A Continuous-Time Average-Cost Flexible Manufacturing and Operator Scheduling Model Solved by Deconvexification Over Time*

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Dedicated to Alan J. Hoffman, who has inspired us by his love of life and mathematics.

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ABSTRACT

A continuous-time flexible manufacturing and operator scheduling problem is introduced and solved. The principle concern is with scheduling operators over time to various activities of a manufacturing system with the purpose of optimizing some steady-state criterion. In mathematical terms the problem is modeled as a deterministic, infinite-horizon, continuous-time discrete dynamic program. Our solution procedure is to convexify the problem to obtain a linear program and then to deconvexify the solution of the linear program over time to arrive at an optimal solution which is periodic and piecewise constant. Apparent loss in object value due to the deconvexifications is circumvented with buffer inventories. The procedure can be reduced to

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solving a sequence of linear programs, and the complexity can be stated in these terms.

1. INTRODUCTION

In a continuous-time flexible manufacturing system our concern is with scheduling operators of different abilities to the activities over time in such a way as to optimize some steady-state criterion such as minimizing average costs or maximizing average throughput. Our focus is on the solution of a general decision problem which is entitled "flexible manufacturing and operator scheduling model," abbreviated FMOS. A discrete-time variant of FMOS was studied in Eaves and Rothblum (1987), whereas the current paper examines a continuous-time variant. As with the discrete-time model in Eaves and Rothblum (1987), our solution here for the continuous-time model proceeds by convexifying the discrete operator assignment decision set to form a linear program and by deconvexifying over time a solution of the linear program to obtain a periodic, piecewise constant optimal manufacturing and operator schedule.

Definition of FMOS. Unfinished goods, perhaps of different types such as parts, raw materials, etc., enter a system. In the system, various divisible in-progress goods are transformed into other in-progress goods through the execution of activities. The rate and character of these activities depend upon the assigned operator, if any, as well as the availability of inventories of constituent in-progress goods. The operator pool may be composed of, for example, skilled and unskilled laborers, robots, and machines. The function of an activity with an assigned operator is to transform in-progress goods into in-progress goods in fixed proportions and at a rate not exceeding a bound which is determined by both the activity and operator in question. An activity can be manned by at most one operator at any given time, and an operator can be assigned at most one activity at any given time. Certain collections of activities are conducted at a single workstation, which may further limit the number of operators present. It is assumed that operators can be moved from activity to activity with negligible time delay or setup cost. Further, it is assumed that in-progress goods can be stored on the spot with negligible cost or time delay. Finally, finished goods, perhaps of different types such as assembled parts, processed materials, rejections, etc., exit the production system. The rates and proportions of unfinished and finished goods entering and exiting the system can vary and depend upon operator assignments and activity rate settings. The objective is to assign operators to activities and to set rates of activities, continuously in time, in order to optimize some steady-state criterion as minimizing average cost or maximizing average throughput.

The restrictive assumptions of FMOS, which limit applicability of the model, are those concerned with moving operators without delays or setups and with storing on the spot without delays or costs. See the discussion in Eaves and Rothblum (1987) regarding the value of FMOS vis-a-vis these assumptions.

To solve continuous-time FMOS we introduce a deterministic infinite-horizon continuous-time discrete dynamic program, which we refer to as MICP. We solve MICP by convexifying the discrete decision set, then applying linear programming, and finally deconvexifying the linear-programming solution over time to regain the discreteness and thereby obtain an optimal solution. As costs are convex in the variables that are deconvexified, there is an apparent loss in objective value in deconvexification; however, this loss is circumvented with buffer inventories. Our solution procedure for MICP reduces to the solution of a sequence of linear programs.

As discussed in Eaves and Rothblum (1987), certain elements of the idea of deconvexification over time have been used previously: see the "folk theorem" for a cooperative game in Aumann (1981) and the rounding in a single-commodity flow problem of Orlin (1984); in these references the objective is linear over the relevant domain and, unlike the situation encountered in FMOS, no device is needed to compensate for objective loss through this process.

The motivation for this research was a fiber-optics assembly-line scheduling problem encountered in a field case study at Hewlett-Packard by students in the Stanford University Operations Research M.S. program; see Brown, Holmquist, Muir, Porter, and Prusak (1986). On the other hand, FMOS fits comfortably with decision models found in Stecke and Suri (1986), for example.

