1. Use the same aircraft plant data from homework 3, build a longitudinal dynamics model that has states $x = \begin{bmatrix} A_z & q & \delta_e & \dot{\delta}_e \end{bmatrix}^T$.

$$\begin{bmatrix} \dot{A}_z \\ \dot{q} \\ \dot{\delta}_e \\ \ddot{\delta}_e \end{bmatrix} = \begin{bmatrix} Z_\alpha/V & Z_\alpha & 0 & Z_\delta \\ M_\alpha/Z_\alpha & 0 & \left(M_\delta - M_\alpha Z_\delta/Z_\alpha\right) & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} A_z \\ q \\ \delta_e \\ \dot{\delta}_e \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \omega_n^2 \end{bmatrix} \delta_c$$

Using this longitudinal dynamics model, design a RSLQR to track a constant acceleration command A_{zc}. The control is $u = -K_{x_{lqr}} \begin{bmatrix} e_I & A_z & q & \delta_e & \dot{\delta}_e \end{bmatrix}^T$. Match the <u>acceleration rise time</u> from homework 3 RSLQR.

```
Ap =[Za_V   Za   0. Zd;
     Ma_Za   0.  M23  0;
     0.     0.   0.  1.;
     0.     0.   -w_act*w_act -2*z_act*w_act]

Bp =[0.; 0.; 0.; w_act*w_act]

Cp=[1. 0. 0. 0.;
    eye(4)]
Dp=[ 0*Cp*Bp]
```

```
Ap =

  -1.3046e+00  -1.1569e+03            0  -1.8995e+02
  -4.1241e-02            0  -1.1266e+02            0
            0            0            0   1.0000e+00
            0            0  -4.7769e+03  -9.7729e+01
```

```
Bp =

            0
            0
            0
   4.7769e+03
```

```
Cp =

     1     0     0     0
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
```

```
Dp =

     0
     0
     0
     0
     0
```

1.1 List out your relevant RSLQR design matrices = Awiggle, Bwiggle, Q_lqr, R_lqr, Kx_lqr.

```
Aw =

           0   1.0000e+00            0            0            0
           0  -1.3046e+00  -1.1569e+03            0  -1.8995e+02
           0  -4.1241e-02            0  -1.1266e+02            0
           0            0            0            0   1.0000e+00
           0            0            0  -4.7769e+03  -9.7729e+01

Bw =

           0
           0
           0
           0
  4.7769e+03

ip =

    15

Q =

  2.5000e-06            0            0            0            0
           0            0            0            0            0
           0            0            0            0            0
           0            0            0            0            0
           0            0            0            0            0

R =

     1

Kx_lqr =

  1.5811e-03   1.1525e-03  -1.7641e-01   5.6755e-01   3.3010e-03
```

1.2 List the controller matrices $\left(A_c, B_{c_1}, B_{c_2}, C_c, D_{c_1}, D_{c_2}\right)$ implementing the 5-state RSLQR controller.

$$\dot{x}_c = A_c x_c + B_{c_1} y + B_{c_2} r$$
$$u = C_c x_c + D_{c_1} y + D_{c_2} r$$

```
Ac = 0.;
Bc1 = [1. 0. 0. 0. 0.];
Bc2 = -1;
Cc = -Kx_lqr(1);
Dc1 = [0. -Kx_lqr(2:3) 0. 0.];
Dc2 = 0.;
```

```
Ac =

     0


Bc1 =

     1     0     0     0     0


Bc2 =

    -1


Cc =

  -1.5811e-03


Dc1 =

              0   -1.1525e-03    1.7641e-01              0              0


Dc2 =

     0
```

```
GM =                    wc_GM =

   2.0899e+00              4.2878e+01


PM_deg =                wc_Pm =

   3.2926e+01              1.9548e+01

Classical Margins

ans =

  struct with fields:

     GainMargin: [3.3048e-01 2.0899e+00]
    GMFrequency: [3.3392e+00 4.2878e+01]
    PhaseMargin: 3.2926e+01
    PMFrequency: 1.9548e+01
    DelayMargin: 2.9398e-02
    DMFrequency: 1.9548e+01
         Stable: 1

SV Margins
RDu_nGM RDu_pGM RDu_Pha
0.69697 1.7692 0.43828
-3.1357 4.9557 25.1117
SRu_nGM SRu_pGM SRu_Pha
0.43351 1.5665 0.57435
-7.26 3.8986 32.908
```

1.3 List out the closed loop eigenvalues and eigenvectors (5-state model).

```
cl_EigVec =

  Columns 1 through 4

  -9.4013e-01 + 0.0000e+00i   9.9226e-01 + 0.0000e+00i   9.9226e-01 + 0.0000e+00i  -9.9075e-01 + 0.0000e+00i
   1.1668e-02 + 0.0000e+00i  -1.9169e-03 - 1.1438e-02i  -1.9169e-03 + 1.1438e-02i  -5.1033e-03 + 0.0000e+00i
   6.1098e-03 + 0.0000e+00i  -3.4697e-03 - 1.2413e-03i  -3.4697e-03 + 1.2413e-03i   3.1633e-05 + 0.0000e+00i
  -3.4016e-01 + 0.0000e+00i   9.3040e-02 - 7.5399e-02i   9.3040e-02 + 7.5399e-02i  -2.3118e-04 + 0.0000e+00i
   1.6886e-02 + 0.0000e+00i  -1.5860e-02 - 2.6091e-02i  -1.5860e-02 + 2.6091e-02i   1.3557e-01 + 0.0000e+00i

  Column 5

  -9.1650e-01 + 0.0000e+00i
  -6.6036e-04 + 0.0000e+00i
   3.2206e-04 + 0.0000e+00i
  -7.3786e-04 + 0.0000e+00i
   4.0003e-01 + 0.0000e+00i
```

```
cl_eigVal =

  -4.8864e+01 + 4.8879e+01i
  -4.8864e+01 - 4.8879e+01i
  -4.2914e+00 + 2.4199e+00i
  -4.2914e+00 - 2.4199e+00i
  -8.4907e+00 + 0.0000e+00i
```

2. Design a SPC output feedback controller to project out the actuator. Using the 5-state RSLQR state feedback controller from 1) above, design a SPC controller using $y = [e_I \quad A_z \quad q]^T$, to keep the dominant RSLQR eigenstructure. (This design includes the int-error state which will then be put in the controller state space model). The control law is $u = -\bar{K}[e_I \quad A_z \quad q]^T$.

