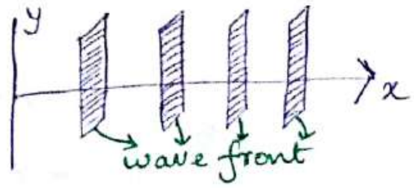
Wave Motion

Equation of a progressive wave propagating through a continuous medium where the particles of the medium execute SHM is

$$y = A\sin(\omega t - kx) = A\sin\frac{2\pi}{\lambda}(vt - x)$$



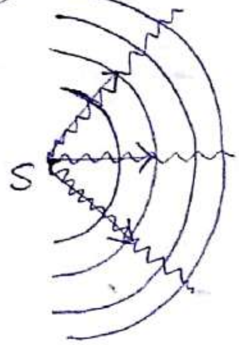
for propagation along +ive x-direction.

Similarly $y = A\sin(\omega t + kx) = A\sin\frac{2\pi}{\lambda}(vt + x)$

for propagation along −ive x-direction.

Note that if t increases by Δt and x by $v\Delta t$, then y will be restored to initial value. Thus a "disturbance" at one place is repeated at a position $v\Delta t$ after time Δt, with propagating velocity v, that propagates without damping. In reality, its amplitude gradually diminishes due to resistive forces of the viscous medium

Equation of such wave is $y = Ae^{-\gamma x}\sin\frac{2\pi}{\lambda}(vt - x)$ which is plane progressive wave.

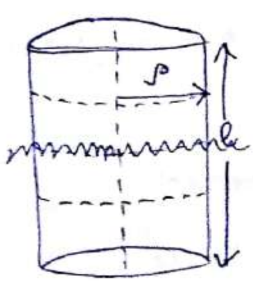


If the wave is diverging then without any dissipation of energy (no damping), the amplitude falls of. In case of ~~spherical~~ spherical progressive wave, Intensity $I \propto \frac{W}{4\pi r^2}$

where W is the energy emitted from source. $\therefore$ Amplitude$^2 \propto \frac{1}{r^2}$

or Amplitude $\propto 1/r$. Such wave is represented as

$$y = \frac{A}{|r|}\sin\frac{2\pi}{\lambda}(vt - |r|) \qquad |r| = \sqrt{x^2+y^2+z^2} \text{ (Expanding wave)}$$



For cylindrical progressive wave $I = \frac{W}{2\pi\rho l}$ or $I \propto \frac{1}{\rho}$

So $A^2 \propto \frac{1}{\rho}$ or $A \propto \frac{1}{\sqrt{\rho}}$

Such wave is represented by $y = \frac{A}{\sqrt{\rho}}\sin\frac{2\pi}{\lambda}(vt - \rho)$

The velocity $v = \dot{x}$ is called the wave velocity as the disturbance propagates with this velocity. This is also called phase velocity as during motion of the wave, phase of the motion of the particles move with this velocity. But as the wave move onward, particles vibrate about their mean position of rest which is called the particle velocity. V

Now $y = A\sin(\omega t - kx)$ gives $V = \frac{dy}{dt} = A\omega\cos(\omega t - kx)$

and $\frac{dy}{dx}$ = slope of the displacement of the particle

$$= -AK\cos(\omega t - kx) = -AK\frac{V}{A\omega} = -\frac{K}{\omega}V$$

$$\therefore \boxed{V = -\frac{\omega}{K}\frac{dy}{dx} = -v\frac{dy}{dx}}$$ → negative gradient of displacement.

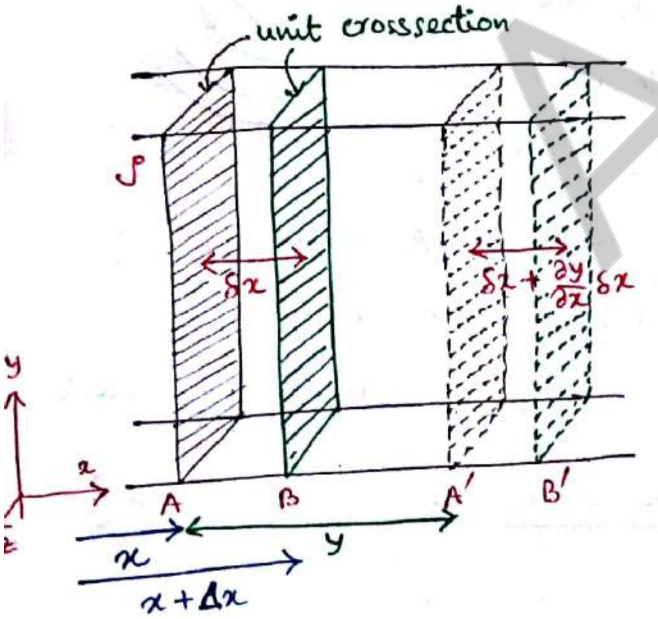
particle velocity   phase velocity

Also, $\frac{\partial^2 y}{\partial t^2} = -A\omega^2\sin(\omega t - kx)$, $\frac{\partial^2 y}{\partial x^2} = -AK^2\sin(\omega t - kx)$

