2024

PHYSICS — HONOURS — PRACTICAL

Paper : DSCC-4P

(Mathematical Physics - I)

Full Marks : 25

Time : 3 Hours

The figures in the margin indicate full marks.

[Program : 15, CNB : 5, Viva : 5]

Set - I

Answer *any one* question.

1. (a) Solve the following system of linear equations using Gauss-Seidel method :

$$\begin{cases} 4x_1 - x_2 + 2x_3 = 3 \\ 3x_1 + 6x_2 - 4x_3 = 2 \\ 2x_1 - 3x_2 + 5x_3 = -2 \end{cases}$$

Verify your result using the NumPy linalg module. 10

(b) Create a NumPy array of 20 elements between 0 and 10 using linspace(). Compute the cumulative sum of the array. Plot the cumulative sum array with respect to the original array. Put appropriate x label, y label and title. 5

2. (a) What is the advantage of RK4 method over RK2? Solve the following differential equation

$$\frac{dy}{dx} = \frac{1}{xy}$$

numerically by fourth order Runge-Kutta method with initial conditions $x_0 = 1$, $y_0 = 1$ in the range $x = 1$ to 5. The analytical solution of this equation is $y = \sqrt{1 + 2\log x}$. Plot Numerical solution as points and Analytical solution as line in a same graph by matplotlib. Put appropriate title, x-label, y-label and legend. 10

(b) Create a NumPy array A containing 10 evenly spaced values from 1 to 10. Reshape this into a 5×2 array B and print it. Compute the sum and product of elements along the columns as well as along the rows. Find the mean, maximum, minimum, standard deviation and variance of the values in B array. 5

3. (a) The following data represents the temperature variation over time :

Time (t)	0	1	2	3	4	5
Temperature (T)	30	45	50	52	48	40

Use Lagrange interpolation to estimate the temperature at $t = 2.5$ and print it. Plot the given data and the interpolated point with different marker and style.   10

(b) Create a 12 element NumPy array using 'NumPy arange'. Reshape it to 3×4 matrix, name it as matrix A. Make a Transpose of it, name it as matrix B. Multiply A and B. Print the matrix A, B and the product AB.   5

4. (a) Evaluate the complete elliptic integral of first kind

$$K(m) = \int_{0}^{\frac{\pi}{2}} \frac{dx}{\sqrt{1 - m\sin^2 x}}$$

by Trapezoidal method in the range $m = [0, 1)$ with step size 0.1. Plot $K(m)$ against m using matplotlib for the given range of m with line and bullets in a rectangular grid.   10

(b) The current (I) and voltage (V) data across a resistance is given by :

Current (I) [A]	0.1	0.2	0.3	0.5	0.7	1.0	1.2	1.5
Voltage (V) [V]	1.2	2.5	3.7	6.2	8.5	12.3	14.8	18.5

Plot the data, fit them using NumPy function and plot the fitted line. Calculate the resistance from the fitted parameters and print it.   5

5. (a) Using Simpson's 1/3 rule, compute and print

$$I = \int_0^\pi \sin x \, dx$$

for 3 different step sizes. Comment on the result.   10

(b) Save the two given matrices A and B as NumPy arrays. Print them. Convert them to 1D arrays using suitable NumPy methods. Square all elements of the 1D array corresponding to B then plot it with respect to the 1D array corresponding to A. Put appropriate axis labels.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, B = \begin{bmatrix} 10 & 11 & 12 \\ 13 & 14 & 15 \\ 16 & 17 & 18 \end{bmatrix}$$

5

6. (a) Solve the following system of linear equations using Gauss-Elimination (with partial pivoting) method :

$$\begin{cases} 2x + y - z + w = 5 \\ 3x - 2y + 4z - w = -3 \\ x + 3y - 2z + 2w = 4 \\ 4x - y + 5z - 3w = 6 \end{cases}$$

Verify your result using the NumPy linalg module.   10

(b) Plot $2x\sin 2x, \dfrac{1}{2}x\sinh\dfrac{x}{2}$ and $\dfrac{1}{2}x\cosh\dfrac{x}{2}$ in subplots arranged in a row. Set the range of x values from $-\pi$ to π. Label x and y axes. Put title as "My Plot".   5

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[Program : 15, CNB : 5, Viva : 5]

Set - II

Answer *any one* question.

1. (a) Solve the following system of linear equations using Gauss-Seidel method :

$$4x + y + 2z = 4$$
$$3x + 5y + z = 7$$
$$x + y + 3z = 3$$

Verify your result using the NumPy linalg module.

10

(b) Create a NumPy array of 20 elements between 21 and 40 using arange(). Reshape the array into a 10×2 matrix. Store the first and second columns of the matrix in two different arrays using array slicing. Plot the second array against first array.

5

2. (a) The second order ODE for vertically hanging mass spring system

$$m\frac{d^2x}{dt^2} = -kx - mg$$

where x is the displacement of the spring that varies with time t. Reduce it assuming $g = 1$ and $\frac{k}{m} = 1$. Solve it numerically by 4th order Runge-Kutta method with initial condition $t = 0$, $x = 0$, $v = 0.3$ up to $t = 30$. Plot the displacement versus velocity of the mass using matplotlib. Put title, legend, x-label, y-label.

10

(b) A vector [5, 2, 6] in cartesian co-ordinate system is rotated by an angle $\theta = 180°$ with the rotation

operator $R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$. Find the new vector and its magnitude.

5

3. (a) Generate a set of (x, y) data where x values are from 0 to 2.0 in a step 0.2 and y values follow $y = \sin(x^2)$. Now find the value of $\sin(0.9^2)$ using Newton's forward interpolation or Lagrange interpolation method. Plot the given data as line and markers and the interpolated data as a marker with different style. Put title, legend, x-label and y-label.

10

(b) Write a 20 element NumPy array using 'NumPy arange'. Reshape it to 5×4 matrix, name it as matrix A. Make a Transpose of it, name it as matrix B. Multiply A and B. Print the matrix A, B and the result.

5

4. (a) Using Simpson's $\frac{1}{3}$ rule, find the value of error function

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

for x in the range $[-3, 3]$. Plot erf(x) against x in the given range with rectangular grid. Draw X-axis and Y-axis. Put a suitable title and axis labels.   10

(b) The velocity (v) and the momentum (p) data of a moving particle of mass (m) is given by :

Velocity (v) $[ms^{-1}]$	1.1	2.1	3.1	4.1	5.1	6.1	7.1	8.1
Momentum (p) $[kg.ms^{-1}]$	5.6	10.4	15.7	20.4	25.5	30.3	35.6	40.5

Plot the data, fit them with NumPy function and plot the fitted line. Calculate the mass (m) from the fitted parameters and print it.   5

5. (a) Using Trapezoidal rule, compute and print

$$I = \int_1^4 \left(4x^3 + 1\right) dx$$

for 4 different values of step sizes.   10

(b) Save the two given matrices, A and B as NumPy arrays. Print them. Compute $C = A + B$. Convert A and C to 1D arrays using suitable NumPy methods. Plot the 1D array corresponding to C with respect to the 1D array corresponding to A. Put appropriate axis labels and title.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, B = \begin{bmatrix} 8 & 3 & 11 \\ 7 & 4 & 2 \\ 9 & 7 & 8 \end{bmatrix}$$

5

6. (a) Solve the following system of linear equations using Gauss Elimination Method :

$$\begin{cases} 3x + 2y - z = 5 \\ 2x - 3y + 4z = -3 \\ x + 4y + 3z = 7 \end{cases}$$

Verify your result using NumPy linalg module. 10

(b) Define a function $f(x) = x^2 + x$. Create an array for x consisting of numbers between 1 to 10 with interval 0.5. Create and print an array for the function $f(x)$. Plot $f(x)$ against x by matplotlib as line through scatter data. Label the axes. 5

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PHYSICS — HONOURS — PRACTICAL

Paper : DSCC-4P

(Mathematical Physics - I)

Full Marks : 25

Time : 3 Hours

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[Program : 15, CNB : 5, Viva : 5]

Set - III

Answer *any one* question.

1. (a) Solve the following system of linear equations using Gauss-Seidel method :

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

   Verify your result using the NumPy linalg module. 10

   (b) Use NumPy to create an array A of 9 evenly spaced values between 0 and 2π (inclusive) and then calculate the cosine of each of these values, storing the results in array B. Print the size of B. Plot the data in B against the data in A. Put a title and label the axes. 5

2. (a) Solve the first-order differential equation using the RK2/RK4 method :

$$\frac{dy}{dx} = x^2 + y, \, y(0) = 1$$

   Find $y(x)$ for $0 \le x \le 2$ with step size $h = 0.05$. Plot the solution curve. Plot the exact solution in the same window. The exact solution is given by

$$y(x) = -x^2 - 2x - 2 + 3e^x$$

   10

   (b) Create an array of 9 equispaced numbers from 1 to 5. Create another array containing 9 integers between 1 and 9. Reshape these two arrays into two 3×3 matrices A and B. Show that $(AB)^T = B^T A^T$. 5

3. (a) Consider the following dataset :

x	1	3	5	7	8	9	10	12	13
y	50	−30	−20	20	5	1	30	80	−10

Use Lagrange interpolation to find y at x = 6. Plot the given datapoints distinctly and the interpolated point in the same graph with different point style.   10

(b) Write a 24 element NumPy array using 'NumPy arange' starting from 1. Reshape it to 4×6 matrix, name it as matrix A. Print matrix A. Find the maximum and minimum of the first row and find the corresponding index.   5

4. (a) Estimate the value of π from the formula $\dfrac{\pi}{4} = \displaystyle\int_{0}^{1} \dfrac{dx}{1+x^2}$ using numerical integration by Trapezoidal method, for 4 different step sizes. Comment on the result.   10

(b) Consider the following dataset from a measurement :

x	0	1	2	3	4	5
y	0	0.8	0.9	0.1	−0.8	−1.0

Fit the data using NumPy polyfit module with a polynomial of degree 3. Plot the discrete data points and the fitted line by matplotlib and print the coefficients. Label X and Y axis. Put title as 'NumPy polyfit'. Put legends as 'Cubic Fit' to the line and 'Data Points' to the points.   5

5. (a) Using Simpson's 1/3 rule, compute and print

$$I = \int_{0}^{2} \left(x^3 + 2x^2 + 3\right) dx$$

for 4 different values of step sizes.   10

(b) Plot $f(x) = \cos^2(x^2) + 0.5x^2$ and $g(x) = \sin^2\left(x^2 + \dfrac{\pi}{2}\right) + x^2$ in same graph with two different linestyles with x-axis from −5 to 5. Put appropriate legend and title.   5

6. (a) Solve the following system of linear equations using Gauss Elimination and print the solution.

$$\begin{cases} 2x + y - z + 3w = 8 \\ x - 2y + 4z - w = -3 \\ 3x + 3y + z + 2w = 10 \\ 4x - y + 5z - 3w = 6 \end{cases}$$

Verify your result using the NumPy linalg module.

10

(b) Create a NumPy array of even numbers from 2 to 18. Create another array of odd numbers from 1 to 17. Reshape these two arrays into 3×3 matrices. Calculate and print the commutator between these two matrices.

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2023

PHYSICS — HONOURS — PRACTICAL

Paper : CC-5P

(Mathematical Physics - II)

Full Marks : 30

SET - I

[Program : 20, CNB : 5, Viva voce : 5]

Answer *any one* question.

1. (a) Write a Python code to solve the following system of linear equations using Gauss elimination method :

$$x + y + z = 3$$
$$x + 2y + 3z = 0$$
$$x + 3y + 2z = 3$$

Verify your result using corresponding function from NumPy linalg module. If both the results are same, explain why Gauss elimination preserves the determinant. Print the upper triangular matrix (transformed). [Code : 7+1+2, Output : 2]

(b) Compute $C = AB - BA$, where $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 9 & 8 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 & 4 \\ 4 & 5 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. Use suitable NumPy's linear algebra module. [Program : 7, Output : 1]

2. (a) Write a Python code to solve $y'' - y' - 2y = 4x^2$, $y(0) = 1$, $y'(0) = 4$ by using fourth-order Runge-Kutta (RK-4) Method. Plot numerical $y(x)$ vs. x using matplotlib and compare that with exact results: $y(x) = 2e^{-x} + 2e^{2x} - 2x^2 + 2x - 3$ by plotting in the same plot. Choose appropriate labels and legends for this. [Code : 10, Plot : 2+2+1]

(b) Given a NumPy array [[1,2,3,4,5], [1,4,9,16,25]], treat the first element as x-data and the second element as y-data. Plot y^2 vs. x through matplotlib. Label the axes as 'X-data' and 'Y-data' and set title as 'Experimental plot'. [Program : 3+2]

3. (a) Consider the given data set

x	0	2	4	6	8	10	12	14	16	18
y	3.6	15.3	29.7	54.6	70	91.8	120.6	152.5	163	174.9

Now plot the above data points using matplotlib. Using Lagrange interpolation method find the value of y for $x = 9.2$ and $x = 15.1$. Plot the interpolated points. Label the axes and set appropriate legend at suitable position.
[Code : 6+1+1, Plot : 3+3]

(b) Create an one-dimensional array of 9 elements. Reshape the array to (3×3) matrix 'A' form. Print the matrix 'A'. Find eigenvalues (λ_i) and corresponding eigenvectors (X_i) using NumPy linalg module.
[Code : 1+1+2+2]

4. (a) Using Simpson's one-third rule write a Python code to evaluate $\int_{-4}^{4} e^{-x^2} \cos x \ dx$ verify your results using corresponding function from Scipy library.
[Program : 8+2, Output : 1+1]

(b) Create an array 'XS' of $N = 100$ equally spaced numbers $x_0, x_1, ..., x_{N-1}$ between 0 and 1. Create another array 'YS' of $y_0, y_1, ..., y_{N-1}$, where $y_i = x_i(1 - x_i)$ for all i. Modify the array 'YS' by the rule $y_i = \mu(y_{i+1} + y_{i-1}) + (1 - 2\mu)y_i$ for $i = 1, ..., N - 2$, where $\mu = 10^{-6}$.
[Code : 2+2+4]

5. (a) Using trapezoidal rule, write a Python code to calculate (up to 3 decimal place accurate) :

$$\int_{0}^{1} \frac{4}{1+x^2} dx$$

Verify your result using the corresponding function from Scipy library.
[Program : 8+2, Output : 1+1]

(b) Given two matrices, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$. Write a Python code to confirm that $(AB)^T = B^T A^T$.
[Code : 8]

6. (a) Consider the following data set

$X=$ [−10.0, −9.0, −8.0, −7.0, −6.0, −5.0, −4.0, −3.0, −2.0, −1.0, 0.0,1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0]

$Y=$ [1.2, 4.2, 6.7, 8.3, 10.6, 11.7, 13.5, 14.5, 15.7, 16.1, 16.6, 16.0, 15.4, 14.4, 14.2, 12.7, 10.3, 8.6, 6.1, 3.9, 2.1]

Now fit a Gaussian curve $f(x) = ae^{-bx^2}$ to the given data set using curve fit function from scipy.optimize module. Hence find the parameters a and b. Plot the fitted curve along with data points using matplotlib. Label x and y axis. Set the appropriate legend at suitable location for fitted curve and data points. [Code : 6, Output : 2, Plot : 2+2]

(b) If your (x, y) data is given in the form : [(0,1), (1,3), (2,2), (3,5), (4,7), (5,8), (6,8)], using NumPy *extract the data in the form of x-list and y-list* and plot through matplotlib. Set x-limit on the x-axis between −1 and 7 and y-limit on the y-axis between 0 and 10. [Code : 5, Plot : 3]

2023

PHYSICS — HONOURS — PRACTICAL

Paper : CC-5P

(Mathematical Physics - II)

Full Marks : 30

SET - II

[Program : 20, CNB : 5, Viva voce : 5]

Answer *any one* question.

