Fluid Motion

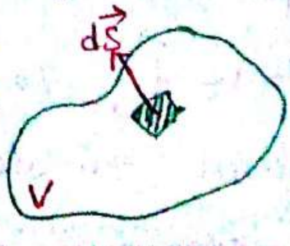
Ideal Fluids - The flow of "dry" Water

Fluid dynamics concerns with the motion of fluids (liquid & gas) in a macroscopic sense to regard as a continuous medium. Infinitely small elements of volume - fluid particle & point in a fluid means very small compared to volume of body but large compared to the molecular distance.

The Equation of Continuity: Mathematically, the state of a moving fluid is given by the fluid velocity distribution over space & time, $\vec{v} = \vec{v}(x,y,z,t)$ and of any two thermodynamic quantities, say, pressure $p(x,y,z,t)$ and density $\rho(x,y,z,t)$. So if given 3-components of velocity, pressure & density, state of the fluid is completely determined. Additionally, a conducting fluid will carry an electric current whose density $\vec{j} = \vec{j}(x,y,z,t)$. Similarly temperature or Magnetic field have similar effect.

We neglect first EM field, temperature variation & assume that density is constant or variation in pressure is very small (or the fluid is incompressible). So if the flow velocity is much less than the speed of sound wave in the fluid, density variation can be neglected. $\boxed{\rho = \text{constant}}$

Conservation of Mass: If matter flows away then there must be decrease in the amount of matter left behind. The mass of fluid flowing in unit time through a surface element $d\vec{S}$ bounding the volume is $\rho\vec{v}\cdot d\vec{S}$ & its positive if flowing out (negative otherwise), so that the total mass is $\oint_S \rho\vec{v}\cdot d\vec{S}$



Decrease in fluid mass per unit time is $-\frac{\partial}{\partial t}\int_V \rho dV$.

Therefore $\frac{\partial}{\partial t}\int_V \rho dV = -\oint_S \rho\vec{v}\cdot d\vec{S} = \int \vec{\nabla}\cdot(\rho\vec{v})\, dV$ using Green's theorem. $\therefore \int_V \left[\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v})\right] dV = 0$. Since this equation must hold for any volume V, the integrand must vanish. So $\boxed{\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0}$ $\vec{j} = \rho\vec{v}$ = mass flux density.

This is the hydrodynamic equation of continuity leading to conservation of mass. For incompressible fluid ρ = constant & so $\vec{\nabla}\cdot\vec{v} = 0$. Like Magnetic field $\vec{B}$, fluid velocity has zero divergence.

<u>Euler's equation of motion</u>: Change of velocity due to forces, torques, so that Newton's 2^{nd} law become,

Rate of increase of momentum of fluid particle = Sum of forces on fluid particle.

There are 2 types of forces on fluid particles,

- <u>Surface forces</u> $\Rightarrow$ - pressure force, viscous force, gravity force
- <u>Body forces</u> $\Rightarrow$ - centrifugal force, Coriolis force, EM force.

So total force acting on the volume $= -\oint_S P\, d\vec{S} = -\int_V \vec{\nabla}P\, dV$

So fluid surrounding any volume element dV exerts a force $-\vec{\nabla}P\, dV$ or $-\vec{\nabla}P$ per unit volume. There are external forces like electromagnetic, gravity. For conservative force with ϕ = potential per unit mass, $-\rho\vec{\nabla}\phi$ = force density, otherwise for non-conservative force $\vec{f}_{ext}$ has to be taken care. Due to shearing stress in a flowing fluid, there are internal force per unit volume $\vec{f}_{visc}$, so that Newton's law become,

$$\rho \frac{d\vec{v}}{dt} = -\vec{\nabla} p - \rho \vec{\nabla}\phi + \cancel{f_{visc}}$$ dry water/ thin liquid (inviscid flow)

The derivative $\frac{d\vec{v}}{dt}$ denotes not the change of rate of the fluid velocity at a fixed point in space but the rate of change of the velocity of a given fluid particle as it moves about in space. So to express $\frac{d\vec{v}}{dt}$ in terms of quantities referring to fixed in space we see composition from two parts, 1 change during dt in the velocity at a fixed point in space, 2 velocity difference at same instant at two points $d\vec{r}$ apart.

