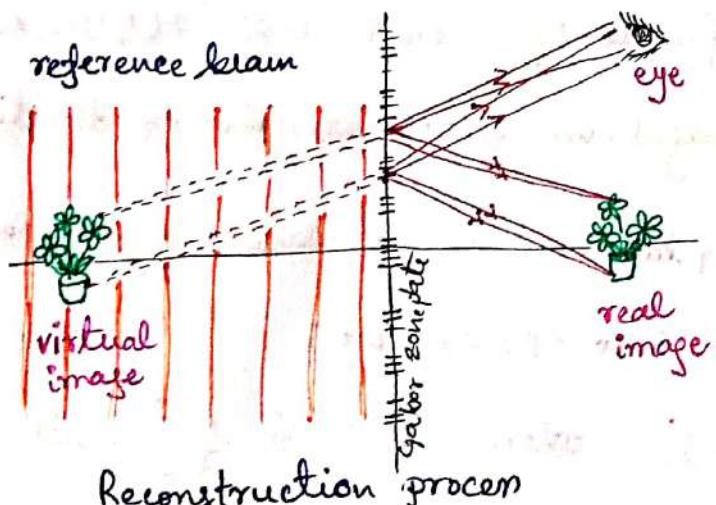
$$d = R - D$$

$$d = n\lambda = \frac{\lambda^2}{2R - D}$$

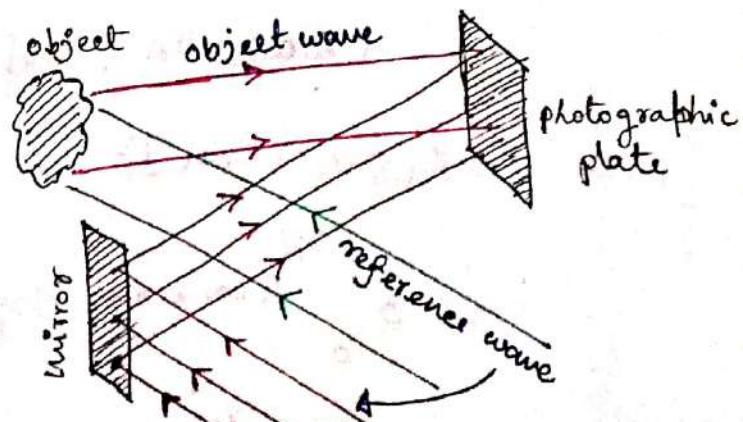
This plate is then illuminated by plane coherent waves in the absence of scatterer S. Light formed by interference between light and dark bands will now produce first order interference maxima at O that corresponds to  $\Delta r \sin\theta = \lambda$ . This light will appear to diverge from S, so a virtual image can be seen from the right side of hologram.

Suppose now two scattering centers  $S_1, S_2$  are present on left & each will record a Gabor zone plate on plate. The reconstruction will produce a virtual image of both scatterers. Extending this argument to continuum of scattering centres, hologram will consist of superposed Gabor zone plates. Upon reconstruction,

the distributed virtual image appears like the real image. This is called the on-axis holography.



In 1962, Leith & Upatneik developed off-axis hologram. Due to increased coherence length of the laser beam, the reference & scattered beam were separated so that hologram could be illuminated from either side & not necessarily the reference beam is plain.



## Point Holography - Basic Theory

Recording of the phase distribution of an object wave which is a point scatterer of form  $\frac{A}{r} \cos(kr - wt + \phi)$  is the basic problem.

Recording process : Suppose object wave is  $O(x, y) = a(x, y) \times \cos[\phi(x, y) - wt]$

due to all point scatterers at the object in plane of photographic plate at  $z=0$ . Suppose reference wave is plane wave propagating in  $xz$  plane inclined at  $\theta$  with  $z$  axis. So,

$$r(x, y, z) = A \cos(\bar{k} \cdot \bar{r} - wt) = A \cos(kx \sin\theta + kz \cos\theta - wt)$$

$$\text{So this field at } z=0 \text{ is } r(x, y) = A \cos(kx \sin\theta - wt) \\ = A \cos(2\pi\alpha x - wt)$$

where  $\alpha = \frac{\sin\theta}{\lambda}$  = spatial frequency. Note that  $r(x, y) = r(x)$  no  $y$  dependence because its propagation vector was in  $xz$  direction.

$\therefore$  Total field at photographic plate = object wave + reference wave

$$u(x, y, t) = a(x, y) \cos[\phi(x, y) - wt] + A \cos(2\pi\alpha x - wt)$$

$$\therefore I(x, y) = \langle u^*(x, y, t) u(x, y, t) \rangle \text{ where } \langle \rangle \text{ is } \frac{1}{T} \int_0^T \langle \rangle dt.$$

$$= \langle [a(x, y) \cos[\phi(x, y) - wt] + A \cos(2\pi\alpha x - wt)]^2 \rangle$$

$$= a^2(x, y) \langle \cos^2[\phi(x, y) - wt] \rangle + A^2 \langle \cos^2(2\pi\alpha x - wt) \rangle$$

$$+ 2a(x, y)A \langle \cos[\phi(x, y) - wt] \cos(2\pi\alpha x - wt) \rangle$$

$$[ \text{Now } \langle \cos^2[\phi(x, y) - wt] \rangle = \frac{1}{2\pi} \int_0^{2\pi} \cos^2(wt - \phi(x, y)) d(wt) ]$$