2.1 List out your projective control design matrices $C, X_r, \bar{K}$.

```
C =

     1     0     0     0     0
     0     1     0     0     0
     0     0     1     0     0
```

Xr =

```
ans =

   1.6971e-01 + 9.8834e-02i   1.6971e-01 - 9.8834e-02i   1.1655e-01 + 0.0000e+00i
  -9.8052e-01 + 0.0000e+00i  -9.8052e-01 + 0.0000e+00i  -9.9316e-01 + 0.0000e+00i
  -2.2714e-03 + 2.0908e-03i  -2.2714e-03 - 2.0908e-03i  -6.3600e-03 + 0.0000e+00i
   3.1857e-04 + 1.3072e-04i   3.1857e-04 - 1.3072e-04i  -1.1750e-04 + 0.0000e+00i
  -1.7029e-03 + 2.3646e-04i  -1.7029e-03 - 2.3646e-04i   1.0013e-03 + 0.0000e+00i
```

Corresponding to dominant eigen values, which are:

```
eigVal =

  Columns 1 through 4

  -4.8864e+01 + 4.8879e+01i   0.0000e+00 + 0.0000e+00i   0.0000e+00 + 0.0000e+00i   0.0000e+00 + 0.0000e+00i
   0.0000e+00 + 0.0000e+00i  -4.8864e+01 - 4.8879e+01i   0.0000e+00 + 0.0000e+00i   0.0000e+00 + 0.0000e+00i
   0.0000e+00 + 0.0000e+00i   0.0000e+00 + 0.0000e+00i  -4.3144e+00 + 2.5126e+00i   0.0000e+00 + 0.0000e+00i
   0.0000e+00 + 0.0000e+00i   0.0000e+00 + 0.0000e+00i   0.0000e+00 + 0.0000e+00i  -4.3144e+00 - 2.5126e+00i
   0.0000e+00 + 0.0000e+00i   0.0000e+00 + 0.0000e+00i   0.0000e+00 + 0.0000e+00i   0.0000e+00 + 0.0000e+00i

  Column 5

   0.0000e+00 + 0.0000e+00i
   0.0000e+00 + 0.0000e+00i
   0.0000e+00 + 0.0000e+00i
   0.0000e+00 + 0.0000e+00i
  -8.5215e+00 + 0.0000e+00i
```

$\bar{K}$ = Ky

```
Ky =

   9.2703e-04 - 4.3368e-19i   7.0174e-04 - 1.0842e-19i  -1.0813e-01 + 1.3878e-17i
```

2.2 List the controller matrices $\left(A_c, B_{c_1}, B_{c_2}, C_c, D_{c_1}, D_{c_2}\right)$ implementing the output feedback SPC controller.

$$\dot{x}_c = A_c x_c + B_{c_1} y + B_{c_2} r$$
$$u = C_c x_c + D_{c_1} y + D_{c_2} r$$

```
Ac = 0.;
Bc1 = [1. 0.];
Bc2 = -1;
Cc = -Ky(1);
Dc1 = [-Ky(2:3)];
Dc2 = 0.;
```

```
Ac =

     0


Bc1 =

     1     0


Bc2 =

    -1


Cc =

  -9.2703e-04 + 4.3368e-19i


Dc1 =

  -7.0174e-04 + 1.0842e-19i   1.0813e-01 - 1.3878e-17i


Dc2 =

     0
```