$$\therefore \rho \frac{d\vec{v}}{dt} = \rho \frac{\partial \vec{v}}{\partial t} + \frac{\rho}{dt}\left(dx \frac{\partial \vec{v}}{\partial x} + dy \frac{\partial \vec{v}}{\partial y} + dz \frac{\partial \vec{v}}{\partial z} \right)$$

$$= \rho \frac{\partial \vec{v}}{\partial t} + \rho\left(\frac{d\vec{r}}{dt}\cdot \vec{\nabla}\right)\vec{v} = \rho\, \partial_t \vec{v} + \rho(\vec{v}\cdot\vec{\nabla})\vec{v}$$

Note that there can be acceleration even though $\frac{\partial \vec{v}}{\partial t} = 0$ so that velocity at a given point is not changing, e.g. water flowing in a circle at constant speed is accelerating due to change in direction of the centripetal acceleration.

$$\therefore \boxed{\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{\vec{\nabla} p}{\rho} - \vec{\nabla}\phi}$$ Euler's Equation

If we define $\vec{\Omega} = \vec{\nabla}\times\vec{v}$, then using the vector identity

$$(\vec{v}\cdot\vec{\nabla})\vec{v} = (\vec{\nabla}\times\vec{v})\times\vec{v} + \frac{1}{2}\vec{\nabla}(\vec{v}\cdot\vec{v}) = \vec{\Omega}\times\vec{v} + \frac{1}{2}\vec{\nabla}v^2$$

$$\therefore \frac{\partial \vec{v}}{\partial t} + \vec{\Omega}\times\vec{v} + \frac{1}{2}\vec{\nabla}v^2 = -\frac{\vec{\nabla} p}{\rho} - \vec{\nabla}\phi$$

$\vec{\Omega}$ is called the "vorticity" & for an irrotational flow, $|\vec{\Omega}| = 0$

Circulation of a vector field around any arbitrary closed loop in a fluid at a given instant is

Circulation $\Gamma = \oint_c \vec{v}\cdot d\vec{l}$ (line integral)

Circulation $\Gamma = \int_S \vec{\nabla} \times \vec{v} \cdot d\vec{S}$ (using Stoke's theorem)

$= \int_S \vec{\Omega} \cdot d\vec{S}$. So vorticity $\vec{\Omega}$ is the circulation around a unit area & perpendicular to the direction of $\vec{\Omega}$.

Conservation of Circulation:

Change in circulation around a "fluid contour" moving over space $= \frac{d}{dt} \oint_C \vec{v} \cdot \delta\vec{l} = \oint_C \frac{d\vec{v}}{dt} \cdot \delta\vec{l} + \oint_C \vec{v} \cdot \frac{d\delta\vec{l}}{dt}$

Now $\vec{v} \cdot \frac{d\delta\vec{l}}{dt} = \vec{v} \cdot \delta \frac{d\vec{l}}{dt} = \vec{v} \cdot \delta\vec{v} = \frac{1}{2}\delta(v^2)$ and then

$\oint_C \frac{1}{2}\delta(v^2) = 0$ as total differential along closed contour $= 0$.

$$\therefore \quad \frac{d\Gamma}{dt} = \oint_C \frac{d\vec{v}}{dt} \cdot \delta\vec{l} = \oint_C \left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{v}\right) \cdot \delta\vec{l}$$

$$= -\oint_C \vec{\nabla}\left(\frac{P+\phi}{\rho}\right) \cdot \delta\vec{l} \quad \text{(Using Euler's equation)}$$

$$= -\int_S \cancelto{0}{\vec{\nabla} \times \vec{\nabla}\left(\frac{P+\phi}{\rho}\right)} \cdot \delta\vec{S} = 0.$$

$\therefore \oint \vec{v} \cdot d\vec{l} =$ constant (Kelvin's theorem of conservation of circulation)

So for irrotational flow, $\frac{\partial \Omega}{\partial t} = 0$ & so $\vec{\nabla} \cdot \vec{v} = 0$ & $\vec{\nabla} \times \vec{v} = 0$

It is also called "Potential flow". As $\vec{\nabla} \times \vec{v} = 0$ on streamlines, steady flow past any body with a uniform incident flow at infinity must be a potential flow.

