2024

PHYSICS — HONOURS — PRACTICAL

Paper : CC-12P

(Statistical Mechanics)

Full Marks : 30

Programming Language : Python

The figures in the margin indicate full marks.

[LNB : 05, Viva voce : 05, Experiment : 20]

Day - 1

Answer ***any one*** question.

1. (a) Using appropriate *NumPy function,* draw a random sample of size 200 having Uniform Distribution in the interval [0, 1) and show the randomness in a scatter plot by dividing the sample into two subsamples of equal size. Also plot the correlogram for the sample drawn as a time series.

   (b) A radioactive sample, having half-life of 500 seconds, initially (at $t = 0$) contains 10000 nuclei. Simulate the radioactive decay experiment to numerically estimate the number of parent nuclei (N) present in the sample at an arbitrary time t (in seconds) and plot the result as $N(t)$ vs. t graph. Fit the simulated data with the analytical result and show the fitted graph in the same plot. Find the fitted values of the initial number of parent nuclei and the decay probability per second per nuclei. 8+12

2. (a) Write a Python program to generate N uniform random numbers $X = \{x_0, x_1, \ldots, x_{N-1}\}$ between [0, 1) using any Python module. Obtain the distribution of X for 20 bins. Plot the distribution histogram.

   Transform the numbers X to obtain the set of numbers Y according to the following rules.

   (i) $Y = \sqrt{X}$

   (ii) $Y = X^2$

   Plot the distribution of each Y separately.

   (b) Simulate an unbiased random walk in two-dimension and plot the RMS value of end-to-end distance as a function of time with symbols. Fit the data and show that it follows a power law. Hence find the exponent of the power law. Plot the fitted curve in same figure with line. 8+12

3. (a) A single trial of a random experiment is tossing an unbiased coin 6 times. Simulate an experiment consisting of 10000 such trials. Plot the distribution of total number of heads obtained per trial in histogram and comment on the nature of the distribution. Find the probability of getting exactly 2 heads in a single trial and compare with its theoretical value.

(b) Generate 1000 uniform random number between 0 and 1. Convert them to exponential distribution using a suitable transformation and plot them as normalized histogram. For the generated exponential distribution, find and print the second central moment about mean and third central moment about mean. What are the significances of these two moments? 10+10

4. (a) Plot Fermi-Dirac Energy Distribution function $f(E)$ as a function of $\frac{E}{E_F}$ in the range 0 to 2 for temperatures T = 3K and 300K. Use legends, label the axes and set a proper title. Save the graph in suitable format. For particles obeying the given distribution, find the probability of occupation of states below Fermi energy E_F for both the temperatures and comment on the result. [Given : $E_F = 0.15$ eV and $k = 8.617 \times 10^{-5}$ eV/K]

(b) Write a Python program to perform the following integration using Monte Carlo integration method

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

for $-3 \le x \le 3$ and $\delta x = 0.1$. Plot $erf(x)$ vs. x within the given range. 10+10

5. (a) Using appropriate *NumPy function,* draw a random sample of size 10000 having Standard Normal Distribution and plot the corresponding distribution as normalized histogram. Determine the mean and variance of the sample. Find the fractions out of this sample that are in the range [–1, 1] and [–3, 3] and comment on the result.

(b) Two persons are playing a simple coin-flip game. They flip a coin in turn, and whoever first gets a head wins the game. Write a program to model M(= 10000) such games and estimate the probability for the person who starts flipping to win the game. 8+12

6. (a) Given the molar specific heat of solids according to

(i) Dulong-Petit law : $C_{vP} = 3R$ and

(ii) Debye model : $$C_{vD} = 9R\left(\frac{T}{T_D}\right)^3 \int_0^{\frac{T_D}{T}} \frac{x^4 \exp(x)}{[\exp(x)-1]^2}\,dx.$$

Plot $\frac{C_{vD}}{C_{vP}}$ as a function of temperature T in the range 1 to 50K for $T_D = 40$K. For evaluating the integral, use Monte-Carlo method. Use legends, label the axes and set a proper title. Save the graph in suitable format.

(b) Using central limit theorem (CLT), generate a standard normal distribution of random numbers from a uniform distribution. Design the range [a, b) and sample size N of the uniform distribution such that the generated distribution is standard normal. Determine the values of mean and variance of the distribution so obtained. Plot the normalized histogram of the distribution. [Given: The mean and variance of a continuous uniform distribution in the range $[a,b)$ are $\frac{(a+b)}{2}$ and $\frac{(b-a)^2}{12}$]

10+10

2024

PHYSICS — HONOURS — PRACTICAL

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(Statistical Mechanics)

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[LNB : 05, Viva voce : 05, Experiment : 20]

Day - 3

Answer ***any one*** question.

1. (a) Using appropriate *NumPy function,* draw a random sample of size 100 having Uniform Distribution in the interval (0,1] and plot the corresponding distribution as histogram. Calculate the autocorrelation function (r_k) of the random sample as a time series for time lag $k = 10$ and $k = 50$.

   (b) Define the Gaussian Error function in the real domain $erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$ using Monte Carlo method of geometric area estimation in rectangular grid over uniform random sampling. Hence plot $erf(x)$ for $-3 \le x \le 3$. What is the significance of error function? 8+12

2. (a) Using appropriate *NumPy function*, draw a random sample (y) of size 10000 that is normally distributed with mean value 0 and standard deviation $\frac{1}{\sqrt{2}}$. Define a function $f(x)$ as the probability that y fall in the range $[-x, x]$. Hence plot $f(x)$ for $0 \le x \le 3$. What is the significance of the function $f(x)$?

   (b) Using central limit theorem (CLT), generate a Gaussian distribution from the uniform distribution [0,1) of random numbers of sample size 1000. Determine the values of mean and variance of the distribution so obtained. Plot the normalized histogram of this distribution, along with corresponding theoretical probability density function with same mean and variance. 12+8

3. (a) A single trial of a random experiment is tossing a biased coin (p = probability of getting head in a toss) m times. Simulate an experiment consisting of N such trials. Plot the total number of Heads obtained per trial in a histogram. Also plot the corresponding binomial distribution in the same graph. (Take the values of $p = 0.4$, $m = 20$ and $N = 1000$ as runtime parameter.)

