EL 7133 (DSP II)
Spring 2016
Homework Assignment - 08

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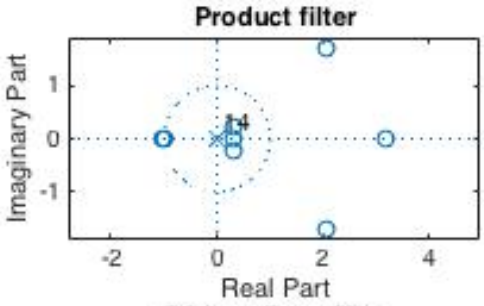
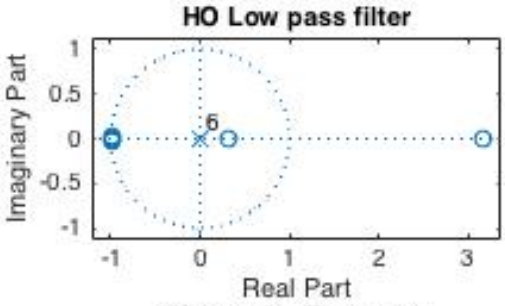
Question 1:

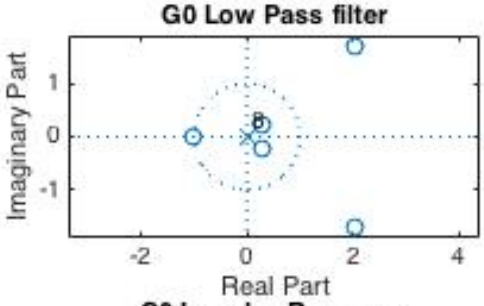
```
% Use Matlab to make the 7/9 symmetric biorthogonal Daubechies filters. Use the
% same product filter P(z) as for the design of orthonormal wavelet filters (i.e.,
a half-band  Type 1 FIR filter of length 2N-1 with N zeros at z = -1). Use N = 8 to
get the product filter P(z) needed here.
%  Implement a perfect reconstruction filter bank using the filter you create.
Verify the filter bank has the perfect reconstruction propertyclc;

clear all;
close all;
Qz=[0.0024 -0.0195 0.0640 -0.1016 0.0640 -0.0195 0.0024];
hp=poly([-1 -1 -1 -1 -1 -1 -1 -1]);
p=conv(Qz,hp);
r=roots(p);
r0=zeros(6,1);
j=1;
for i=1:length(r)
if 0.9< abs(r(i,1)) && abs(r(i,1))<1
    r0(j,1)=r(i,1);
    j=j+1;
end
if imag(r(i,1))==0
    r0(j,1)=r(i,1);
    j=j+1;
end
end
h0=-poly(r0);
r1=setdiff(r,r0);
g0=poly(r1);
figure,
subplot(3,2,1),zplane(p),title('Product filter');
subplot(3,2,2),zplane(h0),title('HO Low pass filter');
subplot(3,2,3),zplane(g0),title('G0 Low Pass filter');
subplot(3,2,4),stem(h0),title('H0 Impulse Response');
subplot(3,2,5),stem(g0),title('G0 Impulse Response');

%% For high pass filter

rh=-r;
ph=poly(rh);
r2=zeros(6,1);
j=1;
for i=1:length(rh)
if 0.9< abs(rh(i,1)) && abs(rh(i,1))<1
    r2(j,1)=rh(i,1);
    j=j+1;
end
if imag(rh(i,1))==0
```

