

### Sem-I - Newtonian Mechanics

(Instructor: AKB, Department of Physics, Asutosh College)

#### Assignment II: Non-inertial Frame of Reference

Submission due date: 16/02/2021

**Q.1)** Show that for a single particle with constant mass, the equation of motion implies the following differential equation for the kinetic energy:

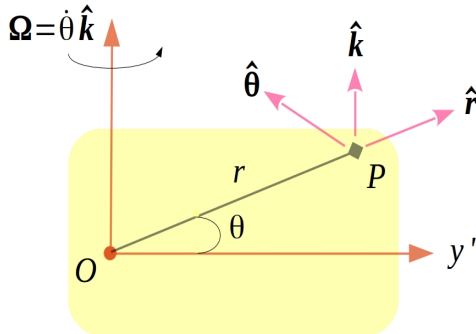
$$\frac{dE}{dt} = \mathbf{F} \cdot \mathbf{v},$$

while if the mass varies with time, the corresponding equation is

$$\frac{d(mE)}{dt} = \mathbf{F} \cdot \mathbf{p}.$$

**Q.2)** Calculate the rate of rotation of the plane of oscillation of a Foucault pendulum at latitude 30 degrees and hence determine the time it will take to turn through a complete right angle.

**Q.3)**



In figure beside, the polar coordinates  $(r, \theta)$  are measured relative to an inertial frame  $\mathcal{F}$ , while  $\mathcal{F}'$  is a frame rotating about the  $z$ -axis passing through  $O$  with angular velocity  $\Omega$ . By using the velocity and acceleration transformation formula between inertial and rotating coordinate system, show that

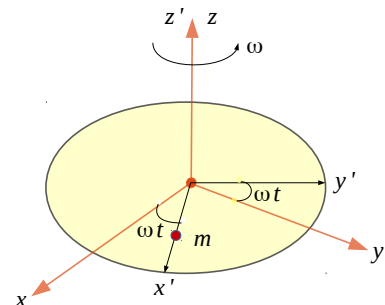
$$\mathbf{v} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}}, \quad \mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\boldsymbol{\theta}}.$$

[Note here as  $\dot{\Omega} \neq 0$ , therefore unlike our derivation in class, you also have to consider  $\dot{\Omega} \times \mathbf{r}'$  term along with fictitious acceleration terms in the rotating coordinate system.]

**Q.4)** An ocean current circulating counter-clockwise when viewed from directly overhead was discovered in a well-isolated layer beneath the surface. The period of rotation was 14 hours ( $\omega = \frac{2\pi}{14} \text{ hr}^{-1}$ ). In Earth's surface fixed coordinate frame, the ocean current is  $x$ -axis pointing south,  $y$ -axis pointing east,  $z$ -axis pointing vertically upward and  $\Omega = -\Omega \sin \lambda \hat{\mathbf{i}} + \Omega \cos \lambda \hat{\mathbf{k}}$  where  $\lambda$  is colatitude ( $\Omega = \frac{2\pi}{24} \text{ hr}^{-1}$ ). At what latitude and in which hemisphere was the current detected?

**Q.5)**

A turn-table marked with three orthogonal axes  $(x', y', z')$  is rotating on the Earth, assumed to be an inertial frame  $(x, y, z)$ , with constant angular velocity  $\omega$  about  $z$ -axis. At  $t = 0$ , both frames coincide with each other. A ball of mass  $m$  is rolling outward without slipping along the  $x'$ -axis with a constant velocity  $v$ . What is the total force exerted by the turn-table on



the ball? Find out the total force as measured in the Earth frame coordinates  $(x, y, z)$ .