EL 7133 (DSP II)
Spring 2016

Homework Assignment - HW 06

Reading: Sparse Signal Restoration (sparse_signal_restoration.pdf)

1) Exercise 9 (Sect. 9) in the notes *Least Squares with Examples in Signal Processing.* [Suppose the available data is noisy and that some samples are missing....]. Also try Exercises 5, 6, and 8 (but not to submit).

2) The Landweber iteration to solve Ax = b is a *matrix-free* algorithm. It requires a sub-program to implement multiplication by A and its transpose. See Section 3 of the notes, *Sparse Signal Restoration.* In this exercise, use the Landweber iteration to perform the deconvolution illustrated in Figure 6 of the notes *Least Squares with Examples in Signal Processing:*

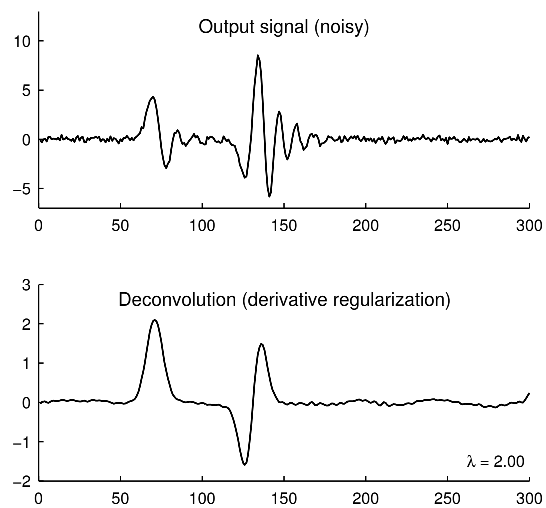

Figure 6: Deconvolution of noisy data by derivative regularization.

Matlab code for this example is available at:

http://eeweb.poly.edu/iselesni/lecture_notes/least_squares/LeastSquares_SPdemos/deconvolution/html/deconv_demo.html

Perform deconvolution by solving the problem:

$$\min_{\mathbf{x}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|_2^2 + \lambda \|\mathbf{D}\mathbf{x}\|_2^2$$

but instead of using the formula $\mathbf{x} = (\mathbf{H}^\mathsf{T}\mathbf{H} + \lambda \mathbf{D}^\mathsf{T}\mathbf{D})^{-1}\,\mathbf{H}^\mathsf{T}\mathbf{y}$. use the Landweber algorithm. You should use functions for H and its transpose (note that H is just convolution so you can use the conv function in Matlab for H). Using the Landweber algorithm, are you able to get the same solution as using the formula? Plot the value of the objective function versus iteration. Comment on your observations.

3) The Landweber iteration does not require that H or H^T be stored as matrices. Only programs to implement H and H^T are needed. Therefore, to perform image deconvolution

using the Landweber iteration, we need functions to implement 2D convolution and its transpose. When H represents 2D convolution, then we can implement H using the Matlab conv2 function. Consider the transpose of 2D convolution. (In the notes *Sparse Signal Restoration*, the transpose of the 1D convolution matrix is presented.)

Describe the transpose of 2D convolution and write a Matlab function to implement it.

4) Implement and demonstrate sparse deconvolution like the example in Section 6 in the notes *Sparse Signal Restoration.*

5) Find a couple of the papers listed in the reference list of the notes *Sparse Signal Restoration.* Comment on the problem the papers solve and how they solve it.

The following exercises are optional

6) The ISTA approach requires that the parameter alpha be specified. When using ISTA to perform deconvolution, where the system is a 5-point moving average (h = ones(1,5)/5), what value should alpha be? Why?

7) The parameter in ISTA should be chosen as the maximum eigenvalue of H^T H. Let H represent convolution and consider choosing alpha as the maximum value of |H(om)|^2 where H(om) is the frequency response of the LTI system. Is this approach to set alpha correct or partially correct? Explain. Support your explanation with numerical evidence in Matlab. (Hint: what are the eigenvectors of H^T H?)

Due date: Due through NYU Classes Tuesday, March 29.