Sem-IV - Special Relativity

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Assignment III: Poincarian Relativity

Submission due date: 01/05/2020

Q.1) (a) A coordinate transformation from $S(x, y, z)$ to $S'(x', y', z')$ frame is defined as the following:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Components of two contravariant vectors $\mathbf{A}$ and $\mathbf{B}$ do transform in the same fashion as that of the coordinates. Examine whether the cross product of $\mathbf{A}$ and $\mathbf{B}$ can be reckoned as a vector.

(b) Show that the inner product of tensors A^{ij}_k and B^p_r is a tensor of rank three.

(c) A contravariant tensor has components $xy, 2y - z^2, xz$ in rectangular coordinates. Find its covariant components in cylindrical coordinates.

(d) Show that $d\mathbf{r}$ transforms like a contravariant vector while $\boldsymbol{\nabla}\phi$ transforms like a covariant vector, ϕ is a scalar function.

Q.2) (a) Defining $A_j = g_{jk}A^k$ and $A^k = g^{jk}A_j$ where symbols bear usual meaning, write down the mathematical relatioship between g^{jk} and g_{jk}. What are the signed values of g^{ij} and g_{ij} in case of a 3-dimensional flat space time (2 space and 1 time dimension)?

(b) Find the metric tensor of a coordinate system (a, b, c) which is related to the Cartesian coordinates by $x = bc, y = ca, z = ab$.

(c) Defining dS^2 (dS being an elementary length) as $g_{ij}dx^i dx^j$ and if $dS^2 = (dx^1)^2 + (dx^2)^2 + 4(dx^1)(dx^2)$, find g^{ij}.

Q.3) (a) Show that the scalar product of two 4 vectors $A_\mu B^\mu$ is invariant under LT & for two tensors $A^{\mu\nu}, B^{\mu\nu}$, $A^{\mu\nu}B_{\mu\nu} = A_{\mu\nu}B^{\mu\nu}$.

(b) In Minkowski space time, let u^μ be a *time-like* 4-vector. Prove that there always exists a *space-like* 4-vector v^μ such that $u_\mu v^\mu = 0$. You may use Schwarz inequality $|\mathbf{P} \cdot \mathbf{Q}| \leq |\mathbf{P}||\mathbf{Q}|$ for any two vectors $\mathbf{P}$ and $\mathbf{Q}$ in ordinary 3-dimensional space. Hence find the linear momentum of photon.