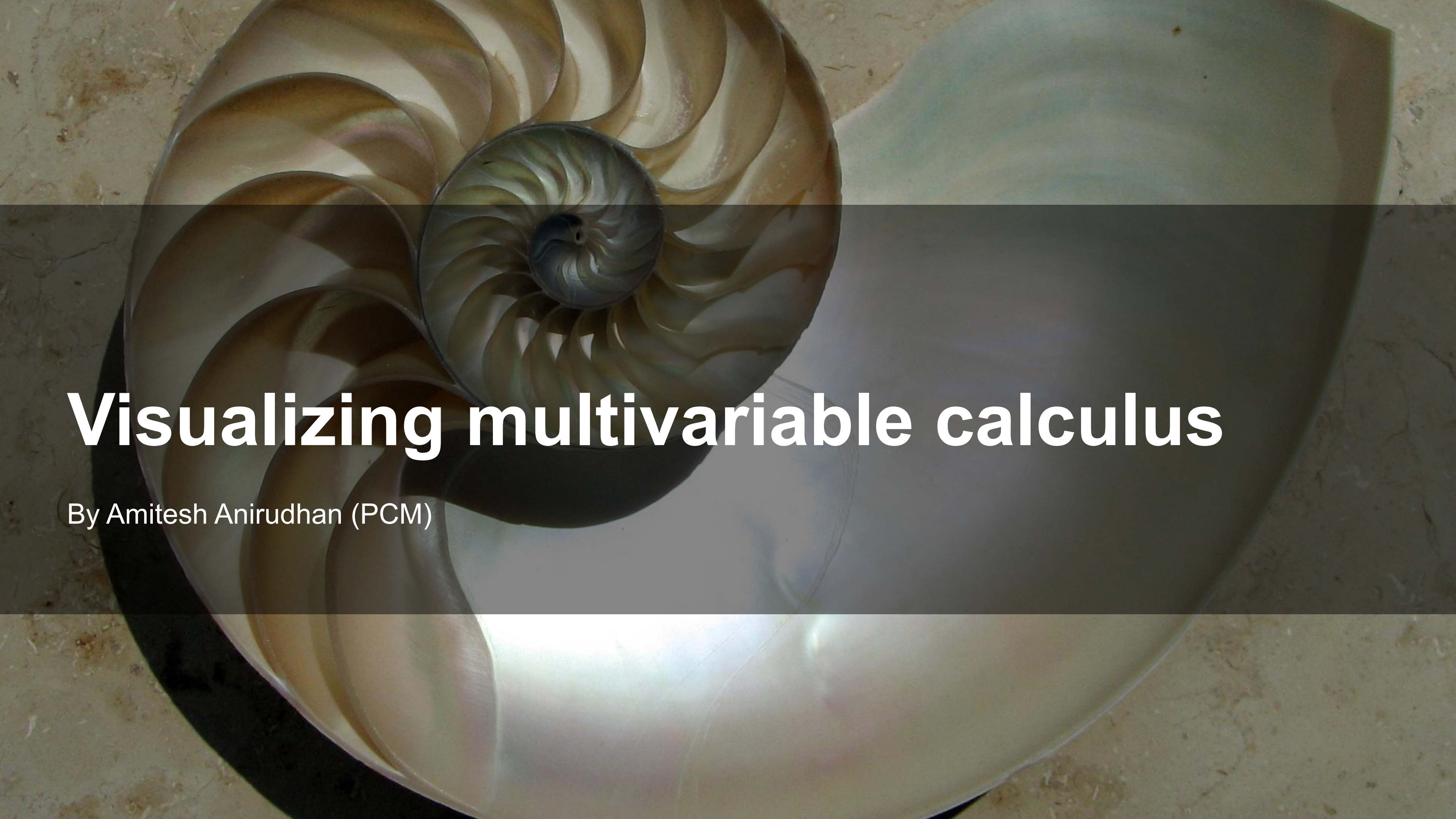


Visualizing multivariable calculus

By Amitesh Anirudhan (PCM)

Topics and objectives :