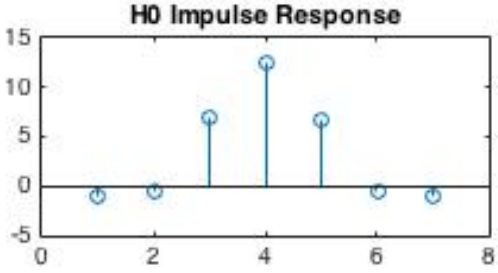
```
    r2(j,1)=rh(i,1);
    j=j+1;
end
end
g1=poly(r2);
r3=setdiff(rh,r2);
h1=poly(r3);
figure,
subplot(3,2,1),zplane(ph),title('Product filter');
subplot(3,2,2),zplane(h1),title('H1 High pass filter');
subplot(3,2,3),zplane(g1),title('G1 High Pass filter');
subplot(3,2,4),stem(h1),title('H1 Impulse Response');
subplot(3,2,5),stem(g1),title('G1 Impulse Response');

%% Verifying perfect reconstruction property
x=[1 2 3 4];
y1=conv(x,h0);
y1=downsample(y1,2);
y1=upsample(y1,2);
y1=conv(y1,g0);
y2=conv(x,h1);
y2=downsample(y2,2);
y2=upsample(y2,2);
y2=conv(y2,g1);
y=(y1+y2)/414;
figure,
subplot(1,2,1),stem(x);title('input signal');xlim([0 6]);
subplot(1,2,2),stem(y);title('output signal scaled down');
```

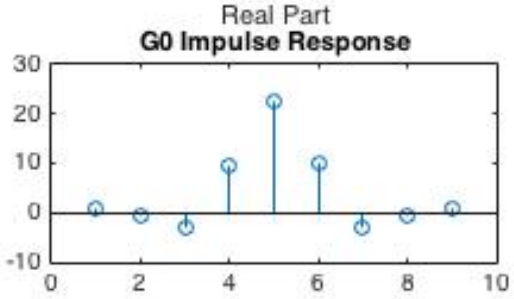
Output:

