

Linear Algebra Applications Project

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Triangle Decomposition

$$\phi A_L = B_L + A_L + B_S$$

$$\phi B_L = B_L + A_L + A_S$$

$$\phi A_S = A_L + A_S$$

$$\phi B_S = B_L + B_S$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} = \begin{bmatrix} 1-\lambda & 1 & 1 & 0 \\ 1 & 1-\lambda & 0 & 1 \\ 0 & 1 & 1-\lambda & 0 \\ 1 & 0 & 0 & 1-\lambda \end{bmatrix}$$

$$\lambda^4 - 4\lambda^3 + 5\lambda^2 - 4\lambda + 1 = 0$$

$$(\lambda^2 - 3\lambda + 1)(\lambda^2 - \lambda + 1)$$

$$\lambda = \frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}, \frac{1}{2} + i\frac{\sqrt{3}}{2}, \frac{1}{2} - i\frac{\sqrt{3}}{2}$$

$$\begin{bmatrix} 1-\phi^2 & 1 & 1 & 0 & 0 \\ 1 & 1-\phi^2 & 0 & 1 & 0 \\ 0 & 1 & 1-\phi^2 & 0 & 0 \\ 1 & 0 & 0 & 1-\phi^2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1-\phi^2 & 0 \\ 0 & 1 & 0 & 1-\phi^2 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_L = (\phi^2 - 1)B_S$$

$$B_L = (\phi^2 - 1)B_S$$

$$A_S = B_S$$

$$B_S = B_S$$

$$Span \left\{ \begin{bmatrix} \phi^2 - 1 \\ \phi^2 - 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$