$$= \frac{1}{2\pi} \frac{1}{2} \int_0^{2\pi} 2 \cos^2 x dx = \frac{1}{4\pi} \int_0^{2\pi} (1 + \cos 2x) dx = \frac{1}{2} . ]$$

$$\langle \cos[\phi(x,y) - wt] \cos(2\pi dx - wt) \rangle$$

$$= \frac{1}{2} \langle \cos[\phi(x,y) - 2wt + 2\pi dx] \rangle + \frac{1}{2} \langle \cos[\phi(x,y) - 2\pi dx] \rangle$$

$$= \frac{1}{2} \langle \cos[\phi(x,y) - 2\pi dx] \rangle = \frac{1}{2} \cos[\phi(x,y) - 2\pi dx]$$

$$\text{So } I(x,y) = \frac{1}{2} a^2(x,y) + \frac{1}{2} A^2 + A a(x,y) \cos[\phi(x,y) - 2\pi dx].$$

↳ phase of object wave

↳ Intensity pattern

Reconstruction process: By suitable process, the photo plate is developed to obtain hologram such that transmittance is linearly related to  $I(x,y)$ . So if  $R(x,y)$  represents the reconstruction wave (identical to the reference wave  $r(x,y)$ ), then

$$r(x,y)|_{z=0} = R(x,y) I(x,y) = \left[ \frac{1}{2} a^2(x,y) + \frac{1}{2} A^2 \right] R(x,y) + A a(x,y) R(x,y) \cos[\phi(x,y) - 2\pi dx]$$

$R(x,y) = r(x,y)$

↓ transmitted field      ↓ reconstructed wave

$$= \left[ \frac{1}{2} a^2(x,y) + \frac{1}{2} A^2 \right] A \cos(2\pi dx - wt) + A^2 a(x,y) \cos[2\pi dx - wt] \overset{\cos}{\underset{(\phi(x,y) - 2\pi dx)}{\cos}}$$

$$= \left[ \frac{1}{2} a^2(x,y) + \frac{1}{2} A^2 \right] A \cos(2\pi dx - wt) + \frac{1}{2} A^2 a(x,y) \cos[\phi(x,y) - wt] \quad (2)$$

①

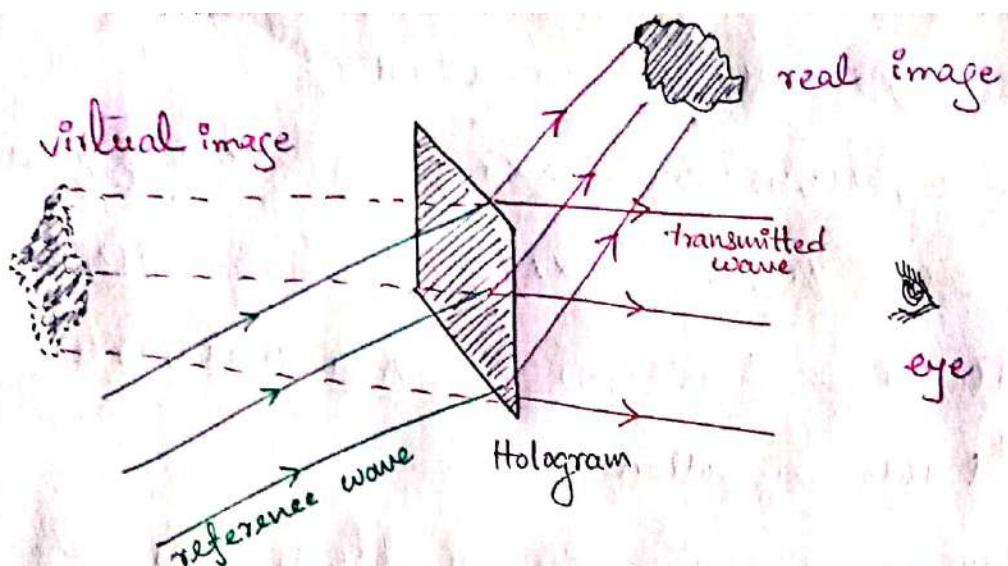
$$+ \frac{1}{2} A^2 a(x,y) \cos[4\pi dx - \phi(x,y) - wt] \quad (3)$$

①  $\Rightarrow$  Reconstruction wave with modulated amplitude due to  $a^2(x,y)$ .

So this part of total field is travelling in the direction of reconstructed wave.

②  $\Rightarrow$  Object wave  $O(x,y)$  that gives a virtual image. The reconstructed object wave travels in same direction as original object wave.

③  $\Rightarrow$  Notice that identical to  $O(x,y)$  in addition to  $4\pi dx$  term phase term  $\phi(x,y)$  has a negative sign, meaning this wave has opposite curvature than the object wave, so if object wave is diverging out then this wave is converging in spherical wave.



Effect of  $4\pi dx$  is understood as follows. If we consider object wave is plane wave propagating along  $z$  direction then  $\phi(x,y)=0$

$$\text{So } \frac{1}{2} A^2 \alpha(x,y) \cos[4\pi dx - wt] \sim \cos(k'x - wt)$$

where previously  $ksin\theta = 2\pi d$ . so that

$$2ksin\theta = 4\pi d = k' = k'sin\theta'$$

$$\text{or } \theta' = \sin^{-1}(2sin\theta)$$

So ③ represents a plane propagating wave along direction  $\theta'$ , so  $4\pi dx$  rotates the wave direction. so this represents the conjugate of object wave propagating along a different direction from reconstruction wave & object wave. All ①, ②, ③ propagate in three different directions, they separate after travelling a distance so that observer can view the virtual image without any disturbance.