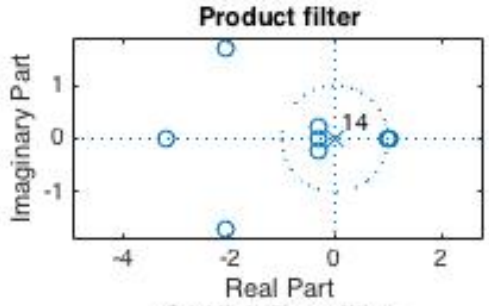

Product filter
Imaginary Part
Real Part
14

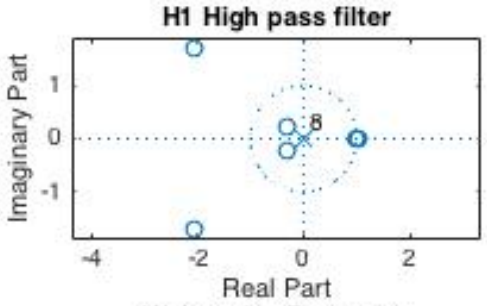

HO Low pass filter
Imaginary Part
Real Part
6

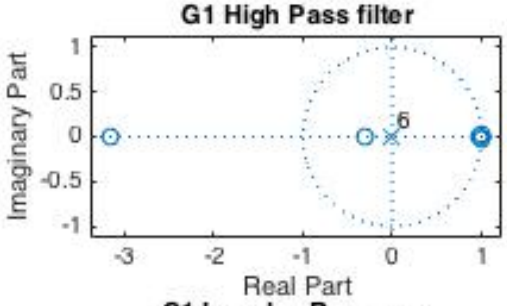

G0 Low Pass filter
Imaginary Part
Real Part
6

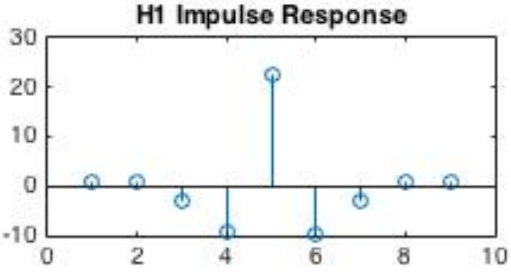

H0 Impulse Response

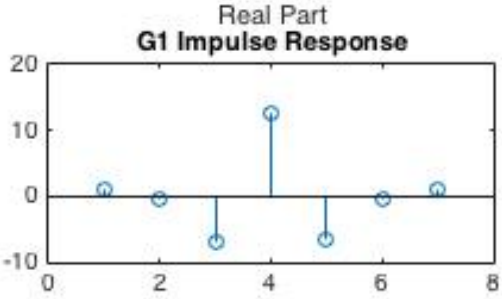

G0 Impulse Response

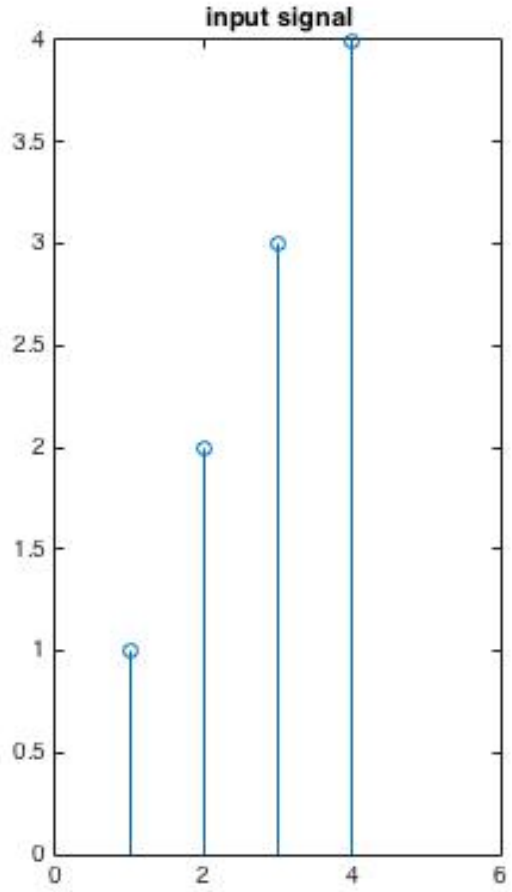

Product filter
Imaginary Part
Real Part
14

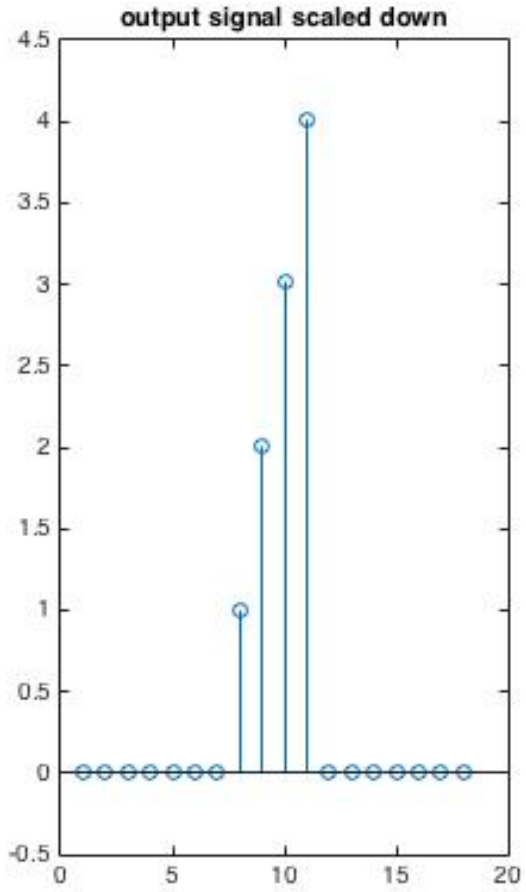

H1 High pass filter
Imaginary Part
Real Part
8


G1 High Pass filter
Imaginary Part
Real Part
6


H1 Impulse Response


G1 Impulse Response






```
Daubechies 7/9 filter
```

h0 =

-1.0000 -0.4859 6.8719 12.5762 6.6330 -0.5451 -0.9598

>> h1

h1 =

1.0000 0.6109 -3.1645 -9.4157 22.2948 -9.9336 -3.1558 0.7220 1.0419

>> g0

g0 =

1.0000 -0.6109 -3.1645 9.4157 22.2948 9.9336 -3.1558 -0.7220 1.0419

>> g1

g1 =

1.0000 -0.4859 -6.8719 12.5762 -6.6330 -0.5451 0.9598

Result

7/9 symmetric biorthogonal Daubechies filters is designed in Matlab. When I gave an input signal, I got the dame output signal, but with shift and scaled. This verifies the filter bank has the perfect reconstruction property.