1. (a) Solve the following system of linear equations by Gauss elimination method :

   $10x_1 - 7x_2 + 3x_3 + 5x_4 = 6$

   $-6x_1 + 8x_2 - x_3 - 4x_4 = 5$

   $3x_1 + x_2 + 4x_3 - 11x_4 = 2$

   $5x_1 - 9x_2 - 2x_3 + 4x_4 = 7$

   Verify your result using corresponding function from NumPy linalg module.

   [Program : 9, Output : 2, Verification : 1]

   (b) Create a NumPy array within [−25, 25] having 11 elements. Omit the middle element i.e., '0'. Using matplotlib plot the following function in the same graph with appropriate legends :

   (i) $x + 2x^2 + 3x^3$ (ii) $e^x \big/ x$. Label the horizontal axis as 'X' and vertical axis as 'Y'. Put title of the graph as 'Test functions'.

   [Code : 2+2, Plot : 4]

2. (a) Write a Python code to solve the following differential equation by RK4 method

$\dfrac{d^2y}{dx^2} + y = 0$ with the boundary condition at $x = 0$, $y = 0$, and $\dfrac{dy}{dx} = 0.4.$.

Find the solution in the range $0 \le x \le 30$ for step size $h = 0.01$.

Plot (i) $y(x)$ vs. x

(ii) $\dfrac{dy(x)}{dx}$ vs. x.

Use appropriate labels and legends.   [Program : 8, Plot : 2+2]

(b) Using NumPy create a (4×4) matrix of 16 elements. Print (i) the matrix (ii) an array containing the sum of the elements of each row (iii) an array containing maximum of the elements from each column, using appropriate code.   [Code : 2+3+3]

3. (a) The speed of a train, running between two stations is measured at different distances from the starting station. If x is the distance in km. from the starting station, then $v(x)$, the speed (in km/hr) of the train at the distance x is given by the following table :

x	0	50	100	150	200	250
$v(x)$	0	60	100	120	80	0

Find the speed of the train at $x = 125$ by using Newton's forward interpolation or Lagrange interpolation method. Plot the 'position-velocity' data with matplotlib. Also show the interpolated point distinctly in the same graph. Label the axes and write 'Position-Velocity graph' as legend inside the graph.   [Code : 10, Output : 1, Plot : 2+1]

(b) Compute the integral, $I = \displaystyle\int_{-10}^{10} e^{-x^2} dx$, using the quad function from scipy package.

[Program : 5, Output : 1]

4. (a) Using Trapezoidal rule, write a Python code to calculate :

   (i) $\displaystyle\int_0^{0.5} \frac{1}{2}\left(5x^3 - 3x\right)dx$ (ii) $\displaystyle\int_{-2}^{+2} x^2 e^{-\frac{x^3}{3}}dx$

   taking the number of intervals as runtime input. For each case, verify your results using corresponding function from SciPy library. Also plot the integrand functions as $f(x)$ vs. x. in the range -10 to 10 using matplotlib in separate figures.

   [Code : 4, Verification : 2+2, Plot : 2+2]

   (b) Show using Python NumPy Linear algebra module $A = \begin{bmatrix} 2 & 4 \\ 0 & -2 \end{bmatrix}$ is similar to $B = \begin{bmatrix} +2 & 0 \\ -4 & -2 \end{bmatrix}$ using

   invertible matrix $P = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$ [Code : 6, Verification : 2]

5. (a) Using Simpson's one-third rule, write a Python code to find $I = \displaystyle\int_0^7 \sqrt{\dfrac{a + \ln x}{a - \ln x}}\,dx$ for three different

   values of a ={7, 5, 2}. Compute the integral using quad function from SciPy package and compare your result. Plot the integrand in the integration limit with appropriate label and legend.

   [Code : 8, Output : 2, Plot : 2]

   (b) The norm of a real valued square matrix A is defined as $\|A\| = \sqrt{Tr\left(A^T A\right)}$. Write a code that given

   two $n \times n$ matrices A and B will calculate their 'distance' $d(A, B) = \|A - B\|$. Use appropriate linalg module from NumPy.   [Program : 5, Output : 3]

6. (a) The pressure of a gas p and its volume v are related by an equation of the type $pv^\gamma = c$, where γ and c are constants. In an experiment the pressure of a certain mass of gas was kept fixed at certain levels and the corresponding volumes of the gas were measured. The results are tabulated below.

p	0.5	1.0	1.5	2.0	2.5	3.0
v	1.63	0.99	0.74	0.64	0.53	0.45

   Determine the value of γ and c by using curvefit function from scipy.optimize module. Plot the fitted curve along with data points using matplotlib. Set the appropriate legend at suitable location for fitted curve and data points.   [Code : 6, Output : 1+1, Plot : 2+2]

   (b) The (x, y) data is given in the form : [(12,23), (82,43), (13,54), (35,64), (63,25)]. Extract the data using NumPy in the form of x-list and y-list and plot by using matplotlib. Use appropriate labels.   [Program : 4, Plot : 2+2]

2023

PHYSICS — HONOURS — PRACTICAL

Paper : CC-5P

(Mathematical Physics - II)

Full Marks : 30

SET - III

[Program : 20, CNB : 5, Viva voce : 5]

Answer *any one* question.

1. (a) Write a Python code to solve the differential equation using fourth-order Runge-Kutta (RK4) method : $2xt\frac{dx}{dt} = t^2 + x^2$ with the initial condition $x(t = 1) = 0$. Plot your result using matplotlib and compare that with its analytical solution $x^2 = t^2 - t$ in the same plot. Choose appropriate labels and legends for this. [Code : 9, Plot : 3]

   (b) Create a NumPy array of 50 integers in the interval 100 to 300.

   (i) Print the array (ii) Replace elements with '0' which are divisible by 3 and print the new array (iii) Now slice the array into five smaller arrays with 10 elements and print the arrays (iv) Arrange the elements of the array in reverse order and find their sum. [Code : 4, Output : 4]

2. (a) Given the data

x	1.0	2.3	3.1	4.8	5.6	6.3
y	2.6	2.8	3.1	4.7	5.1	5.3

   Write a Python code to fit a linear function $y = ax + b$ against those data points using suitable function from SciPy. Optimize module. Print the best-fit values of the parameters a and b. Plot the data points and your best-fit function using matplotlib. [Code : 5, Output : 1+1, Plot : 3]

   (b) Given the matrix, $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. Find its eigenvalues and corresponding eingenvectors using

   NumPy's linalg module. [Program : 6, Output : 2+2]

3. (a) Using Simpson's 1/3 rule, write a Python code to compute (up to 4 decimal place) :

$$\int_0^3 \frac{x}{1+x^5} dx$$

Verify your result using the corresponding function from SciPy library.

[Program : 8+2, Output : 1+1]

(b) Create a one-dimensional NumPy array of numbers between [1 – 15] in integer steps. Reshape this into a (5, 3) 2D array. Treat this as a 5 × 3 matrix. Transpose the matrix and take product between the two and print the trace of the product matrix. Use appropriate linear algebra module from NumPy package. [Program : 2+2+2+2]

4. (a) Write a Python code to estimate the integral

$$I_{Trap} = \int_0^1 \sin^2 x \ dx$$

for number of intervals $N = 1, 10, 100, 10^3, 10^4, 10^5, 10^6$ using trapezoidal method. Now find the value of the integral (I_{quad}) using quad function from SciPy.

Estimate $\varepsilon = |I_{Trap} - I_{quad}|$ for different values of N.   [Program : 8, Output : 2+2+2]

(b) Find eigenvalues and eigenvectors of the matrix, $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. Use appropriate linear algebra module

from NumPy package.   [Code : 6]

5. (a) Write a Python script to solve for x_1, x_2, and x_3 following Gaussian elimination method.

$$\begin{bmatrix} 3 & 2 & -1 \\ 2 & -2 & 4 \\ -1 & 0.5 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$$

Hence solve the system of equations by matrix inversion. Use appropriate linear algebra module from NumPy package. [Program : 10, Output : 1+1]

(b) Given a set of data (in appropriate units) from a measurement. Plot the Temperature-Pressure data with matplotlib.

Time	0	20	40	60	80	100
Temperature	0.0002	0.0012	0.0060	0.0300	0.0900	0.2700

Label the axes and write 'Temperature-Pressure graph' as legend inside the graph.

[Code : 5+3]

6. (a) Using Lagrange or Newton Interpolation formula, write a Python code to find the value of $f(t = 50)$ from the following table :

t	15	30	45	60	75
$f(t)$	0.268	0.577	1	1.732	3.732

Compare your result using any one-dimensional interpolating function from NumPy or SciPy. Using matplotlib, plot the data as $f(t)$ vs. t as red rings. Also show the interpolating function in the same plot. Use appropriate labels and legends. Mention the value of $f(t)$ when $t \to 90$.

[Code : 8, Output : 2, Plot : 2]

(b) Create a linearly-spaced array within $[-10, 10]$. Plot the function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

using matplotlib for $\mu = -5, 5$ and $\sigma = 0.5$ in the same graph. Use appropriate legends and axis labels.

[Program : 6, Plot : 2]

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2023

PHYSICS — HONOURS — PRACTICAL

Paper : CC-5P

(Mathematical Physics - II)

Full Marks : 30

SET - IV

[Program : 20, CNB : 5, Viva voce : 5]

Answer *any one* question.

1. (a) For the data points

x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$f(x)$	−1.26	−1.10	−0.91	−0.67	−0.54	−0.32	−0.10	0.08	0.33	0.51

Find the value of $f(x)$ for $x = 0.58$ and 0.78 using Lagrange's interpolation formula and also the value of x for which $f(x) = -0.20$ using Python code. [Program : 8, Output : 1+1+2]

(b) Create a one-dimensional NumPy array of numbers between [0 - 14] in integer steps. Reshape this into a (5,3) 2D array. Treat this as a 5×3 matrix. Transpose the matrix and take product between the two and print the trace of the product matrix. Use appropriate linear algebra module from NumPy package. [Program : 2+2+2+2]

2. (a) Write a Python code (RK4) to solve the differential equation for the L-R series circuit having an impressed emf $E_0 \sin \omega t$.

$$L \frac{di}{dt} + R_i = E_0 \sin \omega t$$

where $L = 0.9, 3.0$; $R = 1, 10$; $E_0 = 10, 1$; and $\omega = 100, 314$, with the initial condition $i(t = 0) = 0$. Plot your solution as $i(t)$ vs. t ($0 \le t \le 10$) for those parameters using matplotlib in separate graph with proper labels and legends.   [Code : 8, Plot : 4]

(b) Create a linearly-spaced array within $[-5, 5]$. Plot the function

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

using matplotlib for $\mu = 0$ and $\sigma = 1, 0.5, 2$ in the same graph. Use appropriate legends and axis labels.   [Program : 6, Plot : 2]

3. (a) Compute the value of π from the formula : $\dfrac{\pi}{4} = \displaystyle\int_0^1 \dfrac{dx}{1+x^2}$ using Simpson's one-third rule to evaluate

with an accuracy of the order of 10^{-5}.   [Code : 4, Verify : 2, Accuracy : 2]

(b) Compute the following four functions analytically and plot them using matplotlib in the same plot.

$$f_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}\left(e^{-x^2}\right) \text{ for } n = 0, 1, 2, 3.$$

Take $x \in [-1, 1]$. (i) Label the axes as 'x' and '$f(x)$' (ii) Write title '$f(x)$ vs. x' (iii) Use different linestyle, line color and marker for different functions, (iv) Write legend for each functions at the appropriate location.   [Calculation : 2, Code : 4, Plot : 1+1+(1+1+1)+1]

4. (a) Compute the integral using Gaussian Quadrature (16 points)

$$\int_1^2 \frac{dx}{x} dx$$

Compute the integral using quad function from SciPy package and compare your result.

