Advanced Robust Control | amithr3

Robust Servo LQR

1. Using the aircraft pitch-axis plant data from Example 5.2 (Eq. 5.58 page 119) without an actuator, DESIGN a RSLQR to command Az using a state feedback controller.

```
Ap =[Za_V   1.  Zd_V  0.;
     Ma     0.  Md   0;
     0.     0.  0.   1.;
     0.     0.  -w_act*w_act -2*z_act*w_act]

Bp =[0.; 0.; 0.; w_act*w_act]

Cp=[Za 0. Zd 0.;
    eye(4)]

Dp=[ 0*Cp*Bp]
```

```
%Wiggle Model
Aw = [0. Za   0.;
      0. Za_V 1.;
      0. Ma   0.]
Bw = [Zd;
      Zd_V;
      Md]

qq=logspace(-6,0,100);
```

```
Ap =

  -1.3046e+00   1.0000e+00  -2.1420e-01            0
   4.7711e+01            0  -1.0483e+02            0
            0            0           0   1.0000e+00
            0            0  -4.7769e+03  -9.7729e+01


Bp =

            0
            0
            0
   4.7769e+03

Cp =

  -1.1569e+03            0  -1.8995e+02            0
   1.0000e+00            0           0            0
            0   1.0000e+00           0            0
            0            0   1.0000e+00            0
            0            0           0   1.0000e+00

Dp =

     0
     0
     0
     0
     0
```

```
Aw =

            0  -1.1569e+03            0
            0  -1.3046e+00   1.0000e+00
            0   4.7711e+01            0
```

```
Bw =

  -1.8995e+02
  -2.1420e-01
  -1.0483e+02
```

```
ip =

    30
```

```
Q =

   5.7224e-05            0            0
            0            0            0
            0            0            0
```

Eigen Values of Closed Loop System

```
  -3.1297e+01 + 3.3953e+01i
  -3.1297e+01 - 3.3953e+01i
  -1.9517e+01 + 0.0000e+00i
  -8.4613e+00 + 6.4480e+00i
  -8.4613e+00 - 6.4480e+00i
```

```
cl_EigVec =

  -8.6318e-04 + 5.2392e-04i  -8.6318e-04 - 5.2392e-04i  -1.4435e-02 + 0.0000e+00i  -5.9798e-03 + 6.1886e-03i  -5.9798e-03 - 6.1886e-03i
   4.9751e-03 - 4.8411e-02i   4.9751e-03 + 4.8411e-02i   2.7236e-01 + 0.0000e+00i   1.2269e-03 - 8.3708e-02i   1.2269e-03 + 8.3708e-02i
  -1.4587e-02 - 1.5826e-02i  -1.4587e-02 + 1.5826e-02i   4.4138e-02 + 0.0000e+00i  -7.7714e-03 - 4.0153e-03i  -7.7714e-03 + 4.0153e-03i
   9.9388e-01 + 0.0000e+00i   9.9388e-01 + 0.0000e+00i  -8.6147e-01 + 0.0000e+00i   9.1647e-02 - 1.6135e-02i   9.1647e-02 + 1.6135e-02i
  -1.7110e-02 - 9.5248e-02i  -1.7110e-02 + 9.5248e-02i  -4.2607e-01 + 0.0000e+00i  -9.9206e-01 + 0.0000e+00i  -9.9206e-01 + 0.0000e+00i
```

```
Classical Margins

ans =

  struct with fields:

     GainMargin: [3.5217e-01 3.4661e+00]
    GMFrequency: [6.9773e+00 6.0700e+01]
    PhaseMargin: 3.4716e+01
    PMFrequency: 2.1433e+01
    DelayMargin: 2.8269e-02
    DMFrequency: 2.1433e+01
         Stable: 1
```

```
SV Margins
RDu_nGM RDu_pGM RDu_Pha
0.6484 2.1846 0.54913
-3.7631 6.7876 31.463
SRu_nGM SRu_pGM SRu_Pha
0.47596 1.524 0.53023
-6.4487 3.6599 30.3801
```

LQR Charts

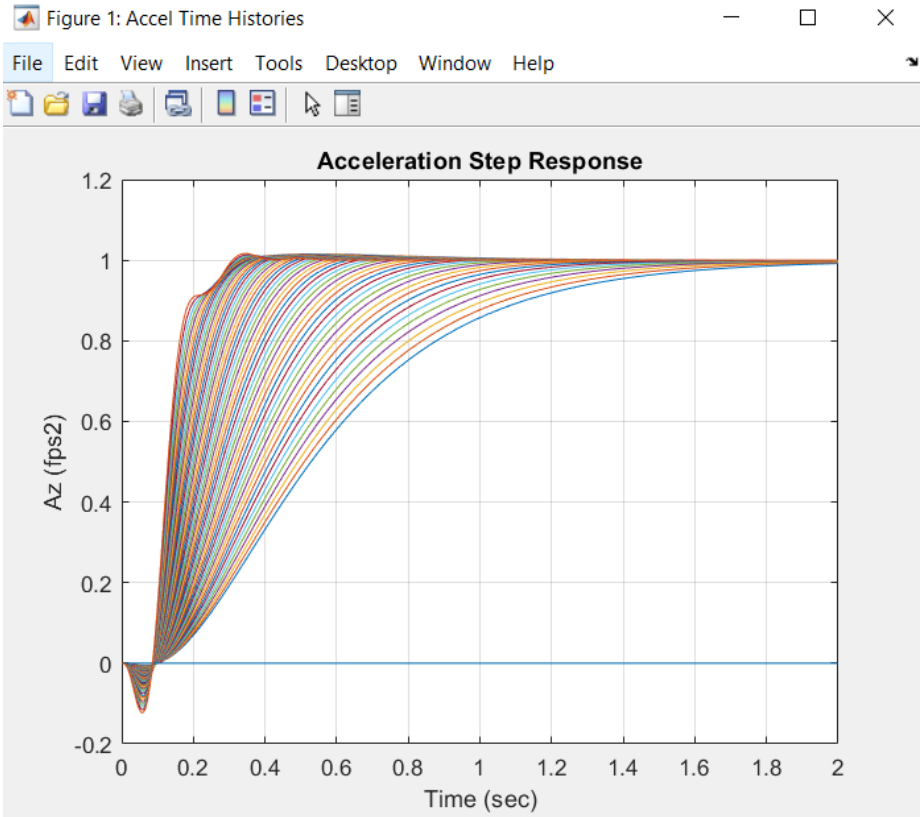


Figure 1: Accel Time Histories
File Edit View Insert Tools Desktop Window Help
Acceleration Step Response
Az (fps2)
Time (sec)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2