Question 2

```
% Read DFT
% Construct the transform matrix for a 'real' 8-point DFT in Matlab. The matrix
% should be real-valued and orthonormal. Verify that the matrix is orthonormal
(i.e., verify
% that the inverse matrix is the transpose matrix). Display the basis vectors
(i.e., the rows
% of the forward transform or the columns of the inverse transform). The basis
vectors of
% the 'real' DFT should be the real and imaginary parts of the basis vectors of the
DFT
% with appropriate scaling constants for normalization.

% Real valued DFT is obtained by mirroring N length sequence to form 2N-1 sequence.
% DFT=X1(k)=e^(-j*2*pi*-0.5*k/2N)*sum(x(n)*cos(pi/N *k*(n+0.5)) from 0 to N-1.
% Transform matrix is D(k,n)=sqrt(2/N)*cos(pi/N *k*(n-0.5)) . First row is
sqrt(1/N)
clc;clear all;close all
N=8;
D=zeros(N,N);
for k=1:N-1
    for n=1:N
D(k+1,n)=sqrt(2/N)*cos(pi/N *k*(n-0.5));
    end
end
D(1,:)=sqrt(1/N);

if inv(D)==transpose(D)
    disp('Orthonormal Matrix')
end

%% Basis Vector
b1=D(1,:)
disp('Basis Vector 1');

b2=D(2,:)
disp('Basis Vector 2');

b3=D(3,:)
disp('Basis Vector 3');

b4=D(4,:)
disp('Basis Vector 4');

b5=D(5,:)
disp('Basis Vector 5');
```

```
b6=D(6,:)
disp('Basis Vector 6');

b7=D(7,:)
disp('Basis Vector 7');

b8=D(8,:)
disp('Basis Vector 8');
```

Result:

```
Transform Matrix for a real 8-point DFT
D =

    0.3536    0.3536    0.3536    0.3536    0.3536    0.3536
0.3536    0.3536
    0.4904    0.4157    0.2778    0.0975   -0.0975   -0.2778   -
0.4157   -0.4904
    0.4619    0.1913   -0.1913   -0.4619   -0.4619   -0.1913
0.1913    0.4619
    0.4157   -0.0975   -0.4904   -0.2778    0.2778    0.4904
0.0975   -0.4157
    0.3536   -0.3536   -0.3536    0.3536    0.3536   -0.3536   -
0.3536    0.3536
    0.2778   -0.4904    0.0975    0.4157   -0.4157   -0.0975
0.4904   -0.2778
    0.1913   -0.4619    0.4619   -0.1913   -0.1913    0.4619   -
0.4619    0.1913
    0.0975   -0.2778    0.4157   -0.4904    0.4904   -0.4157
0.2778   -0.0975



>> inv(D)

ans =

    0.3536    0.4904    0.4619    0.4157    0.3536    0.2778
0.1913    0.0975
    0.3536    0.4157    0.1913   -0.0975   -0.3536   -0.4904   -
0.4619   -0.2778
    0.3536    0.2778   -0.1913   -0.4904   -0.3536    0.0975
0.4619    0.4157
    0.3536    0.0975   -0.4619   -0.2778    0.3536    0.4157   -
0.1913   -0.4904
    0.3536   -0.0975   -0.4619    0.2778    0.3536   -0.4157   -
0.1913    0.4904
    0.3536   -0.2778   -0.1913    0.4904   -0.3536   -0.0975
0.4619   -0.4157
```

```
    0.3536   -0.4157    0.1913    0.0975   -0.3536    0.4904   -
0.4619    0.2778
    0.3536   -0.4904    0.4619   -0.4157    0.3536   -0.2778
0.1913   -0.0975
```