$$\therefore \frac{\partial^2 y}{\partial x^2} = \left(\frac{K}{\omega}\right)^2\frac{\partial^2 y}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

$$\therefore \boxed{\frac{\partial^2 y}{\partial t^2} = v^2\frac{\partial^2 y}{\partial x^2}}$$ is the differential equation for a progressive wave.

Plane longitudinal wave through an elastic fluid medium



Transverse wave cannot resist any shearing stress just like a fluid. So no transverse wave is formed in a fluid. Consider a unit crosssectional area in a fluid medium with its axis in the direction of propagation of the soundwave. Let A & B are two normal planar sections at x & $x+\Delta x$ from some arbitrary origin.

In progression, let at an instant plane A is displaced by y to A' and at the same time the plane B is displaced to B' by an amount $y + \frac{\partial y}{\partial x}\delta x$.

$\therefore$ The actual position of A' plane is $(x+y)$ & B' plane is $(x + y + \delta x + \frac{\partial y}{\partial x}\delta x)$.

Considering the tube has unit cross section, volume of fluid between A & B is δx & that between A' & B' is $\delta x + \frac{\partial y}{\partial x}\delta x$. ∴ Change in volume due to displacement $\Delta V = \frac{\partial y}{\partial x}\delta x$ & hence the volume strain $\frac{\Delta V}{V} = \frac{\frac{\partial y}{\partial x}\delta x}{\delta x} = \frac{\partial y}{\partial x}$.

If δP be the excess pressure over the normal pressure on plane A that is transferred to A', then the Bulk modulus of the medium is

$$B = -\frac{\delta P}{\Delta V/V} = -\frac{\delta P}{\frac{\partial y}{\partial x}}, \text{ or } \delta P = -B\frac{\partial y}{\partial x}.$$

Negative sign indicating that the volume decreases with the increase in pressure.

Now the excess pressure over face B which is transferred to B' is

$$\delta P + \frac{\partial(\delta P)}{\partial x}\delta x = -B\frac{\partial y}{\partial x} + \frac{\partial}{\partial x}\left(-B\frac{\partial y}{\partial x}\right)\delta x$$

$$= -B\frac{\partial y}{\partial x} - B\frac{\partial^2 y}{\partial x^2}\delta x \qquad [B \neq B(x)]$$

∴ Excess pressure on the volume element is

$$-B\frac{\partial y}{\partial x} - \left(-B\frac{\partial y}{\partial x} - B\frac{\partial^2 y}{\partial x^2}\delta x\right) = B\frac{\partial^2 y}{\partial x^2}\delta x.$$

This pressure will create an accelaration along the direction of force within the volume element. If ρ is the density (medium) then using Newton's 2nd law

$$\rho\,\delta x\frac{\partial^2 y}{\partial t^2} = B\frac{\partial^2 y}{\partial x^2}\delta x \quad \text{or} \quad \boxed{\frac{\partial^2 y}{\partial t^2} = \frac{B}{\rho}\frac{\partial^2 y}{\partial x^2} = v^2\frac{\partial^2 y}{\partial x^2}}$$

Compressional wave due to longitudinal vibrations of a long thin rod:

Consider two normal crosssection A & B at a distance x & $x+\delta x$ from some arbitrary origin on a uniform thin rod whose length is large compared to its lateral dimension. Due to flow of compressional wave along the length, A is displaced to A' at a distance y & B is displaced to B' at a distance $(y + \frac{\partial y}{\partial x}\delta x)$. The actual position of A' and B' are $(x+y)$ and $(x+\delta x + y + \frac{\partial y}{\partial x}\delta x)$. Thickness of the slice AB is δx and after time t, thickness of A'B' is $\delta x + \frac{\partial y}{\partial x}\delta x$.

$\therefore$ Change in length of the slice is $\frac{\partial y}{\partial x}\delta x$ & hence the longitudinal strain $= \frac{\frac{\partial y}{\partial x}\delta x}{\delta x} = \frac{\partial y}{\partial x}$. If F is the stretching force per unit crosssectional area (or stress) of the plane A & Y is the Young's modulus of the material then $Y = \frac{F}{\partial y/\partial x}$, $\therefore$ $F = Y\frac{\partial y}{\partial x}$.