(b) Using appropriate *NumPy function,* draw a random sample of size 1000 having Exponential Distribution $f(x;\lambda) = \lambda e^{-\lambda x}$ in the interval $[0, \infty)$ with rate parameter $\lambda = 0.1$ and plot the corresponding distribution as normalized histogram. Determine the mean and variance of the sample. Find how many numbers out of this sample are in the range [5, 10). 12+8

4. (a) The energy distribution functions of particles are given by

$$f(\epsilon) = \frac{1}{\exp\left(\frac{\epsilon - \mu}{k_B T}\right) + a}$$

Here, for $a = -1, 0, 1$ the $f(\epsilon)$ are called as BE, MB and FD distribution functions respectively.

Plot three distribution functions with respect to $x = \frac{\epsilon - \mu}{k_B T}$ for the ranges $0.35 \le x \le 3.0$, $-1 \le x \le 3$ and $-3.0 \le x \le 3.0$ respectively with $\delta x = 0.05$ in single graph. Show x and y axes.

(b) Considering constant probability of decay (λ) of a single radio-active nuclei per unit time interval, simulate a decay experiment for N_0 number of parent nuclei at $t = 0$. Consider the measurement interval to be Δt with total duration of measurement to be T. Experimentally determine the fraction of nuclei that survived at time t and plot a graph. Fit the data with the result obtained using the standard analytical solution. Use $\lambda = 0.0001$ sec^{-1}, $N_0 = 50000$, $\Delta t = 10$ sec and $T = 500$ sec. 8+12

5. (a) Plot Fermi Dirac Distribution function $f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1}$ as a function of $\frac{E}{E_F}$ in the range 0 to 2 for temperature $T = 300$ K. Use legends, label the axes and set a proper title. Save the graph in suitable format. Show that the probability of occupation of a state with energy ΔE above the Fermi energy E_F is equal to the probability that the state with energy ΔE below the Fermi energy E_F is empty, upto a certain numerical error.
[Given: $k = 8.617 \times 10^{-5}$ eV/K, $E_F = 0.2$ eV and $\Delta E = 0.002$ eV]

(b) Consider a random walk problem in one dimension with equal probability of movement in either direction by a step of one unit distance. Construct 10000 such random walk problems and find the probability that the walker is exactly at the starting position after 1000 random steps. 10+10

6. (a) Using appropriate transformation, generate a random sample having exponential distribution from a uniform random sample of size 1000 in the interval (0,1]. Hence plot the histogram of the exponential distribution so obtained. Use legends, label the axes and set a proper title. Save the graph in suitable format.

(b) Consider a random walk problem in one dimension with equal probability of movement in either direction. Construct the above random walk problem with 100 steps and ensemble size of 10000. Find the probability that the random walker will reach a distance of 10 unit in either direction at any time during its motion. Also find the average time to reach there. 8+12

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2024

PHYSICS — HONOURS — PRACTICAL

Paper : CC-12P

(Statistical Mechanics)

Full Marks : 30

Programming Language : Python

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[LNB : 05, Viva voce : 05, Experiment : 20]

Day -2

Answer ***any one*** question.

1. (a) Using appropriate *NumPy function*, draw a random sample of size 1000 having standard normal distribution. Find the fraction of random numbers out of this sample that do not fall in the range $[-2, 2]$. Divide the sample into two randomly selected subsamples x and y of equal size and show qualitative randomness of the original sample via y vs. x graph in a scatter plot.

(b) A single trial of a random experiment is tossing a biased coin m times with 60% probability of getting Head. Simulate an experiment consisting of N such trials. Plot the total number of Tails obtained per trial in a normalized histogram. Also plot the corresponding theoretical binomial distribution in the same plot. (Consider m = 10 and N = 1000) 8+12

2. (a) Generate a sample of N exponentially distributed random integers in the range $[0, \infty)$ with mean value of 10. Find the mean and standard deviation of the sample. Create and print the frequency distribution of the above sample with class intervals [0,10), [10, 20), [20,30),... and plot them in a histogram.

(b) Consider a random walk problem in one dimension with equal probability of movement in either direction by a step of one unit distance. Construct 10000 such random walk problems and find the probability that the walker is exactly at a distance of 10 steps from the starting position in either direction after 1000 random steps. 10+10

3. (a) Generate $N = 1000$ random numbers between 0 to 1. Generate exponential distribution from the above numbers for scale parameter, $\lambda = 2.0$. Plot the normalized distribution of the above numbers. Plot the theoretical curve corresponding to the distribution with line in the same graph.

(b) Two persons A and B are playing a two-coin flip game. They flip two coins in turn, and whoever first get heads for both the coin wins the game. Write a program to model M = 10000 such games. Estimate the probability for the person who starts flipping to win the game. 8+12

4. (a) Define $A(\alpha) = \int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{1-\alpha^2 \sin^2 x}}$. Evaluate $A(\alpha)$ using Monte-Carlo method of averaging over uniform random sampling varying α between 0 to 1 and plot $A(\alpha)$ vs. α.

(b) Simulate a nuclear decay experiment counting the number of decays in a measurement interval of 10 second for a radioactive sample containing 10^5 parent nuclei having constant decay probability of 10^{-5} per nuclei per second. Perform 1000 such Experiments with same number of initial parent nuclei and plot the distribution of measured decay counts as a normalized histogram. In the same graph, plot the corresponding Poisson distribution with line. 8+12

5. (a) Given the molar specific heat of solids, according to Debye model,

$$C_{vD} = 9R\left(\frac{T}{T_D}\right)^3 \int_0^{\frac{T_D}{T}} \frac{x^4 \exp(x)}{[\exp(x)-1]^2} dx.$$ Plot $\frac{C_{vD}}{3R}$ as a function of $\frac{T}{T_D}$ in the range 0.001 to 0.020.

For integration, use Monte-Carlo method of averaging over uniform random sampling. Using curve-fitting procedure, show that in the given temperature range $\frac{C_{vD}}{3R}$ varies as $\left(\frac{T}{T_D}\right)^3$ and find the constant of proportionality.

