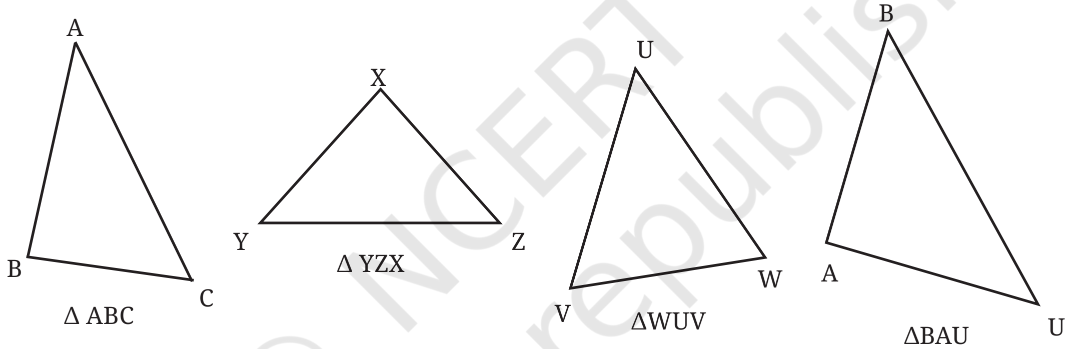
7 A TALE OF THREE INTERSECTING LINES

0774CH07

A triangle is the most basic closed shape. As we know, it consists of:

- three corner points, that we call the vertex of the triangle, and
- three line segments or the sides of the triangle that join the pairs of vertices.

Triangles come in various shapes. Some of them are shown below.



Observe the symbol used to denote a triangle and how the triangles are named using their vertices. While naming a triangle, the vertices can come in any order.

The three sides meeting at the corners give rise to three angles that we call the angles of the triangle. For example, in ΔABC, these angles are ∠CAB, ∠ABC, ∠BCA, which we simply denote as ∠A, ∠B and ∠C, respectively.

? What happens when the three vertices lie on a straight line?

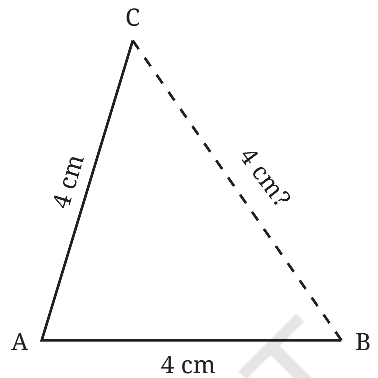
7.1 Equilateral Triangles

Among all the triangles, the equilateral triangles are the most symmetric ones. These are triangles in which all the sides are of equal lengths. Let us try constructing them.

? Construct a triangle in which all the sides are of length 4 cm.

How did you construct this triangle and what tools did you use? Can this construction be done only using a marked ruler (and a pencil)?

Constructing this triangle using just a ruler is certainly possible. But this might require several trials. Say we draw the base — let us call it AB — of length 4 cm (see the figure below), and mark the third point C using a ruler such that AC = 4 cm. This may not lead to BC also having a length of 4 cm. If this happens, we will have to keep making attempts to mark C till we get BC to be 4 cm long.

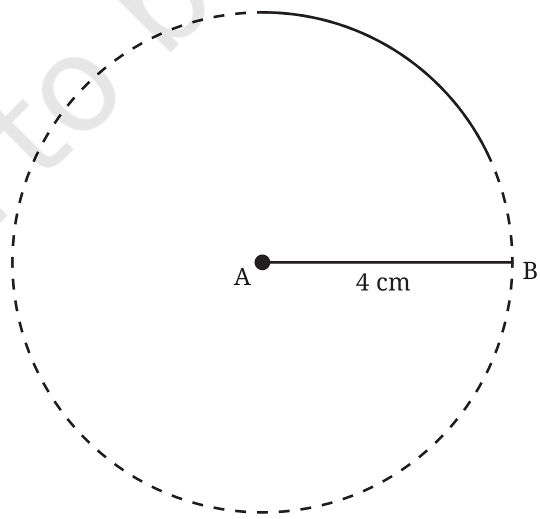


? How do we make this construction more efficient?

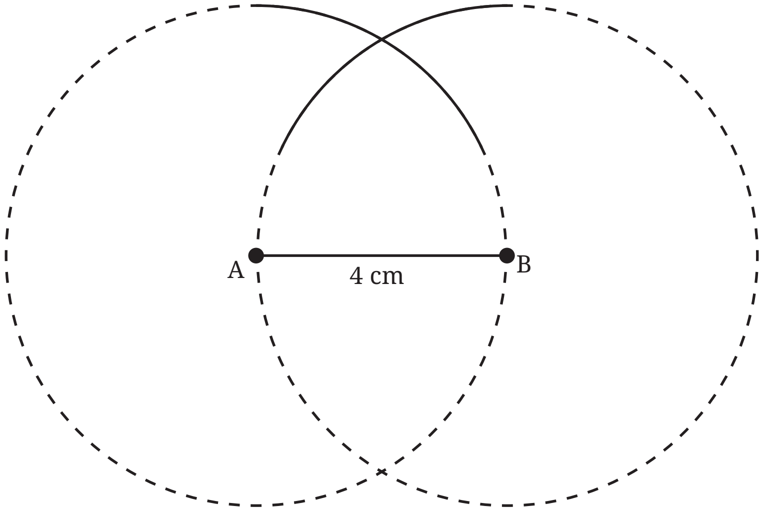
Recall solving a similar problem in the previous year using a compass (in the Chapter ‘Playing with Constructions’). We had to mark the top point of a ‘house’ which is 5 cm from two other points. The method we used to get that point can also be used here.

After constructing AB = 4 cm, we can do the following.

Step 1: Using a compass, construct a sufficiently long arc of radius 4 cm from A, as shown in the figure. The point C is somewhere on this arc. How do we mark it?



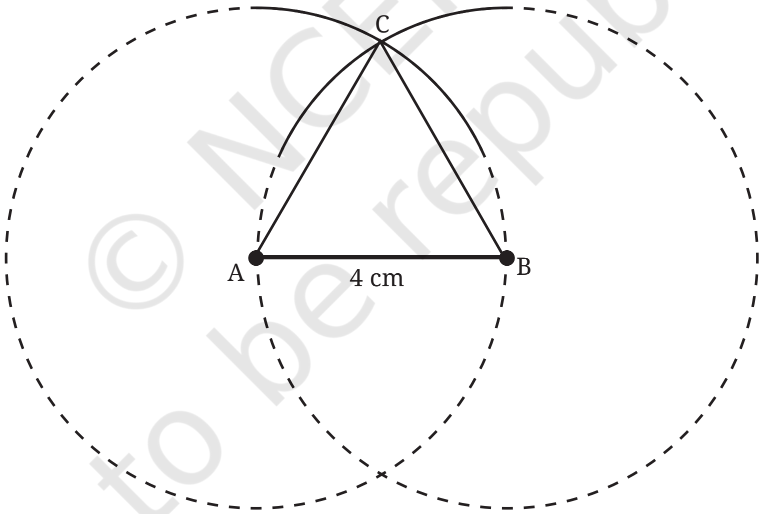
Step 2: Construct another arc of radius 4 cm from B.



Let C be the point of intersection of the arcs.

? The construction ensures that both AC and BC are of length 4 cm. Can you see why?

Step 3: Join AC and BC to get the required equilateral triangle.



7.2 Constructing a Triangle When its Sides are Given

How do we construct triangles that are not equilateral?

? Construct a triangle of sidelength 4 cm, 5 cm and 6 cm.

As in the previous case, this triangle can also be constructed using just a marked ruler. But it will involve several trials.

? How do we construct this triangle more efficiently?

Choose one of the given lengths to be the base of the triangle: say 4 cm. Draw the base. Let A and B be the base vertices, and call the third vertex C. Let AC = 5 cm and BC = 6 cm.

Fig. 7.1

Like we did in the case of equilateral triangles, let us first get all the points that are at a 5 cm distance from A. These points lie on the circle whose centre is A and has radius 5 cm. The point C must lie somewhere on this circle. How do we find it?

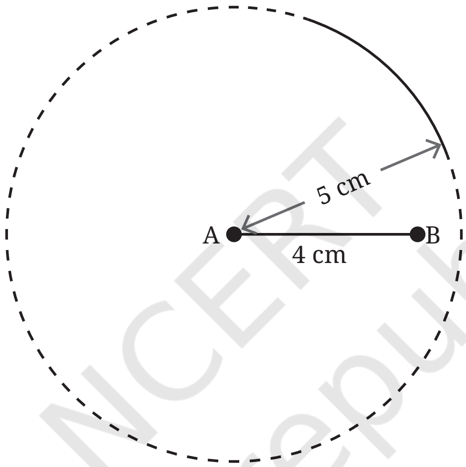


Fig. 7.2

We will make use of the fact that the point C is 6 cm away from B. Construct an arc of radius 6 cm from B.

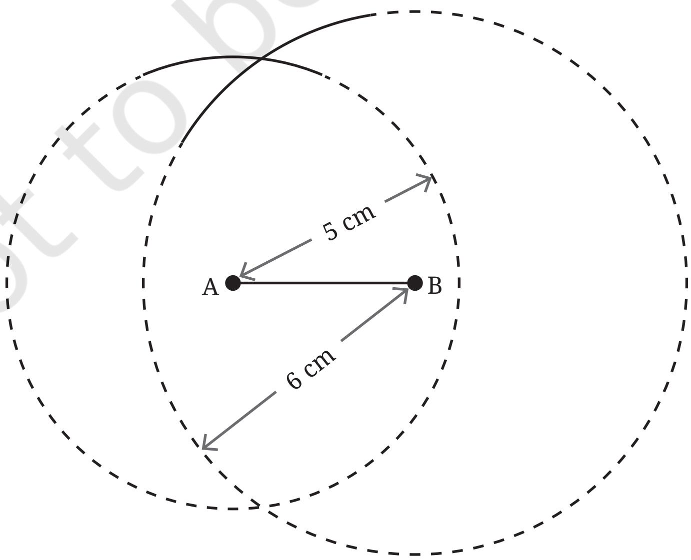
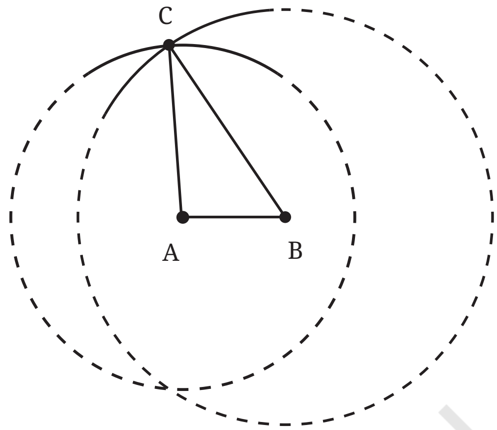


Fig. 7.3

The required point C is one of the points of intersection of the two circles.



The reason why the point of intersection is the third vertex is the same as for equilateral triangles. This point lies on both the circles. Hence its distance from A is the radius of the circle centred at A (5 cm) and its distance from B is the same as the radius of the circle centred at B (6 cm).

Let us summarise the steps of construction, noting that constructing full circles is not necessary to get the third vertex (see Fig. 7.2 and 7.3).

Step 1: Construct the base AB with one of the side lengths. Let us choose AB = 4 cm (see Fig. 7.1).

Step 2: From A, construct a sufficiently long arc of radius 5 cm (see Fig. 7.2).

Step 3: From B, construct an arc of radius 6 cm such that it intersects the first arc (see Fig. 7.3).

Step 4: The point where both the arcs meet is the required third vertex C. Join AC and BC to get ΔABC.

Construct

? Construct triangles having the following sidelengths (all the units are in cm):

(a) 4, 4, 6

(b) 3, 4, 5

(c) 1, 5, 5

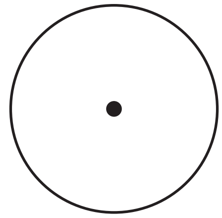
(d) 4, 6, 8

(e) 3.5, 3.5, 3.5

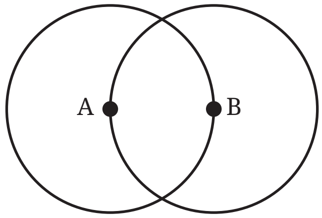
We have seen that triangles having all three equal sides are called equilateral triangles. Those having two equal sides are called **isosceles triangles**.

? Figure it Out

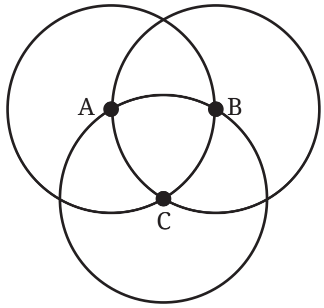
1. Use the points on the circle and/or the centre to form isosceles triangles.



2. Use the points on the circles and/or their centres to form isosceles and equilateral triangles. The circles are of the same size.



A and B are the centres of circles of the same size



A, B, and C are the centres of circles of the same size

Are Triangles Possible for any Lengths?

Can one construct triangles having any given sidelengths? Are there lengths for which it is impossible to construct a triangle? Let us explore this.

? Construct a triangle with sidelengths 3 cm, 4 cm, and 8 cm.

What is happening? Are you able to construct the triangle?

? Here is another set of lengths: 2 cm, 3 cm, and 6 cm. Check if a triangle is possible for these sidelengths.

? Try to find more sets of lengths for which a triangle construction is impossible. See if you can find any pattern in them.

Math Talk

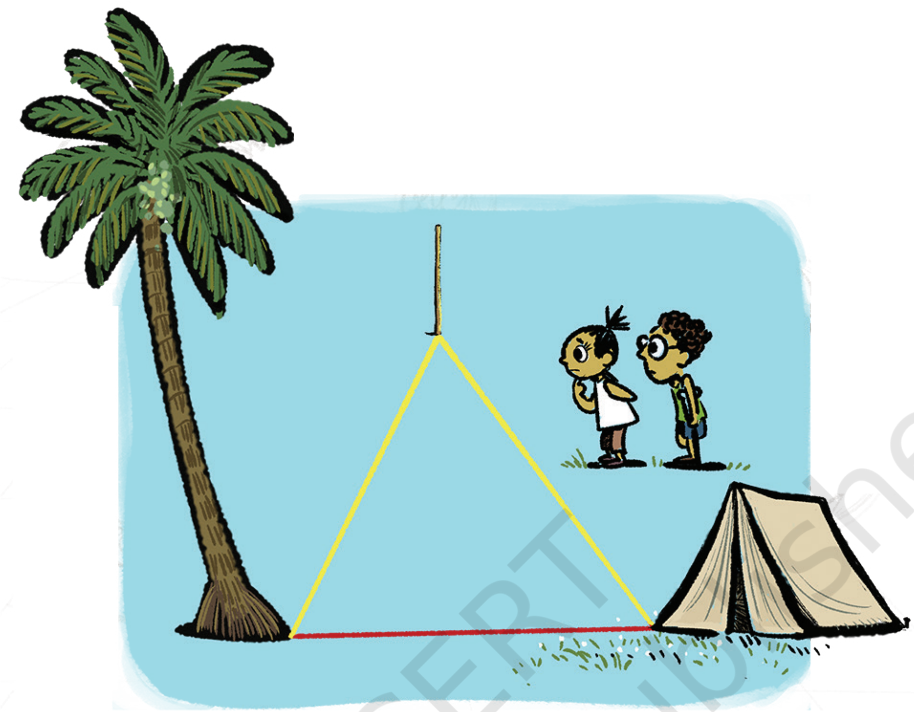
We see that a triangle is possible for some sets of lengths and not possible for others. How do we check if a triangle exists for a given set of lengths? One way is to actually try to construct the triangle and check if it is possible. Is there a more efficient way to check this?

Triangle Inequality

Consider the lengths 10 cm, 15 cm and 30 cm. Does there exist a triangle having these as sidelengths?

To tackle this question, let us study a property of triangles. Imagine a small plot of plain land having a tent, a tree, and a pole. Imagine you are at the entrance of the tent and want to go to the tree. Which is the shorter path: (i) the straight-line path to the tree (the red path) or (ii) the straight-line path from the tent to the pole, followed by the straight-line path from the pole to the tree (the yellow path)?

Clearly, the direct straight-line path from the tent to the tree is shorter than the roundabout path via the pole. In fact, the direct straight-line path is the shortest possible path to the tree from the tent.



Will the direct path between any two points be shorter than the roundabout path via a third point? Clearly, the answer is yes.

? Can this understanding be used to tell something about the existence of a triangle having sidelengths 10 cm, 15 cm and 30 cm?

Let us suppose that there is a triangle for this set of lengths. *Remember that at this point we are not sure about the existence of the triangle but we are only supposing that it exists*. Let us draw a rough diagram.

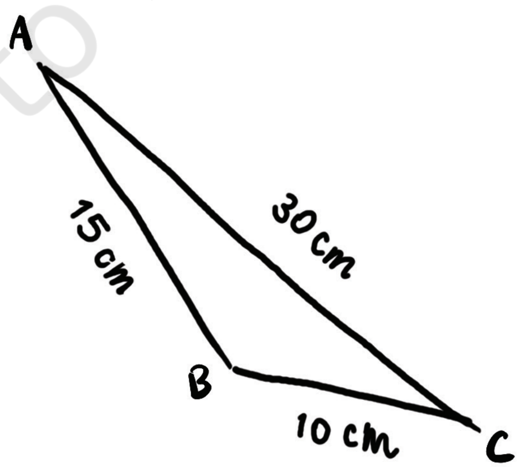


Fig. 7.4

Does everything look right with this triangle?

If this triangle were possible, then the direct path between any two vertices should be shorter than the roundabout path via the third vertex. Is this true for our rough diagram?

Let us consider the paths between B and C.

Direct path length = BC = 10 cm

What is the length of the roundabout path via the vertex A? It is the sum of the lengths of line segments BA and AC.

Roundabout path length = BA + AC = 15 cm + 30 cm = 45 cm

Is the direct path length shorter than the roundabout path length? Yes.

Let us now consider the paths between A and B.

Direct path length = AB = 15 cm

Finding the length of the roundabout path via the vertex C, we get

Roundabout path length = AC + CB = 30 cm + 10 cm = 40 cm

Is the direct path length shorter than the roundabout path length? Yes.
Finally, consider paths between C and A.

Direct path length = CA = 30 cm

Roundabout path length = CB + BA = 10 cm + 15 cm = 25 cm

Is the direct path length shorter than the roundabout path length? In this case, the direct path is longer, which is absurd. Can such a triangle exist? No.

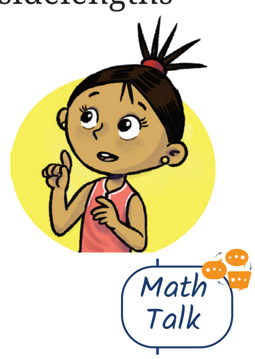
Therefore, a triangle having sidelengths 10 cm, 15 cm and 30 cm cannot exist.

We are thus able to see without construction why a triangle for the set of lengths 10 cm, 15 cm and 30 cm cannot exist. We have been able to figure this out through spatial intuition and reasoning.

Recall how we used similar intuition and reasoning to discover properties of intersecting and parallel lines. We will continue to do this as we explore geometry.

? Can we say anything about the existence of a triangle having sidelengths 3 cm, 3 cm and 7 cm? Verify your answer by construction.

? "In the rough diagram in Fig. 7.4, is it possible to assign lengths in a different order such that the direct paths are always coming out to be shorter than the roundabout paths? If this is possible, then a triangle might exist."



? Is such rearrangement of lengths possible in the triangle?

Figure it Out

1. We checked by construction that there are no triangles having sidelengths 3 cm, 4 cm and 8 cm; and 2 cm, 3 cm and 6 cm. Check if you could have found this without trying to construct the triangle.
2. Can we say anything about the existence of a triangle for each of the following sets of lengths?
   (a) 10 km, 10 km and 25 km
   (b) 5 mm, 10 mm and 20 mm
   (c) 12 cm, 20 cm and 40 cm

You would have realised that using a rough figure and comparing the direct path lengths with their corresponding roundabout path lengths is the same as comparing each length with the sum of the other two lengths. There are three such comparisons to be made.

Try This

3. For each set of lengths seen so far, you might have noticed that in at least two of the comparisons, the direct length was less than the sum of the other two (if not, check again!). For example, for the set of lengths 10 cm, 15 cm and 30 cm, there are two comparisons where this happens:

$$10 < 15 + 30$$

$$15 < 10 + 30$$

But this doesn't happen for the third length: $30 > 10 + 15$.

Will this always happen? That is, for any set of lengths, will there be at least two comparisons where the direct length is less than the sum of the other two? Explore for different sets of lengths.

Further, for a given set of lengths, is it possible to identify which lengths will immediately be less than the sum of the other two, without calculations?

[**Hint:** Consider the direct lengths in the increasing order.]

Given three sidelengths, what do we need to compare to check for the existence of a triangle?

When each length is smaller than the sum of the other two, we say that the lengths satisfy the **triangle inequality**. For example, the set 3, 4, 5 satisfies the triangle inequality whereas, the set 10, 15, 30 does not satisfy the triangle inequality.

We have seen that lengths such as 10, 15, 30 that do not satisfy the triangle inequality cannot be the sidelengths of a triangle.

Does a triangle exist with sidelengths 4 cm, 5 cm and 8 cm?

This satisfies the triangle inequality:

$$8 < 4 + 5 = 9$$

? Why do we not need to check the other two sides?

This means that all the direct path lengths are less than the roundabout path lengths. Does this confirm the existence of a triangle?

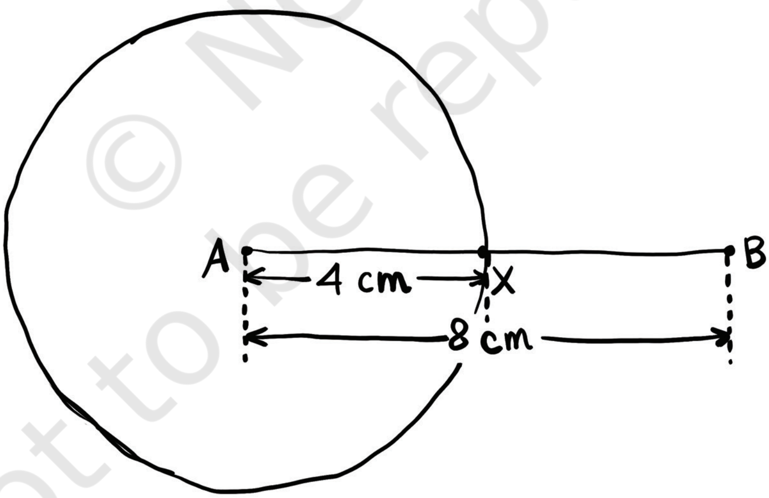
If one of the direct path lengths had been longer, we could have concluded that a triangle would surely not exist. But in this case, we can only say that a triangle may or may not exist.

For the triangle to exist, the arcs that we construct to get the third vertex must intersect. Is it possible to determine that this will happen without actually carrying out the construction?

Visualising the construction of circles

Let us imagine that we start the construction by constructing the longest side as the base. Let AB be the base of length 8 cm. The next step is the construction of sufficiently long arcs corresponding to the other two lengths: 4 cm and 5 cm.

Instead of just constructing the arcs, let us complete the full circles. Suppose, we construct a circle of radius 4 cm with A as the centre.



? Now, suppose that a circle of radius 5 cm is constructed, centred at B. Can you draw a rough diagram of the resulting figure?

Note that in the figure below, AX = 4 cm and AB = 8 cm. So, what is BX? Does this length help in visualising the resulting figure?

Since BX = 4 cm, and the radius of the circle centred at B is 5 cm, it is clear that the circles will intersect each other at two points.

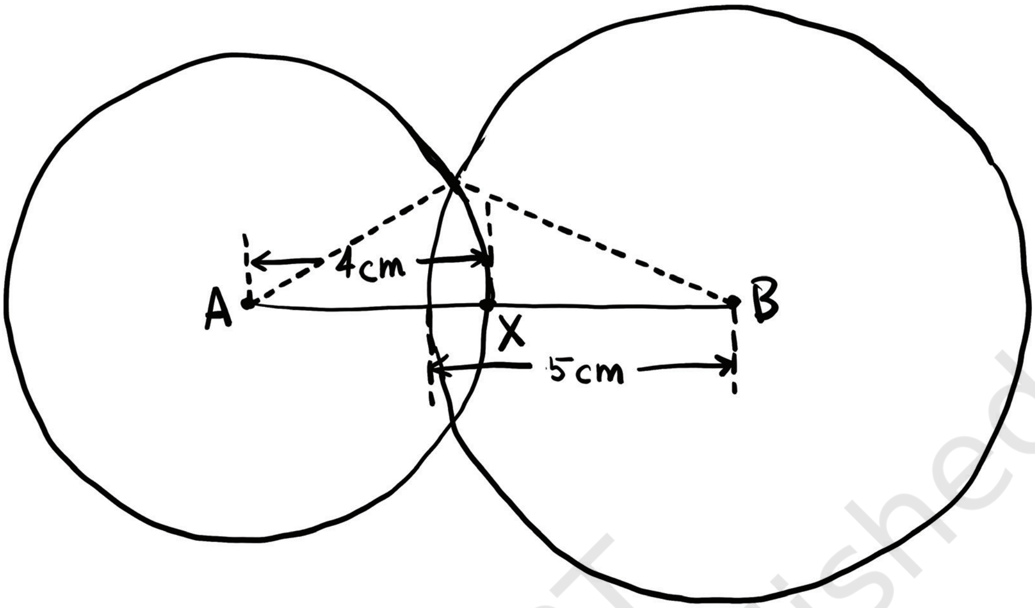


Fig. 7.5: Circles intersecting each other at two points

What does this tell us about the existence of a triangle? The points A and B along with either of the points of intersection of the circles will give us the required triangle. Thus, there exists a triangle having sidelengths 4 cm, 5 cm and 8 cm.

Figure it Out

1. Which of the following lengths can be the sidelengths of a triangle? Explain your answers. Note that for each set, the three lengths have the same unit of measure.

   (a) 2, 2, 5

   (b) 3, 4, 6

   (c) 2, 4, 8

   (d) 5, 5, 8

   (e) 10, 20, 25

   (f) 10, 20, 35

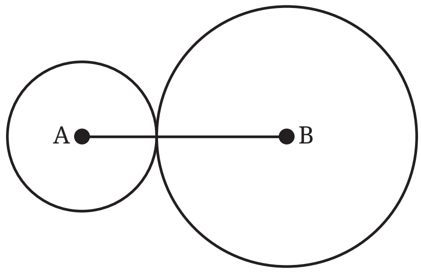
   (g) 24, 26, 28

We observe from the previous problems that **whenever there is a set of lengths satisfying the triangle inequality (each length < sum of the other two lengths), there is a triangle with those three lengths as sidelengths**.

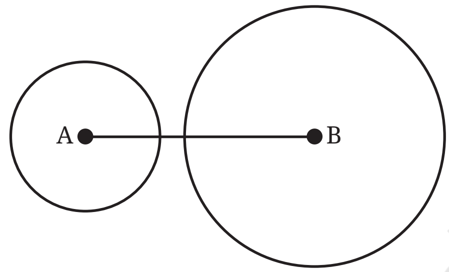
Will triangles always exist when a set of lengths satisfies the triangle inequality? How can we be sure?

We can be sure of the existence of a triangle only if we can show that the circles intersect internally (as in Fig. 7.5) whenever the triangle inequality is satisfied. But are there other possibilities when the two circles are constructed? Let us visualise and study them.

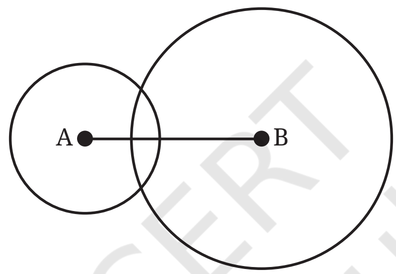
The following different cases can be conceived:



Case 1: Circles touch each other



Case 2: Circles do not intersect



Case 3: Circles intersect each other internally

Note that while constructing the circles, we take

(a) the length of the base AB = longest of the given length

(b) the radii of the circles to be the smaller two lengths.

Which of the above-mentioned cases will lead to the formation of a triangle? Clearly, triangles are formed only when the circles intersect each other internally (Case 3).

Let us study each of these cases by finding the relation between the radii (the smaller two lengths) and AB (longest length).

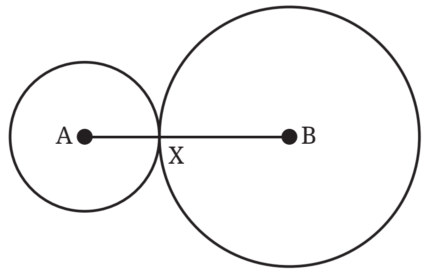
Case 1: Circles touch each other at a point

For this case to happen,

sum of the two radii = AB

or

sum of the two smaller lengths = longest length



Case 2: Circles do not intersect internally

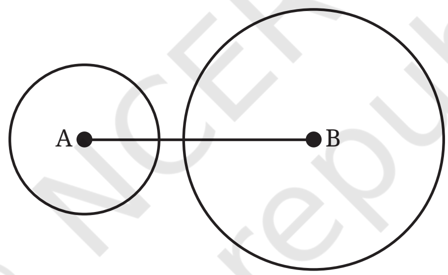
? For this case to happen, what should be the relation between the radii and AB?

It can be seen from the figure that,

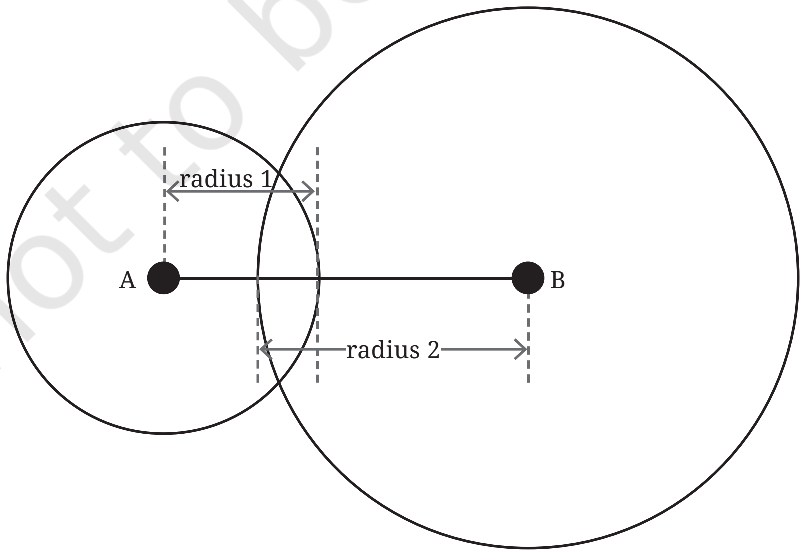
sum of the two radii < AB

or

sum of the the two smaller lengths < longest length



Case 3: Circles intersect each other



AB is composed of one radius and a part of the other. So,

sum of the two radii > AB,

or

sum of the two smaller lengths > longest length

Can we use this analysis to tell if a triangle exists when the lengths satisfy the triangle inequality?

If the given lengths satisfy the triangle inequality, then the sum of the two smaller lengths is greater than the longest length. This means that this will lead to Case 3 where the circles intersect internally, and so a triangle exists.

How will the two circles turn out for a set of lengths that do not satisfy the triangle inequality? Find 3 examples of sets of lengths for which the circles:

(a) touch each other at a point,
(b) do not intersect.

Frame a complete procedure that can be used to check the existence of a triangle.

Conclusion

If a given set of three lengths satisfies the triangle inequality, then a triangle exists having those as sidelengths. If the set does not satisfy the triangle inequality, then a triangle with those sidelengths does not exist.

Figure it Out

1. Check if a triangle exists for each of the following set of lengths:
   (a) 1, 100, 100 (b) 3, 6, 9
   (c) 1, 1, 5 (d) 5, 10, 12
2. Does there exist an equilateral triangle with sides 50, 50, 50? In general, does there exist an equilateral triangle of any sidelength? Justify your answer.
3. For each of the following, give at least 5 possible values for the third length so there exists a triangle having these as sidelengths (decimal values could also be chosen):
   (a) 1, 100
   (b) 5, 5
   (c) 3, 7

Try This

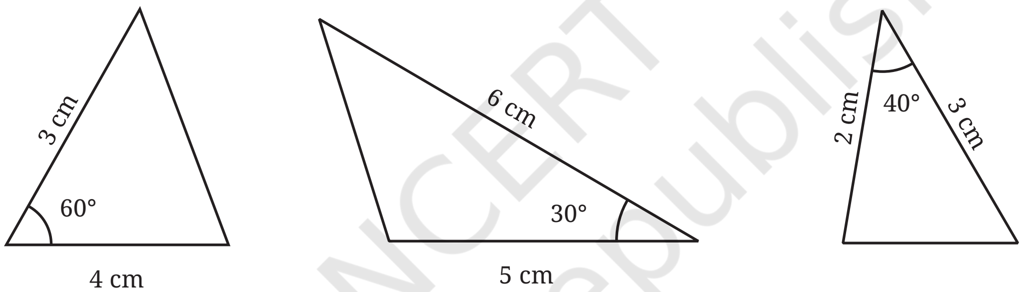
See if you can describe all possible lengths of the third side in each case, so that a triangle exists with those sidelengths. For example, in case (a), all numbers strictly between 99 and 101 would be possible.

7.3 Construction of Triangles When Some Sides and Angles are Given

We have seen how to construct triangles when their sidelengths are given. Now, we will take up constructions where in place of some sidelengths, angle measures are given.

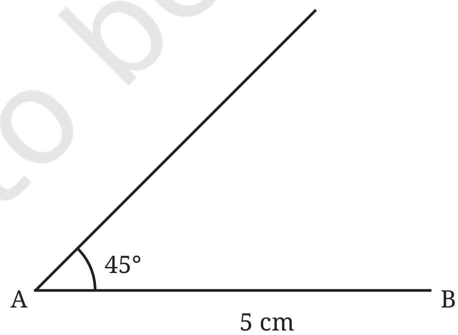
Two Sides and the Included Angle

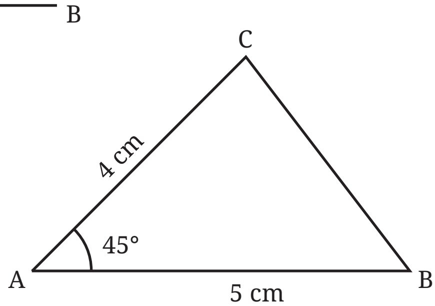
How do we construct a triangle if two sides and the angle included between them are given? Here are some examples of measurements showing the included angle.



? Construct a triangle ABC with AB = 5 cm, AC = 4 cm and ∠A= 45°.

Let us take one of the given sides, AB, as the base of the triangle.





Step 1: Construct a side AB of length 5 cm.
Step 2: Construct ∠A = 45° by drawing the other arm of the angle.
Step 3: Mark the point C on the other arm such that AC = 4 cm.
Step 4: Join BC to get the required triangle.

Figure it Out

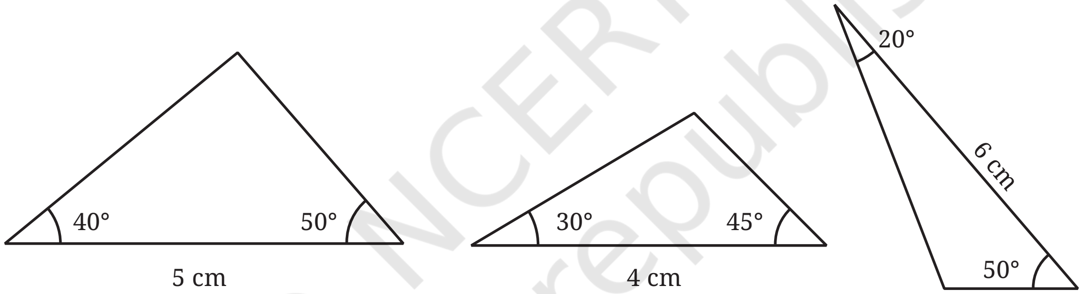
1. Construct triangles for the following measurements where the angle is included between the sides:
   (a) 3 cm, 75°, 7 cm
   (b) 6 cm, 25°, 3 cm
   (c) 3 cm, 120°, 8 cm

We have seen that triangles do not exist for all sets of sidelengths. Is there a combination of measurements in the case of two sides and the included angle where a triangle is not possible? Justify your answer using what you observe during construction.

Math Talk

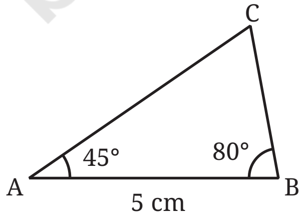
Two Angles and the Included Side

In this case, we are given two angles and the side that is a part of both angles, which we call the included side. Here are some examples of such measurements:



Construct a triangle ABC where AB = 5 cm, $\angle A = 45°$ and $\angle B = 80°$.

Let us take the given side as the base.



Step 1: Draw the base AB of length 5 cm.
Step 2: Draw $\angle A$ and $\angle B$ of measures 45°, and 80° respectively.
Step 3: The point of intersection of the two new line segments is the third vertex C.

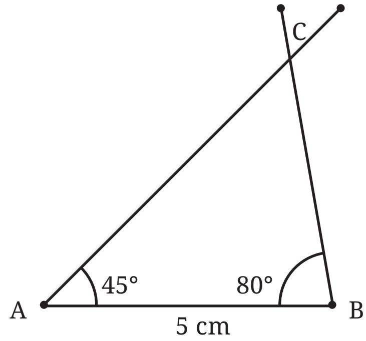


Figure it Out

1. Construct triangles for the following measurements:
   (a) 75°, 5 cm, 75°
   (b) 25°, 3 cm, 60°
   (c) 120°, 6 cm, 30°

Do triangles always exist?

Do triangles exist for every combination of two angles and their included side? Explore.

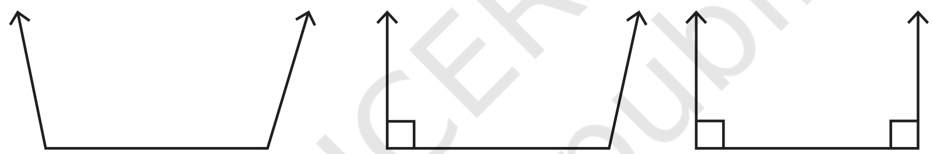
Math Talk

As in the case when we are given all three sides, it turns out that there is not always a triangle for every combination of two angles and the included side.

Find examples of measurements of two angles with the included side where a triangle is not possible.

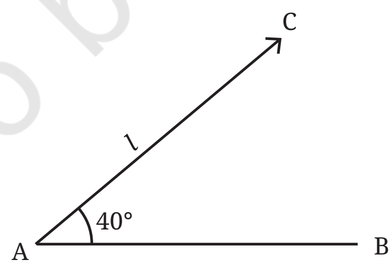
Let us try to visualise such a situation. Once the base is drawn, try to imagine how the other sides should be so that they do not meet.

Here are some obvious examples.



If the two angles are greater than or equal to a right angle (90°), then it is clear that a triangle is not possible.

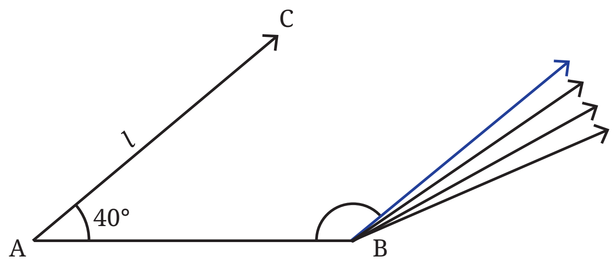
Now we make one of the base angles an acute angle, say 40°. What are the possible values that the other angle should take so that the lines don't meet?



It is clear that if the line from B is "inclined" sufficiently to the right, then it will not meet the line *l*.

(a) Try to find a possible ∠B (marked in the figure) for this to happen.

(b) What could be smallest value of ∠B for the lines to not meet?



The blue line is the line with the least rightward bend that doesn't meet the line '*l*'

Visually, it is clear that the line that creates the smallest ∠B has to be the one parallel to *l*. Let us call this parallel line m.

Can you tell the actual value of ∠B be in this case?

[**Hint:** Note that AB is the transversal.]

We have seen that when two lines are parallel, the internal angles on the same side of the transversal add up to 180°. So ∠B = 140°.

So, for what values of ∠B, does a triangle not exist? Does the length AB play any part here?

From the discussion above, it can be seen that the length AB does not play any part in deciding the existence of a triangle. We can say that a triangle does not exist when ∠B is greater than or equal to 140°.

Figure it Out

1. For each of the following angles, find another angle for which a triangle is (a) possible, (b) not possible. Find at least two different angles for each category:
   (a) 30°
   (b) 70°
   (c) 54°
   (d) 144°
2. Determine which of the following pairs can be the angles of a triangle and which cannot:
   (a) 35°, 150°
   (b) 70°, 30°
   (c) 90°, 85°
   (d) 50°, 150°

Like the triangle inequality, can you form a rule that describes the two angles for which a triangle is possible?

Can the sum of the two angles be used for framing this rule?

When the sum of two given angles is less than 180°, a triangle exists with these angles. If the sum is greater than or equal to 180°, there is no triangle with these angles.

Let us take two angles, say 60° and 70°, whose sum is less than 180°. Let the included side be 5 cm.

? What could the measure of the third angle be? Does this measure change if the base length is changed to some other value, say 7 cm? Construct and find out.

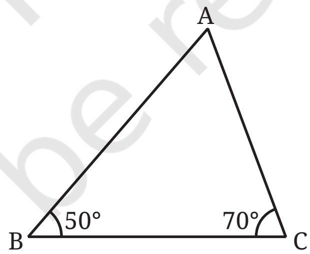
? In general, once the two angles are fixed, does the third angle depend on the included sidelength? Try with different pairs of angles and lengths.

The measurements might show that the sidelength has no effect (or a very small effect) on the third angle. With this observation, let us see if we can find the third angle without carrying out the construction and measurement.

? Try experimenting with different triangles to see if there is a relation between any two angles and the third one. To find this relation, what data will you keep track of and how will you organise the data you collect?

Math Talk

? Consider a triangle ABC with $\angle B = 50°$ and $\angle C = 70°$. Let us see how we can find $\angle A$ without construction.



We saw that the notion of parallel lines was useful to determine that the sum of any two angles of a triangle is less than 180°. Parallel lines can be used to find the third angle, $\angle BAC$ as well.

Let us suppose we construct a line XY parallel to BC through vertex A.

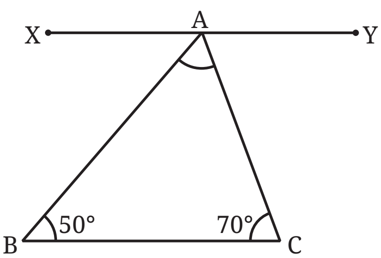


Fig. 7.6

? We can see new angles being formed here: $\angle XAB$, and $\angle YAC$. What are their values?

Since the line XY is parallel to BC,

$\angle XAB = \angle B$ and $\angle YAC = \angle C$, because they are alternate angles of the transversals AB and AC.

Therefore, $\angle XAB = 50°$, and $\angle YAC = 70°$. Can we find $\angle BAC$ from this? We know that $\angle XAB$, $\angle YAC$ and $\angle BAC$ together form 180°. So

$$\angle XAB + \angle YAC + \angle BAC = 180°$$

$$50° + \angle BAC + 70° = 180°$$

$$120° + \angle BAC = 180°$$

$$\text{Thus, } \angle BAC = 60°$$

Now construct a triangle (taking BC to be of any suitable length) and verify if this is indeed the case.

Figure it Out

1. Find the third angle of a triangle (using a parallel line) when two of the angles are:

   (a) 36°, 72°

   (b) 150°, 15°

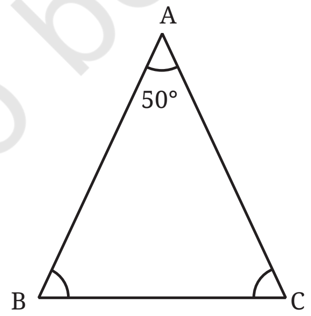
   (c) 90°, 30°

   (d) 75°, 45°

2. Can you construct a triangle all of whose angles are equal to 70°? If two of the angles are 70° what would the third angle be? If all the angles in a triangle have to be equal, then what must its measure be? Explore and find out.

3. Here is a triangle in which we know $\angle B = \angle C$ and $\angle A = 50°$. Can you find $\angle B$ and $\angle C$?

Try This



Angle Sum Property

What can we say about the sum of the angles of any triangle?

Consider a triangle ABC. To find the sum of its angles, we can use the same method of drawing a line parallel to the base: construct a line through A that is parallel to BC.

We need to find $\angle A + \angle B + \angle C$.

We know that $\angle B = \angle XAB$, $\angle C = \angle YAC$.

So, $\angle A + \angle B + \angle C = \angle A + \angle XAB + \angle YAC$

$= 180°$ as together they form a straight angle.

Thus we have proved that the sum of the three angles in any triangle is 180°! This rather surprising result is called the **angle sum property of triangles**.

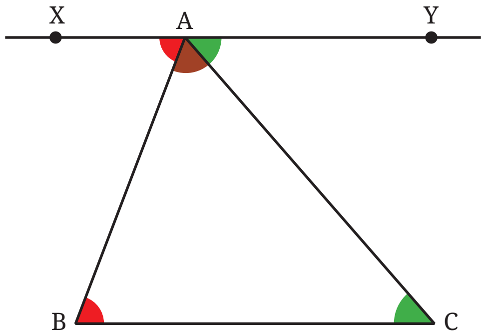
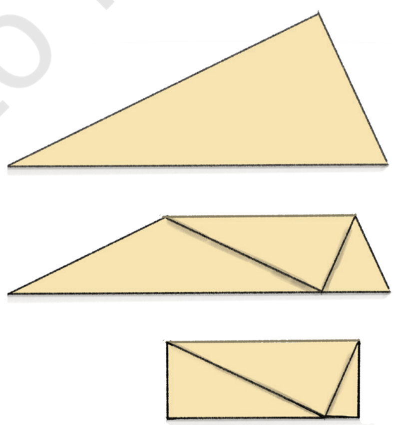


Fig. 7.7

Take some time to reflect upon how we figured out the angle sum property. In the beginning, the relationship between the third angle and the other two angles was not at all clear. However, a simple idea of drawing a line parallel to the base through the top vertex (as in Fig. 7.7) suddenly made the relationship obvious. This ingenious idea can be found in a very influential book in the history of mathematics called 'The Elements'. This book is attributed to the Greek mathematician Euclid, who lived around 300 BCE.

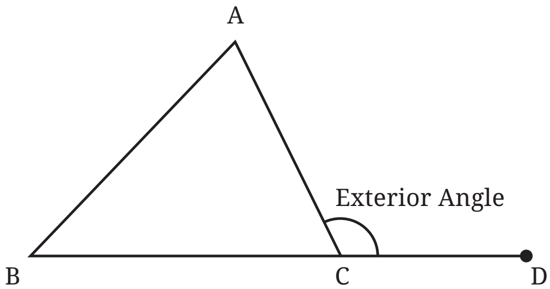
This solution is yet another example of how creative thinking plays a key role in mathematics!

There is a convenient way of verifying the angle sum property by folding a triangular cut-out of a paper. Do you see how this shows that the sum of the angles in this triangle is 180°?



Exterior Angles

The angle formed between the extension of a side of a triangle and the other side is called an **exterior angle** of the triangle. In this figure, $\angle ACD$ is an exterior angle.



Find $\angle ACD$, if $\angle A = 50°$, and $\angle B = 60°$.

From the angle sum property, we know that

$$50° + 60° + \angle ACB = 180°$$

$$110° + \angle ACB = 180°$$

So, $\angle ACB = 70°$
So, $\angle ACD = 180° - 70° = 110°$,
since, $\angle ACB$ and $\angle ACD$ together form a straight angle.

Find the exterior angle for different measures of $\angle A$ and $\angle B$. Do you see any relation between the exterior angle and these two angles?
[Hint: From angle sum property, we have $\angle A + \angle B + \angle ACB = 180°$.]
We also have $\angle ACD + \angle ACB = 180°$, since they form a straight angle.
What does this show?

7.4 Constructions Related to Altitudes of Triangles

There is another set of useful measurements with respect to a triangle — the height of each of its vertices with respect to the opposite sides.

In the world around us, we talk of the heights of various objects: the height of a person, the height of a tree, the height of a building, etc. What do we mean by the word 'height'?

Consider a triangle ABC. What is the height of the vertex A from its opposite side BC, and how can it be measured?

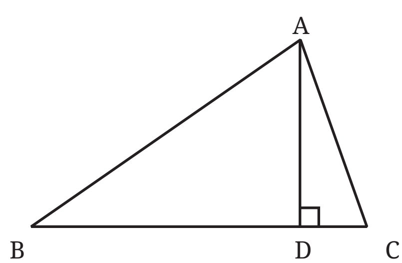
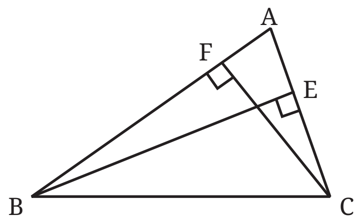


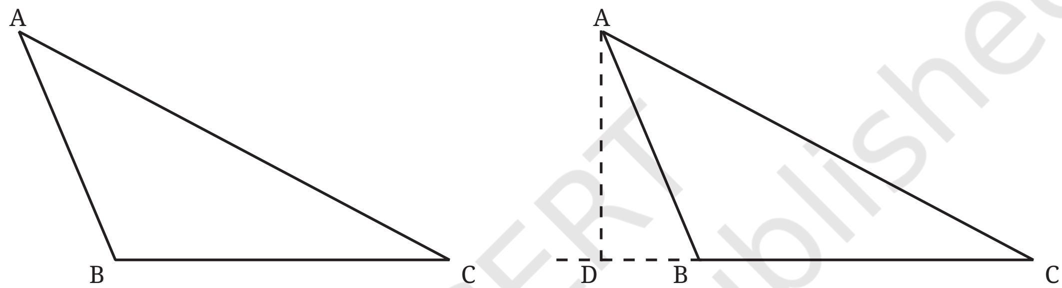
Fig. 7.8

Let AD be the line segment from A drawn perpendicular to BC. The length of AD is the height of the vertex A from BC. The line segment AD is said to be one of the 'altitudes' of the triangle. The other altitudes are BE and CF in the figure below: the perpendiculars drawn from the other vertices to their respective opposite sides.



Whenever we use the word height of the triangle, we generally refer to the length of the altitude to whatever side we take as base (this altitude is AD in the case of Fig. 7.8).

What would the altitude from A to BC be in this triangle?



We extend BC and then drop the perpendicular from A to this line.

Altitudes Using Paper Folding

? Cut out a paper triangle. Fix one of the sides as the base. Fold it in such a way that the resulting crease is an altitude from the top vertex to the base. Justify why the crease formed should be perpendicular to the base.

