Sem-III - Thermal Physics

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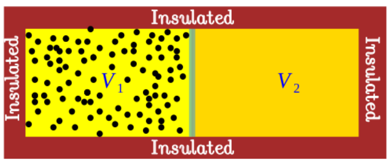
Assignment I: Thermodynamic Systems & 0^{th} law

Submission due date: 31/10/2023

Q.1) (a) Consider a thermally insulated and partitioned container (as shown) of volume $V_1 + V_2$ in which an ideal gas at temperature T_1 is confined to V_1 and V_2 remained empty.



A small hole is made in the partition (equivalently partition is then removed), such that the gas expands to fill the entire container. What is the final temperature of the gas?

(b) For an ideal gas with ambient gas pressure P and the corresponding gas density ρ, we know $P \propto \rho$ holds. The speed of longitudinal waves of small amplitude is defined as $c_s = \sqrt{dP/d\rho}$. Show that the speed of sound in a gas for which the compressions and rarefactions are isothermal is $c_s^I = \sqrt{RT/M}$ and that in adibatic is $c_s^A = \sqrt{\gamma RT/M}$ where $\gamma(= C_P/C_V)$ is the ratio of heat capacities at constant pressure and volume, M is the molecular weight, R is the universal gas constant and T is the absolute temperature.

Q.2) Consider two systems A and B with heat capacities C_A and C_B interact thermally to settle at a temperature T_f. The total energy of the combined system remains constant. Show that if the initial temperature of system A was T_A, then the initial temperature of system B was

$$T_B = \frac{C_A}{C_B}(T_f - T_A) + T_f \, .$$

Q.3) (a) The equation of state of an ideal gas is $PV = RT$. Show that $\beta = T^{-1}$ and $\kappa = P^{-1}$.
(b) The equation of state of a real gas at moderate pressure is $P(V - b) = RT$. Show that

$$\beta = T^{-1}/\{1 + bP/RT\}, \text{ and } \kappa = P^{-1}/\{1 + bP/RT\} \, .$$

(c) The equation of state of a real gas at moderate pressure is $PV = RT(1 + B/V)$ with $B = B(T)$. Show that

$$\beta = T^{-1}\{V + B + T(dB/dT)\}/\{V + 2B\}, \text{ and } \kappa = P^{-1}/\{1 + BRT/PV^2\} \, .$$

Q.4) Systems $A, B,$ and C are gases with coordinates $(P, V), (P', V'), (P'', V'')$. When A and C are in thermal equilibrium, the equation

$$PV - nbP - P''V'' = 0$$

is found to be satisfied. When B and C are in thermal equilibrium, the relation

$$P'V' - P''V'' + \frac{nB'P''V''}{V'} = 0$$

holds. The symbols n, b and B' are constants. **(a)** What are the three functions which are equal to one another at thermal equilibrium and each of which is equal to an empirical temperature T? **(b)** What is the relation expressing thermal equilibrium between A and B?

Q.5) Systems A and B are paramagnetic salts with coordinates $\mathcal{H}, M$ and $\mathcal{H}', M'$ respectively. System C is a gas with coordinates P, V. When A and C are in thermal equilibrium, the equation

$$4\pi nRC_c\mathcal{H} - MPV = 0$$

is found to hold. When B and C are in thermal equilibrium, we get

$$nR\Theta M' + 4\pi nRC_c'\mathcal{H}' - M'PV = 0,$$

where n, R, C_c, C_c' and Θ are constants. **(a)** What are the three functions that are equal to one another at thermal equilibrium? **(b)** Set each of these functions equal to the ideal-gas temperature T and see whether any of these equations are equation of state for paramagnetic substance (Curie's law $M = C_c'\frac{\mathcal{H}}{T}$).

Q.6) The equation of state of an ideal elastic substance is $\Im = KT(\frac{L}{L_0} - \frac{L_0^2}{L^2})$ where K is a constant and zero tension value of $L = L_0(T)$. **(a)** Show that the isothermal Young's modulus Y and at zero tension Y_0 are given by

$$Y = \frac{kT}{A}\Big(\frac{L}{L_0} + \frac{2L_0^2}{L^2}\Big),\ Y_0 = \frac{3KT}{A}.$$

(b) Show that the linear expansivity is given by

$$\alpha = \alpha_0 - \frac{\Im}{AYT} = \alpha_0 - \frac{L^3/L_0^3 - 1}{T(L^3/L_0^3 + 2)},$$

where $\alpha_0 = \frac{1}{L_0}\frac{dL_0}{dT}$ is the linear expansivity at zero tension.