Advanced Robust Control | HW4 | amithr3

1. Using the pitch axis aircraft plant data (same as RSLQR design model in homework 3) design a Hinf state feedback controller to command Az. Include the 2nd order actuator in the design model as presented in class. Tune the Hinf controller to yield a similar rise time as your RSLQR controller.

List out all matrices used in the Hinf design, closed loop eigenvalues, and eigenvectors.

List the controller matrices $\left(A_c, B_{c_1}, B_{c_2}, C_c, D_{c_1}, D_{c_2}\right)$ implementing the Hinf control.

$$\dot{x}_c = A_c x_c + B_{c_1} y + B_{c_2} r$$
$$u = C_c x_c + D_{c_1} y + D_{c_2} r$$

where $y = x_p$.

```
Plant Model
xpdot = Ap*xp + Bp*u
    y = Cp*xp + Dp*u
   xp = AOA (rad), pitch rate q (rps), dele (rad), deledot (rps)
    u =  delec (rad)
    y =  Az (fps2), AOA (rad), pitch rate q (rps), dele (rad), deledot (rps)


Target lgcf = 1.2 Hz
```

```
WS SS Model
[ A B ]
[ C D ]

WS =

            0    1.0000e+00
   3.7699e+00    5.0000e-01
```

```
WT SS Model
[ A B ]
[ C D ]

WT =

  -2.0000e+02    1.0000e+00
  -9.3024e+02    5.3582e+00
```

```
WT SS Model
[ A B ]
[ C D ]
```

```
WT SS Model
[ A B ]
[ C D ]
```

```
WC =

           0             0
           0    1.0000e-01
```

```
Gamma optimal = 1
K optimal = [ 11.3599     0.794031     -1.22049    -0.0106452    -0.0532245     -0.131007 ]
Norm EE opt = 5.0787e-06

gamma =

   1.0020e+00
```

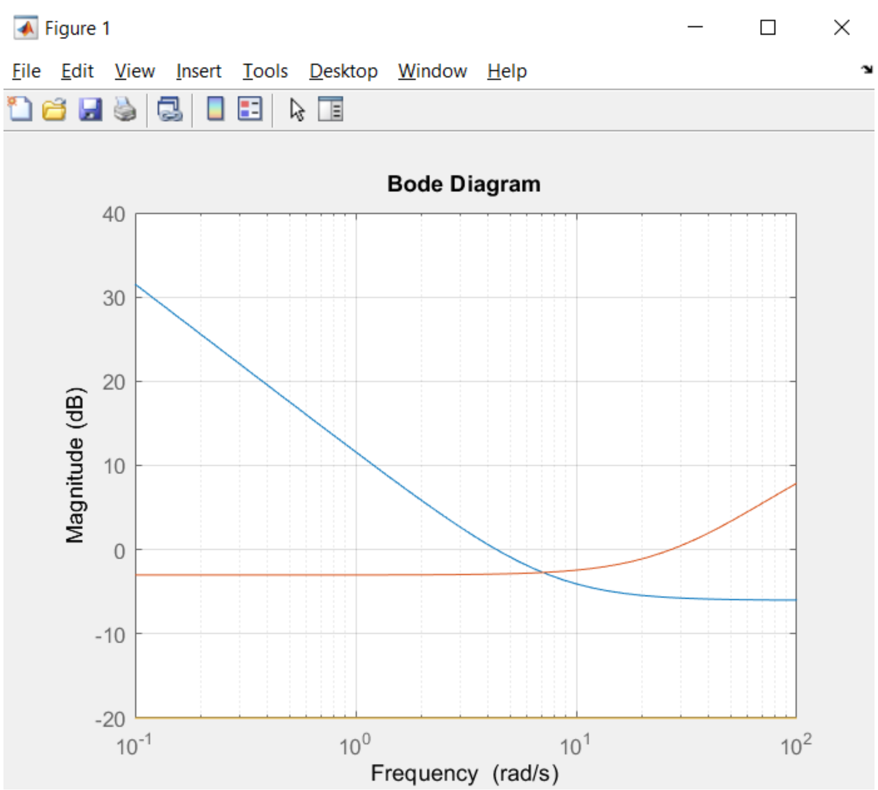
Controller Matrices:

```
Ac = [ Ws.A         0.;
           0.     Wt.A];
Bc1 = [ Ws.B*[1 0 0 0 0]
       Wt.B*[1 0 0 0 0]];
Bc2 = [-Ws.B
             0.];
Cc   = [ HinfSF.Kxref(:,5:6) ];
Dc1 = [ 0. HinfSF.Kxref(:,1:4)];
Dc2 = [ 0. ];
```