Construction of the Altitudes of a Triangle

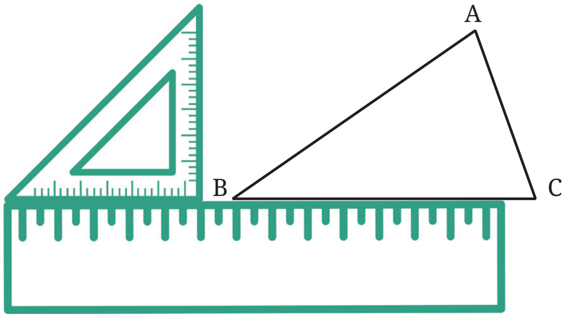
Construct an arbitrary triangle. Label the vertices A, B, C taking BC to be the base.

? Construct the altitude from A to BC,

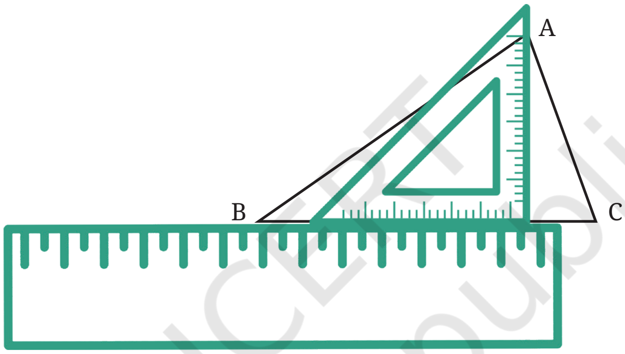
Constructing the altitude using just a ruler is not accurate. To get a more precise angle of 90°, we use a set square along with a ruler.

? Can you see how to do this?

Step 1: Keep the ruler aligned to the base. Place the set square on the ruler as shown, such that one of the edges of the right angle touches the ruler.



Step 2: Slide the set square along the ruler till the vertical edge of the set square touches the vertex A.



Step 3: Draw the altitude to BC through A using the vertical edge of the set square.

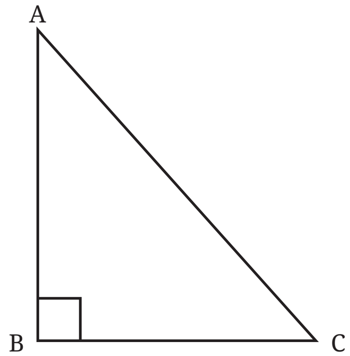
? Does there exist a triangle in which a side is also an altitude?

Visualise such a triangle and draw a rough diagram.

We see that this happens in triangles where one of the angles is a right angle.

Triangles having one right angle are called **right-angled triangles** or simply **right triangles**.

Altitude from A to BC



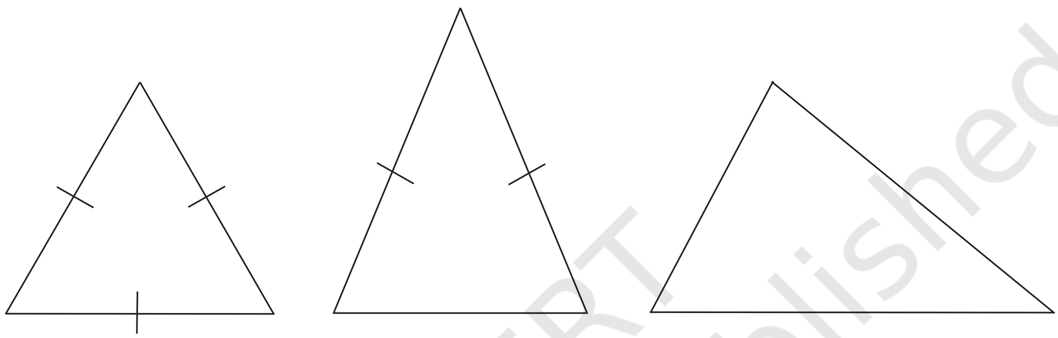
7.5 Types of Triangles

In our study of triangles, we have encountered the following types of triangles; equilateral, isosceles, scalene and right-angled triangles.

? Did you spot any other type of triangle?

The classification of triangles as equilateral and isosceles was based on equality of sides.

Equilateral triangles have sides of equal length. Isosceles triangles have two sides of equal length. Scalene triangles have sides of three different lengths.



Equilateral Triangle   Isosceles Triangle   Scalene Triangle

Can a similar classification be done based on equality of angles? Is there any relation between these two classifications? We will answer these questions in a later chapter.

We used angle measures when classifying a triangle as a right-angled triangle.

? What are the other types of triangles based on angle measures?

A classification of triangles based on their angle measures is acute-angled, right-angled and obtuse-angled triangles. We have already seen what a right-angled triangle is. It is a triangle with one right angle. Similarly, an obtuse-angled triangle has one obtuse angle.

? What could an acute-angled triangle be? Can we define it as a triangle with one acute angle? Why not?

Math Talk

In an **acute-angled** triangle, all three angles are acute angles.

? Figure it Out

1. Construct a triangle ABC with BC = 5 cm, AB = 6 cm, CA = 5 cm. Construct an altitude from A to BC.
2. Construct a triangle TRY with RY = 4 cm, TR = 7 cm, $\angle R = 140°$. Construct an altitude from T to RY.

3. Construct a right-angled triangle ΔABC with $\angle B = 90°$, $AC = 5$ cm. How many different triangles exist with these measurements?

Try This

[**Hint:** Note that the other measurements can take any values. Take AC as the base. What values can $\angle A$ and $\angle C$ take so that the other angle is 90°?]

4. Through construction, explore if it is possible to construct an equilateral triangle that is (i) right-angled (ii) obtuse-angled. Also construct an isosceles triangle that is (i) right-angled (ii) obtuse-angled.

SUMMARY

- Use of a compass simplifies the construction of triangles when the sidelengths are given.
- A set of three lengths where the length of each is smaller than the sum of the other two is said to satisfy the triangle inequality.
- Sidelengths of a triangle satisfy triangle inequality, and, if a given set of lengths satisfy the triangle inequality, a triangle can be constructed with those sidelengths.
- Triangles can be constructed when the following measurements are given:
  (a) two of the sides and their included angle
  (b) two angles and the included side
- The sum of the angles of a triangle is always 180°.
- An altitude of a triangle is a perpendicular line segment from a vertex to its opposite side.
- Equilateral triangles have sides of equal length. Isosceles triangles have two sides of equal length. Scalene triangles have sides of three different lengths.
- Triangles are classified based on their angle measures as acute-angled, right-angled and obtuse-angled triangles.

Shortest Path in a Box!

There is a spider in a corner of a box. It wants to reach the farthest opposite corner (marked in the figure). Since it cannot fly, it can reach the opposite point only by walking on the surfaces of the box. What is the shortest path it can take?

Take a cardboard box and mark the path that you think is the shortest from one corner to its opposite corner. Compare the length of this path with that of the paths made by your friends.

Hint:

2 ARITHMETIC EXPRESSIONS

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2.1 Simple Expressions

You may have seen mathematical phrases like 13 + 2, 20 – 4, 12 × 5, and 18 ÷ 3. Such phrases are called **arithmetic expressions**.

Every arithmetic expression has a value which is the number it evaluates to. For example, the value of the expression 13 + 2 is 15. This expression can be read as '13 plus 2' or 'the sum of 13 and 2'.

We use the equality sign '=' to denote the relationship between an arithmetic expression and its value. For example:

$$13 + 2 = 15.$$

Example 1: Mallika spends ₹25 every day for lunch at school. Write the expression for the total amount she spends on lunch in a week from Monday to Friday.

The expression for the total amount is 5 × 25.
5 × 25 is "5 times 25" or "the product of 5 and 25".

Different expressions can have the same value. Here are multiple ways to express the number 12, using two numbers and any of the four operations +, – , × and ÷:

$$10 + 2,\ 15 - 3,\ 3 \times 4,\ 24 \div 2.$$

Choose your favourite number and write as many expressions as you can having that value.

Comparing Expressions

As we compare numbers using '=', '<' and '>' signs, we can also compare expressions. We compare expressions based on their values and write the 'equal to', 'greater than' or 'less than' sign accordingly. For example,

$$10 + 2 > 7 + 1$$

because the value of 10 + 2 = 12 is greater than the value of 7 + 1 = 8. Similarly,

$$13 - 2 < 4 \times 3.$$

Figure it Out

1. Fill in the blanks to make the expressions equal on both sides of the = sign:

   (a) 13 + 4 = ____ + 6   (b) 22 + ____ = 6 × 5

   (c) 8 × ____ = 64 ÷ 2   (d) 34 – ____ = 25

2. Arrange the following expressions in ascending (increasing) order of their values.

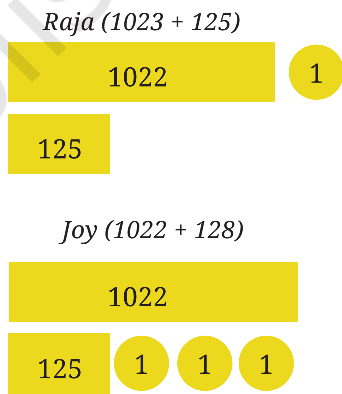
   (a) 67 – 19   (b) 67 – 20

   (c) 35 + 25   (d) 5 × 11

   (e) 120 ÷ 3

Example 2: Which is greater? 1023 + 125 *or* 1022 + 128?

Imagining a situation could help us answer this without finding the values. Raja had 1023 marbles and got 125 more today. Now he has 1023 + 125 marbles. Joy had 1022 marbles and got 128 more today. Now he has 1022 + 128 marbles. Who has more?

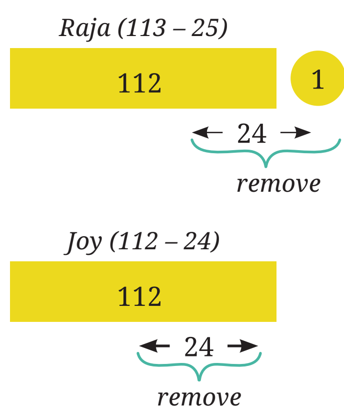


This situation can be represented as shown in the picture on the right. To begin with, Raja had 1 more marble than Joy. But Joy got 3 more marbles than Raja today. We can see that Joy has (two) more marbles than Raja now.
That is,

$$1023 + 125 < 1022 + 128.$$

Example 3: Which is greater? 113 – 25 *or* 112 – 24?

Imagine a situation, Raja had 113 marbles and lost 25 of them. He has 113 – 25 marbles. Joy had 112 marbles and lost 24 today. He has 112 – 24 marbles. Who has more marbles left with them?



Raja had 1 marble more than Joy. But he also lost 1 marble more than Joy did. Therefore, they have an equal number of marbles now.
That is,

$$113 - 25 = 112 - 24.$$

Use '>' *or* '<' or '=' in each of the following expressions to compare them. Can you do it without complicated calculations? Explain your thinking in each case.

(a) 245 + 289 ☐ 246 + 285

(b) 273 – 145 ☐ 272 – 144

(c) 364 + 587 ☐ 363 + 589

(d) 124 + 245 ☐ 129 + 245

(e) 213 – 77 ☐ 214 – 76

2.2 Reading and Evaluating Complex Expressions

Sometimes, when an expression is not accompanied by a context, there can be more than one way of evaluating its value. In such cases, we need some tools and rules to specify how exactly the expression has to be evaluated.

To give an example with language, look at the following sentences:

(a) Sentence: "Shalini sat next to a friend with toys".

Meaning: The friend has toys and Shalini sat next to her.

(b) Sentence: "Shalini sat next to a friend, with toys".

Meaning: Shalini has the toys and she sat with them next to her friend.

This sentence without the punctuation could have been interpreted in two different ways. The appropriate use of a comma specifies how the sentence has to be understood.

Let us see an expression that can be evaluated in more than one way.

Example 4: Mallesh brought 30 marbles to the playground. Arun brought 5 bags of marbles with 4 marbles in each bag. How many marbles did Mallesh and Arun bring to the playground?

Mallesh summarized this by writing the mathematical expression —

$$30 + 5 \times 4.$$

Without knowing the context behind this expression, Purna found the value of this expression to be 140. He added 30 and 5 first, to get 35, and then multiplied 35 by 4 to get 140.

Mallesh found the value of this expression to be 50. He multiplied 5 and 4 first to get 20 and added 20 to 30 to get 50.

In this case, Mallesh is right. But why did Purna get it wrong?

Just looking at the expression 30 + 5 × 4, it is not clear whether we should do the addition first or multiplication.

Just as punctuation marks are used to resolve confusions in language, brackets and the notion of terms are used in mathematics to resolve confusions in evaluating expressions.

Brackets in Expressions

In the expression to find the number of marbles — 30 + 5 × 4 — we had to first multiply 5 and 4, and then add this product to 30. This order of operations is clarified by the use of brackets as follows:

$$30 + (5 \times 4).$$

When evaluating an expression having brackets, we need to first find the values of the expressions inside the brackets before performing other operations. So, in the above expression, we first find the value of 5 × 4, and then do the addition. Thus, this expression describes the number of marbles:

$$30 + (5 \times 4) = 30 + 20 = 50.$$

Example 5: Irfan bought a pack of biscuits for ₹15 and a packet of *toor dal* for ₹56. He gave the shopkeeper ₹100. Write an expression that can help us calculate the change Irfan will get back from the shopkeeper.

Irfan spent ₹15 on a biscuit packet and ₹56 on *toor dal*. So, the total cost in rupees is 15 + 56. He gave ₹100 to the shopkeeper. So, he should get back 100 minus the total cost. Can we write that expression as—

$$100 - 15 + 56\ ?$$

Can we first subtract 15 from 100 and then add 56 to the result? We will get 141. It is absurd that he gets more money than he paid the shopkeeper!

We can use brackets in this case:

$$100 - (15 + 56).$$

Evaluating the expression within the brackets first, we get 100 minus 71, which is 29. So, Irfan will get back ₹29.

Terms in Expressions

Suppose we have the expression 30 + 5 × 4 without any brackets. Does it have no meaning?

When there are expressions having multiple operations, and the order of operations is not specified by the brackets, we use the notion of terms to determine the order.

Terms are the parts of an expression separated by a '+' sign. For example, in 12+7, the terms are 12 and 7, as marked below.

$$12 + 7 = 12 + 7$$

We will keep marking each term of an expression as above. Note that this way of marking the terms is not a usual practice. This will be done until you become familiar with this concept.

Now, what are the terms in 83 – 14? We know that subtracting a number is the same as adding the inverse of the number. Recall that the inverse of a given number has the sign opposite to it. For example, the inverse of 14 is –14, and the inverse of –14 is 14. Thus, subtracting 14 from 83 is the same as adding –14 to 83. That is,

$$83 - 14 = 83 + -14$$

Thus, the terms of the expression 83 – 14 are 83 and –14.

? Check if replacing subtraction by addition in this way does not change the value of the expression, by taking different examples.

? Can you explain why subtracting a number is the same as adding its inverse, using the Token Model of integers that we saw in the Class 6 textbook of mathematics?

Try This

All subtractions in an expression are converted to additions in this manner to identify the terms.

Here are some more examples of expressions and their terms:

$$-18 - 3 = -18 + -3$$

$$6 \times 5 + 3 = 6 \times 5 + 3$$

$$2 - 10 + 4 \times 6 = 2 + -10 + 4 \times 6$$

Note that 6 × 5, 4 × 6 are single terms as they do not have any '+' sign.

In the following table, some expressions are given. Complete the table.

Expression	Expression as the sum of its terms	Terms
13 – 2 + 6	13 + – 2 + 6	13, – 2, 6
5 + 6 × 3	5 + 6 × 3	
4 + 15 – 9	+ +	
23 – 2 × 4 + 16	+ +	
28 + 19 – 8	+ +	

Now we will see how terms are used to determine the order of operations to find the value of an expression.

We will start with expressions having only additions (with all the subtractions suitably converted into additions).

? Does changing the order in which the terms are added give different values?

Swapping and Grouping

Let us consider a simple expression having only two terms.

? **Example 6:** Madhu is flying a drone from a terrace. The drone goes 6 m up and then 4 m down. Write an expression to show how high the final position of the drone is from the terrace.

The drone is 6 – 4 = 2 m above the terrace. Writing it as sum of terms:

$$6 + (-4) = 2$$

Will the sum change if we swap the terms?

$$(-4) + 6 = 2$$

It doesn't in this case.

We already know that swapping the terms does not change the sum when both the terms are positive numbers.

? Will this also hold when there are terms having negative numbers as well? Take some more expressions and check.

? Can you explain why this is happening using the Token Model of integers that we saw in the Class 6 textbook of mathematics? *Try This*

Thus, in an expression having two terms, swapping them does not change the value.

Term 1 + Term 2 = Term 2 + Term 1

Now consider an expression having three terms: (–7) + 10 + (–11). Let us add these terms in the following two different orders:

– 7 + 10 + – 11

(adding the first two terms and then adding their sum to the third term)

– 7 + 10 + – 11

(adding the last two terms and then adding their sum to the first term)

What do you see? The sums are the same in both cases.

Again, we know that while adding positive numbers, grouping them in any of the above two ways gives the same sum.

? Will this also hold when there are terms having negative numbers as well? Take some more expressions and check.

? Can you explain why this is happening using the Token Model of integers that we saw in the Class 6 textbook of mathematics? *Try This*

Thus, grouping the terms of an expression in either of the following ways gives the same value.

Term 1 + Term 2 + Term 3 = Term 1 + Term 2 + Term 3

Let us consider the expression (–7) + 10 + (–11) again. What happens when we change the order and add –7 and –11 first, and then add this sum to 10? Will we get the same sum as before?

We see that adding the terms of the expression (–7) + 10 + (–11) in any order gives the same sum of –8.

Does adding the terms of an expression in any order give the same value? Take some more expressions and check. Consider expressions with more than 3 terms also.

Can you explain why this is happening using the Token Model of integers that we saw in the Class 6 textbook of mathematics?

Try This

Thus, the addition of terms in any order gives the same value. Therefore, in an expression having only additions, it does not matter in what order the terms are added: they all give the same value.

Now let us consider expressions having multiplication and division also, without the order of operations specified by the brackets. The values of such expressions are found by first evaluating the terms. Once all the terms are evaluated, they are added.

For example, the expression 30 + 5 × 4 is evaluated as follows:

$$30 + 5 \times 4 = 30 + 5 \times 4 = 30 + 20 = 50$$

The expression 5 × (3 + 2) + 78 + 3 is evaluated as follows:

$$5 \times (3 + 2) + 78 + 3 = 5 \times (3 + 2) + 7 \times 8 + 3$$

Where (3+2) is first evaluated and this sum is multiplied by 5 (= 25). The expression 7 × 8 is evaluated (= 56). This simplifies to 25 + 56 + 3 = 84.

Manasa is adding a long list of numbers. It took her five minutes to add them all and she got the answer 11749. Then she realised that she had forgotten to include the fourth number 9055. Does she have to start all over again?

1342
774
8611
9055
1022

In mathematics we use the phrase **commutative property** of **addition** instead of saying "swapping terms does not change the sum". Similarly, "grouping does not change the sum" is called the **associative property of addition**.

Swapping the Order of Things in Everyday Life

Manasa is going outside to play. Her mother says, "Wear your hat and shoes!" Which one should she wear first? She can wear her hat first and then her shoes. Or she can wear her shoes first and then her hat.

Manasa will look exactly the same in both cases. Imagine a different situation: Manasa's mother says "Wear your socks and shoes!" Now the order matters. She should wear socks and then shoes. If she wears shoes and then socks, Manasa will feel very uncomfortable and look very different.

More Expressions and Their Terms

Example 7: Amu, Charan, Madhu, and John went to a hotel and ordered four dosas. Each dosa cost ₹23, and they wish to thank the waiter by tipping ₹5. Write an expression describing the total cost.

Cost of 4 dosas = 4×23

Can the total amount with tip be written as $4 \times 23 + 5$? Evaluating it, we get

$$4 \times 23 + 5 = 4 \times 23 + 5 = 92 + 5 = 97$$

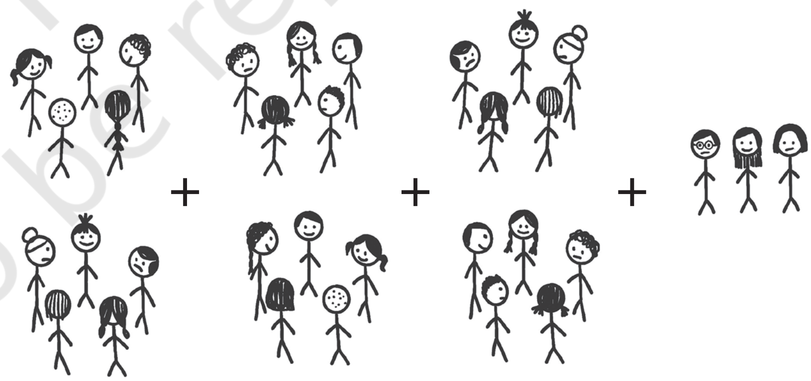
Thus, $4 \times 23 + 5$ is a correct way of writing the expression.

If the total number of friends goes up to 7 and the tip remains the same, how much will they have to pay? Write an expression for this situation and identify its terms.

Example 8: Children in a class are playing "Fire in the mountain, run, run, run!". Whenever the teacher calls out a number, students are supposed to arrange themselves in groups of that number. Whoever is not part of the announced group size, is out.

Ruby wanted to rest and sat on one side. The other 33 students were playing the game in the class.

The teacher called out '5'. Once children settled, Ruby wrote $6 \times 5 + 3$ (understood as 3 more than 6×5)



Think and discuss why she wrote this.

The expression written as a sum of terms is—

$$6 \times 5 + 3.$$

For each of the cases below, write the expression and identify its terms:

If the teacher had called out '4', Ruby would write ____________

If the teacher had called out '7', Ruby would write ____________

Write expressions like the above for your class size.

Example 9: Raghu bought 100 kg of rice from the wholesale market and packed them into 2 kg packets. He already had four 2 kg packets. Write an expression for the number of 2 kg packets of rice he has now and identify the terms.

He had 4 packets. The number of new 2 kg packets of rice is 100 ÷ 2, which we also write as $\frac{100}{2}$.

The number of 2 kg packets he has now is $4 + \frac{100}{2}$. The terms are—

$$4 + \frac{100}{2}.$$

Example 10: Kannan has to pay ₹432 to a shopkeeper using coins of ₹1 and ₹5, and notes of ₹10, ₹20, ₹50 and ₹100. How can he do it?

There is more than one possibility. For example,

$$432 = 4 \times 100 + 1 \times 20 + 1 \times 10 + 2 \times 1$$

Meaning: 4 notes of ₹100, 1 note of ₹20, 1 note of ₹10 and 2 notes of ₹1

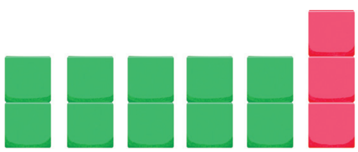
$$432 = 8 \times 50 + 1 \times 10 + 4 \times 5 + 2 \times 1$$

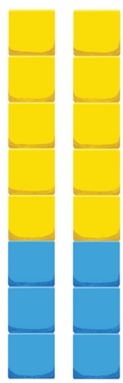
Meaning: 8 notes of ₹50, 1 note of ₹10, 4 notes of ₹5 and 2 notes of ₹1

Identify the terms in the two expressions above.

Can you think of some more ways of giving ₹432 to someone?

Example 11: Here are two pictures. Which of these two arrangements matches with the expression 5 × 2 + 3?





Let us write this expression as a sum of terms.

$$5 \times 2 + 3 = 10 + 3 = 13$$

This expression $5 \times 2 + 3$ can be understood as 3 more than 5×2, which describes the arrangement on the left.

? What is the expression for the arrangement in the right making use of the number of yellow and blue squares?

Do you recall the use of brackets? We need to use brackets for this.

$$2 \times (5 + 3)$$

Notice that this arrangement can also be described using—

$$5 + 3 + 5 + 3$$

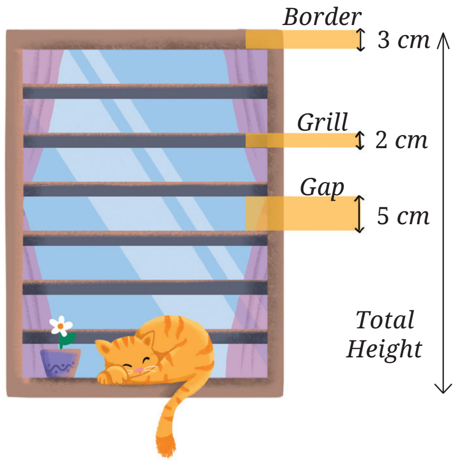
OR

$$5 \times 2 + 3 \times 2$$

? Figure it Out

1. Find the values of the following expressions by writing the terms in each case.
   (a) $28 - 7 + 8$ (b) $39 - 2 \times 6 + 11$
   (c) $40 - 10 + 10 + 10$ (d) $48 - 10 \times 2 + 16 \div 2$
   (e) $6 \times 3 - 4 \times 8 \times 5$
2. Write a story/situation for each of the following expressions and find their values.
   (a) $89 + 21 - 10$ (b) $5 \times 12 - 6$
   (c) $4 \times 9 + 2 \times 6$
3. For each of the following situations, write the expression describing the situation, identify its terms and find the value of the expression.
   (a) Queen Alia gave 100 gold coins to Princess Elsa and 100 gold coins to Princess Anna last year. Princess Elsa used the coins to start a business and doubled her coins. Princess Anna bought jewellery and has only half of the coins left. Write an expression describing how many gold coins Princess Elsa and Princess Anna together have.
   (b) A metro train ticket between two stations is ₹40 for an adult and ₹20 for a child. What is the total cost of tickets:
      (i) for four adults and three children?
      (ii) for two groups having three adults each?

(c) Find the total height of the window by writing an expression describing the relationship among the measurements shown in the picture.



Removing Brackets — I

Let us find the value of this expression,

$$200 - (40 + 3).$$

We first evaluate the expression inside the bracket to 43 and then subtract it from 200. But it is simpler to first subtract 40 from 200:

$$200 - 40 = 160.$$

And then subtract 3 from 160:

$$160 - 3 = 157.$$

What we did here was 200 – 40 – 3. Notice, that we did not do

$$200 - 40 + 3.$$

So,

$$200 - (40 + 3) = 200 - 40 - 3.$$

Example 12: We also saw this earlier in the case of Irfan purchasing a biscuit packet (₹15) and a *toor dal* packet (₹56). When he paid ₹100, the change he gets in rupees is:

$$100 - (15 + 56) = 29.$$

The change could also have been calculated as follows:

(a) First subtract the cost of the biscuit packet (15) from 100:

$$100 - 15 = 85.$$

This is the amount the shopkeeper owes Irfan if he had purchased only the biscuits. As he has purchased *toor dal* also, its cost is taken from this remaining amount of 85.

(b) So, to find the change, we need to subtract the cost of *toor dal* from 85.

$$85 - 56 = 29.$$

What we have done here is 100 – 15 – 56. So,

$$100 - (15 + 56) = 100 - 15 - 56.$$

Notice how upon **removing the brackets preceded by a negative sign**, the signs of the terms inside the brackets change. Observe the signs of 40 and 3 in the first example, and that of 15 and 56 in the second.

Example 13: Consider the expression 500 – (250 – 100). Is it possible to write this expression without the brackets?

To evaluate this expression, we need to subtract 250 – 100 = 150 from 500:

$$500 - (250 - 100) = 500 - 150 = 350.$$

If we were to directly subtract 250 from 500, then we would have subtracted 100 more than what we needed to. So, we should add back that 100 to 500 – 250 to make the expression take the same value as 500 – (250 – 100). This sequence of operations is 500 – 250+100. Thus,

$$500 - (250 - 100) = 500 - 250 + 100.$$

Check that 500 – (250 – 100) is not equal to 500 – 250 – 100.

Notice again that **when the brackets preceded by a negative sign** are removed, the signs of the terms inside the brackets change. In this case, the signs of 250 and – 100 change to – 250 and 100.

Example 14: Hira has a rare coin collection. She has 28 coins in one bag and 35 coins in another. She gifts her friend 10 coins from the second bag. Write an expression for the number of coins left with Hira.

This can be expressed by 28 + (35 – 10).

We know that this is the same as 28 + (35 + (–10)). Since the terms can be added in any order, this expression can simply be written as 28 + 35 + (–10), or 28 + 35 – 10. Thus,

$$28 + (35 - 10) = 28 + 35 - 10 = 53.$$

When the brackets are NOT preceded by a negative sign, the terms within them do not change their signs upon removing the brackets. Notice the sign of the terms 35 and – 10 in the above expression.

Rather than simply remembering rules for when to change the sign and when not to, you can figure it out for yourself by thinking about the meanings of the expressions.

Tinker the Terms I

What happens to the value of an expression if we increase or decrease the value of one of its terms?

Some expressions are given in following three columns. In each column, one or more terms are changed from the first expression. Go through the example (in the first column) and fill the blanks, doing as little computation as possible.

?

53 + (– 16) = 37

54 + (– 16) = 38

54 is one more than 53, so the value will be 1 more than 37.

53 + (– 15) = ___

Is –15 one more or one less than –16?

53 + (– 16) = 37

52 + (– 16) = ___

52 is one less than 53, so the value will be 1 less than 37.

53 + (– 17) = ___

Is –17 one more or one less than –16?

– 87 + (– 16) = ___

– 88 + (– 15) = ___

– 86 + (– 18) = ___

– 97 + (– 26) = ___

? Figure it Out

1. Fill in the blanks with numbers, and boxes with operation signs such that the expressions on both sides are equal.
   (a) 24 + (6 – 4) = 24 + 6 ☐ _____
   (b) 38 + (_____ ☐ _____) = 38 + 9 – 4
   (c) 24 – (6 +4) = 24 ☐ 6 – 4
   (d) 24 – 6 – 4 = 24 – 6 ☐ _____
   (e) 27 – (8 + 3) = 27 8 3
   (f) 27– (_____ ☐ _____) = 27 – 8 + 3
2. Remove the brackets and write the expression having the same value.
   (a) 14 + (12 + 10) (b) 14 – (12 + 10)
   (c) 14 + (12 – 10) (d) 14 – (12 – 10)
   (e) –14 + 12 – 10 (f) 14 – (–12 – 10)
3. Find the values of the following expressions. For each pair, first try to guess whether they have the same value. When are the two expressions equal?
   (a) (6 + 10) – 2 and 6 + (10 – 2)
   (b) 16 – (8 – 3) and (16 – 8) – 3
   (c) 27 – (18 + 4) and 27 + (–18 – 4)
4. In each of the sets of expressions below, identify those that have the same value. Do not evaluate them, but rather use your understanding of terms.

(a) 319 + 537, 319 – 537, – 537 + 319, 537 – 319

(b) 87 + 46 – 109, 87 + 46 – 109, 87 + 46 – 109, 87 – 46 + 109, 87 – (46 + 109), (87 – 46) + 109

5. Add brackets at appropriate places in the expressions such that they lead to the values indicated.

   (a) 34 – 9 + 12 = 13

   (b) 56 – 14 – 8 = 34

   (c) –22 – 12 + 10 + 22 = – 22

6. Using only reasoning of how terms change their values, fill the blanks to make the expressions on either side of the equality (=) equal.

   (a) 423 + ______= 419 + ______

   (b) 207 – 68 = 210 – ______

7. Using the numbers 2, 3 and 5, and the operators '+' and '–', and brackets, as necessary, generate expressions to give as many different values as possible. For example, 2 – 3 + 5 = 4 and 3 – (5 – 2) = 0.

8. Whenever Jasoda has to subtract 9 from a number, she subtracts 10 and adds 1 to it. For example, 36 – 9 = 26 + 1.

   (a) Do you think she always gets the correct answer? Why?

   (b) Can you think of other similar strategies? Give some examples.

   Math Talk

9. Consider the two expressions: a) 73 – 14 + 1, b) 73 – 14 – 1. For each of these expressions, identify the expressions from the following collection that are equal to it.

   (a) 73 – (14 + 1) b) 73 – (14 – 1)

   (c) 73 + (– 14 + 1) d) 73 + (– 14 – 1)

Removing Brackets—II

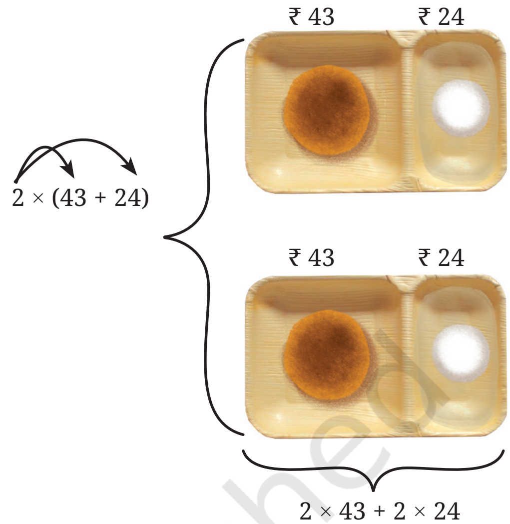
Example 15: Lhamo and Norbu went to a hotel. Each of them ordered a vegetable cutlet and a *rasgulla*. A vegetable cutlet costs ₹43 and a *rasgulla* costs ₹24. Write an expression for the amount they will have to pay.

As each of them had one vegetable cutlet and one *rasagulla*, each of their shares can be represented by 43 + 24.

What about the total amount they have to pay? Can it be described by the expression: 2 × 43 + 24?

Writing it as sum of terms gives:

$$2 \times 43 + 24$$



This expression means 24 more than 2 × 43. But, we want an expression which means twice or double of 43 + 24.

We can make use of brackets to write such an expression:

$$2 \times (43 + 24).$$

So, we can say that together they have to pay 2 × (43 + 24). This is also the same as paying for two vegetable cutlets and two *rasgullas*:

$$2 \times 43 + 2 \times 24.$$

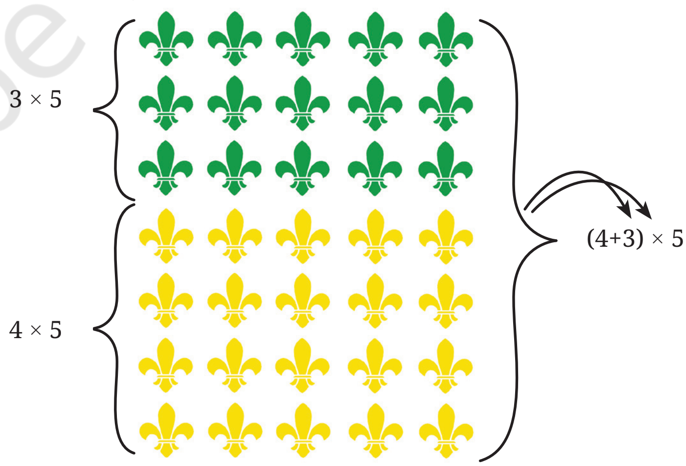
Therefore,

$$2 \times (43 + 24) = 2 \times 43 + 2 \times 24.$$

? If another friend, Sangmu, joins them and orders the same items, what will be the expression for the total amount to be paid?

? **Example 16:** In the Republic Day parade, there are boy scouts and girl guides marching together. The scouts march in 4 rows with 5 scouts in each row. The guides march in 3 rows with 5 guides in each row (see the figure below). How many scouts and guides are marching in this parade?

The number of boy scouts marching is 4 × 5. The number of girl guides marching is 3 × 5.



The total number of scouts and guides will be 4 × 5 + 3 × 5.

This can also be found by first finding the total number of rows, i.e., 4 + 3, and then multiplying their sum by the number of children in each row. Thus, the number of boys and girls can be found by (4 + 3) × 5.

Therefore, 4 × 5 + 3 × 5 = (4 + 3) × 5.

Computing these expressions, we get

$$4 \times 5 + 3 \times 5 = 4 \times 5 + 3 \times 5 = 20 + 15 = 35$$

$(4 + 3) \times 5 = 7 \times 5 = 35$

? $5 \times 4 + 3 \neq 5 \times (4 + 3)$. Can you explain why?

Is $5 \times (4 + 3) = 5 \times (3 + 4) = (3 + 4) \times 5$?

The observations that we have made in the previous two examples can be seen in a general way as follows.

Consider $10 \times 98 + 3 \times 98$. This means taking the sum of 10 times 98 and 3 times 98.

$$\underbrace{98 + 98 + 98 + 98 + 98 + 98 + 98 + 98 + 98 + 98}_{\text{10 times}} + \underbrace{98 + 98 + 98}_{\text{3 times}}$$

Clearly, this is the same as $10 + 3 = 13$ times 98. Thus,

$$10 \times 98 + 3 \times 98 = (10 + 3) \times 98.$$

Writing this equality the other way, we get

$$(10 + 3)\ 98 = 10 \times 98 + 3 \times 98.$$

Swapping the numbers in the products above, this property can be seen in the following form:

$$98 \times 10 + 9 \times 83 = 98\ (10 + 3), \text{ and}$$

$$98\ (10 + 3) = 98 \times 10 + 98 \times 3.$$

Similarly, let us consider the expression $14 \times 10 - 6 \times 10$. This means subtracting 6 times 10 from 14 times 10.

$$\overbrace{10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + \cancel{10 + 10 + 10 + 10 + 10 + 10}}^{\text{14 times}}$$

Clearly, this is $14 - 6 = 8$ times 10. Thus,

$$14 \times 10 - 6 \times 10 = (14 - 6) \times 10,$$

or

$$(14 - 6) \times 10 = 14 \times 10 - 6 \times 10$$

This property can be nicely summed up as follows:

The multiple of a sum (difference) is the same as the sum (difference) of the multiples.

Tinker the Terms II

Let us understand what happens when we change the numbers occurring in a product.

Example 17: Given $53 \times 18 = 954$. Find out 63×18.

As 63×18 means 63 times 18,

$$\begin{aligned} 63 \times 18 &= (53 + 10) \times 18 \\ &= 53 \times 18 + 10 \times 18 \\ &= 954 + 180 \\ &= 1134. \end{aligned}$$

Example 18: Find an effective way of evaluating 97×25.

97×25 means 97 times 25.

We can write it as $(100 - 3) \times 25$

We know that this is the same as the difference of 100 times 25 and 3 times 25:

$$97 \times 25 = 100 \times 25 - 3 \times 25$$

Find this value.

Use this method to find the following products:

(a) 95×8

(b) 104×15

(c) 49×50

Is this quicker than the multiplication procedure you use generally?

Which other products might be quicker to find like the ones above?

Figure it Out

1. Fill in the blanks with numbers, and boxes by signs, so that the expressions on both sides are equal.

(a) $3 \times (6 + 7) = 3 \times 6 + 3 \times 7$

(b) $(8 + 3) \times 4 = 8 \times 4 + 3 \times 4$

(c) $3 \times (5 + 8) = 3 \times 5 \boxed{} 3 \times$ ____

(d) $(9 + 2) \times 4 = 9 \times 4 \boxed{} 2 \times$ ____

(e) $3 \times ($ ____ $+ 4) = 3$ ____ + ____

(f) (____ $+ 6) \times 4 = 13 \times 4 +$ ____

(g) $3 \times ($ ____ + ____ $) = 3 \times 5 + 3 \times 2$

(h) (____ + ____)× ____ $= 2 \times 4 + 3 \times 4$

(i) $5 \times (9 - 2) = 5 \times 9 - 5 \times$ ____

(j) $(5 - 2) \times 7 = 5 \times 7 - 2 \times$ ____

(k) $5 \times (8 - 3) = 5 \times 8 \boxed{} 5 \times$ ____

(l) $(8 - 3) \times 7 = 8 \times 7 \boxed{} 3 \times 7$

(m) 5 × (12 – ____) = ____ $\square$ 5 × ____

(n) (15 – ____) × 7 = ____ $\square$ 6 × 7

(o) 5 × (____ – ____) = 5 × 9 – 5 × 4

(p) (____ – ____) × ____ = 17 × 7 – 9 × 7

2. In the boxes below, fill '<', '>' or '=' after analysing the expressions on the LHS and RHS. Use reasoning and understanding of terms and brackets to figure this out and not by evaluating the expressions.

(a) (8 – 3) × 29 $\square$ (3 – 8) × 29

(b) 15 + 9 × 18 $\square$ (15 + 9) × 18

(c) 23 × (17 – 9) $\square$ 23 × 17 + 23 × 9

(d) (34 – 28) × 42 $\square$ 34 × 42 – 28 × 42

3. Here is one way to make 14: <u>2</u> × (<u>1</u> + <u>6</u>) = 14. Are there other ways of getting 14? Fill them out below:

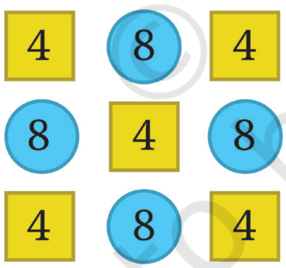
(a) ____ × (____ + ____) = 14

(b) ____ × (____ + ____) = 14

(c) ____ × (____ + ____) = 14

(d) ____ × (____ + ____) = 14

4. Find out the sum of the numbers given in each picture below in at least two different ways. Describe how you solved it through expressions.



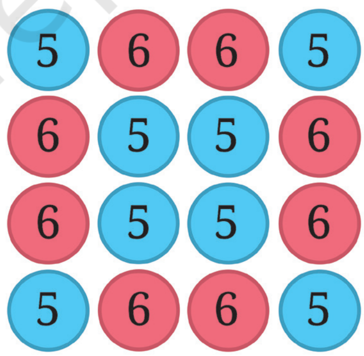


Figure it Out

1. Read the situations given below. Write appropriate expressions for each of them and find their values.

(a) The district market in Begur operates on all seven days of a week. Rahim supplies 9 kg of mangoes each day from his orchard and Shyam supplies 11 kg of mangoes each day from his orchard to this market. Find the amount of mangoes supplied by them in a week to the local district market.

(b) Binu earns ₹20,000 per month. She spends ₹5,000 on rent, ₹5,000 on food, and ₹2,000 on other expenses every month. What is the amount Binu will save by the end of a year?

(c) During the daytime a snail climbs 3 cm up a post, and during the night while asleep, accidentally slips down by 2 cm. The post is 10 cm high, and a delicious treat is on its top. In how many days will the snail get the treat?

2. Melvin reads a two-page story every day except on Tuesdays and Saturdays. How many stories would he complete reading in 8 weeks? Which of the expressions below describes this scenario?

   (a) $5 \times 2 \times 8$

   (b) $(7 - 2) \times 8$

   (c) 8×7

   (d) $7 \times 2 \times 8$

   (e) $7 \times 5 - 2$

   (f) $(7 + 2) \times 8$

   (g) $7 \times 8 - 2 \times 8$

   (h) $(7 - 5) \times 8$

3. Find different ways of evaluating the following expressions:

   (a) $1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + 9 - 10$

   (b) $1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1$

4. Compare the following pairs of expressions using '<', '>' or '=' or by reasoning.

   (a) $49 - 7 + 8$ $\boxed{}$ $49 - 7 + 8$

   (b) $83 \times 42 - 18$ $\boxed{}$ $83 \times 40 - 18$

   (c) $145 - 17 \times 8$ $\boxed{}$ $145 - 17 \times 6$

   (d) $23 \times 48 - 35$ $\boxed{}$ $23 \times (48 - 35)$

   (e) $(16 - 11) \times 12$ $\boxed{}$ $-11 \times 12 + 16 \times 12$

   (f) $(76 - 53) \times 88$ $\boxed{}$ $88 \times (53 - 76)$

   (g) $25 \times (42 + 16)$ $\boxed{}$ $25 \times (43 + 15)$

   (h) $36 \times (28 - 16)$ $\boxed{}$ $35 \times (27 - 15)$

5. Identify which of the following expressions are equal to the given expression without computation. You may rewrite the expressions using terms or removing brackets. There can be more than one expression which is equal to the given expression.
   (a) 83 – 37 – 12
      (i) 84 – 38 – 12
      (ii) 84 – (37 + 12)
      (iii) 83 – 38 – 13
      (iv) – 37 + 83 –12
   (b) 93 + 37 × 44 + 76
      (i) 37 + 93 × 44 + 76
      (ii) 93 + 37 × 76 + 44
      (iii) (93 + 37) × (44 + 76)
      (iv) 37 × 44 + 93 + 76
5. Choose a number and create ten different expressions having that value.

SUMMARY

- We have been reading and evaluating simple expressions for quite some time now. Here we started by revising the meaning of some simple expressions and their values.
- We learnt how to compare certain expressions through reasoning instead of bluntly evaluating them.
- To help read and evaluate complex expressions without confusion, we use terms and brackets.
- When an expression is written as a sum of terms, changing the order of the terms or grouping the terms does not change the value of the expression. This is because the "commutative property of addition" and the "associative property of addition", respectively.
- To evaluate expressions within brackets, we saw that when we remove brackets preceded by a negative sign, the terms within the bracket change their sign.
- We also learnt about the "distributive property" — multiplying a number with an expression inside brackets is equal to the multiplying the number with each term in the bracket.

Expression Engineer!

Using three 3's along with the four operations (addition, subtraction, multiplication, and division) and brackets as needed we can create several expressions. For example, $(3 + 3)/3 = 2$, $3 + 3 - 3 = 3$, $3 \times 3 + 3 = 12$, and so on.

Using four 4's, create expressions to get all values from 1 to 20.

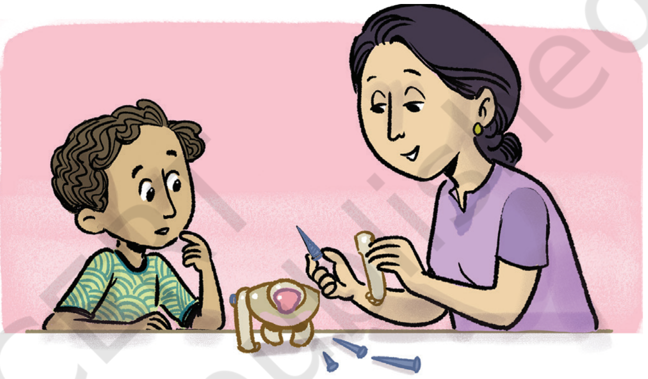
Using the numbers 1, 2, 3, 4, and 5 exactly once in any order get as many values as possible between -10 and $+10$.

Using the numbers 0 to 9 exactly once in any order, make an expression with a value 100.

What other similar interesting questions can you ask?

3 A PEEK BEYOND THE POINT

3.1 The Need for Smaller Units

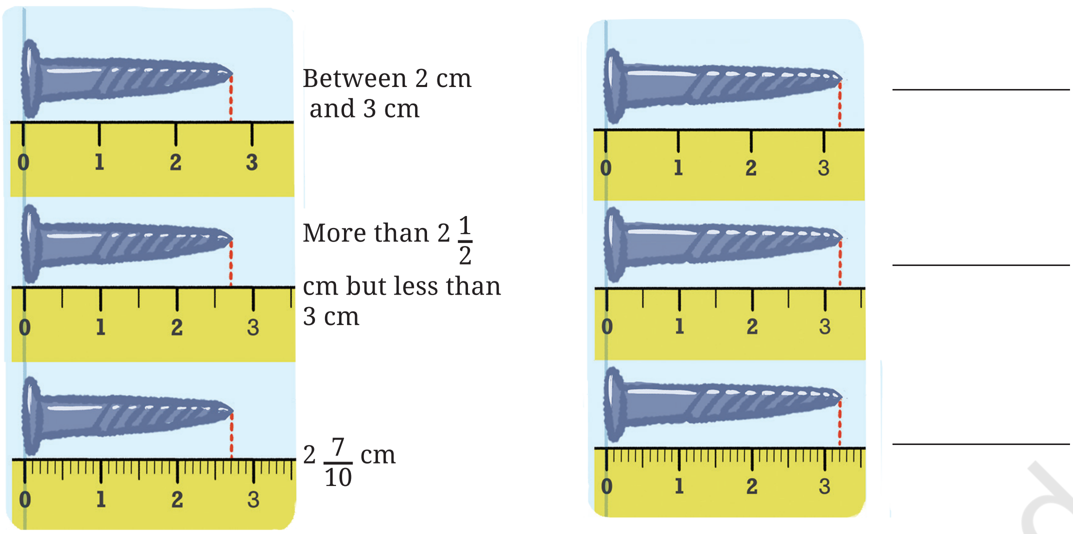


Sonu's mother was fixing a toy. She was trying to join two pieces with the help of a screw. Sonu was watching his mother with great curiosity. His mother was unable to join the pieces. Sonu asked why. His mother said that the screw was not of the right size.

She brought another screw from the box and was able to fix the toy. The two screws looked the same to Sonu. But when he observed them closely, he saw they were of slightly different lengths.

Sonu was fascinated by how such a small difference in lengths could matter so much. He was curious to know the difference in lengths. He was also curious to know how little the difference was because the screws looked nearly the same.

In the following figure, screws are placed above a scale. Measure them and write their length in the space provided.



Which scale helped you measure the length of the screws accurately? Why?

What is the meaning of 2 $\frac{7}{10}$ cm (the length of the first screw)?

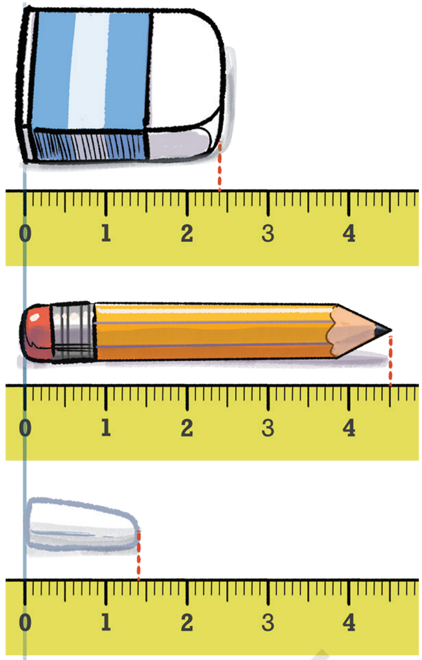
As seen on the ruler, the unit length between two consecutive numbers is divided into 10 equal parts. To get the length 2 $\frac{7}{10}$ cm, we go from 0 to 2 and then take seven parts of $\frac{1}{10}$. The length of the screw is 2 cm and $\frac{7}{10}$ cm. Similarly, we can make sense of the length 3 $\frac{2}{10}$ cm.

We read 2 $\frac{7}{10}$ cm as **two and seven-tenth centimeters**, and 3 $\frac{2}{10}$ cm as **three and two-tenth centimeters**.

Can you explain why the unit was divided into smaller parts to measure the screws?

Measure the following objects using a scale and write their measurements in centimeters (as shown earlier for the lengths of the screws): pen, sharpener, and any other object of your choice.

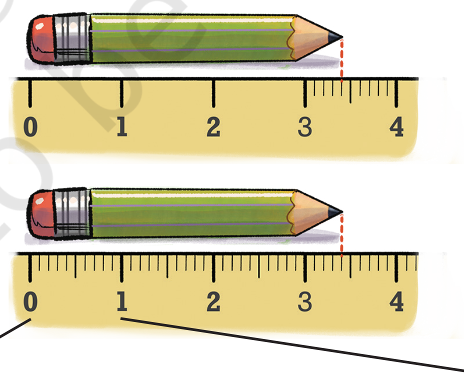
Write the measurements of the objects shown in the picture:



As seen here, when exact measures are required we can make use of smaller units of measurement.