(i) Multivariable function

(ii)Gradient

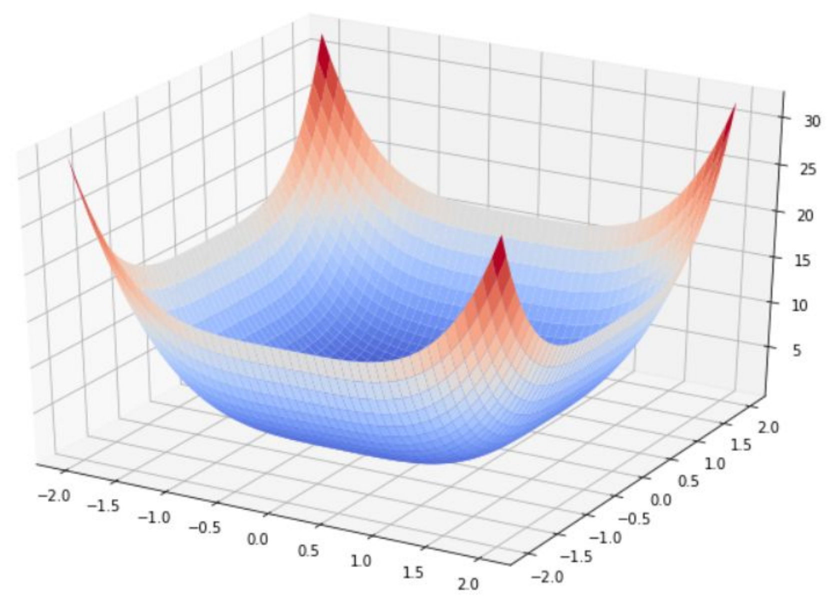
(iii) Laplacian

(iv) Divergence

(Topic discussed are referenced from mathematical methods for physical sciences by ML Boas.)

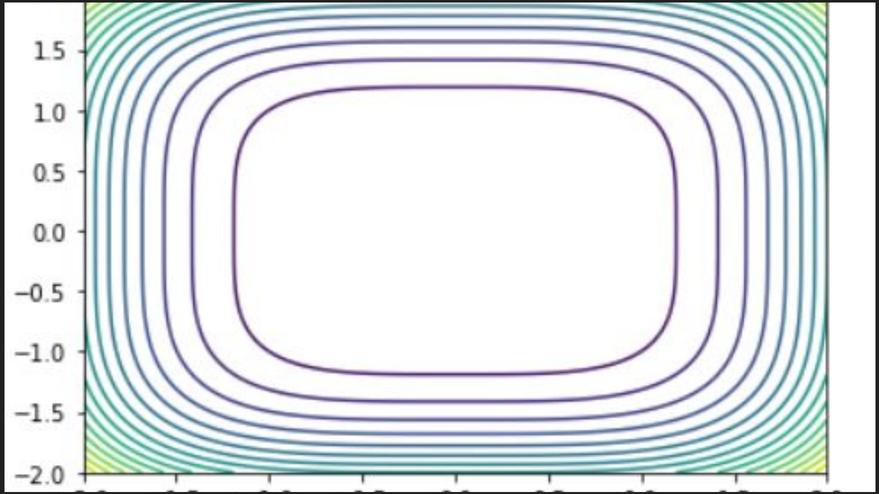
$$f(x, y)$$

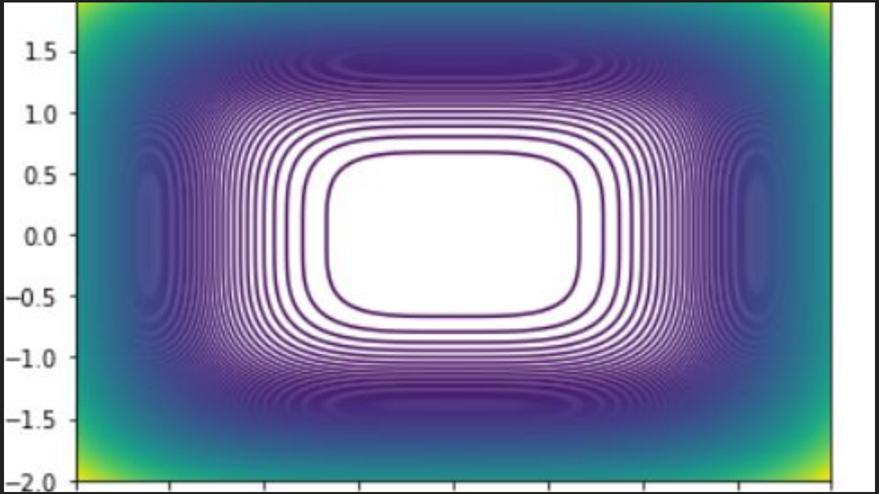
Multivariable function :



