2022

PHYSICS — HONOURS — PRACTICAL

Paper : CC-8P

[Mathematical Physics-II]

(Syllabus : 2019-2020)

Full Marks : 30

The figures in the margin indicate full marks.

[Distribution of Marks : LNB - 5, Viva - 5, Experiment - 20, Total - 30]

1. (a) Consider the following Gaussian integral :

$$\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-(x-\mu)^2/2\sigma^2}\,dx$$

Take suitable values of σ and μ and plot the function. Find the value of the integral.

(b) Suppose $F(t) = -at^2$ and $G(t) = -bt^2$, are two Gaussians, where $a, b > 0$. Taking suitable values of a and b, show that the convolution of these two Gaussians is also a Gaussian. Plot them.

8+12

2. (a) Consider the following damped harmonic motion :

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = 0$$

Take $\gamma = 0.4$ and $k = 1$. Solve the differential equation by using a suitable function from scipy module and plot the decay of x versus t for a definite range.

(b) Consider the Wave functions for 1D quantum harmonic oscillator :

$$\psi_n(x) = \frac{1}{\sqrt{2^n n!}}\left(\frac{1}{\pi}\right)^{\frac{1}{4}} \exp\left(-\frac{x^2}{2}\right) H_n(x)$$

where H_n is the Hermite polynomial and we consider $\frac{m\omega}{h} = 1$. Plot the wave functions for $n = 0$, 1 and 2 in a single graph.

12+8

3. (a) Consider the following forced harmonic motion :

$$\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}+kx=F\cos\omega t$$

where the exact solution of the amplitude of the steady state $= F/\sqrt{(k-\omega)^2+\gamma^2\omega^2}$. Take $\gamma=0.5$, $k=5$ and $F=1$. Solve the differential equation by using a suitable function from scipy module and plot the amplitude resonance curve. Compare with the exact formula.

(b) Prove the orthogonality relation :

$$\int_{-1}^{1}P_m(x)P_n(x)=\frac{2}{2n+1}\delta_{mn}$$

where P_m is the m-th order Legendre polynomial. 12+8

4. (a) Analyse in a Fourier series of a Sawtooth wave whose window size is 6 and frequency is 2. Evaluate the Fourier coefficients and plot the constructed series with the given periodic function.

(b) Consider a rod (length L) of uniform cross-section and made of homogeneous material which is perfectly insulated so that heat flows only along the length. Two ends of the rod are inserted in two ice baths to maintain fixed temperature at both the ends and heat at the middle. Consider the following diffusion equation :

$$\frac{\partial u}{\partial t}=D^2\frac{\partial^2 u}{\partial x^2}$$

where D is the diffusion coefficient and may be set to unity. Forming one initial condition and two boundary conditions, solve this partial differential equation and plot the temperature profile (temperature along the length of the rod). 8+12

5. (a) Evaluate the Fresnel Integral :

$$\int_0^\infty \sin x^2 dx$$

Plot the solution up to a certain value of the upper limit. Comment on the plot.

(b) Write a python program to analyse in a Fourier series of a full-wave rectifier signal :

$$f(x)=|\sin(x)|$$

Evaluate the Fourier coefficients and plot the constructed series with the given periodic function. 10+10

3. (a) Consider the following radioactive decay equation :

$$\frac{dx}{dt} = -kx$$

Solve the differential equation by using a suitable function from scipy module and plot decay of x versus t.

(b) Prove the orthogonality relation :

$$\int_{-1}^{1} P_m(x) P_n(x) = \frac{2}{2n+1} \delta_{mn}$$

where P_m is the m-th order Legendre polynomial. 12+8

4. (a) Analyse in a Fourier series of a square wave whose window size is 6 and frequency is 2. Evaluate the Fourier coefficients and plot the constructed series with the given periodic function.