b1 =

0.3536 0.3536 0.3536 0.3536 0.3536 0.3536 0.3536 0.3536

Basis Vector 1

b2 =

0.4904 0.4157 0.2778 0.0975 -0.0975 -0.2778 -0.4157 -0.4904

Basis Vector 2

b3 =

0.4619 0.1913 -0.1913 -0.4619 -0.4619 -0.1913 0.1913 0.4619

Basis Vector 3

b4 =

0.4157 -0.0975 -0.4904 -0.2778 0.2778 0.4904 0.0975 -0.4157

Basis Vector 4

b5 =

0.3536 -0.3536 -0.3536 0.3536 0.3536 -0.3536 -0.3536 0.3536

Basis Vector 5

b6 =

0.2778 -0.4904 0.0975 0.4157 -0.4157 -0.0975 0.4904 -0.2778

Basis Vector 6

b7 =

0.1913 -0.4619 0.4619 -0.1913 -0.1913 0.4619 -0.4619 0.1913

Basis Vector 7

b8 =

0.0975 -0.2778 0.4157 -0.4904 0.4904 -0.4157 0.2778 -0.0975

Basis Vector 8

Transpose of D is equal to inverse of D , hence it is Orthonormal Matrix. I have also shown all the 8 basis vector.

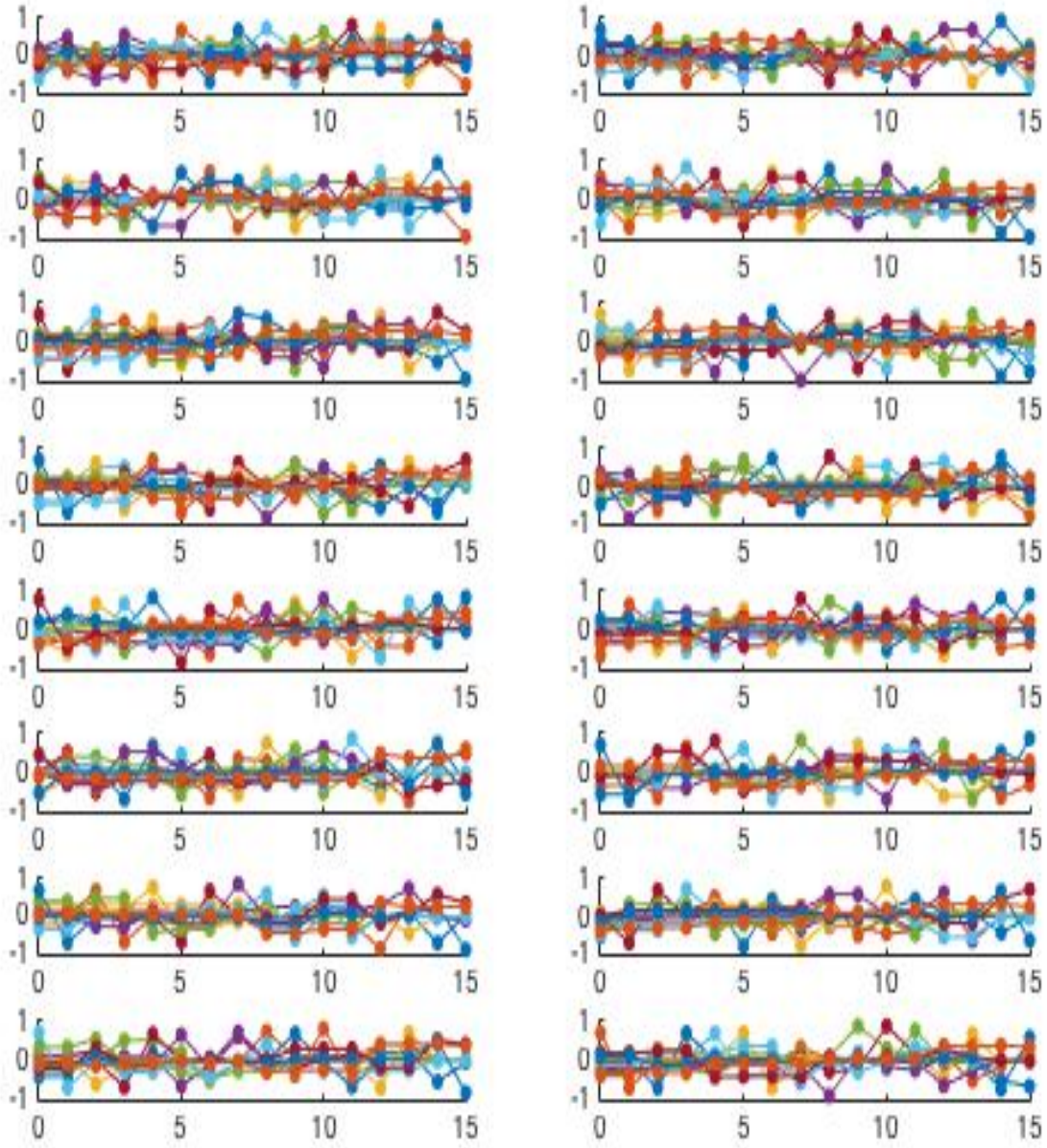
Question 3

```
% Principle component analysis (PCA)
% Use PCA to calculate an optimal orthogonal (non-separable) transform for 4x4
% image blocks. To do this, take all 4x4 image blocks from an image, create a data
matrix,
% then create a covariance matrix, and then perform eigenvector decomposition. The
% covariance matrix should be of size 16x16 and the eigenvectors should be of
length 16.
% Each eigenvector is a vectorized set of 16 pixels in block of size 4x4. For
reference, see
% the Matlab demo from class PCA_from_image_data.m which calculated an optimal
% transform for 8x1 image vectors.
%  Show the two-dimensional basis functions of 4x4 blocks. For example, my result
is
% shown in the file two-dimensional PCA.pdf