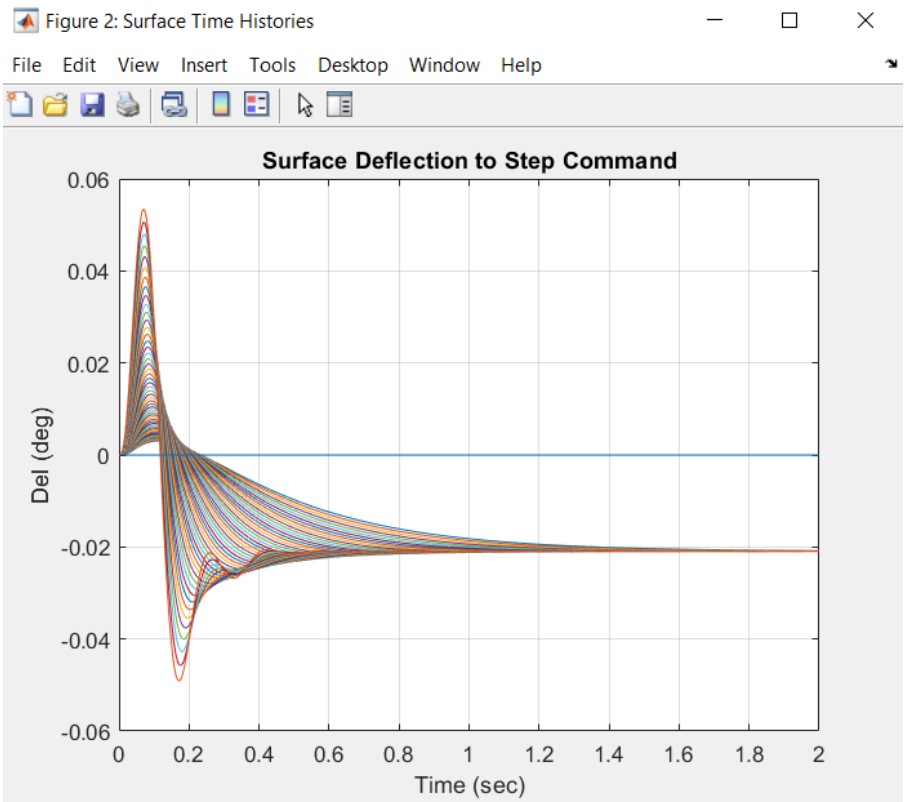


Figure 2: Surface Time Histories
File Edit View Insert Tools Desktop Window Help
Surface Deflection to Step Command
Del (deg)
Time (sec)
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2

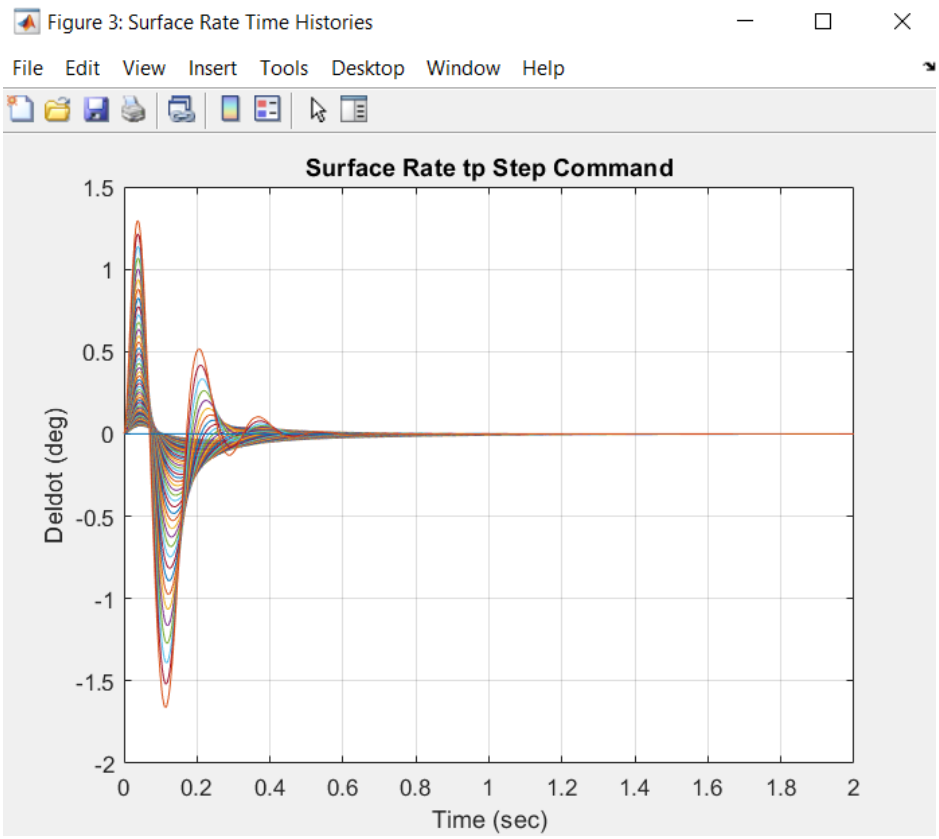


Figure 3: Surface Rate Time Histories
File Edit View Insert Tools Desktop Window Help
Surface Rate tp Step Command
Deldot (deg)
Time (sec)
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2

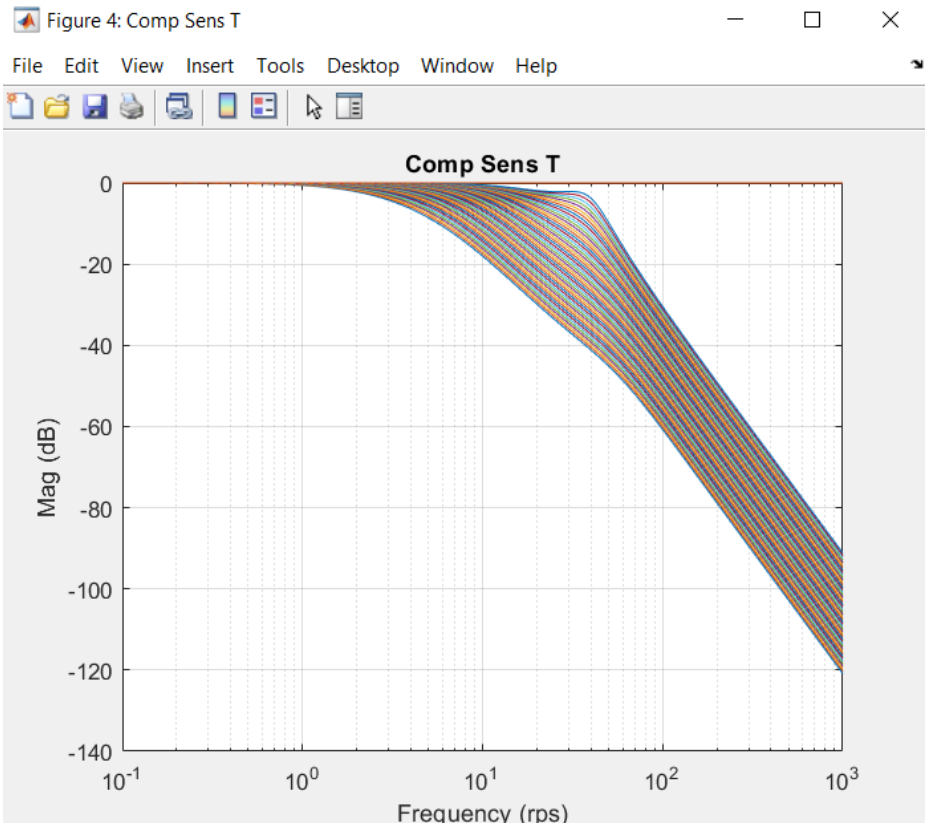


Figure 4: Comp Sens T
File Edit View Insert Tools Desktop Window Help
Comp Sens T
Mag (dB)
Frequency (rps)
0
-20
-40
-60
-80
-100
-120
-140
10^-1 10^0 10^1 10^2 10^3