(b) Generate N exponential random numbers (with mean value 10) and store them in an array. Draw n samples from the array, each sample containing s random numbers. Compute the mean of these n samples and store the means in another array. Plot the histogram of the original array and the array of means in same graph. Take $N = 10000$, $n = 1000$ and $s = 50$ as runtime inputs. Comment on the nature of the distribution that the means follow. 10+10

6. (a) Plot Maxwell-Boltzmann energy Distribution $f(E) = \frac{2}{\sqrt{\pi}} \frac{1}{(kT)^{3/2}} E^{1/2} \exp\left(-\frac{E}{kT}\right)$ in the range 0 to 0.5 eV for temperatures T = 100K, 200K and 300K. Use legends, label the axes and set a proper title. Save the graph in suitable format. Comment on the nature of the plots at different temperatures. [Given: $k = 8.617 \times 10^{-5}$ eV/K.]

(b) Consider a random walk problem in one dimension with equal probability of movement in either direction where x is the distance of the walker from starting position at a time t after a large number of steps. Construct the above random walk problem in python with 1,000 steps and ensemble size of 10,000 and verify the diffusion relation,

$$\overline{x^2} = 2Dt$$

plotting both theoretical and simulated curves in same figure. Here $D = \frac{l^2}{2\tau}$ is the diffusion constant, l = length of each random step and τ = time taken for each step. 8+12

2023

PHYSICS — HONOURS — PRACTICAL

Paper : CC-12P

(Statistical Physics)

Full Marks : 30

The figures in the margin indicate full marks.

[LNB : 05, Viva Voce : 05, Experiment : 20]

Programming Language : Python

Day - 1

Answer *any one* question.

1. (a) Using appropriate *NumPy function,* draw a random sample of size 10000 having Exponential Distribution $f(x;\lambda)=\lambda e^{-\lambda x}$ in the interval $[0,\infty)$ with rate parameter $\lambda=0.1$ and plot the corresponding distribution as normalized histogram. Determine the mean and variance of the sample. Find how many numbers out of this sample are in the range [0, 10).

   (b) A radioactive sample, having constant decay probability of 0.001 per nuclei per second, initially contains 10000 nuclei. Define a function to calculate statistically the number of parent nuclei (N) present in the sample after a time of t second. Plot the numerical solution i.e., $N(t)$ vs. t graph. Fit the plot (in log-log scale) with the analytical result and show the fitted graph in the same plot. Find fitted values of the initial number of parent nuclei and the decay probability per second per nuclei.

   8+12

2. (a) Using appropriate transformation, generate an exponential distribution from a uniform distribution [2, 5) of 10000 random numbers and plot the histogram of the exponential distribution so obtained. With the help of *NumPy exponential function* generate a random sample having the same mean value and sample size and plot the histogram along with the earlier one.

   (b) Simulate an unbiased random walk in one-dimension and plot the RMS value of end-to-end distance as a function of time in log-log scale with symbols. Fit the data and show that it follows a power law. Hence find the exponent of the power law. Plot the fitted curve in same figure with line.

   8+12

3. (a) A single trial of a random experiment is tossing a biased coin (having 70% chance of getting head) m times. Simulate an experiment consisting of N trials. Plot the distribution of total number of heads obtained per trial in histogram. Set a proper title. Save the graph in suitable format. Also find the probability of getting exactly 10 heads in a single trial. [Take $m = 20$ and $N = 10000$ during runtime.]

(b) Generate 1000 uniform random number between 0 and 1. Store them in an array. Generate exponential distribution from the above array using a suitable transformation and store the data in another array. For the generated exponential distribution, find the first raw moment about zero and second central moment about mean and print. 10+10

4. (a) Plot the following energy distribution functions for $T = 10$ K and 300 K :

(i) Bose-Einstein distribution : $f(E)=\dfrac{1}{e^{\frac{E-\mu}{kT}}-1}$ with $\mu = 0.1$ eV and

(ii) Fermi-Dirac distribution : $f(E)=\dfrac{1}{e^{\frac{E-E_F}{kT}}+1}$ with $E_F = 1$ eV

[Given: Boltzmann constant $k = 8.617\times10^{-5}$ eV/K]

(b) Evaluate the value of π using Monte-Carlo integration method of geometric area estimation in rectangular grid over uniform sampling 1000 times and plot the corresponding distribution as normalized histogram. Print the mean value of π obtained with standard error of mean. 8+12

5. (a) Given the molar specific heat of solids according to (i) Dulong-Petit law : $C_{vP} = 3R$ and

(ii) Debye model : $C_{vD} = 9R\left(\dfrac{T}{T_D}\right)^3 \displaystyle\int_0^{\frac{T_D}{T}} \dfrac{x^4 \exp(x)}{[\exp(x)-1]^2}\,dx$. Plot $\dfrac{C_{vD}}{C_{vP}}$ as a function of temperature T in the range 1 to 50 K for $T_D = 40$ K. Use legends, label the axes and set a proper title. Save the graph in suitable format.

(b) Using central limit theorem (CLT), generate a Gaussian distribution from the uniform distribution [0,1) of random numbers of sample size 1000. Determine the values of mean and variance of the distribution so obtained. Plot the normalized histogram of this distribution, along with corresponding theoretical probability density function of a Gaussian distribution with same mean and variance. 8+12

6. (a) Generate N exponential random numbers (with mean m) and store them in an array. Evaluate $e^{-z/m}$ for each of the generated exponential random numbers (z) and store them in a seprarate array. Plot the two arrays in two histogram using subplots. Evaluate mean and s.d. for the transformed distribution. From the plot comment on the nature of the transformed distribution. Take m and N as runtime input.

(b) Plot the BE distribution $f(E)$ vs. E, where $f(E)=\frac{1}{e^{\frac{E-\mu}{kT}}-1}; k=8.617\times10^{-5}\,\mathrm{eV/K}, \mu=0.2\,\mathrm{eV}$.

Take the range of E from 0.201 to 0.26 eV. Take $T=20$ K, $T=100$ K and $T=300$ K. Use legend and label the axes of the graph. Save the graph in suitable format. 12+8

2023

PHYSICS — HONOURS — PRACTICAL

Paper : CC-12P

(Statistical Physics)

Full Marks : 30

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[LNB : 05, Viva Voce : 05, Experiment : 20]

Programming Language : Python

Day - 2

Answer ***any one*** question.