%  Use your PCA to reconstruct an image from only four coefficients in each 4x4
block
% (i.e., set 12 of the 16 coefficients to zero).
% Reading: 'A Tutorial on Principal Component Analysis' by Jonathon Shlens
% http://arxiv.org/abs/1404.1100

clc;
clear all; close all
a=imread('lena_gray.bmp');
a = double(a);
[r,c]=size(a);

%% Creating 4*4 blocks
p=1;q=1;
for i=1:4:r
    for j=1:4:c
d{p,q,:}=a(i:4,j:4);
q=q+1;
    end
p=p+1;
end

%% Center the data (remove mean from each component)
p=1;q=1;
for i=1:4:r
    for j=1:4:c
res=a(i:i+4-1,j:j+4-1)-mean(mean(a(i:i+4-1,j:j+4-1)));
x3{p,q,:}=reshape(res,16,1);
q=q+1;
```

```
    end
p=p+1;
end


%% Compute covariance matrix
p=1;q=1;
for i=1:4:r
    for j=1:4:c
        z=x3{p,q,:};
N = size(z,2);
R{p,q,:} = (1/N) * z * (z');
q=q+1;
    end
p=p+1;
end
size(R{1,1,:})


%% Compute covariance matrix
p=1;q=1;
for i=1:4:r
    for j=1:4:c
        R1=R{p,q,:};
        [V, D] = eig(R1);
        % eigenvalues
        D1{p,q,:}=diag(D);
        V1{p,q,:}=V;
        q=q+1;
    end
    p=p+1;
end

%% Display PCA basis vectors
% Notice how similar the PCA basis vectors are to the DCT basis vectors!
% The first vector is a constant vector (approximately);
% the 2nd vector is like a half-cycle of a cosine waveform;
% the 3rd vector is like a whole cycle of a cosine waveform; etc.
%%
figure(4)
clf
p=1;q=1;
for k = 1:16
    subplot(8,2,k)
    plot(0:15, V1{p,q,:}, '.-', 'markersize', 12)
    box off
    xlim([0 15])
    ylim([-1 1])
    q=q+1;

end
orient landscape
print -dpdf PCA_from_image_data
```

Result

Here, I just displayed the 1st 16 PCA vector.


1
0
-1
0
5
10
15