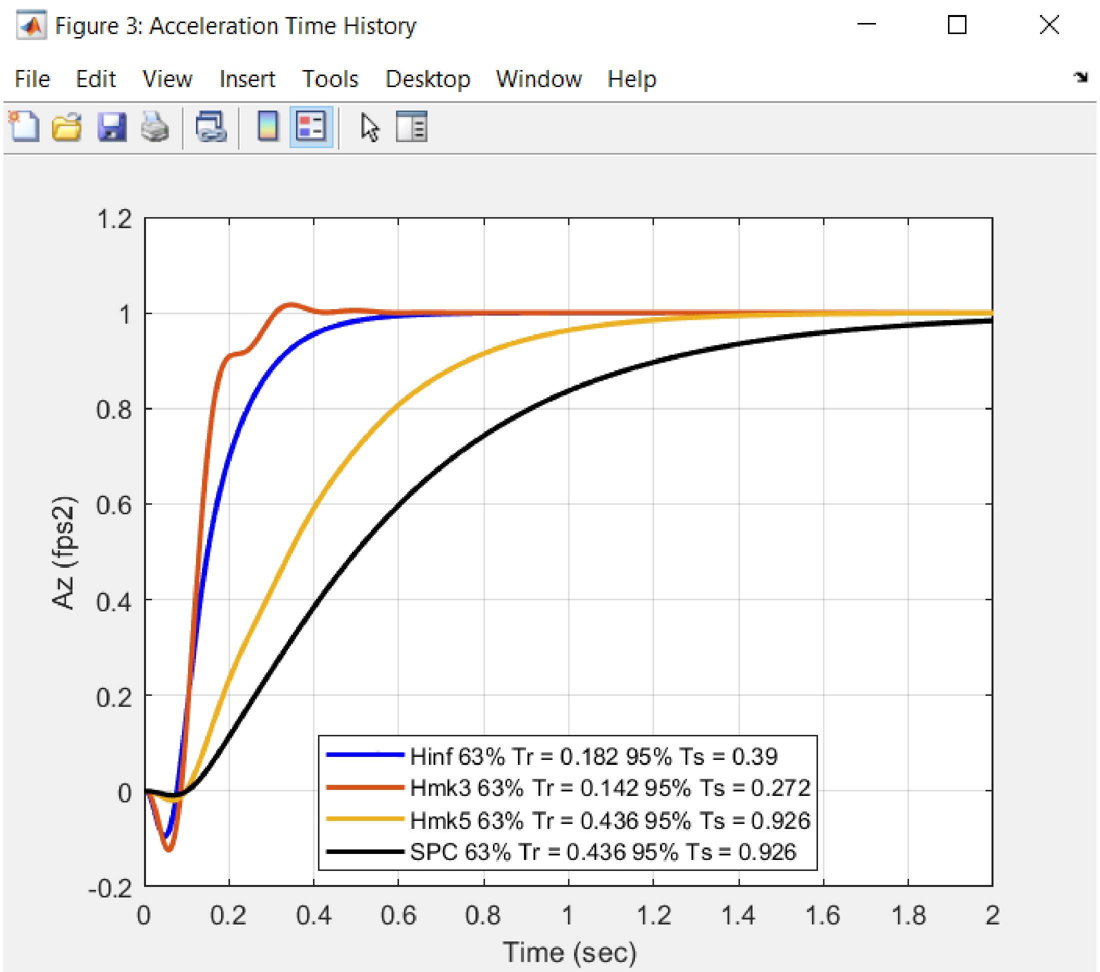
2.3 List out the closed loop eigenvalues and eigenvectors (5-state model still includes actuator) using the output feedback SPC controller.

```
eigval_Acl =

  Columns 1 through 4

  -4.0941e+01 - 3.2264e+01i   0.0000e+00 + 0.0000e+00i   0.0000e+00 + 0.0000e+00i   0.0000e+00 + 0.0000e+00i
   0.0000e+00 + 0.0000e+00i  -4.0941e+01 + 3.2264e+01i   0.0000e+00 + 0.0000e+00i   0.0000e+00 + 0.0000e+00i
   0.0000e+00 + 0.0000e+00i   0.0000e+00 + 0.0000e+00i  -4.3144e+00 - 2.5126e+00i   0.0000e+00 + 0.0000e+00i
   0.0000e+00 + 0.0000e+00i   0.0000e+00 + 0.0000e+00i   0.0000e+00 + 0.0000e+00i  -4.3144e+00 + 2.5126e+00i
   0.0000e+00 + 0.0000e+00i   0.0000e+00 + 0.0000e+00i   0.0000e+00 + 0.0000e+00i   0.0000e+00 + 0.0000e+00i

  Column 5

   0.0000e+00 + 0.0000e+00i
   0.0000e+00 + 0.0000e+00i
   0.0000e+00 + 0.0000e+00i
   0.0000e+00 + 0.0000e+00i
  -8.5215e+00 + 2.9929e-15i

eigvec_Acl =

  Columns 1 through 4

  -1.4523e-02 + 1.1445e-02i  -1.4523e-02 - 1.1445e-02i  -1.6971e-01 + 9.8834e-02i  -1.6971e-01 - 9.8834e-02i
   9.6382e-01 + 0.0000e+00i   9.6382e-01 + 0.0000e+00i   9.8052e-01 + 0.0000e+00i   9.8052e-01 + 0.0000e+00i
  -6.3402e-03 + 8.0832e-03i  -6.3402e-03 - 8.0832e-03i   2.2714e-03 + 2.0908e-03i   2.2714e-03 - 2.0908e-03i
  -4.9716e-03 + 1.1217e-03i  -4.9716e-03 - 1.1217e-03i  -3.1857e-04 + 1.3072e-04i  -3.1857e-04 - 1.3072e-04i
   2.3974e-01 + 1.1448e-01i   2.3974e-01 - 1.1448e-01i   1.7029e-03 + 2.3646e-04i   1.7029e-03 - 2.3646e-04i

  Column 5

  -1.1655e-01 - 5.1396e-17i
   9.9316e-01 + 0.0000e+00i
   6.3600e-03 - 3.2121e-18i
   1.1750e-04 - 2.4326e-19i
  -1.0013e-03 - 1.8963e-18i
```

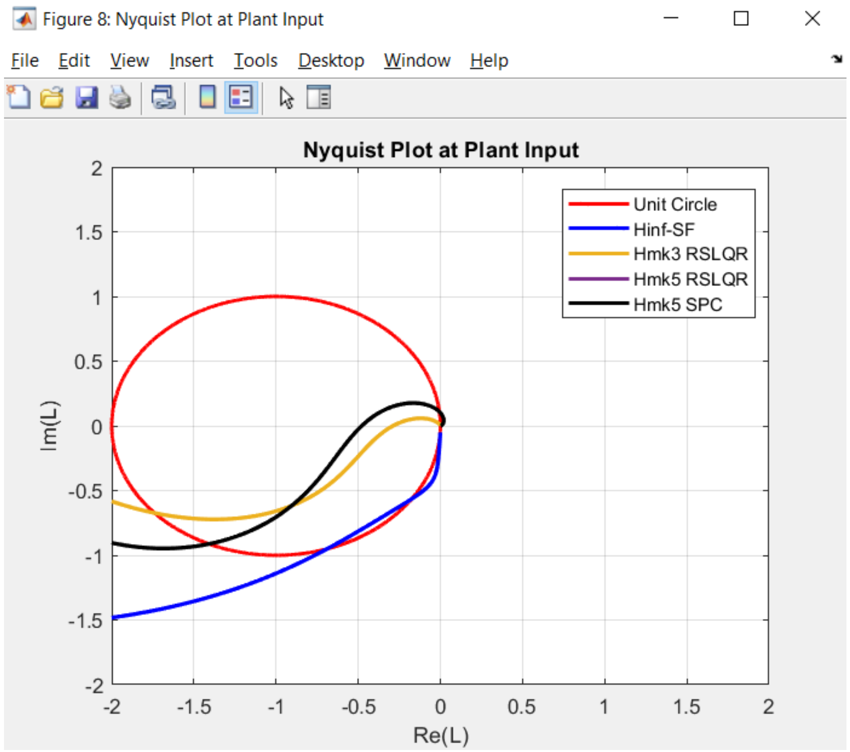
Compare the 5-state RSLQR, SPC controller, homework 3 3-state RSLQR controller, and Homework 4 Hinf controller.

