1. Using 1^{st} law $ds = \frac{dU + PdV}{T}$ and $U = U(V,T)$

$dU = \left(\frac{\partial U}{\partial V}\right)_T dV + \left(\frac{\partial U}{\partial T}\right)_V dT$ we get $ds = \left\{\frac{P}{T} + \frac{1}{T}\left(\frac{\partial U}{\partial V}\right)_T\right\} dV + \frac{1}{T}\left(\frac{\partial U}{\partial T}\right)_V dT$

Now $U = U(V,T) = BT^n \ln(V/V_0) + f(T)$

$$ds = \left(\frac{AT^2}{V} + \frac{V_0 BT^{n-1}}{V}\right) dV + \left[\frac{f'(T)}{T} + nBT^{n-2} \ln\frac{V}{V_0}\right] dT$$

for ds to be exact differential, we have

$$\frac{\partial}{\partial T}\left(\frac{V_0 BT^{n-1} + AT^2}{V}\right) = \frac{\partial}{\partial V}\left(\frac{f'(T)}{T} + nBT^{n-2}\ln\frac{V}{V_0}\right)$$

Entropy

$$\frac{V_0 B(n-1)T^{n-2}}{\cancel{V}} + \frac{2AT}{\cancel{V}} = nBT^{n-2}\frac{V_0}{\cancel{V}}$$

$$\cancel{V_0 nBT^{n-2}} - BT^{n-2} + 2AT = \cancel{V_0 nBT^{n-2}} \quad \therefore \quad 2AT = BT^{n-2}$$

$\therefore$ $B = 2A$, $n = 3$.

2. $$\Delta S_{universe} = \int_{T_1}^{T_c} \frac{C_p dT}{T} + \int_{T_2}^{T_c} \frac{C_p dT}{T} = C_p \ln\frac{T_c^2}{T_1 T_2} \geq 0$$

$\therefore$ $T_c^2 \geq T_1 T_2$ or $T_c \geq \sqrt{T_1 T_2}$.

Maximum work can be obtained using reversible engine $\Delta S = 0$.

$$W_{max} = C_p(T_1 + T_2 - 2T_c^{min}) = C_p(T_1 + T_2 - 2\sqrt{T_1 T_2})$$
$$= C_p(\sqrt{T_1} - \sqrt{T_2})^2$$

① 1^{st} case, $\eta = 30\% = 0.3$, $T_c = 300K$, $T_H = ?$

$\eta = 1 - \frac{T_c}{T_H}$ or $0.3 = 1 - \frac{300}{T_H}$ or $T_H = 428.57\ K$

2^{nd} case, $\eta' = 50\% = 0.5$, $T_c' = 300K$, $T_H' = ?$

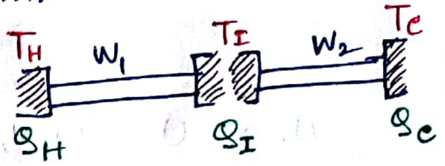
$\eta' = 1 - \frac{T_c'}{T_H'}$ or $0.5 = 1 - \frac{300}{T_H'}$ or $T_H' = 600K$

Carnot

Increase in temperature of source $= 600 - 428.57 = 171.43\ K$

(2) $T_H = 600K$, $T_c = 300K$ $\therefore \eta = 1 - \frac{T_c}{T_H} = 1 - \frac{300}{600} = 0.5 = 50\%$

According to Carnot's theorem, the maximum efficiency is 50% but the claim is 52%. So its not a valid claim.



(3) (i) when work output is equal,

$$W_1 = Q_H - Q_I, \quad W_2 = Q_I - Q_c$$

$$\Rightarrow Q_H - Q_I = Q_I - Q_c \Rightarrow \frac{Q_H}{Q_I} - 1 = 1 - \frac{Q_c}{Q_I}$$

And $\frac{Q_H}{Q_I} = \frac{T_H}{T_I} = \frac{1200}{T_I}$, $\frac{Q_I}{Q_c} = \frac{T_I}{T_c} = \frac{T_I}{300}$

$$\therefore \frac{1200}{T_I} - 1 = 1 - \frac{300}{T_I} \Rightarrow T_I = 750K$$

(ii) when efficiencies are equal, $\eta_1 = 1 - \frac{Q_I}{Q_H}$, $\eta_2 = 1 - \frac{Q_c}{Q_I}$

$$\therefore 1 - \frac{Q_I}{Q_H} = 1 - \frac{Q_c}{Q_I} \Rightarrow 1 - \frac{T_I}{T_H} = 1 - \frac{T_c}{T_I} \Rightarrow T_I^2 = T_c T_H$$

$$\Rightarrow T_I = \sqrt{T_c T_H} = 600K.$$

(4) Here $T_1 = 300K$, $T_2 = 273K$.

Points to remember,
80 cal 1gm ice melting.
80 cal 1gm water freezing
540 cal 1gm water vapourization

$Q_1 = ?$, $Q_2 = 1000 \times 80 = 8 \times 10^4$ cal.

Heat rejected to room $\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \Rightarrow Q_1 = \frac{T_1}{T_2} Q_2 = \frac{300}{273} \times 8 \times 10^4$

$$= 8.79 \times 10^4 \text{ cal.}$$

Work done by Refrigerator $W = Q_1 - Q_2$

$$= (8.79 \times 10^4 - 8 \times 10^4) \times 4.2J = 3.18 \times 10^4 J$$

Coefficient of Performance $P = \frac{Q_2}{Q_1 - Q_2} = \frac{8 \times 10^4}{(8.79 - 8) \times 10^4} = 10.13$