Radiation

According to Maxwell, thermal radiation is defined as the transfer of heat from hot to cold body without any heating of the intervening medium. Thermal radiation, e.g. heat, has the same nature as light with properties:

(i) Electromagnetic wave nature to travel in Ether/vacuum at velocity of light.

(ii) travels in a straight line like light & exhibit reflection, refraction, interference, diffraction & polarisation.

These are not visible radiation but usually in UV, X ray or γ-ray.
These can be detected using Thermopiles, Bolometer etc.

<u>Blackbody radiation</u> If heat is transferred through matter, a fraction reflected
fraction of incident radiation is absorbed (say a), a fraction reflected
(say b) and rest transmitted (say c) then $a+b+c=1$ and if
$b=c=0$ then the body appears black because nothing is reflected
or transmitted. When heated, such blackbody radiates energy of
all wavelengths. These principles are used for instance

(a) White clothes are trendy in summer but dark coloured clothes in winter, as white clothes reflect maximum light & is least warm. Converse is true with dark shades. (b) Utensils are polished atop & blackened bottom so that maximum heat is absorbed & minimal heat flows out from above, (c) Hot water pipes are painted black inside room & white outside to provide heating to room in winter & prevent radiation at outside. (d) thermocouple junctions that has to be heated is painted black and so on.

Spectral Emissive Power e_λ : It is the radiant energy emitted normally from unit area of the black body surface per unit time in unit solid angle within a unit wavelength range.

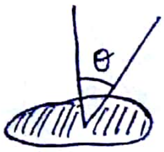
If spectral energy density is u_λ then $u_\lambda d\lambda$ is radiated energy from area dS in solid angle $d\omega$ in time dt, then

$$e_\lambda d\lambda = \frac{u_\lambda d\lambda}{dS\, d\omega\, dt} \longrightarrow [\lambda, \lambda + d\lambda]$$

Absorptive power a_λ : It is the fraction of incident to absorbed radiation, so if Q_λ is incident & Q_λ' is absorbed then

$$a_\lambda = \frac{Q_\lambda'}{Q_\lambda}$$ and total heat absorbed by all $\lambda = \int_0^\infty a_\lambda Q_\lambda d\lambda$.

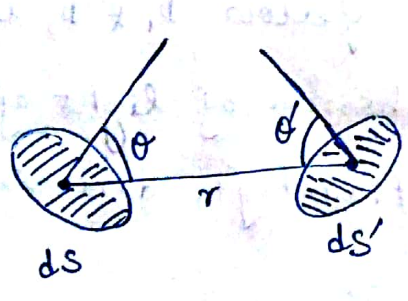
Total Emission from dS



$$dQ_\lambda = \frac{u_\lambda d\lambda}{dt} = e_\lambda d\lambda\, dS\, d\omega.$$ So the total emission on one side $= \int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi} e_\lambda d\lambda\, dS\, d\omega \cos\theta$

$$= e_\lambda d\lambda\, dS \int_0^{\pi/2} \sin\theta\cos\theta\, d\theta \int_0^{2\pi} d\phi = \pi e_\lambda d\lambda\, dS.$$

Mutual radiation between two surfaces



The solid angle subtended by dS' at dS is $d\omega' = \frac{dS'\cos\theta'}{r^2}$. So the amount of radiation incident on dS is $e_\lambda d\lambda\, dS \cos\theta\, d\omega'$

$$= e_\lambda d\lambda\, dS\cos\theta \frac{dS'\cos\theta'}{r^2}$$ and the energy absorbed is $a_\lambda e_\lambda d\lambda\, dS\cos\theta \frac{dS'\cos\theta'}{r^2}$ and energy reflected will be

$$(1 - a_\lambda) e_\lambda d\lambda\, dS\cos\theta \frac{dS'\cos\theta'}{r^2}.$$

Kirchhoff's law Ratio of emissive to absorptive power for a given wavelength at a given temperature for all bodies is same & equal to the emissive power of a perfect black body.