The stretching stress acting on B which is displaced to B' is

$$Y\frac{\partial y}{\partial x} + \frac{\partial}{\partial x}\left(Y\frac{\partial y}{\partial x}\right)\delta x = Y\frac{\partial y}{\partial x} + Y\frac{\partial^2 y}{\partial x^2}\delta x$$

$\therefore$ The total force on AB in the $+x$ direction is $Y\frac{\partial^2 y}{\partial x^2}\delta x$.

If ρ is material density, then $\rho\,\delta x\frac{\partial^2 y}{\partial t^2} = Y\frac{\partial^2 y}{\partial x^2}\delta x$

$$\therefore \boxed{\frac{\partial^2 y}{\partial t^2} = \frac{Y}{\rho}\frac{\partial^2 y}{\partial x^2} = v^2\frac{\partial^2 y}{\partial x^2}} \qquad v = \sqrt{\frac{Y}{\rho}}$$

For iron, $Y = 10^{12}$ dyne/cm^2, $\rho = 8.9$ gm/cc. $v \simeq 10^6$ cm/s.

If lateral strain is also accounted then, $v = \sqrt{\frac{Y + \frac{4}{3}\eta}{\rho}}$, where η = modulus of rigidity. This indicates that consideration of lateral strain <u>enhances</u> the velocity of longitudinal wave through the medium.

<u>Longitudinal waves in a Gas</u> $\frac{\partial^2 y}{\partial t^2} = \frac{E}{\rho}\frac{\partial^2 y}{\partial x^2}$, $v = \sqrt{\frac{E}{\rho}}$

where E is volume elasticity (bulk modulus) of gas, ρ = density.

<u>Newton's Formula</u>: Newton first calculated the velocity of sound wave in a gas, on the assumption that temperature variation is negligible, so in isothermal process, Boyle's law is applicable

PV = constant $\therefore$ $P\delta V + \delta P\, V = 0$ or $P = -\frac{\delta P}{\delta V/V} = E$

$\therefore$ $v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{P}{\rho}}$. For air at standard pressure & temperature (STP), $\rho = 1.29$ kg/m^3. $P = 0.76$m of Hg $= 0.76 \times 13.6 \times 10^3 \times 9.8$ N/m^2

$v = 280$ m/s

But experimental value came as $v = 332$ m/s.

Laplace's correction: Temperature correction, region of compression is heated & region of rarefaction is cooled. Since the thermal conductivity of a gas is small, that these thermal changes occur at much faster time scale that heat developed during compression & cooling due to rarefaction is not transferred out to thermalize. So in adiabatic condition, $PV^\gamma =$ constant, $\gamma = C_P/C_V$

$$\therefore \Delta P V^\gamma + \gamma P V^{\gamma-1} \Delta V = 0 \quad \text{or} \quad \gamma P = -\frac{\Delta P V^\gamma}{V^{\gamma-1}\Delta V} = -\frac{\Delta P}{\Delta V/V} = E.$$

$\therefore v = \sqrt{\frac{\gamma P}{\rho}}$. $\gamma = 1.4$ for air at STP, $v = 331.6$ m/S.

Now $PV = \frac{m}{M} RT$ and $\rho = \frac{m}{V}$ $\therefore v = \sqrt{\frac{\gamma RT}{M}}$.

So velocity is independent of pressure or density, $v \propto \sqrt{T}$, $v \propto \frac{1}{\sqrt{M}}$, $v \propto \sqrt{\gamma}$ ie whether monoatomic, diatomic gas etc.

Energy Transport in Travelling Waves

When a plane progressive harmonic wave passes through a medium, medium particles contain extra energy due to SHM in terms of KE & PE. Total energy is conserved.

$y = a \sin \frac{2\pi}{\lambda}(vt - x)$ is displacement

velocity of each particle $V = \frac{dy}{dt} = \frac{2\pi v a}{\lambda} \cos \frac{2\pi}{\lambda}(vt - x)$

For $\rho =$ density, KE per unit volume $= \frac{1}{2}\rho V^2 = \frac{1}{2}\rho\left(\frac{2\pi v a}{\lambda}\right)^2 \times \cos^2 \frac{2\pi}{\lambda}(vt-x)$

Since this varies over time, average KE density of the medium is $\langle KE \rangle = \frac{\rho}{2} \frac{4\pi^2 v^2 a^2}{\lambda^2} \underbrace{\langle \cos^2 \frac{2\pi}{\lambda}(vt-x)\rangle}_{= 1/2} = \frac{\rho \pi^2 v^2 a^2}{\lambda^2}$

