

## Sem-I - General Properties of Matter

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### Assignment II: Hydrostatics & Hydrodynamics

Submission due date: 07/01/2026

**Q.1)** Water stands at a depth  $h$  in a large open drum whose side-walls are vertical. A hole is made in one of the walls at a depth  $x$  below the water surface. **(i)** Calculate the horizontal distance  $d$  from the wall at which the emerging stream of water strikes the ground. **(ii)** What should be the value of  $x$  which makes  $d$  maximum?

**Q.2)** Two drums A and B contain different liquids and both have a small hole on the vertical wall at a depth  $h$  below the liquid surface. The hole in drum A has half the cross-sectional area of the hole in drum B. **(i)** Using Toricelli's theorem, Calculate the ratio  $\rho_A/\rho_B$  of the densities of the liquids, if the mass flow rate is the same for the two holes. **(ii)** What is the ratio  $V_A/V_B$  of the volume flow rates from the two drums? **(iii)** At one instant, the liquid in drum A is 12.0cm above the hole. If the drums are to have equal volume flow rates, what height above the hole must the liquid in drum B be just then?

**Q.3)** Two streams merge to form a river. One stream has a width of 8.2m, depth of 3.4m, and current speed of 2.3m/s. The other stream is 6.8m wide and 3.2m deep and flows at 2.6m/s. If the river has width of 10.5m and speed at 2.9m/s, what is its depth?

**Q.4)** Dead Sea is about seven times saltier than any ocean due to the high evaporation rate and low rainfall. About one-third of the body of a swimmer floating in the Dead Sea is above the waterline. Assuming that the humanbody density is  $0.98\text{gm/cm}^3$ , find the density of the water in the Dead Sea.

**Q.5)** Water flows along a horizontal tube of which the cross-section is not constant. Using Bernoulli's theorem, calculate the change in pressure when the velocity of flow changes from 10cm/s to 20cm/s.

**Q.6)** A hypothetical material out of which an astronomical object is formed has an equation of state

$$p = \frac{1}{2}K\rho^2,$$

where  $p$  is the pressure and  $\rho$  is the mass density. Show that for this material under conditions of hydrostatic equilibrium, there is a linear relation between the density and the gravitational potential.

**Q.7)** The vorticity  $\boldsymbol{\omega}$  of a flow field is defined as curl of the velocity vector  $\boldsymbol{\omega} = \nabla \times \mathbf{v}$ . Show that for an incompressible fluid, the following relation holds between the velocity and vorticity vector fields,

$$\nabla \cdot [(\mathbf{v} \cdot \nabla) \mathbf{v}] = \frac{1}{2} \nabla^2 (\mathbf{v} \cdot \mathbf{v}) - \mathbf{v} \cdot \nabla^2 \mathbf{v} - \boldsymbol{\omega} \cdot \boldsymbol{\omega}.$$

**Q.8)** An incompressible fluid of mass density  $\rho$ , viscosity  $\eta$  is pumped in steady-state laminar flow through a circular pipe of internal radius  $R$  and length  $L$ . The pressure at the inlet end is  $p_1$  and the pressure at the exit is  $p_2$  with  $p_1 > p_2$ . Let  $Q$  be the mass of fluid that flows through the pipe per unit time. Using steady state limit of the Euler's equation for viscous liquid (or Navier-Stokes equation for “wet water” as in Feynman Lectures)

$$\rho \left[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -\nabla p + \eta \nabla^2 \mathbf{v} + \mathbf{F}_{\text{body}},$$

determine  $Q$ , which is the Poiseuille's equation. *[Hint: Switch to cylindrical coordinates.]*