

1. Write a python code that will generate the minor matrix of a given a  $n \times n$  matrix.
2. The Hungarian Algorithm is used to find the minimum cost when assigning Jobs to machines based on cost, and each machine must be assigned to a different job.

Step 1: Create zero elements in the cost matrix by subtracting the smallest element in each row from the corresponding row.

Step 2: Repeating the same with columns, the final matrix can be obtained.

Step 3: Cover all zeros with the minimum number of lines - Using the smallest number of lines possible, draw lines over rows and columns in order to cover all zeros in the matrix. If the number of lines is equal to the number of rows in your square matrix, stop here. Otherwise, go to step 4.

Step 4: Create additional zeros - Find the smallest element - call it  $c$  - that isn't covered by a line. Subtract  $c$  from all uncovered elements in the matrix and add it to any element that's covered twice. Go back to step 3.

Example:

Let the cost matrix:

1500	4000	4500
2000	6000	3500
2000	4000	2500

**Step 1:** Subtract minimum of every row.

1500, 2000 and 2000 are subtracted from rows 1, 2 and 3 respectively.

0	2500	3000
0	4000	1500
0	2000	500

**Step 2:** Subtract minimum of every column.

0, 2000 and 500 are subtracted from columns 1, 2 and 3 respectively.

0	500	2500
0	2000	1000
0	0	0

**Step 3:** Cover all zeroes with minimum number of horizontal and vertical lines.

0	500	2500
0	2000	1000
0	0	0

**Step 4:** Since we only need 2 lines to cover all zeroes, we have NOT found the optimal assignment.

**Step 5:** We subtract the smallest uncovered entry from all uncovered rows. Smallest entry is 500.

```
-500  0 2000
-500 1500 500
  0   0   0
```

Then we add the smallest entry to all covered columns, we get

```
  0   0 2000
  0 1500 500
500   0   0
```

Now we return to **Step 3**:. Here we cover again using lines. and go to **Step 4**:. Since we need 3 lines to cover, we found the optimal solution.

```
1500 4000 4500
2000 6000 3500
2000 4000 2500
```

So the optimal cost is  $4000 + 2000 + 2500 = 8500$

3. Write a program in python that will determine the inverse mod 26 of  $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ .

The naïve method for calculating multiplicative inverse of 'a' mod m is given below:

```
def modInverse(a, m) :
    a = a % m;
    for x in range(1, m) :
        if ((a * x) % m == 1) :
            return x
    return 1
```

Instead of this you may use Euclid's Extend algorithm to find modular inverse.

Example: If  $A = \begin{bmatrix} 5 & 8 \\ 17 & 3 \end{bmatrix}$ , then find  $A^{-1} \text{ mod } 26$ .

Now  $\det(A) = \{(5 \times 3) - (8 \times 17)\} \text{ mod } 26 = -121 \text{ mod } 26 = 9$

Cofactor matrix of  $A = \begin{bmatrix} 3 & -17 \\ -8 & 5 \end{bmatrix}$  and Adjoint of cofactor matrix is  $\begin{bmatrix} 3 & -8 \\ -17 & 5 \end{bmatrix}$

So  $A^{-1} = (1/\det(A)) \times \text{Adj}(\text{cofactor}(A)) = (1/9) \begin{bmatrix} 3 & -8 \\ -17 & 5 \end{bmatrix} \text{ mod } 26$

$(1/9) \text{ mod } 26 = 9^{-1} \text{ mod } 26 = 3$

So  $A^{-1} = 3 \begin{bmatrix} 3 & -8 \\ -17 & 5 \end{bmatrix} \text{ mod } 26 = 3 \begin{bmatrix} 3 & 18 \\ 9 & 5 \end{bmatrix} \text{ mod } 26 = \begin{bmatrix} 9 & 54 \\ 27 & 15 \end{bmatrix} \text{ mod } 26 = \begin{bmatrix} 9 & 2 \\ 1 & 15 \end{bmatrix}$

4. Write a programme in python that will solve the maximization LPP for two variable using cornet method. The user should input the objective function and the constraints (3 constraints all are  $\leq$  constraint). The assumption that all the decision variables are greater than equal to zero.

#### Corner Point Method to Solve LPP (Linear Programming Problem)

The method includes the following steps

Step 1: Find the feasible region of the LLP.

Step 2: Find the co-ordinates of each vertex of the feasible region.

These co-ordinates can be obtained from the graph or by solving the equation of the lines.

Step 3: At each vertex (corner point) compute the value of the objective function.

Step 4: Identify the corner point at which the value of the objective function is maximum (or minimum depending on the LP)

The co-ordinates of this vertex is the optimal solution and the value of Z is the optimal value

Example:

Find the optimal solution in the above problem of decorative item dealer whose objective function is

$$Z = 50x + 18y.$$

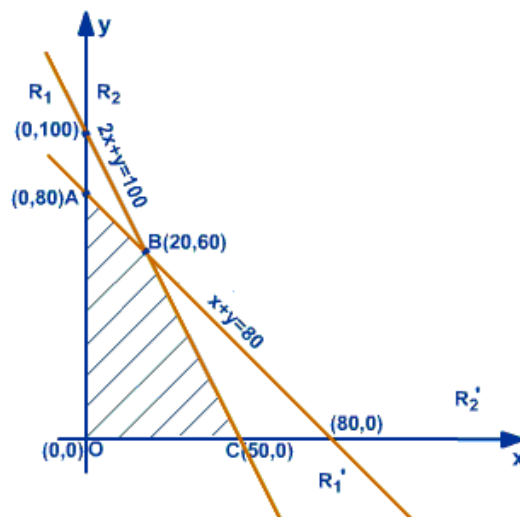
Subject to the constraints

$$2x + y \leq 100$$

$$x + y \leq 80$$

$$x \geq 0, y \geq 0$$

Solution:



In the graph, the corners of the feasible region are

O (0, 0), A (0, 80), B(20, 60), C(50, 0)

At (0, 0)  $Z = 0$

At (0, 80)  $Z = 50(0) + 18(80)$   
 $= 1440$

At (20, 60),  $Z = 50(20) + 18(60)$   
 $= 1000 + 1080 = \text{Rs.}2080$

At (50, 0)  $Z = 50(50) + 18(0)$   
 $= 2500.$

Since our object is to maximize Z and Z has maximum at (50, 0) the optimal solution is  $x = 50$  and  $y = 0$ .

The optimal value is 2500.