Sem-IV - Special Relativity

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Assignment I: Length-contraction, time-dilation, velocity addition

Submission due date: 13/03/2020

Q.1) (a) Two rods of proper length l_0 move lengthwise towards each other parallel to the common axis with the same velocity v relative to the Laboratory frame. Show that the length of each rod in the reference frame fixed to the other rod is

$$l = l_0 \frac{1-\beta^2}{1+\beta^2}, \beta = \frac{v}{c}.$$

(b) Two rockets of rest length l_0 are approaching each other from opposite directions at same speed $\frac{c}{2}$. How long does one of them appear to the other?

(c) In a frame S the following two events occur

$$\texttt{Event 1: } x_1 = x_0,\ t_1 = \frac{x_0}{c} \texttt{ and } y_1 = z_1 = 0,$$
$$\texttt{Event 2: } x_2 = 2x_0,\ t_2 = \frac{x_0}{2c} \texttt{ and } y_2 = z_2 = 0.$$

Show that the velocity of the frame S' (w.r.t. S) at which these two events occur simultaneously is $\beta = -0.5$. Also prove that the time in S' at which these events are simultaneous is $t' = \sqrt{3}x_0/c$. Here, x_0 is a constant and c is the speed of light in free space.

Q.2) (a) An observer in a reference frame S' sees an objective travelling at a velocity $0.8c$ at right angle to x' axis of S'. An observer in a reference frame S whose x-axis is oriented parallel to x' axis of S' observes that S' has a velocity $0.8c$ along the x-axis of S. What is the magnitude of velocity of the object as observed by the observer in S? What is the angle made by the velocity of the object with the x-axis as observed by the observer in S?

(b) Two events occur at the same place in an inertial frame and are separated by a time interval of $4sec$. What is the spatial separation between these two events in an inertial frame in which events are separated by a time interval of $6sec$?

(c) In a certain inertial frame light pulses are emitted by two sources $5km$ apart. Time interval between two pulses is $5\mu s$. An observer moving at a speed v along the line joining these sources notes that the pulses are simultaneous. Find the speed v of the observer.

Q.3) (a) Calculate the velocity at which the relative increase in the mass of relativistic particle is $f\%$.

(b) According to an observer on Earth, a certain star is d light years away. A spaceship travel from the Earth to the star at a uniform speed v and takes d years to get there according to the pilot's measure of time. Show that the speed of the spaceship relative to Earth is $c/\sqrt{2}$, where c is the speed of light. Assume that there are k seconds in a year.

(c) Show that a circle $x^2 + y^2 = a^2$ in frame S appears to be an ellipse with eccentricity $\frac{v}{c}$ in the frame S' which is moving with velocity v along the x-axis relative to S.

Q.4) (a) Two trains A and B each have proper length L and move in the same direction. $A's$ speed is $4c/5$ and $B's$ speed is $3c/5$. A starts behind B. How long, as viewed by a person on the ground, does it take for A to overtake B?

(b) A muon at rest has lifetime $2 \times 10^{-6}s$. What is its lifetime when it travels with a velocity $3c/5$?

(c) Half-life of pions at rest is $1.77 \times 10^{-8}sec$. A collimated pion beam, leaving the accelerator target at a velocity of $0.99c$, is found to drop to half its original intensity. Find the distance travelled by the pions in the laboratory.

Q.5) (a) Suppose that a meter stick pointing in the x-direction moves with the speed $0.8c$ along the x-axis of the reference frame of an observer situated at $x = 0, y = 1m$. The midpoint of the stick passes through the origin at $t = 0$. When does the observer see the mid point pass the origin?

(b) Two particles move in opposite directions, each with a speed of $0.6c$. What is the speed of one particle as seen by the other?

(c) At the time a spaceship moving with speed $v = 0.5c$ passes a space station located near Mars, a radio signal is sent from the station to Earth. This signal is received on Earth $1125s$ later. How long does the spaceship take to reach the Earth according to the observers on Earth? What is the duration according to the crew of the spaceship?