# Physical Optics (Diffraction)

1(a) Given that $12^{th}$ order maximum is seen in a Young's double slit experiment with Na-light of wavelength $\lambda_1 = 6000$ Å

We know the condition for $n$th order interference maxima is $d\sin\theta = n\lambda_1$ for normal incidence, $d$ = distance between two slits & $\theta$ is the angle made by the $n^{th}$ order maxima with the normal of the slit at the middle of the source.

If we replace the monochromatic source with $\lambda_2 = 4800$Å then at the same place say $m^{th}$ order maxima is seen whose condition is $d\sin\theta = m\lambda_2$

$\therefore$ $d\sin\theta = n\lambda_1 = m\lambda_2$ yields $m = \frac{n\lambda_1}{\lambda_2} = \frac{\cancel{12}^{15} \times \cancel{6000}}{\cancel{4800}_{\cancel{4}}}$

$= 15$

So $15^{th}$ order interference maxima will be visible with 4800 Å source at the same place where $12^{th}$ order interference maxima was seen through a 6000 Å source.

(b) Given that the central maxima contains 9 fringes, meaning 4 equidistant bright bands on both side of the central bright line.

As the cetral fringe is bright, we have $\frac{2b}{a} + 1 = 9$

where $d = a+b$ = distance between two slits & $a$ is the slitwidth. $\therefore$ $\frac{2b}{a} = \cancel{8}^{4}$ or $b = 4a$ & $d = 5a$

From the condition of missing order $\frac{d}{a} = \frac{n}{\beta}$, $n = n^{th}$ interference maxima & $\beta$ is $\beta^{th}$ diffraction minima, so $n = 5\beta$,

So 5, 10, 15 .. etc order of interference maxima are absent which corresponds to 1, 2, 3 ... orders of diffraction dark bands. This means that within first & second minima (diffraction dark band) only $6^{th}$, $7^{th}$, $8^{th}$ & $9^{th}$ order interference maxima (fringes) will be visible.

We know the condition for $n^{th}$ order interference maxima is $d\sin\theta_n = n\lambda$. or $\theta_n = \sin^{-1}\left(\frac{n\lambda}{d}\right) = \sin^{-1}\left(\frac{n\lambda}{5a}\right)$.

So the angles where fringes will appear will be

$\theta_6 = \sin^{-1}\left(\frac{6\lambda}{5a}\right)$, $\theta_7 = \sin^{-1}\left(\frac{7\lambda}{5a}\right)$, $\theta_8 = \sin^{-1}\left(\frac{8\lambda}{5a}\right)$ and $\theta_9 = \sin^{-1}\left(\frac{9\lambda}{5a}\right)$

respectively.

(2) (a) Given, for the $m^{th}$ order (say) diffraction spectra of a wavelength $\lambda_1 = 540\,nm = 540\times10^{-9}\,m$ superimposes with the $(m+1)^{th}$ order of another wavelength $\lambda_2 = 405\,nm = 500\times10^{-9}\,m$ at normal incidence of a plane transmission grating. d being the Grating element.



From the condition of secondary maxima, we have

$$d\sin\theta = m\lambda_1 = (m+1)\lambda_2 \quad \text{or} \quad m = \frac{\lambda_2}{\lambda_1 - \lambda_2} = \frac{405}{540-405} = 3.$$

Given, angle of diffraction $\theta = 30°$.

$$\therefore\ d = \frac{m\lambda_1}{\sin\theta} = \frac{3\times 540\times10^{-9}}{1/2} = 324\times10^{-8}\,m$$

So the grating element is $324\times10^{-8}\,m$.

(b) Given for a plane transmission grating,

slit width $a = 0.001\,mm$

opaque space $b = 0.002\,mm$ & wavelength of monochromatic source $\lambda = 500\,nm$.