Bernoulli's theorem from Euler's equation, taking $\vec{v} \cdot$ operation

$\vec{v} \cdot \vec{\Omega} \times \vec{v} = 0$ & so

$\vec{v} \cdot \vec{\nabla}\left(\frac{P}{\rho} + \phi + \frac{1}{2}v^2\right) = 0$ for steady streamline flow $\frac{\partial v}{\partial t} = 0$.

So for a small displacement in the direction of the fluid velocity

$$\boxed{\frac{P}{\rho} + \phi + \frac{1}{2}v^2 = \text{constant}}$$ for all points along a streamline.

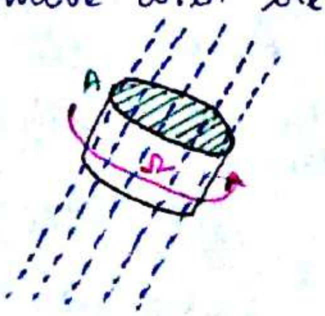
This is called Bernoulli's equation for potential flow. The constant in R.H.S. is constant along any given streamline but is different for different streamlines, while for a potential flow (irrotational), it is constant throughout the fluid.

$$\boxed{\frac{P}{\rho} + \phi + \frac{1}{2}v^2 = \text{constant}}$$ (everywhere).

Vortex lines In terms of vorticity, we have already noted the Euler's equation, $\frac{\partial \vec{v}}{\partial t} + \vec{\Omega} \times \vec{v} + \frac{1}{2}\vec{\nabla} v^2 = -\frac{\vec{\nabla} P}{\rho} - \vec{\nabla}\phi$. By taking a curl, we can permanently eliminate pressure, so that for an incompressible liquid,

$$\boxed{\vec{\nabla}\cdot\vec{v} = 0, \quad \vec{\Omega} = \vec{\nabla}\times\vec{v} \quad \text{and} \quad \frac{\partial \vec{\Omega}}{\partial t} + \vec{\nabla}\times(\vec{\Omega}\times\vec{v}) = 0}$$ describes the velocity field everywhere. Also, if $\vec{\Omega} = 0$ at any time t, $\frac{\partial \vec{\Omega}}{\partial t} = 0$ so at all time $\vec{\Omega} = 0$ or the flow remains permanently irrotational. The equations to be solved are $\vec{\nabla}\cdot\vec{v} = 0$, $\vec{\nabla}\times\vec{v} = 0$

As Helmholtz proposed, imagine that in the fluid we want to draw vortex lines, rather than streamline. Vortex lines are field lines in the direction of $\vec{\Omega}$ and density proportional to the magnitude $|\vec{\Omega}|$. They are similar to magnetic induction $\vec{B}$, without any beginning or end, revolving in closed loops, and move with the fluid. Suppose at time t, a small cylinder of

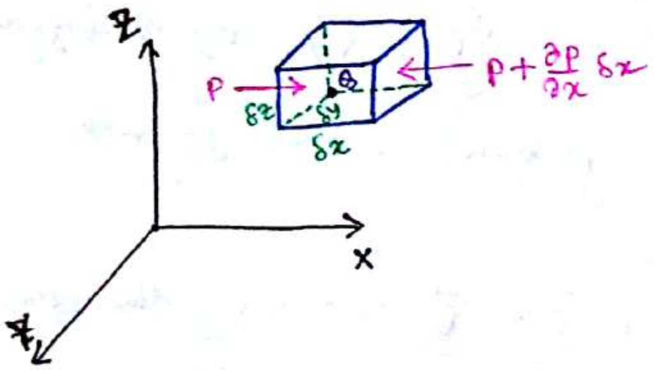


the liquid with axis parallel to vortex lines is transported at $t+dt$ to another cylinder with area A_2.