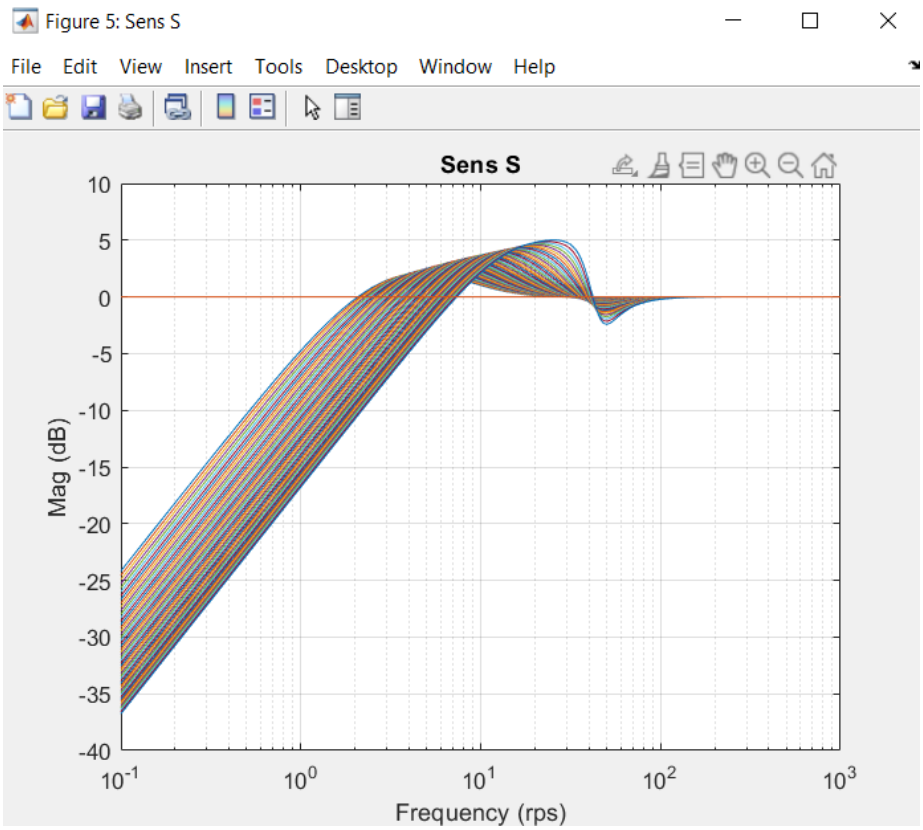


Figure 5: Sens S
File Edit View Insert Tools Desktop Window Help
Sens S
Mag (dB)
Frequency (rps)

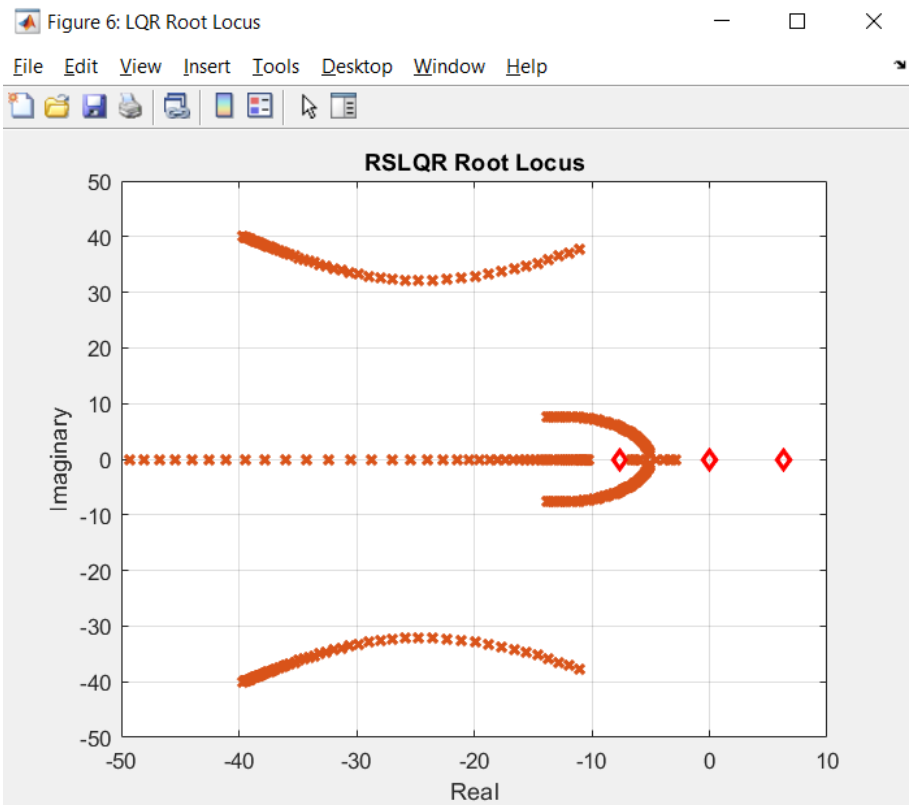


Figure 6: LQR Root Locus
File Edit View Insert Tools Desktop Window Help
RSLQR Root Locus
Imaginary
Real

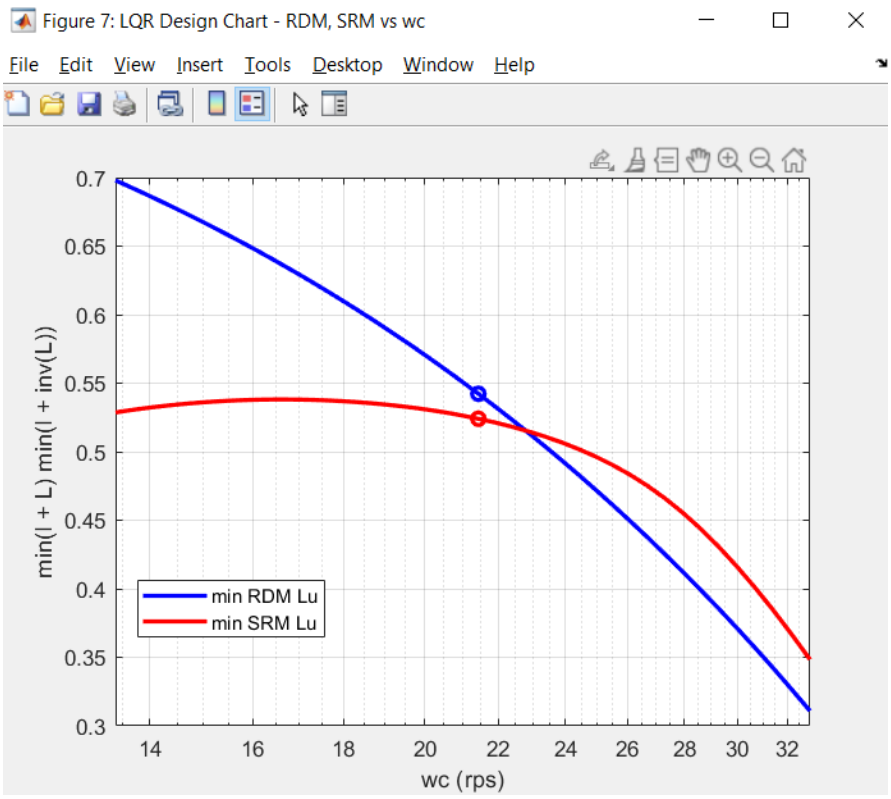


Figure 7: LQR Design Chart - RDM, SRM vs wc
File Edit View Insert Tools Desktop Window Help
min(I + L) min(I + inv(L))
wc (rps)
min RDM Lu
min SRM Lu
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
14
16
18
20
22
24
26
28
30
32