As $\sin 90^\circ = 1$ so the maximum angle of diffraction is $90^\circ$.
Let $m$ be the maximum number of order of spectrum that can be observed.

from the condition of secondary maxima, we have

$d\sin\theta = m\lambda$ where grating element $d = a+b = 0.003$ mm

$$\therefore m = \frac{d}{\lambda} = \frac{0.003\times10^{-3}}{500\times10^{-9}} = \underline{6}.$$

Now we need to account for the missing grating spectra that happens for $m^{th}$ principal maxima (condition $d\sin\theta = m\lambda$) to coincide with $n^{th}$ minimum intensity (condition $a\sin\theta = n\lambda$)

$$\therefore \frac{m\lambda}{d} = \frac{n\lambda}{a} \quad \text{or} \quad m = \frac{a+b}{a}n. = \frac{0.003}{0.001}n = 3n.$$

So 3rd, 6th order will be missing from the grating spectra and only $\underline{\underline{\text{upto } 5^{th}}}$ order spectrum is visible.

(c) Given, the number of rulings of the grating $N = 820$/cm.

$\therefore$ Grating element $d = \frac{1}{N} = \frac{1}{820}$ cm.

Wavelength of Na-D lines $\lambda_{D_1} = 5890\,\text{Å}$, $\lambda_{D_2} = 5896\,\text{Å}$

Let the least-width of the grating be $w$.

We know that the resolving power of a grating is given by

$$\frac{\lambda}{d\lambda} = Mn,$$

$n$ = order of the spectrum; $d\lambda = \lambda_{D_2} - \lambda_{D_1}$ Å.
$= 2.$ $\quad$ $M$ = # of rulings in the grating.

$$\therefore M = \frac{\lambda}{n\,d\lambda} = \frac{(\lambda_{D_1}+\lambda_{D_2})/2}{2(\lambda_{D_2}-\lambda_{D_1})} = \frac{5893}{2\times6} \simeq \underline{\underline{491}}$$

$\therefore$ Least-width of the grating $w = Md = 491\times\frac{1}{820}$ cm

$= \underline{\underline{0.599 \text{ cm}}}.$

to rissolve the Na-D line spectra.

(d) Given, the number of rulings of the grating $N = 3000/cm$

$\therefore$ Grating constant $d = \frac{1}{N} = \frac{1}{3000}\ cm = 3.333 \times 10^{-4}\ cm$

To obtain the direction of the $1^{st}$ order $(m=1)$ of the Na-D lines $\lambda_{D_1} = 5890\,Å$ and $\lambda_{D_2} = 5896\,Å$ with average wavelength $\lambda = \frac{1}{2}(\lambda_{D_1} + \lambda_{D_2}) = 5893\,Å$, we know the condition of secondary maxima $d\sin\theta = m\lambda$, we have

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{1 \times 5893 \times 10^{-8}}{3.333 \times 10^{-4}}\right) = \underline{\underline{10.18^\circ}}.$$

This is the required direction of the $1^{st}$ order grating spectra.

Now from the relation for resolving power $\frac{\lambda}{d\lambda} = Mm$, where $M$ is the number of rulings in the grating and $m$ is the order of the spectrum, we have

$$M = \frac{\lambda}{m\,d\lambda} = \frac{5893}{1 \times 6} = 982.16 \simeq \underline{\underline{983}}.$$

So the least-width of the grating is $Md = \frac{983}{3000}\ cm = \underline{\underline{0.32\,cm}}$.

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(e) Given, least-width of grating $Md = 4$ cm and number of rulings $N = 4000$ lines/cm. Wavelength of monochromatic source $\lambda = 5900\,Å$, $M$ = # of rulings in the grating.

$\therefore$ Grating constant $d = \frac{1}{N} = \frac{1}{4000}$ cm.

$$\therefore\ Md = 4 \Rightarrow \frac{M}{4000} = 4 \Rightarrow M = 16000.$$

While order of spectrum $m=1$, the required resolving power $\frac{\lambda}{d\lambda} = mM = 1 \times 16000 = \underline{\underline{16000}}$.

while $\left(\frac{\lambda}{d\lambda}\right)_{Na} = \frac{5893}{6} = \underline{\underline{982}}$ which is way smaller than the computed above resolving power, so this grating can separate the Na-doublet.