So, $\Omega_1 A_1 = \Omega_2 A_2$ as $\Omega \propto$ density because mass is same in both situation, we see $A_1 = \pi r_1^2$ and $A_2 = \pi r_2^2$ and $M_1 = M_2$ gives.

$$\pi M_1 r_1^2 \Omega_1 = \pi M_2 r_2^2 \Omega_2 \quad \text{i.e.} \quad L_1 = L_2$$ or in the absence of viscosity, angular momentum of an element of the fluid is invariant. This is "ideal" "dry" water case as it means if $\vec{\Omega} = 0$ then $\vec{\Omega}$ cannot be created or there will not be any vorticity.

Fluid Statics: Condition of Equilibrium of a fluid



Consider a container of fluid at rest, & within it an infinitesimal rectangular parallelopiped is taken in which at point Q, we calculate the body force. F_x, F_y, F_z are components of the body force $\vec{F}$ at $Q(x, y, z)$. Now, force due to pressure p on the elemental area $\delta y \delta z$ along x-axis is $p\,\delta y \delta z$. & force on opposite face of the parallelopiped is $-(p + \frac{\partial p}{\partial x}\delta x)\delta y \delta z$.

$$\therefore \text{ The resultant force } = p\,\delta y \delta z - (p + \frac{\partial p}{\partial x}\delta x)\delta y \delta z$$
$$= -\frac{\partial p}{\partial x}\delta x \delta y \delta z.$$

So for equilibrium under the action of the body force

$$F_x \rho\, \delta x \delta y \delta z - \frac{\partial p}{\partial x}\delta x \delta y \delta z = 0 \quad \text{or,} \quad F_x = \frac{1}{\rho}\frac{\partial p}{\partial x}$$

Similarly for y and z direction, $F_y = \frac{1}{\rho}\frac{\partial p}{\partial y}$, $F_z = \frac{1}{\rho}\frac{\partial p}{\partial z}$.

$$\therefore \vec{F} = \frac{1}{\rho}\vec{\nabla}p. \quad \text{and for } \rho = \text{constant}, \quad \vec{\nabla} \times \vec{F} = 0.$$

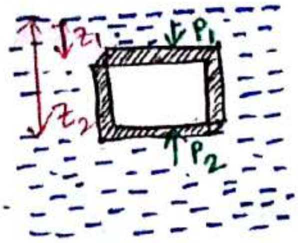
If the force is gravity then $F_z = -g$, $F_x = F_y = 0$.

$$\text{So, } -g - \frac{1}{\rho}\frac{dp}{dz} = 0 \quad \text{i.e.} \quad dp = -g\rho\, dz.$$

$\therefore P = -\rho g z + C$ where at $z=0$, $P = P_0$ gives $C = P_0$

$\therefore \boxed{P = P_0 - \rho g z}$ Equation of hydrostatics for incompressible liquid

As $P - P_0$ does not depend on P_0 or pressure exerted by external forces on the fluid is transmitted equally in all directions. This is Pascal's principle.



Again, consider a body of cross-sectional area A immersed in a fluid with pressure P_1 & P_2 at upper & lower surface, then

$$P_1 = -\rho g z_1 + C_1, \quad P_2 = -\rho g z_2 + C_2$$

$$\therefore (P_1 - P_2) = \rho g (z_2 - z_1).$$

$$\therefore (P_1 - P_2)A = \text{thrust} = \rho g A (z_2 - z_1) = \text{weight of}$$

the fluid dispersed in upward direction.

This is the Archimede's principle.

This can be easily derived from Bernoulli's theorem

$\frac{v^2}{2} + \frac{P}{\rho} + gh$ = constant by substituting $v = 0$, $h = z$.

For compressible gases, Boyle's law give $P \propto \rho$.

$\therefore \frac{P}{P_0} = \frac{\rho}{\rho_0}$. So from $dP = -g\rho dz$ we get $dP = -g dz \frac{\rho_0}{P_0} P$

$$\therefore \frac{dP}{P} = -\frac{g\rho_0}{P_0} dz \quad \therefore \int_{P_0}^{P} \frac{dP}{P} = -\frac{\rho_0 g}{P_0} \int_0^z dz$$

$\therefore \ln \frac{P}{P_0} = -\frac{\rho_0 g z}{P_0}$ [where P_0 = pressure at surface of Earth or $z = 0$.]