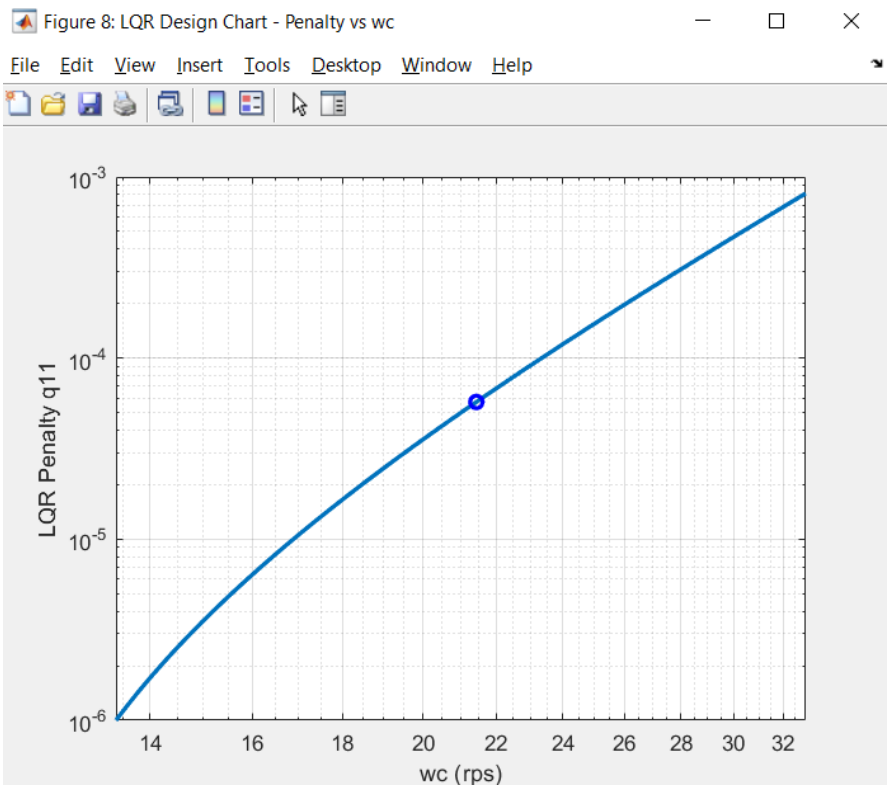


Figure 8: LQR Design Chart - Penalty vs wc
File Edit View Insert Tools Desktop Window Help
LQR Penalty q11
wc (rps)
10^-3
10^-4
10^-5
10^-6
14
16
18
20
22
24
26
28
30
32

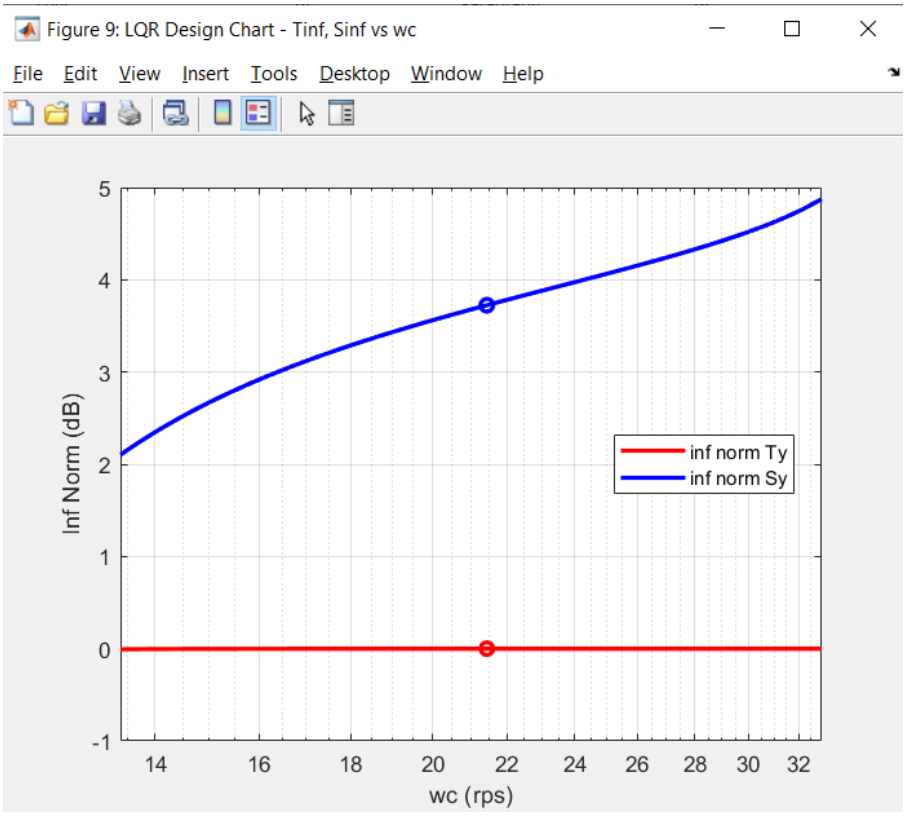


Figure 9: LQR Design Chart - Tinf, Sinf vs wc
File Edit View Insert Tools Desktop Window Help
Inf Norm (dB)
wc (rps)
inf norm Ty
inf norm Sy
5
4
3
2
1
0
-1
14
16
18
20
22
24
26
28
30
32

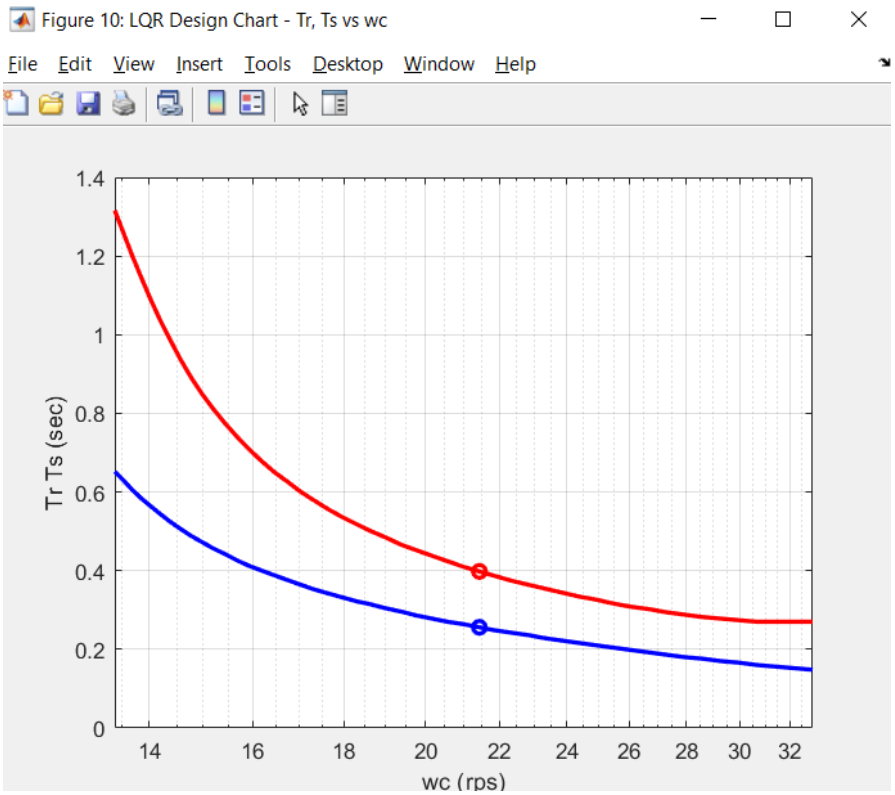


Figure 10: LQR Design Chart - Tr, Ts vs wc
File Edit View Insert Tools Desktop Window Help
Tr Ts (sec)
wc (rps)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
14
16
18
20
22
24
26
28
30
32

