HW 1.

$x = a\sin(\omega t + \phi)$ At $t = 0$, $x = x_0$

$\therefore x_0 = a\sin\phi$, $\dot{x} = a\omega\cos(\omega t + \phi)$, At $t = 0$, $\dot{x} = v_0$

$v_0 = a\omega\cos\phi$.

$$\therefore \tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{x_0/a}{v_0/a\omega} = \frac{x_0\omega}{v_0}$$

also, $a^2\sin^2\phi + a^2\cos^2\phi = x_0^2 + \frac{v_0^2}{\omega^2}$

or $a^2(\sin^2\phi + \cos^2\phi) = x_0^2 + \frac{v_0^2}{\omega^2}$. $\therefore a = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}$.

2. Loss contact = upward force $\geq$ weight mg.

$x = a\sin(\omega t + \phi)$

$\therefore$ Upward force $= m\ddot{x} = ma\omega^2\sin(\omega t + \phi) = m\omega^2 x$.

So loss contact condition, $m\omega^2 x \geq mg$

or $x \geq \frac{g}{\omega^2} \geq \frac{9.81}{4\pi^2 5^2} \geq 10^{-2}$ metre.

3. $v = \omega\sqrt{a^2 - x^2}$

$\therefore 80 = \omega\sqrt{a^2 - 3^2}$ & $60 = \omega\sqrt{a^2 - 4^2}$

or $\frac{80}{60} = \frac{4}{3} = \frac{\sqrt{a^2 - 3^2}}{\sqrt{a^2 - 4^2}}$ or $\frac{16}{9} = \frac{a^2 - 9}{a^2 - 16}$

or $a = 5$ cm.

(4) (a) $T = 2\pi\sqrt{\frac{l}{g}}$. If increased by 44% then new length $l' = l + 0.44l = 1.44l$. So new time period $T' = 2\pi\sqrt{\frac{l'}{g}}$

$\therefore \frac{T'}{T} = \sqrt{\frac{l'}{l}} = \sqrt{\frac{1.44l}{l}} = \sqrt{1.44} = 1.2$, or $T' = 1.2T$

So time period increases by $0.2T$ or % increase $= \frac{0.2T}{T} \times 100$
$= 20\%$.

(b) $T = 2\pi\sqrt{l/g}$. Free fall = weightless, so effective gravity = 0.
So, $T = \infty$, or frequency $= \frac{1}{T} = 0$. It stops oscillating.

(5) (a) $KE = \frac{1}{2}m\omega^2(a^2 - x^2)$, $PE = \frac{1}{2}m\omega^2x^2$, $TE = \frac{1}{2}m\omega^2a^2$

when $x = a/2$, $KE = \frac{1}{2}m\omega^2\left(a^2 - \frac{a^2}{4}\right) = \frac{3}{8}m\omega^2a^2$

$\therefore KE/TE = \frac{3/8\, m\omega^2a^2}{\frac{1}{2}m\omega^2a^2} = \frac{3}{4}$

(b) $\frac{1}{2}m\omega^2(a^2 - x^2) = \frac{1}{2}m\omega^2x^2$ or $a^2 = 2x^2$ or $x = a/\sqrt{2}$

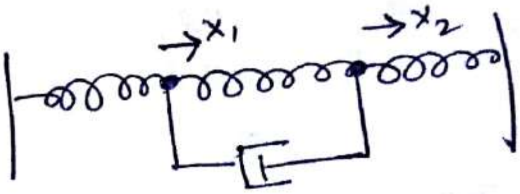
(6) $y_1 = 10\sin(4\pi t + \frac{\pi}{4})$,
$y_2 = 10\left(\frac{1}{2}\sin 3\pi t + \frac{\sqrt{3}}{2}\cos 3\pi t\right) = 10\left(\sin 30^\circ \sin 3\pi t + \cos 30^\circ \cos 3\pi t\right)$
$30^\circ = \frac{\pi}{6}$
$= 10\cos(3\pi t - \frac{\pi}{6})$
So $a_1 : a_2 = 10 : 10 = 1 : 1$.

(7) (a) ~~Wavelength is~~ Sound waves are million times longer than

light waves & light waves are very small, so even a very small mirror can reflect light but not sound. On the other hand, due to large wavelength, sound waves can be reflected even by a rough wall but not light waves.

(b) The mechanical waves are transmitted by the vibration of the particles of an elastic material medium. So, the sound waves which are mechanical waves require a material medium for their propagation. But no medium is required for the propagation of EM waves.

8 (a) Force on mass 1



$$F_1 = -kx_1 - k(x_1-x_2) - b(\dot{x}_1 - \dot{x}_2) \quad -(1)$$

Force on mass 2, $F_2 = -kx_2 + k(x_1-x_2) + b(\dot{x}_1-\dot{x}_2) \quad -(2)$

(b) $(1)-(2) \Rightarrow \quad m(\ddot{x}_1 - \ddot{x}_2) = -k(x_1-x_2) - 2k(x_1-x_2) - 2b(\dot{x}_1-\dot{x}_2)$

$y_1 = x_1 - x_2, \quad m\ddot{y}_1 = -ky_1 - 2ky_1 - 2b\dot{y}_1 \quad = -3ky_1 - 2b\dot{y}_1$

$y_2 = x_1 + x_2, \quad m\ddot{y}_2 = -ky_2 \quad -(3)$

(c) So motion of y_1 is damped & vanishes in time. $\Rightarrow y_1 = 0 \; \epsilon t$. y_2 is SHM

$x_1 = \frac{1}{2}(y_1+y_2)$

$x_2 = -\frac{1}{2}(y_1-y_2)$

initial condition

At $t=0$, $x_1 = x_2 = 0$, $\dot{x}_1 = v_0$, $\dot{x}_2 = 0$

So, $y_1 = y_2 = 0$

So $v_0 = \frac{1}{2}(\dot{y}_1 + \dot{y}_2)$, $0 = -\frac{1}{2}(\dot{y}_1 - \dot{y}_2)$

$\therefore \dot{y}_1 = \dot{y}_2 = v_0$

From (3), $y_2 = A\sin\omega t + B\cos\omega t$, $\boxed{\omega = \sqrt{\frac{k}{m}}}$

$y_2 = 0$ at $t=0$, $B=0$, $v_0 = a\omega$.

$\therefore y_2 = \frac{v_0}{\omega}\sin\omega t$, $y_1 = 0$ $\quad \therefore x_1 = x_2 = \frac{1}{2}y_2$

$= \frac{v_0}{2\omega}\sin\omega t$

$\ddot{y}_2 = -\frac{v_0}{\omega}\omega^2\sin\omega t = -v_0\omega\sin\omega t$