3).

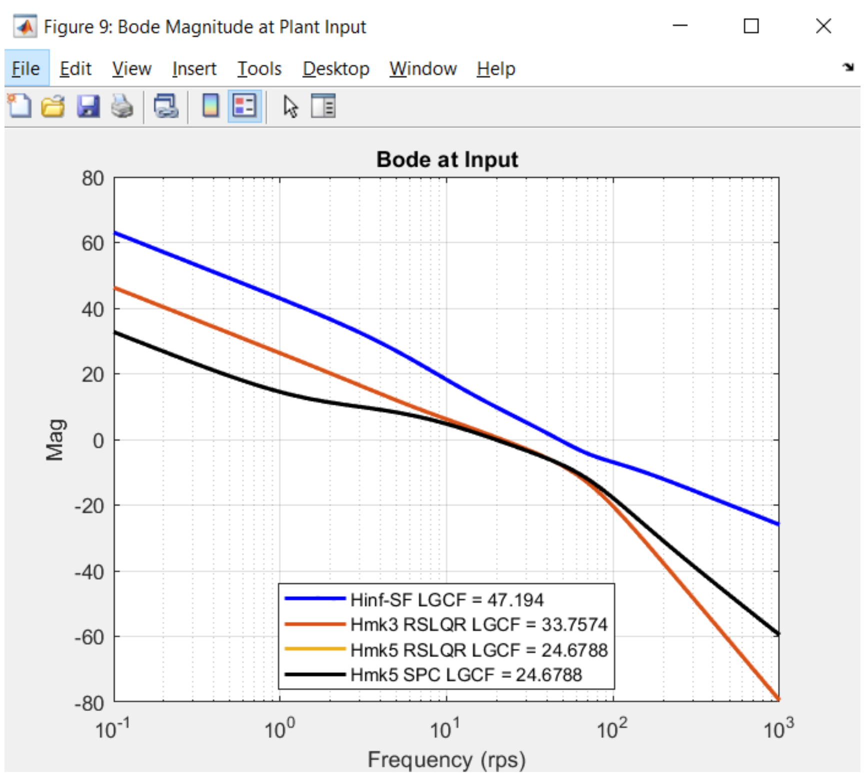
a) Simulate the closed loop system to a unit step Az command. Plot all of the acceleration response from each of the controllers. Compute the rise time and settling time for each. Label this on the plot.



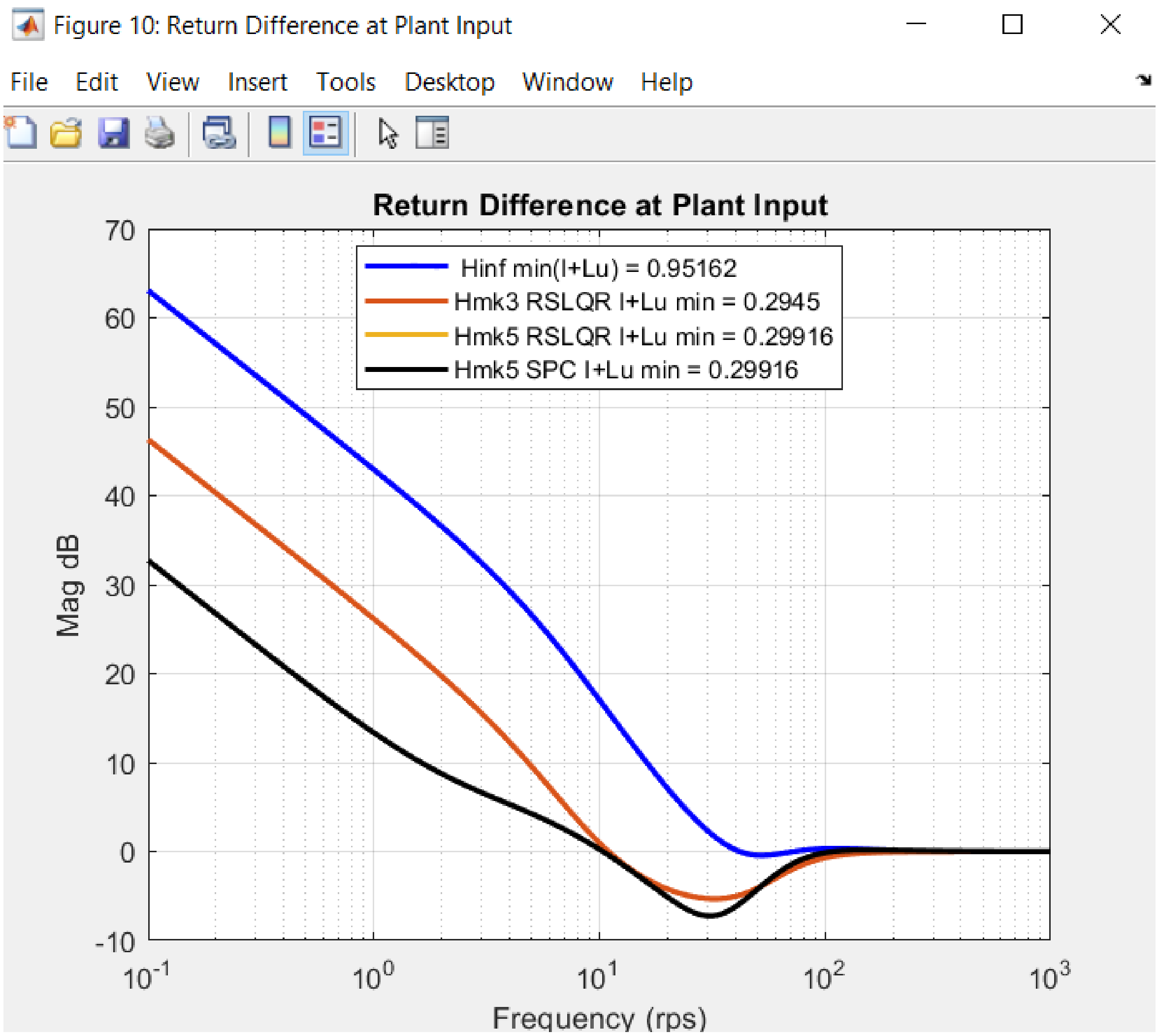
b) Plot a Nyquist plot comparing the frequency response for each of the controllers