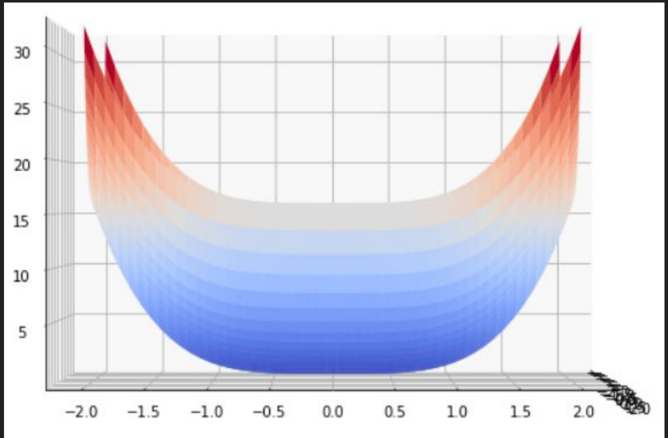
f(x,y) = x^4 + y^4

- A function is called multivariable if its input is made up of multiple numbers.
- Functions whose output is a vector are called vector-valued functions, while functions with a single number as their output are called scalar-valued.
- To model varying temperatures in a large region, you could use a function which takes in two variables, longitude and latitude, maybe even altitude and outputs one variable, the temperature.



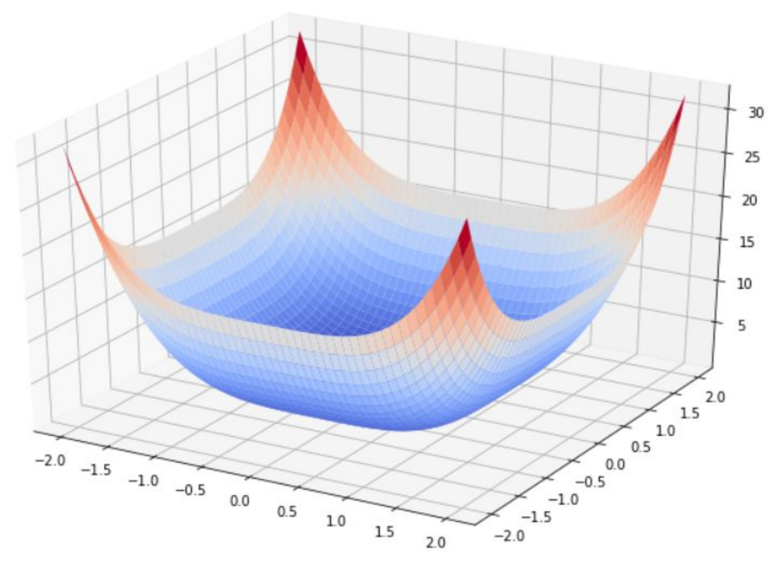


Contour plot :



Contour plots are a way to show a three-dimensional surface on a two-dimensional plane. It graphs two predictor variables X Y on the y-axis and a response variable Z as contours.

Different operations on multivariable functions :

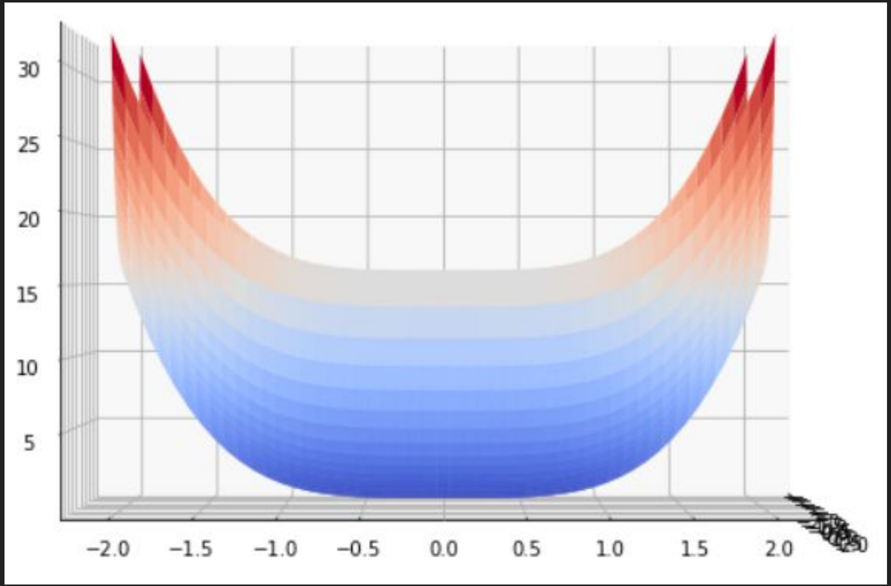


Properties to discuss :

