Data Structure and Algorithm
By
Malay Tripathi

SPARSE MATRIX

A Sparse Matrix → is a two dimensional array where the majority of the elements have the value null. for ex :→

$$\begin{bmatrix} - & \times & - & - & - & - & - \\ \times & - & - & - & - & - & - \\ - & - & \times & - & - & - & - \\ - & - & - & - & \times & - & - \end{bmatrix}$$

where x = represents NON-NULL VALUES.

Memory Representation of Lower Triangular

$$\begin{bmatrix} a_{11} & & & \\ a_{21} & a_{22} & & \\ a_{31} & a_{32} & a_{33} & \\ \vdots & \vdots & \vdots & \\ a_{n1} & a_{n2} & a_{n3} & - a_{mn} \end{bmatrix}_{n \times n}$$

Row-major order.

According to row major order, the address of any element a_{ij}, $1 \le i, j \le n$, can be obtained as :→

$$\begin{aligned} \text{Address}(a_{ij}) &= \text{No. of elements upto the } a_{ij} \text{ elements} \\ &= \text{Total no. of elements in the first } i-1 \text{ rows} \\ &\quad + \text{no. of elements upto } j^{th} \text{ column in } i^{th} \text{ row} \\ &= 1 + 2 + 3 + \cdots + (i-1) + j \\ &= \frac{i(i-1)}{2} + j \end{aligned}$$

$$\text{Address}(a_{ij}) = M + \frac{i(i-1)}{2} + j - 1.$$

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Column Major Order.

According to column-major order, the address of any element a_{ij}, $1 \le i$, $j \le n$ can be obtained as :→

Address (a_{ij}) = No. of elements upto the a_{ij} elements

= Total number of elements in the first $j-1$ columns + number of elements upto the ith row in the jth

$$= [n + (n-1) + (n-2) + \dots + (n-j+2)] + (i-j+1)$$

$$= \{ n \times [j-1] - [1+2+3+\dots+(j-2)+(j-1)] + i \}$$

$$\Rightarrow n * (j-1) - \frac{j(j-1)}{2} + i$$

$$\Rightarrow (j-1)\left[n - \frac{j}{2}\right] + i$$

If the starting location of the first element (that is, of a_{11}) is M then the address of a_{ij}, $1 \le i$, $j \le n$ will be

$$\boxed{\text{Address}(a_{ij}) = M + (j-1) * \left(n - \frac{j}{2}\right) + i - 1}$$

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Memory Representation of an Upper-Triangular

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots & a_{1n} \\ & a_{22} & a_{23} \cdots & a_{2n} \\ & & a_{33} \cdots & a_{3n} \\ & & & a_{nn} \end{bmatrix}_{n \times n}$$

(a). Row-Major Order

Acc. to Row-Major Order, the address of any element a_{ij}, $1 \le i, j \le n$ can be obtained as :→

Address (a_{ij}) = Number of elements upto the a_{ij} element

= Total no. of elements in the first $(i-1)$ rows + number of elements upto the j^{th} column in the i^{th} row

$$= n + (n-1) + (n-2) + (n-3) + \cdots + (j-i+1)$$

$$= n + (n-1) + (n-2) + \cdots + (n-i+2) + (j-i+1)$$

$$= n \times (i-1) - [1+2+3+\cdots+(i-2)+(i-1)] + j$$

$$= n * (i-1) - \frac{i(i-1)}{2} + i$$

$$= (i-1) * \left(n - \frac{i}{2}\right) + j$$

$$\boxed{\text{Address}(a_{ij}) = M + (i-1) * \left(n - \frac{i}{2}\right) + j - 1}$$

Column Major Order.

Acc. to column major order, the address of any element a_{ij}, $1 \le i, j \le n$ can be obtained as :→

Address (a_{ij}) = No. of elements up to the a_{ij} elements
= Total no. of elements in the first $(j-1)$ columns + no. of elements up to the i^{th} row in the j^{th} column

$$= [1+2+3+\cdots+(j-1)]+i$$

$$= \frac{j(j-1)}{2}+i$$

If the starting location of the first element, i.e of a_{11} is M, then the address of a_{ij}, $1 \le i, j \le n$ will be

$$\boxed{\text{Address }(a_{ij}) = M + \frac{j(j-1)}{2} + i - 1}$$

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MEMORY REPRESENTATION OF A TRIDIAGONAL MATRIX

In sparse matrix having the elements only on the diagonal, the following points are evident:

$\boxed{\text{Number of elements in an } n \times n \text{ square matrix} = n}$

Any elements a_{ij} can be referred in memory using the formula

$\boxed{\text{Address } (a_{ij}) = i\,[\text{or } j]}$

$$
\begin{bmatrix}
a_{11} & a_{12} & & & & & \\
a_{21} & a_{22} & a_{23} & & & & \\
 & a_{32} & a_{33} & a_{34} & & & \\
 & & a_{43} & a_{44} & a_{45} & & \\
 & & & & \vdots & & \\
 & & & \cdots & a_{(n-1)(n-2)} & a_{(n-1)(n-1)} & a_{(n-1)n} \\
 & & & & & a_{n(n-1)} & a_{nn}
\end{bmatrix}
$$

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Row Major Order.

The address of the element, a_{ij}, $1 \le i, j \le n$ can be obtained as :-

Address (a_{ij}) = Number of elements up to the a_{ij} element

= Total number of elements in the first $(i-1)$ rows + number of elements up to the j^{th} column in the i^{th} row

$= \{2 + [3+3+3+ \cdots + \text{upto } (i-2) \text{ terms}]\} + (j-i+2)$

$= 2 + 3(i-2) + j - (i-2)$

$= 2 + 2(i-2) + j \Rightarrow 2 + 2(i-2) + j$

If the starting location of the first element, i.e of a_{11} is M, then the address of a_{ij}, $1 \le i, j \le n$ will be

$$\boxed{\text{Address } (a_{ij}) = M + 2(i-2) + j + 1.}$$

Column Major Order

According to column major order, the address of any element a_{ij}, $1 \le i, j \le n$

Address (a_{ij}) = Number of elements upto the a_{ij} element

= Total no. of elements in the first $(j-1)$ columns + number of elements upto the i^{th} row in the j^{th} column.

$= \{2 + [3+3+ \cdots + \text{upto } (j-2) \text{ terms}]\} + (i-j+2)$

$= 2 + (j-2) \times 3 + i - (j-2)$

$= 2 + 2 * (j-2) + i$

If the Starting location of the first element, i.e of a_{11} is M, then the address of a_{ij}, $1 \le i, j \le n$ will be

$$\boxed{\text{Address } (a_{ij}) = M + 2 * (j-2) + i + 1}$$