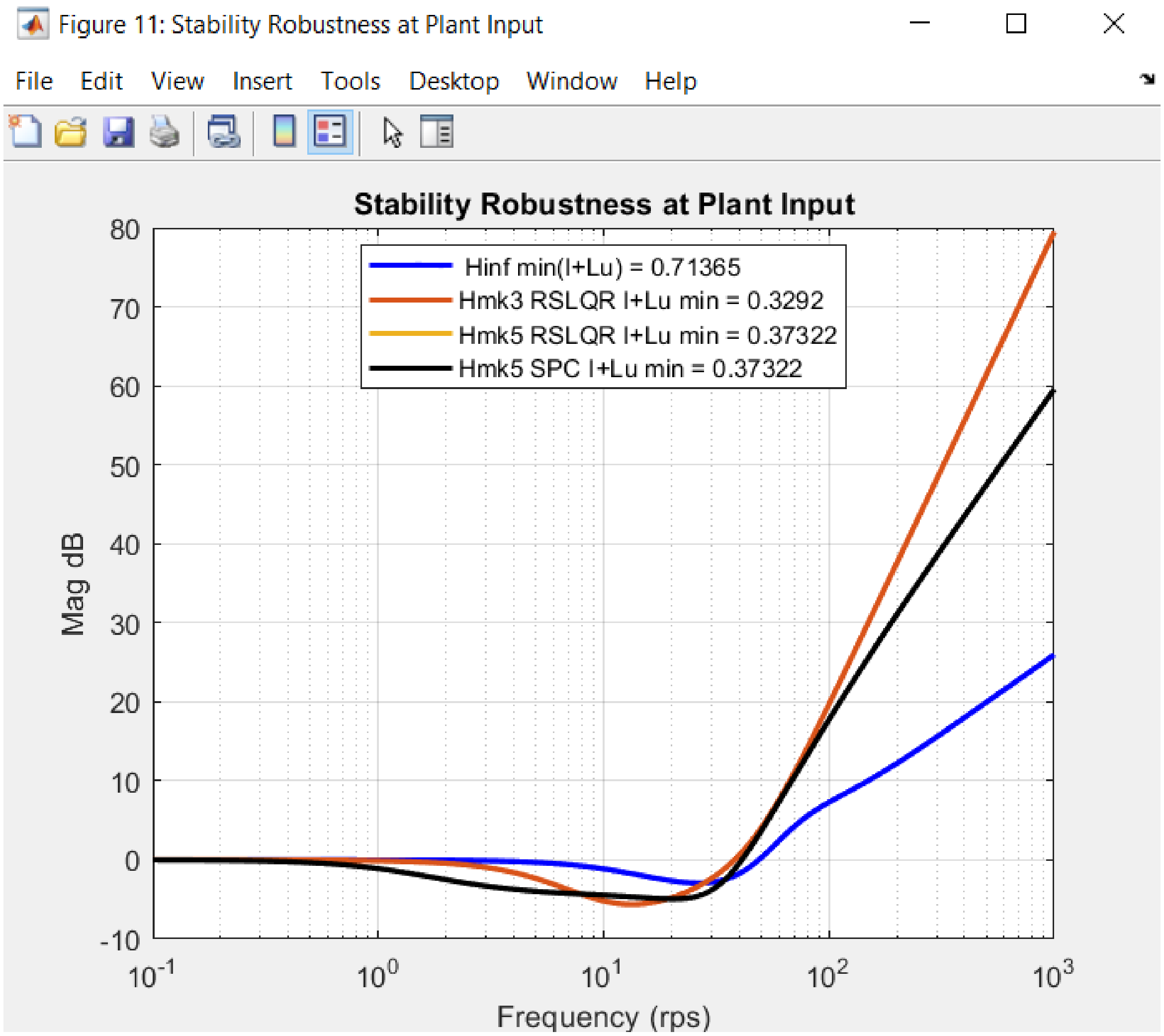
c) Plot a Bode gain plot comparing the frequency response for each of the controllers



d) Plot the minimum singular value of the return difference matrix in dB vs frequency comparing each controller. Identify on the plot the minimum value of the return difference matrix (not in dB). Plot the RSLQR $\underline{\sigma}(I+L_u)$ including the actuator in the plant model with the Hinf $\underline{\sigma}(I+L_u)$ (both plant models should have the actuator in them).