The nodes (x_i) and weights (w_i) are given below for 16-point Gaussian quadrature :

x_i	-0.989401	-0.944575	-0.865631	-0.755404	-0.617876	-0.458017	-0.281604	-0.095013
	0.0950125	0.2816	0.45802	0.617876	0.755404	0.865631	0.944575	0.989401
w_i	0.027152	0.062254	0.095159	0.124629	0.149596	0.169157	0.182603	0.18945
	0.189451	0.1826	0.169157	0.149596	0.12463	0.09516	0.062254	0.027152

[Program : 10+2, Output : 1+1]

(b) Let $A = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}$ the characteristic or eigenvalue equation is $x^2 - 6x + 13 = 0$. It is said according to Caley-Hamilton theorem that the matrix A satisfies this characteristic equation. Establish this numerically. Use appropriate linear algebra module from NumPy. [Program : 6]

5. (a) Write a Python code to solve the following system of linear equations using Gauss elimination method $2x_1 - x_2 + x_3 = -1$

$4x_1 + 2x_2 + x_3 = 4$

$6x_1 - 4x_2 + 2x_3 = -2$

Verify your result using corresponding function from NumPy module from linear algebra module.

[Program : 10, Output : 1+1]

(b) Given a NumPy array : $A = $ np.array([[0.5, 1, 1.5, 2, 2.5], [0.25, 1, 2.25, 4, 6.25]]), treat the first element as x-data and the second element as y-data. Plot y^3 vs. x through matplotlib. Label the axes as 'X-data' and 'Y-data' and set title as 'My plot'. [Program : 4+2+2]

6. (a) R is the resistance to motion of a train at speed V; fit a function of the type $R = a + bV^2$ connecting R and V, using the following data :

V(km/hr) :	10	20	30	40	50
R(kg/ton) :	8	10	15	21	30

Use curve fit function from scipy.optimize. Find the values of a and b. Plot the fitted curve along with data points using matplotlib module. Hence find the parameters a and b. Label x and y axes. Set the appropriate legend at the suitable location for fitted curve and data points.

[Program : 9, Output : 1+1, Plot : 4]

(b) Given a set of x-y data in the form of an array [1, 0.5, 2, 3.8, 3, 7.9, 4, 16.5, 5, 27.5] where the alternate elements are x and y. Prepare two separate lists for x and y and plot y against x with x-label as 'Time' and y-label as 'Temperature.

[Program : 3, Plot : 2]

2022

PHYSICS — HONOURS — PRACTICAL

[Syllabus : 2019-2020] (UR-47/19)

Paper : CC-5P

(Mathematical Physics - II)

Full Marks : 30

The figures in the margin indicate full marks.

Set - 1

[Program : 20, CNB : 5, Viva : 5]

Answer *any one* question.

1. (a) Write a Python code to solve the following system of linear equations using *Gauss elimination* method:

$$x_1 - x_2 + x_3 = 3$$
$$2x_1 + x_2 + 8x_3 = 18$$
$$4x_1 + 2x_2 - 3x_3 = -2$$

Verify your result using corresponding function from NumPy linalg module. Suppose the suitable initial matrix form for the above system is Ax = b and after applying the Gauss elimination, the final form becomes Ux = c, where U is the upper triangular matrix. Print the matrix U.

[Code : 7+1+2, Output : 3]

(b) Generate a NumPy array x within [−10,10]. Using matplotlib plot the following three function in the same graph with appropriate legends: (a) $x^2 + x^3$, (b) $x^5 e^x$ and (c) $x^5 e^{-x}$. Take the range of Y-axis as [−100, 100]. Label the X- and Y-axis as 'x' and 'test functions' respectively.

[Code: 3, Plot: 4]

2. (a) Given the data

x	1	3	5	7	9
$f(x)$	1	21	65	133	225

write a Python code to find the value of $f(x)$ for $x = 6$ using *Lagrange* or *Newton Forward Interpolation* formula using all the points. Using matplotlib plot the data points as $f(x)$ vs. x and label them as 'original data'. Also show the value of $f(6)$ as filled circle inside the same plot with a label 'test point'. Find out the polynomial for those data points using a Lagrange interpolating polynomial function from SciPy. Also plot that polynomial in the same graph (you can use numpy .polyval ( ) function for this) and label it as 'scipy interpolation'.

[Code : 8, Output : 3, Plot : 5]

(b) Using NumPy create a matrix

$$A = \begin{pmatrix} 3 & -2 & 1 \\ 1 & 0 & -3 \\ 4 & 2 & -3 \end{pmatrix}$$

Print (a) an array containing the sum of the elements of each column and (b) an array containing the minimum of the elements from each row. [Don't do these by hand, you need to use appropriate commands.]

[Code: 2+2]

3. (a) Using *trapezoidal* rule, write a Python code to calculate :

(i) $\int_{0}^{2.5} \left(1 + x^2 + x^4\right)^{1/4} dx$

(ii) $\int_{0}^{\pi/4} \sqrt{\tan x} \, dx$

taking the number of intervals as runtime input. For each case, verify your results using corresponding function from SciPy library. Also plot the integrand functions as $f(x)$ vs. x in the range of the given integration limits using matplotlib in separate figures.

[Code: 8, Output: 3, Verification: 2, Plot: 4]

(b) Solve the system of linear equations: $x + 2y - 7 = 0$ and $2x - 5y + 13 = 0$ using a suitable function from NumPy's linear algebra module. Verify your result using numpy. dot function.

[Code: 2, Verification: 1]

4. (a) Using *Simpson's one-third* rule, write a Python code to verify

$$\int_0^{2\pi} \frac{dx}{1 - 2a \cos x + a^2} = \frac{2\pi}{1 - a^2} \quad \text{for } 0 < a < 1$$

for three different values of $a = \{0.1, 0.5, 0.9\}$. For each case, compare your result with the exact value and verify that using corresponding function from SciPy library. Also plot the integrand functions as $f(x)$ vs. x in the range of the given integration limits using matplotlib for $a = 0.5$.

[Code: 8, Output: 3, Verification: 3, Plot: 2]

(b) Given a matrix

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

find its eigenvalues (λ_i) and corresponding eingenvectors (v_i) using NumPy linalg module. For any eigenvalue value, also verify that $Av = \lambda v$.

[Code: 3, Verification: 1]

5. (a) Using *fourth order Runge-Kutta* (RK4) method, write a Python code to solve the first-order differential equation

$$\frac{dy}{dt} = e^{-2t} - 3y$$

with the initial condition $y(0) = 5$. Plot your result using matplotlib and compare that with its analytical solution $y(t) = e^{-2t} + 4e^{-3t}$ in the same plot. Choose appropriate labels and legends for this.

[Code: 9, Plot: 3]

(b) Given the data

x	0	1	2	3	4	5	6
$y(x)$	1	3	2	5	7	8	8

write a Python code to fit a linear function $y = ax + b$ against those data points using suitable function from scipy . optimize module. Print the best-fit values of the parameters a and b. Plot the data points and your best-fit function using matplotlib.

[Code: 5, Output: 1, Plot: 2]

6. Write a Python code to solve the second-order differential equation for the periodically undamped forced harmonic oscillator using *fourth order Runge-Kutta* (RK4) method

$$\frac{d^2y}{dt^2} + ky = F\cos(\omega t)$$

with the initial conditions $y(0) = 0$ and $\frac{dy}{dt}(0) = 0$. Choose the parameters as $k = \omega_0^2 = 9$, $F = 1$. Plot your results as $y(t)$ vs. t ($0 \le t \le 60$) for four different values of $\omega = 3.5, 3.3, 3.1$ and 3 to see the beats and resonance. You should plot these four cases in separate figures with proper labels and legends.

[Code: 12, Plot: 8]

7. (a) Plot the following five functions in the same graph using matplotlib.

$$f_1(x) = x$$

$$f_2(x) = \frac{1}{2}(3x^2 - 1)$$

$$f_3(x) = \frac{1}{2}(5x^3 - 3x)$$

$$f_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$$

$$f_5(x) = \frac{1}{8}(63x^5 - 70x^3 + 15x)$$

Take $x \in [-1, 1]$. (i) Label the axes as 'x' and '$f(x)$'. (ii) Write title '$f(x)$ vs. x'. (iii) Use different linestyle, linecolor and marker for three different functions. (iv) Write legend for each functions at the appropriate location.

program : 6, Output : 1+1+1+1

(b) Given a list of θ-values (in radian) in [0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5] and the corresponding $\sin\theta$ values in [0, 0.48, 0.84, 1.0, 0.91, 0.60, 0.14, –0.35], use any kind of interpolation method to find out sin (1.8). Plot the data using matplotlib and label the axes.

program : 7+2, Output : 1

8. (a)

$$I = \int\limits_0^1 \frac{dx}{\sqrt{-\ln(x)}}.$$

Use Simpson's 1/3rd rule taking number of intervals as runtime input. Compute the integral using quad function from scipy package and compare your result. Plot the integrand.

Code : 7+2, Output : 1, Plot : 2

(b) Using an appropriate numpy function, create a one-dimensional array of 9 numbers from 1 to 2 and then reshape the array to make it a 3×3 array. Now treat this as a 'matrix'. Using linear algebra module in numpy, obtain the eigenvalues and eigenvectors.

Program : 3+3+2

2022

PHYSICS — HONOURS — PRACTICAL

[Syllabus : 2019-2020] (UR-47/19)

Paper : CC-5P

(Mathematical Physics - II)

Full Marks : 30

The figures in the margin indicate full marks.

$$\boxed{\text{Set - 2}}$$

[Program : 20, CNB : 5, Viva : 5]

Answer *any one* question.

1. (a) Using *Gauss elimination* method, write a Python code to calculate the determinant of the given matrix :

$$\begin{pmatrix} 1 & 2 & 1 \\ 3 & -1 & -3 \\ 2 & 3 & 1 \end{pmatrix}$$

Compare your result with the determinant calculated using corresponding function from NumPy linalg module i.e. linalg.det(). If both results are same, explain why Gauss elimination preserves the determinant.

[Code: 7+1+2, Output: 3]

(b) Suppose a survey has been done to calculate the average weight (in kg.) of kids belonging to two different groups (say, Group A and Group B). Below are the findings of that survey in terms of their average ages :

Average age (in yrs.)	5	6	7	8	9	10	11	12	13	14	15
Avg. weight (Gr. A)	16.6	17.8	18.1	18.7	19.3	21.5	23.6	25.8	29.0	32.3	33.5
Avg. weight (Gr. B)	18.7	20.7	22.9	25.3	28.1	31.4	32.2	37.0	40.9	47.0	52.6

Using matplotlib plot average weight vs. average age as 'scatter plot' with different colors and/ or markers for those two different groups (use appropriate legends). Label both axes accordingly.

[Plot: 5, Label: 2]

2. (a) Using *Lagrange* or *Newton Forward Interpolation* formula, write a Python code to find the value of cos(50°) from the following table :

θ	30°	45°	60°	75°	90°
cos θ	0.866	0.707	0.5	0.259	0

Compare your result using any one-dimensional interpolating function from NumPy or SciPy. Using matplotlib, plot the data as cosθ vs. θ as filled red circles. Also show the interpolating function in the same plot. Use appropriate labels and legends. Mention one limitation where we *cannot* use Newton Forward Interpolation method.

[Code: 8, Output: 3, Plot: 4]

(b) Using NumPy, create an one-dimensional array A of size 10 with values ranging from 0 to 1, **both excluded**. Find out the arithmetic mean (average), standard deviation and variance of those array elements. Also write a command to print the array A in *reverse* order.

[Code: 1+1+1+1+1]

3. (a) Using *trapezoidal* rule, write a Python code to calculate the value of A from

$$A \int_{-\infty}^{\infty} e^{-\frac{(x-1)^2}{2}} dx = 1.$$

For ∞ limits, you can take large numbers such as -10 to 10. Verify your result using a function from SciPy library which implements *Gaussian quadrature* method for integration. Also compare your result with the exact value of that integration. Plot the integrand functions as $f(x)$ vs. x in the range of the given integration limits using matplotlib.

[Code: 6, Output: 2, Verification: 2, Plot: 2]

(b) **Matrix diagonalization:** Given a matrix

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & 0 \\ 3 & 0 & 1 \end{pmatrix}$$

print its eigenvalues (λ_i) and corresponding eigenvectors (v_i) using NumPy linalg module. Construct a matrix S that has the eigenvectors v_i as its columns, such as

$$S = \begin{pmatrix} \vdots & \vdots & \vdots \\ v_1 & v_2 & v_3 \\ \vdots & \vdots & \vdots \end{pmatrix}.$$

For this step, you can use numpy.vstack $((v_1, v_2, v_3))$ function and then transpose it. Find the S^{-1} matrix using linalg module. Finally calculate $S^{-1} AS$ and check whether it is diagonal or not. For the matrix multiplication you can use @ operator (valid for Python version 3.5 or higher) or any suitable function from NumPy.

[Code: 1+1+1+2, Output: 3]

4. (a) Using *Simpson's one-third* rule, write a Python code to calculate

$$\text{(a)} \int_0^{2\pi} x^2 \sin(2x)dx, \quad \text{(b)} \int_{-5}^{5} e^{-x^2/2}dx$$

taking the number of interval as runtime input. For each case, verify your results using corresponding function from SciPy library. Also plot integrand functions as $f(x)$ vs. x in the range of the given integration limits using matplotlib in separate figures.

[Code: 8, Output: 3, Verification: 2, Plot: 4]

(b) Write a code using NumPy to create a diagonal matrix whose diagonal elements are 1, 3, 5, 7. Find the trace of that matrix.

[Code: 2+1]

5. (a) Following is a table of data of how a typical virus spreads from one person to many over few days since the first infection detected :

No. of days since 1st detection	0	20	40	60	80	100	120	140	160	180	200
Total no. of infected people	1	2	4	8	20	40	60	70	75	76	77

Our job here is to find a fitting model to gain knowledge on the nature of evolution of this infection and predict the future scenarios. For that, take a model named *Logistic model* which has been widely used to describe the growth of a population:

$$F(x,a,b,c) = \frac{c}{1+e^{-(x-b)/a}}.$$

It is very likely that the infection should end someday in the future after most of the people develop proper immunity and that's why this model seems to be more realistic as it follows a rapid growth at early stages and then saturates. Write a Python code to fit this function $F(x, a, b, c)$ as total number of infected people with x being the number of days since first detection against those data points using scipy. optimize module. Print the best-fit values of the parameters a, b and c. Plot the data points and your best-fit function using matplotlib.