3.2 A Tenth Part

The length of the pencil shown in the figure below is $3\frac{4}{10}$ units, which can also be read as 3 units and four one-tenths, i.e., $(3 \times 1) + \left(4 \times \frac{1}{10}\right)$ units.



$$\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}$$

$$= 10 \text{ times } \frac{1}{10} = 10 \times \frac{1}{10} = 1 \text{ unit}$$

This length is the same as 34 one-tenths units because 10 one-tenths units make one unit.

$$34 \times \frac{1}{10} = \frac{34}{10} = \frac{10}{10} + \frac{10}{10} + \frac{10}{10} + \frac{4}{10} \text{ (34 one-tenths)}$$

$$= 1 + 1 + 1 + \frac{4}{10} \text{ (3 and 4 one-tenths)}$$

A few numbers with fractional units are shown below along with how to read them.

$4\frac{1}{10} \longrightarrow$ **'four and one-tenth'**

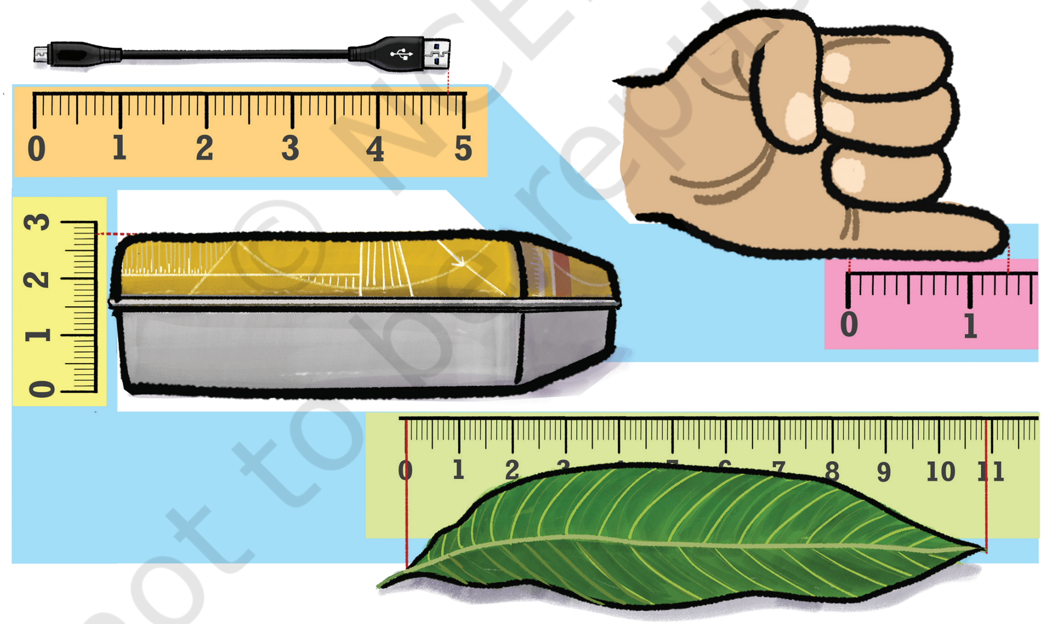
$\frac{4}{10} \longrightarrow$ **'four one-tenths'** or **'four-tenths'**

$\frac{41}{10} \longrightarrow$ **'forty-one one-tenths'** or **'forty-one tenths'**

$41\frac{1}{10} \longrightarrow$ **'forty-one and one-tenth'**

For the objects shown below, write their lengths in two ways and read them aloud. An example is given for the USB cable. (Note that the unit length used in each diagram is not the same).

The length of the USB cable is 4 and $\frac{8}{10}$ units or $\frac{48}{10}$ units.



Arrange these lengths in increasing order:

(a) $\frac{9}{10}$ (b) $1\frac{7}{10}$ (c) $\frac{130}{10}$ (d) $13\frac{1}{10}$

(e) $10\frac{5}{10}$ (f) $7\frac{6}{10}$ (g) $6\frac{7}{10}$ (h) $\frac{4}{10}$

? Arrange the following lengths in increasing order: $4\frac{1}{10}, \frac{4}{10}, \frac{41}{10}, 41\frac{1}{10}$.

? Sonu is measuring some of his body parts. The length of Sonu's lower arm is $2\frac{7}{10}$ units, and that of his upper arm is $3\frac{6}{10}$ units. What is the total length of his arm?

To get the total length, let us see the lower and upper arm length as 2 units and 7 one-tenths, and 3 units and 6 one-tenths, respectively.

So, there are (2 + 3) units and (7 + 6) one-tenths. Together, they make 5 units and 13 one-tenths. But 13 one-tenths is 1 unit and 3 one-tenths. So, the total length is 6 units and 3 one-tenths.

(a)

$$
\begin{aligned}
& (2+3) + \left(\frac{7}{10} + \frac{6}{10}\right) \\
&= (2+3) + \left(\frac{13}{10}\right) \\
&= 5 + \frac{13}{10} \\
&= 5 + \frac{10}{10} + \frac{3}{10} = 5 + 1 + \frac{3}{10} \\
&= 6 + \frac{3}{10} \\
&= 6\frac{3}{10}
\end{aligned}
$$

(b)

$$
\begin{array}{rl}
 & 2\,\frac{7}{10} \\
+ & 3\,\frac{6}{10} \\
\hline
= & 5\,\frac{13}{10} \\
= & 6\,\frac{3}{10}
\end{array}
$$

Or, both the lengths can be converted to tenths and then added:

(c) 27 one-tenths and 35 one-tenths is 62 one-tenths

$$\frac{27}{10} + \frac{35}{10} = \frac{62}{10}$$

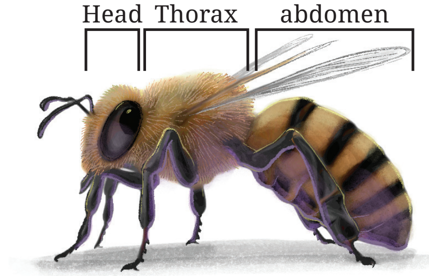
$\frac{62}{10}$ is the same as 60 one-tenths $\left(\frac{60}{10}\right)$ and 2 one-tenths $\left(\frac{2}{10}\right)$, which is equal to 6 units and 2 one-tenths, i.e., $6\frac{2}{10}$.

? The lengths of the body parts of a honeybee are given. Find its total length.

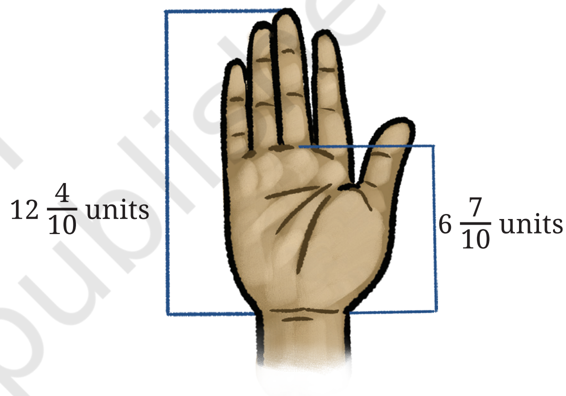
Head: $2\frac{3}{10}$ units

Thorax: $5\frac{4}{10}$ units

Abdomen: $7\frac{5}{10}$ units



? The length of Shylaja's hand is $12\frac{4}{10}$ units, and her palm is $6\frac{7}{10}$ units, as shown in the picture. What is the length of the longest (middle) finger?



The length of the finger can be found by evaluating $\left(12+\frac{4}{10}\right)-\left(6+\frac{7}{10}\right)$. This can be done in different ways. For example,

(a) $12+\frac{4}{10}-6-\frac{7}{10}$

$= (12-6)+\left(\frac{4}{10}-\frac{7}{10}\right)$

$= 6-\frac{3}{10}$

$= 5+1-\frac{3}{10}$

$= 5+\frac{10}{10}-\frac{3}{10}$

$= 5+\frac{7}{10} = 5\frac{7}{10}$

Discuss what is being done here and why.

(b)
$$\begin{array}{r} 12\frac{4}{10} \\ -\ 6\frac{7}{10} \\ \hline \end{array} \longrightarrow \begin{array}{r} 11\frac{14}{10} \\ -\ 6\frac{7}{10} \\ \hline =5\frac{7}{10} \end{array}$$

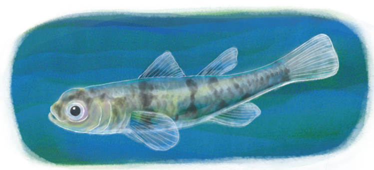
As in the case of counting numbers, it is convenient to start subtraction from the tenths. We cannot remove 7 one-tenths from 4 one-tenths. So we split a unit from 12 and convert it to 10 one-tenths. Now, the number has 11 units and 14 one-tenths. We subtract 7 one-tenths from 14 one-tenths and then subtract 6 units from 11 units.

? Try computing the difference by converting both lengths to tenths.

? A Celestial Pearl Danio's length is $2\frac{4}{10}$ cm, and the length of a Philippine Goby is $\frac{9}{10}$ cm. What is the difference in their lengths?

? How big are these fish compared to your finger?

Celestial Pearl Danio



Philippine Goby

? Observe the given sequences of numbers. Identify the change after each term and extend the pattern:

(a) $4, 4\frac{3}{10}, 4\frac{6}{10},$ ______, ______, ______, ______

(b) $8\frac{2}{10}, 8\frac{7}{10}, 9\frac{2}{10},$ ______, ______, ______, ______

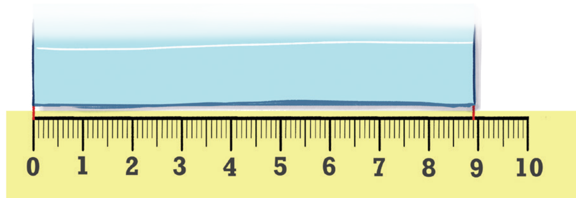
(c) $7\frac{6}{10}, 8\frac{7}{10},$ ______, ______, ______, ______

(d) $5\frac{7}{10}, 5\frac{3}{10},$ ______, ______, ______, ______

(e) $13\frac{5}{10}, 13, 12\frac{5}{10},$ ______, ______, ______, ______

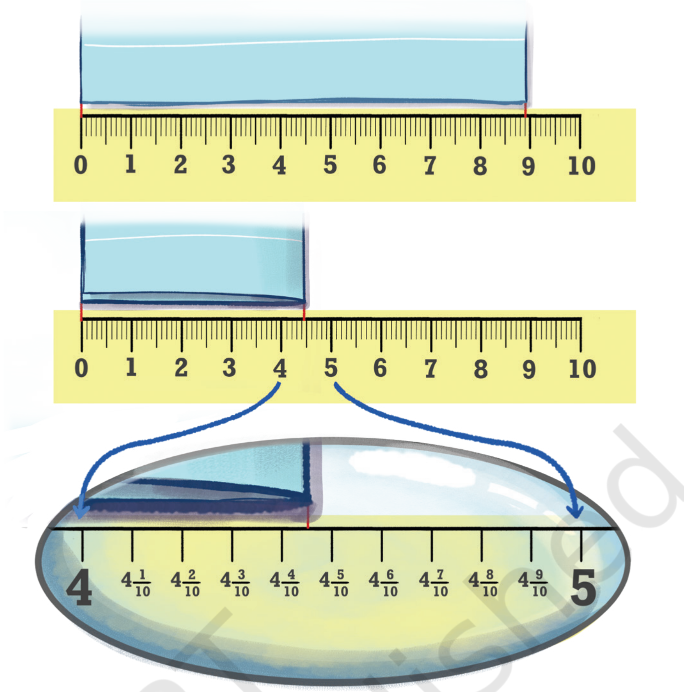
(f) $11\frac{5}{10}, 10\frac{4}{10}, 9\frac{3}{10},$ ______, ______, ______, ______

3.3 A Hundredth Part



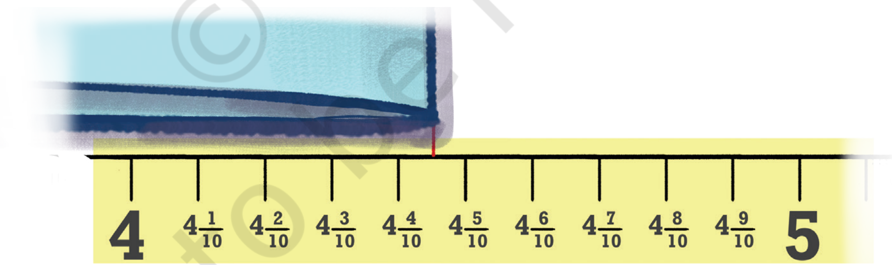
The length of a sheet of paper was $8\frac{9}{10}$ units, which can also be said as 8 units and 9 one-tenths. It is folded in half along its length. What is its length now?

We can say that its length is between $4\frac{4}{10}$ units and $4\frac{5}{10}$ units. But we cannot state its exact measurement, since there are no markings. Earlier, we split a unit into 10 one-tenths to measure smaller lengths. We can do something similar and split each one-tenth into 10 parts.



? What is the length of this smaller part? How many such smaller parts make a unit length?

As shown in the figure below, each one-tenth has 10 smaller parts, and there are 10 one-tenths in a unit; therefore, there will be 100 smaller parts in a unit. Therefore, one part's length will be $\frac{1}{100}$ of a unit.



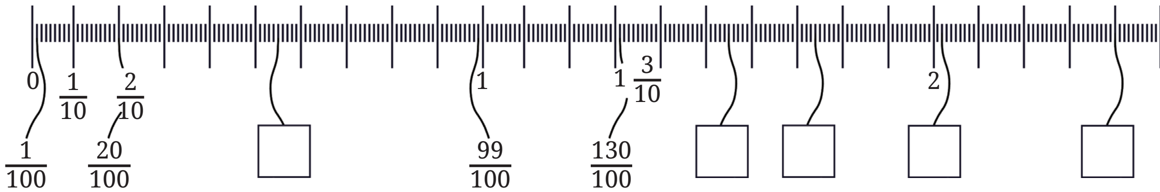
Returning to our question, what is the length of the folded paper?

We can see that it ends at $4\,\frac{4}{10}\,\frac{5}{100}$, read as **4 units and 4 one-tenths and 5 one-hundredths**.

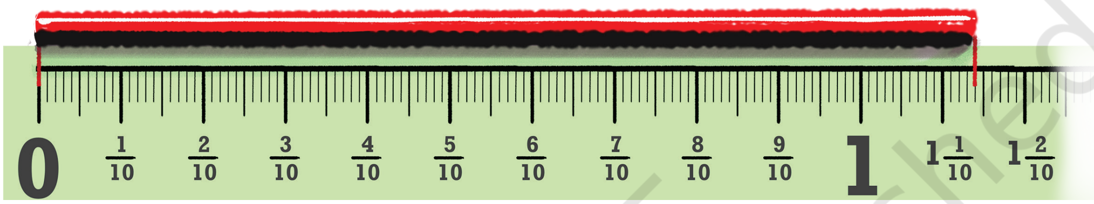
? How many one-hundredths make one-tenth? Can we also say that the length is 4 units and 45 one-hundredths?

Math Talk

? Observe the figure below. Notice the markings and the corresponding lengths written in the boxes when measured from 0. Fill the lengths in the empty boxes.



The length of the wire in the first picture is given in three different ways. Can you see how they denote the same length?

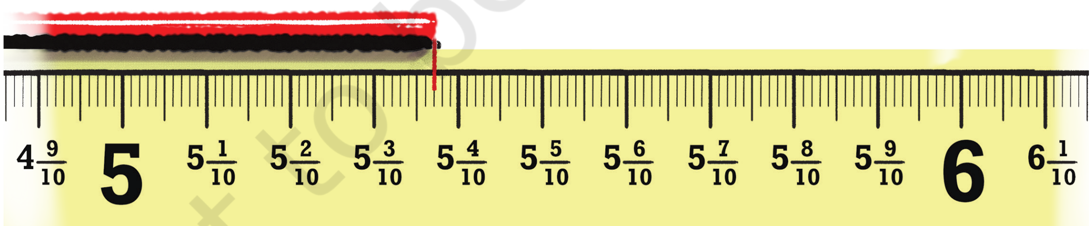


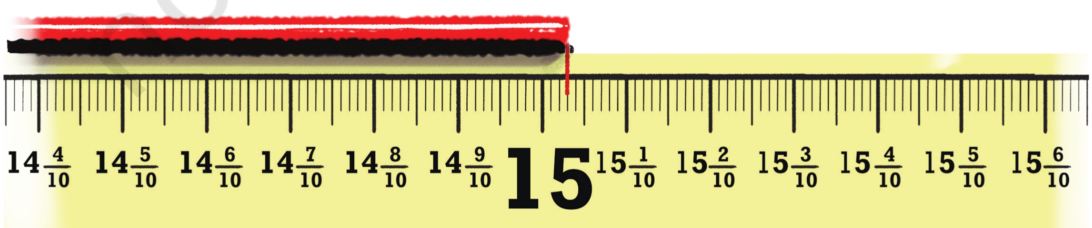
$1 \frac{1}{10} \frac{4}{100}$ **One and one-tenth and four-hundredths**

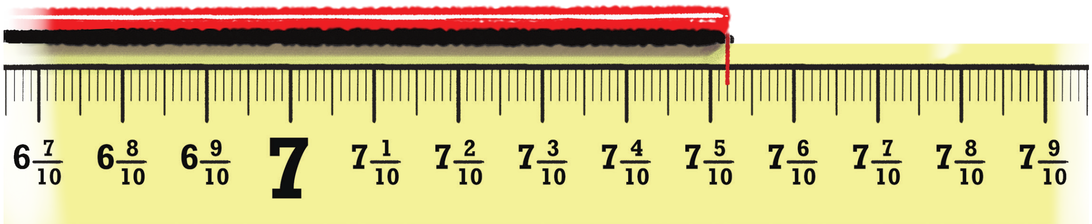
$1 \frac{14}{100}$ **One and fourteen-hundredths**

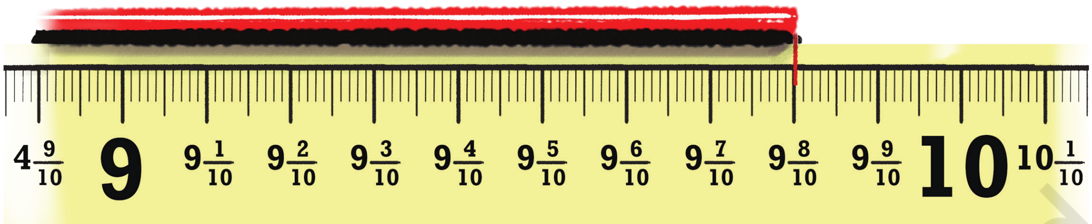
$\frac{114}{100}$ **One Hundred and Fourteen-hundredths**

? For the lengths shown below write the measurements and read out the measures in words.



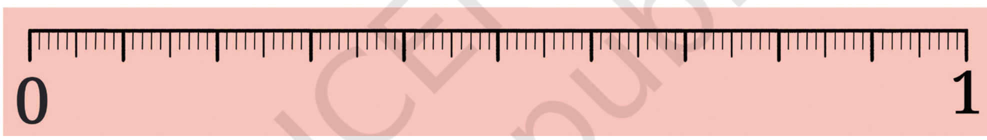




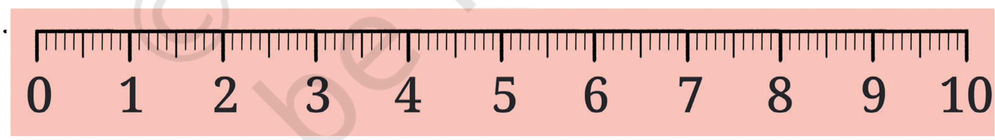


In each group, identify the longest and the shortest lengths. Mark each length on the scale.

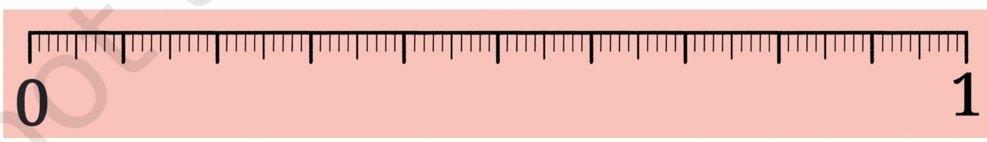
(a) $\frac{3}{10}, \frac{3}{100}, \frac{33}{100}$



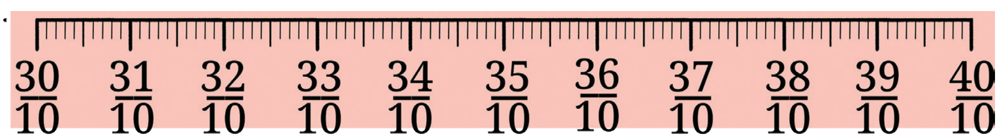
(b) $3\frac{1}{10}, \frac{30}{10}, 1\frac{3}{10}$



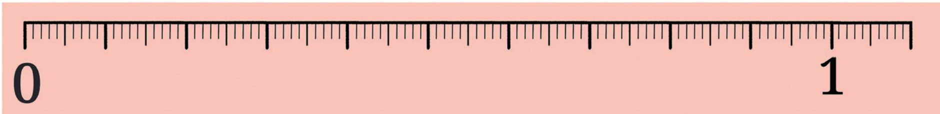
(c) $\frac{45}{100}, \frac{54}{100}, \frac{5}{10}, \frac{4}{10}$



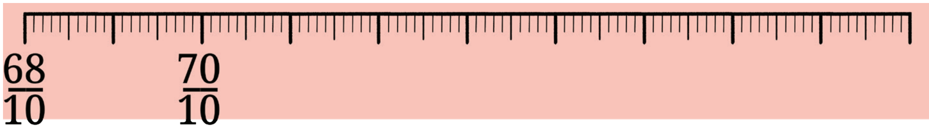
(d) $3\frac{6}{10}, 3\frac{6}{100}, 3\frac{6}{10}\frac{6}{100}$



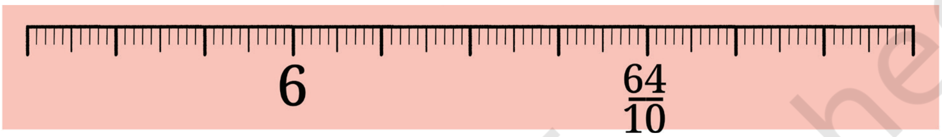
(e) $\frac{8}{10}\ \frac{2}{100}$, $\frac{9}{100}$, $1\ \frac{8}{100}$



(f) $7\ \frac{3}{10}\ \frac{5}{100}$, $7\ \frac{5}{10}$, $7\ \frac{41}{100}$



(g) $\frac{65}{10}\ \frac{15}{100}$, $5\ \frac{87}{100}$, $5\ \frac{7}{100}$



? What will be the sum of $15\ \frac{3}{10}\ \frac{4}{100}$ and $2\ \frac{6}{10}\ \frac{8}{100}$?

This can be solved in different ways. Some are shown below.

(a) Method 1

$$(15 + 2) + \left(\frac{3}{10} + \frac{6}{10}\right) + \left(\frac{4}{100} + \frac{8}{100}\right)$$

$$= 17 + \frac{9}{10} + \frac{12}{100}$$

$$= 17 + \frac{9}{10} + \frac{1}{10} + \frac{2}{100}$$

$$= 17 + \frac{10}{10} + \frac{2}{100}$$

$$= 18\ \frac{2}{100}.$$

10 hundredths is the same as 1 tenth.

(b) Method 2

$$\begin{array}{rrcc} & 15 & \frac{3}{10} & \frac{4}{100} \\ + & 2 & \frac{6}{10} & \frac{8}{100} \\ \hline = & 17 & \frac{9}{10} & \frac{12}{100} \\ = & 17 & \frac{10}{10} & \frac{2}{100} \\ = & 18 & & \frac{2}{100} \end{array}$$

Are both these methods different?

Observe the addition done below for 483 + 268. Do you see any similarities between the methods shown above?

Math Talk

$$\begin{aligned}
&(400 + 80 + 3) + (200 + 60 + 8)\\
&= (400 + 200) + (80 + 60) + (3 + 8)\\
&= 600 + 140 + 11\\
&= 600 + 150 + 1\\
&= 700 + 50 + 1\\
&= 751
\end{aligned}$$

One can also find the sum $15\,\frac{3}{10}\,\frac{4}{100} + 2\,\frac{6}{10}\,\frac{8}{100}$ by converting to hundredths, as follows.

(c) $(15 + 2) + \left(\frac{34}{100} + \frac{68}{100}\right)$

$= 17 + \frac{102}{100}$

$= 17 + 1 + \frac{2}{100}$

$= 18\,\frac{2}{100}$

100 hundredths is same as 1 unit.

(d) $\left(\frac{1534}{100}\right) + \left(\frac{268}{100}\right)$

$= \frac{1802}{100}$

$= \frac{1802}{100} + \frac{2}{100}$

$= 18\,\frac{2}{100}$

15 is the same as 1500 hundredths and 2 is the same as 200 hundredths.

What is the difference: $25\,\frac{9}{10} - 6\,\frac{4}{10}\,\frac{7}{100}$?

One way to solve this is as follows.

$$\begin{array}{r} 25\,\frac{9}{10} \\ -\,6\,\frac{4}{10} \\ \hline \end{array} \longrightarrow \begin{array}{r} 25\,\frac{8}{10}\,\frac{10}{100} \\ -\,6\,\frac{4}{10}\,\frac{7}{100} \\ \hline \end{array} \longrightarrow \begin{array}{r} 25\,\frac{8}{10}\,\frac{10}{100} \\ -\,6\,\frac{4}{10}\,\frac{7}{100} \\ \hline =19\,\frac{4}{10}\,\frac{3}{100} \end{array}$$

Solve this by converting to hundredths.

What is the difference $15\,\frac{3}{10}\,\frac{4}{100} - 2\,\frac{6}{10}\,\frac{8}{100}$?

One way to solve this is as follows:

$$15\,\frac{3}{10}\,\frac{4}{100} \qquad 15\,\frac{2}{10}\,\frac{14}{100} \qquad 14\,\frac{12}{10}\,\frac{14}{100}$$

$$-\,2\,\frac{6}{10}\,\frac{8}{100} \qquad -\,2\,\frac{6}{10}\,\frac{8}{100} \qquad -\,2\,\frac{6}{10}\,\frac{8}{100}$$

$$= 2\,\frac{6}{10}\,\frac{6}{100}$$

Observe the subtraction done below for 653 – 268. Do you see any similarities with the methods shown above?

$$
\begin{aligned}
& (600 + 50 + 3) - (200 + 60 + 8) \\
&= (600 - 200) + (50 - 60) + (3 - 8) \\
&= (600 - 200) + (40 - 60) + (13 - 8) \\
&= (600 - 200) + (40 - 60) + 5 \\
&= (500 - 200) + (140 - 60) + 5 \\
&= 300 + 80 + 5 \\
&= 385
\end{aligned}
$$

Figure it Out

Find the sums and differences:

(a) $\frac{3}{10} + 3\,\frac{4}{100}$

(b) $9\,\frac{5}{10}\,\frac{7}{100} + 2\,\frac{1}{10}\,\frac{3}{100}$

(c) $15\,\frac{6}{10}\,\frac{4}{100} + 14\,\frac{3}{10}\,\frac{6}{100}$

(d) $7\,\frac{7}{100} - 4\,\frac{4}{100}$

(e) $8\,\frac{6}{100} - 5\,\frac{3}{100}$

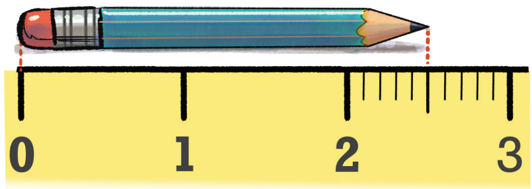
(f) $12\,\frac{6}{100}\,\frac{2}{100} - \frac{9}{10}\,\frac{9}{100}$

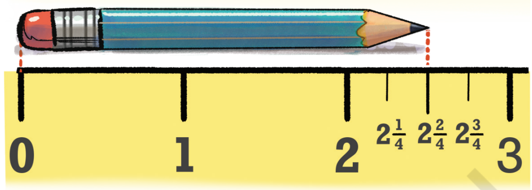
3.4 Decimal Place Value

You may have noticed that whenever we need to measure something more accurately, we split a part into 10 (smaller) equal parts — we split a unit into 10 one-tenths and then split each one-tenth into 10, one-hundredths and then we use these smaller parts to measure.

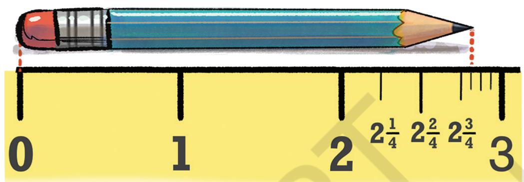
? Can we not split a unit into 4 equal parts, 5 equal parts, 8 equal parts, or any other number of equal parts instead?

Yes, we can. The example below compares how the same length is represented when the unit is split into 10 equal parts and when the unit is split into 4 equal parts.





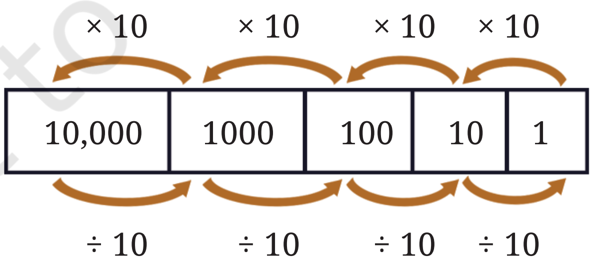
If an even more precise measure is needed, each quarter can further be split into four equal parts. Each part then measures $\frac{1}{16}$ of a unit, i.e., 16 such parts make 1 unit.



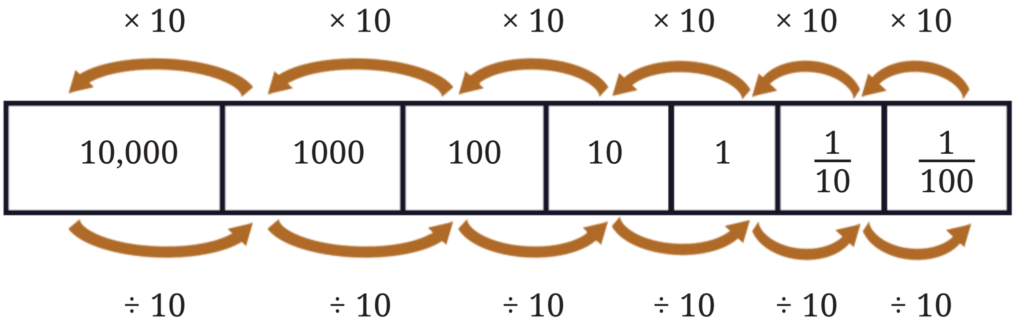
? Then why split a unit into 10 parts every time?

The reason is the special role that 10 plays in the Indian place value system. For a whole number written in the Indian place value system — for example, 281 — the place value of 2 is hundreds (100), that of 8 is tens (10), and that of 4 is one (1). Each place value is 10 times bigger than the one immediately to its right. Equivalently, each place value is 10 times smaller than the one immediately to its left:

10 ones make 1 ten,
10 tens make 1 hundred,
10 hundreds make 1 thousand, and so on.



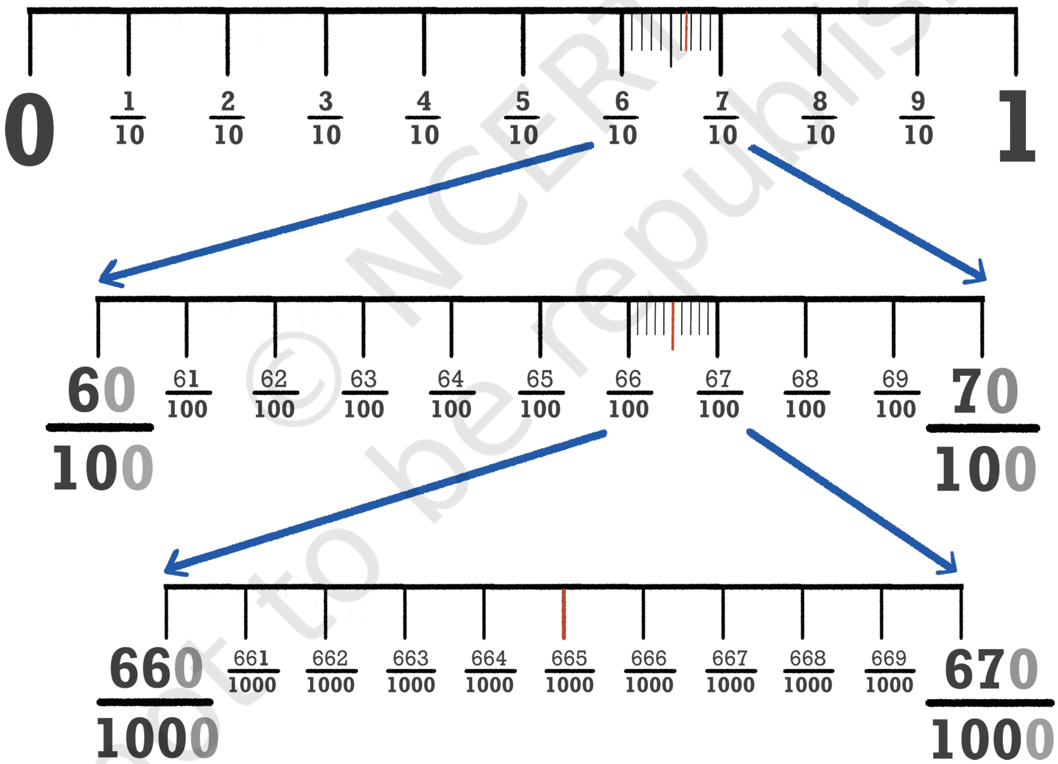
In order to extend this system of writing numbers to quantities smaller than one, we divide one into 10 equal parts. What does this give? It gives one-tenth. Further dividing it into 10 parts gives one-hundredth, and so on.



? Can we extend this further?

? What will the fraction be when $\frac{1}{100}$ is split into 10 equal parts?

It will be $\frac{1}{1000}$, i.e., a thousand such parts make up a unit.



Just as when we extend to the left of 10,000, we get bigger place values at each step, we can also extend to the right of $\frac{1}{1000}$, getting smaller place values at each step.

× 10	× 10	× 10	× 10	× 10	× 10	× 10	× 10	× 10	
1,00,000	10,000	1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10,000}$
÷ 10	÷ 10	÷ 10	÷ 10	÷ 10	÷ 10	÷ 10	÷ 10	÷ 10	

This way of writing numbers is called the "decimal system" since it is based on the number 10; "decem" means ten in Latin, which in turn is cognate to the Sanskrit *daśha* meaning 10, with similar words for 10 occurring across many Indian languages including Odia, Konkani, Marathi, Gujarati, Hindi, Kashmiri, Bodo, and Assamese. We shall learn about other ways of writing numbers in later grades.

How Big?

We already know that a hundred 10s make 1000, and a hundred 100s make 10000.

? We can ask similar questions about fractional parts:

(a) How many thousandths make one unit?
(b) How many thousandths make one tenth?
(c) How many thousandths make one hundredth?
(d) How many tenths make one ten?
(e) How many hundredths make one ten?

? Make a few more questions of this kind and answer them.

Notation, Writing and Reading of Numbers

We have been writing numbers in a particular way, say 456, instead of writing them as 4 × 100 (4 hundreds) + 5 × 10 (5 tens) + 6 × 1 (6 ones). Similarly, can we skip writing tenths and hundredths?

Can the quantity $4\frac{2}{10}$ be written as 42 $\left(\text{skipping the } \frac{1}{10} \text{ in } 2 \times \frac{1}{10}\right)$? Math Talk

If yes, how would we know if 42 means 4 tens and 2 units or it means 4 units and 2 tenths?

Similarly, 705 could mean:

(a) 7 hundreds, and 0 tens and 5 ones (700 + 0 + 5)

(b) 7 tens and 0 units and 5 tenths $\left(70 + 0 + \frac{5}{10}\right)$

(c) 7 units and 0 tenths and 5 hundredths $\left(7 + \frac{0}{10} + \frac{5}{100}\right)$

Since these are different quantities, we need to have distinct ways of writing them.

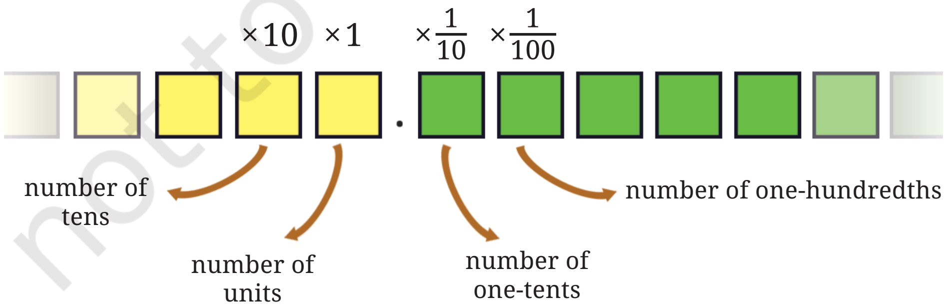
To identify the place value where integers end and the fractional parts start, we use a point or period ('.') as a separator, called a **decimal point**.

The above quantities in decimal notation are then:

Quantity	Decimal Notation
7 hundreds and 5 ones (700 + 0 + 5)	705
7 tens and 5 tenths $\left(70 + 0 + \frac{5}{10}\right)$	70.5
7 units and 5 hundredths $\left(7 + 0 + \frac{5}{100}\right)$	7.05

These numbers, when shown through place value, are as follows:

Decimal number	Hundreds	Tens	Units	• Tenths	Hundredths
705	7 × 100	0 × 10	5 × 1	•	
70.5		7 × 10	0 × 1	• $5 \times \frac{1}{10}$	
7.05			7 × 1	• $0 \times \frac{1}{10}$	$5 \times \frac{1}{100}$



Thus decimal notation is a natural extension of the Indian place value system to numbers also having fractional parts. Just as 705 means $7 \times 100 + 5 \times 1$, the number 70.5 means $7 \times 10 + 5 \times \frac{1}{10}$, and 7.05 means $7 \times 1 + 5 \times \frac{1}{100}$.

We have seen how to write numbers using the decimal point ('.'). But how do we read/say these numbers?

We know that 705 is read as **seven hundred and five**.

70.5 is read as **seventy point five**, short for **seventy and five-tenths**.

7.05 is read as **seven point zero five**, short for **seven and five hundredths**.

0.274 is read as **zero point two seven four**. We don't read it as **zero point two hundred and seventy four** as 0.274 means **2 one-tenths and 7 one-hundredths and 4 one-thousandths**.

? Make a place value table similar to the one above. Write each quantity in decimal form and in terms of place value, and read the number:

(a) 2 ones, 3 tenths and 5 hundredths

(b) 1 ten and 5 tenths

(c) 4 ones and 6 hundredths

(d) 1 hundred, 1 one and 1 hundredth

(e) $\frac{8}{100}$ and $\frac{9}{10}$

(f) $\frac{5}{100}$

(g) $\frac{1}{10}$

(h) $2\frac{1}{100}$, $4\frac{1}{10}$ and $7\frac{7}{1000}$

In the chapter on large numbers, we learned how to write 23 hundreds.

23 hundreds = $23 \times 100 = 2000 + 300 = 2300$.

Thousands	Hundreds	Tens	Units
	23		
2	3	0	0

Similarly, 23 tens would be:

23 tens = $23 \times 10 = 200 + 30 = 230$.

Thousands	Hundreds	Tens	Units
		23	
	2	3	0

How can we write 234 tenths in decimal form?

$$234 \text{ tenths} = \frac{234}{10}$$

$$= \frac{200}{10} + \frac{30}{10} + \frac{4}{10}$$

$$= 20 + 3 + \frac{4}{10}$$

$$= 23.4.$$

Hundreds	Tens	Units	Tenths	Hundredths
			234	
	2	3 •	4	

Write these quantities in decimal form: (a) 234 hundredths, (b) 105 tenths.

3.5 Units of Measurement

Length Conversion

We have been using a scale to measure length for a few years. We already know that 1 cm = 10 mm (millimeters).

How many cm is 1 mm?

$$1 \text{ mm} = \frac{1}{10} \text{ cm} = 0.1 \text{ cm (i.e., one-tenth of a cm).}$$

How many cm is (a) 5 mm? (b) 12 mm?

$$5 \text{ mm} = \frac{5}{10} \text{ cm} = 0.5 \text{ cm}$$

$$12 \text{ mm} = 10 \text{ mm} + 2 \text{ mm}$$

$$= 1 \text{ cm} + \frac{2}{10} \text{ cm}$$

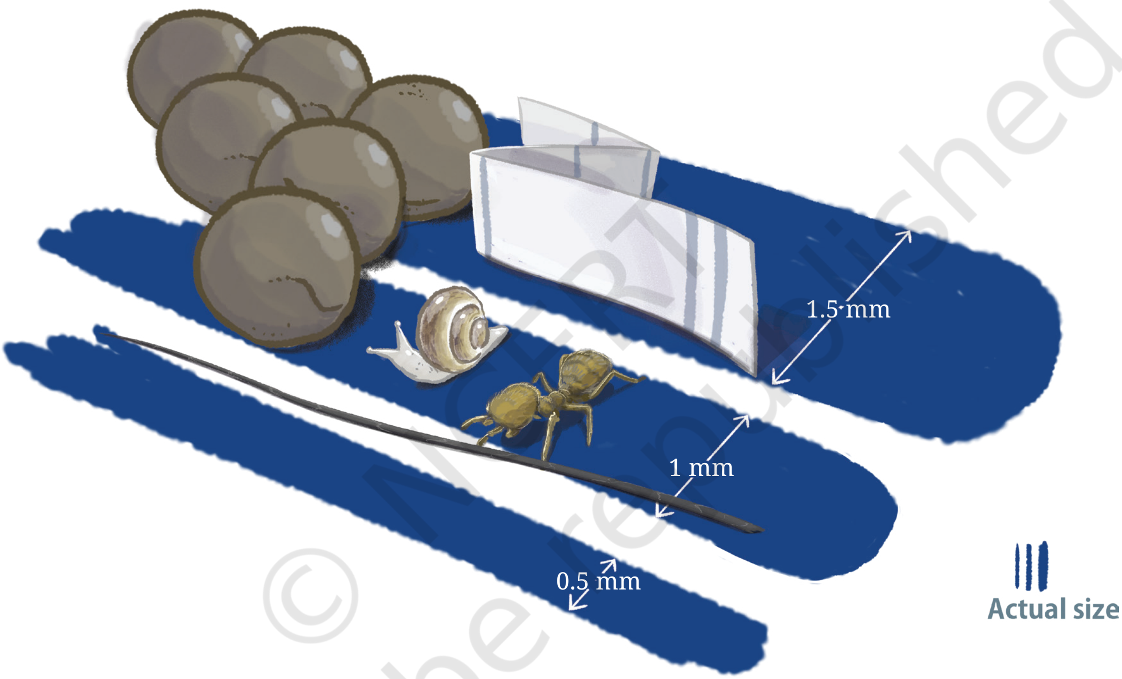
$$= 1.2 \text{ cm.}$$

How many mm is 5.6 cm? Since each cm has 10 mm, 5.6 cm (5 cm + 0.6 cm) is 56 mm.

? Fill in the blanks below (mm <–> cm)

12 mm = 1.2 cm	56 mm = 5.6 cm	70 mm = ______
______ = 0.9 cm	134 mm =______	______ = 203.6 cm

The illustration below shows how small some things are! Try taking an approximate measurement of each.



- The three blue stripes represent the typical relative sizes of pen strokes: fine stroke, medium stroke, and bold stroke.
- A human hair is about 0.1 mm in thickness.
- The thickness of a newspaper can range from 0.05 to 0.08 mm.
- Mustard seeds have a thickness of 1 – 2 mm.
- The smallest ant species discovered so far, Carabera Bruni, has a total length of 0.8 – 1 mm. They are found in Sri Lanka and China.
- The smallest land snail species discovered so far, Acmella Nana, has a shell diameter of 0.7 mm. They are found in Malaysia.

We also know that 1 m = 100 cm. Based on this, we can say that

$$1 \text{ cm} = \frac{1}{100} \text{ m} = 0.01 \text{ m}.$$

? How many m is (a) 10 cm? (b) 15 cm?

$10 \text{ cm} = \frac{1}{10} \text{ m} = 0.1 \text{ m}$

Since each cm is one-hundredth of a meter, 15 cm can be written as

$15 \text{ cm} = \frac{15}{100} \text{ m}$

$= \frac{10}{100} \text{ m} + \frac{5}{100} \text{ m}$

$= \frac{1}{10} \text{ m} + \frac{5}{100} \text{ m}$

$= 0.15$ m.

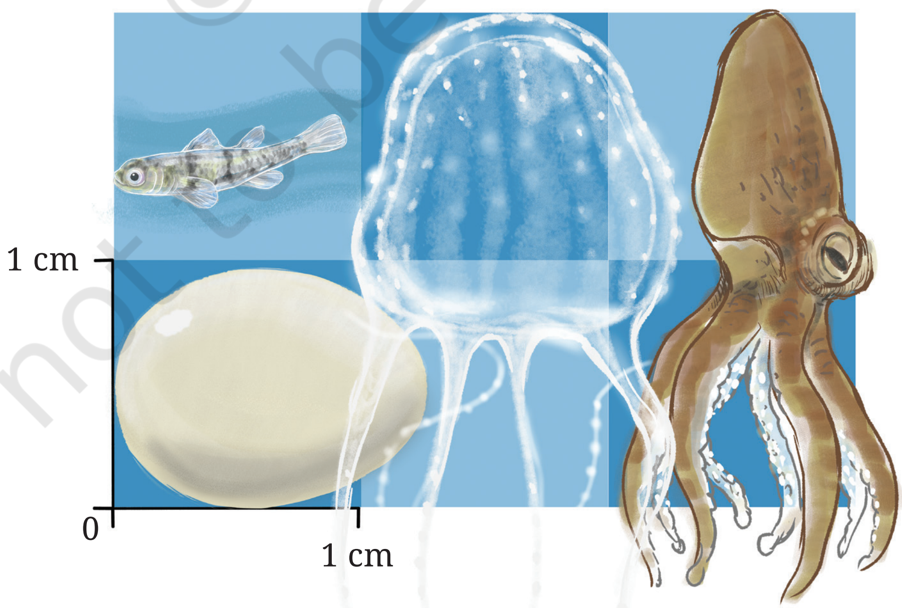
? Fill in the blanks below (cm <–> m):

36 cm = _______	50 cm = _______	_______ = 0.89 m
4 cm = _______	325 cm = _______	________ = 2.07 m

? How many mm does 1 meter have?

? Can we write $1 \text{ mm} = \frac{1}{1000} \text{ m}$?

Here, we have some more interesting facts about small things in nature!



- The egg of a hummingbird typically is 1.3 cm long and 0.9 cm wide.
- The Philippine Goby is about 0.9 cm long. It can be found in the Philippines and other Southeast Asian countries.
- The smallest known jellyfish, Irukandji, has a bell size of 0.5 – 2.5 cm. Its tentacles can be as long as 1 m. They are found in Australia. Its venom can be fatal to humans.
- The Wolfi octopus, also known as the Star-sucker Pygmy Octopus, is the smallest known octopus in the world. Their typical size is around 1 – 2.5 cm and they weigh less than 1 gm. They are found in the Pacific Ocean.

Weight Conversion

Let us look at kilograms (kg). We know that 1 kg = 1000 gram (g). We can say that

$$1 \text{ g} = \frac{1}{1000} \text{ kg} = 0.001 \text{ kg}.$$

? How many kilograms is 5 g?

$$5 \text{ g} = \frac{5}{1000} \text{ kg} = 0.005 \text{ kg}.$$

? How many kilograms is 10 g?

$$10 \text{ g} = \frac{10}{1000} \text{ kg} = \frac{1}{100} \text{ kg} = 0.010 \text{ kg}.$$

As each gram is one-thousandth of a kg, 254 g can be written as

$$254 \text{ g} = \frac{254}{1000} \text{ kg}$$

$$= \left(\frac{200}{1000} + \frac{50}{1000} + \frac{4}{1000}\right) \text{ kg}$$

$$= \left(\frac{2}{10} + \frac{5}{100} + \frac{4}{1000}\right) \text{ kg}$$

$$= 0.254 \text{ kg}.$$

? Fill in the blanks below (g <–> kg)

465 g = ______	68 g = ________	1560 g = _______
704 g = ______	_______ = 0.56 kg	______ = 2.5 kg

Look at the picture below showing different quantities of rice. Starting from the 1g heap, subsequent heaps can be found that are 10 times heavier than the previous heap/packets. The combined weight of rice in this picture is 11.111 kg.

Also,

$$1 \text{ gram} = 1000 \text{ milligrams (mg). So, } 1 \text{ mg} = \frac{1}{1000} \text{ g} = 0.001 \text{ g.}$$

Rupee—Paise conversion

You may have heard of 'paisa'. 100 paise is equal to 1 rupee. As we have coins and notes for rupees, coins for paise were also used commonly until recently. There were coins for 1 paisa, 2 paise, 3 paise, 5 paise, 10 paise, 20 paise, 25 paise, and 50 paise. All denominations of 25 paise and less were removed from use in the year 2011. But we still see paise in bills, account statements, etc.

$$1 \text{ rupee} = 100 \text{ paise}$$

$$1 \text{ paisa} = \frac{1}{100} \text{ rupee} = 0.01 \text{ rupee}$$

As each paisa is one-hundredth of a rupee,

$$75 \text{ paise} = \frac{75}{100} \text{ rupee}$$

$$= \left(\frac{70}{100} + \frac{5}{100}\right) \text{rupee}$$

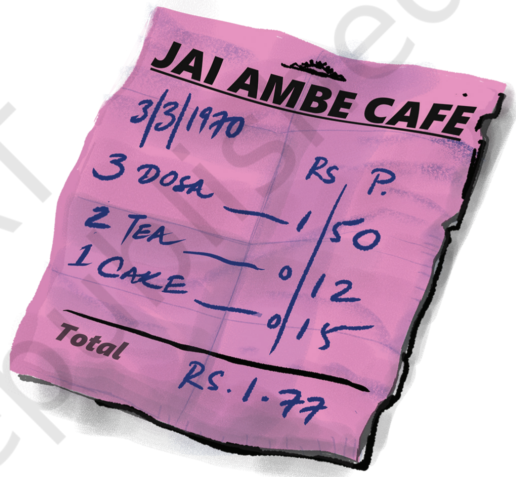
$$= \left(\frac{7}{100} + \frac{5}{100}\right) \text{rupee}$$

$$= 0.75 \text{ rupee.}$$

? Fill in the blanks below (rupee <–> paise)

10 p = ________	______p = ₹ 0.05	______p = ₹ 0.36
________ = ₹ 0.50	99 p = ________	250 p = ________

During the 1970s, a masala dosa cost just 50 paise, one could buy a banana for 20-25 paise, a handful of peppermints were available for 2 paise or 3 paise, and a kg of rice cost ₹2.45.