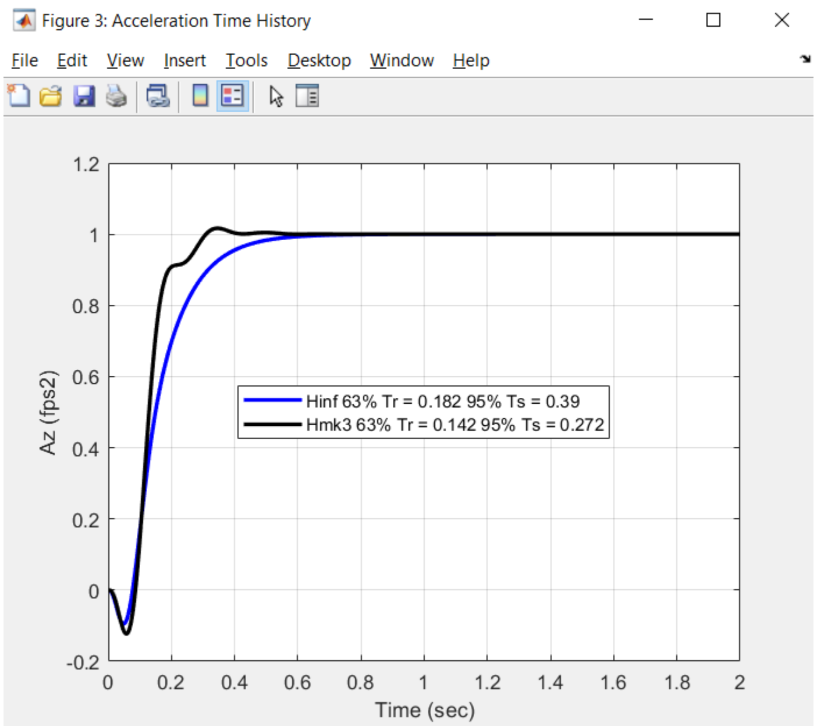
```
Ac =

     0     0
     0  -200


Bc1 =

     1     0     0     0     0
     1     0     0     0     0


Bc2 =

    -1
     0


Cc =

  -5.2052e-02  -1.3157e-01


Dc1 =

            0   1.1136e+01   7.8214e-01  -1.2022e+00  -1.0492e-02

Dc2 =

     0


Controller  Model
[ Ac Bc1 Bc2 ]

HH =

     0     0     1     0     0     0     0    -1
     0  -200     1     0     0     0     0     0

[ Cc Dc1 Dc2 ]

HH =

  -5.2052e-02  -1.3157e-01            0   1.1136e+01   7.8214e-01  -1.2022e+00  -1.0492e-02            0
```

```
Closed Loop Eigenvalues

        Pole                  Damping       Frequency        Time Constan
                                           (rad/TimeUnit)     (TimeUnit)

 -1.93e+02                   1.00e+00       1.93e+02          5.17e-03
 -6.02e+01                   1.00e+00       6.02e+01          1.66e-02
 -2.99e+01 + 3.63e+01i       6.36e-01       4.71e+01          3.34e-02
 -2.99e+01 - 3.63e+01i       6.36e-01       4.71e+01          3.34e-02
 -2.62e+01                   1.00e+00       2.62e+01          3.82e-02
 -9.60e+00                   1.00e+00       9.60e+00          1.04e-01
```