$\therefore P = P_0 e^{-\frac{\rho_0 g}{P_0} z}$ This expression correctly shows exponential fall of pressure with distance but flawed as temperature variation is not accounted for. But from $\frac{dP}{dz} = -\rho g$, using Clausius-Clapeyron's equation

$P = n k_B T$ (k_B = Boltzmann's constant = 1.38×10^{-23} J/k = $\frac{R}{N}$

$= \frac{\text{universal gas constant}}{\text{Avogadro Number}} = \frac{8.314 \text{ J/mol K}}{6.023\times10^{23}}$), M = molecular weight of gas.

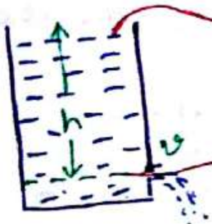
So $\frac{dP}{dz} = -\rho g = -\frac{Mg}{RT}P$ [As, $mn=\rho$, $mN = M$, $K_B = \frac{R}{N}$] [As $P = nk_BT = \frac{\rho}{m}\frac{R}{N}T = \frac{\rho R}{M}T$]

or $\frac{dP}{P} = -\frac{Mg}{RT}dz$

or $\ln P = -\frac{Mg}{RT}z + \ln P_0$

or $\boxed{P = P_0 e^{-Mgz/RT}}$ This is called "law of atmosphere."

Torricelli's theorem Velocity of efflux of a liquid through an orifice is equal to the velocity attained by a body in falling freely from the surface of the liquid to the orifice.



→ Total energy = KE + PE + pressure = $0 + gh + 0 = gh$.

→ Total energy = $\frac{1}{2}v^2 + 0 + 0$. or $\frac{1}{2}v^2 = gh$ or $v = \sqrt{2gh}$

Eulerian and Lagrangian description of conservation laws

The rate of change of a field variable $\phi(t, \bar{x})$ with respect to fixed position of space is called Eulerian derivative $\frac{\partial\phi}{\partial t}$ while derivative following a moving parcel is called Lagrangian derivative or substantial derivative or material derivative $\frac{D\phi}{Dt}$

$$\frac{D\phi}{Dt} = \underbrace{\frac{\partial\phi}{\partial t}}_{\text{local rate of change}} + \underbrace{\vec{v}\cdot\vec{\nabla}\phi}_{\text{convective rate of change}}$$

So changes in the properties of a moving fluid can be measured either on a fixed point in space while fluid particles are crossing it (Eulerian) or by following a fluid parcel along its path (Lagrangian)

Eulerian $v(t, \vec{x}(\vec{x}_0, t)) = \frac{\partial}{\partial t}\vec{X}(t, \vec{x}_0)$

Reynold's transport theorem As we have defined our conservation laws in Lagrangian description, Reynold's transport theorem gives the Eulerian equivalent of the integral taken over a moving material volume of a fluid.

$$\left(\frac{d\phi}{dt}\right)_{\substack{\text{material}\\ \text{volume}}} = \frac{d}{dt}\int_{V(t)} \psi \rho \, dV + \iint_{S(t)} \psi \rho \vec{v}.\hat{n} \, ds \, , \left[\psi = \frac{d\phi}{dm} = \text{amount of } \phi \text{ per unit mass}\right]$$

Divergence theorem

$$= \int_V \left[\frac{\partial}{\partial t}(\psi \rho) + \vec{\nabla}.(\rho \vec{v} \psi)\right] dV = \int_V \left[\frac{D}{Dt}(\rho\psi) + \rho\psi \vec{\nabla}.\vec{v}\right] dV$$

Notice that conservation law indicate no source or sink meaning

$\cancel{\frac{d\phi}{dm}}$ ~~[illegible]~~ $\frac{dm}{dt} = 0$, meaning $\psi = \frac{d\phi}{dm} = 1$ when $\phi = m$.

$$\boxed{\frac{D\rho}{Dt} + \rho \vec{\nabla}.\vec{v} = 0}$$ mass conservation law in Eulerian coordinate system.