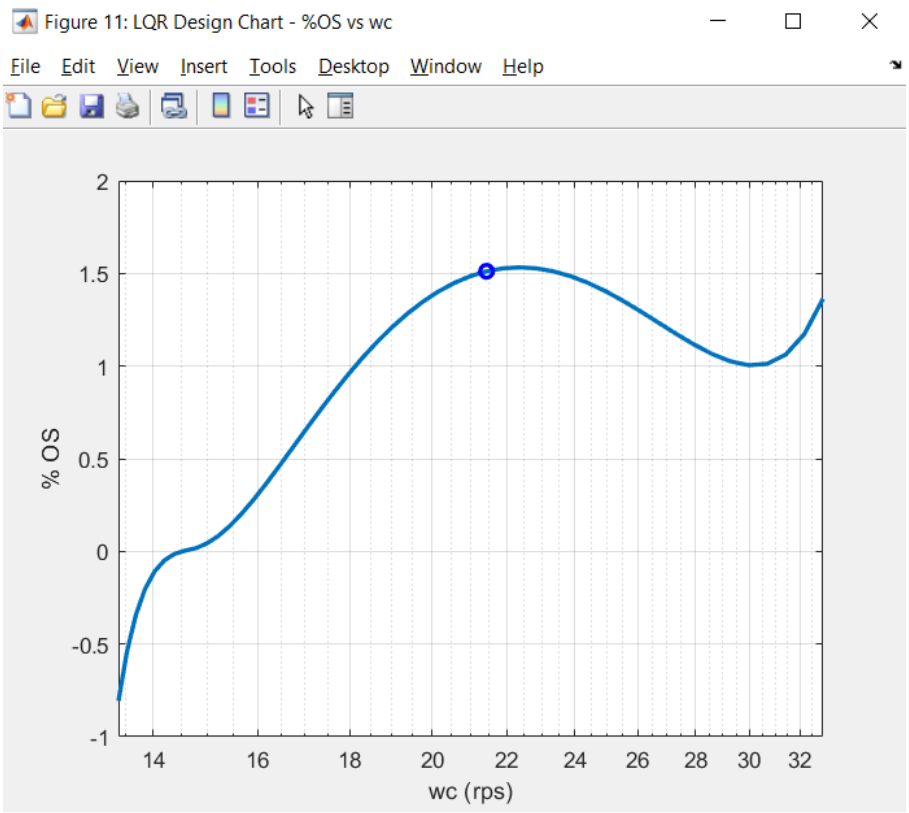


Figure 11: LQR Design Chart - %OS vs wc
File Edit View Insert Tools Desktop Window Help
% OS
wc (rps)
2
1.5
1
0.5
0
-0.5
-1
14 16 18 20 22 24 26 28 30 32

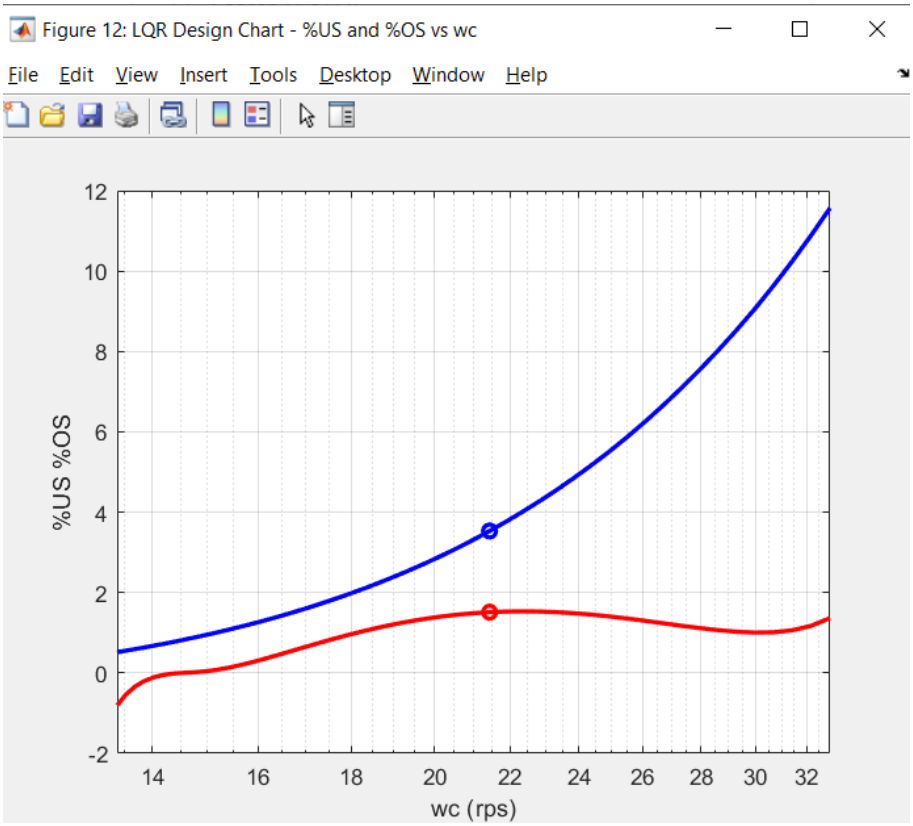


Figure 12: LQR Design Chart - %US and %OS vs wc
File Edit View Insert Tools Desktop Window Help
%US %OS
wc (rps)
12
10
8
6
4
2
0
-2
14 16 18 20 22 24 26 28 30 32

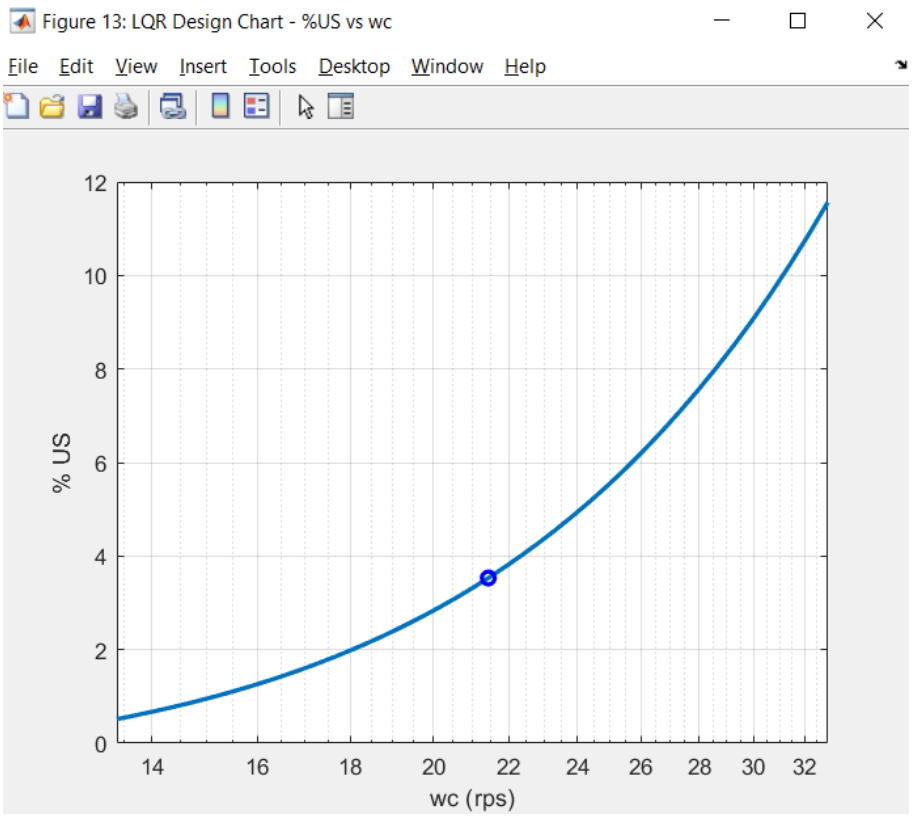


Figure 13: LQR Design Chart - %US vs wc
File Edit View Insert Tools Desktop Window Help
% US
wc (rps)
0 2 4 6 8 10 12
14 16 18 20 22 24 26 28 30 32