e) Plot the minimum singular value of the stability robustness matrix in dB vs frequency comparing each controller. Identify on the plot the minimum value of the stability robustness matrix (not in dB). Plot the RSLQR $\underline{\sigma}\left(I+L_u^{-1}\right)$ including the actuator in the plant model with the Hinf $\underline{\sigma}\left(I+L_u^{-1}\right)$ (both plant models should have the actuator in them).



f) Compute the singular value gain and phase margins for the system (at the plant input) and compare them with the other controllers

```
Homework 3 RSLQR
Singular value margins
Min Singular value I+Lu =    0.2945
Min Singular value I+invLu = 0.3292
Singular value gain margins = [-3.4681 dB,3.0301 dB ]
Singular value phase margins = [ +/-18.9479 deg ]
```

```
Homework 4 Hinf SF
Singular value margins
Min Singular value I+Lu =    0.95167
Min Singular value I+invLu = 0.71364
Singular value gain margins = [-10.8619 dB,26.3152 dB ]
Singular value phase margins = [ +/-56.8273 deg ]
```

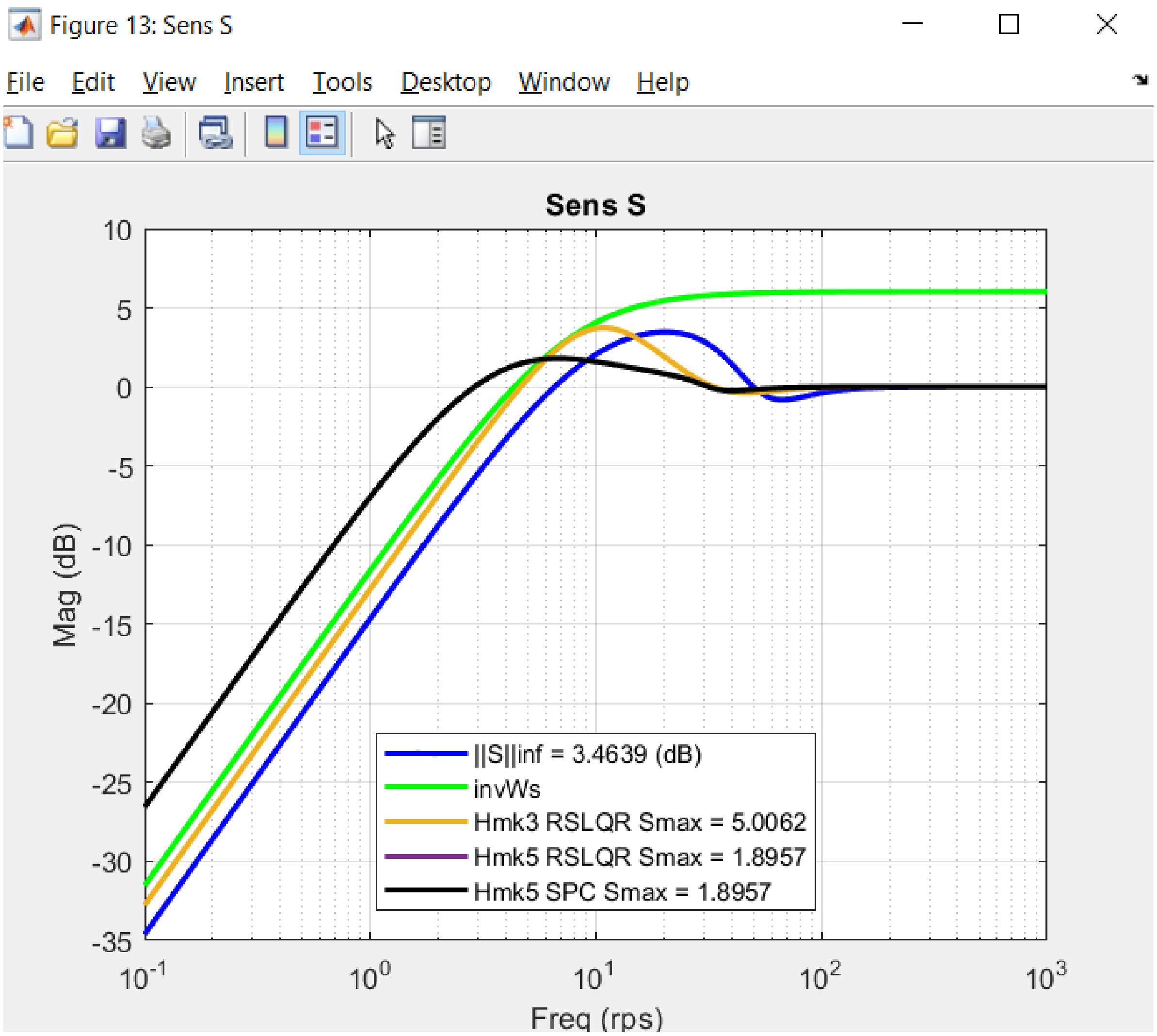
Singular Value Margins: HW5 RSLQR (5 States)

```
SV Margins
RDu_nGM RDu_pGM RDu_Pha
0.69697 1.7692 0.43828
-3.1357 4.9557 25.1117
SRu_nGM SRu_pGM SRu_Pha
0.43351 1.5665 0.57435
-7.26 3.8986 32.908
```