So incompressibility ($\vec{\nabla}.\vec{v} = 0$) means $\frac{D\rho}{Dt} = 0$. or ρ is not a constant but ρ does not change along a streamline.

In presence of external force $\vec{f}$ per unit volume, the non-conservative form with $\psi = \vec{v}$ is

$$\frac{D}{Dt}(\rho\vec{v}) + \rho\vec{v}\,\vec{\nabla}.\vec{v} = \vec{f}$$

check!

$$\Rightarrow \rho\frac{D\vec{v}}{Dt} + v\Big(\underbrace{\frac{D\rho}{Dt} + \rho\vec{\nabla}.\vec{v}}_{=0 \text{ (continuity)}}\Big) = \vec{f} \Rightarrow \boxed{\rho\frac{D\vec{v}}{Dt} = \vec{f}}$$

Using Reynold's transport theorem, we find the conservative form

$$\frac{\partial}{\partial t}(\rho\vec{v}) + \vec{\nabla}.(\underbrace{\rho\vec{v}\vec{v}}_{\text{Dyadic product}}) = \vec{f} = \vec{f}_{surface} + \vec{f}_{body}$$

Now $\int_V \vec{f}_s \, dV = \int_S \overset{=}{\sigma}.\hat{n} \, ds = \int_V \vec{\nabla}.\overset{=}{\sigma} \, dV$ where $\overset{=}{\sigma}$ = Stress tensor or stress matrix.

$$\overset{=}{\sigma} = -\begin{pmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{pmatrix} + \begin{pmatrix} \sigma_{xx}+p & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy}+p & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz}+p \end{pmatrix}$$

$= -p\,\overline{\overline{\mathbb{I}}} + \overline{\overline{\tau}}$ ↳ Deviatoric / viscous stress tensor.

↳ Thermodynamic pressure ($= nk_BT$). So $\vec{\nabla}\cdot\overline{\overline{\sigma}} = -\vec{\nabla}p + \vec{\nabla}\cdot\overline{\overline{\tau}}$.

$$\vec{f}_b = \underbrace{\rho\vec{g}}_{\text{gravity}} - \underbrace{2\rho\vec{\omega}\times\vec{v}}_{\text{coriolis force}} - \underbrace{\rho\vec{\omega}\times(\vec{\omega}\times\vec{r})}_{\text{centrifugal force}}.$$

As gravitation & centrifugal forces are dependent on position but not velocity, so they can be absorbed into a modified pressure & hence effectively ignored. Coriolis force however has to be treated explicitly.

So conservation of momentum equation becomes,

$$\boxed{\frac{\partial}{\partial t}(\rho\vec{v}) + \vec{\nabla}\cdot(\rho\vec{v}\vec{v}) = -\vec{\nabla}p + \vec{\nabla}\cdot\vec{\tau} + \vec{f}_b}$$

Stress tensor for Newtonian fluid

$$\vec{\vec{\tau}} = \underbrace{\eta}_{\substack{\text{viscosity}\\\text{coefficient}\\\text{(molecular)}}}\{\vec{\nabla}\vec{v} + (\vec{\nabla}\vec{v})^T\} + \underbrace{\lambda}_{\substack{\text{viscosity}\\\text{coefficient}\\\text{(bulk)}}}(\vec{\nabla}\cdot\vec{v})\,\mathbb{I} \qquad \left[\lambda = -\tfrac{2}{3}\mu\right]$$

So

$$\frac{\partial}{\partial t}(\rho\vec{v}) + \vec{\nabla}\cdot(\rho\vec{v}\vec{v}) = -\vec{\nabla}p + \vec{\nabla}\cdot[\eta\{\vec{\nabla}\vec{v} + (\vec{\nabla}\vec{v})^T\}] + \vec{\nabla}(\underbrace{\cancel{\lambda\vec{\nabla}\cdot\vec{v}}}_{\substack{\text{incompressible}\\\text{flow}}})^{0} + \vec{f}_b$$