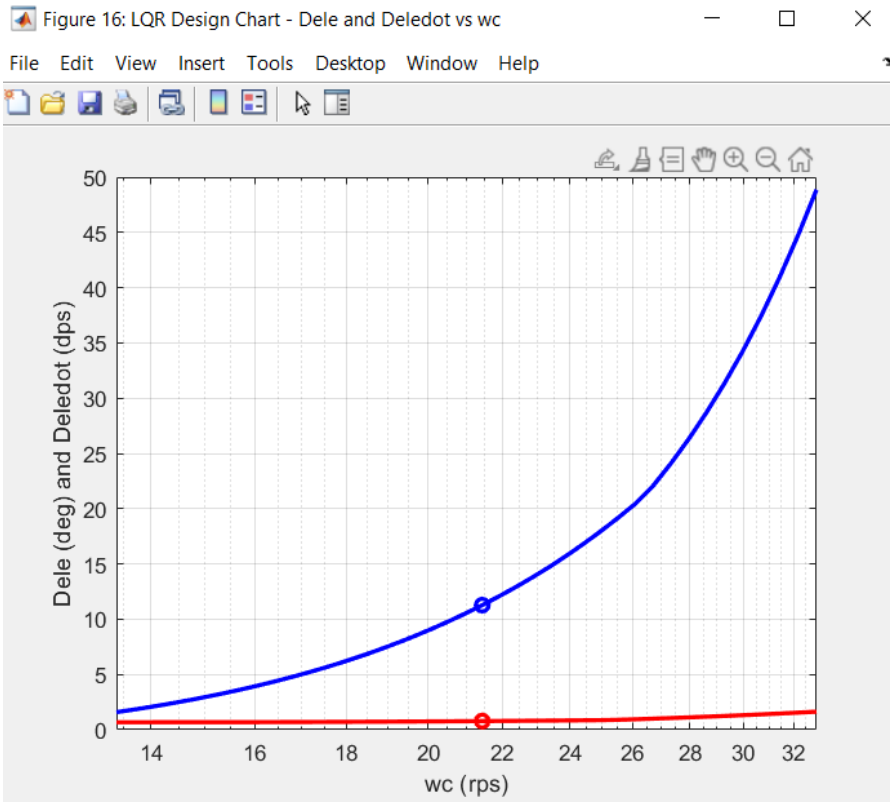


Figure 16: LQR Design Chart - Dele and Deledot vs wc
File Edit View Insert Tools Desktop Window Help
Dele (deg) and Deledot (dps)
wc (rps)
0 5 10 15 20 25 30 35 40 45 50
14 16 18 20 22 24 26 28 30 32

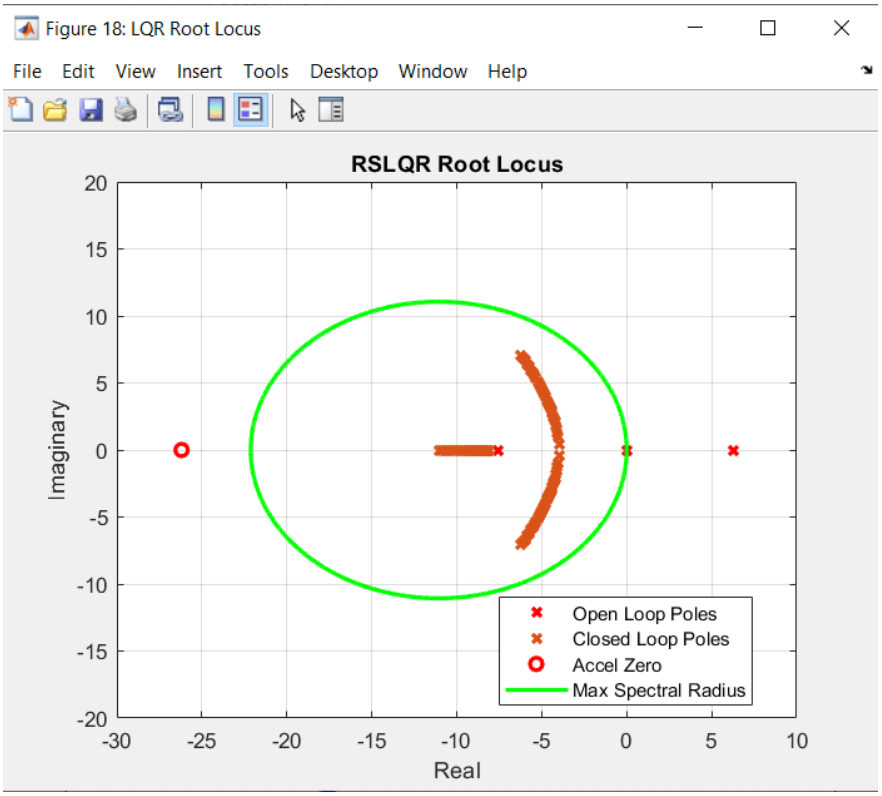


Figure 18: LQR Root Locus
File Edit View Insert Tools Desktop Window Help
RSLQR Root Locus
Imaginary
Real
Open Loop Poles
Closed Loop Poles
Accel Zero
Max Spectral Radius