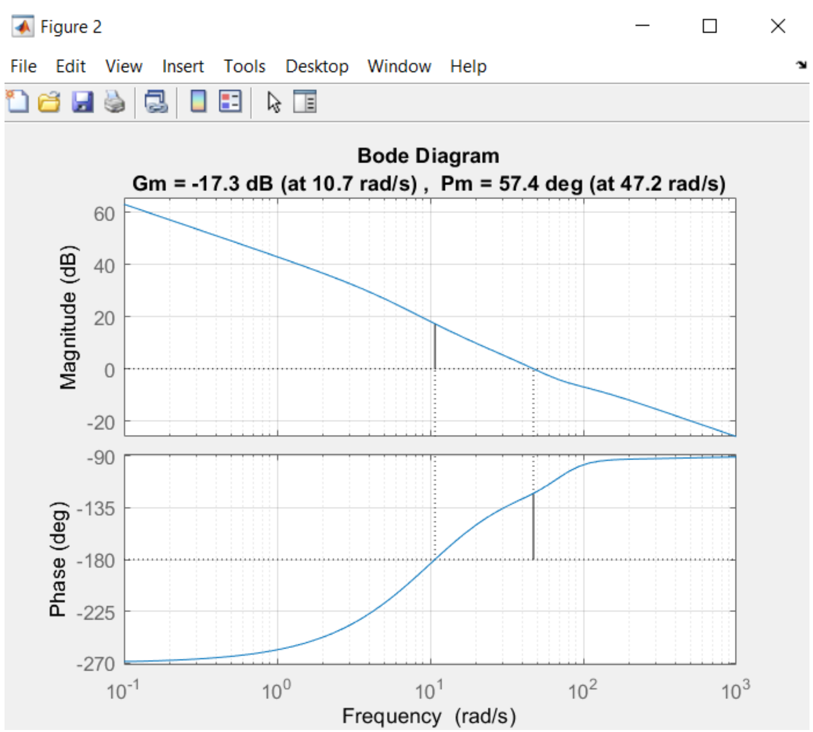
```
V =

  Columns 1 through 5

   8.7622e-06 + 0.0000e+00i  -4.3496e-04 + 0.0000e+00i  -8.8610e-04 + 3.4807e-04i  -8.8610e-04 - 3.4807e-04i  -6.2125e-03 + 0.0000e+00i
  -2.7789e-03 + 0.0000e+00i   2.9184e-02 + 0.0000e+00i   9.8450e-03 - 4.5657e-02i   9.8450e-03 + 4.5657e-02i   1.6245e-01 + 0.0000e+00i
  -5.1199e-03 + 0.0000e+00i   1.6572e-02 + 0.0000e+00i  -1.3420e-02 - 1.6292e-02i  -1.3420e-02 + 1.6292e-02i   3.7700e-02 + 0.0000e+00i
   9.8963e-01 + 0.0000e+00i  -9.9829e-01 + 0.0000e+00i   9.9382e-01 + 0.0000e+00i   9.9382e-01 + 0.0000e+00i  -9.8598e-01 + 0.0000e+00i
  -4.9789e-03 + 0.0000e+00i   4.3903e-02 + 0.0000e+00i  -4.1331e-03 - 9.4947e-02i  -4.1331e-03 + 9.4947e-02i  -1.0003e-03 + 0.0000e+00i
   1.4344e-01 + 0.0000e+00i  -1.8923e-02 + 0.0000e+00i   2.3334e-02 + 1.0843e-02i   2.3334e-02 - 1.0843e-02i   1.5048e-04 + 0.0000e+00i

  Column 6

  -8.7084e-03 + 0.0000e+00i
   7.2834e-02 + 0.0000e+00i
   2.7076e-03 + 0.0000e+00i
  -2.5998e-02 + 0.0000e+00i
  -9.9570e-01 + 0.0000e+00i
   5.0212e-02 + 0.0000e+00i
```



a) Simulate the closed loop system to a unit step Az command. Plot the RSLQR response with the Hinf response. Compute the rise time and settling time. Label on the plot.



b) Plot a Nyquist plot and identify on the plot the gain and phase margins, loop gain and phase crossover frequencies. Plot the RSLQR Nyquist with the Hinf Nyquist.