[Code: 7, Output: 1, Plot: 3]

(b) Numerically solve the first-order linear differential equation using *fourth order Runge-Kutta* (RK4) method

$$\frac{dy}{dx} + 2y = 0$$

with the initial condition $y(0) = 3$. Plot your result and compare that with its analytical solution $y(x) = 3e^{-2x}$ in the same graph using matplotlib.

[Code: 7, Plot: 2]

6. Write a Python code to solve the second-order differential equation for the damped harmonic oscillator using *fourth order Runge-Kutta* (RK4) method

$$\frac{d^2 y}{dt^2} + \lambda \frac{dy}{dt} + ky = 0$$

with the initial conditions $y(0) = 0$ and $\frac{dy}{dt}(0) = 1$. Choose the parameters as $k = 2$ and three different values of $\lambda = \{0.1, 0.5, 2\}$. Plot your solution as $y(t)$ vs. t $(0 \leq t \leq 20)$ for those parameter sets using matplotlib in separate graphs with proper labels and legends.

[Code: 12, Plot: 8]

7. (a) Write a Python script to solve for x_1, x_2 and x_3 following Gaussian elimination method.

$$\begin{bmatrix} 2 & 3 & -2 \\ 1 & -6 & 4 \\ 4 & -1 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

Also solve the system of equations by matrix inversion. Use appropriate linear algebra module from Numpy package.

Gaussian Elimination: 8, Numpy Code: 4

(b) Create a one-dimensional numpy array of numbers between [0 - 14] in integer steps. Reshape this into a (5, 3) 2D array. Treat this as a 5×3 matrix. Transpose the matrix and take product between the two and print the trace of the product matrix. Use appropriate linear algebra module from Nympy package.

Program: 2+2+2+2

8. (a) The speed of a train, running between two station is measured at different distances from the starting station. If x is the distance in *km* from the starting station, then $v(x)$, the speed (in *km/hr.*) of the train at the distance x is given by the following table :

x	0	50	100	150	200	250
$v(x)$	0	60	80	110	90	0

Find the approximate speed of the train at the mid point between the two stations by using Newton's forward interpolating polynomial. Plot the 'position-velocity' data (including the interpolated point) with matplotlib with data-points as discrete symbols and lines. Label the axes and write 'Position-Velocity graph' as legend inside the graph.

Main Code: 9+1+2+1+1+1

(b) It is given that $P = \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}$ and $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. Do a similarity transformation [Similarity transform: $B = P^{-1}AP$] on A by P and find the new matrix B. Use Numpy linear algebra module.

Main Code: 4, Output: 1

2022

PHYSICS — HONOURS — PRACTICAL

[Syllabus : 2019-2020] (UR-47/19)

Paper : CC-5P

(Mathematical Physics - II)

Full Marks : 30

The figures in the margin indicate full marks.

Set - 3

[Program : 20, CNB : 5, Viva : 5]

Answer *any one* question.

1. (a) Write a Python code to solve the following system of linear equations using *Gauss elimination* method :

$$10x_1 - x_2 + 2x_3 = 6$$
$$-x_1 + 11x_2 - x_3 + 3x_4 = 25$$
$$2x_1 - x_2 + 10x_3 - x_4 = -11$$
$$3x_2 - x_3 + 8x_4 = 15$$

Verify your result using corresponding function from NumPy linalg module. Mention one scenario where Gauss elimination method cannot be used.

[Code: 7+1+1, Output: 3]

(b) Create a linearly-spaced NumPy array within [0.01, 3]. Using matplotlib plot the function

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right)$$

for $\mu = 0$ and 3 different $\sigma = 0.25, 0.5, 1$. Superpose these 3 curves in a single figure with different colors and linestyles. Use appropriate legends and axis labels. Next, plot the same 3 curves but now with the X-axis in log-scale. Can you identify these curves?

[Plot: 6, Label: 2]

2. (a) Consider a function $f(x) = e^{-x/2}$. Generate 6 data points i.e. $f(x_i)$ from this function at $x_i = \{0, 2, 4, 6, 8, 10\}$. Write a Python code to find the value of $f(5)$ using *Lagrange* or *Newton Forward Interpolation* formula using all those data points. Compare your result using any one-dimensional interpolating function from NumPy or SciPy. Using matplotlib, plot the data points as well as that interpolating function in a same graph with proper labels and legends. Also show the value of $f(5)$ in that graph.

[Code: 8, Output: 3, Plot: 3]

(b) Create an one-dimensional array of size 12 with the integers ranging from 1 to 12 (both included). Reshape the array to form a 3 × 4 matrix A. Print the matrices A and A^T. Print their shapes, sizes and data-types. Then find the matrix $C = AA^T$ (you can use numpy.matmul() function). Finally flatten the matrix C into an one-dimensional array and print it.

[Code: 1+1+1+1+1+1]

3. (a) Using *trapezoidal* rule, write a Python code to calculate

$$\text{(a)} \int_{-4}^{4} e^{-x^2} \cos x \, dx, \quad \text{(b)} \int_{0}^{10} \frac{\ln(1+x)}{1+x} dx$$

taking the number of interval as runtime input. For each case, verify your results using corresponding function from SciPy library. Also plot the integrand functions as $f(x)$ vs. x in the range of the given integration limits using matplotlib in separate figures.

[Code: 8, Output: 3, Verification: 2, Plot: 4]

(b) Using matplotlib, plot the function

$$f(t) = \begin{cases} 1-|t|, & \text{if } |t| \le 1 \\ 0, & \text{otherwise} \end{cases}$$

in the range $-2 \le t \le 2$. Label the axes accordingly.

[Plot: 2+1]

4. (a) Using *Simpson's one-third* rule, write a Python code to verify

$$\int_0^\pi \sin(mx)\cos(nx)dx = \begin{cases} 0 & \text{if } m+n \text{ is even} \\ \dfrac{2m}{m^2-n^2} & \text{if } m+n \text{ is odd} \end{cases}$$

for three different sets of m, n (take m, n to be positive integers). For each case, verify your results using corresponding function from SciPy library. Also plot the integrand functions as $f(x)$ vs. x in the range of the given integration limits using Matplotlib for all three cases in separate figures.

[Code: 7, Output: 3, Verification: 2, Plot: 3]

(b) Given two matrices $A = \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 2 \\ 1 & -4 \end{pmatrix}$, write a Python code to verify that $(AB)^T = B^T A^T$

(T denotes the transpose of matrix).

[Code: 3, Output: 2]

5. (a) Write a Python code to solve the first order differential equation using *fourth order Runge-Kutta* (RK4) method

$$\frac{dy}{dx} - y(1-2x) = 0$$

with the initial condition $y(0) = 1$. Plot your result and compare that with its analytical solution $y(x) = e^{x-x^2}$ in the same graph using matplotlib.

[Code: 9, Plot: 3]

(b) Given a matrix

$$A = \begin{pmatrix} 0 & 1 & 3 & 0 \\ -2 & 3 & 0 & 4 \\ 0 & 0 & 6 & 1 \\ 0 & 0 & 1 & 6 \end{pmatrix}$$

write a code to print its eigenvalues (λ_i) and corresponding eigenvectors (v_i) using NumPy linalg module. Also verify that

$$\sum_i \lambda_i = \text{sum of eigenvalues} = \text{Trace}(A) \text{ and } \prod_i \lambda_i = \text{product of eigenvalues} = \det(A).$$

[Code: 5, Verification: 3]

6. (a) Consider the given data set

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f(x)$	31.3	19.8	10.9	4.64	1.09	0.00	0.890	3.04	5.49	7.04	6.25

Use Lagrange interpolation method to find out $f(2.5)$. Plot the data along with the interpolated points using matplotlib and label the axes. Set appropriate legend at suitable location.

Program: 9+1+1+1+1

(b) If your (x, y) data is given in the form : [(1,2), (2,3), (3,8), (4,12), (5,20)], using numpy extract the data in the form of x-list and y-list and plot through matplotlib. Set x-limit on the x-axis between 0 and 6 and y-limit on the y-axis between 0 and 25.

Program : 5+2

7. (a)

$$I = \int_{-1}^{1} \sqrt{\frac{1+x}{1-x}}\,dx.$$

Use Simpson's 1/3rd rule taking number of intervals as input. Compute the integral using quad function from scipy package and compare your result. Plot the integrand and shade the region $x \in [-1, 1]$ with color.

Code: 7+2, Output: 1, Plot: 2

(b) Given the array of numbers [2.0, -1.2, 3.4, 9.1, 0.1, -5.8, -4.2, 3.9, 10.4, 1.9, -3.8, -9.6], take numbers with index no. 4 to 8 (end elements included) by slicing. Check if the sum of this slice array of numbers is more or less than the sum of all the numbers in the original array.

Code: 7, Output: 1

8. (a) Consider the following data:

x	0	1	2	3	4	5	6	7	8	9	10
y	4.2	17.6	31.2	57.4	71.8	94.1	122.5	129.2	151.7	179.3	190.2

Now fit a linear equation $f(x) = ax + b$ to the given data set using curve fit function from scipy.optimize module. Hence find the parameters a and b. Plot the fitted curve along with data points using matplotlib. Set the appropriate legend at suitable location for fitted curve and data points.

program: 5+1+1+1+1+1

(b) Consider the integral

$$E(x) = \int_0^x e^{-t^2} \, dt.$$

With a program a calculate $E(x)$ for value of x from 0 to 3 in steps of 0.1. Choose the trapezoidal method for performing the integral and choose a suitable number of slices. Compute the integral using quad function from scipy package and compare your result. Make a graph of $E(x)$ as a function of x. Label x and y axis. Set appropriate legend at suitable location.

program: 5+1+1+1+1+1

2019

PHYSICS — HONOURS — PRACTICAL

Paper : CC-5-P

Full Marks : 30

Set-1

Date : 26/11/2019

[Program = 20, CNB = 5, Viva = 5]

Answer *any one* question.

1. (i) Create a one-dimensional array of 50 random numbers in the interval : [0, 2] with the help of an appropriate function from random module in numpy. Compute the standard deviation (σ) of those numbers. Use the formula $\sigma = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$

    (ii) Let an ellipse be given by $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, where we assume that $a < b$. Compute the parameter (L) of

    the ellipse where, $L = 4b \displaystyle\int_{0}^{\pi/2} \sqrt{1 - k^2 \sin^2 \theta} \, d\theta$, where $k^2 = 1 - \dfrac{a^2}{b^2}$. The values of a and b may be

    supplied or chosen by you. Use composite Simpson's 1/3rd rule to evaluate.

    (i) Program : 4, Output : 1

    (ii) Algorithm : 3, Program : 10, Output : 2

2. (i) Plot the following three functions in the same graph using matplotlib : $y = x^2, y = e^{x/2}, y = 5x$. Take $x \in [0, 10]$.

    Label the axes as 'X' and 'Y' and write title 'My Graph'.

    (ii) Write a Python script to determine one root of the equation :

    $x^4 - 1.99x^3 - 1.76x^2 + 5.22x - 2.23 = 0$ which is close to $x = 1.5$. Use bisection method.

    (i) Program : 6, Output : 2

    (ii) Algorithm : 3, Program : 7, Output : 2

3. (i) Create a one-dimensional numpy array with 20 random numbers between $[-1, 1]$. Reshape this into a $(4, 5)$ 2D array. Treat this as a 4×5 matrix. Transpose the matrix and take product between the two. Print the product matrix.

   (ii) Solve : $\frac{dy}{dx} + y = x$, $y(0) = 1$ by Euler method to find $y(10)$.

   Plot y vs. x in the range $[0, 10]$ using matplotlib. Label the axes as 'X-axis' and 'Y-axis'.

      (i) Program : 5, Output : 1

      (ii) Algorithm : 2, Program : 8, Output : 1, Plot : 3

4. (i) Given the array of numbers $[2.0, -1.2, 3.4, 9.1, 0.1, -5.8, -4.2, 3.9, 10.4, 1.9, -3.8, -9.6]$, take numbers with index no. 4 to 8 (end elements included) by slicing. Check if the sum of this slice array of numbers is more or less than the sum of all the numbers in the original array.

   (ii) Solve the following 1st order differential equation by Euler method :

$$\dot{u} = \left(1 - \frac{4}{3}t\right)u, \quad u(0) = 1.$$

   Plot the solution by matplotlib along with the exact result : $u(t) = \exp\left(t - \frac{2}{3}t^2\right)$.

      (i) Program : 4, Output : 2

      (ii) Program : 8, Plot : 3+3

5. (i) Given a numpy array $[[1, 2, 3, 4, 5], [0, 1, 2, 3, 4]]$ treat the first element as x-data and the second element as y-data. Plot y^2 vs. x through matplotlib. Label the axes as 'X-Data' and 'Y-Data' and set title as 'Experimental plot'.

   (ii) A ball at 1200 K is allowed to cool down in air at an ambient temperature of 300 K. Assuming heat is lost only due to radiation, the differential equation for the temperature of the ball is given by

$$\frac{d\theta}{dt} = -3.58 \times 10^{-12}\left(\theta^4 - 81 \times 10^8\right), \quad \theta(0) = 1200\text{K}$$

   where θ is in K and t in seconds. Find the temperature at $t = 480$ seconds using Euler method. Assume a step size $h = 10$ seconds.

      (i) Program : 4, Plot : 2

      (ii) Algorithm : 2, Program : 10, Output : 2

6. (i) Use the function, simps() or quad() to find out the following integral :

$$I = \int_0^{4.5} J_{2.5}(x)\,dx$$

The integrand is a Bessel function which you can obtain from importing the scipy.special module. Check the result and compare with the following true value,

$$I = \sqrt{\frac{2}{\pi}} \left(\frac{18}{27} \sqrt{2} \cos(4.5) - \frac{4}{27} \sqrt{2} \sin(4.5) + \sqrt{2\pi}.\, Si\left(\frac{3}{\sqrt{\pi}}\right) \right), \quad \text{where}$$

$Si(x) = \int_0^x \sin\left(\frac{\pi}{2} t^2\right) dt$ is the Fresnel Sine Integral, using the functions in special module.

