(1) $n = 2.5\times 10^{28}\ m^{-3}$, $h = 6.625\times10^{-34}$ Js, $m = 9.1\times10^{-31}$ kg

Fermi Energy $E_f = \frac{h^2}{2m}\left(\frac{3n}{8\pi}\right)^{2/3}$

$$= \frac{(6.625\times10^{-34})^2}{2\times9.1\times10^{-31}}\left[\frac{3\times2.5\times10^{28}}{8\times3.142}\right]^{2/3}$$

$$= 4.97\times10^{-19}\ J = \frac{4.97\times10^{-19}}{1.6\times10^{-19}} = \underline{3.1\ eV}.$$

Fermi velocity $v_F = \frac{\hbar}{m}(3\pi^2 n)^{1/3} = \frac{h}{2\pi m}(3\pi^2 n)^{1/3}$

$$= \frac{h}{2m}\left(\frac{3n}{\pi}\right)^{1/3} = \frac{6.625\times10^{-34}}{2\times9.1\times10^{-31}}\left[\frac{3\times2.5\times10^{28}}{3.142}\right]^{1/3}$$

$$= \underline{1.048\times10^{6}\ m/s}.$$

Free electron theory

(2) Volume of 1 gm mole of silver $V = \frac{108\ gm}{10.5\ gm/cc}$ &

$N = 6.02\times10^{23}$

$\therefore\ n = \frac{6.02\times10^{23}\times10.5}{108}$

As 1 electron per atom, no. of electron / unit volume

$$n = \frac{6.02\times10^{23}\times10.5}{108}\times1 = 5.85\times10^{22}\ \text{electrons/cc}$$

$$= 5.85\times10^{28}\ /m^3.$$

$\therefore$ Fermi Energy $E_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi}\right)^{2/3} = \frac{(6.625\times10^{-34})^2}{2\times9.1\times10^{-31}}\left(\frac{3\times5.85\times10^{28}}{8\times3.14}\right)^{2/3}$

$$= 8.77\times10^{-19}\ J = \frac{8.77\times10^{-19}}{1.6\times10^{-19}} = \underline{5.48\ eV}$$

(3) No. of atoms / unit cell for fcc $= 4 =$ no. of electron / unit cell because each atom contributes 1 electron.

volume $V = a^3 = (4\ \text{Å})^3 = (4\times10^{-8})^3\ cc = 64\times10^{-24}\ cc$

$\therefore$ no. density $n = \frac{N}{V} = \frac{4}{64\times10^{-24}} = 6.25\times10^{22}\ /cc$

Fermi energy $E_f = \frac{h^2}{2m}\left(\frac{3n}{8\pi}\right)^{2/3} = 9.19 \times 10^{-12}$ erg

$= \frac{9.19 \times 10^{-12}}{1.6 \times 10^{-12}} = 5.74$ eV.

Fermi vector $k_F = (3\pi^2 n)^{1/3} = (3 \times \pi^2 \times 6.25 \times 10^{22})^{1/3}$

$= 1.23 \times 10^8$ /cm.

Total KE/unit volume at OK = Average KE/electron at OK multiplied by no of electrons/unit volume

$= \frac{3}{5} E_F \times n = \frac{3}{5} \times 5.74 \times 6.25 \times 10^{22} = 21.52 \times 10^{22}$ eV.