To evaluate the average P.E. we calculate work done for the decrease of volume δV against an average pressure $\frac{P + (P+\delta P)}{2}$ is

$dW = (P + \frac{\delta P}{2})\delta V$. But $B = -\frac{\delta P}{\delta V/V}$ and $\delta V = -\frac{\partial y}{\partial x}\delta x$

$\therefore \delta P = -\frac{B}{V}\delta V = -B\frac{\partial y}{\partial x}$

$= \left(P - \frac{B}{2}\frac{\partial y}{\partial x}\right)\left(-\frac{\partial y}{\partial x}\delta x\right)$

as $\delta V/V = \frac{\partial y}{\partial x}$.

$$= -P\frac{\partial y}{\partial x}\,\delta x + \frac{B}{2}\left(\frac{\partial y}{\partial x}\right)^2 \delta x$$

$\therefore$ The work done per unit volume $W = -P\frac{\partial y}{\partial x} + \frac{B}{2}\left(\frac{\partial y}{\partial x}\right)^2$

$$= P\,\frac{2\pi a}{\lambda}\cos\frac{2\pi}{\lambda}(vt-x) + \frac{B}{2}\left(\frac{2\pi a}{\lambda}\right)^2 \cos^2\frac{2\pi}{\lambda}(vt-x)$$

$\therefore$ Time averaged P.E. is $\langle P.E.\rangle = \frac{2\pi a P}{\lambda}\underbrace{\left\langle \cos\frac{2\pi}{\lambda}(vt-x)\right\rangle}_{\to 0} + \frac{4\pi^2 a^2 B}{2\lambda^2}\underbrace{\left\langle \cos^2\frac{2\pi}{\lambda}(vt-x)\right\rangle}_{1/2}$

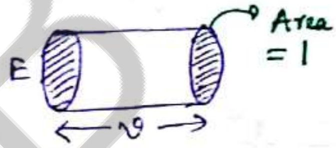
$$= \frac{\pi^2 a^2 B}{\lambda^2}$$

as $v = \sqrt{B/\rho}$

$$\langle PE\rangle = \frac{\pi^2 \rho v^2 a^2}{\lambda^2} = \langle KE\rangle .$$

So average value of PE & KE are same. Therefore total energy of the medium is $\boxed{E = \frac{2\pi^2 \rho v^2 a^2}{\lambda^2}}$ which is the energy crossing unit area per unit time or intensity. $I \propto a^2, \propto \rho, \propto 1/\lambda^2, \propto v^3$

$$I = vE = \frac{2\pi^2 \rho v^3 a^2}{\lambda^2}$$



Unit of Intensity : Bel & Decibel

The ratio of sound intensity from low to high in the detectable range is $1 : 10^{14}$. So logarithmic ratio is 14 : 1 for high to low. For all practical purposes, absolute intensity is unnecessary & relative values have more practical significance.

Bel & Decibel are logarithmic units of relative intensity. If ratio of sound intensity is 10 : 1 then difference of intensity is "1 Bell". $N = \log_{10} I_1/I_2$ where N = number of bels and I_1 & I_2 are intensity of two sounds. A "decibel (db)" is $\frac{1}{10}$th of Bel. $n = 10\log_{10} I_1/I_2$ where n = no. of decibels.

1 bel $= 10$ dB $= 10^1 : 1$ $\qquad \therefore 1\,dB = 10^{0.1} : 1$
$\qquad\qquad\qquad\qquad\qquad\qquad \simeq 1.26 : 1$

$\therefore$ N bels $= 10N$ dB $= 10^N : 1$

$0.1n$ bels $= n$ dB $= 10^{0.1n} : 1$

Intensity level (IL) IL is ratio of intensity to standard intensity I_0 where threshold is 10^{-12} watt/m^2 or 10^{-16} watt/cm^2 which corresponds to lower limit of intensity for audability.

$$\therefore \quad (IL)_{bel} = \log_{10}\left(\frac{I}{I_0}\right), \quad (IL)_{db} = 10\log_{10}\left(\frac{I}{I_0}\right)$$

This means $I = 10^2 I_0$, $(IL)_{bel} = 2$ bels $= 20$ decibels

$I_{max} = 10^{14} I_0$, $(IL)_{bel} = 14$ bels $= 140$ decibels.