(i) Gradient : $\nabla f = f\ (\partial/\partial x\ i + \partial/\partial y\ j + \partial/\partial z\ k)$

(ii) Laplacian : $\nabla(\nabla . f) = \nabla\ (\ \partial f/\partial x + \partial f/\partial y + \partial f/\partial z\ )$

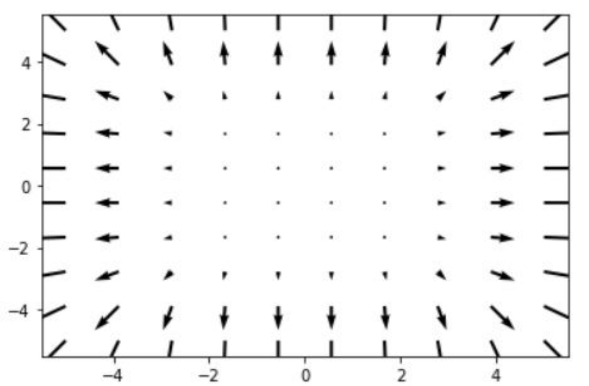
Gradient :



The gradient points in the direction of steepest ascent.

∇f = f (∂/∂x i + ∂/∂y j + ∂/∂z k)

∇f = 4x^3 i + 4y^3 j



Laplacian :

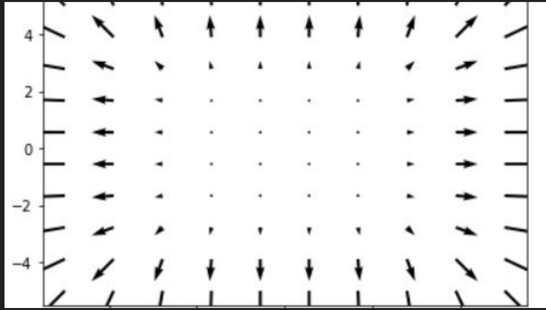
The Laplacian operator is the measure of how much minimum or maximum is a function at point (x,y).

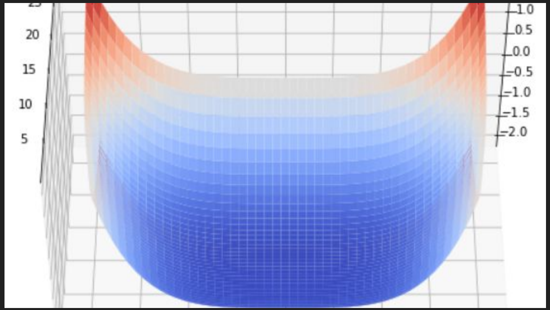
$\triangle f = \nabla.(\nabla f) = \nabla \, . \, (\partial f/\partial x \; i + \partial f/\partial y \; j + \partial f/\partial z \; k)$

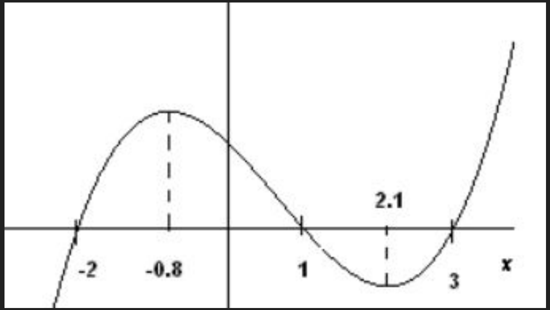
$= \partial^2 f/\partial x^2 + \partial^2 f/\partial y^2 + \partial^2 f/\partial z^2$

$\nabla.(\nabla f) = 12x^2 + 12y^2$

$\triangle f > 0, \; \triangle f < 0$

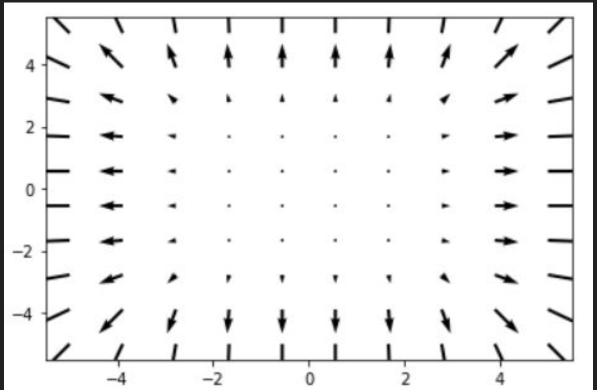






Divergence :

The divergence is an operator, which takes in the vector-valued function defining this vector field, and outputs a scalar-valued function measuring the change in density of the function at each point.



Thank you!