1. $f_c = 4\pi \left(\frac{m}{2\pi K_B T}\right)^{3/2} c^2 e^{-\frac{mc^2}{2K_BT}}$

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$\therefore \left(\frac{m_1}{\cancel{2\pi k_B T}}\right)^{3/2} \cancel{4\pi c^2} e^{-\frac{m_1 c^2}{2K_BT}} = \left(\frac{m_2}{\cancel{2\pi k_B T}}\right)^{3/2} \cancel{4\pi c^2} e^{-\frac{m_2 c^2}{2K_BT}}$

$\therefore e^{\frac{(m_2-m_1)c^2}{2K_BT}} = \left(\frac{m_2}{m_1}\right)^{3/2}$

or $\frac{(m_2-m_1)c^2}{2K_BT} = \frac{3}{2} \ln\left(\frac{m_2}{m_1}\right)$

or $c^2 = \frac{3K_BT \ln\left(\frac{m_2}{m_1}\right)}{m_2 - m_1}$

$3k_BT = 3\times1.38\times10^{-16}\times300$ ergs/mole

$= 3\times1.38\times10^{-16}\times300\times 6.023\times10^{23}$ ergs

$c = \sqrt{\frac{3\times1.38\times10^{-16}\times300\times6.023\times10^{23}\times \ln(2)}{1}}$ cm/s

$= \cancel{7121\times10}$ 2.276 Km/s

2. $$\langle c^{-1}\rangle = \frac{\int_0^\infty c^{-1}\, dN_c}{N}$$

$$\frac{mc^2}{2K_BT} = z$$

$$\therefore mc\,dc = K_BT\,dz$$

$$\therefore c\,dc = \frac{K_BT}{m}\,dz$$

$$= 4\pi\int_0^\infty \left(\frac{m}{2\pi K_BT}\right)^{3/2} c\, e^{-\frac{mc^2}{2K_BT}}\, dc$$

$$= 4\pi\left(\frac{m}{2\pi K_BT}\right)^{3/2}\int_0^\infty e^{-z}\,\frac{K_BT}{m}\,dz$$

$$= \cancel{4}^{2}\pi\,\frac{\cancel{m}}{\cancel{2}\pi \cancel{K_BT}}\sqrt{\frac{m}{2\pi K_BT}}\,\frac{\cancel{K_BT}}{\cancel{m}} = \sqrt{\frac{2m}{\pi K_BT}}.$$

3. $$C_m = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2\times 8.3\times 10^7\times 300}{32}} = 3.94\times 10^4 \text{ cm/s}$$

$$\bar{c} = \sqrt{\frac{8RT}{M\pi}} = \sqrt{\frac{8\times 8.3\times 10^7\times 300}{32\times\pi}} = 4.45\times 10^4 \text{ cm/s}$$

$$C_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\times 8.3\times 10^7\times 300}{32}} = 4.83\times 10^4 \text{ cm/s}$$

check : $C_{rms} > \bar{c} > C_m$

4. $$C_m = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2P}{\rho}} = \sqrt{\frac{2\times 1.013\times 10^5}{1.293}} = 3.96\times 10^2 \text{ m/s}$$

$$\bar{c} = \sqrt{\frac{8RT}{M\pi}} = \sqrt{\frac{8P}{\rho\pi}} = \sqrt{\frac{8\times 1.013\times 10^5}{1.293\times\pi}} = 4.46\times 10^2 \text{ m/s}$$

$$C_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3\times 1.013\times 10^5}{1.293}} = 4.85\times 10^2 \text{ m/s}$$

remember, $P = 0.76\times 13.6\times 10^3\times 9.8 = 1.013\times 10^5 \text{ N/m}^2$

$\rho = 1.293\times 10^{-3}$ gm/cc $= 1.293$ kg/m^3.

5. $$\overline{dc^2} = \frac{1}{N}\int_0^\infty (c-\bar{c})^2\, dN_c = \frac{1}{N}\int_0^\infty c^2\, dN_c - \frac{2\bar{c}}{N}\int_0^\infty c\, dN_c + \frac{\bar{c}^2}{N}\int_0^\infty dN_c$$

$$= \frac{3RT}{M} - 2\sqrt{\frac{8RT}{M\pi}}\sqrt{\frac{8RT}{M\pi}} + \frac{8RT}{M\pi}\,\frac{1}{N}N = \frac{3RT}{M} - \frac{8RT}{M\pi}$$

$$= \left(3-\frac{8}{\pi}\right)\frac{RT}{M} = \left(3-\frac{8}{\pi}\right)\frac{K_BT}{m}.$$

$$\therefore \sqrt{\overline{dc^2}} = \sqrt{\left(3-\frac{8}{\pi}\right)\frac{K_BT}{m}}.$$

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