We formulate MICP in Section 2, and show in Section 3 how FMOS fits within this structure. MICP is analyzed in Section 4, where the existence of rational periodic optimal policies is established constructively.

2. FORMULATION OF THE CONTINUOUS-TIME MODEL.

In this section we define a deterministic infinite-horizon continuous-time discrete dynamic decision problem to which we refer as MICP. We shall use MICP to model continuous-time FMOS, that is, the flexible manufacturing and operator scheduling problem where decisions concerning the activity levels and operator assignments are taken continuously over time. In particular, we study MICP under a long-term average-cost optimality criterion.

Let R be the set of reals, and R_+ the set of nonnegative reals. The *data* for MICP are three positive integers m, n, and k, matrices $A \in R^{m \times n}$, $B \in R^{n \times k}$, and $L \in R^{m \times n}$, vectors $b \in R^{1 \times k}$ and $c \in R^{1 \times n}$, and a finite set $X \subseteq R^k$. A function $\pi = (x(\cdot), y(\cdot), z(\cdot))$ mapping R_+ into R^{k+n+m}, where $x(\cdot): R_+ \to R^k$, $y(\cdot): R_+ \to R^m$, and $z(\cdot): R_+ \to R^n$ are integrable on every compact interval, is defined to be a *policy* for MICP if for all $t \in R_+$

$$0 \leqslant y(t) \leqslant Bx(t), \tag{2.1a}$$

$$x(t) \in X, \tag{2.1b}$$

$$z(t) = z(0) + \int_0^t Ay(r)\, dr, \tag{2.1c}$$

and

$$z(t) \geqslant Ly(t). \tag{2.1d}$$

Evidently, with (2.1a) it is sufficient to restrict attention to those x in X with $Bx \geqslant 0$; henceforth, we assume that $Bx \geqslant 0$ for all $x \in X$.

To formulate our optimality criterion, define for each $0 \neq t \in R_+$ and policy $\pi = (x(\cdot), y(\cdot), z(\cdot))$ the *t-period average cost associated with the use of* π by

$$V_\pi^t \equiv t^{-1} \int_0^t [bx(\tau) + cy(\tau)]\, d\tau \tag{2.2}$$

if $t > 0$ and $V_\pi^0 \equiv 0$. A policy π is called *average-optimal* if for every policy ρ

$$\limsup_{t \to \infty} V_\pi^t \leqslant \liminf_{t \to \infty} V_\rho^t. \tag{2.3}$$

Of course, the above definition is equivalent to requiring that the limit of V_π^t as $t \to \infty$ exists and that for every policy ρ

$$\lim_{t \to \infty} V_\pi^t \leqslant \limsup_{t \to \infty} V_\rho^t. \tag{2.3'}$$

We shall establish the existence of and compute average-optimal policies π

which are periodic and piecewise-constant and for which quality in (2.3) is attained for an infinite sequence of time periods.

3. IDENTIFICATION OF FMOS AS MICP

In this section we show that FMOS fits within the MICP formulation. The discussion follows that of Eaves and Rothblum (1987) for the discrete-time FMOS.

The first step is to index in-progress goods and activities, terms we use in a broad sense. In particular, unfinished goods and finished goods are regarded as in-progress goods; further, for example, identical types of in-progress goods at different locations can be regarded as distinct. Similarly, activities having the same characteristics but executed at different workstations can be viewed as distinct. Let $1,\ldots,m$ index the in-progress goods, and let $1,\ldots,n$ index the activities.

The execution of activities consumes and produces in-progress goods. The amount of in-progress goods used and produced by an activity is proportional to the rate at which that activity is executed and to the duration of its execution. Specifically, we have an $m \times n$ matrix A such that for in-progress good $i = 1,\ldots,m$ the execution of activity $j = 1,\ldots,n$ at rate $y_j \geqslant 0$ results a net gain or loss of $A_{ij}y_j$ units of in-progress good i per unit time. The matrix A is called the *netput matrix*. In particular, if the rates of the activities for the period $t \leqslant \tau \leqslant s$ are given by the coordinates of a vector $y(\tau) \in R^m_+$, then the net change of in-progress good i that occurs is $\int_t^s (Ay(\tau))_i \, d\tau$. Thus, if the levels of inventories of the in-progress good at time t are the coordinates of the vector $z(t)$, we have that

$$z(t) = z(0) + \int_0^t Ay(\tau)\, d\tau. \tag{3.1}$$

Also, we have that

$$z(t) \geqslant 0. \tag{3.2}$$

For emphasis, we note that MICP permits the purchase and receipt of raw materials as well as the selling and shipping of finished goods; see Eaves and Rothblum (1987) for details.

