

## THE UTOPIAN TREE

The Utopian problem is a common problem where a tree grows in cycles of two phases each year.

1. **SPRING (DOUBLE HEIGHT)**: The tree doubles its height

2. **SUMMER (INCR. BY 1m)**: The tree grows by 1m.

The Growth pattern starts with a tree of height 1m. Given a number of cycles, you need to calculate the final height of the tree.

EX:-

if no. of cycle =  $n$

- cycle 0: Initial height = 1
  - cycle 1: (Spring): Height doubles  $\rightarrow 1 \times 2 = 2$
  - cycle 2: (Summer): Height increases by 1  $\rightarrow 2 + 1 = 3$
  - cycle 3 (Spring): Height doubles  $\rightarrow 3 \times 2 = 6$
  - cycle 4 (Summer): Height increases by 1  $\rightarrow 6 + 1 = 7$
- FINAL HEIGHT AFTER 4 cycles = 7.

C++

```
int utopianTree (int n) {  
    int height = 1; // initial height of the tree  
    for (int i = 1; i <= n; i++) {  
        if (i % 2 == 1) {  
            height = height * 2; // Spring: double the  
                                // height.  
        } else {  
            height = height + 1;  
        }  
    }  
    return height;  
}  
  
int main () {  
    int cycles;  
    printf ("Enter the no. of cycles:");  
    scanf ("%d", &cycle);  
    int result = utopianTree (cycles);  
    printf ("height of UtopianTree : ", cycles, result);  
    return 0;  
}
```

TIME COMPLEXITY.

T.C - Iterative -  $O(n)$   
S.C -  $O(1)$

optimized Approach :- (for Specific case) →

• odd cycles: Height is  $2^K - 1$ , where K is no. of complete odd cycles.

• Even cycles: Height can be derived as  $2^{K+1} - 1$

FORMULA:  $O(1)$