1. (a) Using appropriate *NumPy function,* draw a random sample of size 10000 having Gaussian Distribution of mean 1.0 and variance 4.0, and plot the corresponding distribution as normalized histogram. Find the fraction of random numbers out of this sample that fall in the range [–2, 2].

   (b) A single trial of a random experiment is tossing an unbiased coin m times. Simulate an experiment consisting of N trials. Plot the total number of Tails obtained per trial in a normalized histogram. Also plot the corresponding theoretical binomial distribution in the same plot. (Take the values $m = 10$ and $N = 1000$ as runtime parameter.) 8+12

2. (a) Generate N uniform random numbers between –1 and 1. Store them in an array. Select n = 1000 samples of sample size m and find the means of each sample and stored the means in an array. Find the standard deviation of the array of means. Vary m (keeping n constant) from 10 to 300 and evaluate standard deviation of the array of means for each value of m. Plot the sample size vs. the standard deviation. Take N = 100000 as runtime input.

   (b) Consider a random walk problem in one dimension with equal probability of movement in either direction by a step of one unit distance. Construct 10000 such random walk problems and find the probability that the walker is exactly 10 step away from the starting position in the positive direction after 1000 random steps. 10+10

3. (a) Using appropriate *NumPy function*, draw a random sample of size 1000 having Exponential Distribution $f(x;\beta)=\frac{1}{\beta}e^{-\frac{x}{\beta}}$ in the interval $[0, \infty)$ with scale parameter $\beta=5$ and plot the corresponding distribution as normalized histogram. Determine the mean and standard deviation of the sample. Find the fraction of random numbers out of this sample having values more than 10.

(b) Plot Fermi-Dirac Distribution function $f(E)=\frac{1}{e^{\frac{E-E_F}{kT}}+1}$ as a function of $\frac{E}{E_F}$ in the range 0 to 2 for temperatures $T=1$ K and 300 K. Use legends, label the axes and set a proper title. Save the graph in suitable format. For particles obeying the given distribution, find the probability of occupation of states below Fermi energy E_F for both the temperatures and comment on the result. [Given : $E_F=0.15$ eV and $k=8.617\times10^{-5}$ eV/K]

8+12

4. (a) Plot Maxwell-Boltzmann energy Distribution $f(E)=\frac{2}{\sqrt{\pi}}\frac{1}{(kT)^{3/2}}E^{1/2}\exp\left(-\frac{E}{kT}\right)$ in the range 0 to 0.5 eV for temperatures $T=100$ K, 200 K and 300 K. Use legends, label the axes and set a proper title. Save the graph in suitable format. Comment on the nature of the plots at different temperatures. [Given : $K=8.617\times10^{-5}$ eV/K.]

(b) A radioactive sample, having constant decay probability of 0.001 per nuclei per second, initially contains 10000 nuclei. Define a function to calculate statistically the number of parent nuclei (N) present in the sample after a time of t second. Plot the numerical solution i.e., $N(t)$ vs. t graph. Also find the half-life of the radioactive sample interpolating the $N(t)$ vs. t data and compare it with its theoretical value.

8+12

5. (a) Plot molar specific heat of solids by comparing (i) Einstein model : $C_{vE} = 3R\left(\frac{T_E}{T}\right)^2 \frac{\exp\left(\frac{T_E}{T}\right)}{\left[\exp\left(\frac{T_E}{T}\right)-1\right]}$

and (ii) Debye model : $C_{vD} = 9R\left(\frac{T}{T_D}\right)^3 \int_0^{\frac{T_D}{T}} \frac{x^4 \exp(x)}{[\exp(x)-1]^2} dx$ for temperatures ranging between 1 to T_D K. Use legends, label the axes and set a proper title. Save the graph in suitable format. [Given : Einstein temperature $T_E = 175$ K, Debye temperature $T_D = 225$ K and $R = 8.31$ J mol^{-1}K^{-1}.]

(b) Simulate a two-dimensional uniform random walk and plot the RMS value of end-to-end distance (R) as a function of time step (t). Fit the data and show that it follows the power law $R \sim t^{0.5}$. Plot the fitted curve in the same graph. 8+12

6. (a) Generate a sample of 10000 normal random numbers with mean ($\mu = 10$) and standard deviation ($\sigma = 5$) and store it in an array. Using appropriate *NumPy function* determine mean (m) and standard deviation (s) for a randomly selected subsample of 50 numbers from the array. Print the values of mean and standard deviation of the sample and subsample generated. For the subsample, evaluate skewness, defined as $\mu_3 = \frac{m_3}{s^3}$, where m_k is the k-th central moment about mean.

(b) Evaluate the integral $\int_0^1 \sqrt{1-x^2}\, dx$ using Monte-Carlo Integration method of averaging over uniform random sampling and repeat the process 1000 times. Hence find the mean value of π with standard error of mean. 10+10

2023

PHYSICS — HONOURS — PRACTICAL

Paper : CC-12P

(Statistical Physics)

Full Marks : 30

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[LNB : 05, Viva Voce : 05, Experiment : 20]

Programming Language : Python

Day - 3

Answer *any one* question.

1. (a) Using appropriate *NumPy function,* draw a random sample of size 100 having Uniform Distribution in the interval (0, 1] and plot the corresponding distribution as histogram. Calculate the autocorrelation function (r_k) of the random sample as a time series for time lag $k = 10$ and $k = 20$.

(b) Given the molar specific heat of solids according to Debye model,

$C_{vD} = 9R\left(\frac{T}{T_D}\right)^3 \int_0^{\frac{T_D}{T}} \frac{x^4 \exp(x)}{[\exp(x)-1]^2} dx$. Plot $\frac{C_{vD}}{3R}$ as a function of $\frac{T}{T_D}$ in the range 0.001 to 0.02.

Show that in the given temperature range $\frac{C_{vD}}{3R}$ varies as $\left(\frac{T}{T_D}\right)^3$ and find the constant of proportionality. Use legends, label the axes and set a proper title. Save the graph in suitable format. [Hint : Use curve fitting procedure plotting the fitted curve in the same graph]. 8+12

2. (a) Evaluate the value of π using Monte-Carlo Integration method of geometric area estimation in rectangular grid over uniform sampling. Plot the geometric curve and the points accumulated under the curve in the Monte-Carlo method involved.