[Hint : Use $jn(n, x)$, $n = 2.5..$]

(ii) Compute the value of π from the formula :

$$\frac{\pi}{4} = \int_0^1 \frac{dx}{1+x^2}\,.$$

Use Simpson's $1/3^{rd}$ rule to evaluate with an accuracy of 10^{-4}.

(i) Program : 8, Output : 2

(ii) Algorithm : 2, Program : 7, Output : 1

7. (i) For a random variate x, generate an array of 50 random values between 0 and 1 using numpy array. Find the relative fluctuation $\sqrt{\langle x^2 \rangle - \langle x \rangle^2} \big/ \langle x \rangle$.

(ii) Generate (x, y) data where x-values are from 0 up to 2.0 in the equal interval of $\Delta x = 0.2$ and y-values follow $y = \sin(x^2)$. Now you have to find out $\sin(0.9^2)$ from the data set through Lagrange interpolation.

(i) Program : 4, Output : 2

(ii) Algorithm : 2, Program : 10, Output : 2

8. (i) Consider a vector, $r = (2, 1, 3)$ in the XYZ coordinate system. The vector is rotated by $\theta = 35°$ with

the following rotational operator : $R = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$. Find the new vector.

(ii) Using Newton-Raphson method, find the root of the equation $x^3 + (x + 1)^2 + 4x = 20$ that lies in the range $1.0 < x < 2.5$, correct up to 4 decimal places. Comment on the other roots by plotting the function using matplotlib.

   (i) Program : 4, Output : 2

   (ii) Algorithm : 2, Program : 8, Output : 2, Comment : 2

2019

PHYSICS — HONOURS — PRACTICAL

Paper : CC-5-P

Full Marks : 30

Set-2

Date : 27/11/2019

[Program = 20, CNB = 5, Viva = 5]

Answer *any one* question.

1. (i) Using an appropriate numpy function, create a one dimensional array of 9 numbers from 1 to 2 and then reshape the array to make it a 3 × 3 array. Now treat this as a 'matrix'. Obtain a symmetric matrix of the same order using this matrix. Using linear algebra module in numpy, obtain the eigenvalues and eigenvectors. Check, if the eigenvectors are orthogonal to each other or not.

   (ii) The equation for radioactive decay : $\frac{dM}{dt} = -\lambda M$, with $\lambda = 2$, $M(0) = 100$. Solve this by Euler method. Store the output in lists and plot the solution through matplotlib. Label the axes as 'Time' and 'Mass'.

   (i) Program : 6, Output : 2

   (ii) Algorithm : 2, Program : 6, Output : 1, Plot : 3

2. (i) Given two matrices, $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$, write steps to confirm that $(AB)^T = B^T A^T$.

   (ii) Compute the following integral to verify the expression : $\int_0^\pi \frac{x}{x^2+1} \cos\left(10x^2\right) dx = 0.0003156$.

   Use composite Simpson's 1/3rd rule. Comment on how your solution can be improved.

   (i) Program : 6, Output : 2

   (ii) Algorithm : 2, Program : 7, Output + Comment : 2+1

3. (i) Compute $C = AB - BA$, where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \ B = \begin{pmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{pmatrix}$$

   (ii) Use the Newton-Raphson method to find the smallest and the second smallest positive roots of the equation $\tan\theta = 4\theta$, correct up to 4 decimal places.

      (i) Program : 5, Output : 1

      (ii) Algorithm : 2, Program : 8, Output : 2+2

4. (i) Create a numpy array of 100 random numbers between [0, 1] and arrange the numbers in ascending order. Isolate the first 50 numbers to a separate array and the last 50 numbers to another array. Now compute the difference of the averages of the two arrays.

   (ii) Approximate $\int_{1}^{3} e^{x^2} dx$ using Simpson's 1/3rd rule for $n = 8$, where

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
$x =$	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3

      (i) Program : 5, output : 2

      (ii) Algorithm : 2, Program : 8, Output : 3

5. (i) Numerically show that the following identity holds :

$$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$$

   Here $P_n(x)$ is n-th order Legendre polynomial. Use appropriate modules and functions in Python to show this for $n = 4$ and 5.

   (ii) Evaluate error function $erf(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-t^2} dt$ for a set of x-values between [− 4, 4] and plot. Use

   Simpson's 1/3rd rule explicitly to evaluate the integral.

      (i) Program : 4, Output : 2

      (ii) Algorithm : 2, Program : 10, Output : 2

6. (i) Consider a vector, $r = (2, 1, 3)$ in the XYZ coordinate system. The vector is rotated by $\theta = 45°$ with

the following rotational operator : $R = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$. Find the normalized new vector.

(ii) Solve : $y' + 2y = 2 - e^{-4t}$, $y(0) = 1$ by Euler method.

Exact Solution $y(t) = 1 + \frac{1}{2} e^{-4t} - \frac{1}{2} e^{-2t}$.

Plot the numerical solution and the exact solution on the same graph by matplotlib. Label the axes as 'Time' and 'Displacement'.

(i) Program : 5, Output : 1

(ii) Algorithm : 2, Program : 8, Plot : 4

7. (i) Let $A = \begin{pmatrix} 1 & -2 \\ 4 & 5 \end{pmatrix}$, where the characteristic (eigenvalue) equation is $x^2 - 6x + 13 = 0$. It is said (by

Caley-Hamilton theorem) that the matrix A satisfies this characteristic equation. Establish this numerically.

(ii) Find $\sqrt{10}$ by bisection method.

(i) Program : 5, Output : 1

(ii) Formulation : 2, Algorithm : 2, Program : 8, Output : 2

8. (i) Establish the following identity numerically :

$$1 = \left[J_0(x)\right]^2 + 2 \sum_{k=1}^{\infty} \left[J_k(x)\right]^2,$$

where J_k's are Bessel functions of first kind. Use scipy.special module.

(ii) Estimate the integral with accuracy level of 0.0001 by Simpson's $1/3^{rd}$ method :

$$I = \int_0^1 \left(\frac{3}{x^2\sqrt{2\pi}}\right) e^{-9/(2x^2)} dx$$

(i) Program : 4, Output : 2

(ii) Algorithm : 3, Program : 9, Output : 2

2019

PHYSICS — HONOURS — PRACTICAL

Paper : CC-5-P

Full Marks : 30

Set-3

Date : 28/11/2019

[Program = 20, CNB = 5, Viva = 5]

Answer *any one* question.

1. (i) A generating function of Legendre polynomial $P_n(x)$ is given by

$$g(t, x) = \left(1 - 2xt + t^2\right)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)t^n$$

Numerically and approximately verify this by taking one chosen set of values of x and t. Use the scipy.special module to obtain Legendre polynomials.

(ii) Estimate $f(15)$ from the following table of values :

x	10	12	14	16	18	20	22
$y = f(x)$	46	66	81	93	77	53	36

Use Lagrange interpolation formula.

(i) Program : 6, Output : 2

(ii) Algorithm : 2, Program : 8, Output : 2

2. (i) Create a one-dimensional numpy array with 15 random numbers between [– 2, 2]. Reshape this into a (5, 3) 2D array. Treat this as a 5 × 3 matrix. Transpose the matrix and take product between the two and print the trace of the product matrix.

(ii) The temperature θ of a well stirred liquid by the isothermal heating coil is given by the equation :

$\frac{d\theta}{dt} = K(100 - \theta)$, where K is a constant of the system. Write a computer program to solve the equation by Euler method to find θ at $t = 1.0$ sec for $K = 2.5$. Initial condition : $\theta = 25°$ C at $t = 0$ sec.

(i) Program : 7, Output : 1

(ii) Algorithm : 2, Program : 8, Output : 2

3. (i) Create two 1D random arrays to length 10 each, through numpy. Print the two arrays. Compare the two arrays and check if they are same. Find the mean values of the two arrays of numbers.

(ii) Evaluate the following differential equation by Euler method :

$$e^y \frac{dy}{dx} + x^2 y^2 = 2\sin(3x), \ y(0) = 5$$

Print the output (x, y) in separate lists, taking the range of x in [0, 5] with step 0.2.

   (i) Program : 4, Output : 2
   (ii) Algorithm : 2, Program : 10, Output : 2

4. (i) Generate 100 random integers between 1 to 100 from an appropriate function in random module in numpy. Now treat the first 10 values as x-data and the last 10 values as y-data. Plot x-y through matplotlib as scattered points with symbols. Set the symbol size as 14 point.

(ii) Calculate the value of the elliptic integral of the first kind :

$$K(0.25) = \int\limits_0^{\pi/2} \frac{dx}{\sqrt{1 - 0.25\sin^2 x}}$$

Divide the interval [0, $\pi/2$] into 1000 equal parts and use composite Simpson's 1/3rd rule to evaluate the integral.

   (i) Program : 4, Plot : 2
   (ii) Algorithm : 2, Program : 10, Output : 2

5. (i) Establish the following identity numerically,

$$\int\limits_{-1}^{1} P_n(x)P_m(x)dx = \frac{2}{2n+1}\delta_{mn},$$

where P_n's are the Legendre polynomials. Utilize the Legendre polynomials and integration function from the suitable modules.

(ii) The period of a pendulum of length L oscillating at a large angle α is given by

$$T = \frac{2T_0}{\pi} \int\limits_0^{\pi/2} \frac{d\varphi}{\left(1 - \sin^2 \alpha/2 \sin^2 \varphi\right)^{1/2}}.$$

Now, write a script to solve with Simpson's 1/3rd rule. Calculate the ratio T/T_0 for $\alpha = 45°$.

   (i) Program : 6, Output : 2
   (ii) Algorithm : 2, Program : 8, Output : 2

6. (i) Generate 12 random numbers between 0 to 1 using numpy, reshape it into a 3 × 4 array, call it A. Now reshape the same set of numbers into a 4 × 3 array, call it B. Now treat A and B as matrices. Compute $C = AB$.

   (ii) Given the system of equations :

$$4x + y + 2z = 4$$
$$3x + 5y + z = 7$$
$$x + y + 3z = 3$$

   Write a Python script to solve for x, y, z following Gauss-Seidel method.

   (i) Program : 4, Output : 2

   (ii) Algorithm : 2, Program : 10, Output : 2

7. (i) Given a set of data (in appropriate units) from a measurement,

Temperature	0	20	40	60	80	100
Pressure	0.0002	0.0012	0.0060	0.0300	0.0900	0.2700

   Plot the Temperature-Pressure data with matplotlib with data-points as discrete symbols and lines. Label the axes and write 'Temperature-Pressure graph' as legend inside the graph.

   (ii) Create 10 x-data values in the interval [0, 1] by numpy, with equal interval. Now, consider y-values are given by $y = 4x^3$. Using forward difference formulas,

   $y' = (f(x + \Delta x) - f(x - \Delta x))/(2\Delta x)$ and

   $y'' = (f(x + \Delta x) - 2f(x) + f(x - \Delta x))/\Delta x^2$, obtain the arrays for 1st and 2nd order differentials and plot them in separate graphs by matplotlib.

   (i) Program : 4, Plot : 2

   (ii) Program : 8, Plot : 6

8. (i) It is given that $P = \begin{pmatrix} 1 & 4 \\ -1 & 1 \end{pmatrix}$ and $A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$. Do a similarity transformation on A by P and find the new matrix B. [Similarity Transform : $B = P^{-1} AP$]

(ii) Calculate the following integral by composite Trapezoidal rule : $\int_0^5 \frac{1}{1+x^2}\, dx.$

Verify that the answer is $\arctan(5)$.

(i) Program : 4, Output : 2

(ii) Algorithm : 2, Program : 8, Output : 4

2019

PHYSICS — HONOURS — PRACTICAL

Paper : CC-5-P

Full Marks : 30

Set-4

Date : 29/11/2019

[Program = 20, CNB = 5, Viva = 5]

Answer *any one* question.

1. (i) If your (x, y) data is given in the form : $[(1, 2), (2, 3), (3, 8), (4, 12), (5, 20)]$, using numpy extract the data in the form of x-list and y-list and plot through matplotlib. Set x-limit on the x-axis between 0 and 6 and y-limit on the y-axis between 0 and 25.

   (ii) A population growth model is given by $\frac{du}{dt} = \alpha u(1 - u/R)$, where $\alpha > 0$ and R is the maximum possible value of u. Set the values of α and R yourself and solve the equation by Euler method to print $u - t$ in a file.

      (i) Program : 8, Plot : 2

      (ii) Algorithm : 2, Program : 6, Data File : 2

2. (i) The set $O = \{u_1, u_2, u_3\} = \left\{ \left(\frac{1}{\sqrt{2}}, 0, 1/\sqrt{2} \right), (0, 1, 0), \left(\frac{1}{\sqrt{2}}, 0, -1/\sqrt{2} \right) \right\}$ is an orthonormal basis set for 3D Euclidean space. Check this numerically. Use Numpy.

   (ii) Use the following discrete differentiation with forward differences :

   $$f'(x) = \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x}$$

   $$f''(x) = \frac{f(x + \Delta x) - 2f(x) + f(x - \Delta x)}{\Delta x^2}$$

   Calculate $f'(x)$ and $f''(x)$ at all possible points where
   $x = [0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9]$ and
   $y = [0, 0.004, 0.032, 0.108, 0.256, 0.5, 0.864, 1.372, 2.048, 2.916]$.

   Plot $(x, f'(x))$ and $(x, f''(x))$ in two different graphs with matplotlib.

      (i) Program : 4, Output : 2

      (ii) Program : 8, Plot : 3+3

3. (i) Plot $y = x^2 \sin x$, where x ranges from 0 to 6π with spacing 0.15. Use plotting function from matplotlib package. Label the axes and give a suitable title for the plot.

   (ii) Given a list of θ-values (in radian) in [0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5] and the corresponding $\sin(\theta)$ values in [0, 0.48, 0.84, 1.0, 0.91, 0.60, 0.14, – 0.35], use any kind of interpolation method to find out $\sin(1.8)$.

      (i) Program : 4, Plot : 2

      (ii) Algorithm : 3, Program : 10, Output : 1

4. (i) Given a matrix $A = \begin{pmatrix} 5 & 1 \\ 3 & 3 \end{pmatrix}$,

   Find out the eigenvalues and eigenvectors and print them. Form the Diagonal matrix, D with the eigenvalues. Consider the matrix V whose columns are the eigenvectors. Now show that $V.D.V^{-1} = A$.