Conversely intensity 140 db means $140 = 10\log_{10}\frac{I}{I_0}$

or $I = I_0 \times 10^{14} = 10^{-16} \times 10^{14} = 10^{-2}$ watt/cm^2

Relation between wave intensity & mean square of excess pressure

The excess pressure due to propagation of sound wave is

$$p = -B\frac{\partial y}{\partial x} \quad \text{where } B = \rho v^2 = \text{Bulk modulus of the medium}$$

displacement $y = a\sin\frac{2\pi}{\lambda}(vt - x)$

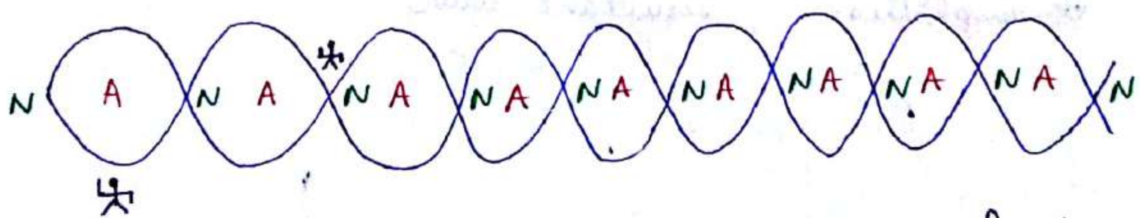
$$\therefore p = v^2\rho a\frac{2\pi}{\lambda}\cos\frac{2\pi}{\lambda}(vt-x) \quad \text{or} \quad p^2 = \frac{4\pi^2 v^4\rho^2 a^2}{\lambda^2}\cos^2\frac{2\pi}{\lambda}(vt-x)$$

$$\text{or} \quad \langle p^2\rangle = \frac{4\pi^2 v^4\rho^2 a^2}{\lambda^2}\left\langle\cos^2\frac{2\pi}{\lambda}(vt-x)\right\rangle = \frac{2\pi^2 v^4\rho^2 a^2}{\lambda^2}$$

But we know $I = \frac{2\pi^2 v^3\rho a^2}{\lambda^2} = \frac{\langle p^2\rangle}{\rho v}$

So whenever pressure is maximum, I is maximum → loud sound (node)

pressure is minimum, I is minimal → very low sound (antinode)



$y \propto \sin(\ )$, $p \propto \cos(\ )$ . . p and y are $\frac{\lambda}{2}$ phase different

so whenever y is minimum, p is maximum and vice versa.

Phase velocity and Group velocity

A single progressive wave along +ive x axis is represented by $x = a\sin(\omega t - Kx) = a\sin\frac{2\pi}{\lambda}(vt - x)$ where v is the "**phase velocity**" of the wave, as with this velocity phase of a wave moves from one point to another point.

But when two or more such harmonic waves of slightly different frequencies are superposed, the anharmonic phenomena of beat occurs. These beats are generally known as **modulations** and such anharmonic motion has modulated amplitude that repeats at a frequency called beat frequency or modulation frequency. It carries energy from one point to another with a velocity different from those of the harmonic waves. These travelling modulations that consist of group of harmonic waves are called wave packets or wave groups. The velocity with which this modulated amplitude moves is called "**group velocity**."

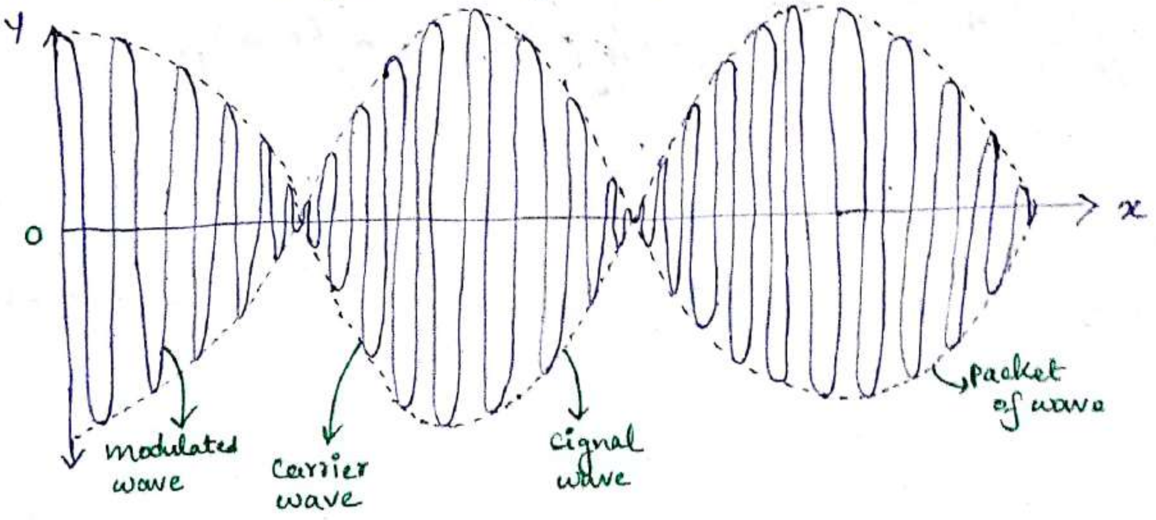
If we superpose two different waves that are slightly different in frequency/wavelength, then resultant amplitude is