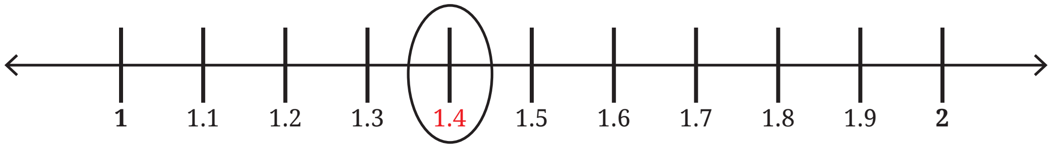


Discuss with adults at home/school the prices of different products and services during their childhood. Try to find old coins and stamps.

Try This

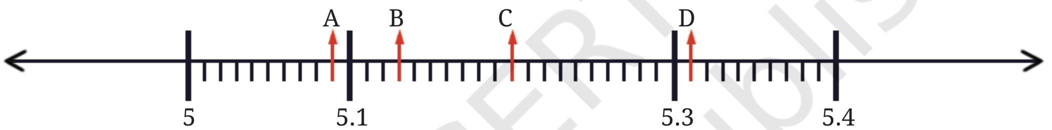
3.6 Locating and Comparing Decimals

Let us consider the decimal number 1.4. It is equal to 1 unit and 4 tenths. This means that the unit between 1 and 2 is divided into 10 equal parts, and 4 such parts are taken. Hence, 1.4 lies between 1 and 2. Draw the number line and divide the unit between 1 and 2 into 10 equal parts. Take the fourth part, and we have 1.4 on the number line.



? Name all the divisions between 1 and 1.1 on the number line.

? Identify and write the decimal numbers against the letters.



There is Zero Dilemma!

? Sonu says that 0.2 can also be written as 0.20, 0.200; Zara thinks that putting zeros on the right side may alter the value of the decimal number. What do you think?

We can figure this out by looking at the quantities these numbers represent using place value.

Decimal number	Units	Tenths	Hundredths	Thousandths
0.2	0	2		
0.20	0	2	0	
0.200	0	2	0	0
0.02	0	0	2	
0.002	0	0	0	2

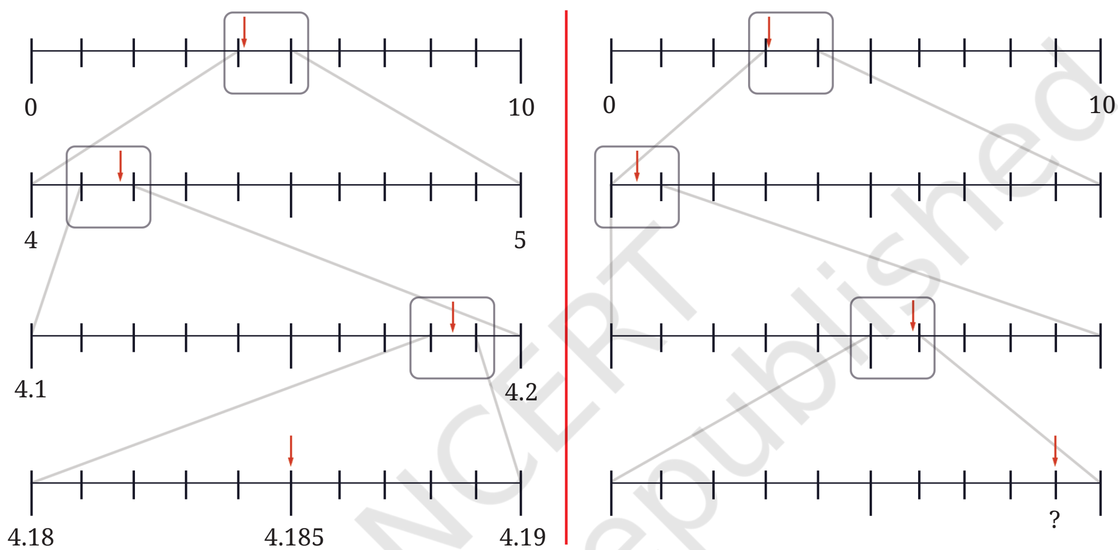
We can see that 0.2, 0.20, and 0.200 are all equal as they represent the same quantity, i.e., 2 tenths. But 0.2, 0.02, and 0.002 are different.

? Can you tell which of these is the smallest and which is the largest?

? Which of these are the same: 4.5, 4.05, 0.405, 4.050, 4.50, 4.005, 04.50?

Observe the number lines in Figure (a) below. At each level, a particular segment of the number line is magnified to locate the number 4.185.

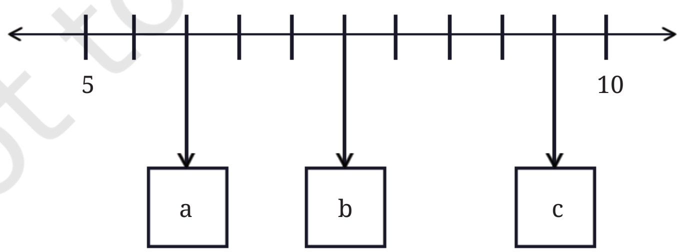
? Identify the decimal number in the last number line in Figure (b) denoted by '?'.



(a) (b)

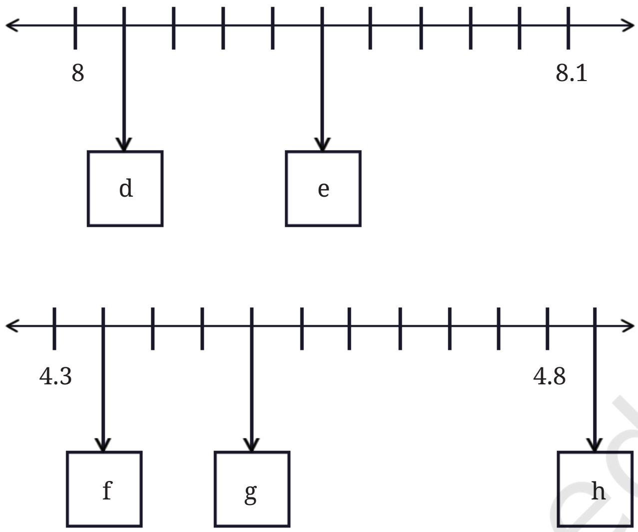
? Make such number lines for the decimal numbers: (a) 9.876 (b) 0.407.

? In the number line shown below, what decimal numbers do the boxes labelled 'a', 'b', and 'c' denote?



The box with 'b' corresponds to the decimal number 7.5; are you able to see how? There are 5 units between 5 and 10, divided into 10 equal parts. Hence, every 2 divisions make a unit, and so every division is $\frac{1}{2}$ unit. What numbers do 'a' and 'c' denote?

? Using similar reasoning find out the decimal numbers in the boxes below.

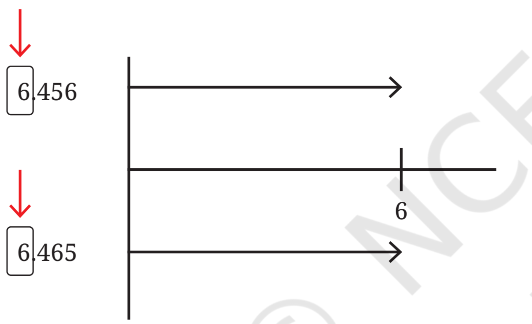
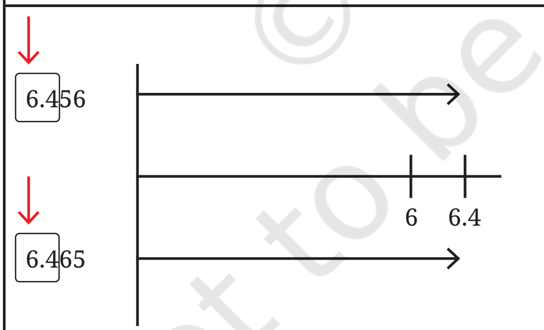
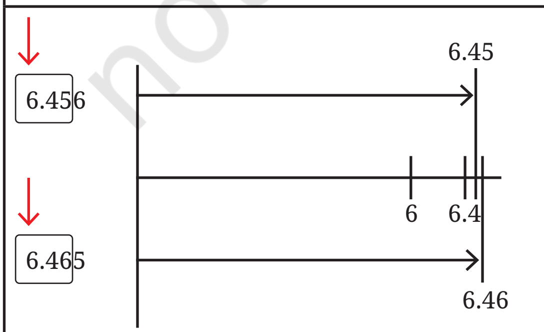


? Which is larger: 6.456 or 6.465?

To answer this, we can use the number line to locate both decimal numbers and show which is larger.

This can also be done by comparing the corresponding digits at each place value, as we do with whole numbers.

This comparison is visualised step by step below. Note that the visualisation below is not to scale.

	Both numbers have 6 units.
	Both numbers have 6 units and 4 tenths.
	Both numbers have 6 units and 4 tenths, but the first number has only 5 hundredths, whereas the second number has 6 hundredths.

We start by comparing the most significant digits (digits with the highest place value) of the two numbers. If the digits are the same, we compare the next smaller place value. We keep going till we find a position where the digits are not equal. The number with the larger digit at this position is the greater of the two.

? Why can we stop comparing at this point? Can we be sure that whatever digits are there after this will not affect our conclusion?

Math Talk

Which decimal number is greater?

(a) 1.23 or 1.32

(b) 3.81 or 13.800

(c) 1.009 or 1.090

Closest Decimals

Consider the decimal numbers 0.9, 1.1, 1.01, and 1.11. Identify the decimal number that is closest to 1.

Let us compare the decimal numbers. Arranging these in ascending order, we get $0.9 < 1 < 1.01 < 1.1 < 1.11$. Among the neighbours of 1, 1.01 is 1/100 away from 1 whereas 0.9 is 10/100 away from 1. Therefore, 1.01 is closest to 1.

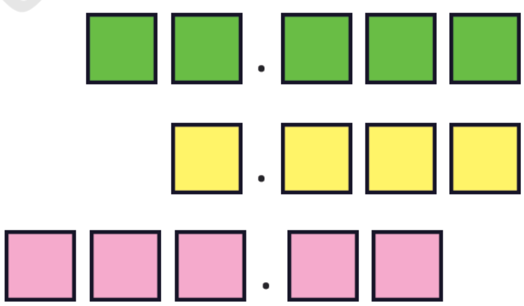
? Which of the above is closest to 1.09?

? Which among these is closest to 4: 3.56, 3.65, 3.099?

? Which among these is closest to 1: 0.8, 0.69, 1.08?

? In each case below use the digits 4, 1, 8, 2, and 5 exactly once and try to make a decimal number as close as possible to 25.

Math Talk



3.7 Addition and Subtraction of Decimals

? Priya requires 2.7 m of cloth for her skirt, and Shylaja requires 3.5m for her kurti. What is the total quantity of cloth needed?

We have to find the sum of 2.7m + 3.5m.

Earlier, we saw how to add $2\frac{7}{10} + 3\frac{5}{10}$ (also shown below). Can you carry out the same addition using decimal notation? It is shown below.

Share your observations.
The total quantity of cloth needed is 6.2 m.

$$\begin{array}{rl} & 2\frac{7}{10} \\ + & 3\frac{5}{10} \\ \hline = & 5\frac{12}{10} \\ = & 6\frac{2}{10} \end{array} \qquad \begin{array}{rl} & \overset{1}{2}.7 \\ + & 3.5 \\ \hline = & 6.2 \end{array}$$

? How much longer is Shylaja's cloth compared to Priya's?

We have to find the difference of 3.5m – 2.7m. Again, observe how the differences $3\frac{5}{10} - 2\frac{7}{10}$ and 3.5m – 2.7m are computed.

$$\begin{array}{rl} & 3\frac{5}{10} \\ - & 2\frac{7}{10} \\ \hline & \end{array} \longrightarrow \begin{array}{rl} & 2\frac{15}{10} \\ - & 2\frac{7}{10} \\ \hline = & 0\frac{8}{10} \end{array} \qquad \begin{array}{rl} & 3.5 \\ - & 2.7 \\ \hline & \end{array} \longrightarrow \begin{array}{rl} & \cancel{3}^{2}.5^{1} \\ - & 2.7 \\ \hline = & 0.8 \end{array}$$

As you can see, the standard procedure for adding and subtracting whole numbers can be used to add and subtract decimals.

A detailed view of the underlying place value calculation is shown below for the sum 75.345 + 86.691. Its compact form is shown next to it.

$$
\begin{array}{lllllll}
 & 1 \times 100 & 1 \times 10 & 1 \times 1 & 1 \times \frac{1}{10} & & \\
 & & 7 \times 10 & 5 \times 1 & 3 \times \frac{1}{10} & 4 \times \frac{1}{100} & 5 \times \frac{1}{1000} \\
+ & & 8 \times 10 & 6 \times 1 & 6 \times \frac{1}{10} & 9 \times \frac{1}{100} & 1 \times \frac{1}{1000} \\
\hline
= & 1 \times 100 & 16 \times 10 & 12 \times 1 & 10 \times \frac{1}{10} & 13 \times \frac{1}{100} & 6 \times \frac{1}{1000}
\end{array}
$$

$$
\begin{array}{r}
11\ 1 \\
75.345 \\
+86.691 \\
\hline
=162.036
\end{array}
$$

? Write the detailed place value computation for 84.691 – 77.345, and its compact form.

Try This

? Figure it Out

1. Find the sums

   (a) 5.3 + 2.6 (b) 18 + 8.8

   (c) 2.15 + 5.26 (d) 9.01 + 9.10

   (e) 29.19 + 9.91 (f) 0.934 + 0.6

   (g) 0.75 + 0.03 (h) 6.236 + 0.487

2. Find the differences

   (a) 5.6 – 2.3 (b) 18 – 8.8

   (c) 10.4 – 4.5 (d) 17 – 16.198

   (e) 17 – 0.05 (f) 34.505 – 18.1

   (g) 9.9 – 9.09 (h) 6.236 – 0.487

Decimal Sequences

Observe this sequence of decimal numbers and identify the change after each term.

4.4, 4.8. 5.2, 5.6, 6.0, ...

We can see that 0.4 is being added to a term to get the next term.

? Continue this sequence and write the next 3 terms.

? Similarly, identify the change and write the next 3 terms for each sequence given below. Try to do this computation mentally.

(a) 4.4, 4.45, 4.5, ...

(b) 25.75, 26.25, 26.75, ...

(c) 10.56, 10.67, 10.78, ...

(d) 13.5, 16, 18.5, ...

(e) 8.5, 9.4, 10.3, ...

(f) 5, 4.95, 4.90, ...

(g) 12.45, 11.95, 11.45, ...

(h) 36.5, 33, 29.5, ...

? Make your own sequences and challenge your classmates to extend the pattern.

Estimating Sums and Differences

Sonu has observed sums and differences of decimal numbers and says, "If we add two decimal numbers, then the sum will always be greater than the sum of their whole number parts. Also, the sum will always be less than 2 more than the sum of their whole number parts."

Let us use an example to understand what his claim means:

If the two numbers to be added are 25.936 and 8.202, the claim is that their sum will be greater than 25 + 8 (whole number parts) and will be less than 25 + 1 + 8 + 1.

? What do you think about this claim? Verify if this is true for these numbers. Will it work for any 2 decimal numbers?

Math Talk

? What about for the sum of 25.93603259 and 8.202?

? Similarly, come up with a way to narrow down the range of whole numbers within which the difference of two decimal numbers will lie.

Try This

Note to the Teacher: Estimating the result before computing may help in identifying if a mistake happens with the calculation.

3.8 More on the Decimal System

Decimal and Measurement Disasters

Decimal point and unit conversion mistakes may seem minor sometimes but they can lead to serious problems. Here are some actual incidents in which such errors caused major issues.

- In 2013, the finance office of Amsterdam City Council (Netherlands) mistakenly sent out €188 million in housing benefits instead of the intended €1.8 million due to a programming error that processed payments in euro cents instead of euros. (1 euro-cent = 1/100 euro).
- In 1983, a decimal error nearly caused a disaster for an Air Canada Boeing 767. The ground staff miscalculated the fuel, loading 22,300 pounds instead of kilograms—about half of what was needed (1 pound ~ 0.453 kg). The plane ran out of fuel mid-air, forcing the pilots to make an emergency landing at an abandoned airfield. Fortunately, everyone survived.

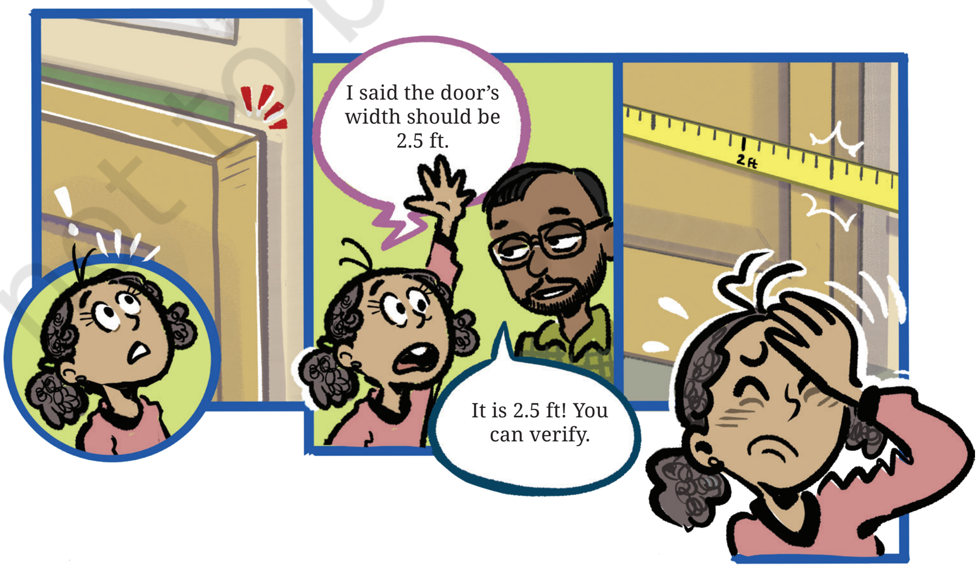
Several incidents have occurred due to incorrect reading of decimal numbers while giving medication. For example, reading 0.05 mg as 0.5 mg can lead to using a medicine 10 times more than the prescribed quantity. It is therefore important to pay attention to units and the location of the decimal point.

Deceptive Decimal Notation

Sarayu gets a message: "The bus will reach the station 4.5 hours post noon." When will the bus reach the station: 4:05 p.m., 4:50 p.m., 4:25 p.m.?

None of these! Here, 0.5 hours means splitting an hour into 10 equal parts and taking 5 parts out of it. Each part will be 6 minutes (60 minutes/10) long. 5 such parts make 30 minutes. So, the bus will reach the station at 4:30.

Here is a short-story of a decimal mishap: A girl measures the width of an opening as 2 ft 5 inches but conveys to the carpenter to make a door 2.5 ft wide. The carpenter makes a door of width 2 ft 6 inches (since 1 ft = 12 inches, 0.5 ft = 6 inches), and it wouldn't close fully.



If you watch cricket, you might have noticed decimal-looking numbers like 'Overs left: 5.5'. Does this mean 5 overs and 5 balls or 5 overs and 3 balls? Here, 5.5 overs means $5\frac{5}{6}$ overs (as 1 over = 6 balls), i.e., 5 overs and 5 balls.

Where else can we see such 'non-decimals' with a decimal-like notation?

Math Talk

A Pinch of History – Decimal Notation Over Time

Decimal fractions (i.e., fractions with denominators like $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, and so on) are used in the works of a number of ancient Indian astronomers and mathematicians, including in the important 8th century works of Śhrīdharāchārya on arithmetic and algebra. Decimal notation, in essentially its modern form, was described in detail in *Kitāb al-Fuṣūl fī al-Ḥisāb al Hindī* (The Book of Chapters on Indian Arithmetic) by Abūl Ḥassan al-Uqlīdisī, an Arab mathematician, in around 950 CE. He represented the number 0.059375 as $\overset{|}{0}059375$.

In the 15th century, to separate whole numbers from fractional parts, a number of different notations were used:

- a vertical mark on the last digit of the whole number part (as shown above),
- use of different colours and
- a numerical superscript giving the number of fractional decimal places (0.36 would be written as $3\overset{2}{6}$).

In the 16th century, John Napier, a Scottish mathematician, and Christopher Clavius, a German mathematician, used the point/period ('.') to separate the whole number and the fractional parts, while François Viète, a French mathematician, used the comma (',') instead.

Currently, several countries use the comma to separate the integer part and the fractional part. In these countries, the number 1,000.5 is written as 1 000,5 (space as a thousand separator). But the decimal point has endured as the most popular notation for writing numbers having fractional parts in the Indian place value system.

Figure it Out

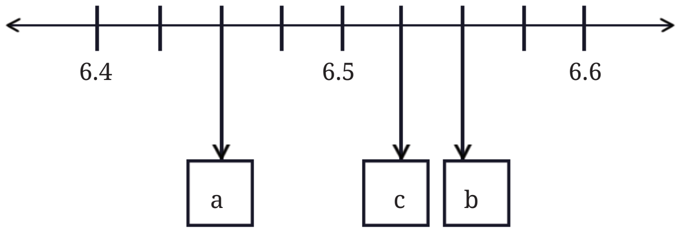
1. Convert the following fractions into decimals:

(a) $\frac{5}{100}$ (b) $\frac{16}{1000}$ (c) $\frac{12}{10}$ (d) $\frac{254}{1000}$

2. Convert the following decimals into a sum of tenths, hundredths and thousandths:

   (a) 0.34 (b) 1.02 (c) 0.8 (d) 0.362

3. What decimal number does each letter represent in the number line below?



4. Arrange the following quantities in descending order:
   (a) 11.01, 1.011, 1.101, 11.10, 1.01
   (b) 2.567, 2.675, 2.768, 2.499, 2.698
   (c) 4.678 g, 4.595 g, 4.600 g, 4.656 g, 4.666 g
   (d) 33.13 m, 33.31 m, 33.133 m, 33.331 m, 33.313 m
5. Using the digits 1, 4, 0, 8, and 6 make:
   (a) the decimal number closest to 30
   (b) the smallest possible decimal number between 100 and 1000.
6. Will a decimal number with more digits be greater than a decimal number with fewer digits?
7. Mahi purchases 0.25 kg of beans, 0.3 kg of carrots, 0.5 kg of potatoes, 0.2 kg of capsicums, and 0.05 kg of ginger. Calculate the total weight of the items she bought.
8. Pinto supplies 3.79 L, 4.2 L, and 4.25 L of milk to a milk dairy in the first three days. In 6 days, he supplies 25 litres of milk. Find the total quantity of milk supplied to the dairy in the last three days.
9. Tinku weighed 35.75 kg in January and 34.50 kg in February. Has he gained or lost weight? How much is the change?
10. Extend the pattern: 5.5, 6.4, 6.39, 7.29, 7.28, 6.18, 6.17, ____, _____
11. How many millimeters make 1 kilometer?
12. Indian Railways offers optional travel insurance for passengers who book e-tickets. It costs 45 paise per passenger. If 1 lakh people opt for insurance in a day, what is the total insurance fee paid?
13. Which is greater?

   (a) $\frac{10}{1000}$ or $\frac{1}{10}$?

(b) One-hundredth or 90 thousandths?

(c) One-thousandth or 90 hundredths?

14. Write the decimal forms of the quantities mentioned (an example is given):

    (a) 87 ones, 5 tenths and 60 hundredths = 88.10

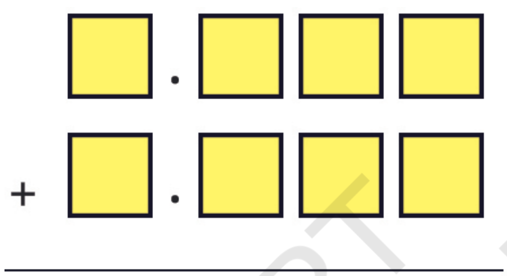
    (b) 12 tens and 12 tenths

    (c) 10 tens, 10 ones, 10 tenths, and 10 hundredths

    (d) 25 tens, 25 ones, 25 tenths, and 25 hundredths

Try This

15. Using each digit 0 – 9 not more than once, fill the boxes below so that the sum is closest to 10.5:



16. Write the following fractions in decimal form:

    (a) $\frac{1}{2}$ (b) $\frac{3}{2}$

    (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

    (e) $\frac{1}{5}$ (f) $\frac{4}{5}$

SUMMARY

- We can split a unit into smaller parts to get more exact/accurate measurements.
- We extended the Indian place value system and saw that
  - » 1 unit = 10 one-tenths,
  - » 1 tenth = 10 one-hundredths,
  - » 1 hundredth = 10 one-thousandths,
  - » 10 one-hundredths = 1 tenth,
  - » 100 one-hundredths = 1 unit.
- A decimal point ('.') is used in the Indian place value system to separate the whole number part of a number from its fractional part.
- We also learnt how to compare decimal numbers, locate them on the number line, and perform addition and subtraction on them.

4 EXPRESSIONS USING LETTER-NUMBERS

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4.1 The Notion of Letter-Numbers

In this chapter we shall look at a concise way of expressing mathematical relations and patterns. We shall see how this helps us in thinking about these relationships and patterns, and in explaining why they may hold true.

Example 1: Shabnam is 3 years older than Aftab. When Aftab's age 10 years, Shabnam's age will be 13 years. Now Aftab's age is 18 years, what will Shabnam's age be? _______

Given Aftab's age, how will you find out Shabnam's age?

Easy: We add 3 to Aftab's age to get Shabnam's age.

Can we write this as an expression?

Shabnam's age is 3 years more than Aftab's. In short, this can be written as:

Shabnam's age = **Aftab's age** + **3.**

Such mathematical relations are generally represented in a shorthand form. In the relation above, instead of writing the phrase 'Aftab's Age', the convention is to use a convenient symbol. Usually, letters or short phrases are used for this purpose.

Let us say we use the letter a to denote Aftab's age (we could have used any other letter), and s to denote Shabnam's age. Then the expression to find Shabnam's age will be $a + 3$, which can be written as

Aftab's age	Expression for Shabnam's age
4	4 + 3
10	10 + 3
23	23 + 3
?	? + 3
a	a + 3

Fig. 4.1

$$s = a + 3.$$

If a is 23 (Aftab's age in years), then what is Shabnam's age?

Replacing a by 23 in the expression $a + 3$, we get, $s = 23 + 3 = 26$ years.

Letters such as a and s that are used to represent numbers are called **letter-numbers**. Mathematical expressions containing letter-numbers, such as the expression $a + 3$, are called **algebraic expressions**.

? Given the age of Shabnam, write an expression to find Aftab's age.

We know that Aftab is 3 years younger than Shabnam. So, Aftab's age will be 3 less than Shabnam's. This can be described as

Aftab's age = Shabnam's age – 3.

If we again use the letter a to denote Aftab's age and the letter s to denote Shabnam's age, then the algebraic expression would be: $a = s - 3$, meaning 3 less than s.

? Use this expression to find Aftab's age if Shabnam's age is 20.

? **Example 2:** Parthiv is making matchstick patterns. He repeatedly places Ls next to each other. Each L has two matchsticks as shown in Figure 4.2.

Fig. 4.2

How many matchsticks are needed to make 5 Ls? It will be 5×2.

How many matchsticks are needed to make 7 Ls? It will be 7×2.

How many matchsticks are needed to make 45 Ls? It will be 45×2.

Now, what is the relation between the number of Ls and the number of sticks?

First, let us describe the relationship or the pattern here. Every L needs 2 matchsticks. So **the number of matchsticks needed will be 2 times the number of L's**. This can be written as:

Number of matchsticks = 2 × Number of L's

Now, we can use any letter to denote the number of L's. Let's use n. The algebraic expression for the number of matchsticks will be:

$$2 \times n.$$

This expression tells us how many matchsticks are needed to make n L's. To find the number of matchsticks, we just replace n by the number of L.

? **Example 3:** Ketaki prepares and supplies coconut-jaggery laddus. The price of a coconut is ₹35 and the price of 1 kg jaggery is ₹60.

? How much should she pay if she buys 10 coconuts and 5 kg jaggery?

Cost of 10 coconuts = 10 × ₹35
Cost of 5 kg jaggery = 5 × ₹60
Total cost = 10 × ₹35 + 5 × ₹60 = ₹350 + ₹300 = ₹650.

? How much should she pay if she buys 8 coconuts and 9 kg jaggery?

? Write an algebraic expression to find the total amount to be paid for a given number of coconuts and quantity of jaggery.

Let us identify the relationships and then write the expressions.

Quantity needed	Relationship	Expression
Cost of coconuts	Number of coconuts × 35	$c \times 35$
Cost of jaggery	Number of kgs of jaggery × 60	$j \times 60$

Here, '*c*' represents the number of coconuts and '*j*' represents the number of kgs of jaggery. The total amount to be paid will be:

Cost of coconuts + Cost of jaggery.

The corresponding algebraic expression can be written as:

$$c \times 35 + j \times 60$$

? Use this expression (or formula) to find the total amount to be paid for 7 coconuts and 4 kg jaggery.

Notice that for different values of '*c*' and '*j*', the value of the expression also changes.

Writing this expression as a sum of terms we get:

$$\boxed{c \times 35} + \boxed{j \times 60}$$

? **Example 4:** We are familiar with calculating the perimeters of simple shapes. Write expressions for perimeters.

The perimeter of a square is **4 times the length of its side**. This can be written as the expression: **4 ×** $\boldsymbol{q}$, where q stands for the sidelength.

? What is the perimeter of a square with sidelength 7 cm? Use the expression to find out.

You must have realised how the use of letter-numbers and algebraic expressions allows us to express general mathematical relations in a concise way. Mathematical relations expressed this way are often called formulas.

Figure it Out

1. Write formulas for the perimeter of:
   - (a) triangle with all sides equal.
   - (b) a regular pentagon (as we have learnt last year, we use the word 'regular' to say that all sidelengths and angle measures are equal)
   - (c) a regular hexagon
2. Munirathna has a 20 m long pipe. However, he wants a longer watering pipe for his garden. He joins another pipe of some length to this one. Give the expression for the combined length of the pipe. Use the letter-number '*k*' to denote the length in meters of the other pipe.
3. What is the total amount Krithika has, if she has the following numbers of notes of ₹100, ₹20 and ₹5? Complete the following table:

No. of ₹100 notes	No. of ₹20 notes	No. of ₹5 notes	Expression and total amount
3	5	6	
			$6 \times 100 + 4 \times 20 + 3 \times 5 = 695$
8	4	z	
x	y	z	

4. Venkatalakshmi owns a flour mill. It takes 10 seconds for the roller mill to start running. Once it is running, each kg of grain takes 8 seconds to grind into powder. Which of the expressions below describes the time taken to complete grind '*y*' kg of grain, assuming the machine is off initially?
   - (a) $10 + 8 + y$
   - (b) $(10 + 8) \times y$
   - (c) $10 \times 8 \times y$
   - (d) $10 + 8 \times y$
   - (e) $10 \times y + 8$
5. Write algebraic expressions using letters of your choice.
   - (a) 5 more than a number
   - (b) 4 less than a number

(c) 2 less than 13 times a number

(d) 13 less than 2 times a number

6. Describe situations corresponding to the following algebraic expressions:

(a) $8 \times x + 3 \times y$

(b) $15 \times j - 2 \times k$

7. In a calendar month, if any 2 × 3 grid full of dates is chosen as shown in the picture, write expressions for the dates in the blank cells if the bottom middle cell has date ‘*w*’.

November 2024

Mon	Tue	Wed	Thu	Fri	Sat	Sun
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

w – 1	w	

4.2 Revisiting Arithmetic Expressions

We learnt to write expressions as sums of terms and it became easy for us to read arithmetic expressions. Many times they could have been read in multiple ways and it was confusing. We used **swapping** (adding two numbers in any order) and **grouping** (adding numbers by grouping them conveniently) to find easy ways of evaluating expressions. Swapping and grouping terms does not change the value of the expression. We also learnt to use brackets in expressions, including brackets with a negative sign outside. We learnt the **distributive property** (multiple of a sum is the same as sum of multiples).

Let us revise these concepts and find the values of the following expressions:

1. $23 - 10 \times 2$
2. $83 + 28 - 13 + 32$
3. $34 - 14 + 20$
4. $42 + 15 - (8 - 7)$
5. $68 - (18 + 13)$
6. $7 \times 4 + 9 \times 6$
7. $20 + 8 \times (16 - 6)$

Let us evaluate the first expression, 23 – 10 × 2. First we shall write the terms of the expression. Notice that one of the terms is a number, while the other one has to be converted to a number before adding the two terms.

$$23 - 10 \times 2 = 23 + -10 \times 2 = 23 + -20 = 3$$

Let us now evaluate the second one. All the terms of this expression are numbers. If we notice the terms, we find that it will be easier to evaluate if we swap and group the terms.

$$83 + 28 - 13 + 32 =$$

$$\boxed{83} + \boxed{28} + \boxed{-13} + \boxed{32}$$

$$= \boxed{70} + \boxed{60} = \boxed{130}$$

Let us now look at the fifth expression. It has brackets with a negative sign outside. This can be evaluated in two ways — by solving the bracket first (like the solution on the left side) or by removing the brackets appropriately (as on the right side).

$$= \boxed{68} + \boxed{-(18 + 13)}$$

$$= \boxed{68} + \boxed{-31}$$

$$= \boxed{37}$$

OR

$$= \boxed{68} + \boxed{-(18 + 13)}$$

$$= \boxed{68} + \boxed{-18} + \boxed{-13}$$

$$= \boxed{50} + \boxed{-13} = \boxed{37}$$

Now, find the values of the other arithmetic expressions.

Algebraic expressions also take number values when the letter-numbers they contain are replaced by numbers. In Example 1, for finding Shabnam's age when Aftab is 23 years old, we replaced the letter-number a in the expression $a + 3$ by 23, and it took the value 26.

4.3 Omission of the Multiplication Symbol in Algebraic Expressions

Look at this number sequence:

4, 8, 12, 16, 20, 24, 28, ...

How can we describe this sequence or pattern? Easy: These are the numbers appearing in the multiplication table of 4 (multiples of 4 in an increasing order).

What is the third term of this sequence? It is 4×3.

What is the 29th term of this sequence? It is 4×29.

? Find an algebraic expression to get the nth term of this sequence.

Note that here 'n' is a letter-number that denotes a position in the sequence.

As it is the sequence of multiples of 4, it can be seen that the nth term will be 4 times n:

$$4 \times n$$

As a standard practice, we shorten $4 \times n$ to $4n$ by skipping the multiplication sign. We write the number first, followed by the letter(s). Find the value of the expression $7k$ when $k = 4$. The value is $7 \times 4 = 28$.

Find the value that the expression $5m + 3$ takes when $m = 2$.

As $5m$ stands for $5 \times m$, the value of the expression when $m = 2$ is $5 \times 2 + 3 = 13$.

Mind the Mistake, Mend the Mistake

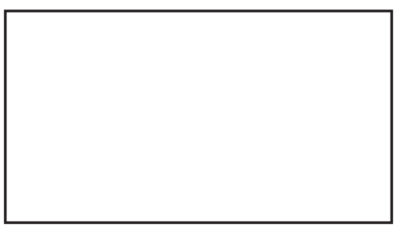
Some simplifications are shown below where the letter-numbers are replaced by numbers and the value of the expression is obtained.

1. Observe each of them and identify if there is a mistake.
2. If you think there is a mistake, try to explain what might have gone wrong.
3. Then, correct it and give the value of the expression.

1 If $a = -4$, then $10 - a = 6$.	**2** If $d = 6$, then $3d = 36$.	**3** If $s = 7$, then $3s - 2 = 15$.
4 If $r = 8$, then $2r + 1 = 29$.	**5** If $j = 5$, then $2j = 10$.	**6** If $m = -6$, then $3(m + 1) = 19$.
7 If $f = 3$, $g = 1$ then $2f - 2g = 2$.	**8** If $t = 4$, $b = 3$ then $2t + b = 24$.	**9** If $h = 5$, $n = 6$ then $h - (3 - n) = 4$.

4.4 Simplification of Algebraic Expressions

Earlier we found expressions to find perimeters of different regular figures in terms of their sides. Let us now find an expression to find the perimeter of a rectangle.



As in the previous cases, we will first describe how to get the perimeter when the length and the breadth of the rectangle are known:

Find the sum of length + breadth + length + breadth.

Let us use the letter-numbers l and b in place of length and breadth respectively. Let p denote the perimeter of the rectangle. Then we have

$$p = l + b + l + b$$

As we know, these represent numbers, and so the terms of an expression can be added in any order. Hence the above expression can be written as:

$$= l + l + b + b$$

As $l + l = 2 \times l = 2l$, and $b + b = 2 \times b = 2b$, we have

$$p = 2l + 2b.$$

Notice that the initial expression ($l + b + l + b$) and the final expression ($2l + 2b$) that we got for the perimeter look different. However, they are equal since the expression was obtained from the initial one by applying the same rules and operations we do for numbers; they are equal in the sense that they both take the same values when letter-numbers are replaced by numbers.

For example, if we assign $l = 3$, $b = 4$, we get

$$l + b + l + b = 3 + 4 + 3 + 4 = 14, \text{ and}$$

$$2l + 2b = 2 \times 3 + 2 \times 4 = 14.$$

We call the expression $2l + 2b$ the **simplified form** of $l + b + l + b$.

Let us see some more examples of simplification.

Example 5: Here is a table showing the number of pencils and erasers sold in a shop. The price per pencil is c, and the price per eraser is d. Find the total money earned by the shopkeeper during these three days.

	Day 1	Day 2	Day 3
Pencils (Price 'c')	5	3	10
Erasers (Price 'd')	4	6	1

Let us first find the money earned by the sale of pencils.

The money earned by selling pencils on Day 1 is $5c$. Similarly, the money earned by selling pencils on Day 2 is _____, and Day 3 is ______.

The total money earned by the sale of pencils is $5c + 3c + 10c$. Can we simplify this expression further and reduce the number of terms?

The expression means 5 times c is added to 3 times c is added to 10 times c. So in total, the letter-number c is added (5 + 3 + 10) times. This is what we have seen as the distributive property of numbers. Thus,

$$5 \times c + 3 \times c + 10 \times c = (5 + 3 + 10) \times c$$

$(5 + 3 + 10) \times c$ can be simplified to $18 \times c = 18c$.

? If c = ₹50, find the total amount earned by the scale of pencils.

? Write the expression for the total money earned by selling erasers. Then, simplify the expression.

The expression for the total money earned by selling pencils and erasers during these three days is $18c + 11d$.

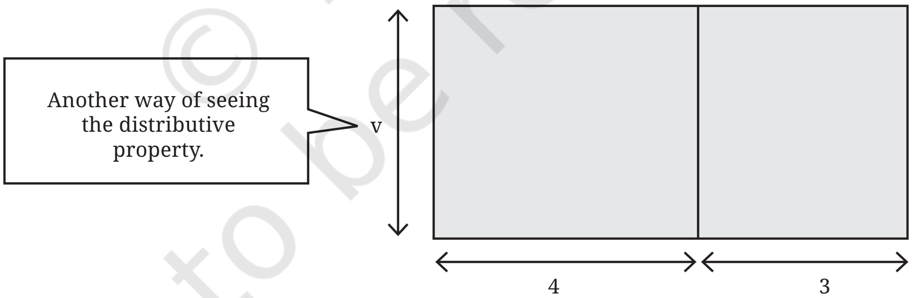
? Can the expression $18c + 11d$ be simplified further?

There is no way of further simplifying this expression as it contains different letter-numbers. It is in its simplest form.

In this problem, we saw the expression $5c + 3c + 10c$ getting simplified to the expression $18c$.

? Check that both expressions take the same value when c is replaced by different numbers.

? **Example 6:** A big rectangle is split into two smaller rectangles as shown. Write an expression describing the area of the bigger rectangle.



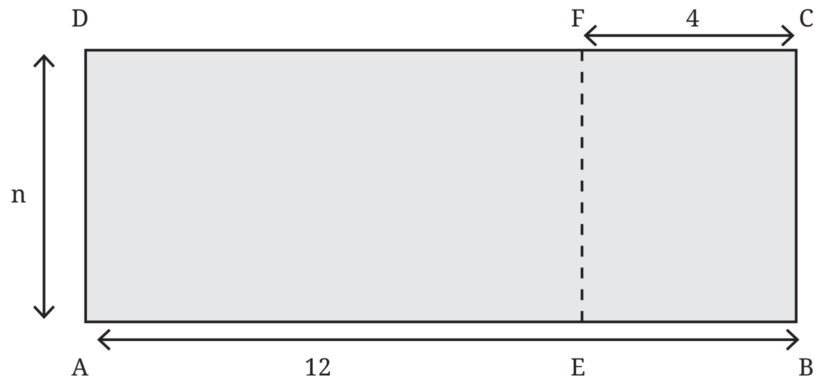
The areas of the smaller rectangles are $4v$ sq. units and $3v$ sq. units.

The area of the bigger rectangle can be found in two ways: (i) by directly using its side lengths v and $(4 + 3)$, or (ii) by adding the areas of the smaller rectangles.

The first way gives $7v$, and the second way gives $4v + 3v$. We know that they are equal: $4v + 3v = 7v$, and this is the required expression for the area of the bigger rectangle.

As earlier, a big rectangle is split into two smaller rectangles as shown below. Write an expression to find the area of the rectangle AEFD.

Even in this case, the area of rectangle AEFD can be found in two ways: (i) by directly using the side lengths n and (12 – 4), or (ii) subtracting the area of the rectangle EBCF from that of ABCD.



The first method gives us 8n, and the second method gives us $12n - 4n$, and they are equal, since $12n - 4n = 8n$. This is the expression for the area of the rectangle AEFD.

Sets of terms such as ($5c$, c, $10c$), ($12n$, $-4n$) that involve the same letter-numbers are called **like terms**. Sets of terms such as {$18c$, $11d$} are called **unlike terms** as they have different letter-numbers.

As we have seen, like terms can be added together and simplified into a single term.

Example 7: A shop rents out chairs and tables for a day's use. To rent them, one has to first pay the following amount per piece.

Item	Amount
Chair	₹40
Table	₹75

When the furniture is returned, the shopkeeper pays back some amount as follows.

	Amount returned
Chair	₹6
Table	₹10

Write an expression for the total number of rupees paid if x chairs and y tables are rented.

For x chairs and y tables, let us find the total amount paid at the beginning and the amount one gets back after returning the furniture.

Describe the procedure to get these amounts.

The total amount in rupees paid at the beginning is $40x + 75y$, and the total amount returned is $6x + 10y$.

So, the total amount paid = $(40x + 75y) - (6x + 10y)$.

Can we simplify this expression? If yes, how? If not, why not?

Math Talk

Recalling how we open brackets in an arithmetic expression, we get

$$(40x + 75y) - (6x + 10y) = (40x + 75y) - 6x - 10y$$

Since the terms can be added in any order, the remaining bracket can be opened and the expression becomes $40x + 75y + -6x + -10y$

We can group the like terms together, This results in

$$40x + -6x + 75y + -10y$$

$$= (40 - 6)x + (75 - 10)y$$

$$= 34x + 65y.$$

The expression $(40x + 75y) - (6x + 10y)$ is simplified to $34x + 65y$, which is the total amount paid in rupees.

? Could we have written the initial expression as $(40x + 75y) + (-6x - 10y)$?

Math Talk

? **Example 8:** Charu has been through three rounds of a quiz. Her scores in the three rounds are $7p - 3q$, $8p - 4q$, and $6p - 2q$. Here, p represents the score for a correct answer and q represents the penalty for an incorrect answer.

? What do each of the expressions mean?

If the score for a correct answer is 4 ($p = 4$) and the penalty for a wrong answer is 1 ($q = 1$), find Charu's score in the first round.

Charu's score is $7 \times 4 - 3 \times 1$. We can evaluate this expression by writing it as a sum of terms.

$7 \times 4 - 3 \times 1 = 7 \times 4 + -3 \times 1 = 28 + -3 = 25$

What are her scores in the second and third rounds?

What if there is no penalty? What will be the value of q in that situation?

What is her final score after the three rounds?

Her final score will be the sum of the three scores: $(7p - 3q) + (8p - 4q) + (6p - 2q)$.

Since the terms can be added in any order, we can remove the brackets and write

$7p + -3q + 8p + -4q + 6p + -2q$

$= 7p + 8p + 6p + -(3q) + -(4q) + -(2q)$ (by swapping and grouping)

$= (7 + 8 + 6)p + -(3 + 4 + 2)q$

$= 21p + -9q$

$= 21p - 9q.$

Charu's total score after three rounds is $21p - 9q$. Her friend Krishita's score after three rounds is $23p - 7q$.

Give some possible scores for Krishita in the three rounds so that they add up to give $23p - 7q$.

Can we say who scored more? Can you explain why?

How much more has Krishita scored than Charu? This can be found by finding the difference between the two scores.

$23p - 7q - (21p - 9q)$

Simplify this expression further.

Example 9: Simplify the expression $4\,(x + y) - y$

Using the distributive property, this expression can be simplified to

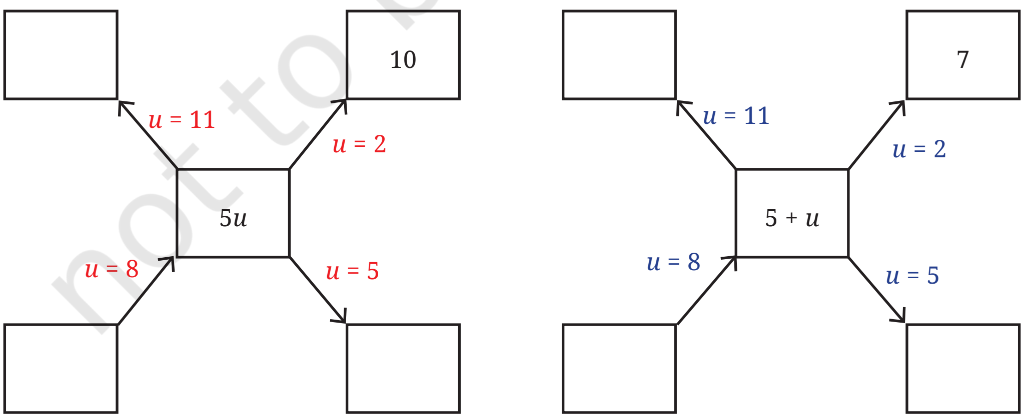
$$4\,(x + y) - y = 4x + 4y - y$$
$$= 4x + 4y + -y$$
$$= 4x + (4 - 1)y$$
$$= 4x + 3y.$$

Example 10: Are the expressions $5u$ and $5 + u$ equal to each other?

The expression $5u$ means 5 times the number u, and the expression $5 + u$ means 5 more than the number u. These two being different operations, they should give different values for most values of u.

Let us check this.

Fill the blanks below by replacing the letter-numbers by numbers; an example is shown. Then compare the values that $5u$ and $5 + u$ take.



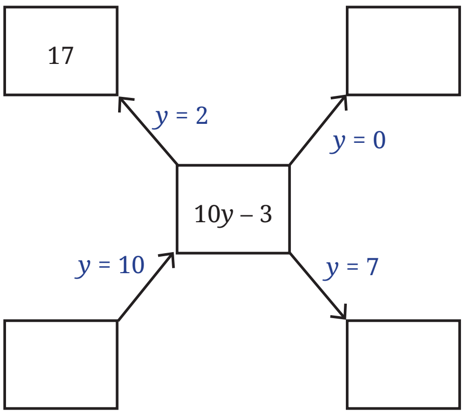
If the expressions $5u$ and $5 + u$ are equal, then they should take the same values for any given value of u. But we can see that they do not. So, these two expressions are not equal.

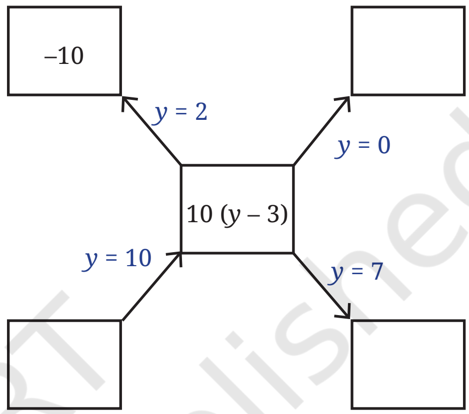
Are the expressions $10y - 3$ and $10(y - 3)$ equal?

Math Talk

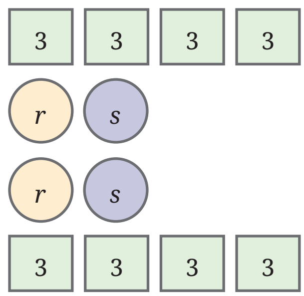
$10y - 3$, short for $10 \times y - 3$, means 3 less than 10 times y,
$10(y - 3)$, short for $10 \times (y - 3)$, means 10 times (3 less than y).

Let us compare the values that these expressions take for different values of y.





After filling in the two diagrams, do you think the two expressions are equal?



Example 11: What is the sum of the numbers in the picture (unknown values are denoted by letter-numbers)?

There are many ways to go about it. Here, we show some of them.

1. Adding row wise gives:

$$(4 \times 3) + (r + s) + (r + s) + (4 \times 3)$$

2. Adding like terms together gives:

$$(8 \times 3) + (r + r) + (s + s)$$

3. Adding the upper half and doubling gives:

$$2 \times (4 \times 3 + r + s)$$

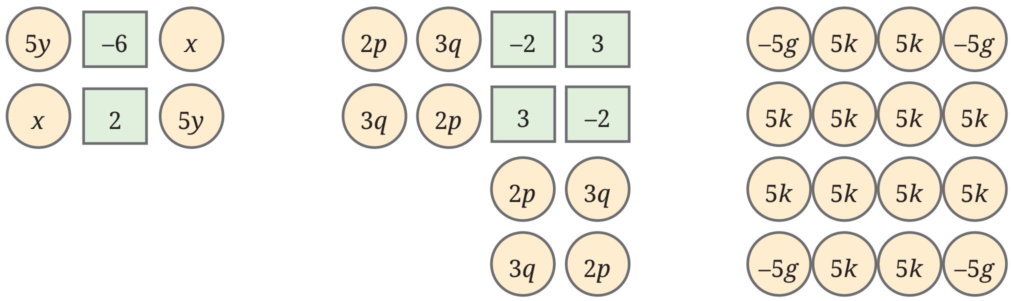
The three expressions might seem different. We can simplify each one and see that they all are the same: $2r + 2s + 24$.

Figure it Out

Math Talk

1. Add the numbers in each picture below. Write their corresponding expressions and simplify them. Try adding the numbers in each picture in a couple different ways and see

that you get the same thing.



2. Simplify each of the following expressions:

   (a) $p + p + p + p$, $p + p + p + q$, $p + q + p - q$,

   (b) $p - q + p - q$, $p + q - p + q$,

   (c) $p + q - (p + q)$, $p - q - p - q$

   (d) $2d - d - d - d$, $2d - d - d - c$,

   (e) $2d - d - (d - c)$, $2d - (d - d) - c$,

   (f) $2d - d - c - c$

Mind the Mistake, Mend the Mistake

Some simplifications of algebraic expressions are done below. The expression on the right-hand side should be in its simplest form.

- Observe each of them and see if there is a mistake.
- If you think there is a mistake, try to explain what might have gone wrong.
- Then, simplify it correctly.