By definition, a_λ = absorptive power of a body, if dQ heat is incident on unit area in unit time within λ & $\lambda + d\lambda$, then heat absorbed $= a_\lambda dQ$ and $dQ - a_\lambda dQ$ will be transmitted or reflected. If e_λ = emissive power then $e_\lambda d\lambda$ is the energy emitted per unit area per unit time within λ & $\lambda + d\lambda$.

$\therefore$ Total emitted energy $= (1-a_\lambda)dQ + e_\lambda d\lambda$ and in equilibrium, $\cancel{dQ} = (\cancel{1} - a_\lambda)dQ + e_\lambda d\lambda \Rightarrow a_\lambda dQ = e_\lambda d\lambda$.

For a perfect blackbody $e_\lambda = E_\lambda$ (notation) & $a_\lambda = 1$.

$\therefore dQ = E_\lambda d\lambda$. $\therefore a_\lambda E_\lambda d\lambda = e_\lambda d\lambda$

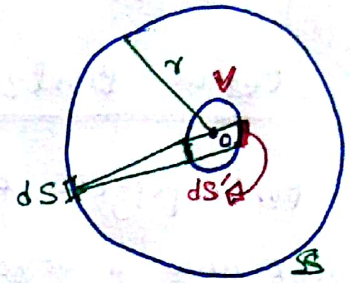
or $\boxed{E_\lambda = \dfrac{e_\lambda}{a_\lambda}}$ Kirchhoff's law

It also means not only radiation is independent on the shape or nature of wall of a hollow radiator & good absorbers are also good emitters. Na vapour that emit yellow D_1 & D_2 lines of λ 5890 Å & 5896 Å is also a good absorber of light of these two wavelengths. This explains the Fraunhofer dark lines in Sun's spectrum.

Pressure of diffuse radiation

When radiation falls normally to a surface then the radiation pressure is the sum of incident & reflected waves energy density. Larmor calculated it using Kepler's observation of radiation pressure of tail of comets rotating around so as to be always opposite to sun. Inside a heated container such radiation is diffuse.

Consider volume V at a very large distance from container wall so that radiation through V is the radiation coming from surface of sphere of radius r. As by construction $ds << V$, we can divide solid angle subtended by V at ds into many cones of solid angle $d\omega$ with area ds', so that $d\omega = \frac{ds'}{r^2}$.



If the volume V intersects infinitesimal cone of length l then time taken by radiation to travel is $dt = \frac{l}{c}$. If K is specific intensity or radiation emitted per unit area per unit time per unit solid angle then energy coming from $ds = K\,ds\,d\omega\,dt$

$$= K\,ds\,\frac{ds'}{r^2}\,\frac{l}{c}$$

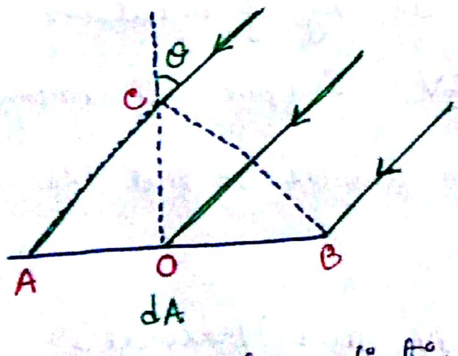
So the radiation contained in $V = \sum_V K ds \frac{ds'}{r^2}\frac{l}{c} = \frac{K ds}{c r^2}\sum_V l\,ds'$

$$= \frac{K ds}{c r^2} V$$

$\therefore$ Total radiation in V from the whole surface is $= \sum_S \frac{K ds}{c r^2} V$

$$= \frac{KV}{cr^2}\sum_S ds = \frac{KV}{cr^2}4\pi r^2 = \frac{4\pi K}{c}V, \qquad E = \frac{4\pi K}{c}$$

or energy density of diffuse radiation is $\frac{4\pi K}{c}$. Suppose a parallel beam of radiation is incident on dA at angle θ so that, pressure on BC is



$$p = \frac{\text{intensity of radiation}}{\text{speed of radiation}} = \frac{K}{c}\sin\theta\,d\theta\,d\phi$$

Force due to radiation on $BC = \frac{K}{c}\sin\theta d\theta d\phi\, BC$ & the normal component is $= \frac{K}{c}\sin\theta\, d\theta\, d\phi\; BC\cos\theta = \frac{K}{c}\sin\theta d\theta d\phi\; AB\cos^2\theta$