(b) A single trial of a random experiment is tossing a biased coin (p = probability of obtaining head in a toss) m times. Simulate an experiment consisting of N such trials. Plot the total number of Heads obtained per trial in normalized histogram. Also plot the corresponding binomial distribution in the same plot. [Take the values of $p = 0.2$, $m = 20$ and $N = 1000$ as runtime parameter.] 8+12

3. (a) A single trial of a random experiment is tossing a biased coin (with 20% chance of getting Tail) 20 times. Simulate an experiment consisting of 1000 such trials. Find the probability of getting at least 6 but not more than 10 heads in a trial.

(b) Using central limit theorem (CLT), generate a standard normal distribution of random numbers from uniform distribution. Design the range $[a, b)$ and sample size N of the uniform distribution such that the generated distribution is standard normal. Determine the values of mean and variance of the distribution so obtained. Plot the normalized histogram of the normal distribution.
[Given : The mean and variance of a continuous uniform distribution in the range $[(a,b)$ are $\frac{(a+b)}{2}$ and $\frac{(b-a)^2}{12}$]

8+12

4. (a) The mean occupation number of a single particle state of energy ϵ is given by $\langle n_\epsilon \rangle = \frac{1}{e^{\frac{\epsilon-\mu}{kT}} + a}$ with $a = 0, -1, +1$ according to MB, BE and FD statistics, respectively. Plot $\langle n_\epsilon \rangle$ as a function of $\frac{\epsilon - \mu}{kT}$ in the range (–2, 2). Comment on the physical implication of the divergence of $\langle n_\epsilon \rangle$ near $\epsilon = \mu$. Use legends, label the axes and set a proper title. Save the graph in suitable format.

(b) Simulate a nuclear decay experiment counting the number of decays in a measurement interval of 10 second for a radioactive sample containing 10^5 parent nuclei having constant decay probability of 10^{-5} per nuclei per second. Perform 1000 such measurements and plot the distribution of measured decay counts as a normalized histogram. In the same graph, plot the distribution function $f(x;\mu) = \frac{\mu^x}{x!} e^{-\mu}$, for $\mu = 10$; and comment. What is the significance of μ?

8+12

5. (a) Plot Fermi-Dirac Distribution function $f(E)=\frac{1}{e^{\frac{E-E_F}{kT}}+1}$ as a function of $\frac{E}{E_F}$ in the range 0 to 2 for temperature $T=300$ K. Use legends, label the axes and set a proper title. Save the graph in suitable format. Show that the probability of occupation of a state with energy ΔE above the Fermi energy E_F is equal to the probability that the state with energy ΔE below the Fermi energy E_F is empty, upto a certain numerical error. [Given: $k = 8.617\times10^{-5}$ eV/K and $\Delta E = 0.02\ E_F$]

(b) Define $A(\alpha)=\int_0^{\pi/2}\frac{dx}{\sqrt{1-\alpha^2\sin^2 x}}$. Evaluate $A(\alpha)$ using Monte-Carlo method of Integration by averaging over uniform random sampling varying α between 0 to 1. Plot $A(\alpha)$ vs. α and save the plot in suitable format.

10+10

6. (a) Using appropriate transformation, generate a random sample having exponential distribution from a uniform random sample of size 1000 in the interval (0,1]. Hence plot the histogram of the exponential distribution so obtained. Use legends, label the axes and set a proper title. Save the graph in suitable format. Evaluate first moment of the exponential distribution about zero and print.

(b) Consider a random walk problem in one dimension with equal probability of movement in either direction, where x is the distance of the walker from starting position at time t after a large number of steps. Construct the above random walk problem in python with 1,000 steps and ensemble size of 10,000 and verify the diffusion relation

$$\overline{x^2} = 2Dt$$

plotting both theoretical and simulated curves in same figure. Here $D=\frac{l^2}{2\tau}$ is the diffusion constant, $l=$ length of each random step and $\tau=$ time taken for each step. [Hint: l and τ may conveniently be taken as unity]

8+12

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2022

PHYSICS — HONOURS — PRACTICAL

Paper : CC-12P

(Statistical Physics)

Full Marks : 30

The figures in the margin indicate full marks.

Set-I

LNB	:	05
viva voce	:	05
Experiment	:	20

Answer ***any one*** question.

1. (a) With the help of appropriate *NumPy function* draw random samples (of size 10000) from a normal (Gaussian) distribution and plot the distribution in histogram. Given that the mean and the standard deviation of the distribution are 0 and 0.1 respectively.

   (b) A radioactive sample of Cobalt-60 initially contains 10000 nuclei. The half-life of Cobalt-60 is 5.27 years. Calculate statistically the number of nuclei (N) of Cobalt-60 present in the sample after a time t years. Plot the numerical solution i.e., N(t) vs t graph, with the exact analytical solution on the same graph. 5+15

2. (a) A random walker performs a one-dimensional random walk from origin. Draw a trajectory of the walker for 100 random steps.

   (b) Using central limit theorem (CLT) generate a Gaussian distribution from the uniform distribution [0,1). Determine the values of mean (μ) and standard deviation (σ) of the Gaussian distribution so obtained.

   Plot the histogram of this distribution, along with the probability density function

$$p(x)=\frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$ 5+15

3. (a) A random walker performs a two-dimensional random walk starting from origin. Draw a trajectory of the walker for 1000 random steps.

(b) Using appropriate transformation generate an exponential distribution from the uniform distribution [0,1). Plot the histogram of the exponential distribution so obtained. With the help *of NumPy exponential function* generate a random sample having the same values of scale parameter and sample size and plot the histogram along with the earlier one. 5+15

4. (a) With the help of appropriate *NumPy function* draw random samples (of size 10000) from an exponential distribution and plot the distribution in histogram. Given that the mean of the distribution, $\beta = 5.0$.

(b) Evaluate $\int_0^1 \frac{x}{x^2+1}\,dx$ using Monte-Carlo Integration method. Plot the function and the points accumulated under the curve in this method. 5+15

5. (a) With the help of appropriate *NumPy function* draw random samples (of size 10000) from a normal (Gaussian) distribution and plot the distribution in histogram. Given that the mean and the standard deviation of the distribution are 0.5 and 0.2 respectively.