(f) Width of one transmission grating $G_1$ is $w_1 = M_1 d_1 = 3$ cm with # of rulings $M_1 = 3000$. Width of another grating $G_2$ is $w_2 = M_2 d_2 = 2$ cm with # of rulings $M_2 = 2000$.

Resolving power of a grating $\frac{\lambda}{d\lambda} = mM$

So for the same order ratio of R.P. of $G_1$ & $G_2$ is $mM_1 : mM_2$

$= 3000 : 2000 = \underline{3:2}$.

(3)(a) Given, two points on Moon is resolved by a telescope of diameter $D = 500$ cm. $= 5$ m. Most sensitive wavelength of light to eye is $\lambda = 5500$ Å $= 5500 \times 10^{-10}$ m, and distance of moon to surface of Earth $d = 3.8 \times 10^5$ km $= 3.8 \times 10^8$ m.

∴ The angular separation (inverse of resolving power) of the two points is $d\theta = \frac{1.22\lambda}{D} = \frac{1.22 \times 5500 \times 10^{-10}}{5} = 1.342 \times 10^{-7}$ rad

So from the geometry of arclength, separation between the points on Moon will be $d\,d\theta = 3.8 \times 10^8 \times 1.342 \times 10^{-7}$ m

$= \underline{51}$ m.

(b) Given, the resolving power (R.P.) of eye is $d\theta_e = 1'$ arc. Let $D_o$, $D_{ep}$, $D_e$ be the diameters of the telescope objective, telescope eyepiece and human eye. Wavelength $\lambda = 6000$ Å.

Given, the magnification $\frac{D_o}{D_{ep}} = 80$ where $D_o = 8$ cm

∴ $D_{ep} = 0.1$ cm $= 1$mm. We know R.P. of eye $\frac{1}{d\theta_e} = \frac{D_e}{1.22\lambda}$

or $\frac{D_e}{1.22\lambda} = 1/1' = 1/\frac{\pi}{60 \times 180}$ rad. or $D_e = \frac{1.22 \times 6000 \times 10^{-10}}{\pi/(60 \times 180)}$

$= \underline{2.516 \text{ mm}}$

For telescope, the angular resolving power is $d\theta_T = \frac{1.22\lambda}{D_o}$

$$= \frac{1.22 \times 6000 \times 10^{-8}}{8} \text{ rad} = \frac{1.22 \times 6000 \times 10^{-8}}{8} \times \frac{180}{\pi}^{\circ} =$$

$$\frac{1.22 \times 6000 \times 10^{-8}}{8} \times \frac{180}{\pi} \times 3600 \text{ sec} = \underline{\underline{1.887''}}$$

while $d\theta_e = 1' = 60''$ and magnification $= 80$, so normally human eye's resolving power is $\frac{60''}{80} = \underline{0.75''}$ which is way smaller than the Telescope's resolving power $1.887''$.

Therefore, the resolvable angular separation of stars using this refracting telescope is $1.887''$.

(4) We know that $n^{th}$ dark ring in Newton's ring interference experiment is $r_n = \sqrt{n\lambda R}$ where $\lambda$ is wavelength of monochromatic source and $R$ is the radius of curvature of the plano convex lens $= 2m$.

So radius of the $1^{st}$ dark ring which is also the radius of the $1^{st}$ dark ring of the zone plate is $r_1 = \sqrt{2\lambda}$.

Now we know that the focal length of $m^{th}$ order of a zone-plate is $f_m = \frac{r_m^2}{m\lambda}$. So at $1^{st}$ order, $f_1 = \frac{r_1^2}{\lambda} = \frac{2\lambda}{\lambda} = \underline{\underline{2m}}$.

$\therefore$ The first focal length of the zone plate is 2 metres.