$\therefore$ Total radiation pressure on $AB = 2\int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi}\frac{K}{c}\sin\theta d\theta d\phi\cos^2\theta$

also reaction pressure

$$= \frac{2K}{c}\int_{\theta=0}^{\pi/2} \cos^2\theta \sin\theta d\theta \int_{\phi=0}^{2\pi} d\phi = \frac{1}{3}\frac{4\pi K}{c} = \frac{1}{3}E.$$

$$\therefore \boxed{P = \frac{1}{3}E}$$

Stefan - Boltzmann law

For a perfect blackbody, the rate of emission of radiant energy by unit area is proportional to the fourth power of its absolute temperature.

$$E = \sigma T^4, \quad \sigma = \text{Stefan's constant} = 5.67 \times 10^{-8}\ W m^{-2} K^{-4}$$

In other words, if a black body at absolute temperature T is surrounded by another blackbody at absolute temperature T_0 then the net rate of loss of heat energy per unit area of the surface per unit time is $E = \sigma(T^4 - T_0^4)$.

Note that this is in accordance with "Prevost's theory of heat exchange" that states, "the net loss of heat is the difference in the heat radiated by the hot body and the heat absorbed by it from its surroundings." Stefan's law refers to the emission of heat radiation only by the blackbody and not to the net loss of heat by the blackbody after heat exchange with its surroundings.

In 1884, Boltzmann theoretically proved Stefan's law using Thermodynamics. Suppose an encloser of volume V is filled with radiation at uniform temperature T. E is the energy density of radiation so that total internal energy is $U = EV$. Suppose dQ amount of heat is flowed into the enclosure from outside so that the volume changed to $V + dV$.

Using first law of T.D. $dQ = dU + PdV$ and Maxwell's thermodynamic relation $\left(\frac{\partial Q}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V$ we have

$$\left(\frac{\partial U + PdV}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V$$

$$\therefore \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

using $P = \frac{1}{3}E$ and $U = EV$, we have $\left(\frac{\partial U}{\partial V}\right)_T = E$, $\left(\frac{\partial P}{\partial T}\right)_V = \frac{1}{3}\left(\frac{\partial E}{\partial T}\right)_V$

$$\therefore E = \frac{T}{3}\frac{dE}{dT} - \frac{E}{3} \quad \text{or} \quad \frac{4E}{\cancel{3}} = \frac{T}{\cancel{3}}\frac{dE}{dT}$$

$$\text{or} \int \frac{dE}{E} = 4\int \frac{dT}{T} + \text{constant} \quad \text{or} \quad \ln E = 4\ln T + \ln \sigma$$

$$\text{or} \quad E = \sigma T^4$$

Newton's law of cooling

Stefan's law is applicable for all temperatures but Newton's law is applicable when temperature difference between blackbody & surrounding is small. If T_1 is hotbody's temperature which is placed in an enclosure at T_2 then from Stefan's law

$$E = \sigma(T_1^4 - T_2^4) = \sigma(T_1 - T_2)(T_1^3 + T_1^2T_2 + T_1T_2^2 + T_2^3).$$

while $(T_1 - T_2)$ is small, $T_1 \approx T_2$ so that $T_1^2T_2 \sim T_2^3$ & so on