$$y = y_1 + y_2 = a\sin(\omega t - Kx) + a\sin[(\omega + \Delta\omega)t - (K + \Delta K)x]$$

$$\text{or } y = \underbrace{2a\cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta K}{2}x\right)}\sin\left[\left(\omega + \frac{\Delta\omega}{2}\right)t - \left(K + \frac{\Delta K}{2}\right)x\right]$$

$$\text{or } y = A\sin\left[\left(\omega + \frac{\Delta\omega}{2}\right)t - \left(K + \frac{\Delta K}{2}\right)x\right]$$

$\Downarrow$ amplitude of resultant wave



This represents a travelling wave at a frequency $\omega + \frac{\Delta\omega}{2}$, wave vector $K + \frac{\Delta K}{2}$ and amplitude $2a\cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta K}{2}x\right)$ which is modulated by an envelope of frequency $\Delta\omega$. Thus we can say that velocity of this amplitude is $\frac{\Delta\omega}{\Delta K}$ and in $\lim_{\Delta K \to 0}$, we get $V_g = \frac{d\omega}{dK}$ is the group velocity. Note that the phase velocity is $V_p = \frac{\omega}{K}$.

$$\therefore \frac{d\omega}{dK} = \frac{d}{dK}(KV_p) \quad \text{or} \quad V_g = V_p + K\frac{dV_p}{dK}$$

Using $K = \frac{2\pi}{\lambda}$, $dK = -\frac{2\pi}{\lambda^2}d\lambda$, $V_g = V_p - \frac{2\pi}{\lambda}\frac{\lambda^2}{2\pi}\frac{dV_p}{d\lambda}$

$$\text{or} \quad \boxed{V_g = V_p - \lambda\frac{dV_p}{d\lambda}}$$

<u>Dispersive Medium</u> A medium where the phase velocity of any wave varies with frequency/wavelength is called a dispersive medium. Glass, water & all transparent substances are dispersive for electromagnetic waves. In dispersive medium $\lambda_1 \neq \lambda_2 \neq \lambda_3 \neq \cdots \neq \lambda_n$ and V_g is always greater than V_p. However in nondispersive medium $\lambda_1 = \lambda_2 = \cdots = \lambda_n$ and $V_g = V_p$. Vaccum is nondispersive to EM waves but dispersive for deBroglie wave (matter wave). Note that all mediums are nondispersive for sound waves, therefore all waves of different wavelength move with a constant velocity in a medium.

$$\text{Now} \quad \frac{1}{V_g} = \frac{dK}{d\omega} = \frac{d}{d\omega}\left(\frac{\omega}{V_p}\right) = \frac{1}{V_p} - \frac{\omega}{V_p^2}\frac{dV_p}{d\omega}$$

As $\omega = 2\pi\nu$, $d\omega = 2\pi d\nu$. $\therefore \frac{1}{V_g} = \frac{1}{V_p} - \frac{\nu}{V_p^2}\frac{dV_p}{d\nu}$

But $V_p = \frac{c}{n}$ where n is the refractive index of the medium.

$$\therefore \frac{1}{V_g} = \frac{1}{V_p} - \frac{\nu}{(c/n)^2}\frac{d}{d\nu}\left(\frac{c}{n}\right) = \frac{1}{V_p} + \frac{\cancel{n^2}\nu}{\cancel{c^2}}\frac{\cancel{c}}{\cancel{n^2}}\frac{dn}{d\nu} = \frac{1}{V_p} + \frac{1}{\lambda}\frac{dn}{d\nu}$$

$$\left[\text{as } c = \nu\lambda \quad \text{or} \quad \nu = \frac{c}{\lambda} \quad \text{or} \quad d\nu = -\frac{c}{\lambda^2}d\lambda.\right] \qquad = \frac{1}{V_p} - \frac{\lambda}{c}\frac{dn}{d\lambda}$$

$$\therefore V_g = \left(\frac{1}{V_p} - \frac{\lambda}{c}\frac{dn}{d\lambda}\right)^{-1} = \left(\frac{1}{V_p}\right)^{-1}\left(1 - \frac{\lambda V_p}{c}\frac{dn}{d\lambda}\right)^{-1} \simeq V_p\left(1 + \frac{\lambda}{n}\frac{dn}{d\lambda}\right)$$

$$\therefore \boxed{V_g = V_p\left(1 + \frac{\lambda}{n}\frac{dn}{d\lambda}\right)}.$$