(b) Simulate an unbiased random walk in one-dimension and plot the RMS value of end-to-end distance as a function of time step (you may use a log-log scale). Fit the data and show that it follows a power law. Hence find the exponent of the power law. 5+15

6. (a) Plot the following distribution functions with energy for two temperatures, 20K and 400K:

(i) Bose-Einstein distribution : $f(E) = \frac{1}{e^{(E-\mu)/kT} - 1}$ with $\mu = 0\ eV$ and

(ii) Fermi-Dirac distribution : $f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$ with $E_F = 1\ eV$ and

the Boltzmann constant $k = 8.617 \times 10^{-5}\ eV/K$.

(b) Evaluate the error function $\text{erf}\,(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt$ for the following four values of x : x = 0.25, 0.5, 1.0 and 2.0. 5+15

7. (a) With the help of appropriate *NumPy function* draw random samples (of size 10000) from an exponential distribution and plot the distribution in histogram. Given that the mean of the distribution, $\beta = 1.0$.

(b) Simulate a two-dimensional uniform random walk and plot the RMS value of end-to-end distance (R) as a function of time step (t) (you may use a log-log scale). Fit the data and show that it follows the power law $R \sim t^{1/2}$. 5+15

8. (a) Plot molar specific heat of solids by comparing (i) Einstein model: $C_v = 3R\left(\frac{T_E}{T}\right)^2 \frac{e^{T_E/T}}{\left(e^{T_E/T}-1\right)^2}$ and (ii) Debye model: $C_v = 9R\left(\frac{T}{T_D}\right)^3 \int_0^{T_D/T} \frac{x^4 e^x}{\left(e^x-1\right)^2} dx$ for a temperature range of 1 *to* T_D Kelvin. Take Einstein temperature $T_E = 174\ K$, Debye temperature $T_D = 225\ K$ and $R = 8.31\ J\,\mathrm{mol}^{-1}\,K^{-1}$.

(b) A single trial of a random experiment is tossing an unbiased coin 100 times. Simulate an experiment consisting of such 1000 trials. Plot the total number of Heads obtained per trial in a histogram. 10+10

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2022

PHYSICS — HONOURS — PRACTICAL

Paper : CC-12P

(Statistical Physics)

Full Marks : 30

The figures in the margin indicate full marks.

Set-II

LNB	:	05
viva voce	:	05
Experiment	:	20

1. (a) Generate N uniform random numbers between 0 and 1. Store them in an array. Evaluate the mean and standard deviation of the numbers generated. Calculate the autocorrelation function $r_k = \frac{c_k}{c_0}$ of the series for $k = 2$ and $k = 10$. Print the values of mean, standard deviation and the values of r_2 and r_{10} of the random series.

(b) Plot Fermi Dirac Distribution $f(E) = \frac{1}{e^{\frac{E-E_F}{kT}}+1}$ vs. $\frac{E}{E_F}$. Given $E_F = 0.2\ eV$ and $k = 8.617\times10^{-5} eV/K$. Take the range of $\frac{E}{E_F}$ from 0 to 2. Take $T = 1K$, $T = 50\ K$ and $200\ K$. Use legend and label the axes of the graph. Set a proper title. Save the graph in suitable format. 10+10

2. (a) Generate N uniform random number between 0 and 1. Store them in an array. Generate exponential distribution from the above array using a suitable transformation and store the data in another array. Find the mean m and standard deviation s of the exponential random numbers. Compute k_3, the third order central moment about mean $\left(k_3 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3\right)$ for the created exponential distribution. Define skewness as $\mu_3 = \frac{k_3}{s^3}$. Print the values of m, s, k_3 and μ_3.

(b) Plot the MB distribution $f(E)$ vs E, where $f(E) = \frac{2}{\sqrt{\pi}}\frac{1}{(kT)^{\frac{3}{2}}}\sqrt{E}e^{-\frac{E}{kT}}$, where $k = 8.617\times10^{-5}eV/K$.

Take the range of E from 0 to 0.5 eV. Take T = 100 K, T = 250 K and T = 500 K. Use legend and label the axes of the graph. Set a proper title. Save the graph in suitable format. 10+10

3. (a) Generate N exponential random numbers (with mean 5) and store them in an array. Draw n samples from the array, each sample should contain s random numbers. Compute the mean of these n samples and store the means in another array. Plot the original array and the array of means in separate Histogram using subplot. Take N = 100000, n = 1000 and s = 40 as runtime inputs. Comment on the nature of the distribution that the means follow.

(b) Plot the BE distribution $f(E)$ vs E, where $f(E) = \frac{1}{e^{\frac{E-\mu}{kT}} - 1}$, $k = 8.617\times10^{-5}\ eV/K, \mu = 0.1eV$. Take the range of E from 0.101 to 0.16 eV. Take T = 10 K, T = 50 K and T = 100 K. Use legend and label the axes of the graph. Set a proper title. Save the graph in suitable format. 10+10

4. (a) A single trial of a random experiment is tossing a biased coin (p = probability of obtaining head in a toss) m times. Simulate an experiment consisting of N trials. Plot the total number of heads obtained per trial in histogram. Set a proper title. Save the graph in suitable format. (Take the values *of* p = 0.2, m = 80 and N = 100 during runtime.)

(b) Evaluate the following integral using Monte-Carlo method of integration. $\int_0^4 \frac{dx}{x^2+16}$. Take the number ($n$) of random points as input. Run the program for two different values of n = 100 and n = 1000. Print the results. 10+10

5. (a) Considering decay of a given nuclei to be independent of each other and $\lambda \equiv$ probability of decay of a single nuclei per unit time. Simulate radio active decay for N_0 number of parent nuclei at $t = 0$. Consider the final time to be t_n and number of un-decayed nuclei at time t_n be $N(t_n)$. Estimate the fraction of nuclei that remains at t_n. Also evaluate it directly using the standard relation and compare it with the results obtained from simulation. Use $\lambda = 0.005$, $N_0 = 60000$ and $t_n = 450$.