$$\therefore E = \sigma(T_1 - T_2)(T_2^3 + T_2^3 + T_2^3 + T_2^3)$$

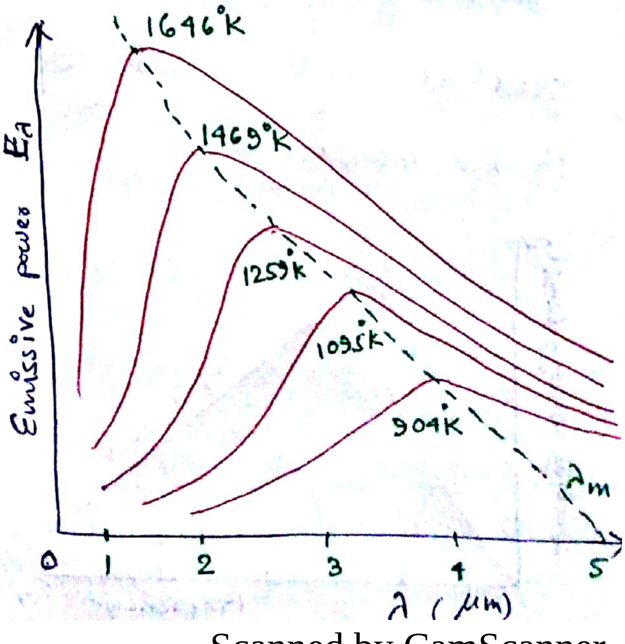
$$= \underbrace{4\sigma T_2^3}_{K}(T_1 - T_2) = K(T_1 - T_2).$$

$$\therefore E \propto (T_1 - T_2)$$

Distribution of Energy in Blackbody Spectrum

Energy is not uniformly distributed for isothermals, and exhibits a maxima at a particular wavelength. If λ_m is the wavelength for which the emitted energy is maximum, then Wien's displacement law states that

$$\lambda_m T = \text{constant}.$$



for all wavelengths, increase in temperature leads to increase in energy emission. Area under each curve represents total energy emitted & is found to be directly proportional to T^4, or $E \propto T^4$ which is the Stefan-Boltzmann's law.

Wien's Displacement law

When an electric wire is heated, at 500°C it is dull red, at 900°C its cherry red, at 1100°C its orange red, at 1250°C its yellow and at > 1600°C becomes white. So as the temperature is raised, the maximum intensity of emission is displaced towards the shorter wavelength. Wien's law is $\lambda_m T = \text{constant} = 0.2892 \text{ cm K}$

$$E_m \propto T^5 \quad \text{or} \quad E_m T^{-5} = \text{constant}$$

This can be combined with Stefan's law in one form as

$$E_\lambda = C \lambda^{-5} f(\lambda T)$$

Wien derived that $E_\lambda d\lambda = K\lambda^{-5} e^{-a/\lambda T} d\lambda$. This law holds good only at shorter wavelength & lower temperature, but do not hold good at longer wavelengths & higher temperature.

Rayleigh - Jeans law & the UV Catastrophe :

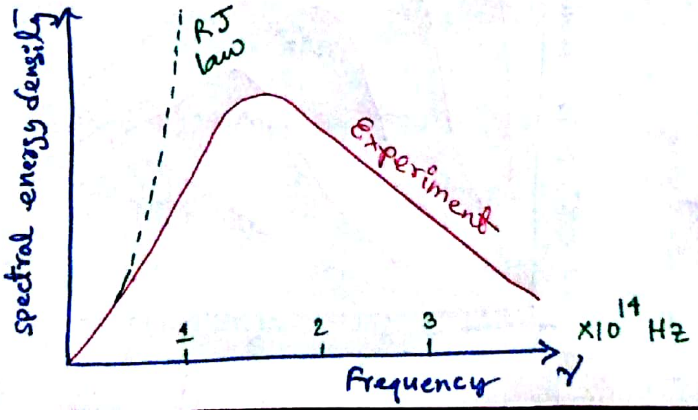
According to RJ law, $E_\lambda d\lambda = \dfrac{8\pi k_B T}{\lambda^4} d\lambda$. This law holds good at longer wavelengths at higher temperatures and not good at shorter wavelengths.