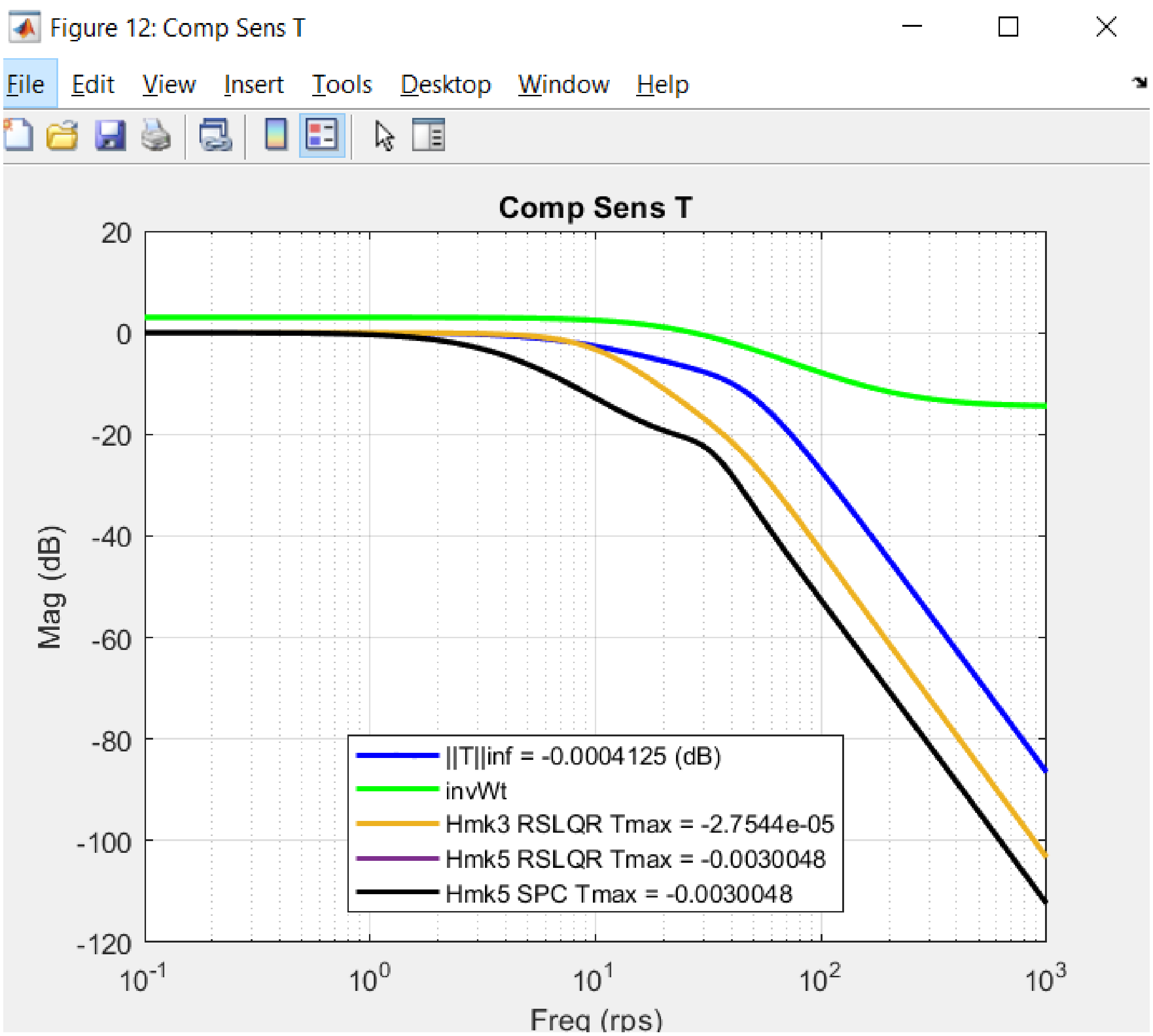
Singular Value Margins: HW5 SPC

```
SV Margins
RDu_nGM RDu_pGM RDu_Pha
0.69886 1.7572 0.43431
-3.1122 4.8963 24.8842
SRu_nGM SRu_pGM SRu_Pha
0.43798 1.562 0.56969
-7.1708 3.8737 32.6407
```

g) Compute the SISO sensitivity function e/r = S (at the output) for the commanded variable comparing each controller. Identify on the plot the $\|S\|_\infty$ for each controller. Plot the RSLQR $e/r = S$ including the actuator in the plant model with the Hinf $e/r = S$ (both plant models should have the actuator in them).

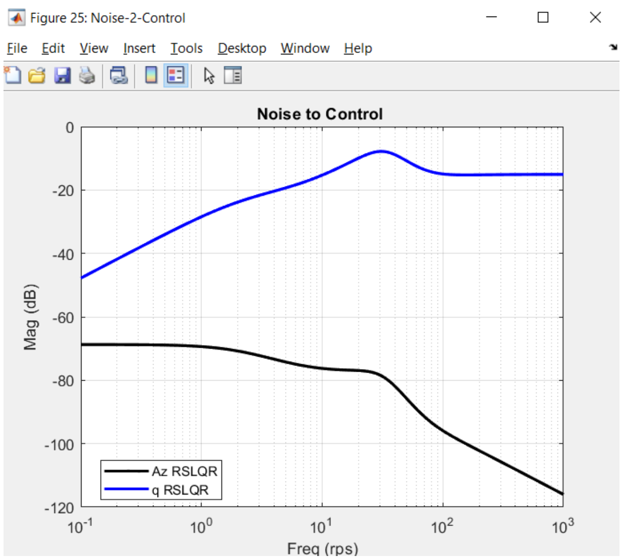


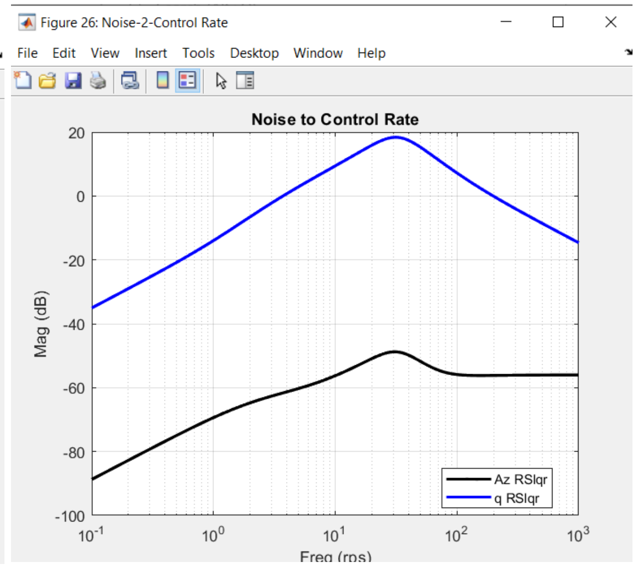
h) Compute the SISO complementary sensitivity y/r = T (at the output) for the commanded variable comparing each controller. Identify on the plot the $\|T\|_\infty$ for each controller. Plot the RSLQR $A_z / A_{zc} = T$ including the actuator in the plant model with the Hinf $A_z / A_{zc} = T$ (both plant models should have the actuator in them). Plot your Weighting filter WT with these two frequency responses.