   (ii) Write a program using the Newton-Raphson method to determine one of the roots of the equation : $f(x) = x^3 - x^2 - 2x + 1$.

      (i) Program : 6, Output : 5

      (ii) Algorithm : 2, Program : 6, Output : 1

5. (i) Consider the logistic equation for a population $N : \frac{dN}{dt} = rN(1 - N/K)$, where r is the rate of growth and K is the carrying capacity. Take initial population of 80 with carrying capacity of 50 and growth rate 4. Solve this equation by Euler method at t = 10 and 100.

   (ii) Given a set of x-y data in the form of an array [1, 0.5, 2, 3.8, 3, 7.9, 4, 16.5, 5, 27.5] where the alternate elements are x and y. Prepare two separate lists for x and y and plot y against x with x-label as 'Time' and y-label as 'Temperature'.

      (i) Program : 4, Output : 2

      (ii) Algorithm : 2, Program : 8, Output : 4

6. (i) Consider the following numpy arrays :

$x = [2, 4, 5, 6, 8, 10]$ and $y = [5, 9, 11, 10, 12, 19]$.

Now calculate the correlation coefficient :

$$r = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2} \cdot \sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

Scatter plot (x, y) with matplotlib. Give title : "x–y Plot" with the axes labelled as 'X' and 'Y'.

(ii) In an electrical circuit, comprising of DC voltage sources and resistor elements, the mesh equations are written (by Kirchhoff's voltage law) as follows.

$$50I_1 - 30I_3 = 80$$
$$40I_2 - 20I_3 = 80$$
$$-30I_1 - 20I_2 + 100I_3 = 0$$

Solve this to find the currents by matrix inversion. [Hint : Use the module for linear algebra from numpy package.]

(i) Program : 7, Output : 2, Plot : 3,

(ii) Program : 6, Output : 2

7. (i) Plot $J_0(x)$, $J_1(x)$, $J_2(x)$, and $J_3(x)$, in four different graphs as subplots (four different graphs in the same plot) through matplotlib. In this $J_n(x)$'s are the Bessel functions of 1st kind which can be obtained from scipy.special module. Take x-scale from 0 to 10. [Hints : Consider $jn(n, x)$ function]

(ii) Generate (x, y) data where x-values are from 0 up to 2.0 in the equal interval of $\Delta x = 0.2$ and y-values follow $y = \sin(x^2)$. Now you have to find out y at $x = 0.3$ from the data set by Lagrange interpolation formula. Write a Python script for that.

(i) Program : 6, Plot : 2

(ii) Algorithm : 2, Program : 8, Output : 2

8. (i) Generate 100 random integers between 10 and 50 from an appropriate random module in numpy. Now treat the first 20 values as X-data and the last 20 values as Y-data. Plot X-Y through matplotlib as scattered points with big symbols.

(ii) Use bisection method to find the root of the equation.

$$\cos^2 x - 5.6\, x^2 + x + 20 = 0$$

that lies in the range $1 < x < 2.5$.

(i) Program : 6, Output : 2

(ii) Algorithm : 3, Program : 7, Output : 2

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2016

PHYSICS — HONOURS — PRACTICAL

Eighth Paper

(Group – B)

Full Marks – 50

The figures in the margin indicate full marks

Programming Language : C or Fortran

Print the output of your programs at the terminals

Group A

1. Given an integer M (say. 3). find the smallest integer n for which the series

$$1 + \frac{1}{2} + \frac{1}{3} \cdots + \frac{1}{n}$$

is larger than M.
(Flow chart/Algorithm - 2, Program - 8, Result - 2)

2. Read 10 numbers and write a program to arrange them in ascending order. Test your program for the following numbers :

1.2, -2.9, 2.1, 6.9, -9.8, 8.7, 5.1, 1.8, -3.5, -4.7
(Flow chart/Algorithm - 2, Program - 8, Result - 2)

3. Each term of a sequence $\{a_1, a_2, a_3, \cdots\}$ is generated by taking the sum of the previous three terms. If the first three terms are 0 and 1 and 2, find the ratio a_{n+1}/a_n correct to three decimal places for $n \rightarrow \infty$.
(Flow chart/Algorithm - 2, Program - 8, Result - 2)

1.839

4. The series expansion for $\log_e(x)$ in the range $x > 1$ is

$$\log_e(x) = \frac{x-1}{x} + \frac{1}{2}\left(\frac{x-1}{x}\right)^2 + \frac{1}{3}\left(\frac{x-1}{x}\right)^3 + \cdots$$

1.1447 ✓

Evaluate $\log_e(x)$ for $x = \pi$ up to three decimal places by using this expansion. ✓
(Flow chart/Algorithm - 2, Program - 8, Result - 2)

5. Find the prime numbers less than or equal to 59.
(Flow chart/Algorithm - 2, Program - 8, Result - 2)

o. Four positive integers i, j, k, l, each ≤ 8, satisfy the condition $i^2 + j^2 + k^2 = l^2$. Write down all possible such sets with a program.
(Flow chart/Algorithm - 2, Program - 8, Result - 2)

(handwritten)
1, 2, 2, 3
2, 1, 2, 3
2, 2, 1, 3
2, 3, 6, 7
2, 6, 3, 7
3, 2, 6, 7
3, 6, 2, 7
6, 2, 3, 7
6, 3, 2, 7
2, 4, 4, 6
4, 2, 4, 6
4, 4, 2, 6 (12)

7. Write a program to compute the matrix

$$A + \frac{1}{2}A^2 + \frac{1}{6}A^3$$

where

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

(Flow chart/Algorithm - 2, Program - 8, Result - 2)

8. Calculate the commutator $[A, B]$, where

(handwritten)
$$\begin{pmatrix} 0 & 0.1162 & 0.1342 \\ 0.1162 & 0 & 0.368 \\ 0.1342 & -0.368 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix}$$

with $\alpha = 40°$ and $\beta = 35°$
(Flow chart/Algorithm - 2, Program - 8, Result - 2)

(handwritten)
$$\begin{pmatrix} 0 & 0.1162 & 0.1342 \\ 0.1162 & 0 & 0.3\overline{5} \\ & 6 & \end{pmatrix}$$

Group B

1. Use the *Gauss-Seidel* method (without rearrangement or refinement) to solve the simultaneous equations

$$9x_1 + x_2 + x_3 + x_4 = 75$$
$$x_1 + 8x_2 + x_3 + x_4 = 54$$
$$x_1 + x_2 + 7x_3 + x_4 = 43$$
$$x_1 + x_2 + x_3 + 6x_4 = 34$$

(handwritten)
$x_1 = 7$
$x_2 = 5$
$x_3 = 4$
$x_4 = 3$

(Flow chart/Algorithm - 4, Program - 10, Result - 4)

2. Given the data

x	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	0.545	0.331	0.275	0.258	0.240	0.235

(handwritten) 0.2928

find the value of $f(x)$ for $x = 0.25$ using *Lagrange's interpolation formula* using all the points.
(Flow chart/Algorithm - 4, Program - 10, Result - 4)

3. Given the data

x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$f(x)$	-1.26	-1.10	-0.91	-0.67	-0.54	-0.32	-0.10	0.08	0.33	0.51

find the value of $f(x)$ for $x = 0.58$ using *Lagrange's interpolation formula* using all the points.
(Flow chart/Algorithm - 4, Program - 10, Result - 4)

-0.369957

4. Using the following data, calculate the values of m and c for *least square fit* to a straight line $y = mx + c$.

x	1	2	3	4	5	6	7	8	9	10
y	-0.94	-0.82	-0.72	-0.58	-0.49	-0.32	-0.21	-0.08	0.06	0.20

(Flow chart/Algorithm - 4, Program - 10, Result - 4)

$m = 0.126667$
$c = -1.086667$

5. Using the *bisection* method, find the root of the equation

$$x^3 - 5.816x^2 + 9.632x - 7.632 = 0$$

correct upto the third decimal place. (This equation has only one root, that lies in the range $0 < x < 5$.)
(Flow chart/Algorithm - 4, Program - 10, Result - 4)

3.816

6. Using the *bisection* method, find the root of the equation

$$20 - 2.5x - 0.01x^3 = 0$$

correct upto 3 significant digits. (This equation has only one root, that lies in the range $0 < x < 10$.)
(Flow chart/Algorithm - 4, Program - 10, Result - 4)

6.76

7. Using the *Newton-Raphson* method, find the root of the equation

$$x^2 \ln x = 5.72$$

correct upto the third decimal place. (This equation has only one root, that lies in the range $2 < x < 3$.)
(Flow chart/Algorithm - 4, Program - 10, Result - 4)

2.499

8. Using *Newton-Raphson* method, find a real root of the equation

$$x^2 - 2\exp(-x) = 0$$

correct upto 3 significant digits. (This equation has only one root, that lies in the range $0 < x < 1$.)
(Flow chart/Algorithm - 4, Program - 10, Result - 4)

0.901

9. Using *trapezoidal* rule, calculate

$$\int_0^\pi \sqrt{x}\exp x \, dx$$

-32.831

correct upto 3 significant digits.
(Flow chart/Algorithm - 4, Program - 10, Result - 4)

10. Using *trapezoidal* rule, calculate

$$\int_0^{\pi/4} \sqrt{1-x^2}\,\cos x \, dx$$

-0.63298

correct upto 3 decimal places.
(Flow chart/Algorithm - 4, Program - 10, Result - 4)

11. Using *Simpson's one-third rule*, calculate

$$\int_0^\pi e^{-x^2}\sin x \, dx$$

0.424

correct upto 3 significant digits.
(Flow chart/Algorithm - 4, Program - 10, Result - 4)

12. Using *Simpson's one-third rule*, calculate

0.878

$$\int_{-1}^1 x^2 e^x \, dx$$

correct upto 2 decimal places.
(Flow chart/Algorithm - 4, Program - 10, Result - 4)

2018

PHYSICS - HONOURS - PRACTICAL

Eighth Paper

(Group B)

Full Marks - 50

Set 1

Date of Examination: 12.03.2018

Programming Language : C or Fortran

Print the output of your programs at the terminals

Group A

A1 Search from 50 onwards and find the first five prime numbers. Store them in an array and calculate the sum of those five numbers.
(Flow chart / Algorithm -2, Program -8, Result -2)

A2 Sort the following ten numbers using any type of sorting algorithm in ascending order :

$1, -4.5, 6.9, -0.1, 2.3, 5, 9, 1.63, 2.76, 8.1$
(Flow chart / Algorithm -2, Program -8, Result -2)

A3 Calculate the value of ln (3) from the series expansion

$$\ln\left(\frac{1+x}{1-x}\right) = 2x + \frac{2}{3}x^3 + \frac{2}{5}x^5 + \frac{2}{7}x^7 + \ldots$$

(for $-1 < x < 1$) up to an accuracy of three decimal places.
(Flow chart / Algorithm -2, Program -8, Result -2)

A4 Find the factors of the number 4158. Separate the prime factors from the list and print them.
(Flow chart / Algorithm -2, Program -8, Result -2)

A5 Consider the series

$$\frac{1}{2\cdot 3} + \frac{2}{3\cdot 4} + \frac{3}{4\cdot 5} + \cdots$$

Calculate the sum of the series up to 8 terms.
(Flow chart / Algorithm -2, Program -8, Result -2)

[Turn Over]

A6 Take the two lists $a(1) = 2, a(2) = 2.3, a(3) = 3, a(4) = 3.4, a(5) = 4$ and $b(1) = 7, b(2) = 7.2, b(3) = 7.3, b(4) = 7.4, b(5) = 7.5$. After taking the input from the screen and storing two lists create a separate (say c) list of 10 elements taking one from the lists a and b alternatively. i.e.,
$c(1) = a(1), c(2) = b(1), c(3) = a(2), c(4) = b(2)\ldots\ldots, c(9) = a(5), c(10) = b(5)$
Print the three lists.

(Flow chart / Algorithm -2, Program -8, Result -2)

A7 It is given that $A = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}$. Find the products AB and BA.

(Flow chart / Algorithm -2, Program -8, Result -2)

A8 If $A = \begin{pmatrix} -3 & 1 \\ -4 & 2 \end{pmatrix}$, calculate $A^2 + A$.