Writing $\lambda = \dfrac{c}{\nu}$ or $d\lambda = \dfrac{c}{\nu^2} d\nu$

using RJ law we can write

$$dE = E_\lambda d\lambda = \frac{8\pi k_B T}{c^4} \nu^4 \frac{c}{\nu^2} d\nu$$

$$= \frac{8\pi \nu^2 k_B T}{c^3} d\nu$$

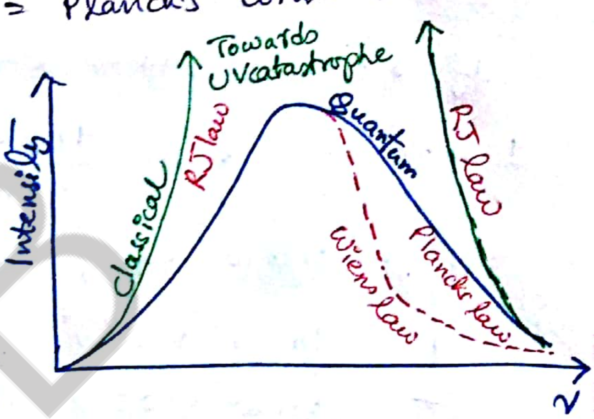


For $\nu \to \infty$, $dE \to \infty$ is a direct contradiction to experimental observations $\rightarrow$ "ultraviolet catastrophe". Again,

$$E = \int_0^{\infty} \frac{8\pi\nu^2 k_B T \, d\nu}{c^3} \to \infty$$ which is contradiction to Stefan's law.

Thus before 1900, UV catastrophe was the biggest failure of classical physics until Max Planck, who used quantum mechanics idea to treat radiation as emitted quanta of energy $h\nu$. According to his treatment

$$\boxed{\begin{aligned} E_\nu \, d\nu &= \frac{8\pi h}{c^3} \frac{\nu^3 d\nu}{e^{h\nu/k_B T} - 1} \\ E_\lambda \, d\lambda &= \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{hc/\lambda k_B T} - 1} \end{aligned}}$$, $h = 6.626 \times 10^{-34}$ Js = Planck's constant.



This is called Planck's radiation law, that agrees perfectly with experiments.

Planck's quantum postulates say that

(a) A radiation enclosure can be imagined as a collection of resonators (simple harmonic oscillators) that can vibrate at all frequencies. (b) Resonators cannot radiate/absorb energy continuously but in the form of quanta-packets (photons). Each photon has energy $h\nu$ so that energy emitted/absorbed is $0, h\nu, 2h\nu, \ldots, nh\nu$ or "quantum".

① $\lambda <<$ (short wavelengths) Planck's Radiation law

$$E_\lambda \, d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{hc/\lambda k_B T} - 1} \simeq \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{hc/\lambda k_B T}}$$

$$= \frac{8\pi hc}{\lambda^5} e^{-hc/\lambda k_B T} d\lambda$$ $\Rightarrow$ "Wien's displacement law."

② $\lambda >>$ (larger wavelengths) $e^{hc/\lambda k_B T} \simeq 1 + \frac{hc}{\lambda k_B T}$

Planck's radiation law $E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{1 + \frac{hc}{\lambda k_B T} - 1} = \frac{8\pi k_B T}{\lambda^4} d\lambda$

$\Rightarrow$ "Rayleigh-Jeans law."

Solar Constant

It is the amount of radiation absorbed per unit time per unit area of a black body placed at a mean distance between sun & earth in the absence of atmosphere with the surface held normal to the sunrays.

If mean distance between sun & earth = R, solar constant = S

∴ Total amount of radiation received by the sphere in 1 minute $= 4\pi R^2 S$.

If r = sun's radius then radiation by 1 square cm surface in 1 minute $E = \dfrac{4\pi R^2 S}{4\pi r^2}$

$r = 6.928 \times 10^5$ Km, $R = 148.48 \times 10^7$ Km, $S = 1.94$ cals/cm²/minute

$$E = \frac{(148.48 \times 10^7)^2}{(6.928 \times 10^5)^2} \times \frac{1.94}{60} \text{ cals/second.}$$

But from Stefan's law $E = \sigma T^4 = \dfrac{5.75 \times 10^{-5}}{4.2 \times 10^7} T^4$ as

$\sigma = 5.75 \times 10^{-5}$ Ergs/cm²/deg⁴ $= \dfrac{5.75 \times 10^{-5}}{4.2 \times 10^7}$ cals/cm²/deg⁴

Equating,
$$\frac{(148.48 \times 10^7)^2}{(6.928 \times 10^5)^2} \cdot \frac{1.94}{60} = \frac{5.75 \times 10^{-5}}{4.2 \times 10^7} T^4$$

$$\therefore T = \underline{5730\,K}.$$

The photosphere of sun (outer surface) is approximately $\underline{6000K}$. Calculated value yields the effective temperature when sun acts as a blackbody radiator.

