Q. 3.17. Given the half life of radioactive K^{40} is 18.3×10^8 years, calculate the number of β-particle emitted per second per kg. *(Bang. U. 1994)*

Ans. Given half life $T = 18.3 \times 10^8$ years $= 18.3 \times 10^8 \times 365 \times 24 \times 60 \times 60$

$= 5.77 \times 10^{16}$ sec.

Radioactive constant $\lambda = \dfrac{0.6931}{T} = \dfrac{0.6931}{5.77 \times 10^{16}} = 1.2 \times 10^{-17}$ sec^{-1}

If N_0 is the initial number of nuclei and N the number remaining after a time t, then Number of atoms decaying during this period

$$\Delta N = N_0 - N = N_0 - N_0 e^{-\lambda t}$$

But λt being a very small quantity $e^{-\lambda t} = 1 - \lambda t$

$$\therefore \quad \Delta N = N_0 - N_0(1 - \lambda t) = N_0 \lambda t$$

$$N_0 = \text{number of atoms in 1 kg of } K^{40} = \frac{6.023 \times 10^{26}}{40}$$

$$= 1.5 \times 10^{25}$$

$$\therefore \quad \Delta N = 1.5 \times 10^{25} \times 1.2 \times 10^{-17} = 1.8 \times 10^8$$

$\therefore$ Number of β-particles emitted per second $= 1.8 \times 10^8$.

Q. 3.18. Natural carbon is 18% of human body weight. The activity of ^{14}C in a person weighing 70 kg is 0.1 micro-curie. What fraction of carbon in the body is ^{14}C? Given one currie is 3.7×10^{10} nuclei disintegration per second and half life of ^{14}C = 5730 years.

Ans. Activity $R = \dfrac{dN}{dt} = -\lambda N$

Half life $T = \dfrac{0.6931}{\lambda} = 5730 \times 365 \times 24 \times 60 \times 60$ sec.

$$\therefore \quad \lambda = \frac{R}{N} = \frac{0.6931}{T} = \frac{0.6931}{5730 \times 365 \times 24 \times 60 \times 60}\ s^{-1}$$

If the body contains m gm of ^{14}C, then

$$N = \frac{6.025 \times 10^{23}}{14} \times m$$

$$R = 0.1 \text{ micro curie}$$

$$= 0.1 \times 10^{-6} \times 3.7 \times 10^{10} = 3.7 \times 10^3 \text{ disint/sec}$$

$$\therefore \quad \frac{R}{N} = \frac{3.7 \times 10^3 \times 14}{6.025 \times 10^{23} \times m} = \frac{0.6931}{5730 \times 365 \times 24 \times 60 \times 60}$$

or $\quad m = 2.242 \times 10^{-8}$ gm

$\therefore$ Percentage of ^{14}C in natural carbon

$$= \frac{2.242 \times 10^{-8} \times 100}{70 \times 1000 \times \frac{18}{100}}$$

$$= 1.78 \times 10^{-10}\ \%$$

Q. 3.19. Calculate the mass of Pb^{214} (RaB) having a radioactivity of 1 curie. Half life of Pb^{214} = 26.8 minutes. *(P.U. 1992)*

Ans. One curie $= 3.7 \times 10^{10}$ disintegrations/sec.

Let a mass m gm of Pb^{214} (RaB) has an activity of one curie, then

No. of atoms in m gm of Pb^{214}

$$N = \frac{6.025 \times 10^{23} \times m}{214}$$

Since one gm atom (214 gm) of Pb^{214} have 6.025×10^{23} atoms (Avogadro's number)

Half-life of Pb^{214} $\quad T = 26.8$ minutes $= 26.8 \times 60$ sec.

$\therefore$ Radioactive constant

$$\lambda = \frac{0.6931}{T} = \frac{0.6931}{26.8 \times 60}$$

Now activity $\quad R = -\frac{dN}{dt} = \lambda N$

or $\quad 3.7 \times 10^{10} = \frac{0.6931 \times 6.025 \times 10^{23} \times m}{26.8 \times 60 \times 214}$

or $\quad m = 3.048 \times 10^{-3}$ gm.

Q. 3.20. One gm of Ra^{226} has an activity of one curie. Calculate the mean life and half life of radium. *(P.U. 1996; Luck. U. 1995)*

Ans. Number of atoms of Ra^{226} breaking per second

$R = 1$ Curie $= 3.7 \times 10^{10}$ [1 Curie $= 3.7 \times 10^{10}$ disintegrations per second]

Number of atoms of Ra^{226} present in one gm

$$N = \frac{6.025 \times 10^{23}}{226}$$

as the number of atoms in one gram atom (226 gm) $= 6.025 \times 10^{23}$ (Avogadro's number)

Radioactive constant $\quad \lambda = \frac{R}{N} = \frac{3.7 \times 10^{10} \times 226}{6.025 \times 10^{23}}$

$$= 1.38 \times 10^{-11} \text{ sec}^{-1}$$

Average life $= \frac{1}{\lambda} = \frac{1}{1.38 \times 10^{-11}} = 7.25 \times 10^{10}$ sec $= 2298$ years.