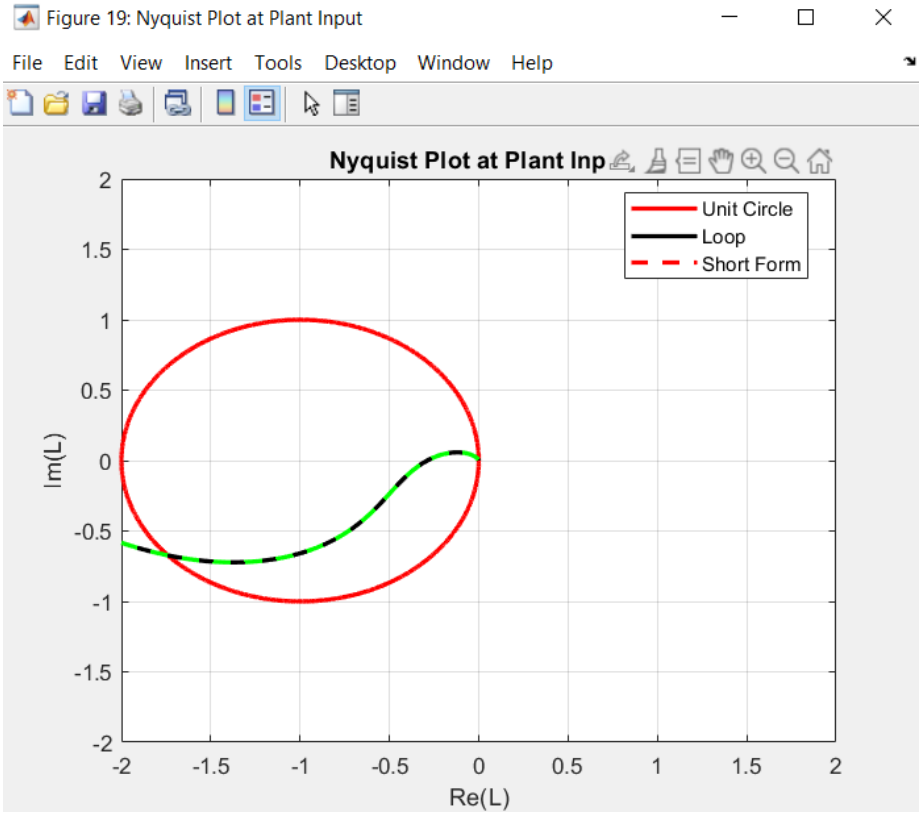
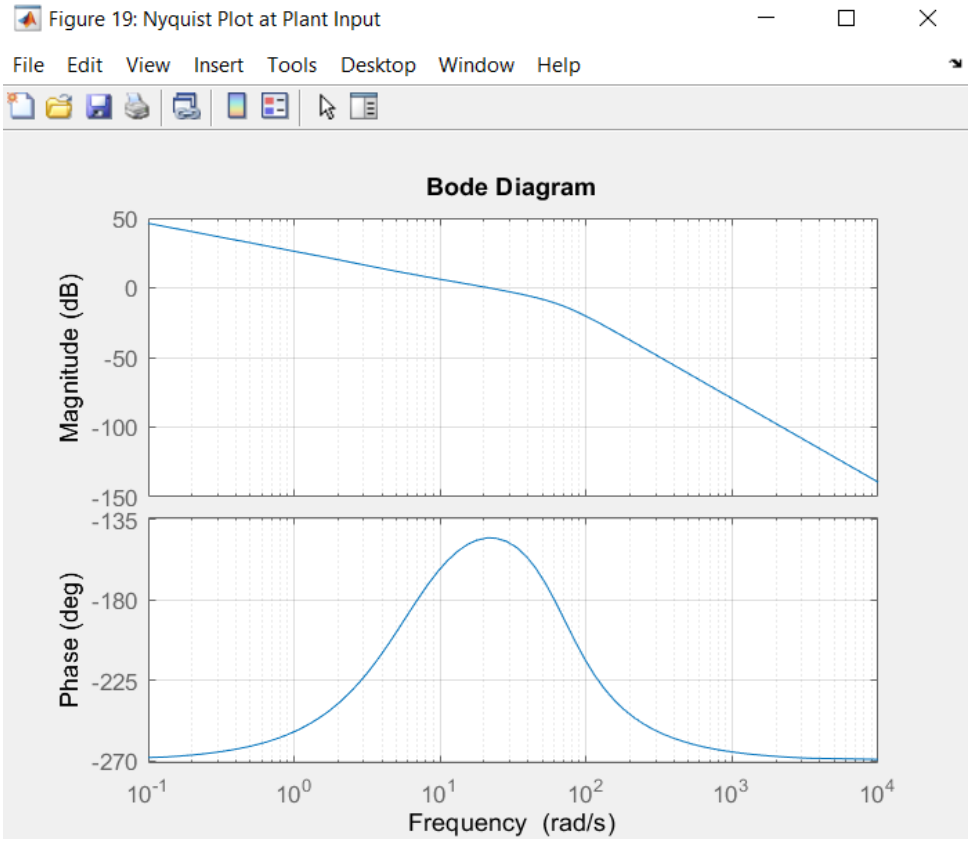


Figure 19: Nyquist Plot at Plant Input
File Edit View Insert Tools Desktop Window Help
Nyquist Plot at Plant Inp
Im(L)
Re(L)
Unit Circle
Loop
Short Form




```
GM =

   3.5217e-01


PM_deg =

   3.4716e+01
```

```
wc_GM =

   6.9773e+00


wc_Pm =

   2.1433e+01
```

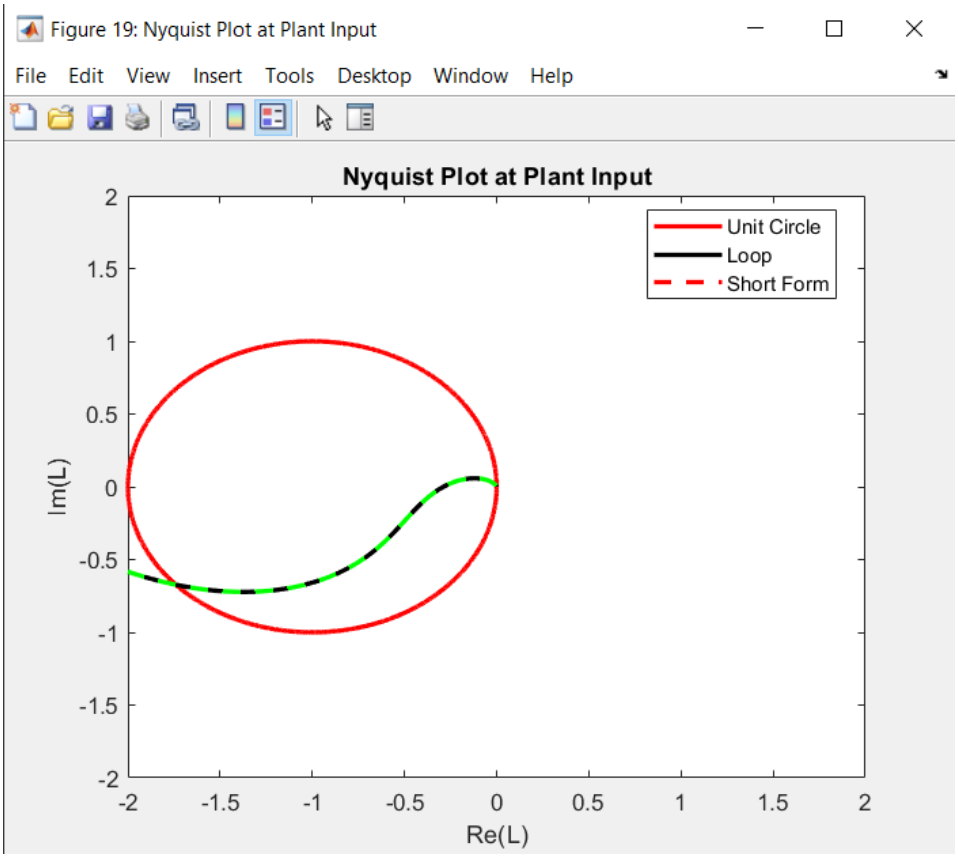


Figure 19: Nyquist Plot at Plant Input
File Edit View Insert Tools Desktop Window Help
Nyquist Plot at Plant Input
Unit Circle
Loop
Short Form
Im(L)
Re(L)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2 -1.5 -1 -0.5 0 0.5 1 1.5 2

