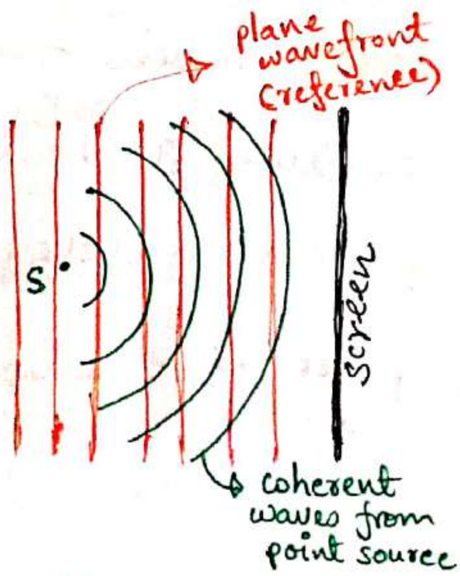
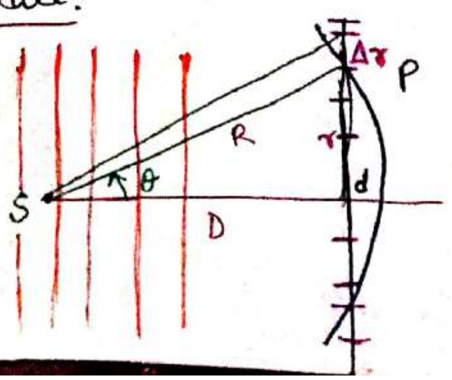
HOLOGRAPHY

In Greek, holography means whole writing. A regular photograph is a two dimensional snapshot of a 3D object whose intensity distribution projected on a plane is recorded. Denis Gabor invented in 1947 a technique in which the lost phase as well as the 3D nature of intensity is recorded. It is a two step process by which (1) an object illuminated by coherent light makes interference fringes in a photographic emulsion & (2) reillumination of the developed interference pattern by light of same wavelength to produce 3D image. Coherent source (Laser) was developed in 1960 & Gabor received Nobel prize in Physics in 1971 for 3D lensless method of photography.

Basic Principles of Holography



Consider the interference pattern caused by coherent monochromatic plane waves incident on a point scatterer S. Concentric bright & dark circles will be formed by superposition of scattered light & reference beam. The developed screen (plate) contain light & dark partially absorbing fringe pattern, called "Gabor zone plate".



Since the reference beam is assumed to be in constant phase across the hologram plate fringes at any point P will be separated by Δr that correspond to path difference of λ.

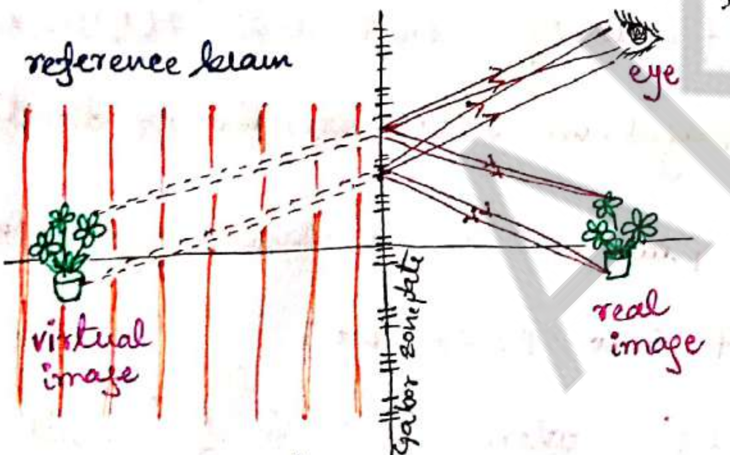
$\Delta r \sin\theta = \lambda.$

$d = R - D$

$d = n\lambda = \frac{r^2}{2R - D}$

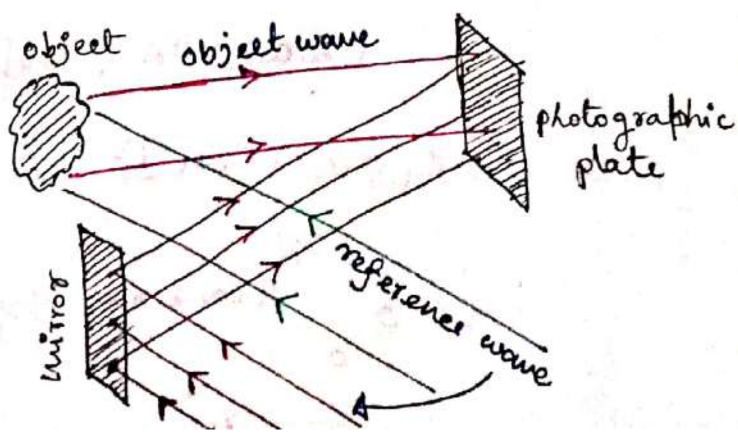
This plate is then illuminated by plane coherent waves in the absence of scatterer S. Light formed by interference between light and dark bands will now produce first order interference maxima at θ that corresponds to $\Delta r \sin\theta = \lambda$. This light will appear to diverge from S, so a virtual image can be seen from the right side of hologram.