(b) Consider a rod (length L) of uniform cross-section and made of homogeneous material which is perfectly insulated so that heat flows only along the length. The right end of the rod is inserted in an ice bath to maintain fixed temperature at that end and heated at the left end. Consider the following diffusion equation :

$$\frac{\partial u}{\partial t} = D^2 \frac{\partial^2 u}{\partial x^2}$$

where D is the diffusion coefficient and may be set to unity. Forming one initial condition and two boundary conditions, solve this partial differential equation and plot the temperature profile (temperature along the length of the rod). 8+12

5. (a) Evaluate the Fresnel Integral :

$$\int_{0}^{\infty} \sin x^2 dx$$

Plot the solution up to a certain value of the upper limit. Comment on the plot.

(b) Write a python program to anlyse in a Fourier series of a symmetric triangular signal. Evaluate the Fourier coefficients and plot the constructed series with the given periodic function. 10+10

6. (a) Write a suitable python program to calculate the integral : $\int_0^\infty e^{-x^2} dx$. Set the accuracy level to 0.0001.

(b) Write a suitable program to solve the following equation :

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \text{ where } 0 < x < \pi,\ t > 0 \text{ with the initial and boundary conditions :}$$

$$u(x,0) = 1,\ \frac{\partial u(x,0)}{\partial t} = 0$$

$$u(0, t) = 0,\ u(\pi, t) = 0.$$

8+12

7. (a) Verify the following identity with a python program :

$$\frac{\sin(n+1)\theta}{\sin\theta} = \sum_{l=0}^{n} P_l(\cos\theta)\, P_{n-l}(\cos\theta),$$ where the symbols have their usual meanings.

(b) Consider the Cauchy problem :

$$\frac{\partial u}{\partial t} = x\frac{\partial u}{\partial x} - u + 1,\ -\infty < x < \infty, t \geq 0.$$

$$u(x, 0) = \sin x,\ -\infty < x < \infty$$

Solve this problem and discuss the behaviour of the solution for large time.

10+10

2022

PHYSICS — HONOURS — PRACTICAL

Paper : CC-10P

(Syllabus : 2019-2020)

[Quantam Mechanics]

Full Marks : 30

The figures in the margin indicate full marks.

[Distribution of marks : LNB – 5, viva – 5, Experiment – 20, Total – 30]

Attempt *any one* question : 20×1

1. Solve the following boundary value problem (BVP) by Shooting method :

   $u'' = 6t,\ \ 0 < t < 1$ with BC $u(0) = 0,\ u(1) = 1$, Plot the solutions.

2. Solve the 1D time independent Schrodinger equation with the finite potential of depth $V_0 = -10$ in the domain $0 < x < 1$. Find out the ground state and the first excited state.

3. Consider 1D Quantum Harmonic Oscillator. Solve the Schrodinger equation numerically and obtain the second excited state (E_2). Take two energy values $= E_2\,(1 \pm 0.02)$ and plot the corresponding solutions.

4. Solve the BVP : $y'' + y = \cos 2x$ with BC : $y(0) = 0,\ y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$

   Plot with exact solution : $\dfrac{(\cos x - \cos 2x)}{3} + \sin x$

5. Solve the Infinite square well problem and plot a graph to show only the eigen energies between 4th to 6th eigen energy values. Plot the three eigen states in a same graph.

6. Consider a double well potential : $V(x) = \lambda(Cx^4 - x^2)$ with $\lambda = 0.0002$ and $C = 0.045$. Find out the two lowest energy states and plot.

7. Solve the radial part of the Hydrogen atom problem. Find out the first three s-states and plot.

8. Find out the first three excited states of a Quantum Harmonic Oscillator (QHO) in 1D and plot the corresponding probability densities in a same graph.

9. Do a graphical presentation to locate the first three excited states for QHO in 1D. Obtain the probability density for the third excited state.

10. Write a Python function to obtain the eigen energies for the radial part of the Hydrogen atom problem. Use this function to obtain first 4 eigen energies.

11. Obtain the eigen energies and eigen states for a screened Coulomb potential $V(r) = -\frac{e^2}{4\pi\epsilon_0}\frac{1}{r}e^{-r/a}$.

[Hint : Change only the line in the Python function for Hydrogen atom problem :
`(L*(L+1)/r**2 - 2/r*np.exp(-r/10)- E)* u`, with $a = 10$]