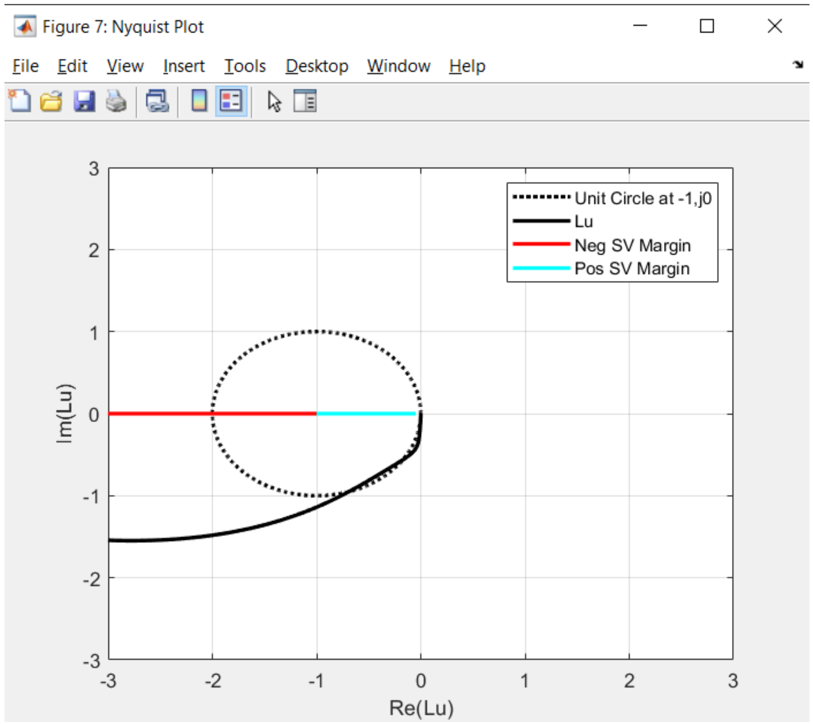


Figure 7: Nyquist Plot
File Edit View Insert Tools Desktop Window Help
Unit Circle at -1,j0
Lu
Neg SV Margin
Pos SV Margin
Im(Lu)
Re(Lu)
3
2
1
0
-1
-2
-3
-3
-2
-1
0
1
2
3