Expression	Simplest Form	Correct Simplest Form
1. $3a + 2b$	5	
2. $3b - 2b - b$	0	
3. $6\ (p + 2)$	$6p + 8$	
4. $(4x + 3y) - (3x + 4y)$	$x + y$	
5. $5 - (2 - 6z)$	$3 - 6z$	
6. $2 + (x + 3)$	$2x - 6$	
7. $2y + (3y - 6)$	$-y + 6$	
8. $7p - p + 5q - 2q$	$7p + 3q$	
9. $5\ (2w + 3x + 4w)$	$10w + 15x + 20w$	

10. $3j + 6k + 9h + 12$ $\quad$ $3\,(j + 2k + 3h + 4)$
11. $4\,(2r + 3s + 5)$ $\quad$ $-\,20 - 8r - 12s$

? Take a look at all the corrected simplest forms (i.e. brackets are removed, like terms are added, and terms with only numbers are also added). Is there any relation between the number of terms and the number of letter-numbers these expressions have?

4.5 Pick Patterns and Reveal Relationships

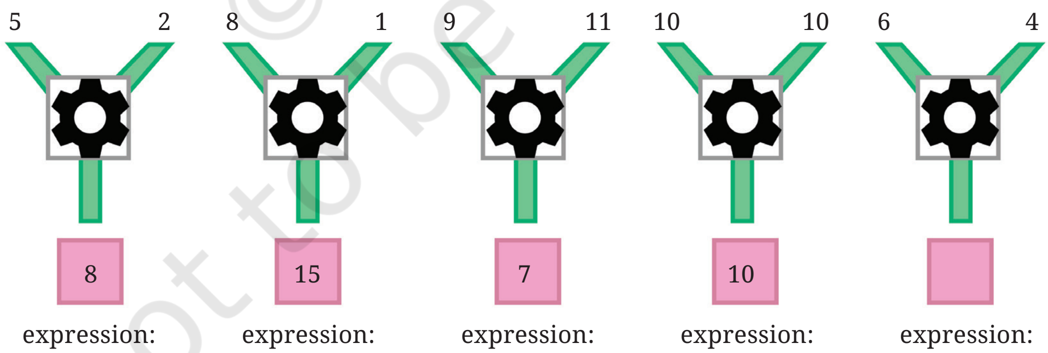
In the first section we got a glimpse of algebraic expressions and how to use them to describe simple patterns and relationships in a concise and elegant manner. Here, we continue to look for general relationships between quantities in different scenarios, find patterns and, interestingly, even explain why these patterns occur.

Remember the importance of describing in simple language, or visualising mathematical relationships, before trying to write them as expressions.

Formula Detective

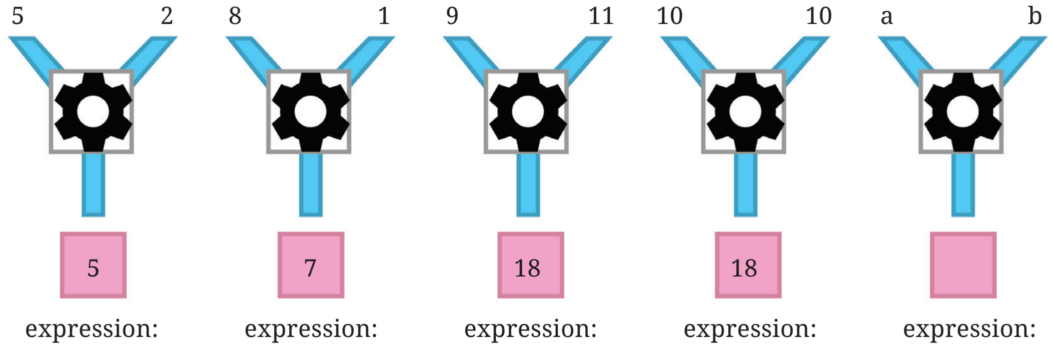
Look at the picture given. In each case, the number machine takes in the 2 numbers at the top of the 'Y' as inputs, performs some operations and produces the result at the bottom. The machine performs the same operations on its inputs in each case.

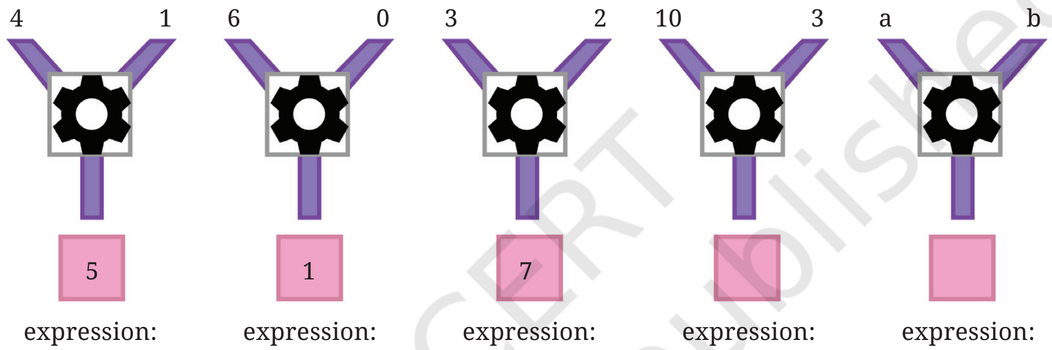
? Find out the formula of this number machine.



The formula for the number machine above is "two times the first number minus the second number". When written as an algebraic expression, the formula is $2a - b$. The expression for the first set of inputs is $2 \times 5 - 2 = 8$. Check that the formula holds true for each set of inputs.

Find the formulas of the number machines below and write the expression for each set of inputs.



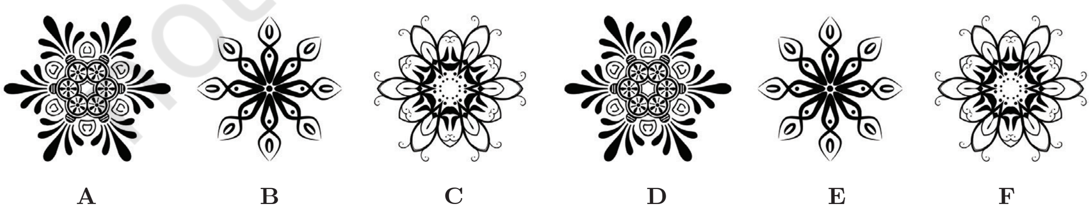


Now, make a formula on your own. Write a few number machines as examples using that formula. Challenge your classmates to figure it out!

> **Note to the Teacher:** Not just solving problems but creating new questions is also very much a part of learning and doing mathematics!

Algebraic Expressions to Describe Patterns

Example 12: Somjit noticed a repeating pattern along the border of a saree.



Somjit wonders if there is a way to describe all the positions where the (i) Design A occurs, (ii) Design B occurs, and (iii) Design C occurs.

Let us start with design C. It appears for the first time at position 3, the second time at position 6.

? Where would design C appear for the n^{th} time?

We can see that this design appears in positions that are multiples of 3. So the n^{th} occurrence of Design C will be at position 3n.

? Similarly, find the formula that gives the position where the other Designs appear for the n^{th} time.

The positions where B occurs are 2, 5, 8, 11, 14, and so on.

We can see that the position of the nth appearance of Design B is one less than the position at which Design C appears for the n^{th} time. Thus, the nth occurrence of Design B is at position:

$$3n - 1$$

Similarly, the expression describing the position at which the design A appears for the nth time is: $3n - 2$.

? Given a position number can we find out the design that appears there? Which Design appears at Position 122?

If the position is a multiple of 3, then clearly we have Design C. As seen earlier, if the position is one less than a multiple of 3, it has Design B, and if it is 2 less than a multiple of 3, then it has Design A.

? Can the remainder obtained by dividing the position number by 3 be used for this? Observe the table below.

Position no.	Quotient on division by 3	Remainder
99	33	0
122	40	2
148	49	1

? Use this to find what design appears at positions 99, 122, and 148.

Patterns in a Calendar

Here is the calendar of November 2024. Consider 2 × 2 squares, as marked in the calendar. The numbers in this square show an interesting property.

November 2024

Mon	Tue	Wed	Thu	Fri	Sat	Sun
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

12	13
19	20

Let us take the marked 2 × 2 square, and consider the numbers lying on the diagonals; 12 and 20; 13 and 19. Find their sums; 12 + 20, 13 + 19. What do you observe?

They are equal.

Let us extend the numbers in the calendar beyond 30, creating endless rows.

November 2024

Mon	Tue	Wed	Thu	Fri	Sat	Sun
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31	32	33	34	35	36	37
38	39	40	41	42	43	44
⋮	⋮	⋮	⋮	⋮	⋮	⋮

? Will the diagonal sums be equal in every 2 × 2 square in this endless grid? How can we be sure?

To be sure of this we cannot check with all 2 × 2 squares as there are an unlimited number of them.

Let us consider a 2 × 2 square. Its top left number can be any number. Let us call it ‘a’.

a	?
?	?

? Given that we know the top left number, how do we find the other numbers in this 2 × 2 square?

As we have been doing, first let us describe the other numbers in words.

- the number to the right of 'a' will be **1 more than it**.
- the number below 'a' will be 7 **more than it**.
- the number diagonal to 'a' will be **8 more than it**.

a	$a + 1$
$a + 7$	$a + 8$

So the other numbers in the 2 × 2 square can be represented as shown in the grid. Let us find the diagonal sums; $a + (a + 8)$, and $(a + 1) + (a + 7)$.

Let us simplify them.

Since the terms can be added in any order, the brackets can be opened.

$a + (a + 8) = a + a + 8 = 2a + 8$

$(a + 1) + (a + 7) = a + 1 + a + 7 = a + a + 1 + 7 = 2a + 8$

We see that both diagonal sums are equal to $2a + 8$ (8 more than 2 times a).

? Verify this expression for diagonal sums by considering any 2 × 2 square and taking its top left number to be 'a'.

Thus, we have shown that diagonal sums are equal for any value of a, i.e., for any 2 × 2 square!

This problem is an example that shows the power of algebraic modelling in verifying whether a pattern will always hold.

Consider a set of numbers from the calendar (having endless rows) forming under the following shape:

	8	
14	15	16
	22	

? Find the sum of all the numbers. Compare it with the number in the centre: 15. Repeat this for another set of numbers that forms this shape. What do you observe?

We see that the total sum is always 5 times the number in the centre.

? Will this always happen? How do you show this?

Math Talk

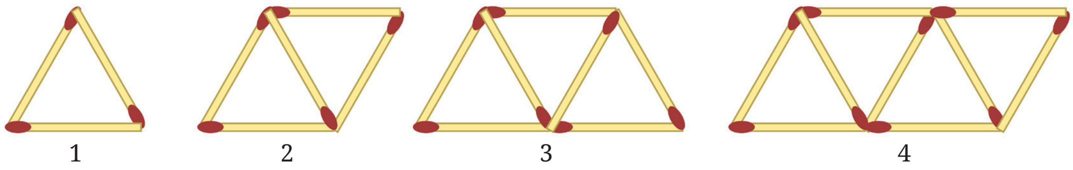
[**Hint:** Consider a general set of numbers that forms this shape. Take the number at the centre to be 'a'. Express the other numbers in terms of 'a'.]

Find other shapes for which the sum of the numbers within the figure is always a multiple of one of the numbers.

Math Talk

Matchstick Patterns

Look at the picture below. It is a pattern using matchsticks. Can you identify what the pattern is?



We can see that Step 1 has 1 triangle, Step 2 has 2 triangles, Step 3 has 3 triangles, and so on.

Can you tell how many matchsticks there will be in the next step, Step 5? It is 11. You can also draw this and see.

? How many matchsticks will there be in Step 33, Step 84, and Step 108? Of course, we can draw and count, but is there a quicker way to find the answers using the pattern present here?

What is the general rule to find the number of matchsticks in the next step? We can see that at each step 2 matchsticks are placed to get the next one, i.e., the number of matchsticks increases by 2 every time.

Step Number	1	2	3	4	5	6
No. of Matchsticks	3	5	7	9	11	13

Think of a way to use this to find out the number of matchsticks in Step 33 (without continuing to write the numbers).

As each time 2 matchsticks are being added, finding out how many 2s will be added in Step 33 will help. Look at the table below and try to find out.

Step Number	1	2	3	4	5	6
No. of Matchsticks	3	5	7	9	11	13
		3 + 2	3 + 2 + 2	3 + 2 + 2 + 2	3 + 2 + 2 + 2 + 2	

The number of matchsticks needed to make 33 triangles (Step 33) is ________. Similarly, find the number of matchsticks needed for Step 84 and Step 108.

What could be an expression describing the rule/formula to find out the number of matchsticks at any step?

The pattern is such that in Step 10, nine 2*s* and an added 3 ($3 + 2 \times 9$) gives the number of matchsticks; in Step 11, ten 2*s* and an added 3 ($3 + 2 \times 10$) gives the number of matchsticks. For step y, what is the expression?

It is: one less than y (i.e. $y - 1$) 2*s* and *a* 3.

Therefore, the expression is

$$3 + 2 \times (y - 1).$$

This expression gives the number of matchsticks in Step y. Now we can find the number of matchsticks at any step quickly.

You might have already noticed that there is a 2 in the first step also, $3 = 1 + 2$. Using this, the expression we get is

$$2y + 1.$$

? Does the above expression also give the number of matchsticks at each step correctly? Are these expressions the same?

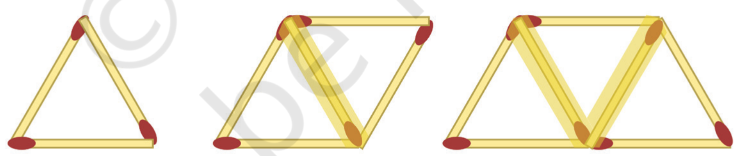
We can check by simplifying the expression $3 + 2 \times (y - 1)$.

$$3 + 2 \times (y - 1) = 3 + 2y - 2$$

$$= 2y + 1.$$

Both expressions are the same.

There is a different way to count, or see the pattern. Let us take a look at the picture again.



Matchsticks are placed in two orientations — (a) horizontal ones at the top and bottom, and (b) the ones placed diagonally in the middle. For example, in step 2 there are 2 matchsticks placed horizontally and 3 matchsticks placed diagonally.

? What are these numbers in Step 3 and Step 4?

? How does the number of matchsticks change in each orientation as the steps increase? Write an expression for the number of matchsticks at Step 'y' in each orientation. Do the two expressions add up to $2y + 1$?

Figure it Out

For the problems asking you to find suitable expression(s), first try to understand the relationship between the different quantities in the situation described. If required, assume some values for the unknowns and try to find the relationship.

1. One plate of *Jowar roti* costs ₹30 and one plate of *Pulao* costs ₹20. If x plates of *Jowar roti* and y plates of *pulao* were ordered in a day, which expression(s) describe the total amount in rupees earned that day?

   (a) $30x + 20y$ (b) $(30 + 20) \times (x + y)$

   (c) $20x + 30y$ (d) $(30 + 20) \times x + y$

   (e) $30x - 20y$

2. Pushpita sells two types of flowers on Independence day: champak and marigold. '*p*' customers only bought champak, '*q*' customers only bought marigold, and '*r*' customers bought both. On the same day, she gave away a tiny national flag to every customer. How many flags did she give away that day?

   (a) $p + q + r$ (b) $p + q + 2r$

   (c) $2 \times (p + q + r)$ (d) $p + q + r + 2$

   (e) $p + q + r + 1$ (f) $2 \times (p + q)$

3. A snail is trying to climb along the wall of a deep well. During the day it climbs up '*u*' cm and during the night it slowly slips down '*d*' cm. This happens for 10 days and 10 nights.

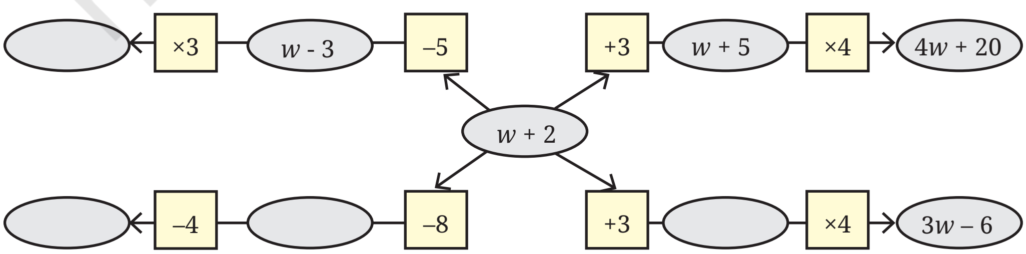
   (a) Write an expression describing how far away the snail is from its starting position.

   (b) What can we say about the snail's movement if $d > u$?

4. Radha is preparing for a cycling race and practices daily. The first week she cycles 5 km every day. Every week she increases the daily distance cycled by '*z*' km. How many kilometers would Radha have cycled after 3 weeks?

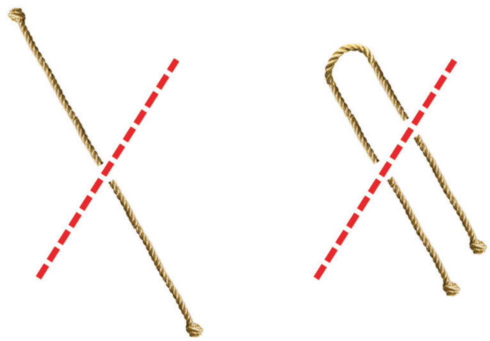
   Try This

5. In the following figure, observe how the expression $w + 2$ becomes $4w + 20$ along one path. Fill in the missing blanks on the remaining paths. The ovals contain expressions and the boxes contain operations.

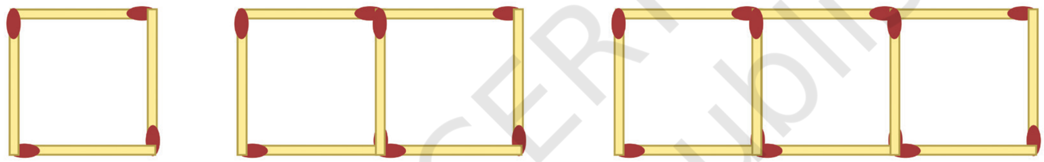


6. A local train from Yahapur to Vahapur stops at three stations at equal distances along the way. The time taken in minutes to travel from one station to the next station is the same and is denoted by t. The train stops for 2 minutes at each of the three stations.
   (a) If $t = 4$, what is the time taken to travel from Yahapur to Vahapur?
   (b) What is the algebraic expression for the time taken to travel from Yahapur to Vahapur? [Hint: Draw a rough diagram to visualise the situation]
7. Simplify the following expressions:
   (a) $3a + 9b - 6 + 8a - 4b - 7a + 16$
   (b) $3\,(3a - 3b) - 8a - 4b - 16$
   (c) $2\,(2x - 3) + 8x + 12$
   (d) $8x - (2x - 3) + 12$
   (e) $8h - (5 + 7h) + 9$
   (f) $23 + 4(6m - 3n) - 8n - 3m - 18$
8. Add the expressions given below:
   (a) $4d - 7c + 9$ and $8c - 11 + 9d$
   (b) $-6f + 19 - 8s$ and $-23 + 13f + 12s$
   (c) $8d - 14c + 9$ and $16c - (11 + 9d)$
   (d) $6f - 20 + 8s$ and $23 - 13f - 12s$
   (e) $13m - 12n$ and $12n - 13m$
   (f) $-26m + 24n$ and $26m - 24n$
9. Subtract the expressions given below:
   (a) $9a - 6b + 14$ from $6a + 9b - 18$
   (b) $-15x + 13 - 9y$ from $7y - 10 + 3x$
   (c) $17g + 9 - 7h$ from $11 - 10g + 3h$
   (d) $9a - 6b + 14$ from $6a - (9b + 18)$
   (e) $10x + 2 + 10y$ from $-3y + 8 - 3x$
   (f) $8g + 4h - 10$ from $7h - 8g + 20$
10. Describe situations corresponding to the following algebraic expressions:
    (a) $8x + 3y$
    (b) $15x - 2x$
11. Imagine a straight rope. If it is cut once as shown in the picture, we get 2 pieces. If the rope is folded once and then cut as shown, we

get 3 pieces. Observe the pattern and find the number of pieces if the rope is folded 10 times and cut. What is the expression for the number of pieces when the rope is folded r times and cut?

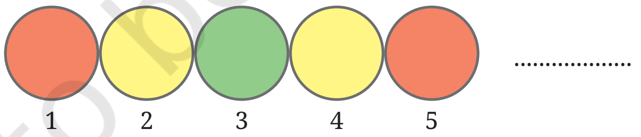


12. Look at the matchstick pattern below. Observe and identify the pattern. How many matchsticks are required to make 10 such squares. How many are required to make w squares?

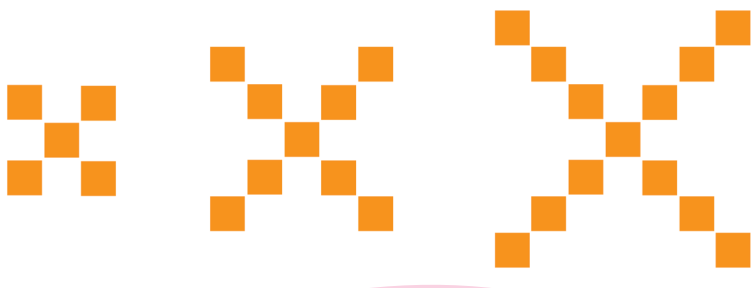


13. Have you noticed how the colours change in a traffic signal? The sequence of colour changes is shown below.

Find the colour at positions 90, 190, and 343. Write expressions to describe the positions for each colour.



14. Observe the pattern below. How many squares will be there in Step 4, Step 10, Step 50? Write a general formula. How would the formula change if we want to count the number of vertices of all the squares?



15. Numbers are written in a particular sequence in this endless 4-column grid.

1	2	3	4
1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

   (a) Give expressions to generate all the numbers in a given column (1, 2, 3, 4).

   (b) In which row and column will the following numbers appear:

      (i) 124

      (ii) 147

      (iii) 201

   (c) What number appears in row r and column c?

   (d) Observe the positions of multiples of 3.

Do you see any pattern in it? List other patterns that you see.

Math Talk

SUMMARY

- Algebraic expressions are used in formulas to model patterns and mathematical relationships between quantities, and to make predictions.
- Algebraic expressions use not only numbers but also letter-numbers. The rules for manipulating arithmetic expressions also apply to algebraic expressions. These rules can be used to reduce algebraic expressions to their simplest forms.
- Algebraic expressions can be described in ordinary language, and vice versa. Patterns or relationships that are easily written using algebra can often be long and complex in ordinary language. This is one of the advantages of algebra.

1 LARGE NUMBERS AROUND US

0774CH01

1.1 A Lakh Varieties!

Eshwarappa is a farmer in Chintamani, a town in Karnataka. He visits the market regularly to buy seeds for his rice field. During one such visit he overheard a conversation between Ramanna and Lakshmamma. Ramanna said, “Earlier our country had about a lakh varieties of rice. Farmers used to preserve different varieties of seeds and use them to grow rice. Now, we only have a handful of varieties. Also, farmers have to come to the market to buy seeds”.

Lakshmamma said, “There is a seed bank near my house. So far, they have collected about a hundred indigenous varieties of rice seeds from different places. You can also buy seeds from there.”

You may have heard the word ‘lakh’ before. Do you know how big one lakh is? Let us find out.

Eshwarappa shared this incident with his daughter Roxie and son Estu .

Estu was surprised to know that there were about one lakh varieties of rice in this country. He wondered “One lakh! So far I have only tasted 3 varieties. If we tried a new variety each day, would we even come close to tasting all the varieties in a lifetime of 100 years?”

What do you think? Guess.

But how much is one lakh? Observe the pattern and fill in the boxes given below.

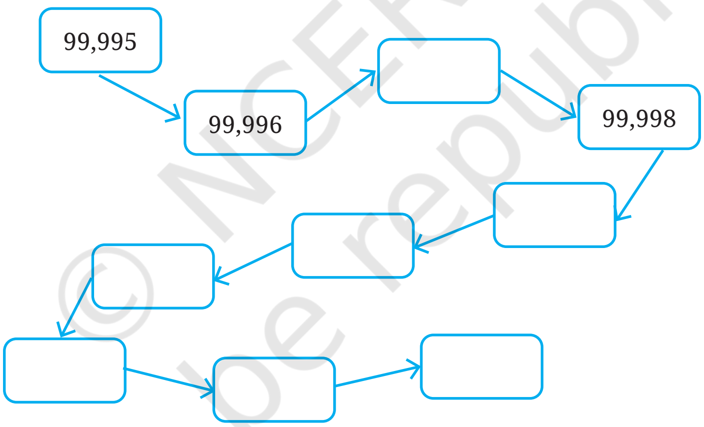
The largest 3-digit number is	999
The smallest 4-digit number is	
The largest 4-digit number is	
The smallest 5-digit number is	
The largest 5-digit number is	
The smallest 6-digit number is	1,00,000

+1

+1

+1

1,00,000 is read as "One Lakh"



Roxie and Estu found that if they ate one variety of rice a day, they would come nowhere close to a lakh in a lifetime! Roxie suggests, "What if we ate 2 varieties of rice every day? Would we then be able to eat 1 lakh varieties of rice in 100 years?"

? What if a person ate 3 varieties of rice every day? Will they be able to taste all the lakh varieties in a 100 year lifetime? Find out.

Estu said, "We know how many days there are in a year — 365, if we ignore leap years. If we live for y years, the number of days in our lifetime will be $365 \times y$."

Choose a number for y. How close to one lakh is the number of days in y years, for the y of your choice?

Figure it Out

1. According to the 2011 Census, the population of the town of *Chintamani* was about 75,000. How much less than one lakh is 75,000?
2. The estimated population of *Chintamani* in the year 2024 is 1,06,000. How much more than one lakh is 1,06,000?
3. By how much did the population of *Chintamani* increase from 2011 to 2024?

Getting a Feel of Large Numbers

You may have come across interesting facts like these:

- The world's tallest statue is the 'Statue of Unity' in Gujarat depicting Sardar Vallabhbhai Patel. Its height is about 180 metres.
- Kunchikal waterfall in Karnataka is said to drop from a height of about 450 metres.

It is not always easy to get a sense of how big these measurements are. But, we can get a better sense of their size when we compare them with something familiar. Let us see an example.

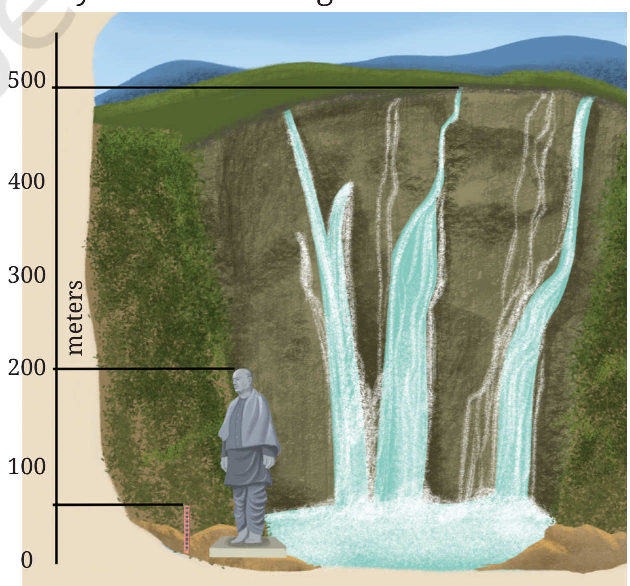
Look at the picture on the right. Somu is 1 metre tall. If each floor is about four times his height, what is the approximate height of the building?

Which is taller — The Statue of Unity or this building? How much taller? ___________m.

We can see that the height of the Statue of Unity is close to 4 times the height of Somu's building.

How much taller is the Kunchikal waterfall than Somu's building? ___________m.

How many floors should Somu's building have to be as high as the waterfall? ___________ .



Is One Lakh a Very Large Number?

Eshwarappa asked Roxie and Estu, "Is a lakh big or small?"

Roxie feels that 1 lakh is a large number:

- "We had one lakh varieties of rice — that is a lot."
- "Living 1 lakh days would mean living for 274 years — that is a really long time!"
- "If 1 lakh people stood shoulder to shoulder in a line they could stretch as far as 38 kilometres."

Estu, however, thinks it is not that big:

- "Do you know that the cricket stadium in Ahmedabad has a seating capacity of more than 1 lakh? One lakh people in such a small area!"
- "Most humans have 80,000 to 1,20,000 hairs on their heads. Imagine, 1 lakh hairs fit in such a tiny space!"
- "I heard that there are some species of fish where a female fish can lay almost one lakh eggs at once very comfortably. Some even lay tens of lakhs of eggs at a time."

? How do you view a lakh — is a lakh big or small?

Reading and Writing Numbers

We have already been using commas for 5-digit numbers like 45,830 in the Indian place value system. As numbers grow bigger, using commas helps in reading the numbers easily. We use a comma in between the digits representing the "ten thousands" place and the "one lakh" place as you have seen just before (1,00,000).

The number name of 12,78,830 is twelve lakh seventy eight thousand eight hundred thirty.

Similarly, the number 15,75,000 in words is fifteen lakh seventy five thousand.

Write each of the numbers given below in words:

(a) 3,00,600

(b) 5,04,085

(c) 27,30,000

(d) 70,53,138

Write the corresponding number in the Indian place value system for each of the following:

(a) One lakh twenty three thousand four hundred and fifty six

(b) Four lakh seven thousand seven hundred and four

(c) Fifty lakhs five thousand and fifty

(d) Ten lakhs two hundred and thirty five

Note to the Teacher: Encourage students to make connections between these facts. For example, can the whole population of Chintamani fit in the stadium? How can we imagine the line of 38 km, having a lakh people, sitting next to each other in the stadium?

1.2 Land of Tens

In the **Land of Tens**, there are special calculators with special buttons.

1. The **Thoughtful Thousands** only has a +1000 button. How many times should it be pressed to show:

   (a) Three thousand? 3 times

   (b) 10,000? ___________

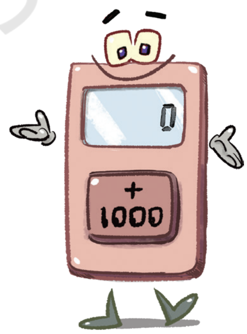
   (c) Fifty three thousand? ___________

   (d) 90,000? ___________

   (e) One Lakh? ___________

   (f) ___________? 153 times

   (g) How many thousands are required to make one lakh?



2. The **Tedious Tens** only has a +10 button. How many times should it be pressed to show:

   (a) Five hundred? ___________

   (b) 780? ________

   (c) 1000? ________

   (d) 3700? ________

   (e) 10,000? ___________

   (f) One lakh? ___________

   (g) ___________? 435 times

3. The **Handy Hundreds** only has a +100 button. How many times should it be pressed to show:

   (a) Four hundred? __________times

   (b) 3,700? __________

(c) 10,000? __________

(d) Fifty three thousand? __________

(e) 90,000? __________

(f) 97,600? __________

(g) 1,00,000? __________

(h) __________? 582 times

(i) How many hundreds are required to make ten thousand?

(j) How many hundreds are required to make one lakh?

(k) Handy Hundreds says, “There are some numbers which Tedious Tens and Thoughtful Thousands can’t show but I can.” Is this statement true? Think and explore.

4. **Creative Chitti** is a different kind of calculator. It has the following buttons: +1, +10, +100, +1000, +10000, +100000 and +1000000. It always has multiple ways of doing things. “How so?”, you might ask. To get the number 321, it presses +10 thirty two times and +1 once. Will it get 321? Alternatively, it can press +100 two times and +10 twelve times and +1 once.

5. Two of the many different ways to get 5072 are shown below:
These two ways can be expressed as:

(a) (50 × 100) + (7 × 10) + (2 × 1) = 5072

(b) (3×1000) + (20×100) + (72 ×1) = 5072

Buttons	5072	
+10,00,000		
+1,00,000		
+10,000		
+1,000		3
+100	50	20
+10	7	
+1	2	72

? Find a different way to get 5072 and write an expression for the same.

? Figure it Out

For each number given below, write expressions for at least two different ways to obtain the number through button clicks. Think like Chitti and be creative.

(a) 8300

(b) 40629

(c) 56354

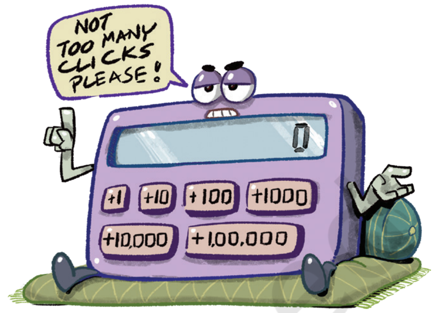
(d) 66666

(e) 367813

Creative Chitti has some questions for you —

(a) You have to make exactly 30 button presses. What is the largest 3-digit number you can make? What is the smallest 3-digit number you can make?

(b) 997 can be made using 25 clicks. Can you make 997 with a different number of clicks?



Create questions like these and challenge your classmates.

Systematic Sippy is a different kind of calculator. It has the following buttons: +1, +10, +100, +1000, +10000, +100000. It wants to be used as minimally as possible.

? How can we get the numbers (a) 5072, (b) 8300 using as few button clicks as possible?

Find out which buttons should be clicked and how many times to get the desired numbers given in the table. The aim is to click as few buttons as possible.

Here is one way to get the number 5072. This method uses 23 button clicks in total.

Is there another way to get 5072 using less than 23 button clicks? Write the expression for the same.

Buttons	5072
+10,00,000	
+1,00,000	
+10,000	
+1,000	5
+100	0
+10	6
+1	12

? Figure it Out

1. For the numbers in the previous exercise, find out how to get each number by making the smallest number of button clicks and write the expression.
2. Do you see any connection between each number and the corresponding smallest number of button clicks?
3. If you notice, the expressions for the least button clicks also give the Indian place value notation of the numbers. Think about why this is so.

Math Talk

What if we press the +10,00,000 button ten times? What number will come up? How many zeroes will it have? What should we call it? The number will be 100 lakhs, which is also called a **crore**. 1 crore is written as 1,00,00,000 — it is 1 followed by seven zeroes.

1.3 Of Crores and Crores!

The table below shows some numbers according to both the Indian system and the American system (also called the International system) of naming numerals and placing commas. Observe the placement of commas in both systems.

Indian System		American System	
1,000	One thousand	1,000	One thousand
10,000	Ten thousand	10,000	Ten thousand
1,00,000	One lakh	100,000	Hundred thousand
10,00,000	Ten lakhs	1,000,000	One million
1,00,00,000	One crore	10,000,000	Ten million
10,00,00,000	Ten crores	100,000,000	Hundred million
1,00,00,00,000	One arab or One hundred crores	1,000,000,000	One billion

Notice that in the Indian system, commas are placed to group the digits in a 3-2-2-2... pattern from right to left (thousands, lakhs, crores, etc.). In the American system, the digits are grouped uniformly in a 3-3-3-3... pattern from right to left (thousands, millions, billions, etc.).

The Indian system of naming numbers is also followed in Bhutan, Nepal, Sri Lanka, Pakistan, Bangladesh, Maldives, Afghanistan, and Myanmar. The words lakh and crore originate from the Sanskrit words *lakṣha* (लक्ष) and *koṭi* (कोटि). The American system is also used in many countries.

Observe the number of zeroes in 1 lakh and 1 crore.

1 lakh, written in numbers would be 1 followed by 5 zeroes.

1 crore, written in numbers would be 1 followed by 7 zeroes.

A lakh is a hundred times a thousand, a crore is a hundred times a lakh and an arab is a hundred times a crore (i.e., a hundred thousand is 1 lakh, 100 lakhs is 1 crore, and 100 crores is 1 arab).

? How many zeros does a thousand lakh have? _____

How many zeros does a hundred thousand have? ____

The number 9876501234 can be easily read by placing commas first:

(a) 9,87,65,01,234 → 9 arab 87 crore 65 lakhs 1 thousand and 234 or 987 crore 65 lakh 1 thousand 234 (in the Indian system).

(b) 9,876,501,234 → 9 billion 876 million 501 thousand 234 (in the American system).

Figure it Out

1. Read the following numbers in Indian place value notation and write their number names in both the Indian and American systems:
   (a) 4050678 (b) 48121620
   (c) 20022002 (d) 246813579
   (e) 345000543 (f) 1020304050
2. Write the following numbers in Indian place value notation:
   (a) One crore one lakh one thousand ten
   (b) One billion one million one thousand one
   (c) Ten crore twenty lakh thirty thousand forty
   (d) Nine billion eighty million seven hundred thousand six hundred
3. Compare and write '<', '>' or '=':
   (a) 30 thousand ____ 3 lakhs
   (b) 500 lakhs ______ 5 million
   (c) 800 thousand ____ 8 million
   (d) 640 crore ______ 60 billion

We shall come across even bigger numbers in later grades.

1.4 Exact and Approximate Values

What do you think of this conversation? Have you read or heard such headlines or statements?

Very often, exact numbers are not required and just an approximation is sufficient. For example, according to the 2011 census, the population of Chintamani town is 76,068. Instead, saying that the population is about 75,000 is enough to give an idea of how big the quantity is.

There are situations where it makes sense to **round up** a number (rounding up is when the approximated number is more than the actual number). For example, if a school has 732 people including students, teachers and staff: the principal might order 750 sweets instead of 700 sweets.

There are situations where it is better to **round down** (rounding down is when the approximated number is less than the actual number). For example, if the cost of an item is ₹470, the shopkeeper may say that the cost is around ₹450 instead of saying it is around ₹500.

? Think and share situations where it is appropriate to (a) round up, (b) round down, (c) either rounding up or rounding down is okay and (d) when exact numbers are needed.

Nearest Neighbours

With large numbers it is useful to know the nearest thousand, lakh or crore. For example, the nearest neighbours of the number 6,72,85,183 are shown in the table below.

Nearest thousand	6,72,85,000
Nearest ten thousand	6,72,90,000
Nearest lakh	6,73,00,000
Nearest ten lakh	6,70,00,000
Nearest crore	7,00,00,000

? Similarly, write the five nearest neighbours for these numbers:

(a) 3,87,69,957

(b) 29,05,32,481

? I have a number for which all five nearest neighbours are 5,00,00,000. What could the number be? How many such numbers are there?

Math Talk

Roxie and Estu are estimating the values of simple expressions.

1. 4,63,128 + 4,19,682,

   Roxie: "The sum is near 8,00,000 and is more than 8,00,000."
   Estu: "The sum is near 9,00,000 and is less than 9,00,000."

   (a) Are these estimates correct? Whose estimate is closer to the sum?

   (b) Will the sum be greater than 8,50,000 or less than 8,50,000? Why do you think so?

   (c) Will the sum be greater than 8,83,128 or less than 8,83,128? Why do you think so?

   (d) Exact value of 4,63,128 + 4,19,682 = ___________

2. 14,63,128 – 4,90,020

   Roxie: "The difference is near 10,00,000 and is less than 10,00,000."
   Estu: "The difference is near 9,00,000 and is more than 9,00,000."

   (a) Are these estimates correct? Whose estimate is closer to the difference?

   (b) Will the difference be greater than 9,50,000 or less than 9,50,000? Why do you think so?

(c) Will the difference be greater than 9,63,128 or less than 9,63,128? Why do you think so?

(d) Exact value of 14,63,128 – 4,90,020 = __________

Note to the Teacher: Ask students questions like — "what could the numbers be if the sum had to be less than 8,50,000."

Populations of Cities

Observe the populations of some Indian cities in the **table below**.

Rank	City	Population (2011)	Population (2001)
1	Mumbai	1,24,42,373	1,19,78,450
2	New Delhi	1,10,07,835	98,79,172
3	Bengaluru	84,25,970	43,01,326
4	Hyderabad	68,09,970	36,37,483
5	Ahmedabad	55,70,585	35,20,085
6	Chennai	46,81,087	43,43,645
7	Kolkata	44,86,679	45,72,876
8	Surat	44,67,797	24,33,835
9	Vadodara	35,52,371	16,90,000
10	Pune	31,15,431	25,38,473
11	Jaipur	30,46,163	23,22,575
12	Lucknow	28,15,601	21,85,927
13	Kanpur	27,67,031	25,51,337
14	Nagpur	24,05,665	20,52,066

15	Indore	19,60,631	14,74,968
16	Thane	18,18,872	12,62,551
17	Bhopal	17,98,218	14,37,354
18	Visakhapatnam	17,28,128	13,45,938
19	Pimpri-Chinchwad	17,27,692	10,12,472
20	Patna	16,84,222	13,66,444

? From the information given in the table, answer the following questions by approximation:

1. What is your general observation about this data? Share it with the class.
2. What is an appropriate title for the above table?
3. How much is the population of Pune in 2011? Approximately, by how much has it increased compared to 2001?
4. Which city's population increased the most between 2001 and 2011?
5. Are there cities whose population has almost doubled? Which are they?
6. By what number should we multiply Patna's population to get a number/population close to that of Mumbai?

1.5 Patterns in Products

Roxie and Estu are playing with multiplication. They encounter an interesting technique for multiplying a number by 10, 100, 1000, and so on.

A Multiplication Shortcut

Roxie evaluated 116 × 5 as follows:

$$\begin{aligned} 116 \times 5 &= \overset{58}{\cancel{116}} \times \frac{10}{\cancel{2}} \\ &= 58 \times 10 \\ &= 580. \end{aligned}$$

Estu evaluated 824 × 25 as follows:

$$824 \times 25 \quad = \overset{206}{\cancel{824}} \times \frac{100}{\cancel{4}}$$
$$= 20600.$$

Using the meaning of multiplication and division, can you explain why multiplying by 5 is the same as dividing by 2 and multiplying by 10?

Math Talk

Figure it Out

1. Find quick ways to calculate these products:
   (a) 2 × 1768 × 50
   (b) 72 × 125 [**Hint:** 125 = $\frac{1000}{8}$]
   (c) 125 × 40 × 8 × 25
2. Calculate these products quickly.
   (a) 25 × 12 = ____________
   (b) 25 × 240 = ____________
   (c) 250 × 120 = ____________
   (d) 2500 × 12 =____________
   (e) ______×______= 120000000

How Long is the Product?

In each of the following boxes, the multiplications produce interesting patterns. Evaluate them to find the pattern. Extend the multiplications based on the observed pattern.

11 × 11 = 111 × 111 = 1111 × 1111 =	66 × 61 = 666 × 661 = 6666 × 6661 =
3 × 5 = 33 × 35 = 333 × 335 =	101 × 101 = 102 × 102 = 103 × 103 =

? Observe the number of digits in the two numbers being multiplied and their product in each case. Is there any connection between the numbers being multiplied and the number of digits in their product?

? Roxie says that the product of two 2-digit numbers can only be a 3- or a 4-digit number. Is she correct?

? Should we try all possible multiplications with 2-digit numbers to tell whether Roxie's claim is true? Or is there a better way to find out?

She explains her reasoning: "We want to know about the number of digits in the product of two 2-digit numbers. To know the smallest such product I took 10 × 10, so all other products will be greater than 100.

To know the greatest such product I multiplied the smallest 3-digit numbers (100 × 100) to get 10,000; so the product of all the 2-digit multiplications will be less than 10,000."

? Can multiplying a 3-digit number with another 3-digit number give a 4-digit number?

? Can multiplying a 4-digit number with a 2-digit number give a 5-digit number?

? Observe the multiplication statements below. Do you notice any patterns? See if this pattern extends for other numbers as well.

1-digit	×	1-digit	=	1-digit	or	2-digit
2-digit	×	1-digit	=	2-digit	or	3-digit
2-digit	×	2-digit	=	3-digit	or	4-digit
3-digit	×	3-digit	=	5-digit	or	6-digit
5-digit	×	5-digit	=		or	
8-digit	×	3-digit	=		or	
12-digit	×	13-digit	=		or	

Fascinating Facts about Large Numbers

Some interesting facts about large numbers are hidden below. Calculate the product or quotient to uncover the facts. Once you find the product or quotient, read the number in both Indian and American naming systems. Share your thoughts and questions about the fact with the class after you discover each number.

1250×380

____

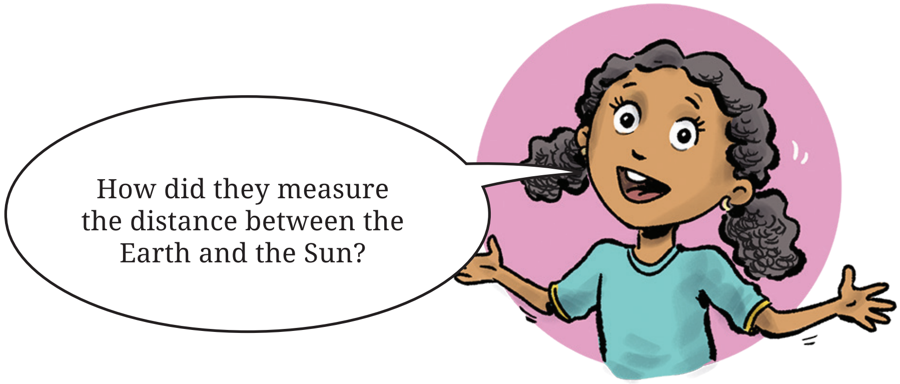
is the number of *kīrtanas* composed by Purandaradāsa according to legends.
Purandaradāsa was a composer and singer in the 15th century. His *kīrtanas* spanned social reform, *bhakti* and spirituality. He systematised methods for teaching Carnatic music which is followed to the present day.

$2100 \times 70{,}000$

____

is the approximate distance in kilometers, between the Earth and the Sun.

This distance keeps varying throughout the year. The farthest distance is about 152 million kilometers.



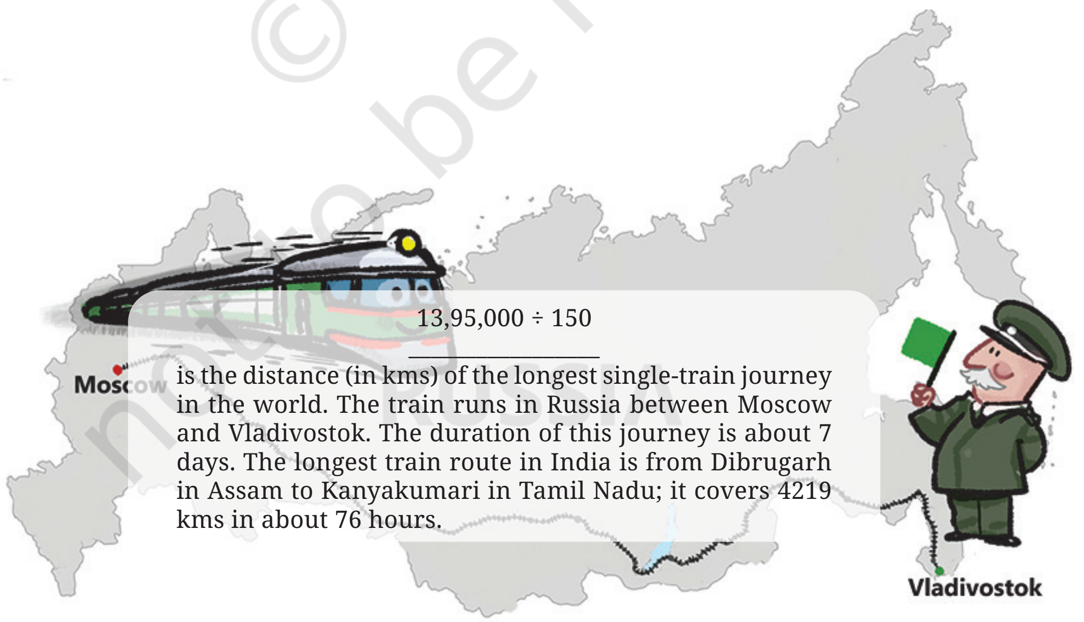
6400 × 62,500

____

is the average number of litres of water the Amazon river discharges into the Atlantic Ocean every second.

The river's flow into the Atlantic is so much that drinkable freshwater is found even 160 kilometers into the open sea.

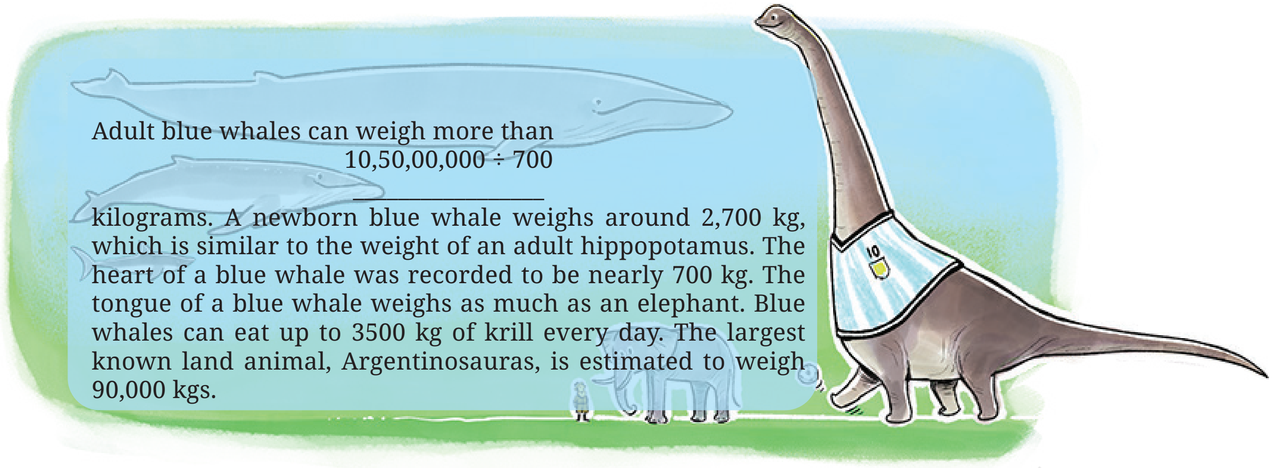
As you did before, divide the given numbers to uncover interesting facts about division. Share your thoughts and questions with the class after you uncover each number.



13,95,000 ÷ 150

____

is the distance (in kms) of the longest single-train journey in the world. The train runs in Russia between Moscow and Vladivostok. The duration of this journey is about 7 days. The longest train route in India is from Dibrugarh in Assam to Kanyakumari in Tamil Nadu; it covers 4219 kms in about 76 hours.



Large Number Fact

In a single gram of healthy soil there can be 100 million to 1 billion bacteria and 1 lakh to 1 million fungi, which can support plants' growth and health.

Share such large-number facts you know / come across with your class.

1.6 Did You Ever Wonder...?

Estu is amused by all these interesting facts about large numbers. While thinking about these, he came up with an unusual question, "Could the entire population of Mumbai fit into 1 lakh buses?"

What do you think?

How can we find out?

Let us assume a bus can accommodate 50 people. Then 1 lakh buses can accommodate 1 lakh × 50 = 50 lakh people.

The population of Mumbai is more than 1 crore 24 lakhs (we saw this in an earlier table). So, the entire population of Mumbai cannot fit in 1 lakh buses.

? The RMS Titanic ship carried about 2500 passengers. Can the population of Mumbai fit into 5000 such ships?

Inspired by this strange question, Roxie wondered, "If I could travel 100 kilometers every day, could I reach the Moon in 10 years?" (The distance between the Earth and the Moon is 3,84,400 km.)

How far would she have travelled in a year?

How far would she have travelled in 10 years?

Is it not easier to perform these calculations in stages? You can use this method for all large calculations.