This can also be calculated from Wien's displacement law

$\lambda_m T = 0.2892$. $\lambda_m = 4900 \times 10^{-8}$ cm (maximum in spectrum)

$\therefore T = \underline{5902\,K}$.

CW 1. (a) Two large closely spaced concentric spheres (blackbody radiator) are kept at temperature 200K & 300K & the in between space is vacuum. Calculate the net rate of energy transfer between the two spheres. (b) Calculate the radiant emittance of a blackbody at temperatures 400K & 4000K. Given $\sigma = 5.672 \times 10^{-8}$ M.K.S. units.

(a) $T_1 = 300K$, $T_2 = 200K$ ∴ from Stefan's law, net rate of energy transfer $E = \sigma(T_1^4 - T_2^4)$

$= 5.672 \times 10^{-8} (300^4 - 200^4) = 368.68$ watts/m^2

(b) for $T = 400K$, $E = 5.672 \times 10^{-8} \times 400^4 = 1452$ watts/m^2

for $T = 4000K$, $E = 5.672 \times 10^{-8} \times 4000^4 = 1452 \times 10^4$ watts/m^2

$= 14520$ Kilowatts/m^2.

2. An aluminium foil is placed between two concentric spheres (blackbody radiators) at temperatures 300K & 200K. Calculate the temperature of the foil in the steady state. Also calculate the rate of energy transfer between one of the spheres and the foil.

If x is the temperature of foil in steady state then we have using Stefan's law, $\sigma(T_1^4 - x^4) = \sigma(x^4 - T_2^4)$

here $T_1 = 300K$, $T_2 = 200K$. or $300^4 - x^4 = x^4 - 200^4$

or $x = 263.8$ K.

∴ Rate of energy transfer $E = \sigma(T_1^4 - x^4)$

$= 5.672 \times 10^{-8} (300^4 - 263.8^4)$

$= 185$ watts/m^2.

[N.B. If relative emittance is mentioned (say $e = 0.1$) then
$E = e\sigma(T_1^4 - x^4) = 18.5$ watts/m^2]

3. Obtain the number of modes of vibration per unit volume in the wavelength range 4990 Å to 5010 Å for a cubic shaped cavity of a blackbody.

Number of modes/volume within λ & $\lambda+d\lambda$

$$n = \frac{E_\lambda d\lambda}{k_B T} = \frac{8\pi d\lambda}{\lambda^4} \quad \text{using Rayleigh-Jean's law.}$$

$\lambda = 5000\,\mathring{A} = 5000\times10^{-8}$ cm, $d\lambda = (5010-4990) = 20\,\mathring{A} = 20\times10^{-8}$ cm

$$\therefore n = \frac{8\times3.14\times20\times10^{-8}}{(5000\times10^{-8})^4} = 8.038\times10^{11}\ /\text{cc}.$$

[Same thing, if asked frequency range say 4×10^{14} & 4.01×10^{14} sec^{-1} for a chamber of volume 50 cc, then $n = \frac{8\pi\nu^2 d\nu}{c^3} = 1.5\times10^{11}$ /cc

and total number of modes in $V = 50$ cc is $= 1.5\times10^{11}\times50$
$= 7.5\times10^{12}$]

HW

1. If a black body at a temperature 6174 K emits 4700 Å with maximum energy, calculate the temperature at which it will emit a wavelength of 1.4×10^{-5} m with maximum energy.

2. Using Stefan's law, calculate the total radiant energy emitted by Sun/second. Also calculate the rate at which energy is reaching the top of earth's atmosphere. Given radius of sun $= 7\times10^{8}$ m & distance of earth's atmosphere from sun $= 1.5\times10^{11}$ m and sun (blackbody) temperature = 5800 K.

3. The order of magnitude of the energy received from sun at earth's surface is 10^{-1} Joule/cm^{2}sec. Calculate the order of magnitude of the total force due to solar radiation on the earth (perfectly absorbing). Given earth's diameter $= 10^7$ metre, & radiation pressure $P = \frac{E}{C}$.