Suppose now two scattering center S_1, S_2 are present on left & each will record a Gabor zone plate on plate. The reconstruction will produce a virtual image of both scatterers. Extending this argument to continuum of scattering centres, hologram will consist of superposed Gabor zone plates. Upon reconstruction, the distributed virtual image appears like the real image. This is called the on-axis holography.



Reconstruction process

In 1962, Leith & Upatneiks developed off-axis hologram. Due to increased coherence length of the laser beam, the reference & scattered beam were separated so that hologram could be illuminated from either side & not necessarily the reference beam is plain.



Point Holography - Basic Theory

Recording of the phase distribution of an object wave which is a point scatterer of form $\frac{A}{r}\cos(kr - \omega t + \phi)$ is the basic problem.

Recording process : Suppose object wave is $O(x,y) = a(x,y) \times \cos[\phi(x,y) - \omega t]$

due to all point scatterers at the object in plane of photographic plate at $z=0$. Suppose reference wave is plane wave propagating in xz plane inclined at θ with z axis. So,

$$r(x,y,z) = A\cos(\bar{k}.\bar{r} - \omega t) = A\cos(kx\sin\theta + kz\cos\theta - \omega t)$$

So this field at $z=0$ is $r(x,y) = A\cos(kx\sin\theta - \omega t)$

$$= A\cos(2\pi\alpha x - \omega t)$$

where $\alpha = \frac{\sin\theta}{\lambda}$ = spatial frequency. Note that $r(x,y) = r(x)$ no y dependence because its propagation vector was in xz direction.

$\therefore$ Total field at photographic plate = object wave + reference wave

$$u(x,y,t) = a(x,y)\cos[\phi(x,y) - \omega t] + A\cos(2\pi\alpha x - \omega t)$$

so $I(x,y) = \langle u^*(x,y,t)\, u(x,y,t)\rangle$ where $\langle\ \rangle$ is $\frac{1}{T}\int_0^T (\ )dt$.

$$= \langle [a(x,y)\cos[\phi(x,y) - \omega t] + A\cos(2\pi\alpha x - \omega t)]^2 \rangle$$

$$= a^2(x,y)\langle \cos^2[\phi(x,y) - \omega t]\rangle + A^2\langle \cos^2(2\pi\alpha x - \omega t)\rangle$$

$$+ 2a(x,y)A\langle \cos[\phi(x,y) - \omega t]\cos(2\pi\alpha x - \omega t)\rangle$$

[Now $\langle \cos^2[\phi(x,y) - \omega t]\rangle = \frac{1}{2\pi}\int_0^{2\pi}\cos^2(\omega t - \phi(x,y))\, d(\omega t)$

$$= \frac{1}{2\pi}\frac{1}{2}\int_0^{2\pi} 2\cos^2 x\, dx = \frac{1}{4\pi}\int_0^{2\pi}(1+\cos 2x)dx = \frac{1}{2}.]$$

$$\langle \cos[\phi(x,y) - \omega t] \cos(2\pi\alpha x - \omega t)\rangle$$

$$= \frac{1}{2}\langle \cos[\phi(x,y) \cancel{- 2\omega t + 2\pi\alpha x}]\rangle + \frac{1}{2}\langle \cos[\phi(x,y) - 2\pi\alpha x]\rangle$$

$$= \frac{1}{2}\langle \cos[\phi(x,y) - 2\pi\alpha x]\rangle = \frac{1}{2}\cos[\phi(x,y) - 2\pi\alpha x]$$

So $I(x,y) = \frac{1}{2}a^2(x,y) + \frac{1}{2}A^2 + A\,a(x,y)\cos[\phi(x,y) - 2\pi\alpha x]$.