? Find out if you can reach the Sun in a lifetime, if you travel 1000 kilometers every day. (You had written down the distance between the Earth and the Sun in a previous exercise.)

? Make necessary reasonable assumptions and answer the questions below:

(a) If a single sheet of paper weighs 5 grams, could you lift one lakh sheets of paper together at the same time?

(b) If 250 babies are born every minute across the world, will a million babies be born in a day?

(c) Can you count 1 million coins in a day? Assume you can count 1 coin every second.

? Think and create more such fun questions and share them with your class.

? Figure it Out

1. Using all digits from 0 – 9 exactly once (the first digit cannot be 0) to create a 10-digit number, write the —

(a) Largest multiple of 5

(b) Smallest even number

2. The number 10,30,285 in words is Ten lakhs thirty thousand two hundred eighty five, which has 43 letters. Give a 7-digit number name which has the maximum number of letters.

3. Write a 9-digit number where exchanging any two digits results in a bigger number. How many such numbers exist?

4. Strike out 10 digits from the number 12345123451234512345 so that the remaining number is as large as possible.

5. The words ‘zero’ and ‘one’ share letters ‘e’ and ‘o’. The words ‘one’ and ‘two’ share a letter ‘o’, and the words ‘two’ and ‘three’ also share a letter ‘t’. How far do you have to count to find two consecutive numbers which do not share an English letter in common?

6. Suppose you write down all the numbers 1, 2, 3, 4, ..., 9, 10, 11, ... The tenth digit you write is ‘1’ and the eleventh digit is ‘0’, as part of the number 10. *Try This*

    (a) What would the 1000th digit be? At which number would it occur?

    (b) What number would contain the millionth digit?

    (c) When would you have written the digit ‘5’ for the 5000th time? *Try This*

7. A calculator has only ‘+10,000’ and ‘+100’ buttons. Write an expression describing the number of button clicks to be made for the following numbers:

    (a) 20,800

    (b) 92,100

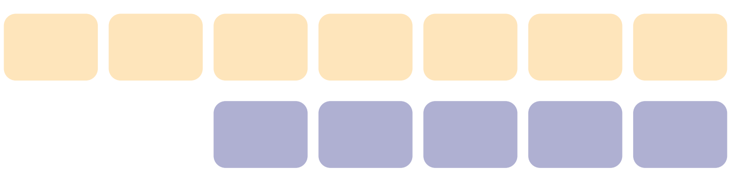
    (c) 1,20,500

    (d) 65,30,000

    (e) 70,25,700

8. How many lakhs make a billion?

9. You are given two sets of number cards numbered from 1 – 9. Place a number card in each box below to get the (a) largest possible sum (b) smallest possible difference of the two resulting numbers.



10. You are given some number cards; 4000, 13000, 300, 70000, 150000, 20, 5. Using the cards get as close as you can to the numbers below using any operation you want. Each card can be used only once for making a particular number.

    (a) 1,10,000: Closest I could make is 4000 × (20 + 5) + 13000 = 1,13,000

    (b) 2,00,000:

    (c) 5,80,000:

    (d) 12,45,000:

    (e) 20,90,800:

11. Find out how many coins should be stacked to match the height of the Statue of Unity. Assume each coin is 1 mm thick.

12. Grey-headed albatrosses have a roughly 7-feet wide wingspan. They are known to migrate across several oceans. Albatrosses can cover about 900 – 1000 km in a day. One of the longest single trips recorded is about 12,000 km. How many days would such a trip take to cross the Pacific Ocean approximately?

13. A bar-tailed godwit holds the record for the longest recorded non-stop flight. It travelled 13,560 km from Alaska to Australia without stopping. Its journey started on 13 October 2022 and continued for about 11 days. Find out the approximate distance it covered every day. Find out the approximate distance it covered every hour.

14. Bald eagles are known to fly as high as 4500 – 6000 m above the ground level. Mount Everest is about 8850 m high. Aeroplanes can fly as high as 10,000 – 12,800 m. How many times bigger are these heights compared to Somu's building?

SUMMARY

- We came across large numbers — lakhs, crores and arabs; millions and billions. We learnt how to read and write these numbers in the Indian and American/International naming systems.
  - (a) 1 lakh is 1 followed by 5 zeroes: 1,00,000
  - (b) 1 crore is 1 followed by 7 zeroes: 1,00,00,000
  - (c) 1 million is 1 followed by 6 zeroes: 1,000,000 (which is also ten lakhs)
  - (d) 1 arab is 1 followed by 9 zeroes: 1,000,000,000 (which is also 100 crore or 1 billion)
- We generally round up or round down large numbers. Many times it is enough just to know roughly how big or small something is.
- To get a sense of large numbers or quantities, we can check how many times bigger they are compared to numbers or quantities that are more familiar.
- We saw how to factorise numbers and regroup them to make multiplications simpler.
- We carried out interesting thought experiments such as — "Would one be able to watch 1000 movies in a year?"

Toothpick Digits

We can write digits as shown in the image below:

You can either use toothpicks or matchsticks, or just write the digits in this way, using lines to represent sticks.

To make the digit 7, three sticks are needed.

Write or make the number 5108. How many sticks are required?

1. Make or write the number 42,019. It would require exactly 23 sticks.
2. Starting with 42,019, add or write two more sticks, and make a bigger number. One example is 42,078. What other numbers bigger than 42,019 can you make in this way?
3. Preetham wants to insert the digit '1' somewhere among the digits '4', '2', '0', '1' and '9'. Where should he place the digit '1' to get the biggest possible number?
4. What other numbers can he make by placing the digit '1'?

---

1. Make or write the number 63,890.
2. Starting with 63,890, rearrange exactly four sticks and make a bigger number. One example is 88,078. What other numbers bigger than 63,890 can you make in this way?

---

1. Make any number using exactly 24 sticks or lines.
2. What is the biggest number that can be made using 24 sticks or lines?
3. What is the smallest number that can be made using 24 sticks or lines?

Make your own questions and challenge each other.

GANITA PRAKASH

TEXTBOOK OF MATHEMATICS

GRADE

7

Part – I

0774

विद्ययाऽमृतमश्नुते

एन सी ई आर टी
NCERT

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
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FOREWORD

The National Education Policy 2020 envisages a system of education in the country that is rooted in an Indian ethos and its civilisational accomplishments in all fields of knowledge and human endeavour. At the same time, it aims to prepare students to engage constructively with the opportunities and challenges of the 21st century. The basis for this aspirational vision has been well laid out by the National Curriculum Framework for School Education (NCF-SE) 2023 across curricular areas at all stages. By nurturing students' inherent abilities across all five planes of human existence (pañchakośhas), the Foundational and Preparatory Stages set the stage for further learning at Middle Stage. Spanning Grades 6 to 8, the Middle Stage serves as a critical three-year bridge between the Preparatory and Secondary Stages.

The NCF-SE 2023, at the Middle Stage, aims to equip students with the skills that are needed to grow, as they advance in their lives. It endeavours to enhance their analytical, descriptive, and narrative capabilities, and to prepare them for the challenges and opportunities that await them. A diverse curriculum, covering nine subjects ranging from three languages — including at least two languages native to India — to Science, Mathematics, Social Science, Art Education, Physical Education and Well-being, and Vocational Education promotes their holistic development.

Such a transformative learning culture requires certain essential conditions. One of them is to have appropriate textbooks in different curricular areas, as these textbooks will play a central role in mediating between content and pedagogy — a role that will strike a judicious balance between direct instruction and opportunities for exploration and inquiry. Among the other conditions, classroom arrangement and teacher preparation are crucial to establish conceptual connections both within and across curricular areas.

The National Council of Educational Research and Training, on its part, is committed to providing students with such high-quality textbooks. Various Curricular Area Groups, which have been constituted for this purpose, comprising notable subject-experts, pedagogues, and practising teachers as their members, have made all possible efforts to develop such textbooks. Ganita Prakash, the textbook of Mathematics for Grade 7, Part I, aligns with the expectations of NEP 2020 and NCF-SE 2023, with respect to creating a spark for initiating mathematical thinking. This textbook designed for Grade 7 students, takes forward its journey through the world of mathematics that started in Grade 6. During this journey the concepts and problems emerge from daily life situations and so it is expected that students will be able to relate to them with ease. The book makes efforts to encourage the students to observe and explore the patterns around them and discover mathematical concepts on their own. The content attempts to integrate mathematics with other subject areas such as science, social science with cross-cutting themes like environmental education, value education, inclusive education, and Indian Knowledge Systems (IKS). Colourful illustrations and interactive exercises form the basis of this textbook that would develop a strong foundation among children in understanding more complex mathematical concepts. Throughout the book, stories, conversations and anecdotes have been incorporated to make abstract mathematical concepts more relatable and accessible to young learners. Puzzles and innovative problems will not only engage the students in thoughtfully relating the mathematical concepts to the world around them and help them in deepening their understanding of mathematics, but also prepare them to understand the concepts of the emerging field of computational thinking. The focus is on collaboration and active engagement through student-centered approach to education.

However, in addition to this textbook, students at this stage should also be encouraged to explore various other learning resources. School libraries play a crucial role in making such resources available. Besides, the role of parents and teachers will also be invaluable in guiding and encouraging students to do so.

With this, I express my gratitude to all those who have been involved in the development of this textbook and hope that it will meet the expectations of all stakeholders. At the same time, I also invite suggestions and feedback from all its users for further improvement in the coming years.

Dinesh Prasad Saklani
Director
National Council of Educational
Research and Training

New Delhi
March 2025

ABOUT THE BOOK

Mathematics helps students develop not only basic arithmetic skills, but also the crucial capacities of logical reasoning, creative problem solving, and clear and precise communication (both oral and written). Mathematical knowledge also plays a crucial role in understanding concepts in other school subjects, such as Science and Social Science, and even Arts, Physical Education, and Vocational Education. Learning Mathematics can also contribute to the development of capacities for making informed choices and decisions. Understanding numbers and quantitative arguments is necessary for effective and meaningful democratic and economic participation. Mathematics thus has an important role to play in achieving the overall aims of school education.

Mathematics at the Middle Stage is a major challenge and has to perform the dual role of being both close to the experience and environment of the child and being abstract. It must perform the dual role of developing intuition while also maintaining and emphasising rigour. It must perform the dual role of enhancing critical and logical thinking while also developing artistry and creativity and a sense of elegance and aesthetics. Finally, Mathematics must perform the dual role of providing students plenty of opportunities for exploration and discovery of concepts on their own while also teaching best-known methods in the global repertoire of mathematics.

The present textbook has made an attempt to address the above mentioned goals and challenges of learning mathematics. The writers of this book have aimed to strike a judicious balance between informal and formal definitions and methods to develop in students both intuition and rigour. The book also provides numerous opportunities for student-student and student–teacher interaction in the classroom to promote active and experiential learning. A number of questions, puzzles, and interactive exercises are posed throughout the book to encourage constant exploration. Many of the questions are open-ended to stimulate in-class discussion.

The first chapter of this book, 'Large Numbers Around Us', is an introduction to the world of lakhs and crores, and millions and billions through engaging explorations and contexts.

Chapter 2, 'Arithmetic Expressions', considers expressions having multiple operations and discusses how they can be written and read without ambiguity. Chapter 3, 'A Peek Beyond the Point', introduces the usage of the decimal point and additions and subtractions involving decimal numbers. Chapter 4, 'Expressions using Letter-Numbers' builds on the chapter on Arithmetic Expressions to guide students to take the first steps into the world of algebra. The fundamental notions of letter-numbers and algebraic expressions are introduced. Chapter 5, 'Parallel and Intersecting Lines', introduces some fundamental building blocks of geometry and has a balance of engaging activities like paper folding and rigorous mathematical reasoning. Chapter 6, 'Number Play', covers aspects of computational thinking and problem solving through puzzles and concepts of parity, the Virahāṅka-Fibonacci sequence, and cryptarithms. Chapter 7, 'A Tale of Three Intersecting Lines', explores, through construction, some of the striking properties of a triangle related to the lengths of their sides and angles. Chapter 8, 'Working with Fractions', builds on the students' understanding of fractions, and explores the multiplication and division of fractions. The methods and formulas of the great Indian mathematician Brahmagupta (628 CE) for performing these operations on fractions is brought alive with fresh examples and interesting puzzles. In all chapters, an attempt has been made to emphasise connections with other subjects including Art, History, and Science.

By weaving together storytelling and hands-on activities, we hope that an immersive learning experience will be created that ignites curiosity and fosters a love for mathematics. It is hoped that teachers would give children the opportunity to discuss, play, engage with each other, provide logical arguments for different ideas, and find loopholes in arguments presented. This is necessary for the learners to eventually develop the ability to understand what it means to prove something and also become confident about underlying concepts. The mathematics classroom should not expect a blind application of algorithms but should rather encourage children to find many different ways to solve problems.

As per the NEP 2020, computational thinking has also been gently introduced through puzzles, games, and interactive exercises that encourage such thinking. Indian rootedness has also been kept in mind while giving contexts for different concepts. The contributions of Indian mathematicians have been given as part of a problem-solving approach to make students aware of India's rich mathematical heritage and its global contributions to mathematics.

The concepts and problems are related to daily life situations. An attempt has been made to use contexts and materials with which the students are familiar. Learning material sheets have been given at the back of the book that may be photocopied and used. At many places, exercises or activities are given to encourage peer group efforts and discussions. The textbook intends to address the learning needs of a diverse group of students in the classroom.

We have tried to link concepts learnt in initial chapters with ideas in subsequent chapters to show the connectedness and unity of mathematics. We hope that teachers will use this as an opportunity to revise these concepts in a spiralling way so that children are able to appreciate the entire conceptual structure of mathematics. We hope that teachers may give more time to the ideas of fractions, negative numbers and other notions that are new to students. Many of these are the basis for further learning in mathematics.

Finally, this book aims to be more than just a textbook—it's a passport to a world of mathematical discovery and exploration. Whether used in the classroom or at home, we hope that it may inspire students to embark on their own mathematical adventures, empowering them to see the beauty and relevance of mathematics in everything around them. With its engaging approach and comprehensive coverage of Grade 7 mathematics concepts, this book hopes and aims to captivate young minds and set them on a lifelong journey of mathematical discovery.

I thank again all the writers of and contributors to this textbook for this important and valuable contribution and service to the nation's mathematics teachers, learners and enthusiasts.

We look forward to your comments and suggestions regarding the book and hope that you will send interesting exercises, activities and tasks that you develop during the course of teaching and learning, to be included in future editions.

Ashutosh Wazalwar
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NOTE TO THE TEACHER

We hope that this book, *Ganita Prakash*, will serve as a strong support and guide to you in achieving the exciting task that you have before you: that of passing on the joy of learning the beautiful subject of Mathematics to the next generation.

This task calls for providing a fertile environment that allows for the flowering of mathematical thinking in the minds of students. Classrooms, where students just listen and write down whatever is being told to them or written on the board, are deficient in the conditions required for learning mathematics. Instead, classrooms need to be places where students are engaged in playing with mathematical concepts, finding and discussing patterns, and developing creative strategies together to solve problems. Students should also be posing problems to each other and discussing possible solutions with each other. In fact, these are the very conditions that have led to the development of the entire field of mathematics so far, and so one cannot expect students to pick up mathematical thinking and understanding without these conditions.

Fortunately, it is not difficult to create such conditions in the classroom. It just requires an interesting question, problem, pattern, or challenge to be thrown open to the students on a regular basis, and sufficient time to be given to them to play with, discuss, and work on it as a class or in pairs or groups.

Along with it, an environment that accepts mistakes and acknowledges their importance in learning needs to be nurtured.

While creating the spark for initiating mathematical thinking in classrooms is not difficult, sustaining it may be challenging and may involve efforts from your side. Nevertheless, even if just the first part of throwing open a question, problem, pattern, or challenge is done at least once or twice a week, accompanied by sufficient waiting time from your side for students to play, discuss, and work on it, it can have a great positive impact on how the students view and approach mathematics.

It should be noted that this positive impact will not happen overnight. That takes time and depends on various factors such as the number of opportunities you give for problem solving, your patience, and the encouragement you give to the students.

To support you in posing problems, all the problems or questions in this book are marked using the icon ?. This icon is an indicator of a potential opportunity to start off a process of problem solving and exploration in the classroom. You will find some of the problems labelled “Math Talk”. Such questions can especially be made as topics for classroom discussion.

An owl mascot appears at various points in the textbook to highlight important mathematical processes, ways of thinking, and problem-solving approaches. These can be brought out during classroom discussions, both where the owl is present and also in other similar situations.

To develop students' mathematical thinking and understanding of concepts, a sufficient number of problems are given. Trying to "cover" all of them must not happen at the cost of students not getting to spend quality time on playing with and discussing them.

It is important to understand that the exploratory problems are not only for promoting problem solving skills; they also serve in strengthening procedural fluency when children start engaging in exploration.

Efforts must be made in making students independent learners. One essential aspect required for this is an ability to read and understand mathematical text. To promote this skill, students should be encouraged to read the book by themselves and in groups. Give opportunities to them to interpret what they read and express it to others. This will also address the big problem that students face in speaking mathematics and interpreting word problems.

This book contains a number of open-ended problems. It also contains new treatments of certain concepts. If you are not able to solve them or follow some of them immediately, it is perfectly okay! Not everyone knows everything. Along with trying to understand and reflect upon such content, it will be very useful to take it to the classroom and open it up for discussion. After the discussion, things that are clear and those that are not yet clear can be clearly summarised. This process itself can throw a lot of light on the content.

In these discussions, you can participate as a fellow seeker, and when students see a teacher seek and think to understand something, it sets a wonderful example for them.

It is hoped that you and your students will have a great and fruitful time using this book!

Summary of Key Points

Time for Exploration

1. It is important to routinely pose new problems, questions, patterns, or challenges to the students and give them sufficient time to play with, discuss, and work on them, individually and in groups.
2. During this time, an environment that accepts mistakes and acknowledges their importance in learning needs to be nurtured.
3. There should be a culture where students pose problems to each other and discuss with each other various ways to approach the problems.

About the Problems in the Book

1. The exploratory problems in the book not only promote problem solving; they also aim to strengthen procedural fluency when children start engaging in exploration.

2. Trying to ‘cover’ all the problems in the book must not happen at the cost of students not getting to spend quality time on playing with, discussing, and solving them.

Reading

1. Encourage students to read the book by themselves and in groups.
2. Give opportunities to them to interpret what they read and to express it to others.

Right of Not Knowing!

1. It is perfectly okay if some of the content is not understood immediately. Along with trying to understand and reflect upon such content, it can also be taken to the classroom and opened up for discussion. After the discussion, things that are clear and those that are not yet clear can be clearly summarised. In these discussions, you can participate as a fellow seeker, and when students see a teacher seek and think to understand something, it sets a wonderful example for them!
2. Learning is a continual process. Indeed, there is so much in mathematics that is still not known and requires further exploration!

CONSTITUTION OF INDIA

Part III (Articles 12 – 35)
(Subject to certain conditions, some exceptions
and reasonable restrictions)
guarantees these

Fundamental Rights

Right to Equality

- before law and equal protection of laws;
- irrespective of religion, race, caste, sex or place of birth;
- of opportunity in public employment;
- by abolition of untouchability and titles.

Right to Freedom

- of expression, assembly, association, movement, residence and profession;
- of certain protections in respect of conviction for offences;
- of protection of life and personal liberty;
- of free and compulsory education for children between the age of six and fourteen years;
- of protection against arrest and detention in certain cases.

Right against Exploitation

- for prohibition of traffic in human beings and forced labour;
- for prohibition of employment of children in hazardous jobs.

Right to Freedom of Religion

- freedom of conscience and free profession, practice and propagation of religion;
- freedom to manage religious affairs;
- freedom as to payment of taxes for promotion of any particular religion;
- freedom as to attendance at religious instruction or religious worship in educational institutions wholly maintained by the State.

Cultural and Educational Rights

- for protection of interests of minorities to conserve their language, script and culture;
- for minorities to establish and administer educational institutions of their choice.

Right to Constitutional Remedies

- by issuance of directions or orders or writs by the Supreme Court and High Courts for enforcement of these Fundamental Rights.

A NOTE TO STUDENTS!

To be able to appreciate the art of mathematics, it is not enough to just be a passive spectator. You need to immerse yourself in its process like a detective getting into action to solve a mystery.

This is especially required when you see a new question or when a question arises from your own sense of wonder, or when you come across a new beautiful pattern. When you encounter these, pause your reading, and use your creativity to work out the question or understand and appreciate the pattern.

You will find that some questions are accompanied by their answers. Even if this is the case, it is worthwhile to work on the problems by yourself or in a group before you see the answer.

This will enrich your experience of going through the book!

Whenever there are questions coming up, you will see this icon: ?. This indicates that it is time for figuring things out!

Sometimes you will find many questions collected together in a single place under the title '**Figure it Out**'.

Some questions are marked Math Talk. These questions are meant to be discussed and worked out with your friends.

Finally, there are questions marked Try This. These questions demand more creativity to be answered, and therefore will also often be more fun to answer as a result!

Constitution of India

Part IV A (Article 51 A)

Fundamental Duties

It shall be the duty of every citizen of India —

(a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;

(b) to cherish and follow the noble ideals which inspired our national struggle for freedom;

(c) to uphold and protect the sovereignty, unity and integrity of India;

(d) to defend the country and render national service when called upon to do so;

(e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;

(f) to value and preserve the rich heritage of our composite culture;

(g) to protect and improve the natural environment including forests, lakes, rivers, wildlife and to have compassion for living creatures;

(h) to develop the scientific temper, humanism and the spirit of inquiry and reform;

(i) to safeguard public property and to abjure violence;

(j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement;

*(k) who is a parent or guardian, to provide opportunities for education to his child or, as the case may be, ward between the age of six and fourteen years.

Note: The Article 51A containing Fundamental Duties was inserted by the Constitution (42nd Amendment) Act, 1976 (with effect from 3 January 1977).

*(k) was inserted by the Constitution (86th Amendment) Act, 2002 (with effect from 1 April 2010).

CONTENTS

THE CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a [1]**[SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC]** and to secure to all its citizens :

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the [2][unity and integrity of the Nation];

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949 do **HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.**

---

1. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Sovereign Democratic Republic" (w.e.f. 3.1.1977)
2. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Unity of the Nation" (w.e.f. 3.1.1977)

8 WORKING WITH FRACTIONS

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8.1 Multiplication of Fractions

Aaron walks 3 kilometres in 1 hour. How far can he walk in 5 hours?

This is a simple question. We know that to find the distance, we need to find the product of 5 and 3, i.e., we multiply 5 and 3.

Distance covered in 1 hour = 3 km.

Therefore,

Distance covered in 5 hours

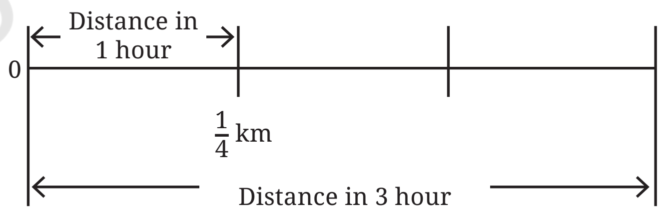
$$= 5 \times 3 \text{ km}$$

$$= 3 + 3 + 3 + 3 + 3 \text{ km}$$

$$= 15 \text{ km.}$$

? Aaron's pet tortoise walks at a much slower pace. It can walk only $\frac{1}{4}$ kilometre in 1 hour. How far can it walk in 3 hours?

Here, the distance covered in an hour is a fraction. This does not matter. The total distance covered is calculated in the same way, as multiplication.



Distance covered in 1 hour = $\frac{1}{4}$ km.

Therefore, distance covered in 3 hours = $3 \times \frac{1}{4}$ km

$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ km

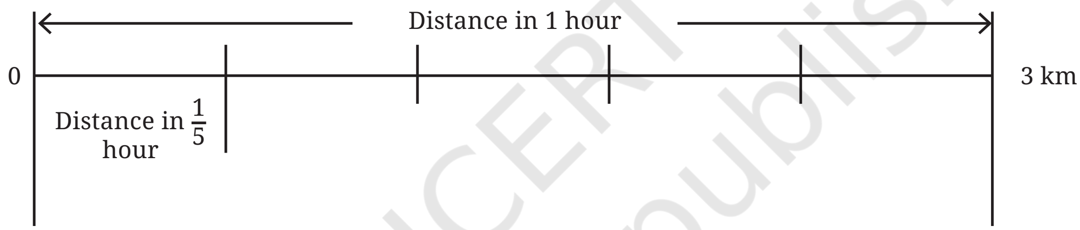
$= \frac{3}{4}$ km.

The tortoise can walk $\frac{3}{4}$ km in 3 hours.

Let us consider a case where the time spent walking is a fraction of an hour.

? We saw that Aaron can walk 3 kilometres in 1 hour. How far can he walk in $\frac{1}{5}$ hours?

We continue to calculate the total distance covered through multiplication.



Distance covered in $\frac{1}{5}$ hours = $\frac{1}{5} \times 3$ km.

Finding the product:

Distance covered in 1 hour = 3 km.

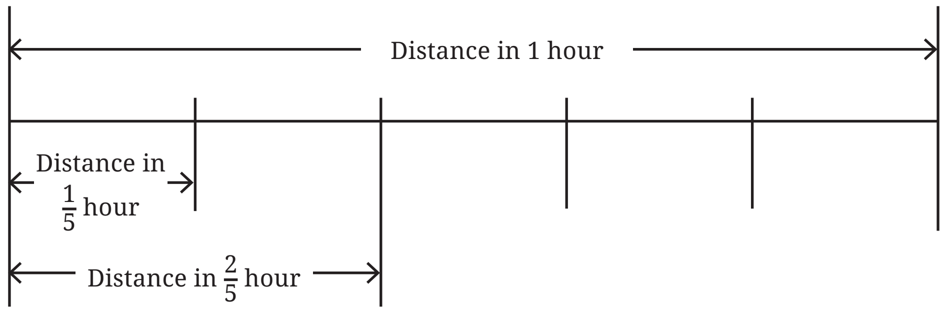
In $\frac{1}{5}$ hours, distance covered is equal to the length we get by dividing 3 km into 5 equal parts, which is $\frac{3}{5}$ km.

This tells us that $\frac{1}{5} \times 3 = \frac{3}{5}$.

? How far can Aaron walk in $\frac{2}{5}$ hours?

Once again, we have—

Distance covered = $\frac{2}{5} \times 3$ km.



Finding the product:

1. We can first find the distance covered in $\frac{1}{5}$ hours.
2. Since, the duration $\frac{2}{5}$ is twice $\frac{1}{5}$, we multiply this distance by 2 to get the total distance covered.

Here is the calculation.

Distance covered in 1 hour = 3 km.

1. Distance covered in $\frac{1}{5}$ hour

   = The length we get by dividing 3 km in 5 equal parts

   = $\frac{3}{5}$ km.

2. Multiplying this distance by 2, we get

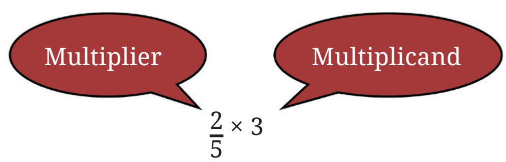
$$2 \times \frac{3}{5} = \frac{6}{5} \text{ km.}$$

From this we can see that

$$\frac{2}{5} \times 3 = \frac{6}{5}.$$

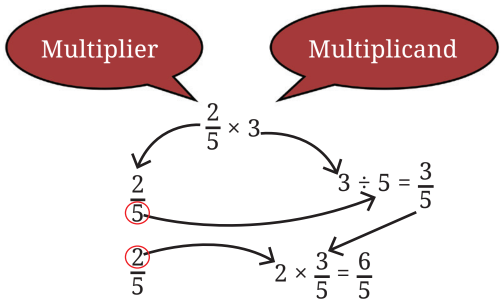
Discussion

We did this multiplication as follows:



- First, we divided the multiplicand, 3 , by the denominator of the multiplier, 5, to get $\frac{3}{5}$.

- We then multiplied the result by the numerator of the multiplier, that is 2, to get $\frac{6}{5}$.



Thus, whenever we need to multiply a fraction and a whole number, we follow the steps above.

Example 1: A farmer had 5 grandchildren. She distributed $\frac{2}{3}$ acre of land to each of her grandchildren. How much land in all did she give to her grandchildren?

$$5 \times \frac{2}{3} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{10}{3}.$$

Example 2: 1 hour of internet time costs ₹8. How much will $1\frac{1}{4}$ hours of internet time cost?

$1\frac{1}{4}$ hours is $\frac{5}{4}$ hours (converting from a mixed fraction).

Cost of $\frac{5}{4}$ hour of internet time $= \frac{5}{4} \times 8$

$$= 5 \times \frac{8}{4}$$
$$= 5 \times 2$$
$$= 10.$$

It costs ₹10 for $1\frac{1}{4}$ hours of internet time.

Figure it Out

1. Tenzin drinks $\frac{1}{2}$ glass of milk every day. How many glasses of milk does he drink in a week? How many glasses of milk did he drink in the month of January?
2. A team of workers can make 1 km of a water canal in 8 days. So, in one day, the team can make ___ km of the water canal. If they work 5 days a week, they can make ___ km of the water canal in a week.
3. Manju and two of her neighbours buy 5 litres of oil every week and share it equally among the 3 families. How much oil does each family get in a week? How much oil will one family get in 4 weeks?
4. Safia saw the Moon setting on Monday at 10 pm. Her mother, who is a scientist, told her that every day the Moon sets $\frac{5}{6}$ hour later than

the previous day. How many hours after 10 pm will the moon set on Thursday?

5. Multiply and then convert it into a mixed fraction:

   (a) $7 \times \frac{3}{5}$

   (b) $4 \times \frac{1}{3}$

   (c) $\frac{9}{7} \times 6$

   (d) $\frac{13}{11} \times 6$

So far, we have learnt multiplication of a whole number with a fraction, and a fraction with a whole number. What happens when both numbers in the multiplication are fractions?

Multiplying Two Fractions

We know, that Aaron's pet tortoise can walk only $\frac{1}{4}$ km in 1 hour. How far can it walk in half an hour?

Hour	Distance
1	$\frac{1}{4}$
$\frac{1}{2}$	?

Following our approach of using multiplication to solve such problems, we have,

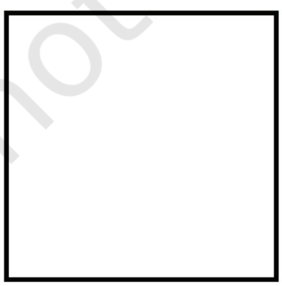
Distance covered in $\frac{1}{2}$ hour = $\frac{1}{2} \times \frac{1}{4}$ km.

Finding the product:

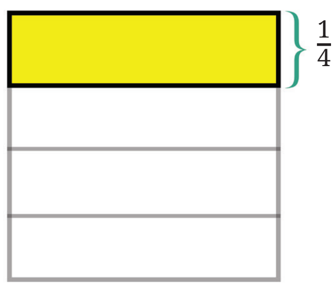
Distance covered in 1 hour = $\frac{1}{4}$ km.

Therefore, the distance covered in $\frac{1}{2}$ an hour is the length we get by dividing $\frac{1}{4}$ into 2 equal parts.

To find this, it is useful to represent fractions using the unit square to stand for a "whole".



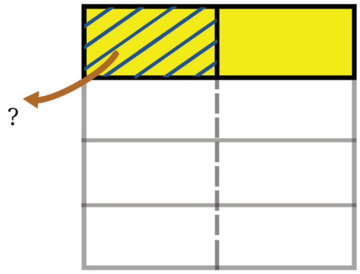
Unit square as a "whole"



$\frac{1}{4}$ of the whole

Now we divide this $\frac{1}{4}$ into 2 equal parts. What do we get?

What fraction of the whole is shaded?



$\frac{1}{4}$ divided into 2 equal parts

Since the whole is divided into 8 equal parts and one of the parts is shaded, we can say that $\frac{1}{8}$ of the whole is shaded. So, the distance covered by the tortoise in half an hour is $\frac{1}{8}$ km.

This tells us that $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$.

? If the tortoise walks faster and it can cover $\frac{2}{5}$ km in 1 hour, how far will it walk in $\frac{3}{4}$ of an hour?

Distance covered = $\frac{3}{4} \times \frac{2}{5}$.

Finding the product:

(i) First find the distance covered in $\frac{1}{4}$ of an hour.

(ii) Multiply the result by 3, to get the distance covered in $\frac{3}{4}$ of an hour.

(i) Distance in km covered in $\frac{1}{4}$ of an hour

= The quantity we get by dividing $\frac{2}{5}$ into 4 equal parts.

Taking the unit square as the whole, the shaded part (in Fig. 8.1) is a region we get when we divide $\frac{2}{5}$ into 4 equal parts.

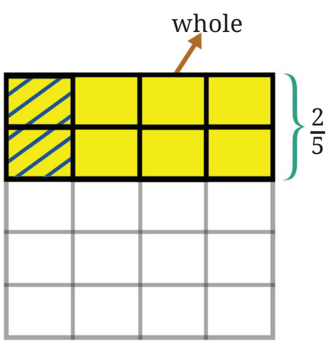


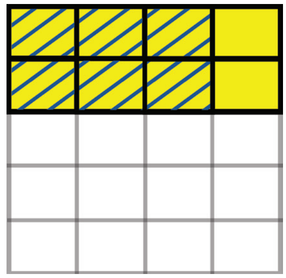
Fig. 8.1

How much of the whole is it?

The whole is divided into
5 rows and 4 columns,
creating 5 × 4 = 20 equal parts.

Number of these parts shaded = 2.

So, the distance covered in $\frac{1}{4}$ of an hour = $\frac{2}{20}$.



(ii) Now, we need to multiply $\frac{2}{20}$ by 3.

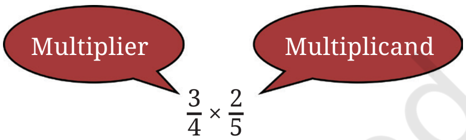
Distance covered in $\frac{3}{4}$ of an hour $= 3 \times \frac{2}{20}$

$= \frac{6}{20}$.

So, $\frac{3}{4} \times \frac{2}{5} = \frac{6}{20} = \frac{3}{10}$.

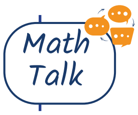
Discussion

In the case of a fraction multiplied by another fraction, we follow a method similar to the one we used, when we multiplied a fraction by a whole number. We multiplied as follows:



$\frac{3}{4}$ $\quad$ $\frac{2}{5} \div 4 = \frac{2}{20}$ Divide the multiplicand by 4.

$\frac{3}{4}$ $\quad$ $3 \times \frac{2}{20} = \frac{6}{20} =$ Divide the multiplicand by 3.



Using this understanding, multiply $\frac{5}{4} \times \frac{3}{2}$.

First, let us represent $\frac{3}{2}$, taking the unit square as the whole. Since, the fraction $\frac{3}{2}$ is one whole and a half, it can be seen as follows:

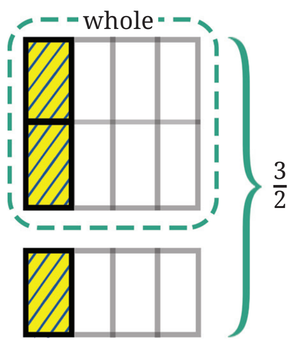


Fig. 8.2

Following the steps of multiplication, we need to first divide this fraction $\frac{3}{2}$ into 4 equal parts. It can be done as shown in the Fig. 8.2 with the yellow shaded region representing the fraction obtained by dividing $\frac{3}{2}$ into 4 equal parts. What is its value?

We see that the whole is divided into —
2 rows and 4 columns,
creating $2 \times 4 = 8$ equal parts.
Number of parts shaded = 3.

So the yellow shaded part $= \frac{3}{8}$.

Now, the next step is multiplying this result by 5. This gives the product of $\frac{5}{4}$ and $\frac{3}{2}$:

$$\frac{5}{4} \times \frac{3}{2} = 5 \times \frac{3}{8} = \frac{15}{8}.$$

Connection between the Area of a Rectangle and Fraction Multiplication

In the Fig. 8.3, what is the length and breadth of the shaded rectangle? Since we started with a unit square (of side 1 unit), the length and breadth are $\frac{1}{2}$ unit and $\frac{1}{4}$ unit.

What is the area of this rectangle? We see that 8 such rectangles give the square of area 1 square unit. So, the area of each rectangle is $\frac{1}{8}$ square units.

Fig. 8.3

? Do you see any relation between the area and the product of length and breadth?

The area of a rectangle of fractional sides equals the product of its sides.

In general, if we want to find the product of two fractions, we can find the area of the rectangle formed with the two fractions as its sides.

? Figure it Out

1. Find the following products. Use a unit square as a whole for representing the fractions:

   (a) $\frac{1}{3} \times \frac{1}{5}$

   (b) $\frac{1}{4} \times \frac{1}{3}$

   (c) $\frac{1}{5} \times \frac{1}{2}$

   (d) $\frac{1}{6} \times \frac{1}{5}$

   Now, find $\frac{1}{12} \times \frac{1}{18}$.

Doing this by representing the fractions using a unit square is cumbersome. Let us find the product by observing what we did in the above cases.

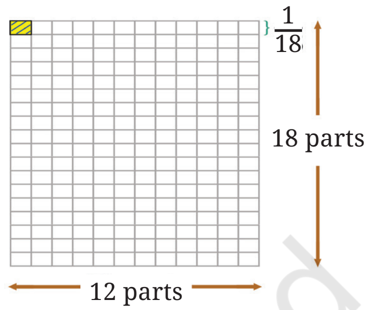
In each case, the whole is divided into rows and columns.

The number of rows is the denominator of the multiplicand, which is 18 in this case.

The number of columns is the denominator of the multiplier, which is 12 in this case.

Thus, the whole is divided into 18 × 12 equal parts.

So, $\frac{1}{18} \times \frac{1}{12} = \frac{1}{(18 \times 12)} = \frac{1}{216}$.



Thus, when two fractional units are multiplied, their product is

$$\frac{1}{\text{(product of denominators)}}.$$

We express this as:

$$\frac{1}{b} \times \frac{1}{d} = \frac{1}{b \times d}.$$

2. Find the following products. Use a unit square as a whole for representing the fractions and carrying out the operations.

(a) $\frac{2}{3} \times \frac{4}{5}$ (b) $\frac{1}{4} \times \frac{2}{3}$

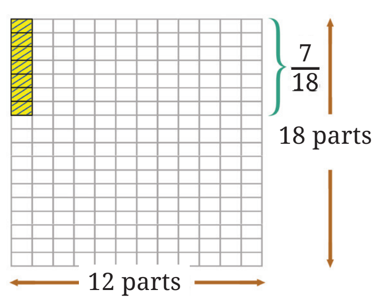
(c) $\frac{3}{5} \times \frac{1}{2}$ (d) $\frac{4}{6} \times \frac{3}{5}$

Multiplying Numerators and Denominators

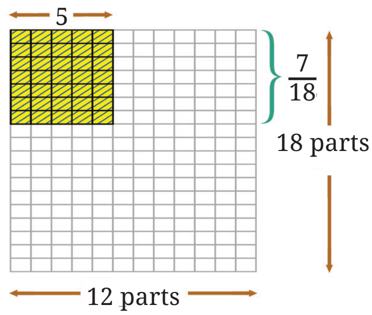
Now, find $\frac{5}{12} \times \frac{7}{18}$.

Like the previous case, let us find the product by performing the multiplication, step by step. First, the whole is divided into 18 rows and 12 columns creating 12 × 18 equal parts.

The value we get by dividing $\frac{7}{18}$ into 12 equal parts is $\frac{7}{(12 \times 18)}$.



Then, we multiply this result by 5 to get the product. This is $\frac{(5 \times 7)}{(12 \times 18)}$.



So, $\frac{5}{12} \times \frac{7}{8} = \frac{(5 \times 7)}{(12 \times 18)} = \frac{35}{216}$.

From this we can see that, in general,

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}.$$

This formula was first stated in this general form by Brahmagupta in his *Brāhmasphuṭasiddhānta* in 628 CE.

The formula above works even when the multiplier or multiplicand is a whole number. We can simply rewrite the whole number as a fraction with denominator 1. For example,

$$3 \times \frac{3}{4} \text{ can be written } \frac{3}{1} \times \frac{3}{4}$$

$$= \frac{3 \times 3}{1 \times 4} = \frac{9}{4}.$$

And,

$$\frac{3}{5} \times 4 \text{ can be written } \frac{3}{5} \times \frac{4}{1}$$

$$= \frac{3 \times 4}{5 \times 1} = \frac{12}{5}.$$

Multiplication of Fractions—Simplifying to Lowest Form

? Multiply the following fractions and express the product in its lowest form:

$$\frac{12}{7} \times \frac{5}{24}$$

Instead of multiplying the numerators (12 and 5) and denominators (7 and 24) first and then simplifying, we could do the following:

$$\frac{12}{7} \times \frac{5}{24} = \frac{12 \times 5}{7 \times 24}$$

We see that both the circled numbers have a common factor of 12. We know that a fraction remains the same when the numerator and denominator are divided by the common factor. In this case, we can divide them by 12.

$$\frac{\overset{1}{\cancel{12}} \times 5}{7 \times \underset{2}{\cancel{24}}} = \frac{1 \times 5}{7 \times 2} = \frac{5}{14}.$$

Let us use the same technique to do one more multiplication.

$$\frac{14}{15} \times \frac{25}{42}$$

$$\frac{\overset{1}{\cancel{14}} \times \overset{5}{\cancel{25}}}{\underset{3}{\cancel{15}} \times \underset{3}{\cancel{42}}} = \frac{1 \times 5}{3 \times 3} = \frac{5}{9}.$$

When multiplying fractions, we can first divide the numerator and denominator by their common factors before multiplying the numerators and denominators. This is called cancelling the common factors.

A Pinch of History

In India, the process of reducing a fraction to its lowest terms — known as *apavartana* — is so well known that it finds mention even in a non-mathematical work. A Jaina scholar Umasvati (c. 150 CE) used it as a simile in a philosophical work.

? Figure it Out

1. A water tank is filled from a tap. If the tap is open for 1 hour, $\frac{7}{10}$ of the tank gets filled. How much of the tank is filled if the tap is open for

   (a) $\frac{1}{3}$ hour ___________

   (b) $\frac{2}{3}$ hour ___________

   (c) $\frac{3}{4}$ hour ___________

   (d) $\frac{7}{10}$ hour ___________

   (e) For the tank to be full, how long should the tap be running? Try This

2. The government has taken $\frac{1}{6}$ of Somu's land to build a road. What part of the land remains with Somu now? She gives half of

the remaining part of the land to her daughter Krishna and $\frac{1}{3}$ of it to her son Bora. After giving them their shares, she keeps the remaining land for herself.

(a) What part of the original land did Krishna get?

(b) What part of the original land did Bora get?

(c) What part of the original land did Somu keep for herself?

3. Find the area of a rectangle of sides $3\frac{3}{4}$ ft and $9\frac{3}{5}$ ft.

4. Tsewang plants four saplings in a row in his garden. The distance between two saplings is $\frac{3}{4}$ m. Find the distance between the first and last sapling. [**Hint:** Draw a rough diagram with four saplings with distance between two saplings as $\frac{3}{4}$ m]

5. Which is heavier: $\frac{12}{15}$ of 500 grams or $\frac{3}{20}$ of 4 kg?

Is the Product Always Greater than the Numbers Multiplied?

Since, we know that when a number is multiplied by 1, the product remains unchanged, we will look at multiplying pairs of numbers where neither of them is 1.

When we multiply two counting numbers greater than 1, say 3 and 5, the product is greater than both the numbers being multiplied.

$$3 \times 5 = 15$$

The product, 15, is more than both 3 and 5.

But what happens when we multiply $\frac{1}{4}$ and 8?

$$\frac{1}{4} \times 8 = 2$$

In the above multiplication the product, 2, is greater than $\frac{1}{4}$, but less than 8.

What happens when we multiply $\frac{3}{4}$ and $\frac{2}{5}$?

$$\frac{3}{4} \times \frac{2}{5} = \frac{6}{20}$$

Let us compare this product $\frac{6}{20}$ with the numbers $\frac{3}{4}$ and $\frac{2}{5}$. For this, let us express $\frac{3}{4}$ as $\frac{15}{20}$ and $\frac{2}{5}$ as $\frac{8}{20}$.

From this we can see that the product is less than both the numbers.

When do you think the product is greater than both the numbers multiplied, when is it in between the two numbers, and when is it smaller than both?

[**Hint:** The relationship between the product and the numbers multiplied depends on whether they are between 0 and 1 or they are greater than 1. Take different pairs of numbers and observe their product. For each multiplication, consider the following questions.]

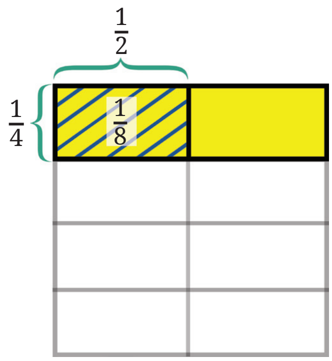
Situation	Multiplication	Relationship
Situation 1	Both numbers are greater than 1 (e.g., $\frac{4}{3} \times 4$)	The product ($\frac{16}{3}$) is greater than both the numbers
Situation 2	Both numbers are between 0 and 1 (e.g., $\frac{3}{4} \times \frac{2}{5}$)	The product ($\frac{3}{10}$) is less than both the numbers
Situation 3	One number is between 0 and 1, and one number is greater than 1 (e.g., $\frac{3}{4} \times 5$)	The product ($\frac{15}{4}$) is less than the number greater than 1 and greater than the number between 0 and 1

Create more such examples for each situation and observe the relationship between the product and the numbers being multiplied.

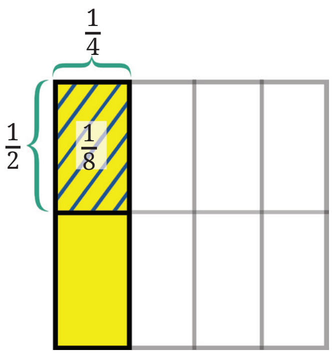
? What can you conclude about the relationship between the numbers multiplied and the product? Fill in the blanks:

- When one of the numbers being multiplied is between 0 and 1, the product is ____________ (greater/less) than the other number.
- When one of the numbers being multiplied is greater than 1, the product is _____________ (greater/less) than the other number.

Order of Multiplication



We know that $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$.



Now, what is $\frac{1}{4} \times \frac{1}{2}$?

That is $\frac{1}{8}$ too.

In general, note that the area of a rectangle remains the same even if the length and breadth are interchanged.

The order of multiplication does not matter. Thus,

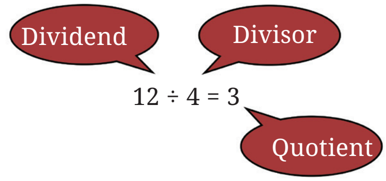
$$\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}.$$

This can also be seen from Brahmagupta's formula for multiplying fractions.

8.2 Division of Fractions

What is 12 ÷ 4? You know this already. But can this problem be restated as a multiplication problem?

What should be multiplied by 4 to get 12? That is,



$$4 \times ? = 12$$

We can use this technique of converting division into multiplication problems to divide fractions.

What is $1 \div \frac{2}{3}$?

Let us rewrite this as a multiplication problem

$$\frac{2}{3} \times ? = 1$$

What should be multiplied by $\frac{2}{3}$ to get the product 1?

If we somehow cancel out the 2 and the 3, we are left with 1.

$$\frac{\cancel{2}}{\cancel{3}} \times \underbrace{\boxed{\frac{\cancel{3}}{\cancel{2}}}}_{\text{Answer}} = 1$$

So,

$$1 \div \frac{2}{3} = \frac{3}{2}.$$

Let us try another problem:

$$3 \div \frac{2}{3}.$$

This is the same as

$$\frac{2}{3} \times ? = 3.$$

Can you find the answer?

We know what to multiply $\frac{2}{3}$ by to get 1. We just need to multiply that by 3 to get 3. So,

$$\frac{\cancel{2}}{\cancel{3}} \times \underbrace{\boxed{\frac{\cancel{3}}{\cancel{2}} \times 3}}_{\text{Answer}} = 3$$

So,

$$3 \div \frac{2}{3} = \frac{3}{2} \times 3 = \frac{9}{2}.$$

What is $\frac{1}{5} \div \frac{1}{2}$?

Rewriting it as a multiplication problem, we have

$$\frac{1}{2} \times ? = \frac{1}{5}.$$

How do we solve this?

$$\frac{1}{\cancel{2}} \times \underbrace{\boxed{\cancel{2} \times \frac{1}{5}}}_{\text{Answer}} = \frac{1}{5}$$

So,

$$\frac{1}{5} \div \frac{1}{2} = 2 \times \frac{1}{5} = \frac{2}{5}.$$

What is $\frac{2}{3} \div \frac{3}{5}$?

Rewriting this as multiplication, we have

$$\frac{3}{5} \times ? = \frac{2}{3}.$$

How will we solve this?

$$\frac{3}{\cancel{5}} \times \underbrace{\boxed{\frac{\cancel{5}}{\cancel{3}} \times \frac{2}{3}}}_{\text{Answer}} = \frac{2}{3}$$

So,

$$\frac{2}{3} \div \frac{3}{5} = \frac{5}{3} \times \frac{2}{3} = \frac{10}{9}.$$

Discussion

In each of the division problems above, observe how we found the answer. Can we frame a rule that tells us how to divide two fractions? Let us consider the previous problem.

In every division problem we have a dividend, divisor and quotient. The technique we have been using to get the quotient is:

$$\underbrace{\frac{2}{3}}_{\text{Dividend}} \div \underbrace{\frac{3}{5}}_{\text{Divisor}}$$

$$= \frac{5}{3} \times \frac{2}{3} = \underbrace{\frac{10}{9}}_{\text{Quotient}}$$

1. First, find the number which gives 1 when multiplied by the divisor. We see that the resulting number is a fraction whose numerator is the divisor's denominator and denominator is the divisor's numerator.

   For the divisor $\frac{3}{5}$ this fraction is $\frac{5}{3}$. We call $\frac{5}{3}$ the **reciprocal** of $\frac{3}{5}$. When we multiply a fraction by its reciprocal, we get 1. So, the first step in our technique is to find the divisor's reciprocal.

2. We then multiply the dividend with this reciprocal to get the quotient.

   Summarising, to divide two fractions:

   - Find the reciprocal of the divisor
   - Multiply this by the dividend to get the quotient.

So, $$\frac{a}{b} \div \frac{c}{d} = \frac{d}{c} \times \frac{a}{b} = \frac{d \times a}{c \times b}.$$

This can be rewritten as:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c}.$$

As with methods and formulas for addition, subtraction, and multiplication of fractions that you learnt earlier, this method and formula for division of fractions, in this general form, was first explicitly stated by Brahmagupta in his *Brāhmasphuṭasiddhānta* (628 CE).

So, to evaluate, for example, $\frac{2}{3} \div \frac{3}{5}$ using Brahmagupta's formula above, we write:

$$\frac{2}{3} \div \frac{3}{5} = \frac{2}{3} \times \frac{5}{3} = \frac{2 \times 5}{3 \times 3} = \frac{10}{9}.$$

Dividend, Divisor and the Quotient

When we divide two whole numbers, say 6 ÷ 3, we get the quotient 2. Here the quotient is less than the dividend.

$$6 \div 3 = 2,\ 2 < 6$$

But what happens when we divide 6 by $\frac{1}{4}$?

$$6 \div \frac{1}{4} = 24.$$

Here the quotient is greater than the dividend!

