Sem-IV - Special Relativity

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Assignment II: Relativistic Dynamics

Submission due date: 30/03/2020

Q.1) (a) A rocket propels itself rectilinearly through the empty space by emitting radiation, whose recoil provides the necessary thrust. If v is the final velocity relative to its initial rest frame, prove that the ratio of the initial and final rest mass of the rocket is

$$\frac{m_i}{m_f} = \sqrt{\frac{1+\beta}{1-\beta}};\ \beta = \frac{v}{c}.$$

(b) The density of a stationary body is ρ_0. Find the velocity (relative to the body) of the reference frame in which the density is 25% greater than ρ_0. (c) Two lumps of clay each of rest mass m_0 move towards each other with equal speed $\frac{3}{5}c$ and stick together. What is the mass of the composite lump?

Q.2) (a) Half life of pions at rest is $1.77 \times 10^{-8} sec$. A collimated pion beam, leaving the accelerator target at a velocity of $0.99c$, is found to drop to half its original intensity. Find the distance travelled by the pions in the laboratory. (b) A pion of mass m_π at rest decays into a muon of mass m_μ and a neutrino of zero mass. Find the energy of the outgoing muon. (c) A kaon decays at rest via $K^+ \rightarrow e^+ + \pi^0 + \nu_e$. Calculate the maximum energy of the positron emitted. Given, mass of kaon $= 494 MeV/c^2$, mass of pion $= 135 MeV/c^2$ and mass of electron $= 0.5 MeV/c^2$.

Q.3) (a) A neutral pion of rest mass m and relativistic momentum $P = \frac{3}{4}mc$ decays into two photons. One of the photons is emitted in the same direction as the original pion, and the other in the opposite direction. Find the relativistic energy of each photon. (b) A pion (π) at rest decays into a muon (μ) and a neutrino (ν), moving in opposite directions. Find the energy of the outgoing muon (μ) in terms of the two masses, m_π and m_μ (assume $m_\nu = 0$). (c) A π^+ meson is created at a height $120 km$ above sea level with a total energy $1.35 \times 10^5 MeV$ and travels vertically downward. In its proper frame it disintegrates in $2 \times 10^{-8} sec$ after its creation. At what altitude does the disintegration occur? Rest energy of π^+ meson is $135 MeV$.

Q.4) (a) A muon at rest has lifetime $2 \times 10^{-6} s$. What is its lifetime when it travels with a velocity $\frac{3}{5}c$? (b) In an accelerator, a particle of mass $1 GeV$ is accelerated to a total energy $5 GeV$. Find out the velocity of the particle in the rest frame of the accelerator. (c) Find the velocity of the electron whose KE is $0.25 MeV$. Rest mass of electron is $0.51 MeV$.

Q.5) (a) Calculate the velocity at which the relative increase in the mass of relativistic particle is f%. (b) A muon at rest has lifetime $2 \times 10^{-6} s$. What is its lifetime when it travels with a velocity $\frac{3}{5}c$? (b) Half life of pions at rest is $1.77 \times 10^{-8} sec$. A collimated pion beam, leaving the accelerator target at a velocity of $0.99c$, is found to drop to half its original intensity. Find the distance travelled by the pions in the laboratory.

Q.6) (a) If the total energy of a particle of mass m is equal to twice its rest energy, then what will be the magnitude of its relativistic momentum? (b) A body of mass m at rest breaks up spontaneously into two parts with masses m_1 and m_2 and speeds v_1 and v_2 respectively. Show that $m > m_1 + m_2$ using conservation of mass-energy. (c) Show that a particle of rest mass zero has to travel with the velocity of light.

Q.7) (a) A particle of rest mass m_0 moving with speed v collides and sticks with a stationary particle of rest mass M_0. Show that the speed of the composite particle is given by $\gamma m_0 v/(M_0 + \gamma m_0)$. (b) A particle moving at speed $0.8c$ collides with another of the same rest mass and they stick together. What is the rest mass and speed of the composite particle?