(Flow chart / Algorithm -2, Program -8, Result -2)

Group B

B1 Use the *Gauss-Seidel* method (without rearrangement or refinement) to solve the simultaneous equations

$$8x_1 + 4x_2 - 2x_3 = 3$$
$$2x_1 - 4x_2 + x_3 = 1$$
$$3x_1 + x_2 + 7x_3 = 11$$

(Flow chart / Algorithm -4, Program -10, Result -4)

B2 Find the value of $f(x)$ for $x = 3.5$ using Lagrange's interpolation formula using all the data.

x	1.3	2.7	3.1	3.9	4.2	5.3
$f(x)$	3.901	60.759	88.763	171.747	213.624	427.541

(Flow chart / Algorithm -4, Program -10, Result -4)

B3 Using the following data, calculate the values of m and c for *least square fit* to a straight line $y = mx + c$.

x	1.5	1.75	2.27	2.81	3.19	4.7	5.32	7.24
y	19.14	19.00	18.70	18.40	18.18	17.32	16.90	15.87

(Flow chart / Algorithm -4, Program -10, Result -4)

B4 Using the following data, calculate the value of m for *least square fit* to a straight line $y = mx$

x	-5.2	-4.1	-3.5	-2.1	1.68	4.6	7.4
y	-27.04	-21.32	-18.2	-10.9	8.74	23.9	38.48

(Flow chart / Algorithm -4, Program -10, Result -4)

B5 Using the *bisection* method, find the root of the equation

$$2(x - 5)^2 - 10x = 11$$

that lies in the range $1.0 < x < 2.5$, correct up to the third decimal place.
(Flow chart / Algorithm -4, Program -10, Result -4)

B6 Using the *Newton-Raphson* method, find the root of the equation

$$x^3 + (x + 1)^2 + 4x = 20$$

that lies in the range $1.0 < x < 2.5$, correct up to three decimal places, by choosing the initial point suitably.
(Flow chart / Algorithm -4, Program -10, Result -4)

B7 Using *trapezoidal* rule, calculate

$$\int_0^5 e^{-x^2} x^2 dx$$

correct up to 2 decimal places.
(Flow chart / Algorithm -4, Program -10, Result -4)

B8 Using *Simpson's one-third rule*, calculate

$$\int_{-2}^{2} x^3 e^{-x} dx$$

correct up to 2 decimal places.
(Flow chart / Algorithm -4, Program -10, Result -4)

2018

PHYSICS - HONOURS - PRACTICAL

Eighth Paper

(Group B)

Full Marks - 50

Set 2

Date of Examination: 13.03.2018

Programming Language : C or Fortran

Print the output of your programs at the terminals

Group A

A1 Start from 3993 and find six consecutive prime numbers less than 3993. Store them in an array and find the sum of those six numbers.
(Flow chart / Algorithm -2, Program -8, Result -2)

A2 Sort the following ten numbers using any type of sorting algorithm in ascending order:
2.3, 5.6, −8.4, 10.6, −2.5, 8.7, 9.44, 11.25, 50.24, 1.5.
(Flow chart / Algorithm -2, Program -8, Result -2)

A3 Generate the Fibonacci sequence $F_{i+1} = F_i + F_{i-1}$. $(i \geq 2, F_2 = 1$ and $F_1 = 1)$. and calculate the reciprocal Fibonacci series

$$S = \sum_i \frac{1}{F_i}$$

up to an accuracy of four decimal places.
(Flow chart / Algorithm -2, Program -8, Result -2)

A4 Factories 168 and calculate the product of all the non-prime factors.
(Flow chart / Algorithm -2, Program -8, Result -2)

A5 It is known that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

Calculate the partial sum S_n and determine the number of terms for which $\frac{\pi^2}{6} - S_n = 0.0001$.
(Flow chart / Algorithm -2, Program -8, Result -2)

[Turn Over]

A6 A list of numbers is given below

3.1, −6.4, 5.21, 7.2, −9.11, −11.1, 3.45, −4.52, −2.53, −8.87

Accept the numbers from the screen and store in an array. Then form two separate lists such that one contains the negative numbers and other contains the positive numbers. (No need to arrange them.)

Print the three lists.

(Flow chart / Algorithm -2, Program -8, Result -2)

A7 A matrix is defined as $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$. where $\theta = 30^0$. Construct $B = A^T$ and find the product BA.

(Flow chart / Algorithm -2, Program -8, Result -2)

A8 If $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$, calculate $A^2 - 4A$.

(Flow chart / Algorithm -2, Program -8, Result -2)

Group B

B1 Use the *Gauss-Seidel* method (without rearrangement or refinement) to solve the simultaneous equations

$$12x_1 + 3x_2 - 5x_3 = 1$$
$$x_1 + 5x_2 + 3x_3 = 28$$
$$3x_1 + 7x_2 + 13x_3 = 76$$

(Flow chart / Algorithm -4, Program -10, Result -4)

B2 Find the value of $f(x)$ for $x = -2.5$ using Lagrange's interpolation formula using all the data.

x	-3.5	-3.0	-2.75	-2.1	-1.9	-1.7
$f(x)$	-3.125	-14.0	-17.703	-22.739	-23.141	-23.087

(Flow chart / Algorithm -4, Program -10, Result -4)

B3 Using the following data, calculate the values of m and c for *least square fit* to a straight line $y = mx + c$.

x	-2.0	-1.5	2.1	4.44	5.17	6.85	7.53
y	6.92	6.86	6.0	5.44	5.25	4.86	4.72

(Flow chart / Algorithm -4, Program -10, Result -4)

B4 Using the following data, calculate the value of m for *least square fit* to a straight line $y = mx$:

x	6.31	7.2	8.3	9.1	10.6	11.2	13.6
y	19.5	22.32	25.73	28.27	32.86	34.72	42.16

(Flow chart / Algorithm -4, Program -10, Result -4)

B5 Using the *bisection* method, find the root of the equation

$$(x+5)^2 + 10x - 11 = 0$$

that lies in the range $-1.5 < x < 0$, correct up to the third decimal place.
(Flow chart / Algorithm -4, Program -10, Result -4)

B6 Using the *Newton-Raphson* method, find the root of the equation

$$e^x + 20x^6 - 25x + 2 = 0$$

that lies in the range $-0.5 < x < 0.5$, correct up to three decimal places, by choosing the initial point suitably.
(Flow chart / Algorithm -4, Program -10, Result -4)

B7 Using *trapezoidal* rule, calculate

$$\int_1^{3\pi} \frac{\sin x}{x^2} dx$$

correct up to 3 decimal places.
(Flow chart / Algorithm -4, Program -10, Result -4)

B8 Using *Simpson's one-third rule*, calculate

$$\int_0^1 x\sqrt{1 - x^2}\, dx$$

correct up to 3 decimal places.
(Flow chart / Algorithm -4, Program -10, Result -4)

2018

PHYSICS - HONOURS - PRACTICAL

Eighth Paper

(Group B)

Full Marks - 50

Set 4
Date of Examination: 15.03.2018

Programming Language : C or Fortran

Print the output of your programs at the terminals

Group A

A1 Find the first three prime numbers n greater than 4 and check whether $2^n - 1$ is prime.
(Flow chart / Algorithm -2, Program -8, Result -2)

A2 Sort the following ten numbers using any type of sorting algorithm in ascending order:
99.23, 44.55, 65.21, 88.44, 23.21, 35.47, 15.46, 111.2, 77.52, 10.24
(Flow chart / Algorithm -2, Program -8, Result -2)

A3 Calculate the sum

$$\sum_{k=0}^{\infty} \frac{1}{n^k}$$

for $n = 2$ with an accuracy of three decimal places.
(Flow chart / Algorithm -2, Program -8, Result -2)

A4 Find the factors of the number 30030 and calculate the sum of all the prime factors.
(Flow chart / Algorithm -2, Program -8, Result -2)

A5 It is known that

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots \cdots = \frac{\pi^2}{12}$$

Calculate the partial sum S_n and determine the number of terms for which $\frac{\pi^2}{12} - S_n = 0.0001$.
(Flow chart / Algorithm -2, Program -8, Result -2)

[Turn Over]

A6 Accept any twelve numbers from the screen and store them in an array [$a(i)$ say].
Create two separate lists $b(i)$ and $c(i)$ in the following way:
$b(1) = a(6)$, $b(2) = a(5)$, ... $b(6) = a(1)$ and
$c(1) = a(12)$, $c(2) = a(11)$, ... $c(6) = a(7)$
Print the three lists.
(Flow chart / Algorithm -2, Program -8, Result -2)

A7 $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 2 & 5 \\ 1 & 5 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 3 & 2 \\ -3 & 1 & -1 \\ -2 & 1 & 1 \end{pmatrix}$. Calculate the sum of all the elements of AB.
(Flow chart / Algorithm -2, Program -8, Result -2)

A8 $A = \frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$, $B = \frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}$. Calculate A^2 and B^2.
(Flow chart / Algorithm -2, Program -8, Result -2)

Group B

B1 Use the *Gauss-Seidel* method (without rearrangement or refinement) to solve the simultaneous equations

$$12x_1 + 3x_2 - 2x_3 = 8$$
$$x_1 - 9x + 2x_3 = 7$$
$$x_1 + 5x_2 + 7x_3 = -5$$

(Flow chart / Algorithm -4, Program -10, Result -4)

B2 Find the value of $f(x)$ for $x = -3.5$ using Lagerange's interpolation formula using all the data.

x	-5	-4.8	-4.2	-3.7	-3.22	-2.95
$f(x)$	57.0	54.968	46.472	37.357	27.609	21.950

(Flow chart / Algorithm -4, Program -10, Result -4)

B3 Using the following data, calculate the values of m and c for *least square fit* to a straight line $y = mx + c$.

x	2	3	4	5	6	7	8
y	-8.7	-10.4	-12.1	-13.8	-15.5	-17.2	-18.9

(Flow chart / Algorithm -4, Program -10, Result -4)

B4

B5

B6

B7

B8

B4 Using the following data, calculate the value of m for *least square fit* to a straight line $y = mx$

x	-16.2	-9.67	-4.3	10.9	18.7	21.3	23.8
y	37.81	22.57	10.04	-25.44	-43.64	-49.72	-55.54

(Flow chart / Algorithm -4, Program -10, Result -4)

B5 Using the *bisection* method, find the root of the equation

$$\cos^2 x - 5.6x^2 + x + 20 = 0$$

that lies in the range $1 < x < 2.5$, correct up to the third decimal place.
(Flow chart / Algorithm -4, Program -10, Result -4)

B6 Using the *Newton-Raphson* method, find the root of the equation

$$\sin^2 x - 5x + 9 = 0$$

that lies in the range $1 < x < 2.5$, correct up to three decimal places, by choosing the initial point suitably.
(Flow chart / Algorithm -4, Program -10, Result -4)

B7 Using *trapezoidal* rule, calculate

$$\int_{-1}^{2} xe^{-x^2} dx$$

correct up to 3 decimal places.
(Flow chart / Algorithm -4, Program -10, Result -4)

B8 Using *Simpson's one-third rule*, calculate

$$\int_{-\pi/3}^{\pi/3} x^3 \tan x \, dx$$

correct up to 3 decimal places.
(Flow chart / Algorithm -4, Program -10, Result -4)

2018

PHYSICS - HONOURS - PRACTICAL

Eighth Paper

(Group B)

Full Marks - 50

Set 3

Date of Examination: 14.03.2018

Programming Language : C or Fortran

Print the output of your programs at the terminals

Group A

A1 Find the first three prime numbers greater than 4000 and the first three prime numbers less than 4000. Add the six numbers.
(Flow chart / Algorithm -2, Program -8, Result -2)

A2 Sort the following ten numbers using any type of sorting algorithm in ascending order.
$-5.6, -4.9, -9.6, 4.56, 6.58, 2.54, -8.95, 19.52, 7.15, 13.45$
(Flow chart / Algorithm -2, Program -8, Result -2)

A3 Evaluate the sum

$$S = \sum_{n=1}^{\infty} \left(\frac{1}{n+0.5} - \frac{1}{n+1.3} \right)$$

correct up to third place of decimal.
(Flow chart / Algorithm -2, Program -8, Result -2)

A4 Factories 168 and calculate the product of all the prime factors.
(Flow chart / Algorithm -2, Program -8, Result -2)

A5 It is known that

$$1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots = \frac{\pi^3}{32}$$

Calculate the partial sum S_n and determine the number of terms for which $\frac{\pi^3}{32} - S_n = 0.0001$.
(Flow chart / Algorithm -2. Program -8, Result -2)

A6 Accept any twelve numbers from the screen and store them in an array [$a(i)$ say].
Create two separate lists $b(i)$ and $c(i)$ in the following way:
b contains the square root of the odd elements i.e., $b(1) = \sqrt{a(1)}$, $b(2) = \sqrt{a(3)}$ ...

...

c contains the square of the even elements i.e., $c(1) = a^2(2)$, $c(2) = a^2(4)$, ...
Print the three lists.
(Flow chart / Algorithm -2, Program -8, Result -2)

A7 $A = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 1 & 5 & 2 \end{pmatrix}$; $B = \begin{pmatrix} 1 & 3 & 2 \\ 4 & 1 & 3 \\ 2 & 1 & 1 \end{pmatrix}$. Calculate the trace of AB.

(Flow chart / Algorithm -2, Program -8, Result -2)

A8 It is given that $P = \begin{pmatrix} 1 & 4 \\ -1 & 1 \end{pmatrix}$, $P^{-1} = \frac{1}{5}\begin{pmatrix} 1 & -4 \\ 1 & 1 \end{pmatrix}$ and $A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$. Evaluate
$P^{-1}AP$.
(Flow chart / Algorithm -2, Program -8, Result -2)

Group B

B1 Use the *Gauss-Seidel* method (without rearrangement or refinement) to solve the
simultaneous equations

$$3x_1 + x_2 - x_3 = 7$$
$$x_1 - 5x_2 + x_3 = -1$$
$$2x_1 + x_2 + 6x_3 = 4$$

(Flow chart / Algorithm -4, Program -10, Result -4)

B2 Find the value of $f(x)$ for $x = 3.5$ using Lagrange's interpolation formula using all
the data:

x	2.5	2.89	3.21	3.67	3.92	4.14
$f(x)$	-1.625	-6.237	-11.976	-23.731	-32.036	-40.558

(Flow chart / Algorithm -4, Program -10, Result -4)

B3 Using the following data, calculate the values of m and c for *least square fit* to a
straight line $y = mx + c$.

x	9.56	10.62	11.45	12.01	13.89	14.23	17.5
y	7.05	6.88	6.75	6.66	6.38	6.33	5.82

(Flow chart / Algorithm -4, Program -10, Result -4)

x	-4.4	-3.3	-2.2	-1.1	1.05	2.4	3.12
y	5.28	3.96	2.64	1.32	-1.26	-2.88	-3.74

(Flow chart / Algorithm -4, Program -10, Result -4)

B5 Using the *bisection* method, find the root of the equation

$$3 \sin x - 3.5 = 5 \cos x$$

that lies in the range $3 < x < 4$, correct up to the third decimal place.
(Flow chart / Algorithm -4, Program -10, Result -4)

B6 Using the *Newton-Raphson* method, find the root of the equation

$$3 \cos x + x^3 = 0$$

that lies in the range $-1.5 < x < -0$, correct up to three decimal places, by choosing the initial point suitably.
(Flow chart / Algorithm -4, Program -10, Result -4)

B7 Using *trapezoidal* rule, calculate

$$\int_2^{3.7} x^4 \ln x \, dx$$

correct up to 3 significant digits.
(Flow chart / Algorithm -4, Program -10, Result -4)

B8 Using *Simpson's one-third rule*, calculate

$$\int_{-0.5}^{0.8} x^3 \sqrt{1 - x^2} \, dx$$

correct up to 3 decimal places.
(Flow chart / Algorithm -4, Program -10, Result -4)

2019

PHYSICS — HONOURS — PRACTICAL

Eighth Paper

(Group – B)

Full Marks : 50

(under 1+1+1 system)

Date of Examination : 05.03.2019

Programming Language : C or Fortran.