What happens when we divide $\frac{1}{8}$ by $\frac{1}{4}$?

$$\frac{1}{8} \div \frac{1}{4} = \frac{1}{2}.$$

Here too the quotient is greater than the dividend.

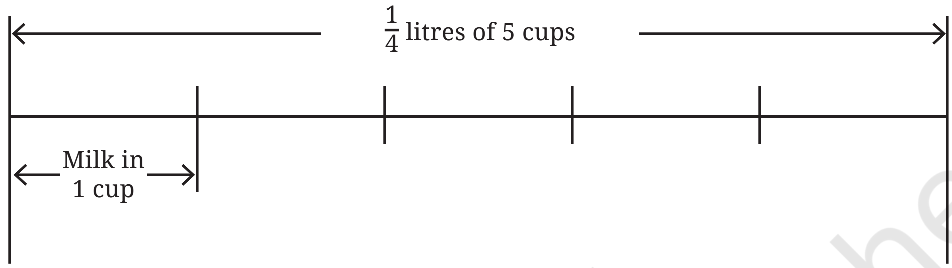
When do you think the quotient is less than the dividend and when is it greater than the dividend?

Is there a similar relationship between the divisor and the quotient?

Use your understanding of such relationships in multiplication to answer the questions above.

8.3 Some Problems Involving Fractions

Example 3: Leena made 5 cups of tea. She used $\frac{1}{4}$ litre of milk for this. How much milk is there in each cup of tea?



Leena used $\frac{1}{4}$ litres of milk in 5 cups of tea. So, in 1 cup of tea the volume of milk should be:

$$\frac{1}{4} \div 5.$$

Writing this as multiplication, we have:

$$5 \times (\text{milk per cup}) = \frac{1}{4}.$$

We perform the division as follows as per Brahmagupta's method:

The reciprocal of 5 (the divisor) is $\frac{1}{5}$.

Multiplying this reciprocal by the dividend ($\frac{1}{4}$), we get

$$\frac{1}{5} \times \frac{1}{4} = \frac{1}{20}.$$

So, each cup of tea has $\frac{1}{20}$ litre of milk.

Example 4: Some of the oldest examples of working with non-unit fractions occur in humanity's oldest geometry texts, the *Śhulbasūtra*. Here is an example from Baudhāyana's *Śhulbasūtra* (c. 800 BCE).

Cover an area of $7\frac{1}{2}$ square units with square bricks each of whose sides is $\frac{1}{5}$ units.

How many such square bricks are needed?

Each square brick has an area of $\frac{1}{5} \times \frac{1}{5} = \frac{1}{25}$ square units.

The total area to be covered is $7\frac{1}{2}$ sq. units = $\frac{15}{2}$ sq. units.

As (Number of bricks) × (Area of a brick) = Total Area,

$$\text{Number of bricks} = \frac{15}{2} \div \frac{1}{25}.$$

The reciprocal of the divisor is 25.
Multiplying the reciprocal by the dividend, we get

$$25 \times \frac{15}{2} = \frac{25 \times 15}{2} = \frac{375}{2}.$$

Example 5: This problem was posed by Chaturveda Pṛithūdakasvāmī (c. 860 CE) in his commentary on Brahmagupta's book *Brāhmasphuṭasiddhānta.*

Four fountains fill a cistern. The first fountain can fill the cistern in a day. The second can fill it in half a day. The third can fill it in a quarter of a day. The fourth can fill the cistern in one fifth of a day. If they all flow together, in how much time will they fill the cistern?

Let us solve this problem step by step.

In a day, the number of times —

- the first fountain will fill the cistern is $1 \div 1 = 1$
- the second fountain will fill the cistern is $1 \div \frac{1}{2} =$ _____
- the third fountain will fill the cistern is $1 \div \frac{1}{4} =$ _____
- the fourth fountain will fill the cistern is $1 \div \frac{1}{5} =$ _____

The number of times the four fountains together will fill the cistern in a day is ___ + ____ + ___ + ____ = 12.

Thus, the total time needed by the four fountains to fill the cistern together is $\frac{1}{12}$ days.

Fractional Relations

Here is a square with some lines drawn inside.

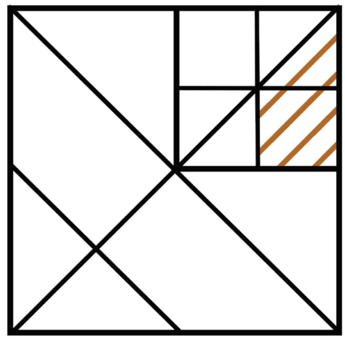


Fig. 8.4

What fraction of area of the whole square does the shaded region occupy? Math Talk

There are different ways to solve this problem. Here is one of them: Let the area of the whole square be 1 square unit.

We can see that the top right square (in Fig. 8.5), occupies $\frac{1}{4}$ of the area of the whole square.

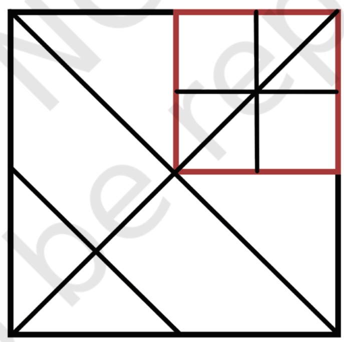
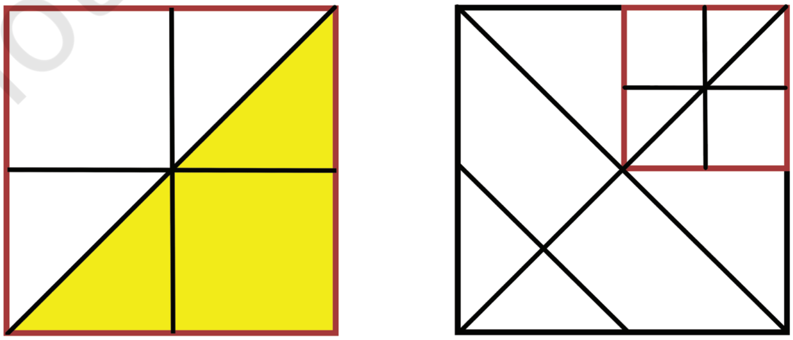


Fig. 8.5

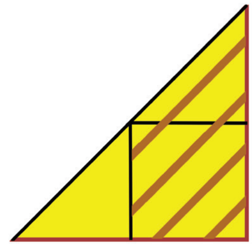
Area of red square = $\frac{1}{4}$ square units.



Fig, 8.6

Let us look at this red square. The area of the triangle inside it (coloured yellow) is half the area of the red square. So,

the area of the yellow triangle = $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ square units.



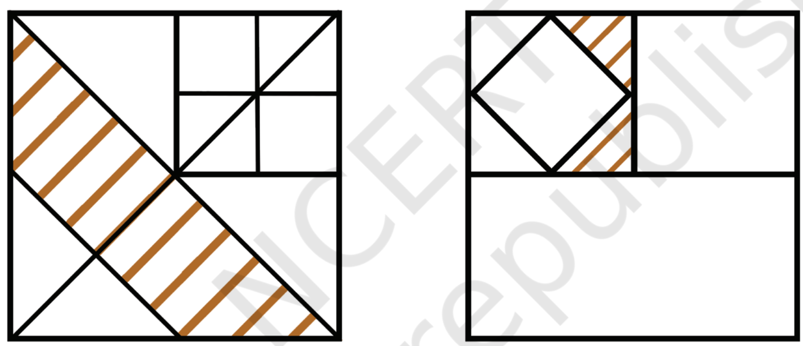
What fraction of this yellow triangle is shaded?

The shaded region occupies $\frac{3}{4}$ of the area of the yellow triangle. Are you able to see why?

The area of shaded part = $\frac{3}{4} \times \frac{1}{8} = \frac{3}{32}$ square units.

Thus, the shaded region occupies $\frac{3}{32}$ of the area of the whole square.

? In each of the figures given below, find the fraction of the big square that the shaded region occupies.



We will solve more interesting problems of this kind in a later chapter.

A Dramma-tic Donation

The following problem is translated from Bhāskarāchārya's (Bhāskara II's) book, *Līlāvatī*, written in 1150 CE.

"O wise one! A miser gave to a beggar $\frac{1}{5}$ of $\frac{1}{16}$ of $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of a dramma. If you know the mathematics of fractions well, tell me O child, how many cowrie shells were given by the miser to the beggar."

Dramma refers to a silver coin used in those times. The tale says that 1 dramma was equivalent to 1280 cowrie shells. Let's see what fraction of a dramma the person gave:

$$\left(\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{1}{5} \times \frac{1}{16} \times \frac{1}{4}\right)^{\text{th}} \text{ part of a dramma.}$$

Evaluating it gives $\frac{6}{7680}$.

Upon simplifying to its lowest form, we get

$$\frac{6}{7680} = \frac{1}{1280}.$$

So, one cowrie shell was given to the beggar.

You can see in the answer Bhāskarāchārya's humour! The miser had given the beggar only one coin of the least value (cowrie).

Around the 12th century, several types of coins were in use in different kingdoms of the Indian subcontinent. Most commonly used were gold coins (called *dinars/gadyanas* and *hunas*), silver coins (called *drammas/tankas*), copper coins (called *kasus/panas* and *mashakas*), and cowrie shells. The exact conversion rates between these coins varied depending on the region, time period, economic conditions, weights of coins and their purity.

Gold coins had high-value and were used in large transactions and to store wealth. Silver coins were more commonly used in everyday transactions. Copper coins had low-value and were used in smaller transactions. Cowrie shells were the lowest denomination and were used in very small transactions and as change.

If we assume 1 gold dinar = 12 silver drammas, 1 silver dramma = 4 copper panas, 1 copper pana = 6 mashakas, and 1 pana = 30 cowrie shells,

$$1 \text{ copper pana} = \frac{1}{48} \text{ gold dinar } \left(\frac{1}{12} \times \frac{1}{4}\right)$$

1 cowrie shell =____ copper panas

1 cowrie shell =____ gold dinar.

A Pinch of History

As you have seen, fractions are an important type of number, playing a critical role in a variety of everyday problems that involve sharing and dividing quantities equally. The general notion of non-unit fractions as we use them today — equipped with the arithmetic operations of addition, subtraction, multiplication, and division — developed largely in India. The ancient Indian geometry texts called the *Śhulbasūtra* — which go back as far as 800 BCE, and were concerned with the construction of fire altars for rituals — used general non-unit fractions extensively, including performing division of such fractions as we saw in Example 3.

Fractions even became commonplace in the popular culture of India as far back as 150 BCE, as evidenced by an offhand reference to the reduction of fractions to lowest terms in the philosophical work of the revered Jain scholar Umasvati.

General rules for performing arithmetic operations on fractions — in essentially the modern form in which we carry them out today — were first codified by Brahmagupta in his *Brāhmasphuṭasiddhānta* in 628 CE. We have already seen his methods for adding and subtracting general fractions. For multiplying general fractions, Brahmagupta wrote:

"Multiplication of two or more fractions is obtained by taking the product of the numerators divided by the product of the denominators." (*Brāhmasphuṭasiddhānta*, Verse 12.1.3)

That is, $$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}.$$

For division of general fractions, Brahmagupta wrote:

"The division of fractions is performed by interchanging the numerator and denominator of the divisor; the numerator of the dividend is then multiplied by the (new) numerator, and the denominator by the (new) denominator."

Bhāskara II in his book *Līlāvatī* in 1150 CE clarifies Brahmagupta's statement further in terms of the notion of reciprocal:

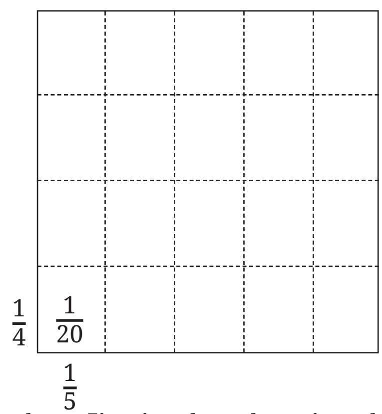
"Division of one fraction by another is equivalent to multiplication of the first fraction by the reciprocal of the second." (*Līlāvatī*, Verse 2.3.40)

Both of these verses are equivalent to the formula:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c}.$$

Bhāskara I, in his 629 CE commentary *Āryabhaṭīyabhāṣhya* on Aryabhata's 499 CE work, described the geometric interpretation of multiplication of fractions (that we saw earlier) in terms of the division of a square into rectangles via equal divisions along the length and breadth.

Many other Indian mathematicians, such as Śhrīdharāchārya (c. 750 CE), Mahāvīrāchārya (c. 850 CE), Caturveda Pṛithūdakasvāmī (c. 860 CE), and Bhāskara II (c. 1150 CE) developed the usage of arithmetic of fractions significantly further.



Bhāskara I's visual explanation that $\frac{1}{5} \times \frac{1}{4} = \frac{1}{20}$

The Indian theory of fractions and arithmetic operations on them was transmitted to, and its usage developed further, by Arab and African mathematicians such as al-Hassâr (c. 1192 CE) of Morocco. The theory was then transmitted to Europe via the Arabs over the next few centuries, and came into general use in Europe in only around the 17th century, after which it spread worldwide. The theory is indeed indispensable today in modern mathematics.

Figure it Out

1. Evaluate the following:

$3 \div \frac{7}{9}$	$\frac{14}{4} \div 2$	$\frac{2}{3} \div \frac{2}{3}$	$\frac{14}{6} \div \frac{7}{3}$
$\frac{4}{3} \div \frac{3}{4}$	$\frac{7}{4} \div \frac{1}{7}$	$\frac{8}{2} \div \frac{4}{15}$	
$\frac{1}{5} \div \frac{1}{9}$	$\frac{1}{6} \div \frac{11}{12}$	$3\frac{2}{3} \div 1\frac{3}{8}$	

2. For each of the questions below, choose the expression that describes the solution. Then simplify it.

   (a) Maria bought 8 m of lace to decorate the bags she made for school. She used $\frac{1}{4}$ m for each bag and finished the lace. How many bags did she decorate?

   (i) $8 \times \frac{1}{4}$ (ii) $\frac{1}{8} \times \frac{1}{4}$

   (iii) $8 \div \frac{1}{4}$ (iv) $\frac{1}{4} \div 8$

   (b) $\frac{1}{2}$ meter of ribbon is used to make 8 badges. What is the length of the ribbon used for each badge?

   (i) $8 \times \frac{1}{2}$ (ii) $\frac{1}{2} \div \frac{1}{8}$

   (iii) $8 \div \frac{1}{2}$ (iv) $\frac{1}{2} \div 8$

   (c) A baker needs $\frac{1}{6}$ kg of flour to make one loaf of bread. He has 5 kg of flour. How many loaves of bread can he make?

   (i) $5 \times \frac{1}{6}$ (ii) $\frac{1}{6} \div 5$

   (iii) $5 \div \frac{1}{6}$ (iv) 5×6

3. If $\frac{1}{4}$ kg of flour is used to make 12 rotis, how much flour is used to make 6 rotis?

4. *Pāṭīgaṇita*, a book written by Sridharacharya in the 9th century CE, mentions this problem: "Friend, after thinking, what sum will be obtained by adding together $1 \div \frac{1}{6}$, $1 \div \frac{1}{10}$, $1 \div \frac{1}{13}$, $1 \div \frac{1}{9}$, and $1 \div \frac{1}{2}$". What should the friend say?

5. Mira is reading a novel that has 400 pages. She read $\frac{1}{5}$ of the pages yesterday and $\frac{3}{10}$ of the pages today. How many more pages does she need to read to finish the novel?

6. A car runs 16 km using 1 litre of petrol. How far will it go using $2\frac{3}{4}$ litres of petrol?

7. Amritpal decides on a destination for his vacation. If he takes a train, it will take him $5\frac{1}{6}$ hours to get there. If he takes a plane, it will take him $\frac{1}{2}$ hour. How many hours does the plane save?

8. Mariam's grandmother baked a cake. Mariam and her cousins finished $\frac{4}{5}$ of the cake. The remaining cake was shared equally by Mariam's three friends. How much of the cake did each friend get?

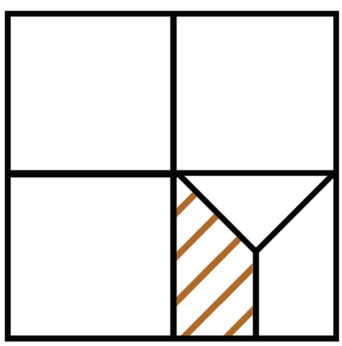
9. Choose the option(s) describing the product of $\left(\frac{565}{465} \times \frac{707}{676}\right)$:

   (a) $> \frac{565}{465}$ (b) $< \frac{565}{465}$

   (c) $> \frac{707}{676}$ (d) $< \frac{707}{676}$

   (e) > 1 (f) < 1

10. What fraction of the whole square is shaded?



11. A colony of ants set out in search of food. As they search, they keep splitting equally at each point (as shown in the Fig. 8.7) and reach two food sources, one near a mango tree and another near a sugarcane field. What fraction of the original group reached each food source?

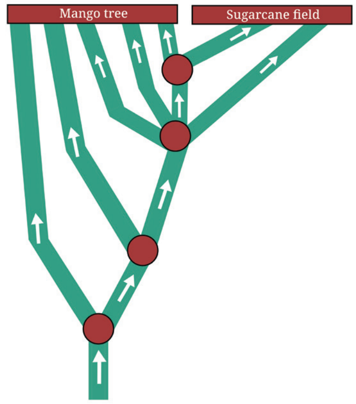


Fig. 8.7

12. What is $1 - \frac{1}{2}$?

$\left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right)$?

$\left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{4}\right) \times \left(1 - \frac{1}{5}\right)$?

$\left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{4}\right) \times \left(1 - \frac{1}{5}\right) \times \left(1 - \frac{1}{6}\right) \times \left(1 - \frac{1}{7}\right) \times \left(1 - \frac{1}{8}\right) \times \left(1 - \frac{1}{9}\right) \times \left(1 - \frac{1}{10}\right)$?

Make a general statement and explain.

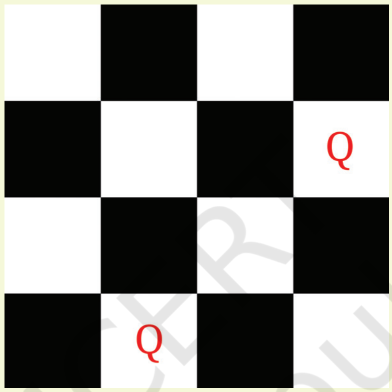
SUMMARY

- Brahmagupta's formula for multiplication of fractions:
$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}.$$
- When multiplying fractions, if the numerators and denominators have some common factors, we can cancel them first before multiplying the numerators and denominators.
- In multiplication — when one of the numbers being multiplied is between 0 and 1, the product is less than the other number. If one of the numbers being multiplied is greater than 1, then the product is greater than the other number.
- The reciprocal of a fraction $\frac{a}{b}$ is $\frac{b}{a}$. When we multiply a fraction by its reciprocal, the product is 1.
- Brahmagupta's formula for division of fractions:
$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c}.$$
- In division — when the divisor is between 0 and 1, the quotient is greater than the dividend. When the divisor is greater than 1, the quotient is less than the dividend.

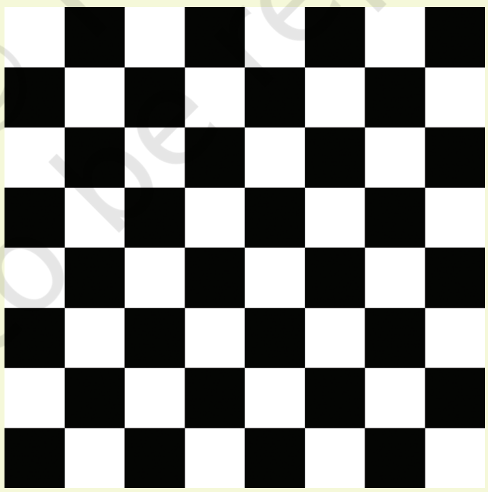
Chess Puzzles — Non-attacking Queens

Chess is a popular 2-player strategy game. This game has its origins in India. It is played on an 8 × 8 chequered grid. There are 2 sets of pieces — black and white — one set for each player. Find out how each piece should move and the rules of the game.

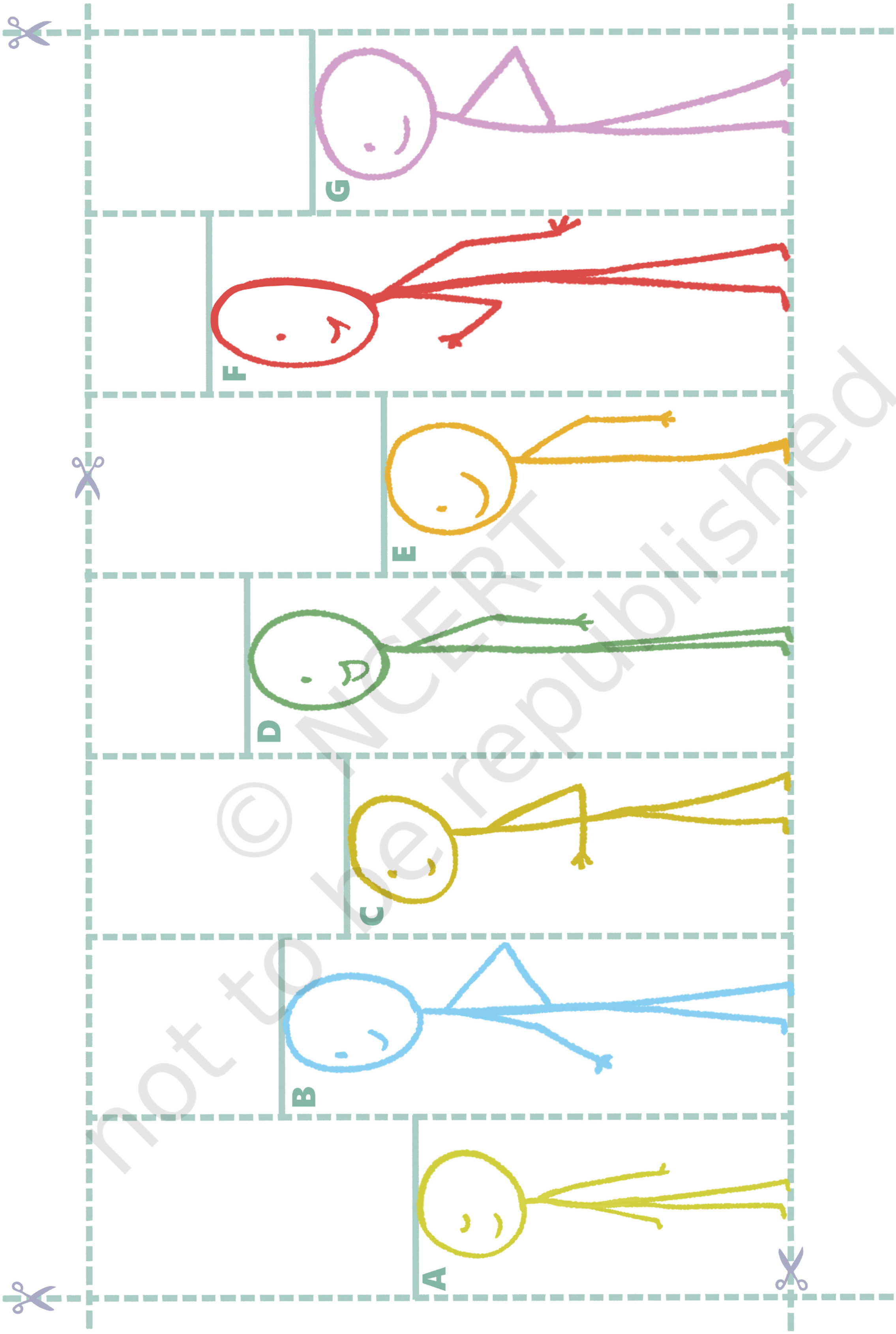
Here is a famous chess-based puzzle. From its current position, a Queen piece can move along the horizontal, vertical or diagonal. Place 4 Queens such that no 2 queens attack each other. For example, the arrangement below is not valid as the queens are in the line of attack of each other.



Now, place 8 queens on this 8 × 8 grid so that no 2 queens attack each other!



Learning Material Sheets


G
F
E
D
C
B
A

Note

5 PARALLEL AND INTERSECTING LINES

0774CH05

5.1 Across the Line

Take a piece of square paper and fold it in different ways. Now, on the creases formed by the folds, draw lines using a pencil and a scale. You will notice different lines on the paper. Take any pair of lines and observe their relationship with each other. Do they meet? If they do not meet within the paper, do you think they would meet if they were extended beyond the paper?

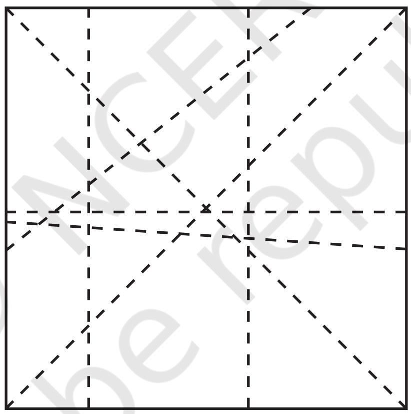


Fig. 5.1

In this chapter, we will explore the relationship between lines on a **plane surface**. The table top, your piece of paper, the blackboard, and the bulletin board are all examples of plane surfaces.

Let us observe a pair of lines that meet each other. You will notice that they meet at a point. When a pair of lines meet each other at a point on a plane surface, we say that the lines **intersect** each other. Let us observe what happens when two lines intersect.

? How many angles do they form?

In Fig. 5.2, where line l intersects line m, we can see that four angles are formed.

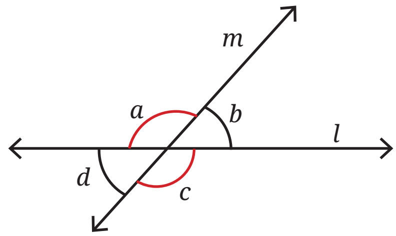


Fig. 5.2

? Can two straight lines intersect at more than one point?

Activity 1

Draw two lines on a plain sheet of paper so that they intersect. Measure the four angles formed with a protractor. Draw four such pairs of intersecting lines and measure the angles formed at the points of intersection.

? What patterns do you observe among these angles?

? In Fig. 5.2, if $\angle a$ is 120°, can you figure out the measurements of $\angle b$, $\angle c$ and $\angle d$, without drawing and measuring them?

We know that $\angle a$ and $\angle b$ together measure 180°, because when they are combined, they form a straight angle which measures 180°. So, if $\angle a$ is 120°, then $\angle b$ must be 60°.

Similarly, $\angle b$ and $\angle c$ together measure 180°. So, if $\angle b$ is 60°, then $\angle c$ must be 120°. And $\angle c$ and $\angle d$ together measure 180°. So, if $\angle c$ is 120°, then $\angle d$ must be 60°.

Therefore, in Fig. 5.2, $\angle a$ and $\angle c$ measure 120°, and $\angle b$ and $\angle d$ measure 60°.

When two lines intersect each other and form four angles, labelled a, b, c and d, as in Fig. 5.2, then $\angle a$ and $\angle c$ are equal, and $\angle b$ and $\angle d$ are equal!

? Is this always true for any pair of intersecting lines?

Check this for different measures of $\angle a$. Using these measurements, can you reason whether this property holds true for any measure of $\angle a$?

We can generalise our reasoning for Fig. 5.2, without assuming the values of $\angle a$.

Since straight angles measure 180°, we must have $\angle a + \angle b = \angle a + \angle d = 180°$. Hence, $\angle b$ and $\angle d$ are always equal. Similarly, $\angle b + \angle a = \angle b + \angle c = 180°$, so $\angle a$ and $\angle c$ must be equal.

Adjacent angles, like $\angle a$ and $\angle b$, formed by two lines intersecting each other, are called **linear pairs**. Linear pairs always add up to 180°.

Opposite angles, like $\angle b$ and $\angle d$, formed by two lines intersecting each other, are called **vertically opposite angles**. Vertically opposite angles are always equal to each other.

From the above reasoning, we conclude that whenever two lines intersect, vertically opposite angles are equal. Such a justification is called a **proof** in mathematics.

Figure it Out

List all the linear pairs and vertically opposite angles you observe in Fig. 5.3:

Linear Pairs	$\angle a$ and $\angle b$, ...
Pairs of Vertically Opposite Angles	$\angle b$ and $\angle d$, ...

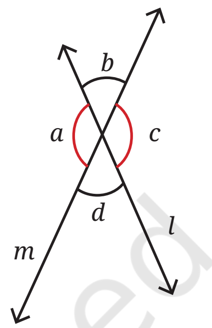


Fig. 5.3

Measurements and Geometry

You might have noticed that when you measure linear pairs, sometimes they may not add up to 180°. Or, when you measure vertically opposite angles they may be unequal sometimes. What are the reasons for this? There could be different reasons:

- Measurement errors because of improper use of measuring instruments — in this case, a protractor
- Variation in the thickness of the lines drawn. The "ideal" line in geometry does not have any thickness! But it is not possible for us to draw lines without any thickness

In geometry, we create ideal versions of "lines" and other shapes we see around us, and analyse the relationships between them. For example, we know that the angle formed by a straight line is 180°. So, if another line divides this angle into two parts, both parts should add up to 180°. We arrive at this simply through reasoning and not by measurement. When we measure, it might not be exactly so, for the reasons mentioned above. Still the measurements come out very close to what we predict, because of which geometry finds widespread application in different disciplines such as physics, art, engineering and architecture.

5.2 Perpendicular Lines

Can you draw a pair of intersecting lines such that all four angles are equal? Can you figure out what will be the measure of each angle?

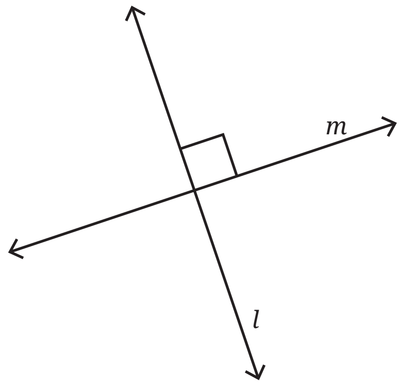


Fig. 5.4

If two lines intersect and all four angles are equal, then each angle must be a right angle (90°).

Perpendicular lines are a pair of lines which intersect each other at right angles (90°). In Fig. 5.4, we can say that lines l and m are perpendicular to each other.

5.3 Between Lines

Observe Fig. 5.5 and describe the way the line segments meet or cross each other in each case, with appropriate mathematical words (a point, an endpoint, the midpoint, meet, intersect) and the degree measure of each angle.

For example, line segments FG and FH meet at the endpoint F at an angle 115.3°.

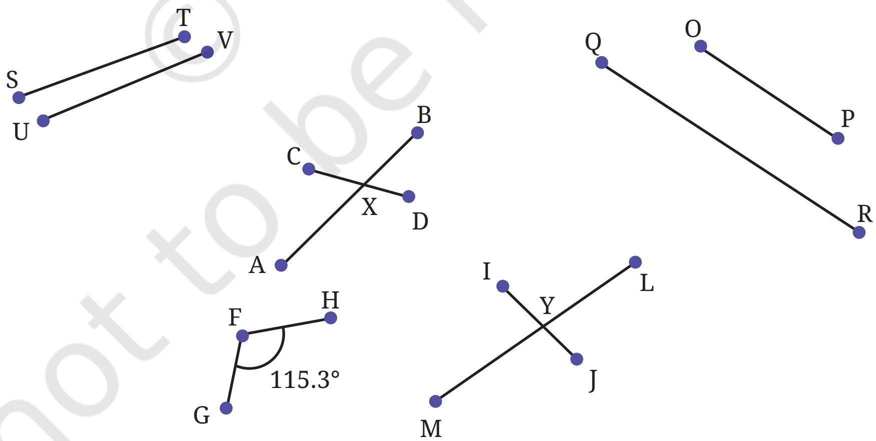
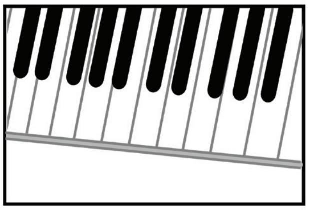


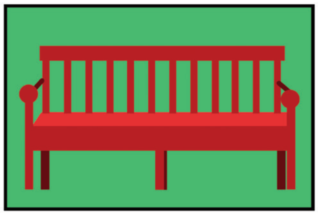
Fig. 5.5

Are line segments ST and UV likely to meet if they are extended?

Are line segments OP and QR likely to meet if they are extended?

Here are some examples of lines we notice around us.

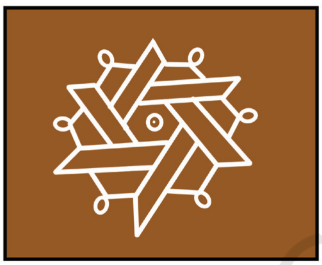


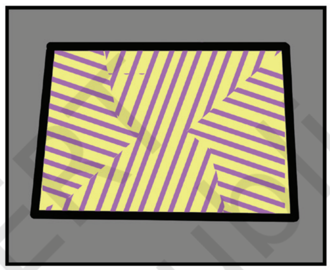


What is common to the lines in the pictures above? They do not seem likely to intersect each other. Such lines are called parallel lines.

Parallel lines are a pair of lines that lie on the same plane, and do not meet however far we extend them at both ends.

Name some parallel lines you can spot in your classroom.





Parallel lines are often used in artwork and shading.

Which pairs of lines appear to be parallel in Fig. 5.6 below?

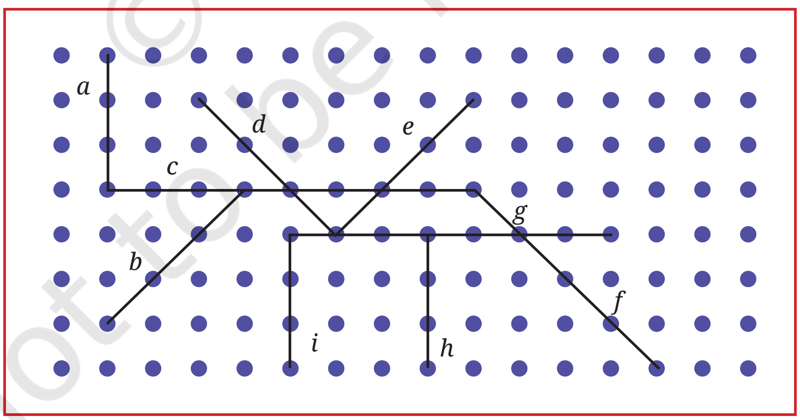


Fig. 5.6

Note to the Teacher: It is important that the lines lie on the same plane. A line drawn on a table and a line drawn on the board may never meet but that does not make them parallel.

5.4 Parallel and Perpendicular Lines in Paper Folding

Activity 2

Take a plain square sheet of paper (use a newspaper for this activity).

- How would you describe the opposite edges of the sheet? They are _______________ to each other.
- How would you describe the adjacent edges of the sheet? The adjacent edges are _______________ to each other. They meet at a point. They form right angles.
- Fold the sheet horizontally in half. A new line is formed (see Fig. 5.7).
- How many parallel lines do you see now? How does the new line segment relate to the vertical sides?

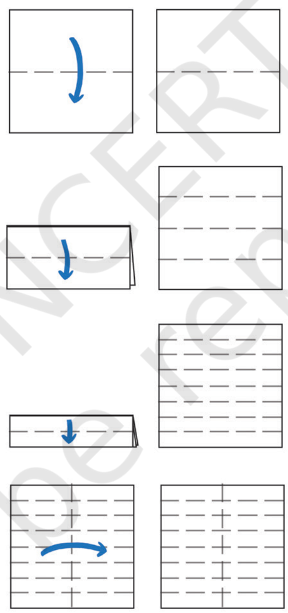


Fig. 5.7

- Make one more horizontal fold in the folded sheet. How many parallel lines do you see now?
- What will happen if you do it once more? How many parallel lines will you get? Is there a pattern? Check if the pattern extends further, if you make another horizontal fold.
- Make a vertical fold in the square sheet. This new vertical line is _______ to the previous horizontal lines.
- Fold the sheet along a diagonal. Can you find a fold that creates a line parallel to the diagonal line?

Here is another activity for you to try.

- Take a square sheet of paper, fold it in the middle and unfold it.
- Fold the edges towards the centre line and unfold them.
- Fold the top right and bottom left corners onto the creased line to create triangles. Refer to Fig. 5.8.
- The triangles should not cross the crease lines.
- Are *a*, *b* and *c* parallel to *p*, *q* and *r* respectively? Why or why not?

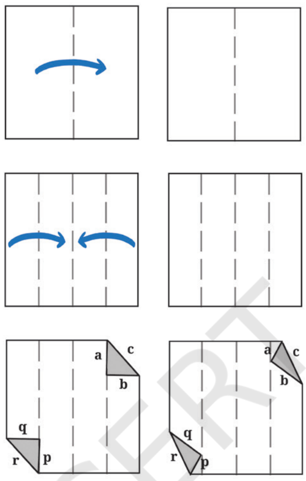


Fig. 5.8

Notations

In mathematics, we use an arrow mark (>) to show that a set of lines is parallel. If there is more than one set of parallel lines (as in Fig. 5. 9), the second set is shown with two arrow marks and so on. Perpendicular lines are marked with a square angle between them.

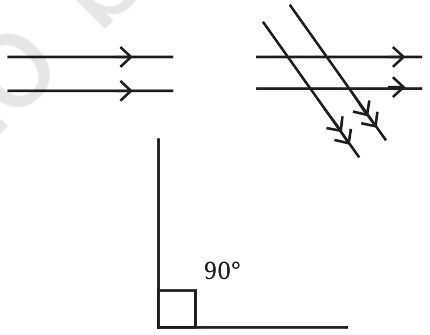


Fig. 5.9

Figure it Out

1. Draw some lines perpendicular to the lines given on the dot paper in Fig. 5.10.

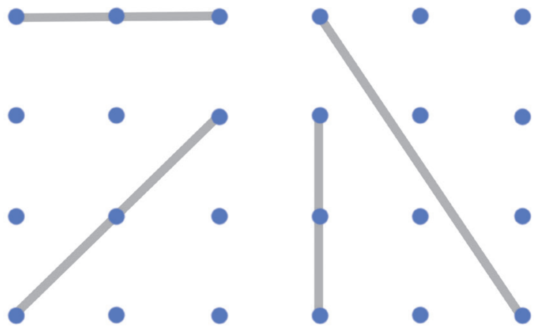


Fig. 5.10

2. In Fig. 5.11, mark the parallel lines using the notation given above (single arrow, double arrow etc.). Mark the angle between perpendicular lines with a square symbol.
   (a) How did you spot the perpendicular lines?
   (b) How did you spot the parallel lines?

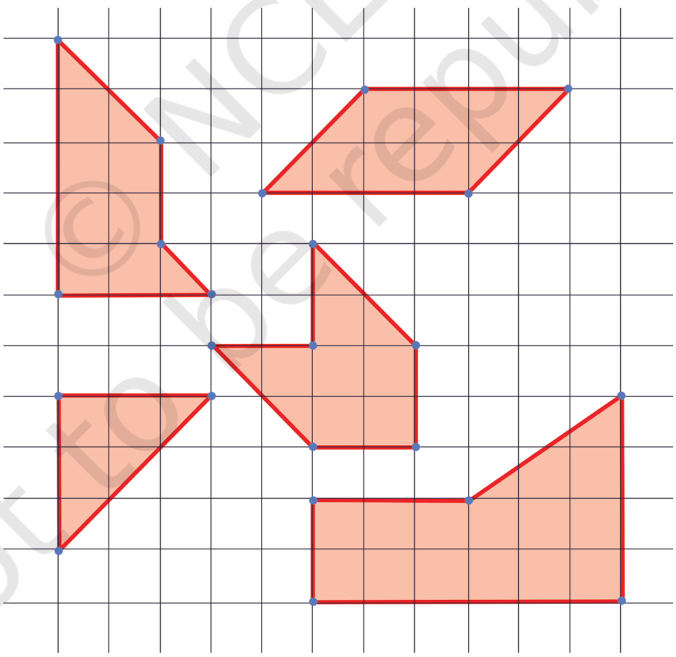


Fig. 5.11

3. In the dot paper following, draw different sets of parallel lines. The line segments can be of different lengths but should have dots as endpoints.

4. Using your sense of how parallel lines look, try to draw lines parallel to the line segments on this dot paper.

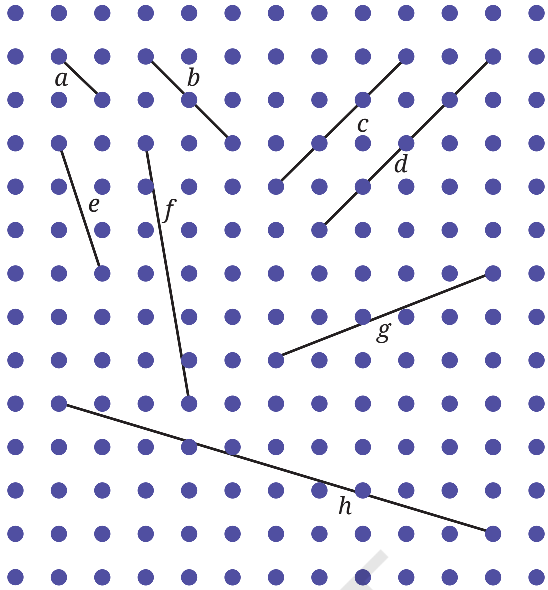


Fig. 5.12

(a) Did you find it challenging to draw some of them?

(b) Which ones?

(c) How did you do it?

5. In Fig. 5.13, which line is parallel to line *a* — line *b* or line *c*? How do you decide this?

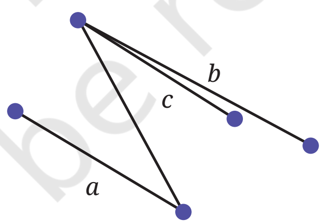


Fig. 5.13

Note to the Teacher: It is easier to draw vertical and horizontal lines and the ones inclined at 45° (on rectangular dot sheets), but drawing a line parallel to one which has a different orientation is slightly harder. Let students use their intuition for this.

From previous exercises we observed that sometimes it is difficult to be sure whether two lines are parallel. To determine this we use the idea of transversals.

5.5 Transversals

We saw what happens when two lines intersect in different ways. Let us explore what happens when one line intersects two different lines.

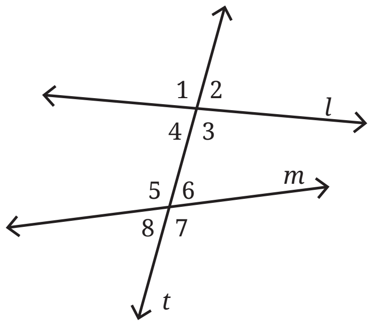


Fig. 5.14

In Fig. 5.14, line t intersects lines l and m. t is called a **transversal**. Notice that 8 angles are formed when a line crosses a pair of lines.

? Is it possible for all the eight angles to have different measurements? Why, why not?

? What about five different angles — 6, 5, 4, 3 and 2?

In Fig. 5.14, since $\angle 1$ and $\angle 3$ are vertically opposite angles, they are equal. Are there other pairs of vertically opposite angles? We can see that there are a total of four pairs of vertically opposite angles and in each pair, the angles are equal to each other.

Thus, when a transversal intersects two lines, it forms eight angles with a maximum of four distinct angle measures.

5.6 Corresponding Angles

In Fig. 5.14, we notice that the transversal t forms two sets of angles — one with line l and another with line m. There are angles in the first set that correspond to angles in the second set based on their position. $\angle 1$ and $\angle 5$ are called **corresponding angles**. Similarly, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$ are the corresponding angles formed when transversal t intersects lines l and m.

? **Activity 3**

Draw a pair of lines and a transversal such that they form two distinct angles.

Step 1: Draw a line *l* and a transversal *t* intersecting it at point X.

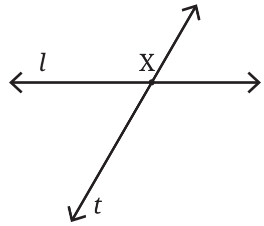


Fig. 5.15

Step 2: Measure $\angle a$ formed by lines *l* and *t* (let us say it is 60°).

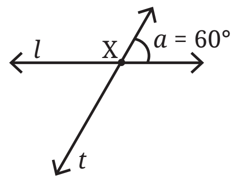


Fig. 5.16

How many distinct angles have formed now?

If one angle is 60°, the other angle of the linear pair should be 120°. So, we already have two distinct angles.

So, when we draw another line intersecting the transversal *t* we wish to form only two angles, 60° and 120°.

Step 3: Mark a point Y on line *t*.

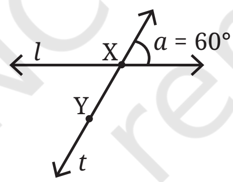


Fig. 5.17

Step 4: Draw a line *m* through point Y that forms a 60° angle to line *t*. This can be done either by copying $\angle a$ with a tracing paper or you can use a protractor to measure the angles.

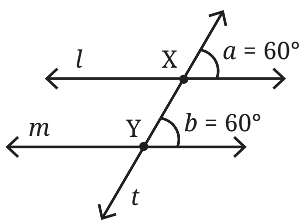


Fig. 5.18

What do you observe about lines *l* and *m*? Do they appear to be parallel to each other?

Yes, they do appear to be parallel to each other.

Angles, $\angle a$ and $\angle b$ are corresponding angles formed by the transversal *t* on lines *l* and *m*. These corresponding angles are equal to each other.

From this we can observe:

When the corresponding angles formed by a transversal on a pair of lines are equal to each other, then the pair of lines are parallel to each other.

Suppose, we have a transversal intersecting two parallel lines. What can be said about the corresponding angles?

Activity 4

Fig. 5.19 has a pair of parallel lines l and m (what is the notation used in the figure to indicate they are parallel?) . Line t is the transversal across these two lines. $\angle a$ and $\angle b$ are corresponding angles. Take a tracing paper and trace $\angle a$ on it. Now place this tracing paper over $\angle b$ and see if the angles align exactly. You will observe that the angles match. Check the other corresponding angles in the figure using a protractor. Are all the corresponding angles equal to each other?

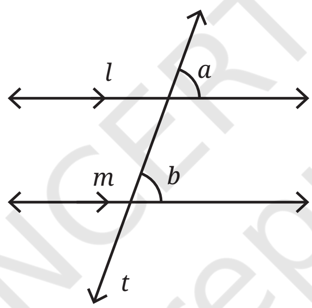


Fig. 5.19

Corresponding angles formed by a transversal intersecting a pair of parallel lines are always equal to each other.

Activity 5

In Fig. 5.20, draw a transversal t to the lines l and m such that one pair of corresponding angles is equal. You can measure the angles with a protractor.

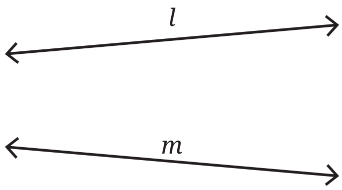


Fig. 5.20

Are you finding it hard to draw a transversal such that the corresponding angles are equal?

When a pair of lines are not parallel to each other, the corresponding angles formed by a transversal can never be equal to each other.

5.7 Drawing Parallel Lines

Can you draw a pair of parallel lines using a ruler and a set square?

Fig. 5.21 shows how you can do it.

Draw a line l with a scale. By sliding your set square you can make two lines perpendicular to line l.

Are these two lines parallel to each other? How are we sure that they are parallel to each other? What angles are formed between these lines and line l?

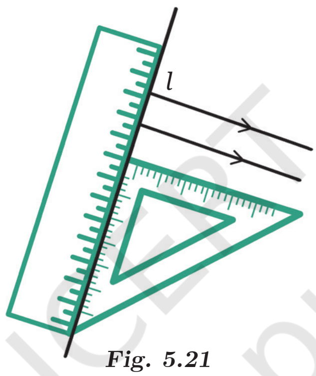


Fig. 5.21

Since we used a set square, the angles measure 90°. The position of the lines is different, but they make the same angle with l. If line l is seen as a transversal to the two new lines, then the corresponding angles measure 90°.

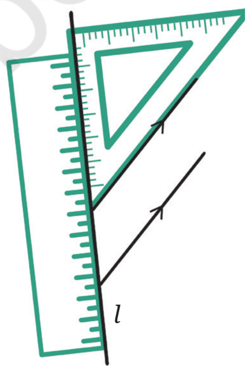


Fig. 5.22

As we know these are corresponding angles and they are equal, we can be sure that the lines are parallel.

Draw two more parallel lines using the long side of the set square as shown in Fig. 5.22.

How do you know these two lines are parallel? Can you check if the corresponding angles are equal?

Note to the Teacher: Students should be encouraged to check the equality of corresponding angles both by using the tracing method and using protractors to measure the angles. Pay attention to the language used to make the relationship between corresponding angles and parallel lines. Equality of corresponding angles is both necessary and sufficient for the pair of lines to be parallel to each other.

Figure it Out

Can you draw a line parallel to *l*, that goes through point A? How will you do it with the tools from your geometry box? Describe your method.

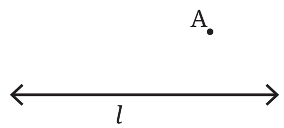


Fig. 5.23

Making Parallel Lines through Paper Folding

Let us try to do the same with paper folding. For a line *l* (given as a crease), how do we make a line parallel to *l* such that it passes through point A?

We know how to fold a piece of paper to get a line perpendicular to *l*. Now, try to fold a perpendicular to *l* such that it passes through point A. Let us call this new crease *t*.

Now, fold a line perpendicular to *t* passing through A again. Let us call this line *m*. The lines *l* and *m* are parallel to each other.

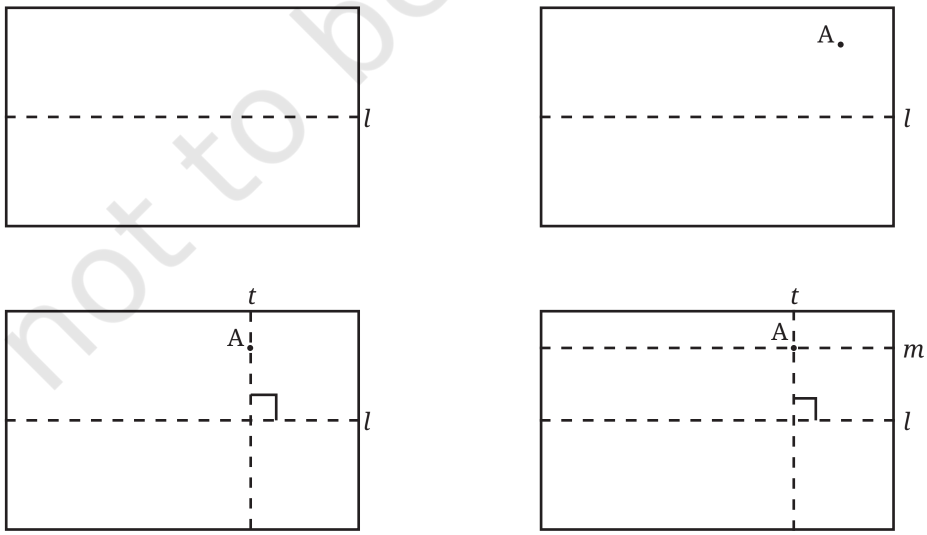


Fig. 5.24

Why are lines l and m parallel to each other?