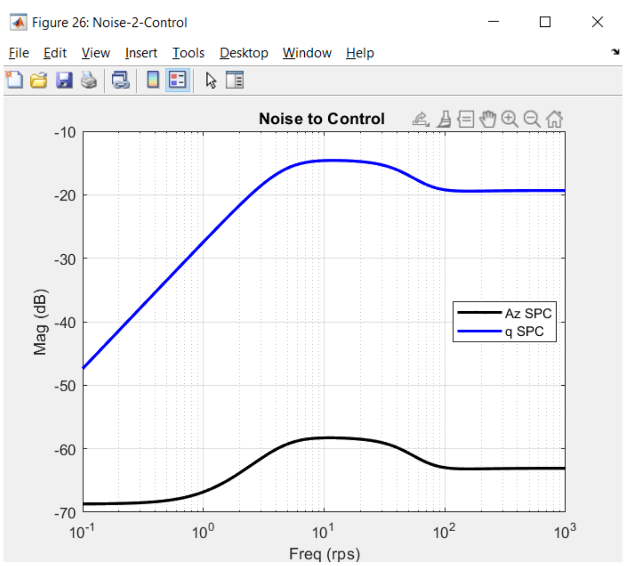
i) Plot the SISO Bode noise-to-control and noise-to-control rate frequency responses for each controller.

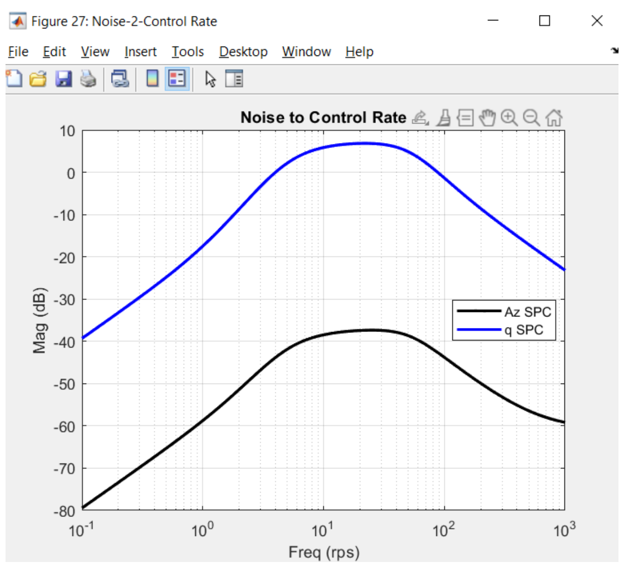
HW5_RSLQR (5 States)



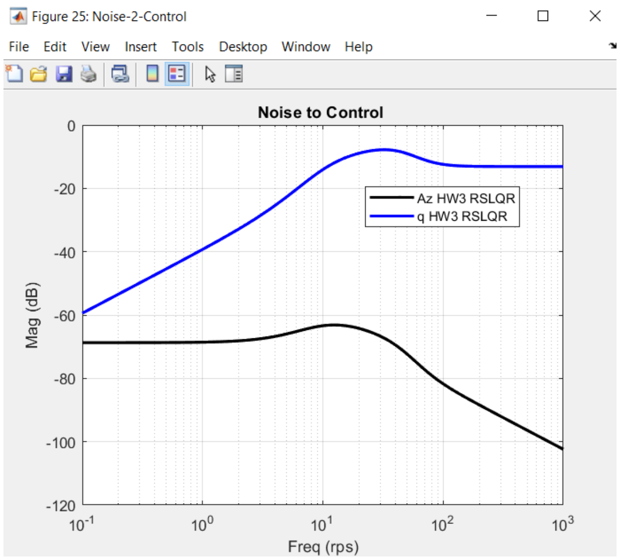


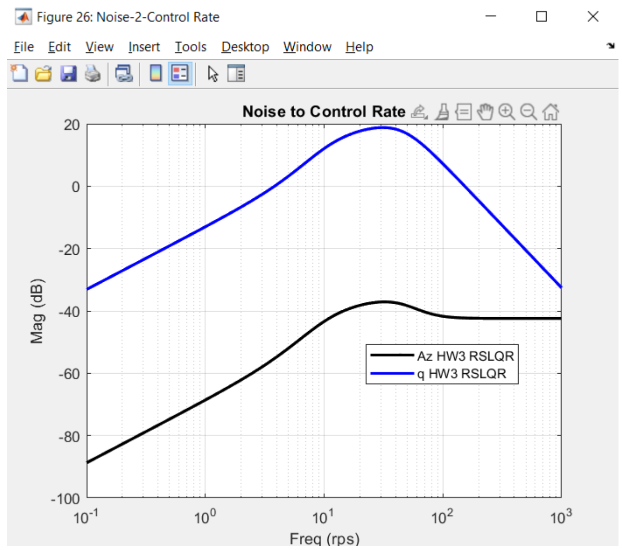
HW5 SPC





HW3_RSLQR (3 States)





HW4 HINF