(b) The mean occupation number of a single particle state of energy ϵ is given by $\langle n_\epsilon \rangle = \dfrac{1}{e^{\frac{\epsilon-\mu}{kT}} + a}$

where $a = +1, -1$ for FD and BE distribution respectively. Plot $\langle n_\epsilon \rangle$ *vs.* $\dfrac{\epsilon - \mu}{kT}$ for the FD and BE distribution. Consider the range of $\dfrac{\epsilon - \mu}{kT}$ within -3 to 3. Use legend and label the axes of the graph. Set a proper title. Save the graph in suitable format. 10+10

6. (a) A single 2D random walk consists of n steps. Simulate a collection of N such random walks. Evaluate the r.m.s. value of end to end distance (r) for different number of steps ((t) = 1 to n). Find the value of r at $t = 9$ and $t = 64$. Fit the following function with the data $r(t) = at^b$. Print the values of a and b. (Use the necessary scipy function for fitting)

(b) Given C_v according to Dulong Petit Law : $C_v = 3R$, and Einstein Theory : $C_v = 3R\dfrac{\left(\frac{\theta_E}{T}\right)^2 e^{\frac{\theta_E}{T}}}{\left(e^{\frac{\theta_E}{T}} - 1\right)^2}$

Assuming $\theta_E = 10\ K$, plot $\dfrac{C_v}{3R}$ vs T for T lying within the range 1 to $100K$ for Dulong Petit's Law and Einstein Theory. Use legend and label the axes of the graph. Set a proper title. Save the graph in suitable format. 10+10

7. (a) Generate N normal random number with mean (μ) and standard deviation (σ). Store it in an array. Evaluate mean and standard deviation for a randomly selected sample of 20 numbers from the array. Print the values of μ and σ used to generate the normal random numbers and also the values of mean and s.d. obtained from the sample drawn from it. Evaluate kurtosis for this selected sample given by $\mu_4 = \frac{k_4}{s^4} - 3$, where s is s.d. of the sample, and k_4 is the fourth order central moment about mean $\left(k_4 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4\right)$ for the sample. Print the value of k_4 and μ_4.

(b) Given $D(T) = \int_0^{\frac{\theta_D}{T}} \frac{x^4 e^x dx}{\left(e^x - 1\right)^2}$ and $\theta_D = 10K$. For T within range 1 K to 50 K evaluate $f(T) = 3\left(\frac{T}{\theta_D}\right)^3 D(T)$ using Monte-Carlo method for evaluating the integral. Plot f(T) vs. T. Label the axes of the graph. Set a proper title. Save the graph in suitable format. 10+10

8. (a) A single 1 D random walk consists of N steps. Evaluate the square of the distance of the particle from the origin after i steps $\{i = 1(1)\ N\}$) and store the result in an array. Print the last 5 elements of the array. Plot the coordinate of the particle after i steps vs i. Label both axes of the graph. Set a proper title. Save the graph in suitable format. (Take $N = 1000$ as runtime input.)

(b) Evaluate the following Integral using Monte-Carlo method of integration. $\int_0^{10} \frac{xdx}{x^2 + 16}$. Take the number ($n$) of random points as input. Run the program for two different values of n. Print the results. 10+10

2022

PHYSICS — HONOURS — PRACTICAL

Paper : CC-12P

(Statistical Physics)

Full Marks : 30

The figures in the margin indicate full marks.

Set-III

LNB	:	05
viva voce	:	05
Experiment	:	20

1. (a) Generate N uniform random numbers between 5 and 10, [5, 10). Store them in an array. Evaluate the mean and standard deviation of the numbers generated. Calculate the autocorrelation function $r_k = \frac{c_k}{c_0}$ of the series for $k = 1$ and 10. Print the values of mean, standard deviation of the random series, and r_1 and r_{10} for the series. Take N as runtime input ($N = 100$ and $N = 1000$).

   (b) Plot Fermi Dirac Distribution $f(E,T) = \frac{1}{e^{\frac{E-E_F}{kT}}+1}$ vs T. Given $E_F = 0.3\ eV$ and $k = 8.617 \times 10^{-5} eV/K$. Take the range of T from 10 to 200 K. Take $E = 0.29$ eV, $E = 0.30\ eV$ and $E = 0.31\ eV$. Use legend and label the axes of the graph. Set a proper title. Save the graph in suitable format. 10+10

2. (a) Generate N uniform random integer between in the range [0, 9]. Store them in an array. Find the mean m and standard deviation s of the random integers. Compute k_4, the fourth order central moment about mean (given by $k_4 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^4$ for the generated random integers. Define kurtosis as $\mu_4 = \frac{k_4}{s^4} - 3$. Print the values of m, s, k_4 and μ_3.

(b) Plot the MB distribution $f(E,T)$ vs T, where $f(E) = \frac{2}{\sqrt{\pi}} \frac{1}{(kT)^{\frac{3}{2}}} \sqrt{E} e^{-\frac{E}{kT}}$. $k = 8.617 \times 10^{-5} eV/K$.

Take the range of T from 100 to 2000 K. Take E = 0.05 eV, E = 0.10 eV and E = 0.15 eV. Use legend and label the axes of the graph. Set a proper title. Save the graph in suitable format.

10+10

3. (a) Generate N uniform random numbers within [0,1) and store them in an array. Draw n samples from the array, each sample should contain s random numbers. Compute the mean of these n samples and store the means in an array. Plot the original array and the array of means in separate Histogram using subplot. Take N = 100000, n = 1000 and s = 40 as runtime inputs. Comment on the nature of the distribution that the means follow.

(b) Plot the BE distribution $f(E, T)$ vs. T, where $f(E) = \frac{1}{e^{\frac{E-\mu}{kT}} - 1}$. $k = 8.617 \times 10^{-5}$ eV/K, $\mu = 0.1\, eV$.

Take the range of T from 10 to 400 K. Take $E = 0.12$ eV $E = 0.15$ eV and E = 0.18 eV. Use legend and label the axes of the graph. Set a proper title. Save the graph in suitable format.

10+10

4. (a) A single trial of a random experiment is tossing an unbiased coin m times. Simulate an experiment consisting of N trials. Plot the total number of heads obtained per trial in histogram. Set a proper title. Save the graph in suitable format. (m = 40 and N = 10000 during runtime.)