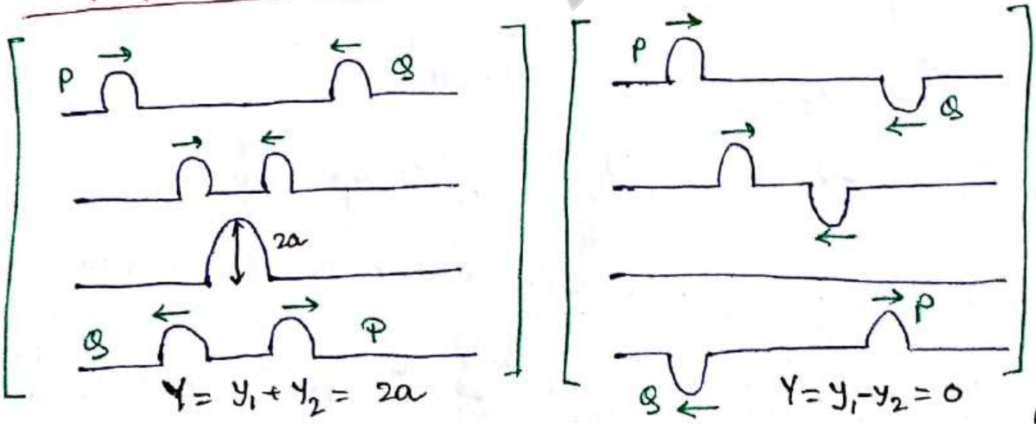
Loudness of Sound : Phon & Sone

Intensity of sound refers to a purely physical aspect of sound which is the amount of energy reaching unit area in unit time. But the loudness of sound is a subjective phenomena & it refers to the sensation produced to ear. Sensation of loudness depends on the intensity of sound, but loudness is not proportional to intensity. According to Waber & Fechner, $SL \propto \frac{\delta I}{I}$ where SL is increment in loudness & δI is that of intensity.

$\therefore \delta L = K \frac{\delta I}{I}$ or $L = K \ln I$ where K is some constant.

The absolute unit of intensity is Watt/cm^2 but it is expressed in decibel. The unit of loudness is "phon". To measure loudness of sound in phons, another pure tone of frequency 1000 cycles per second (C.P.S.) is taken whose intensity is gradually altered till its loudness becomes exactly equal to that of the given sound. If then the intensity level of the pure tone in n decibel, then the loudness of that sound is n phon. Phon is a small unit, another larger unit is "sone" which is equal to 40 times of phon.

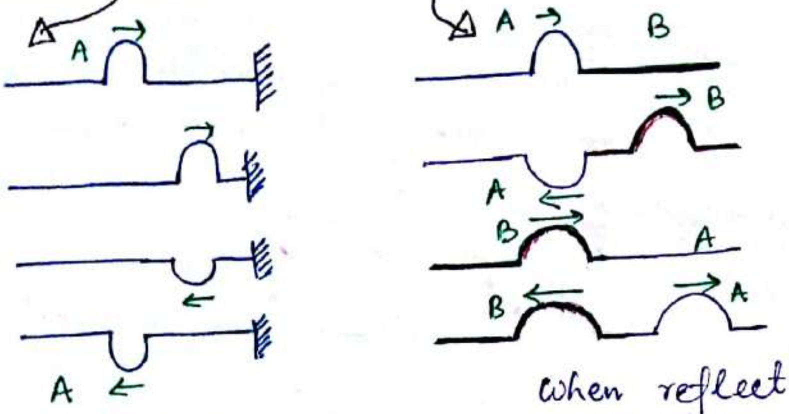
Superposition of Waves



Different waves can pass through each other simultaneously through the same medium. We can hear distinctly the conversation in room, or when dropped a pebble in pond, nice ripples passing (solitonic waves satisfying KdV equation).

"Processes by which different trains of waves travelling through a medium simultaneously overlap into one another without losing their individual nature/shape is called superposition of waves."

