

① From Maxwell's relations we know  $C_p - C_v = \frac{TV\beta^2}{\alpha}$

$$V = 7.06 \text{ cm}^3/\text{mol} = 7.06 \times 10^{-6} \text{ m}^3/\text{mol}$$

$$\therefore C_p - C_v = \frac{300 \times 7.06 \times 10^{-6} \times (50.4 \times 10^{-6})^2}{7.78 \times 10^{-12}} = 0.6915 \text{ J/mol-K}$$

$$\therefore C_v = C_p - 0.6915 = 24.5 - 0.6915 = 23.8085 \text{ J/mol-K}$$

②  $\alpha_s = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_s$ ,  $\alpha_p = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p$   $\therefore \frac{\alpha_s}{\alpha_p} = \frac{\left( \frac{\partial V}{\partial T} \right)_s}{\left( \frac{\partial V}{\partial T} \right)_p} = \frac{1}{\left( \frac{\partial T}{\partial V} \right)_s \left( \frac{\partial V}{\partial T} \right)_p}$

Using Maxwell's relation  $\left( \frac{\partial T}{\partial V} \right)_s = - \left( \frac{\partial P}{\partial S} \right)_V$

$$\therefore \frac{\alpha_s}{\alpha_p} = - \frac{1}{\left( \frac{\partial P}{\partial S} \right)_V \left( \frac{\partial V}{\partial T} \right)_p} = - \frac{1}{\left( \frac{\partial P}{\partial T} \right)_V \left( \frac{\partial T}{\partial S} \right)_V \left( \frac{\partial V}{\partial T} \right)_p} = - \frac{\left( \frac{\partial S}{\partial T} \right)_V}{\left( \frac{\partial P}{\partial T} \right)_V \left( \frac{\partial V}{\partial T} \right)_p}$$

$$= - \frac{T \left( \frac{\partial S}{\partial T} \right)_V}{T \left( \frac{\partial P}{\partial T} \right)_V \left( \frac{\partial V}{\partial T} \right)_p}$$

But  $C_p - C_v = T \left( \frac{\partial P}{\partial T} \right)_V \left( \frac{\partial V}{\partial T} \right)_p$  &  $C_v = T \left( \frac{\partial S}{\partial T} \right)_V = \left( \frac{\partial Q}{\partial T} \right)_V$

$$\therefore \frac{\alpha_s}{\alpha_p} = - \frac{C_v}{C_p - C_v} = \frac{1}{1 - C_p/C_v} = \frac{1}{1 - \gamma} \quad (\text{Proved})$$

Again  $E_T = -V \left( \frac{\partial P}{\partial V} \right)_T$ ,  $E_S = -V \left( \frac{\partial P}{\partial V} \right)_S$   $\therefore \frac{E_S}{E_T} = \frac{\left( \frac{\partial P}{\partial V} \right)_S}{\left( \frac{\partial P}{\partial V} \right)_T}$

$$= \frac{\left( \frac{\partial P}{\partial T} \right)_S \left( \frac{\partial T}{\partial V} \right)_S}{\left( \frac{\partial P}{\partial S} \right)_T \left( \frac{\partial S}{\partial V} \right)_T} = \frac{\left( \frac{\partial S}{\partial P} \right)_T \left( \frac{\partial T}{\partial V} \right)_S}{\left( \frac{\partial T}{\partial P} \right)_S \left( \frac{\partial S}{\partial V} \right)_T}$$

Using Maxwell's relations,  $\left( \frac{\partial S}{\partial P} \right)_T = - \left( \frac{\partial V}{\partial T} \right)_P$ ,  $\left( \frac{\partial T}{\partial V} \right)_S = - \left( \frac{\partial P}{\partial S} \right)_V$

$$\left( \frac{\partial T}{\partial P} \right)_S = \left( \frac{\partial V}{\partial S} \right)_P, \quad \left( \frac{\partial S}{\partial V} \right)_T = \left( \frac{\partial P}{\partial T} \right)_V \text{ we get}$$

$$\frac{E_S}{E_T} = \frac{\left( \frac{\partial V}{\partial T} \right)_P \left( \frac{\partial P}{\partial S} \right)_V}{\left( \frac{\partial V}{\partial S} \right)_P \left( \frac{\partial P}{\partial T} \right)_V} = \frac{\left( \frac{\partial V}{\partial T} \right)_P \left( \frac{\partial S}{\partial V} \right)_P}{\left( \frac{\partial S}{\partial P} \right)_V \left( \frac{\partial P}{\partial T} \right)_V} = \frac{\left( \frac{\partial S}{\partial T} \right)_P}{\left( \frac{\partial S}{\partial T} \right)_V} = \frac{\left( \frac{\partial Q}{\partial T} \right)_P}{\left( \frac{\partial Q}{\partial T} \right)_V} = \frac{C_p}{C_v} = \gamma$$

$$\begin{aligned}
 \textcircled{3} \quad \beta_s &= \frac{1}{P} \left( \frac{\partial P}{\partial T} \right)_s, \quad \beta_v = \frac{1}{P} \left( \frac{\partial P}{\partial T} \right)_v \quad \therefore \quad \frac{\beta_s}{\beta_v} = \frac{\left( \frac{\partial P}{\partial T} \right)_s}{\left( \frac{\partial P}{\partial T} \right)_v} \\
 &= \frac{1}{\left( \frac{\partial T}{\partial P} \right)_s \left( \frac{\partial P}{\partial T} \right)_v} = \frac{1}{\left( \frac{\partial V}{\partial S} \right)_P \left( \frac{\partial P}{\partial T} \right)_v} \quad \left[ \text{as } \left( \frac{\partial T}{\partial P} \right)_s = \left( \frac{\partial V}{\partial S} \right)_P \right] \\
 &= \frac{1}{\left( \frac{\partial V}{\partial T} \right)_P \left( \frac{\partial T}{\partial S} \right)_P \left( \frac{\partial P}{\partial T} \right)_v} = \frac{\left( \frac{\partial S}{\partial T} \right)_P}{\left( \frac{\partial V}{\partial T} \right)_P \left( \frac{\partial P}{\partial T} \right)_v} = \frac{T \left( \frac{\partial S}{\partial T} \right)_P}{T \left( \frac{\partial V}{\partial T} \right)_P \left( \frac{\partial P}{\partial T} \right)_v} \\
 \frac{\beta_s}{\beta_v} &= \frac{C_P}{C_P - C_V} = \frac{C_P/C_V}{C_P/C_V - 1} = \frac{\gamma}{\gamma - 1} \quad \left[ \text{as } C_P - C_V = T \left( \frac{\partial V}{\partial T} \right)_P \left( \frac{\partial P}{\partial T} \right)_v \right]
 \end{aligned}$$