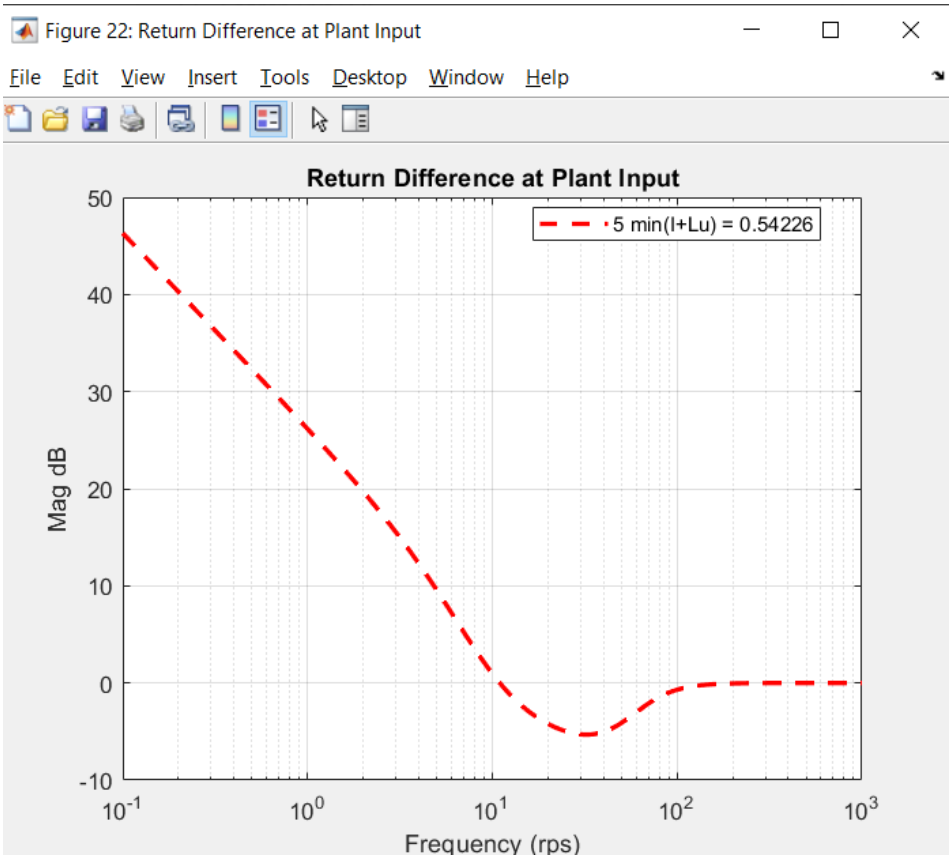
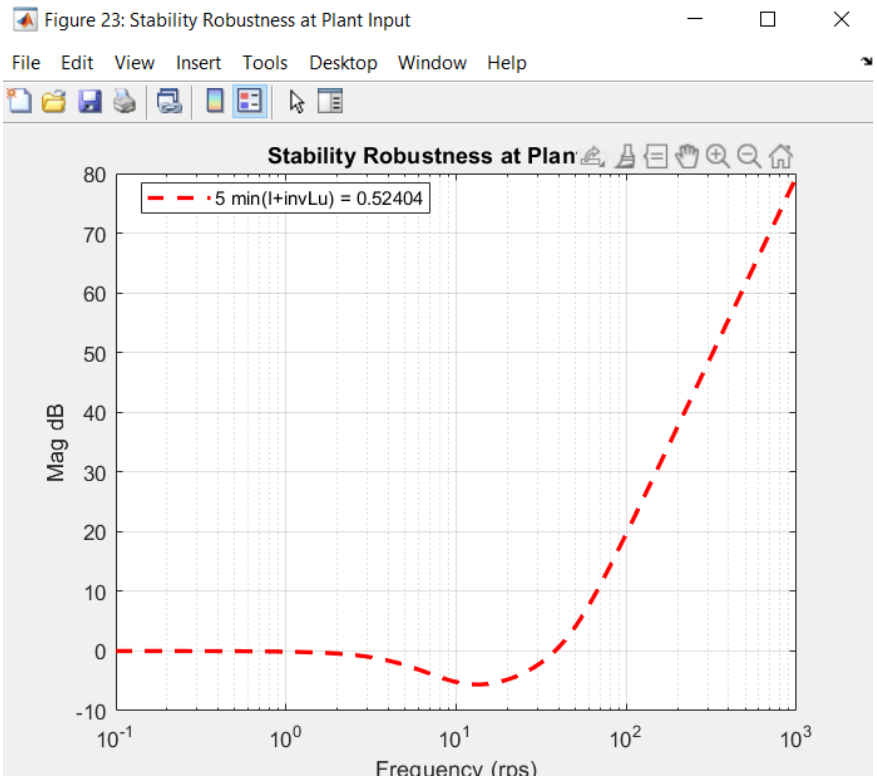
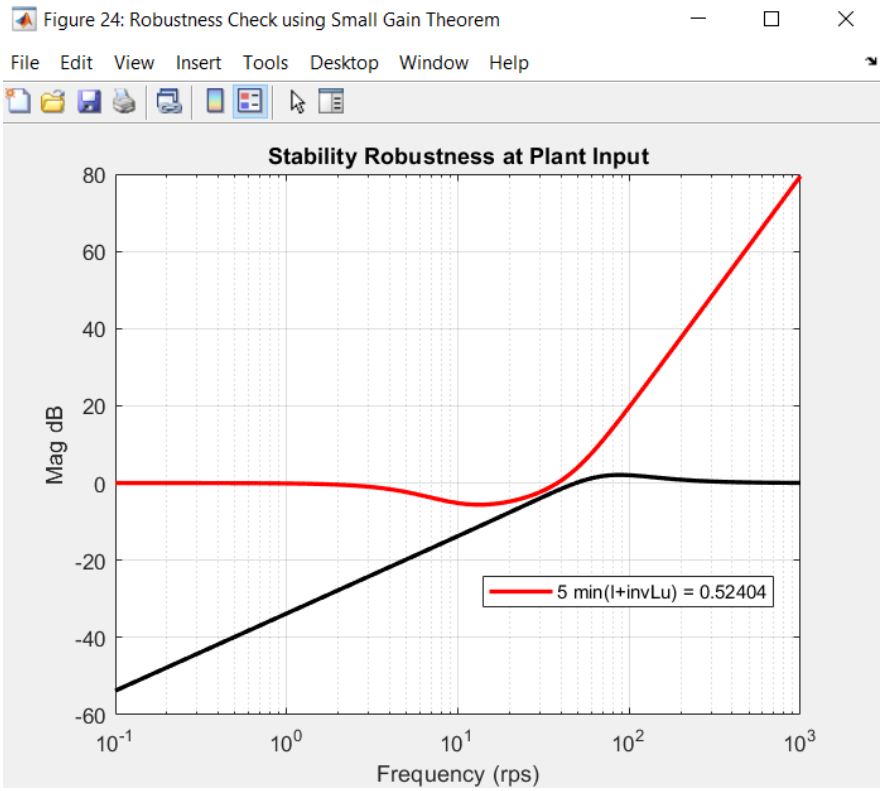


Figure 22: Return Difference at Plant Input
File Edit View Insert Tools Desktop Window Help
Return Difference at Plant Input
5 min(I+Lu) = 0.54226
Mag dB
Frequency (rps)
50
40
30
20
10
0
-10
10^-1 10^0 10^1 10^2 10^3





Robust; using small gain theorem for actuator dynamics (w_act = 11Hz & zeta_act = 0.707)

2.

```
Ac = 0.;
Bc1 = [1. 0. 0. 0. 0.];
Bc2 = -1;
Cc = -Kx_lqr(1);
Dc1 = [0. -Kx_lqr(2:3) 0. 0.];
Dc2 = 0.;
```

```
Ac =

     0


Bc1 =

     1     0     0     0     0


Bc2 =

    -1


Cc =

  -7.5646e-03


Dc1 =

            0   2.3099e+00   2.2086e-01            0            0


Dc2 =

     0
```