Reflection & refraction of waves



rarer to denser

denser to rarer

$$y_p = a\sin\frac{2\pi}{\lambda}(vt-x)$$

$$y_r = -a\sin\frac{2\pi}{\lambda}(vt+x)$$

$$= a\sin\left[\frac{2\pi}{\lambda}(vt+x) \pm \pi\right]$$

When reflected from denser to rarer medium, no phase change occur. But from rarer to denser reflection 180° phase change occur. λ and v of refracted wave different than incident wave. But ν will remain unchanged & according to

Snell's law, $\nu = \frac{v_i}{\lambda_i} = \frac{v_r}{\lambda_r}$

Comparison between Interference & Beats / Progressive & stationary wave

Interference	Beats
1. Two waves must have same frequency.	1. Two waves must have slightly different frequencies.
2. Phase difference of two waves is constant.	2. Phase difference varies from 0 to π with time.
3. Position of maximum & minimum intensity remains unchanged.	3. Positions of maximum & minimum intensity changes continuously.

—X—

Progressive wave	Stationary wave
1. Produced due to continuous periodic vibrations of medium particles.	1. Produced due to superposition of two identical progressive waves (colinear) in opposite directions.
2. Particles execute identical periodic motion about mean position.	2. Except particles at nodes, execute motions of varying amplitudes.
3. wave advances with definite velocity.	3. wave does not advance in medium.
4. wave retains its shape	4. wave change by shrinking to straight line twice in each period.

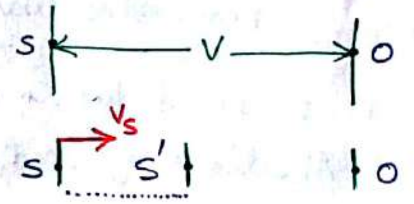
Progressive Waves	Stationary waves
5. Same amplitude.	5. Amplitude varies continuously with maximum at antinode & minimum at node.
6. Phase change with space & time.	6. phase between two nodes is same & changes with time. Phase change by π from one loop to other.
7. In a complete Time period medium particle never came to rest.	7. In a complete time period, all particles come to rest twice together.

Doppler Effect

When there exist a relative motion between the source and observer, then the apparent change in frequency of the sound as perceived by the observer is known as Doppler Effect. Example: sound heard by a fast mail train by an observer at platform.

This is because if ν frequency emitted by source is received by observer then both are stationary, but if they're in motion then received frequency can be less or more. When observer approaches the source, the apparent frequency will increase because observer receives more waves / second.

Let both observer & medium is stationary (no wind) & the soure is moving uniformly towards the observer.



V = velocity of sound (in air)

V_s = velocity of source (train)

ν = frequency of emitted sound

when S is stationary $\nu = \frac{V}{\lambda}$ and $SO = V$. Now source S is moving with velocity V_s, then $SS' = V_s$ and $S'O = V - V_s$. As velocity of sound is constant, ν waves will be contained in $V - V_s$. So the decreased wave length is $\lambda' = \frac{V - V_s}{\nu}$ & so the apparent frequency heard by observer $\nu' = \frac{V}{\lambda'} = \nu \frac{V}{V - V_s}$.

$\therefore$ Increase in frequency $\Delta\nu = \nu' - \nu = \frac{\nu V}{V - V_s} - \nu = \frac{\nu V_s}{V - V_s}$.

If S is moving away from O then, $\nu' = \nu \frac{V}{V + V_s}$ and then decrease in frequency is $\Delta\nu = \nu - \nu' = \frac{\nu V_s}{V + V_s}$.

If wind blows at V_ω towards the direction of sound then effective velocity of sound is $V + V_\omega$, then $\nu' = \nu \frac{V + V_\omega}{V + V_\omega - V_s}$. If direction is opposite then, $\nu' = \nu \frac{V - V_\omega}{V - V_\omega - V_s}$.

CW Two observers A & B have sources of sound of frequency 500 Hz. If A remains stationary while B moves away with velocity 10 m/s find the no. of beats heard by A and B. $V_{sound} = 332$ m/s.

Beats heard by A   A = stationary, B = moving, "away"

$$\nu' = \nu \frac{V}{V + V_s} = 500 \times \frac{332}{332 + 10} = 485.4 \text{ Hz}$$

$\therefore$ frequency of beats heard by A $= 500 - \nu' = 14.6$ Hz.

Beats heard by B   A = moving, B = stationary,

$$\nu' = \nu \frac{V - V_o}{V} = 500 \times \frac{332 - 10}{332} = 485 \text{ Hz}.$$

$\therefore$ Frequency of beats heard by B $= 500 - \nu' = 15$ Hz.

Observer fixed	Source fixed
(a) Towards $\nu' = \nu \frac{V}{V - V_s}$ Away $\nu' = \nu \frac{V}{V + V_s}$	(b) Towards $\nu' = \nu \frac{V + V_o}{V}$ Away $\nu' = \nu \frac{V - V_o}{V}$
Source observer moving	
(c) $\nu' = \nu \frac{V \pm V_o}{V \pm V_s}$	