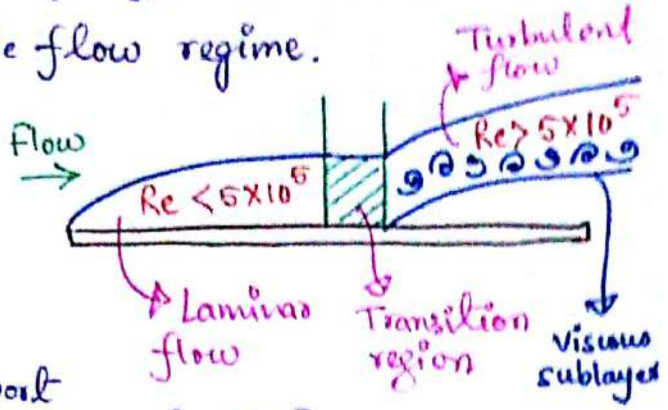
$$= -\vec{\nabla}p + \underbrace{\cancel{\eta\nabla^2\vec{v}}}_{\substack{\text{inviscid}\\\text{flow}}}{}^{0} + \vec{f}_b = -\vec{\nabla}p + \vec{f}_b$$

Similarly energy conservation equation can be derived.

General form

$$\underbrace{\frac{\partial}{\partial t}(\rho\phi)}_{\text{[unsteady term]}} + \underbrace{\vec{\nabla}\cdot(\rho\vec{v}\phi)}_{\text{[convection term]}} = \underbrace{\vec{\nabla}\cdot(\Gamma\vec{\nabla}\phi)}_{\text{[diffusion term]}} + \underbrace{S}_{\text{[source term]}}$$

Like Reynolds number $Re = \dfrac{\rho v L}{\eta} = \dfrac{\text{advection (inertia)}}{\text{diffusion (viscous)}}$ reveals the boundary layer characteristic of the flow if momentum fluxes are in same direction, e.g. if diffusion is in the cross-stream direction then Re conveys the flow regime.



Similarly Schmidt number

$$Sc = \frac{\text{momentum diffusivity}}{\text{mass diffusivity}}$$

Peclet number $\quad Pe = \dfrac{\text{advective transport}}{\text{diffusive transport}} = Re \times Sc$

where Pr = prandtl number $= \dfrac{\text{hydrodynamic boundary layer}}{\text{thermal boundary layer}}$

$$= \frac{\text{convection}}{\text{conduction}}.$$

Grashof number $= \dfrac{\text{buoyancy force}}{\text{Viscous force}}$

Mach number (M) $= \dfrac{\text{object speed}}{\text{speed of sound}}$ where $v_{sound} = \sqrt{\gamma\left(\frac{\partial p}{\partial \rho}\right)_T}$

$= \sqrt{\gamma R T}$ (ideal gas)

$\gamma = C_p/C_v$

$M < 0.2$ is incompressible flow.
$M < 1$ is subsonic flow.
$M = 1$ is sonic flow.
$M < 5$ is supersonic flow
$M > 5$ is hypersonic flow.

flow classification

Euler equation in Lagrangian form

$$\rho\left(\frac{\partial \vec{v}}{\partial t} + (\vec{v}.\vec{\nabla})v\right) = -\vec{\nabla}p - \rho\vec{\nabla}\varphi + \eta\nabla^2\vec{v} \quad \& \quad \vec{\Omega} = \vec{\nabla}\times\vec{v}$$

gives, $\quad \dfrac{\partial\vec{\Omega}}{\partial t} + \vec{\nabla}\times(\vec{\Omega}\times\vec{v}) = \dfrac{\eta}{\rho}\nabla^2\vec{\Omega}$

$\frac{\eta}{\rho}$ ↳ specific viscosity $= \frac{1}{Re}$

If we non-dimensionalize the equation (see Feynman lectures, vol 2 section 41.3), then $\dfrac{\partial\vec{\Omega}}{\partial t} + \vec{\nabla}\times(\vec{\Omega}\times\vec{v}) = \dfrac{1}{Re}\nabla^2\vec{\Omega}$

So if Mach number & Reynold's number are same then velocities near or above sound speed in two scenarios are equivalent.