Print the output of your programs at the terminals.

SET – 1

Group – A

A1. Write a program to find the first N prime numbers (2, 3, 5,...). Write your output for $N = 10, 17, 23$.

(Flowchart / Algorithm – 2, Program – 8, Result – 2)

A2. Sort the following ten numbers using any type of sorting algorithm in ascending order :

1, $(4.5)^2$, 6.9, – 0.1, 2.3, 5, 5, 9, $\sqrt{1.63}$, 2.76, 8.1^3

(Flowchart / Algorithm – 2, Program – 8, Result – 2)

A3. Calculate the value of ln(1.5) from the series expansion

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + ...$$

up to an accuracy of three decimal places.

(Flowchart / Algorithm – 2, Program – 8, Result – 2)

Please Turn Over

A4. Find the largest prime number below 5000.

(Flowchart / Algorithm – 2, Program – 8, Result – 2)

A5. Write a program to compute

$$S = \frac{1.2}{3.4} + \frac{5.6}{7.8} + \frac{9.10}{11.12} + \dots$$

up to n terms. Find the value of S for $n = 10$.

(Flowchart / Algorithm – 2, Program – 8, Result – 2)

A6. Fibonacci numbers F_1, F_2,... are defined by the recursion relation $F_{n+1} = F_n + F_{n-1}$ with the initial values $F_1 = 1$ and $F_2 = 1$. Find the ratio $\lim_{n \to \infty}(F_{n+1} / F_n)$ correct up to second decimal place.

(Flowchart / Algorithm – 2, Program – 8, Result – 2)

A7. Compute and print M^T and Trace $(I - M^2)$ where I is the identity matrix and

$$M = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 1 & 2 \\ 3 & 4 & 1 \end{pmatrix}$$

(Flowchart / Algorithm – 2, Program – 8, Result – 2)

A8. Calculate the commutator $[A, B]$, where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 11 & 12 & 13 \\ 12 & 13 & 14 \\ 13 & 14 & 11 \end{pmatrix}$$

(Flowchart / Algorithm – 2, Program – 8, Result – 2)

Group – B

B1. Use the *Gauss-Seidel* method (without rearrangement or refinement) to solve the simultaneous equations

$$10x_1 - 2x_2 - x_3 - x_4 = 3$$
$$-2x_1 + 10x_2 - x_3 - x_4 = 15$$
$$-x_1 - x_2 + 10x_3 - 2x_4 = 27$$
$$-x_1 - x_2 - 2x_3 + 10x_4 = -9$$

(Flowchart / Algorithm – 4, Program – 10, Result – 4)

B2. Find the value of $f(x)$ for $x = 3.5$ using *Lagrange's interpolation formula* using all the data.

x	1.3	2.7	3.5	3.9	4.4
$f(x)$	3.901	60.759	166.763	171.747	231.92

(Flowchart / Algorithm – 4, Program – 10, Result – 4)

B3. Using the following data, calculate the values of m and c for *least square fit* to a straight line $y = mx + c$.

x	1	2	3	4	5	6
$f(x)$	−2.95	−1.15	0.90	3.15	4.95	7.05

(Flowchart / Algorithm – 4, Program – 10, Result – 4)

B4. Using the following data, calculate the values of m and c for *least square fit* to a straight line $y = mx + c$:

x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y	− 1.26	− 1.10	− 0.91	− 0.67	− 0.54	− 0.32	− 0.10	0.08	0.33	0.51

(Flowchart / Algorithm – 4, Program – 10, Result – 4)

B5. Using the *bisection* method, find the root of the equation

$$x \sin x + (x - 2) \cos x = 0$$

that lies in the range $2 < x < 5$, correct up to the third decimal place.

(Flowchart / Algorithm – 4, Program – 10, Result – 4)

B6. Using the *Newton-Raphson* method, find the root of the equation

$$(1 - x^2) \tan x - x = 0$$

that lies in the range $2 < x < 3$, correct up to the third decimal place.

(Flowchart / Algorithm – 4, Program – 10, Result – 4)

B7. Using *trapezoidal* rule, calculate

$$\int_0^1 \sqrt{1 - x^2}\, dx$$

correct upto 3 decimal places.

(Flowchart / Algorithm – 4, Program – 10, Result – 4)

B8. Using *Simpson's one-third rule*, calculate

$$\int_0^3 \frac{x}{1 + x^5}\, dx$$

correct up to 3 significant digits.

(Flowchart / Algorithm – 4, Program – 10, Result – 4)

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2019

PHYSICS — HONOURS — PRACTICAL

Eighth Paper

(Group – B)

Full Marks : 50

(Under 1+1+1 system)

Date of Examination : 06.03.2019

Programming Language : C or Fortran.

Print the output of your programs at the terminals.

SET – 2

Group – A

A1. Given an integer M (say, 3), find the smallest integer n for which the series

$$1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}$$

is larger than M.

(Flowchart / Algorithm – 2, Program – 8, Result – 2)

A2. Read 10 numbers and write a program to arrange them in descending order. Test your program for the following numbers :

1.2, –2.9, 2.1, 6.9, –9.8, 8.7, 5.1, 1.8, –3.5, –4.7

(Flowchart / Algorithm – 2, Program – 8, Result – 2)

A3. Each term of a sequence $\{a_1, a_2, a_3,...\}$ is generated by taking the sum of the previous three terms. If the first three terms are 0 and 1 and 2, find the ratio a_{n+1} / a_n correct to three decimal places for $n \to \infty$.

(Flowchart / Algorithm – 2, Program – 8, Result – 2)

Please Turn Over

A4. Find the prime numbers less than or equal to 89 and count them.

(Flowchart / Algorithm – 2, Program – 8, Result – 2)

A5. Zeta function is defined as $\zeta(n) = \sum\limits_{k=1}^{\infty} \dfrac{1}{k^n}$

Given that $\zeta(4) = \pi^4/90$ estimate a value of π correct up to 3 significant digits. How many terms are needed in the series for this?

(Flowchart / Algorithm – 2, Program – 8, Result – 2)

A6. Find the *prime number* lying between 3980 and 3990. (There will be only one such number.)

(Flowchart / Algorithm – 2, Program – 8, Result – 2)

A7. Write a program to calculate Trace(M^2) where

$$M = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \end{pmatrix}$$

(Flowchart / Algorithm – 2, Program – 8, Result – 2)

A8. For

$$M = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \end{pmatrix} \quad and \quad V = \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$$

calculate $\dfrac{1}{2} MV + V$.

(Flowchart / Algorithm – 2, Program – 8, Result – 2)

Group – B

B1. Use the *Gauss-Seidel* method (without rearrangement or refinement) to solve the simultaneous equations

$$12x_1 + 3x_2 - 5x_3 = 1$$
$$x_1 + 5x_2 + 3x_3 = 28$$
$$3x_1 + 7x_2 + 13x_3 = 76$$

(Flowchart / Algorithm – 4, Program – 10, Result – 4)

B2. Given the data

x	0.1	0.2	0.3	0.45	0.5	0.6
$f(x)$	0.545	0.333	0.275	0.258	0.242	0.235

find the value of $f(x)$ for $x = 0.25$ using *Lagrange's interpolation formula* using all the points.

(Flowchart / Algorithm – 4, Program – 10, Result – 4)

B3. Using the following data, calculate the values of m and c for *least square fit* to a straight line of $y = mx + c$.

x	1	2	3	4	5	6	7	8	9	10
y	0.94	0.82	0.72	0.58	0.49	0.32	0.21	0.08	0.04	0.01

(Flowchart / Algorithm – 4, Program – 10, Result – 4)

B4. Using the *bisection* method, find the root of the equation

$$(x - 2.1)^{1/4} - (4.1 - x)^{1/3} = 0$$

that lies in the range $2 < x < 4$, correct up to the third decimal place.

(Flowchart / Algorithm – 4, Program – 10, Result – 4)

B5. Using the *bisection* method, find the root of the equation

$$\tan x = \frac{x}{1 - x^2}$$

that lies in the range $5 < x < 7$, correct up to three significant digits.

(Flowchart / Algorithm – 4, Program – 10, Result – 4)

B6. Using the *Newton-Raphson* method, find the root of the equation

$$e^x + 20x^6 - 25x + 2 = 0$$

that lies in the range $-0.5 < x < 0.5$, correct up to three decimal places, by choosing the initial point suitably.

(Flowchart / Algorithm – 4, Program – 10, Result – 4)

B7. Using *trapezoidal* rule, calculate

$$\int_1^{3\pi} \frac{\sin x}{x^2} dx$$

correct up to 3 decimal places.

(Flowchart / Algorithm – 4, Program – 10, Result – 4)

B8. Using *Simpson's one-third rule*, calculate

$$\int_0^1 x\sqrt{1-x^2}\, dx$$

correct up to 3 decimal places.

(Flowchart / Algorithm – 4, Program – 10, Result – 4)

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2019

PHYSICS — HONOURS — PRACTICAL

Eighth Paper

(Group – B)

Full Marks : 50

(Under 1+1+1 system)

Date of Examination : 07.03.2019

Programming Language : C or Fortran

Print the output of your programs at the terminals.

SET – 3

Group – A

A1. Write a program to find the first 20 prime numbers after 97 and find the average of those prime numbers.

(Flowchart / Algorithm – 2, Program – 8, Result – 2)

A2. Read 11 numbers and write a program to *arrange them in descending order*. Test your program for the following numbers :

3.2, 9.8, –5.4, 1.2, 7.6, –8.7, 4.3, 2.5, –0.3, –5.4, 3.2

(Flowchart / Algorithm – 2, Program – 8, Result – 2)

A3. The series expansion for $\log_e(x)$ in the range $x > 1$ is

$$\log_e(x) = \frac{x-1}{x} + \frac{1}{2}\left(\frac{x-1}{x}\right)^2 + \frac{1}{3}\left(\frac{x-1}{x}\right)^3 + \dots$$

Evaluate $\log_e(x)$ for $x = \pi$ up to three decimal places by using this expansion.

(Flowchart / Algorithm – 2, Program – 8, Result – 2)

Please Turn Over

A4. Calculate log(100!) by using the relation

$$\log(n!) = \sum_{i=2}^{n} \log(i)$$

Also print the value obtained from Stirling's approximation

$$\log n! = n\log(n) - n$$

(Flowchart / Algorithm – 2, Program – 8, Result – 2)

A5. Write down a program to find all the *factors* of a given integer N. Check your program for $N = 3604$.

(Flowchart / Algorithm – 2, Program – 8, Result – 2)

A6. The matrix elements $A_{m,n}$ of a 3×3 matrix A are given by the formula

$$A_{mn} = \sqrt{\frac{n}{2}}\delta_{m,n-1} + \sqrt{\frac{(n+1)}{2}}\delta_{m,n+1}$$

Generate and print the matrix.

(Flowchart / Algorithm – 2, Program – 8, Result – 2)

A7. Calculate the matrix $A^T B$, where

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 1.1 & 1.2 & 1.3 & 1.4 \\ 1.2 & 1.3 & 1.4 & 1.1 \\ 1.3 & 1.4 & 1.1 & 1.2 \\ 1.4 & 1.1 & 1.2 & 1.3 \end{pmatrix}$$

(Flowchart / Algorithm – 2, Program – 8, Result – 2)

A8. Compute and print $\text{Trace}(M)I - M + M^T$ where I is the identity matrix and

$$M = \begin{pmatrix} 1 & 3 & 5 \\ 3 & 3 & 9 \\ 11 & 15 & 11 \end{pmatrix}$$

(Flowchart / Algorithm – 2, Program – 8, Result – 2)

Group – B

B1. Use the *Gauss-Seidel* method (without rearrangement or refinement) to solve the simultaneous equations

$$5x + 3y + 2z = 17$$
$$2x + 3y - z = 5$$
$$x - 2y - 3z = -12$$

(Flowchart / Algorithm – 4, Program – 10, Result – 4)

B2. Find the value of $f(x)$ for $x = 3.5$ using Lagrange's interpolation formula using all the data.

x	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	0.545	0.331	0.275	0.258	0.240	0.235

(Flowchart / Algorithm – 4, Program – 10, Result – 4)

B3. Using the following data, calculate the values of m and c for *least square fit* to a straight line $y = mx + c$:

x	1	2	3	4	5	6	7	8	9	10
y	−0.94	−0.82	−0.72	−0.58	−0.49	−0.32	−0.21	−0.08	0.06	0.20

(Flowchart / Algorithm – 4, Program – 10, Result – 4)

B4. Using the *bisection* method, find the root of the equation

$$20 - 2.5x - 0.01x^3 = 0$$

correct up to 3 significant digits. (This equation has only one root, that lies in the range $0 < x < 10$.)

(Flowchart / Algorithm – 4, Program – 10, Result – 4)

B5. Using the *Newton-Raphson* method, find the root of the equation

$$x^2 \ln x = 5.72$$

correct up to the third decimal place.

(Flowchart / Algorithm – 4, Program – 10, Result – 4)

B6. Using the *Newton-Raphson* method, find the root of the equation

$$x^2 - 2\exp(-x) = 0$$

correct up to 3 significant digits. (This equation has only one root, that lies in the range $0 < x < 1$.)

(Flowchart / Algorithm $- 4$, Program $- 10$, Result $- 4$)

B7. Using *trapezoidal* rule, calculate

$$\int_0^\pi \sqrt{x}\,\exp x\,dx$$

correct up to 3 significant digits.

(Flowchart / Algorithm $- 4$, Program $- 10$, Result $- 4$)

B8. Using *Simpson's one-third rule*, calculate

$$\int_0^\pi e^{-x^2} \sin x\,dx$$

correct up to 3 significant digits.

(Flowchart / Algorithm $- 4$, Program $- 10$, Result $- 4$)

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