5.8 Alternate Angles

In Fig. 5.25, $\angle d$ is called the **alternate angle** of $\angle f$, and $\angle c$ is the alternate angle of $\angle e$.

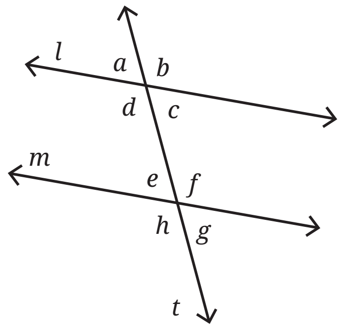


Fig. 5.25

You can find the alternate angle of a given angle, say $\angle f$, by first finding the corresponding angle of $\angle f$, which is $\angle b$ and then finding the vertically opposite angle of $\angle b$, which is $\angle d$.

Activity 6

In Fig. 5.25, if $\angle f$ is 120° what is the measure of its alternate angle $\angle d$?

We can find the measure of $\angle d$ if we know $\angle b$ because they are vertically opposite angles. Remember, vertically opposite angles are equal.

What is the measure of $\angle b$? It is 120° because it is the corresponding angle of $\angle f$.

So, $\angle d$ also measures 120°.

In fact, $\angle f = \angle b$ irrespective of the measure of $\angle f$. Why? Because $\angle b$ is the corresponding angle of $\angle f$.

Similarly, $\angle b = \angle d$ irrespective of the measure of $\angle b$. Why? Because $\angle d$ is the vertically opposite angle of $\angle b$. So, it must always be the case that

$\angle f = \angle d$

Using our understanding of corresponding angles without any measurements, we have justified that alternate angles are always equal.

Alternate angles formed by a transversal intersecting a pair of parallel lines are always equal to each other.

Example 1: In Fig. 5.26, parallel lines l and m are intersected by the transversal t. If $\angle 6$ is 135°, what are the measures of the other angles?

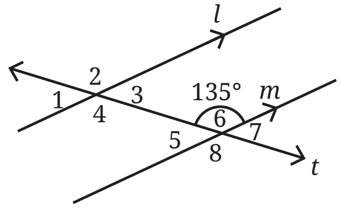


Fig. 5.26

Solution: ∠6 is 135°, so ∠2 is also 135°, because it is the corresponding angle of ∠6 and the lines *l* and m are parallel.

∠8 is 135°, because it is the vertically opposite angle of ∠6. ∠4 is 135° because it is the corresponding angle of ∠8.

∠2 is 135° because is the vertically opposite angle of ∠4. So, ∠2, ∠4, ∠6, and ∠8 are all 135°.

∠5 and ∠6 are a linear pair, together they measure 180°. If ∠6 is 135°, then

∠5 = 180 – 135 = 45°

We can similarly find out that ∠1, ∠3, and ∠7 measure 45°.

Example 2: In Fig. 5.27, lines *l* and m are intersected by the transversal *t*. If ∠*a* is 120° and ∠*f* is 70°, are lines *l* and m parallel to each other?

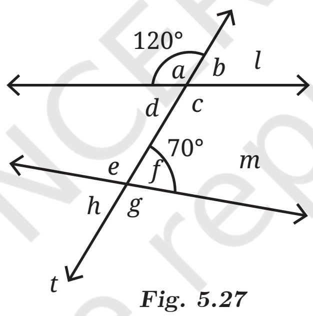


Fig. 5.27

Solution: ∠*a* is 120°, so ∠*b* is 60° because ∠*a* and ∠*b* form a linear pair. ∠*b* is a corresponding angle of ∠*f*. If *l* and *m* are parallel, ∠*b* should be equal to ∠*f*, however, they are not equal.

Therefore, lines *l* and *m* are not parallel to each other as the corresponding angles formed by the transversal *t* are not equal to each other.

Example 3: In Fig. 5.28, parallel lines *l* and *m* are intersected by the transversal *t*. If ∠3 is 50°, what is the measure of ∠6?

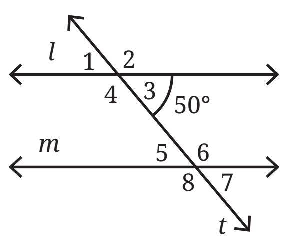


Fig. 5.28

Solution: ∠3 is 50°; therefore, ∠2 is 130°, because ∠2 and ∠3 form a linear pair, and linear pairs always add up to 180°.

∠2 and ∠6 are corresponding angles, and they need to be equal since lines l and m are parallel.

So, ∠6 is 130°.

Angles ∠3 and ∠6 are called **interior angles**.

Is there a relation between ∠3 and ∠6? You could try to find the relationship by taking different values for ∠3 and see what ∠6 is. Once you find a relation, try to justify it or prove that this relation holds always. You will find that the sum of the interior angles on the same side of the transversal always add up to 180°.

Example 4: In Fig. 5.29, line segment AB is parallel to CD and AD is parallel to BC. ∠DAC is 65° and ∠ADC is 60°. What are the measures of angles ∠CAB, ∠ABC, and ∠BCD?

Solution: Let us observe the parallel lines AB and CD. AD is a transversal of these two lines.

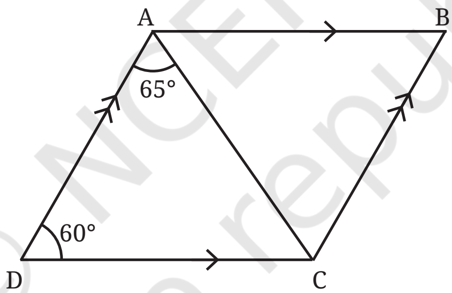


Fig. 5.29

We know that the sum of the interior angles formed by a transversal on a pair of parallel lines adds up to 180°. So

∠ADC + ∠DAB = 180°

60° + ∠DAB = 180°.

So ∠DAB = 120°.

Can we find ∠CAB from this?

∠DAB = ∠DAC + ∠CAB.

So 120° = 65° + ∠CAB.

So ∠CAB = 55°.

Let us observe the parallel line segments AD and BC. They are intersected by a transversal CD. So, ∠ADC + ∠BCD = 180°, because they are interior angles on the same side of the transversal. Since ∠ADC is given as 60°, ∠BCD = 120°

Similarly, we find ∠ABC = 60°.

Therefore, in Fig. 5.29, ∠CAB = 55°, ∠ABC = 60°, and ∠BCD = 120°.

Figure it Out

1. Find the angles marked below.

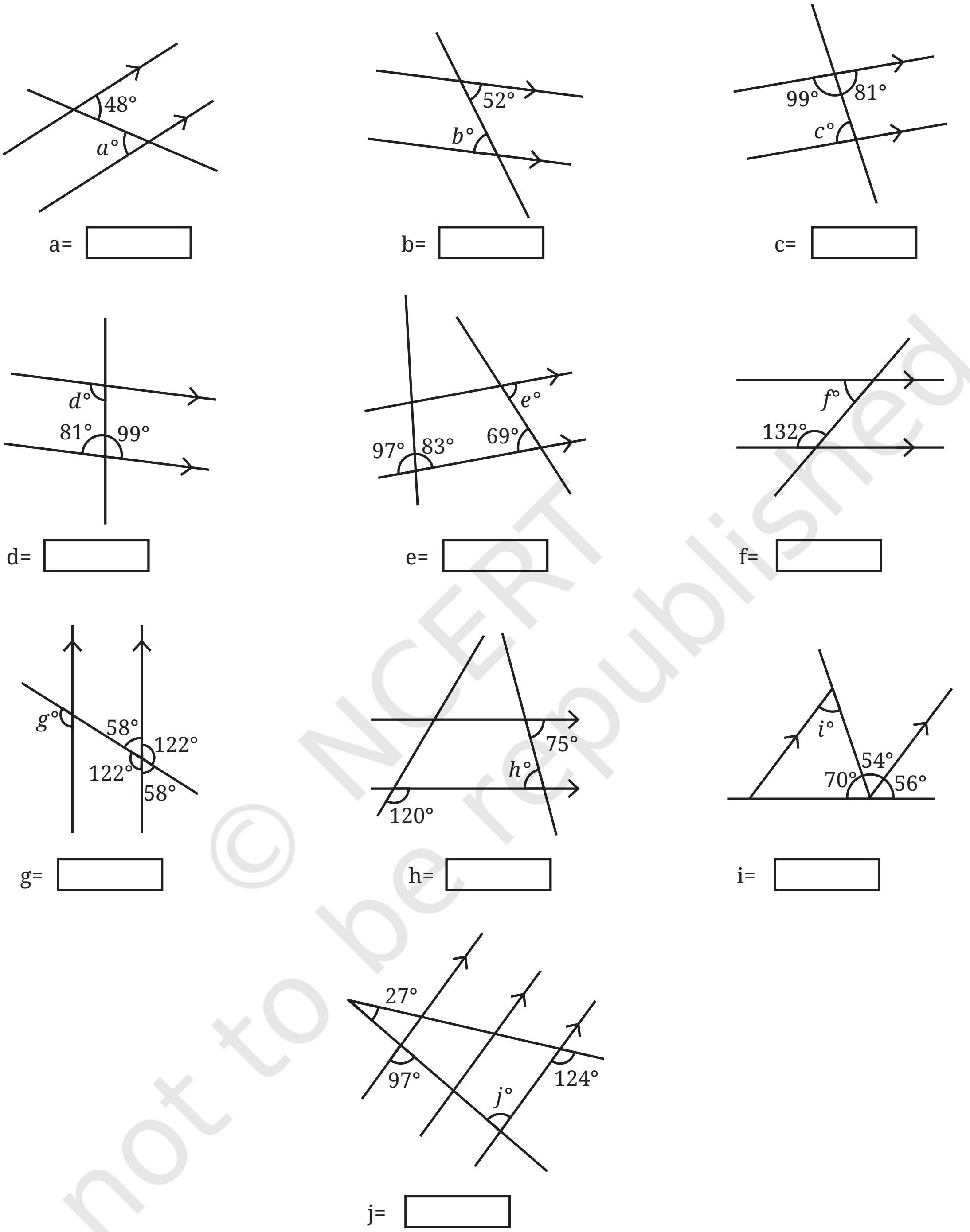


Fig. 5.30

2. Find the angle represented by *a*.

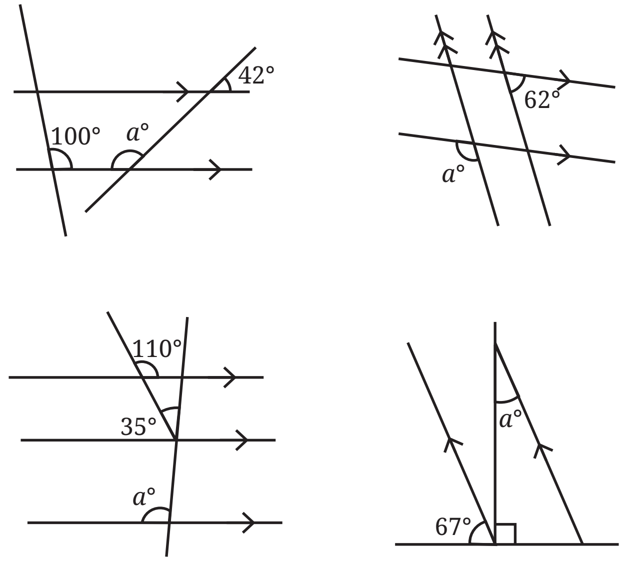


Fig. 5.31

3. In the figures below, what angles do *x* and *y* stand for?

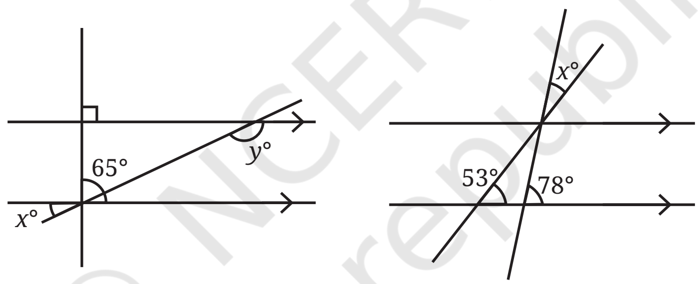


Fig. 5.32

4. In Fig. 5.33, ∠ABC = 45° and ∠IKJ = 78°. Find angles ∠GEH, ∠HEF, ∠FED

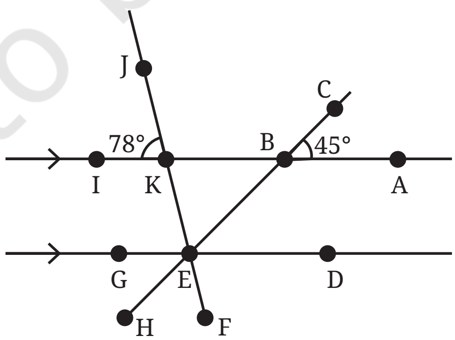


Fig. 5.33

5. In Fig. 5.34, *AB* is parallel to CD and CD is parallel to EF. Also, EA is perpendicular to AB. If ∠BEF = 55°, find the values of *x* and *y*.

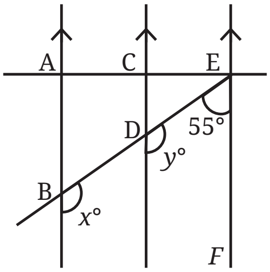


Fig. 5.34

6. What is the measure of angle ∠NOP in Fig. 5.35?

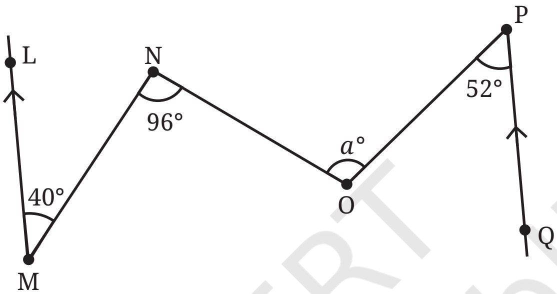
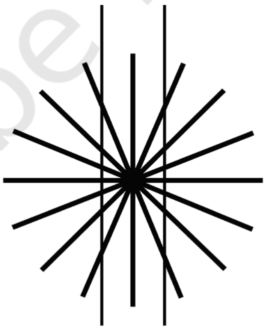


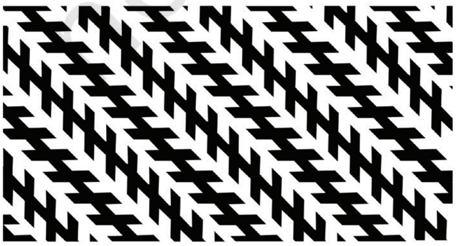
Fig. 5.35

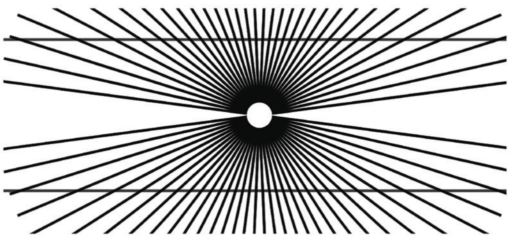
[**Hint:** Draw lines parallel to LM and PQ through points N and O.]

5.9 Parallel Illusions

There do not seem to be any parallel lines here. Or, are there?







What causes these illusions?

SUMMARY

- When two lines intersect, they form four angles. The vertically opposite angles are equal and the linear pairs add up to 180°.
- When two lines intersect and the angles formed are 90° (i.e., all four angles are equal), the lines are said to be perpendicular to each other.
- When two lines never intersect on a plane, they are called parallel lines.
- When a line *t* intersects another pair of lines, it is called a transversal and it forms 2 sets of 4 angles. Each of the 4 angles in the first set has a corresponding angle in the second set.
- When a transversal intersects a pair of parallel lines, the corresponding angles are equal. When a transversal intersects a pair of lines and the corresponding angles are equal, then the pair of lines is parallel.
- When a transversal intersects a pair of parallel lines, the alternate angles are equal.
- The interior angles on the same side formed by a transversal intersecting a pair of parallel lines always add up to 180°.

6 NUMBER PLAY

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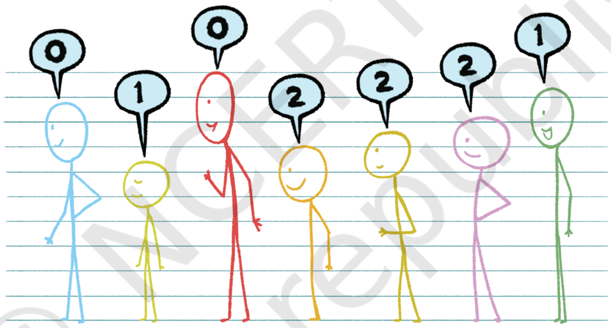
6.1 Numbers Tell us Things

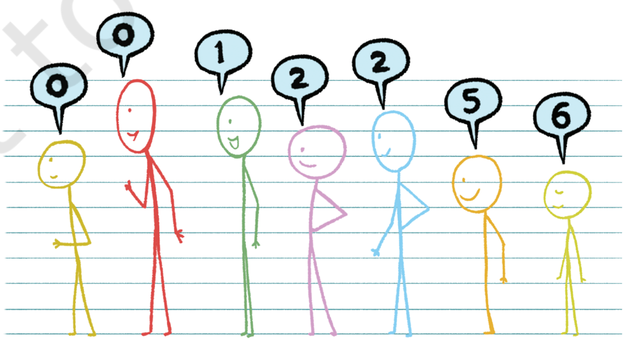
? What do the numbers in the figure below tell us?

Remember the children from the Grade 6 textbook of mathematics? Now, they call out numbers using a different rule.



? What do you think these numbers mean?

The children rearrange themselves and each one says a number based on the new arrangement.



? Could you figure out what these numbers convey? Observe and try to find out.

The rule is — each child calls out the number of children in front of them who are taller than them. Check if the number each child says matches this rule in both the arrangements.

Write down the number each child should say based on this rule for the arrangement shown below.

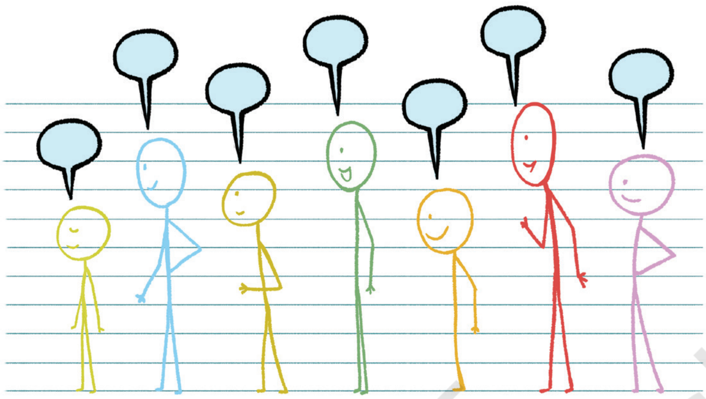


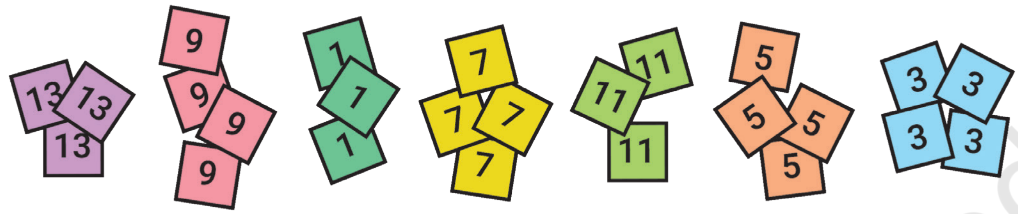
Figure it Out

1. Arrange the stick figure cutouts given at the end of the book or draw a height arrangement such that the sequence reads:
   (a) 0, 1, 1, 2, 4, 1, 5
   (b) 0, 0, 0, 0, 0, 0, 0
   (c) 0, 1, 2, 3, 4, 5, 6
   (d) 0, 1, 0, 1, 0, 1, 0
   (e) 0, 1, 1, 1, 1, 1, 1
   (f) 0, 0, 0, 3, 3, 3, 3
2. For each of the statements given below, think and identify if it is *Always True*, *Only Sometimes True*, or *Never True*. Share your reasoning.
   (a) If a person says '0', then they are the tallest in the group.
   (b) If a person is the tallest, then their number is '0'.
   (c) The first person's number is '0'.
   (d) If a person is not first or last in line (i.e., if they are standing somewhere in between), then they cannot say '0'.
   (e) The person who calls out the largest number is the shortest.
   (f) What is the largest number possible in a group of 8 people?

6.2 Picking Parity

Kishor has some number cards and is working on a puzzle: There are 5 boxes, and each box should contain exactly 1 number card. The numbers in the boxes should sum to 30. Can you help him find a way to do it?

$\square + \square + \square + \square + \square = 30$



Can you figure out which 5 cards add to 30? Is it possible?

There are many ways of choosing 5 cards from this collection.

Is there a way to find a solution without checking all possibilities?

Let us find out.

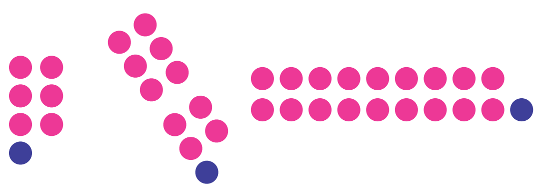
? Add a few even numbers together. What kind of number do you get? Does it matter how many numbers are added?

Any even number can be arranged in pairs without any leftovers. Some even numbers are shown here, arranged in pairs.

As we see in the figure, adding any number of even numbers will result in a number which can still be arranged in pairs without any leftovers. In other words, the sum will always be an even number.

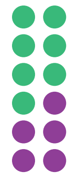
? Now, add a few odd numbers together. What kind of number do you get? Does it matter how many odd numbers are added?

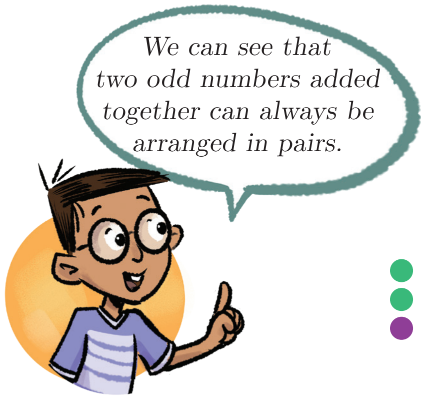
Odd numbers can not be arranged in pairs. An odd number is one more than a collection of pairs. Some odd numbers are shown below:

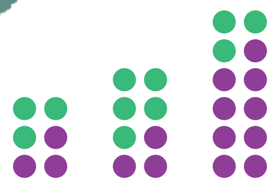


Can we also think of an odd number as one less than a collection of pairs?

This figure shows that the sum of two odd numbers must always be even! This along with the other figures here are more examples of a **proof**!







? What about adding 3 odd numbers? Can the resulting sum be arranged in pairs? No.

? Explore what happens to the sum of (a) 4 odd numbers, (b) 5 odd numbers, and (c) 6 odd numbers.

Let us go back to the puzzle Kishor was trying to solve. There are 5 empty boxes. That means he has an odd number of boxes. All the number cards contain odd numbers.

They should add to 30, which is an even number. Since, adding any 5 odd numbers will never result in an even number, Kishor cannot arrange these cards in the boxes to add up to 30.

? Two siblings, Martin and Maria, were born exactly one year apart. Today they are celebrating their birthday. Maria exclaims that the sum of their ages is 112. Is this possible? Why or why not?

As they were born one year apart, their ages will be (two) consecutive numbers. Can their ages be 51 and 52? 51 + 52 = 103. Try some other consecutive numbers and see if their sum is 112.

The counting numbers 1, 2, 3, 4, 5, ... alternate between even and odd numbers. In any two consecutive numbers, one will always be even and the other will always be odd!

What would be the resulting sum of an even number and an odd number? We can see that their sum can't be arranged in pairs and thus will be an odd number.

Since 112 is an even number, and Martin's and Maria's ages are consecutive numbers, they cannot add up to 112.

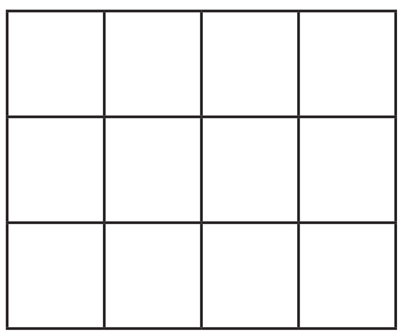
We use the word **parity** to denote the property of being even or odd. For instance, the parity of the sum of any two consecutive numbers is odd. Similarly, the parity of the sum of any two odd numbers is even.

Figure it Out

1. Using your understanding of the pictorial representation of odd and even numbers, find out the parity of the following sums:
   - (a) Sum of 2 even numbers and 2 odd numbers (e.g., even + even + odd + odd)
   - (b) Sum of 2 odd numbers and 3 even numbers
   - (c) Sum of 5 even numbers
   - (d) Sum of 8 odd numbers
2. Lakpa has an odd number of ₹1 coins, an odd number of ₹5 coins and an even number of ₹10 coins in his piggy bank. He calculated the total and got ₹205. Did he make a mistake? If he did, explain why. If he didn't, how many coins of each type could he have?
3. We know that:
   - (a) even + even = even
   - (b) odd + odd = even
   - (c) even + odd = odd

   Similarly, find out the parity for the scenarios below:
   - (d) even – even = ____________
   - (e) odd – odd = ____________
   - (f) even – odd = ____________
   - (g) odd – even = ____________

Small Squares in Grids



In a 3 × 3 grid, there are 9 small squares, which is an odd number. Meanwhile, in a 3 × 4 grid, there are 12 small squares, which is an even number.

Given the dimensions of a grid, can you tell the parity of the number of small squares without calculating the product?

Find the parity of the number of small squares in these grids:

(a) 27×13
(b) 42×78
(c) 135×654

Parity of Expressions

Consider the algebraic expression: $3n + 4$. For different values of n, the expression has different parity:

n	Value of $3n + 4$	Parity of the Value
3	13	odd
8	28	even
10	34	even

Come up with an expression that always has even parity.

Some examples are: $100p$ and $48w - 2$. Try to find more.

Come up with expressions that always have odd parity.

Come up with other expressions, like $3n + 4$, which could have either odd or even parity.

The expression $6k + 2$ evaluates to 8, 14, 20,... (for $k = 1, 2, 3,...$) — many even numbers are missing.

Are there expressions using which we can list all the even numbers?

Hint: All even numbers have a factor 2.

Are there expressions using which we can list all odd numbers?

We saw earlier how to express the n^{th} term of the sequence of multiples of 4, where n is the letter-number that denotes a position in the sequence (e.g., first, twenty third, hundred and seventeenth, etc.).

What would be the n^{th} term for multiples of 2? Or, what is the n^{th} even number?

Let us consider odd numbers.

What is the 100th odd number?

To answer this question, consider the following question:

? What is the 100th even number?

This is 2 × 100 = 200.

Does this help in finding the 100th odd number? Let us compare the sequence of evens and odds term-by-term.

Even Numbers: 2, 4, 6, 8, 10, 12,...

Odd Numbers: 1, 3, 5, 7, 9, 11,...

We see that at any position, the value at the odd number sequence is one less than that in the even number sequence. Thus, the 100th odd number is 200 – 1 = 199.

? Write a formula to find the n^{th} odd number.

Let us first describe the method that we have learnt to find the odd number at a given position:

(a) Find the even number at that position. This is 2 times the position number.

(b) Then subtract 1 from the even number.

Writing this in expressions, we get

(a) $2n$

(b) $2n - 1$

Thus, 2n is the formula that gives the n^{th} even number, and $2n - 1$ is the formula that gives the n^{th} odd number.

6.3 Some Explorations in Grids

Observe this 3 × 3 grid. It is filled following a simple rule — use numbers from 1 – 9 without repeating any of them. There are circled numbers outside the grid.

4	7	5	16
6	1	2	9
3	9	8	20
13	17	15	

? Are you able to see what the circled numbers represent?

The numbers in the yellow circles are the sums of the corresponding rows and columns.

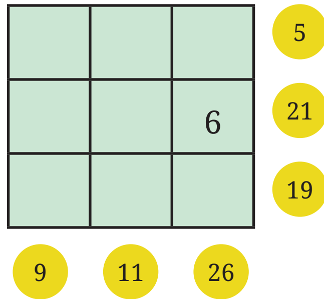
Fill the grids below based on the rule mentioned above:

9			13
			14
		5	18
24	9	12	

			24
4			15
		3	6
12	16	17	

? Make a couple of questions like this on your own and challenge your peers.

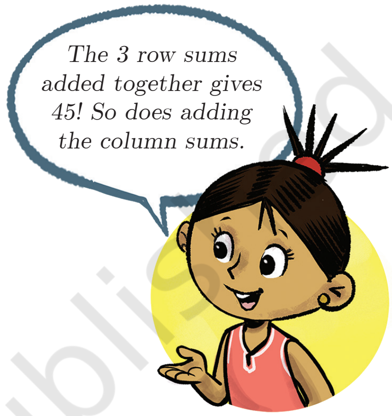
Try solving the problem below.



? You might have realised that it is not possible to find a solution for this grid. Why is this the case?

The smallest sum possible is 6 = 1 + 2 + 3. The largest sum possible is 24 = 9 + 8 + 7. Clearly, any number in a circle cannot be less than 6 or greater than 24. The grid has sums 5 and 26. Therefore, this is impossible!

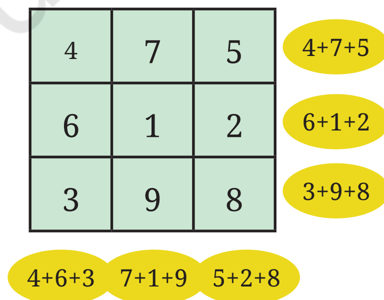
In the earlier grids which we solved, Kishor noticed that the sum of all the numbers in the circles was always 90. Also, Vidya observed that the sum of the circled numbers for all three rows, or for all three columns, was always 45. Check if this is true in the previous grids you have solved.



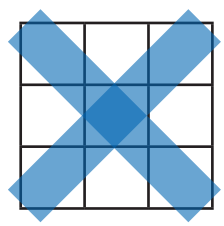
? Why should the row sums and column sums always add to 45?

From this grid, we can see that all the row sums added together will be the same as the sum of the numbers 1 – 9. This can be seen for column sums as well. The sum of the numbers 1 – 9 is

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45.$$



A square grid of numbers is called a **magic square** if each row, each column and each diagonal, add up to the same number. This number is called the **magic sum**. Diagonals are shown in the picture.



Trying to create a magic square by randomly filling the grid with numbers may be difficult! This is because there are a large number of ways of filling a 3 × 3 grid using the numbers 1 – 9 without repetition. In fact, it can be found that there are exactly 3,62,880 such ways. Surprisingly, the number of ways to fill in the grid can be found without listing all of them. We will see in later years how to do this.

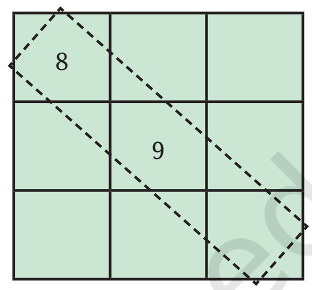
Instead, we should proceed systematically to make a magic square. For this, let us ask ourselves some questions.

1. What can the magic sum be? Can it be any number?

Let us focus, for the moment, only on the row sums. We have seen that in a 3 × 3 grid with numbers 1 – 9, the total of row sums will always be 45. Since in a magic square the row sums are all equal, and they add up to 45, they have to be 15 each. Thus, we have the following observation.

Observation 1: In a magic square made using the numbers 1 – 9, the magic sum must be 15.

2. What are the possible numbers that could occur at the centre of a magic square?

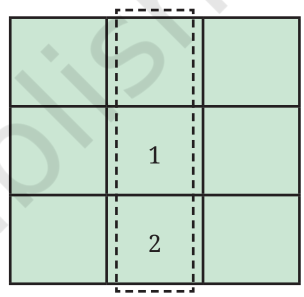


Let us consider the possibilities one by one. Can the central number be 9? If yes, then 8 must come in one of the other squares. For example,

In this, we must have 8 + 9 + other number = 15.

But this is not possible! The same issue will occur no matter where we place 8.

So, 9 cannot be at the centre. Can the central number be 1?



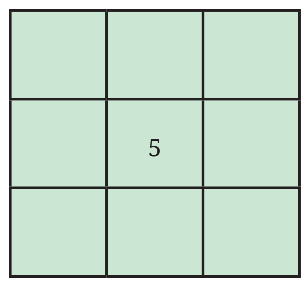
If yes, then 2 should come in one of the other squares.

Here, we must have 2 + 1 + other number = 15.

But this is not possible because we are only using the numbers 1 – 9. The same issue will occur no matter where we place 1.

So, 1 cannot be at the centre, either.

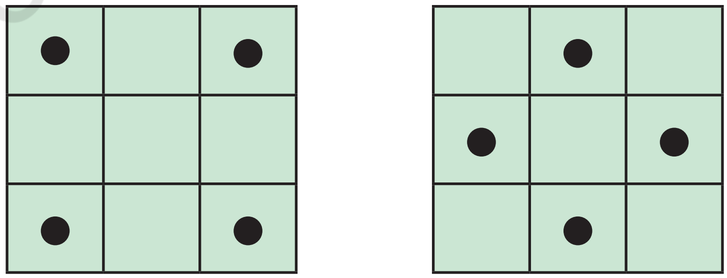
? Using such reasoning, find out which other numbers 1 – 9 cannot occur at the centre.



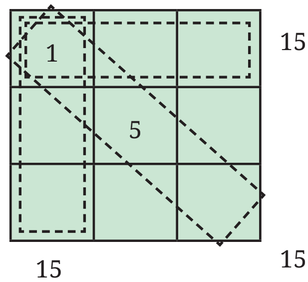
This exploration will lead us to the following interesting observation.

Observation 2: The number occurring at the centre of a magic square, filled using 1 – 9, must be 5.

Let us now see where the smallest number 1 and the largest number 9 should come in a magic square. Our second observation tells us that they will have to come in one of the boundary positions. Let us classify these positions into two categories:



Can 1 occur in a corner position? For example, can it be placed as follows?



If yes, then there should exist three ways of adding 1 with two other numbers to give 15. We have 1 + 5 + 9 = 1 + 6 + 8 = 15. Is any other combination possible?

Similarly, can 9 can be placed in a corner position?

Observation 3: The numbers 1 and 9 cannot occur in any corner, so they should occur in one of the middle positions.

Can you find the other possible positions for 1 and 9?

1	5	9

	1	
	5	
	9	

Now, we have one full row or column of the magic square! Try completing it!
[**Hint:** First fill the row or columns containing 1 and 9]

Figure it Out

1. How many different magic squares can be made using the numbers 1 – 9?
2. Create a magic square using the numbers 2 – 10. What strategy would you use for this? Compare it with the magic squares made using 1 – 9.
3. Take a magic square, and
   (a) increase each number by 1
   (b) double each number
   In each case, is the resulting grid also a magic square? How do the magic sums change in each case?
4. What other operations can be performed on a magic square to yield another magic square?
5. Discuss ways of creating a magic square using any set of 9 consecutive numbers (like 2 – 10, 3 – 11, 9 – 17, etc.).

Math Talk

Generalising a 3 × 3 Magic Square

We can describe how the numbers within the magic square are related to each other, i.e., the structure of the magic square.

Choose any magic square that you have made so far using consecutive numbers. If m is the letter-number of the number in the centre, express how other numbers are related to m, how much more or less than m.

	m	

[**Hint:** Remember, how we described a 2 × 2 grid of a calendar month in the Algebraic Expressions chapter].

Once the generalised form is obtained, share your observations with the class.

Math Talk

Figure it Out

1. Using this generalised form, find a magic square if the centre number is 25.
2. What is the expression obtained by adding the 3 terms of any row, column or diagonal?
3. Write the result obtained by—
   (a) adding 1 to every term in the generalised form.
   (b) doubling every term in the generalised form
4. Create a magic square whose magic sum is 60.
5. Is it possible to get a magic square by filling nine non-consecutive numbers?

Try This

The First-ever 4 × 4 Magic Square

The first ever recorded 4 × 4 magic square is found in a 10th century inscription at the Pārśhvanath Jain temple in Khajuraho, India, and is known as the *Chautīsā Yantra*.

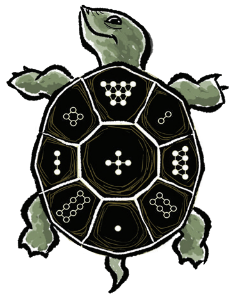
7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

The first ever recorded 4 × 4 magic square, the Chautīsā Yantra, at Khajuraho, India

Chaůtīs means 34. Why do you think they called it the *Chautīsā Yantra*?

Every row, column and diagonal in this magic square adds up to 34. Can you find other patterns of four numbers in the square that add up to 34?

Magic Squares in History and Culture



The first magic square ever recorded, the Lo Shu Square, dates back over 2000 years to ancient China. The legend tells of a catastrophic flood on the Lo River, during which the gods sent a turtle to save the people. The turtle carried a 3 × 3 grid on its back, with the numbers 1 to 9 arranged in a magical pattern.

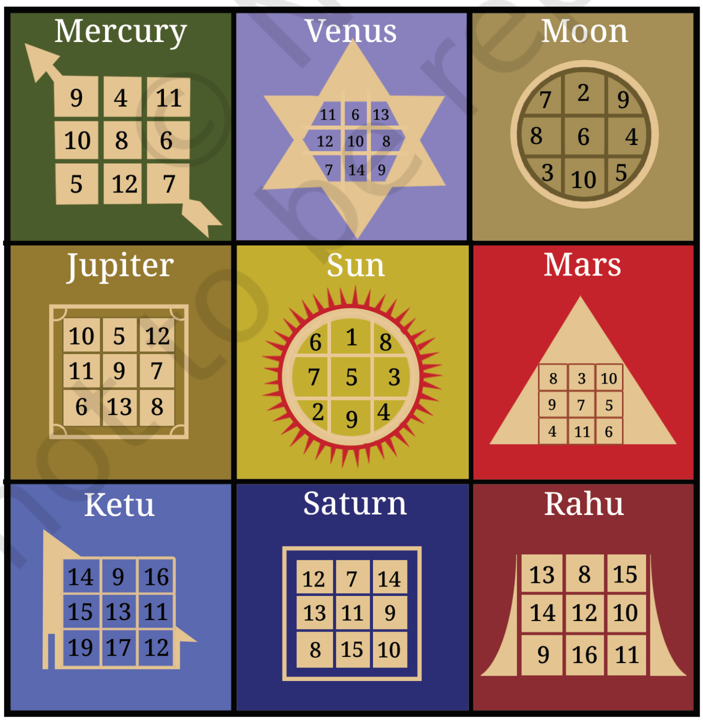
2	7	6
9	5	1
4	3	8

Magic squares were studied in different parts of the world at different points of time including India, Japan, Central Asia, and Europe.

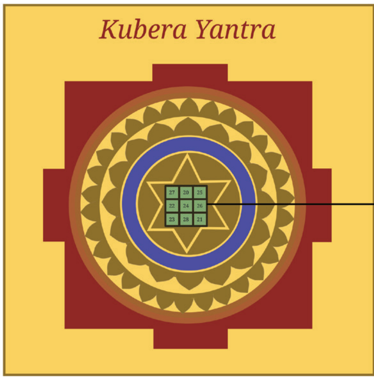
Indian mathematicians have worked extensively on magic squares, describing general methods of constructing them. The work of Indian mathematicians was not just limited to 3 × 3 and 4 × 4 grids, which we considered above, but also extended to 5 × 5 and other larger square grids. We shall learn more about these in later grades.

The occurrence of magic squares is not limited to scholarly mathematical works. They are found in many places in India. The picture to the right is of a 3 × 3 magic square found on a pillar in a temple in Palani, Tamil Nadu. The temple dates back to the 8th century CE.

3 × 3 magic squares can also be found in homes and shops in India. The *Navagraha Yantra* is one such example shown below.



Notice that a different magic sum is associated with each *graha*. A picture of a *Kubera Yantra* is shown below:



27	20	25
22	24	26
23	28	21

6.4 Nature's Favourite Sequence: The Virahāṅka–Fibonacci Numbers!

The sequence 1, 2, 3, 5, 8, 13, 21, 34, . . . (**Virahāṅka–Fibonacci Numbers**) is one of the most celebrated sequences in all of mathematics — it occurs throughout the world of Art, Science, and Mathematics. Even though these numbers are found very frequently in Science, it is remarkable that these numbers were first discovered in the context of Art (specifically, poetry)!

The **Virahāṅka–Fibonacci Numbers** thus provide a beautiful illustration of the close links between Art, Science, and Mathematics.

Discovery of the Virahāṅka Numbers

The Virahāṅka numbers first came up thousands of years ago in the works of Sanskrit and Prakrit linguists in their study of poetry!

In the poetry of many Indian languages, including, Prakrit, Sanskrit, Marathi, Malayalam, Tamil, and Telugu, each syllable is classified as either long or short.

A long syllable is pronounced for a longer duration than a short syllable — in fact, for exactly twice as long. When singing such a poem, a **short syllable** lasts one beat of time, and a **long syllable** lasts two beats of time.

This leads to numerous mathematical questions, which the ancient poets in these languages considered extensively. A number of important mathematical discoveries were made in the process of asking and answering these questions about poetry.

One of these particularly important questions was the following.

How many rhythms are there with 8 beats consisting of short syllables (1 beat) and long syllables (2 beats)? That is, in how many ways can one fill 8 beats with short and long syllables, where a short syllable takes one beat of time and a long syllable takes two beats of time.

Here are some possibilities:

long long long long
short short short short short short short short
short **long long** short **long**
long long short short **long**
⋮

Can you find others?

Phrased more mathematically: In how many different ways can one write a number, say 8, as a sum of **1's** and **2's**?

For example, we have:

$$8 = 2 + 2 + 2 + 2,$$
$$8 = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1,$$
$$8 = 1 + 2 + 2 + 1 + 2,$$
$$8 = 2 + 2 + 1 + 1 + 2,$$
etc.

Do you see other ways?

Here are all the ways of writing each of the numbers 1, 2, 3, and 4, as the sum of **1's** and **2's**.

	Different Ways	Number of Ways
$n = 1$	1	1
$n = 2$	1 + 1 2	2
$n = 3$	1 + 1 + 1 1 + 2 2 + 1	3
$n = 4$	1 + 1 + 1 + 1 1 + 1 + 2 1 + 2 + 1 2 + 1 + 1 2 + 2	5

Try writing the number 5 as a sum of **1*s*** and **2*s*** in all possible ways in your notebook! How many ways did you find? (You should find 8 different ways!) Can you figure out the answer without listing down all the possibilities? Can you try it for n = 8?

Here is a systematic way to write down all rhythms of short and long syllables having 5 beats. Write a **'1+'** in front of all rhythms having 4 beats, and then a **'2+'** in front of all rhythms having 3 beats. This gives us all the rhythms having 5 beats:

$n = 5$	1 + 1 + 1 + 1 + 1 1 + 1 + 1 + 2 1 + 1 + 2 + 1 1 + 2 + 1 + 1 1 + 2 + 2	2 + 1 + 1 + 1 2 + 1 + 2 2 + 2 + 1

Thus, there are 8 rhythms having 5 beats!

The reason this method works is that every 5-beat rhythm must begin with either a '1+' or a '2+'. If it begins with a '1+', then the remaining numbers must give a 4-beat rhythm, and we can write all those down. If it begins with a 2+, then the remaining number must give a 3-beat rhythm, and we can write all those down. Therefore, the number of 5-beat rhythms is the number of 4-beat rhythms, plus the number of 3-beat rhythms.

How many 6-beat rhythms are there? By the same reasoning, it will be the number of 5-beat rhythms plus the number of 4-beat rhythms, i.e., 8 + 5 = 13. Thus, there are 13 rhythms having 6 beats.

? Use the systematic method to write down all 6-beat rhythms, i.e., write 6 as the sum of 1's and 2's in all possible ways. Did you get 13 ways?

This beautiful method for counting all the rhythms of short syllables and long syllables having any given number of beats was first given by the great Prakrit scholar **Virahāṅka** around the year 700 CE. He gave his method in the form of a Prakrit poem! For this reason, the sequence 1, 2, 3, 5, 8, 13, 21, 34, . . . is known as the **Virahāṅka sequence**, and the numbers in the sequence are known as the **Virahāṅka numbers**. **Virahāṅka** was the first known person in history to explicitly consider these important numbers and write down the rule for their formation.

Other scholars in India also considered these numbers in the same poetic context. **Virahāṅka** was inspired by earlier work of the legendary Sanskrit scholar Piṅgala, who lived around 300 BCE. After **Virahāṅka**, these numbers were also written about by Gopala (c. 1135 CE) and then by Hemachandra (c. 1150 CE).

In the West, these numbers have been known as the **Fibonacci numbers**, after the Italian mathematician who wrote about them in the year 1202 CE — about 500 years after **Virahāṅka**. As we can see, Fibonacci was not first, nor the second, not even the third person to write about these numbers! Sometimes the term "**Virahāṅka**–Fibonacci numbers" is used so that everyone understands what is being referred to.

So, how many rhythms of short and long syllables are there having 8 beats? We simply take the 8th element of the **Virahāṅka** sequence:

1, 2, 3, 5, 8, 13, 21, 34, 55, ...

Thus, there are 34 rhythms having 8 beats.

Write the next number in the sequence, after 55.

We have seen that the next number in the sequence is given by adding the two previous numbers. Check that this holds true for the numbers given above. The next number is 34 + 55 = 89.

? Write the next 3 numbers in the sequence:

1, 2, 3, 5, 8, 13, 21, 34 , 55, 89, ____, ____, ____, ...

If you have to write one more number in the sequence above, can you tell whether it will be an odd number or an even number (without adding the two previous numbers)?

? What is the parity of each number in the sequence? Do you notice any pattern in the sequence of parities?

Today, the **Virahāṅka–Fibonacci** numbers form the basis of many mathematical and artistic theories, from poetry to drumming, to visual arts and architecture, to science. Perhaps the most stunning occurrences of these numbers are in nature. For example, the number of petals on a daisy is generally a **Virahāṅka** number.

How many petals do you see on each of these flowers?

A daisy with 13 petals

A daisy with 21 petals

A daisy with 34 petals

There are many other remarkable mathematical properties of the **Virahāṅka–Fibonacci numbers** that we will see later, in mathematics as well as in other subjects.

These numbers truly exemplify the close connections between Art, Science, and Mathematics.

6.5 Digits in Disguise

You have done arithmetic operations with numbers. How about doing the same with letters?

In the calculations below, digits are replaced by letters. Each letter stands for a particular digit (0 – 9). You have to figure out which digit each letter stands for.

$$\begin{array}{r} T \\ T \\ +\quad T \\ \hline UT \end{array}$$

Here, we have a one-digit number that, when added to itself twice, gives a 2-digit sum. The units digit of the sum is the same as the single digit being added.

What could U and T be? Can T be 2? Can it be 3?

$$\begin{array}{r} K2 \\ +\quad K2 \\ \hline HMM \end{array}$$

Once you explore, you will see that T = 5 and UT = 15.

Let us look at one more example shown on the right. Here K2 means that the number is a 2-digit number having the digit '2' in the units place and 'K' in the tens place. K2 is added to itself to give a 3-digit sum HMM. What digit should the letter M correspond to?

Both the tens place and the units place of the sum have the same digit.

What about H? Can it be 2? Can it be 3?

These types of questions can be interesting and fun to solve! Here are some more questions like this for you to try out. Find out what each letter stands for.

Share how you thought about each question with your classmates; you may find some new approaches.

$$\begin{array}{r} YY \\ +\quad Z \\ \hline ZOO \end{array} \qquad \begin{array}{r} B5 \\ +\quad 3D \\ \hline ED5 \end{array} \qquad \begin{array}{r} KP \\ +\quad KP \\ \hline PRR \end{array} \qquad \begin{array}{r} C1 \\ +\quad C \\ \hline 1FF \end{array}$$

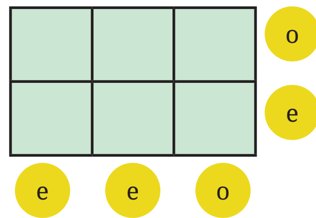
These types of questions are called 'cryptarithms' or 'alphametics'.

Figure it Out

1. A light bulb is ON. Dorjee toggles its switch 77 times. Will the bulb be on or off? Why?

2. Liswini has a large old encyclopaedia. When she opened it, several loose pages fell out of it. She counted 50 sheets in total, each printed on both sides. Can the sum of the page numbers of the loose sheets be 6000? Why or why not?

3. Here is a 2 × 3 grid. For each row and column, the parity of the sum is written in the circle; ‘*e*’ for even and ‘o’ for odd. Fill the 6 boxes with 3 odd numbers (‘*o*’) and 3 even numbers (‘*e*’) to satisfy the parity of the row and column sums.



4. Make a 3 × 3 magic square with 0 as the magic sum. All numbers can not be zero. Use negative numbers, as needed.

5. Fill in the following blanks with ‘odd’ or ‘even’:
   (a) Sum of an odd number of even numbers is ______
   (b) Sum of an even number of odd numbers is ______
   (c) Sum of an even number of even numbers is ______
   (d) Sum of an odd number of odd numbers is ______

6. What is the parity of the sum of the numbers from 1 to 100?

7. Two consecutive numbers in the Virahāṅka sequence are 987 and 1597. What are the next 2 numbers in the sequence? What are the previous 2 numbers in the sequence?

8. Angaan wants to climb an 8-step staircase. His playful rule is that he can take either 1 step or 2 steps at a time. For example, one of his paths is 1, 2, 2, 1, 2. In how many different ways can he reach the top?

9. What is the parity of the 20th term of the Virahāṅka sequence?

10. Identify the statements that are true.
    (a) The expression $4m - 1$ always gives odd numbers.
    (b) All even numbers can be expressed as $6j - 4$.
    (c) Both expressions $2p + 1$ and $2q - 1$ describe all odd numbers.
    (d) The expression $2f + 3$ gives both even and odd numbers.

11. Solve this cryptarithm:

$$\begin{array}{r} \text{UT} \\ +\ \ \text{TA} \\ \hline \text{TAT} \end{array}$$

SUMMARY

In this chapter, we have explored the following:

- In the first activity, we saw how to represent information about how a sequence of numbers (e.g., height measures) is arranged without knowing the actual numbers.
- We learnt the notion of parity — numbers that can be arranged in pairs (even numbers) and numbers that cannot be arranged in pairs (odd numbers).
- We learnt how to determine the parity of sums and products.
- While exploring sums in grids, we could determine whether filling a grid is impossible by looking at the row and column sums. We extended this to construct magic squares.
- We saw how Virahāṅka numbers were first discovered in history through the Arts. The Virahāṅka sequence is 1, 2, 3, 5, 8, 13, 21, 34, 55, ...
- We became math-detectives through cryptarithms, where digits are replaced by letters.