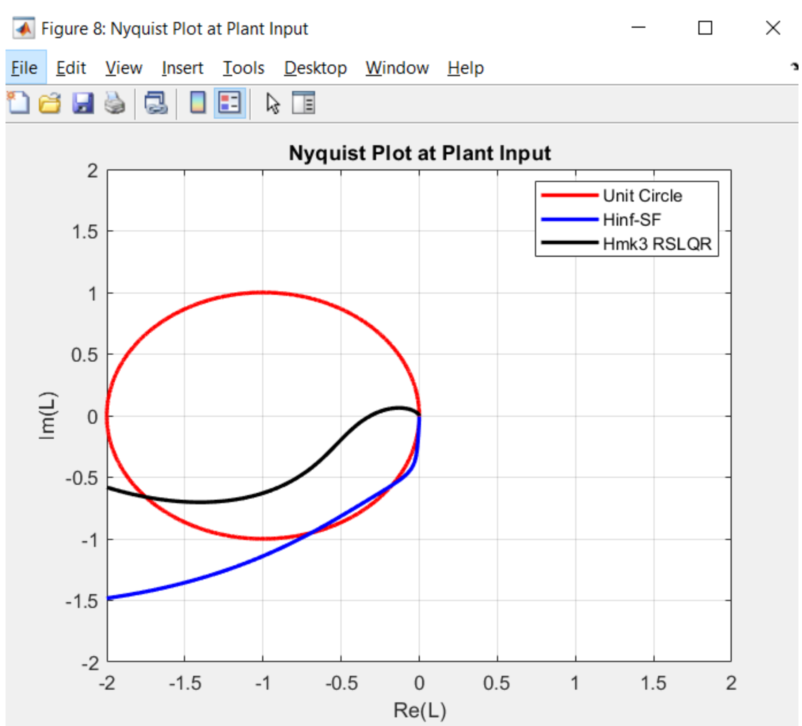
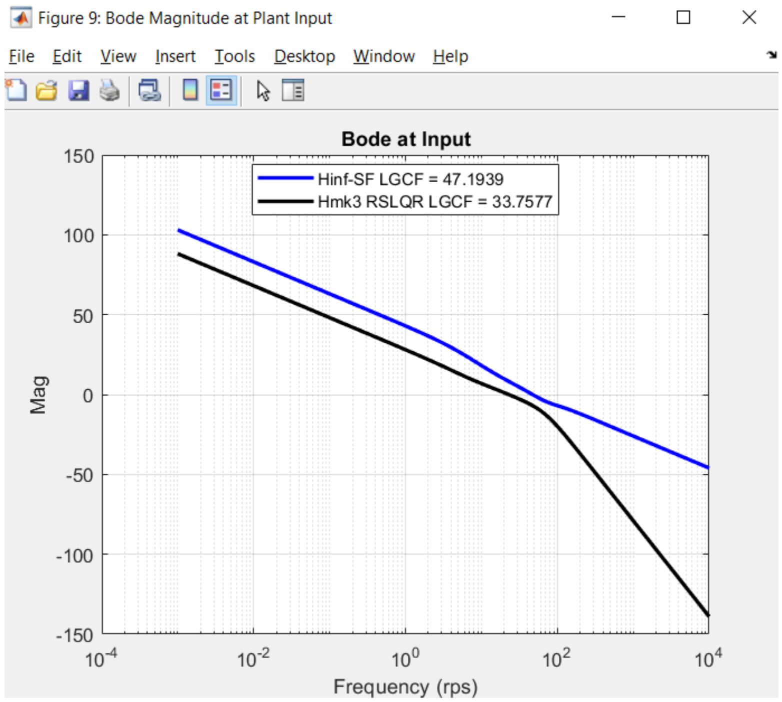
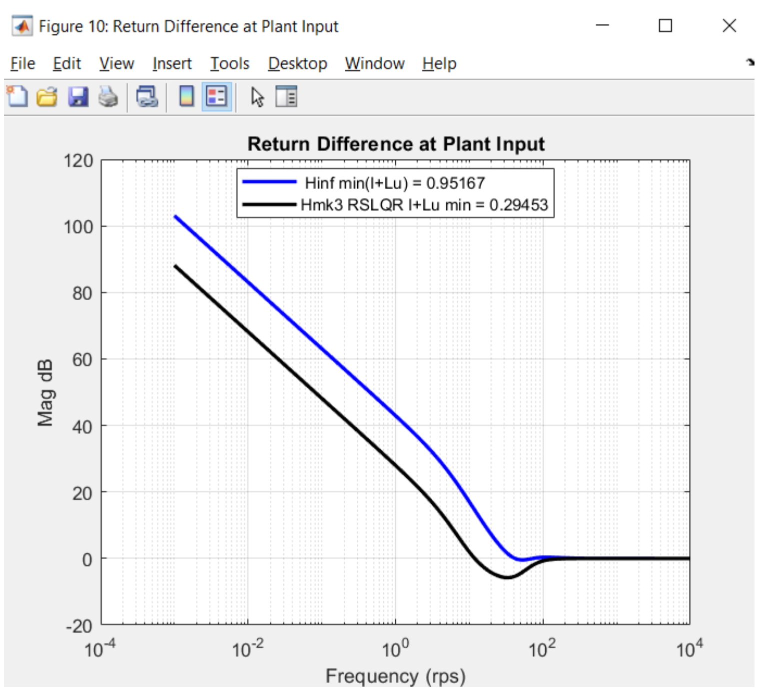


Figure 8: Nyquist Plot at Plant Input
File Edit View Insert Tools Desktop Window Help
Nyquist Plot at Plant Input
Unit Circle
Hinf-SF
Hmk3 RSLQR
Im(L)
Re(L)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

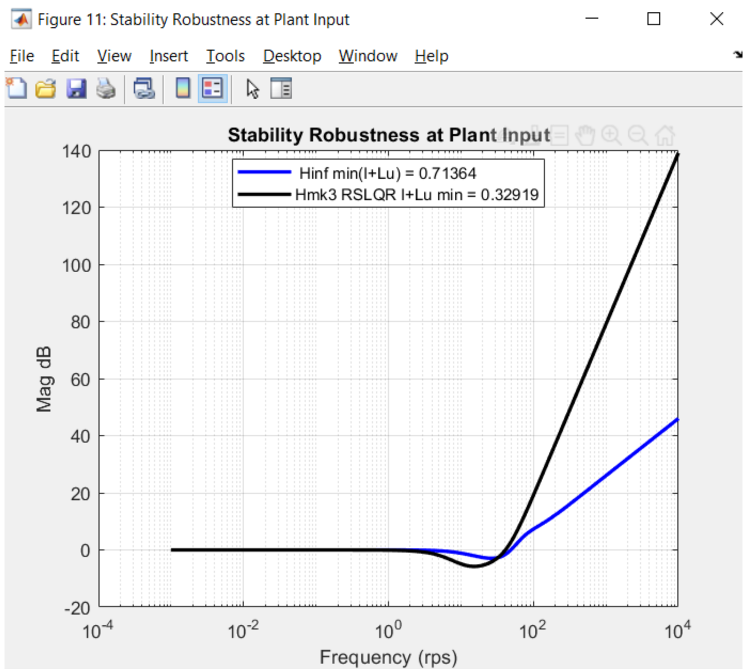
c) Plot a Bode plot and identify on the plot the gain and phase margins, loop gain and phase crossover frequencies. Plot the RSLQR Bode with the Hinf Bode.



