"श्री कृष्ण गोविंद हरे मुरारी, हे नाथ नारायण वासुदेवा"

Data Structure and Algorithm

By

Malay Tripathi

BINARY SEARCH 3

FIRST AND LAST OCCURENCES IN ARRAY + COUNT OCCURENCES

Q. find the first and last occurence of element x.

arr = [2, 4, 6, 8, 8, 8, 11, 13]

ind → 0 1 2 3 4 5 6 7

So, we have to find first and last occurence index of x.

# $x=8$

{3, 5}

first index of occu. — last index of occu.

# $x=10$

{-1, -1}

first index of occu. — last index of occu.

# $x=11$

{6, 6}

first index of occu. — last index of occu.

# LINEAR SEARCH

arr = [2 4 6 8 8 8 11 13]

0 1 2 3 4 5 6 7

→ when we find $x=8$

first = ~~-1~~ last = -1

3 ------> last = ~~-1~~ ~~3~~ ~~4~~ 5

3 ------> 5

{3, 5}.

```
first = -1, last = -1
for(i=0 to n-1)
{
    if (arr[i] == x)
    {
        if (first == -1)
            first = i
        last = i
    }
}
```

$\boxed{\text{TIME COMPLEXITY} = O(N)}$

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As we know that the given array is Sorted, we can apply BinarySearch for first and last Occurence.

```
       0  1 2 3 4 5  6  7
arr=[ 2, 4,6, 8,8,8 ,11, 13]
              ↑      ↑
```

as we know about upperbound and lowerbound.

lower bound → that it is smallest index , A[index] >= x

upper bound → A[index] > x

given me lowerbound of (8) ⟶ A[ind] ⩾ n → it will point at index 3

give me upperbound of (8) → A[index] > n → it will point at index 6

```
arr = [ 2,4, 6, 8,8,8, 11, 13]
                ↑    ↑   ↑
          lowerbound  |  upperbound
        [upperbound(8) -1]
```

# it will not covering all the cases, if the value is not in array

for $\boxed{x=10}$

since 10 is not in array, you will get 11 as the lowerbound.

$\boxed{x=14}$ lb(14)

⟶ it is start pointing hypothetical index of 8.

if((lb == n || (arr[lb] != x)) return {-1, -1};

means element is not present in the array.

return {lb , upperBound (arr, n,k)-1 } ;

TIME COMPLEXITY = $\underset{\text{lower bound}}{\underset{\uparrow}{O(\log_2 n)}} + \underset{\text{upper bound}}{\underset{\uparrow}{O(\log_2 n)}}$

$$\boxed{\text{TIME COMPLEXITY} = 2 * O(\log_2 n)}$$

APPROACH 2

USE SIMPLE BINARY SEARCH

$$arr = \begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ [2, & 8, & 8, & 8, & 8, & 8, & 11, & 13] \\ \uparrow & & & \uparrow & & & & \uparrow \\ & & & mid & & & & \end{array}$$

first = -1 ← - - - - mid

$\therefore$ mid = $\frac{0+7}{2} = 3.5 \simeq 3$

first = ~~-1~~ 3 , so for finding the first index, look for the left hand side. So right search space will be eliminated.

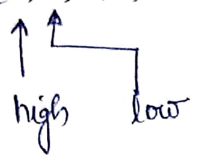
$$arr = \begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ [2, & 8, & 8, & 8, & 8, & 8, & 11, & 13] \\ \uparrow & \uparrow & \uparrow & & & & & \\ low & mid & high & & & & & \end{array}$$

first = ~~-1~~ ~~3~~ 1

$$arr = \begin{array}{cccccccc} [2, & 8, & 8, & 8, & 8, & 8, & 11, & 13] \\ \uparrow\uparrow & & & & & & & \\ low\ high & & & & & & & \end{array}$$

→ this element is not equal to x, thus we have to move in the right search space.

arr = [2, 8, 8, 8, 8, 8, 11, 13]



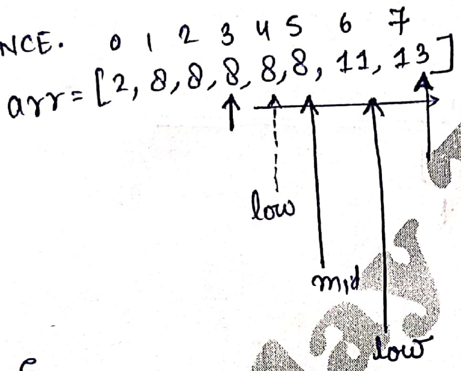
→ low crosses high, thus first = ~~-1~~ ~~3~~ 1

$\boxed{\text{FIRST OCCURENCE} = 1}$

// FIND LAST OCCURENCE.

last = -1

Last = ~~-1~~ 3



* mid = $\frac{4+7}{2}$ = 5

a[mid] = 8 = so, last = ~~-1~~ ~~3~~ 5

* mid = $\frac{6+7}{2}$ = 6

a[mid] = a[6] = 11

8 ≠ 11. So we can't find, 8 on the right.

$\boxed{\text{last occurrence} = 5}$

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Pseudo Code for first Occurence.

```
fn (arr, n, x)
  {
  low=0, high=n-1
   first=-1
  while( low<= high)
    {
     mid = (low + high)/2
    if (a[mid] = = x
        first = mid , high = mid-1

    else if ( a[mid] < n)
          low= mid +1

     else
        high= mid-1
```

Pseudo Code for last Occurence.

```
fn(arr, n, x)
  {
   low =0 , high=n-1;
   last =-1;
  while (low<=high)
   {
   mid=( low+high)/2
   if (arr[mid] = =n)
      last = mid;
      low= mid +1;
   else if ( a[mid]<x)
         low = mid+1
   else
      high= mid-1;
```

```
pair <int,int> firstANDLastPosition (vector<int> & arr , int n, int k)
  { int first = firstOccurence (arr, n, k);
    if (first== -1) return {-1,-1};
    int last = lastOccurence (arr, n, k);
    return ( first, last);
  }
```

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COUNT TOTAL NUMBER OF OCCURENCE OF X.

(last occurence - first occurence) + 1 = total no. of x element in the array.

```
int count (vector <int> & arr, int n, int x) {
    pair <int,int> ans = firstAndLastPosition (arr, n, x);
    if(ans.first == -1) return 0;
    return ans.second - ans.first + 1;
}
```