Half life $= \frac{0.6931}{\lambda} = \frac{0.6931}{1.38 \times 10^{-11}} = 5 \times 10^{10}$ sec $= 1585$ years.

Q. 3.21. Half life of radon is 3.8 days. After how many days will $\frac{1}{10}$th of a radon sample remain behind?

Ans. Half life of radon $T = 3.8$ days.

$\therefore$ Radioactive constant $\lambda = \frac{0.6931}{T} = \frac{0.6931}{3.8} = 0.1824$ days^{-1}

Let t be the time in which $\frac{1}{10}$ of the radon sample remains behind then

$$\frac{N}{N_0} = \frac{1}{10} = e^{-\lambda t}$$

or $\quad 10 = e^{\lambda t}$

or $\quad \log_e 10 = \lambda t$ or $t = \frac{\log_e 10}{\lambda} = \frac{2.3026 \times \log_{10} 10}{0.1824}$

$$= 12.62 \text{ days.}$$

Q. 3.22. Calculate the activity of 1 gm of Bi^{209} with a half life of 2.7×10^7 years, in curies. *(Luck. U. 1995)*

Ans. Half life of B^{209}, $\quad T = 2.7 \times 10^7$ years

$= 2.7 \times 10^7 \times 365 \times 24 \times 60 \times 60 = 8.5 \times 10^{14}$ sec.

Radioactive constant $\lambda = \frac{0.6931}{T} = \frac{0.6931}{8.5 \times 10^{14}} = 8.15 \times 10^{-16}\ \text{sec}^{-1}$

If N_0 is the original number of atoms and N remaining after a time t, then

$$\Delta N = N_0 - N = N_0 - N_0 e^{-\lambda t}$$

But $\lambda = 8.15 \times 10^{-16}\ \text{s}^{-1}$ and $t = 1$ sec, therefore, λt is very small.

Hence $e^{-\lambda t} = 1 - \lambda t$

or $\Delta N = N_0 - N = N_0 - N_0(1 - \lambda t) = N_0 \lambda t$

Now N_0 = number Bi^{209} atoms in 1 gm $= \frac{6.023 \times 10^{23}}{209} = 2.88 \times 10^{21}$ where 6.023×10^{23} is Faraday's number representing the number of atoms in one gram atom *i.e.*, 209 gm of Bi^{209}.

$\therefore \quad \Delta N = 2.88 \times 10^{21} \times 8.15 \times 10^{-16} \times 1 = 23.472 \times 10^5$

or Number of disintegrations per second $= 23.472 \times 10^5$

But one Curie $= 3.7 \times 10^{10}$ disintegrations per second

$\therefore$ Activity in Curies $= \frac{23.472 \times 10^5}{3.7 \times 10^{10}} = 63.6 \times 10^{-6}$

$= 63.6$ micro-curie.

Q. 3.23. Calculate the activity of K^{40} in 100 kg mass, assuming that 0.35% of the total weight is potassium. The abundance of K^{40} is 0.012%, its half life is 1.31×10^9 years.
(Bang. U. 1994)

Ans. Total mass of potassium in 100 kg mass $= 100 \times \frac{0.35}{100} = 0.35$ kg.

Mass of K^{40} in the total mass $= \frac{0.35 \times 0.012}{100} = 4.2 \times 10^{-5}$ kg.

Number of atoms in one kg. atom of a substance $= 6.023 \times 10^{26}$ atoms

$\therefore$ Total number of K^{40} atoms $N_0 = \frac{6.023 \times 10^{26}}{40} \times 4.2 \times 10^{-5}$

$= 6.32425 \times 10^{20}$

Half life of $K^{40} = 1.31 \times 10^9$ years $= 1.31 \times 10^9 \times 365 \times 24 \times 60 \times 60$

$= 4.13 \times 10^{16}$ sec.

$\therefore$ Radioactive constant $\lambda = \frac{0.6931}{4.13 \times 10^{16}} = 1.678 \times 10^{-17}$

If N_0 is the original number of atoms and N that remaining after a time t, then

$$\Delta N = N_0 - N = N_0 - N_0 e^{-\lambda t}$$

As λ is a very small quantity $e^{-\lambda t} = 1 - \lambda t$

$\therefore \quad \Delta N = N_0 - N = N_0 - N_0(1 - \lambda t) = N_0 \lambda t$

$= 6.32425 \times 10^{20} \times 1.678 \times 10^{-17} = 1.061 \times 10^4$ disintegrations/sec

$= \frac{1.061 \times 10^4}{3.7 \times 10^{10}} = 0.287 \times 10^{-6}$ curie $= 0.287$ micro-curie