d) Plot the minimum singular value of the return difference matrix in dB vs frequency. Identify on the plot the minimum value of the return difference matrix (not in dB). Plot the RSLQR $\underline{\sigma}(I+L_u)$ including the actuator in the plant model



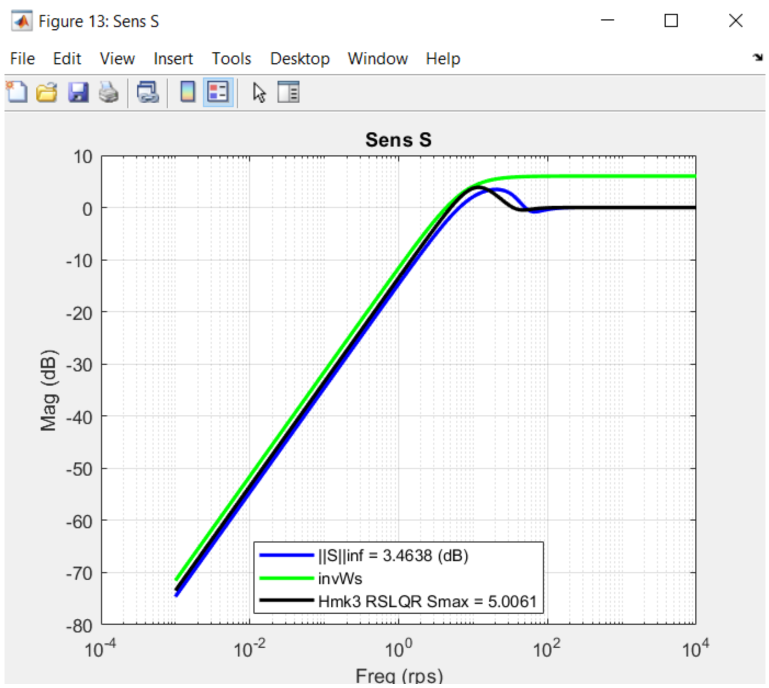
e) Plot the minimum singular value of the stability robustness matrix in dB vs frequency. Identify on the plot the minimum value of the stability robustness matrix (not in dB). Plot the RSLQR $\underline{\sigma}\left(I+L_u^{-1}\right)$ including the actuator in the plant model with the Hinf $\underline{\sigma}\left(I+L_u^{-1}\right)$ (both plant models should have the actuator in them).



f) Compute the singular value gain and phase margins for the system (at the plant input) and compare them with the RSLQR margins (both plant models should have the actuator in them).

```
Homework 4 Hinf SF
Singular value margins
Min Singular value I+Lu =    0.95167
Min Singular value I+invLu = 0.71364
Singular value gain margins = [-10.8619 dB,26.3152 dB ]
Singular value phase margins = [ +/-56.8273 deg ]

Homework 3 RSLQR
Singular value margins
Min Singular value I+Lu =    0.29453
Min Singular value I+invLu = 0.32919
Singular value gain margins = [-3.468 dB,3.0305 dB ]
Singular value phase margins = [ +/-18.9475 deg ]
```

g) Compute the sensitivity function e/r = S (at the output).). Plot the RSLQR $e/r = S$ including the actuator in the plant model with the Hinf $e/r = S$ (both plant models should have the actuator in them). Plot your Weighting filter WS with these two frequency responses.



h) Compute the complementary sensitivity y/r = T (at the output). Plot the RSLQR $A_z / A_{zc} = T$ including the actuator in the plant model with the Hinf $A_z / A_{zc} = T$ (both plant models should have the actuator in them). Plot your Weighting filter WT with these two frequency responses.