(b) Evaluate the following Integral using Monte-Carlo method of integration. $\int_0^{\pi} x^2 \sin\left(\frac{x}{2}\right) dx$. 10+10

5. (a) Considering decay of a given nucleus to be independent of each other and $\lambda \equiv$ probability of decay of a single nucleus per unit time. Simulate radio active decay for N_0 number of parent nuclei at $t = 0$. Consider the final time to be t_n and number of un-decayed nuclei at time t_n be $N(t_n)$. Also evaluate it directly using the standard relation and print the percentage given by $100\times\dfrac{N(t_n)_{simulated} - N(t_n)_{exact}}{N(t_n)_{exact}}$. Use $\lambda = 0.004$, $N_0 = 50000$ and $t_n = 470$.

(b) The mean occupation number of a single particle state of energy ϵ is given by $\langle n_\epsilon \rangle = \dfrac{1}{e^{\frac{\epsilon-\mu}{kT}}+1}$ for fermion. Plot $\langle n_\epsilon \rangle$ vs kT. Consider the range of T within 10 to 1000 K. Use $\epsilon = 0.44$ and $\epsilon = 0.46$, for $\mu = 0.45$ eV. Use legend label the axes of the graph. Set a proper title. Save the graph in suitable format. 10+10

6. (a) A single $2D$ random walk consists of n steps. Simulate a collection of N such random walks. Evaluate the r.m.s. value of end to end distance (r) for different number of steps $(t)t = 1$ to n). Fit the following function with the data $\ln r(t) = A + B \ln t$. Print the values of A and B. Find the value of r at $t = 10$.

(b) Given C_v according to Einstein Theory : $C_v = 3R\dfrac{\left(\dfrac{\theta_E}{T}\right)^2 e^{\frac{\theta_E}{T}}}{\left(e^{\frac{\theta_E}{T}}-1\right)^2}$. Plot $\dfrac{C_v}{3R}$ vs T for T lying within the range 1 to 50 K for Einstein Theory for $\theta_E = 10$ and $\theta_E = 20$ K. Use legend and label the axes of the graph. Set a proper title. Save the graph in suitable format. 10+10

7. (a) Generate N normal random number with mean (μ) and standard deviation (σ). Store it in an array. Evaluate mean and standard deviation for randomly selected 20 numbers from the array. Print the values of μ and σ used generate the normal random numbers and also the values of mean and Standard Deviation(s) obtained from the sample drawn from it. Evaluate skewness for both the array of numbers. $\mu_3 = \frac{k_3}{s^3}$, where $k_3 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^3$.

(b) Given $D(T) = \int_0^{\frac{\theta_D}{T}} \frac{x^4 e^x dx}{\left(e^x - 1\right)^2}$. For T within range 1 K to 40 K evaluate $f(T) = 3\left(\frac{T}{\theta_D}\right)^3 D(T)$.

Plot $f(T)$ vs T. Take $\theta_D = 4$K and $\theta_D = 10$ K. Use legend and label the axes of the graph. Set a proper title. Save the graph in suitable format.

10+10

8. (a) A single 1D random walk consists of n steps. Simulate a collection of N such random walks. Evaluate the r.m.s. value of end to end distance (r) for different number of steps (t = 1 to n). Fit the following function with the data $r(t) = at^b$, Print the values of a and b. Plot the fitted function as well as the actual r.m.s values vs t in the same plot. Use legend label both the axes of the graph. Set a proper title. Save the graph in suitable format.

(b) Evaluate the following Integral using Monte-Carlo method of integration. $\int_1^{10} \frac{dx}{x}$. Take the number (n) of random points as input. Run the program for two different values of n. Print the results. Evaluate the value of e from the result of the integration.

10+10

2022

PHYSICS — HONOURS — PRACTICAL

(Syllabus : 2018 - 2019)

Paper : CC-14-P

(Statistical Physics)

Full Marks : 30

The figures in the margin indicate full marks.

Distribution of Marks : Program - 20, LNB - 5; viva = 5.

Answer ***any one*** question. 20

1. A radioactive sample of C-14 carbon nuclei has initially 1000 nuclei. Half life time of a C-14 carbon nucleus is $t_{\frac{1}{2}} = 5730$ years.

   (a) Statistically calculate the number of nuclei present in the sample at any time t.

   (b) Plot $N(t)$ vs t graph.

   (c) Also plot the exact theoretical result for radioactive decay in the same graph.

2. (a) Generate 10000 random numbers from uniform random number distribution [0, 1).

   (b) Using appropriate transformation generate exponential distribution $f(x) = \lambda e^{-\lambda x}$ from this uniform distribution.

   (c) Plot the histogram of the exponential distribution so obtained.

   (d) Verify this exponential distribution by plotting exponential function $f(x) = \lambda e^{-\lambda x}$ on the same graph.

3. (a) Using Central Limit Theorem (CLT) generate Gaussian distribution $G(X)$ from uniform distribution [0, 1).

   (b) Print the values of mean and standard deviation of the both uniform distribution and Gaussian distribution.

   (c) Plot the histogram of the Gaussian distribution $G(X)$.

   (d) Check whether this distribution is Gaussian or not by plotting a Gaussian function with appropriate μ and σ.

4. Using Monte Carlo Integration method evaluate the following integration $erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$ with $x = 1$. Plot the function and point accumulated under the curve in this method.

5. A random walker performs a one-dimensional random walk from origin. Draw a trajectory of the walker for 50 random steps (the random walk consists of 50 random steps).

(a) Calculate the mean and mean square displacement of the random walker for 50 random steps averaged over 3000 independent random walks.

(b) Plot the mean and mean square displacement vs. the number of steps.

(c) Determine the slopes of the two curves.

(d) Calculate the probability of taking n steps to right out of N steps.

(e) Plot this probability distribution function with n using **plt.bar()** function.

6. Consider a random walker performs two dimensional random walk from the origin.

(a) Calculate mean square displacement (end to end distance) of the random walker for 50 random steps averaged over 1000 independent random walks (one random walk consists of 50 random steps).

(b) Plot the mean square displacement vs number of steps and find the slope.

7. Draw the following distribution functions $f(E)$ vs E for three different temperatures.

(a) Maxwell-Boltzmann distribution function

(b) Bose-Einstein distribution function

(c) Fermi-Dirac distribution function.