↳ phase of object wave

↳ Intensity pattern

<u>Reconstruction process</u>: By suitable process, the photo plate is developed to obtain hologram such that transmittance is linearly related to $I(x,y)$. So if $R(x,y)$ represents the reconstruction wave (identical to the reference wave $r(x,y)$), then

$\boxed{R(x,y) = r(x,y)}$

$$\underset{\text{transmitted field}}{\psi(x,y)}\Big|_{z=0} = \underset{\text{reconstructed wave}}{R(x,y)}\, I(x,y) = \left[\frac{1}{2}a^2(x,y) + \frac{1}{2}A^2\right]R(x,y) + A\,a(x,y)R(x,y)\cos[\phi(x,y) - 2\pi\alpha x]$$

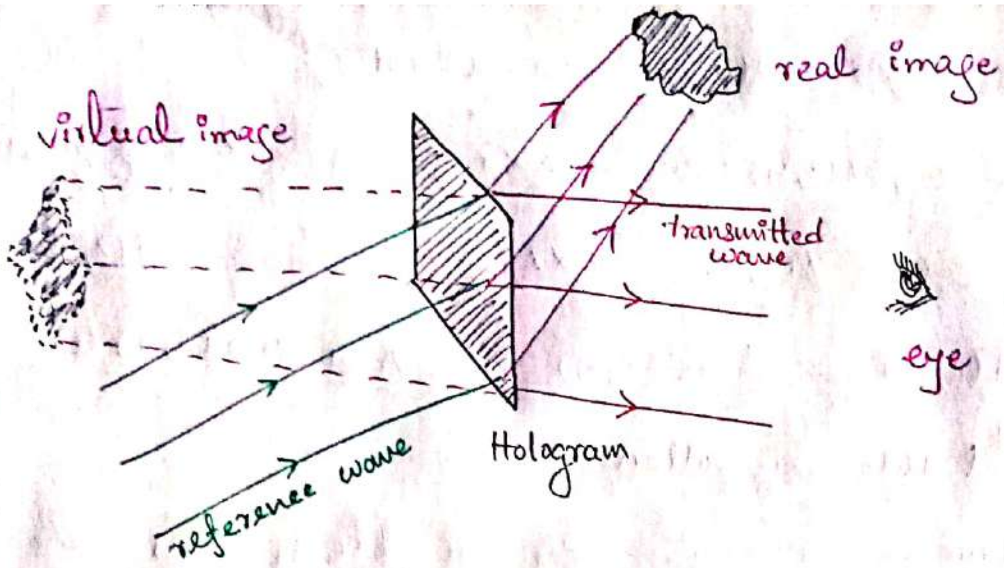
$$= \left[\frac{1}{2}a^2(x,y) + \frac{1}{2}A^2\right]A\cos(2\pi\alpha x - \omega t) + A^2 a(x,y)\cos[2\pi\alpha x - \omega t]\cos[\phi(x,y) - 2\pi\alpha x]$$

$$= \underbrace{\left[\frac{1}{2}a^2(x,y) + \frac{1}{2}A^2\right]A\cos(2\pi\alpha x - \omega t)}_{1} + \underbrace{\frac{1}{2}A^2 a(x,y)\cos[\phi(x,y) - \omega t]}_{2} + \underbrace{\frac{1}{2}A^2 a(x,y)\cos[4\pi\alpha x - \phi(x,y) - \omega t]}_{3}$$

1 ⇒ Reconstruction wave with modulated amplitude due to $a^2(x,y)$. So this part of total field is travelling in the direction of reconstructed wave.

2 ⇒ Object wave $O(x,y)$ that gives a virtual image. The reconstructed object wave travels in same direction as original object wave.

3 ⇒ Notice that identical to $O(x,y)$ in addition to $4\pi\alpha x$ term phase term $\phi(x,y)$ has a negative sign, meaning this wave has opposite curvature than the object wave, so if object wave is diverging out then this wave is converging in spherical wave.



Effect of $4\pi\alpha x$ is understood as follows. If we consider object wave is plane wave propagating along z direction then $\phi(x,y)=0$

So $\frac{1}{2}A^2 a(x,y)\cos[4\pi\alpha x - \omega t] \sim \cos(k'x - \omega t)$

where previously $k\sin\theta = 2\pi\alpha$. so that

$2k\sin\theta = 4\pi\alpha = k' = \cancel{k}\sin\theta'$

or $\theta' = \sin^{-1}(2\sin\theta)$

So (3) ⇒ represents a plane propagating wave along direction θ'. So $4\pi\alpha x$ rotates the wave direction. so this represents the conjugate of object wave propagating along a different direction from reconstruction wave & object wave. All (1), (2), (3) propagate in three different directions, they separate after travelling a distance so that observer can view the virtual imge without any disturbance.