As carefully explained in Eaves and Rothblum (1987), the availability of operators and the constraints on their assignments can be represented by extreme solutions of a system of form

$$Ux \leqslant u,$$

where U is totally unimodular and u is integral. So the assignments of operators to activities at each instance $t \in R_+$ can be represented by a vector $x(t)$ satisfying

$$x(t) \in X, \tag{3.3}$$

where X is the set of extreme solutions to the above system. Let k be the number of columns of the matrix U. Notice that some of the k coordinates of a vector $x(t)$ satisfying (3.3) indicate operator assignments, whereas others are concerned with the numbers of operators at workstations, etc.

The feasible rates of an activity depend upon the assigned operator, if any. If $y(t)$ is the vector of activity rates at time t, we can represent the bounds on the activity rates by

$$0 \leqslant y(t) \leqslant Bx(t), \tag{3.4}$$

where $x(t)$ is the vector representing the assignment of operators to activities and B is a corresponding nonnegative matrix. We refer to B as the *rate bound matrix.*

Finally, in the use of MICP to model FMOS it is natural to select $L = 0$; this is because in a continuous-time model divisible in-progress goods can be used in further production as soon as they become available.

The decision maker for FMOS must select for each instance the assignment of operators to activities, the activity rates and the inventory levels of in-progress goods. Suppose that for each $t \in R_+$, the functions $x(t)$, $y(t)$, and $z(t)$ represent the selected operator assignments, the activity rates, and the inventory levels. We have seen that the conditions (3.1)–(3.4), which coincide with (2.1a)–(2.1d), must be satisfied, where (2.1a) has $L = 0$. Also we observe that both marginal throughput and marginal cost at the instant $t \geqslant 0$ can be represented by $bx(t) + cy(t)$ for corresponding vectors b and c, and the total cost from time 0 till time t is then expressible as $\int_0^t [bx(\tau) + cy(\tau)]\, d\tau$.

4. ANALYSIS OF MICP UNDER THE AVERAGE-COST CRITERION

In this section MICP is analyzed under the long-term average-cost optimality criterion. In particular, MICP is shown to have computable piecewise-constant, periodic, average-optimal policies. The results and their proofs resemble those in Eaves and Rothblum (1987) for the discrete-time FMOS, though some of the arguments required there are not needed here and vice versa. Our solution here is of MICP, not just MICP tailored to FMOS.

The following lemma gives a useful representation of the right-hand side of (2.3) for any given policy π. As usual, the *convex hull* of X, denoted $\operatorname{conv} X$, is the set of all convex combination of elements in X where the weights are *real*.

LEMMA 1. *Let $\pi = (\pi(\cdot), y(\cdot), z(\cdot))$ be a policy. Then there exists vectors $\bar{x} \in R^k$ and $\bar{y} \in R^n$ with*

$$0 \leqslant \bar{y} \leqslant B\bar{x}, \tag{4.1a}$$

$$\bar{x} \in \operatorname{conv} X, \tag{4.1b}$$

$$A\bar{y} \geqslant 0, \tag{4.1c}$$

and

$$\liminf_{t \to \infty} V_\pi^t = c\bar{x} + b\bar{y}. \tag{4.2}$$

Proof. For $0 \neq t \in R_+$, let $\bar{x}^t = t^{-1}\int_0^t x(\tau)\,d\tau$ and $\bar{y}^t = t^{-1}\int_0^t y(\tau)\,d\tau$. Then $V_\pi^t = b\bar{x}^t + c\bar{y}^t$ by conditions (2.1a) and (2.1b) from the definition of a policy, $x(\cdot)$ and $y(\cdot)$ are bounded, and therefore $\bar{x}^t$, $\bar{y}^t$, and V_π^t are bounded in t. Let $v \equiv \liminf_{t \to \infty} V_\pi^t$. It follows that v is finite and that for some increasing sequence $(t_0, t_1, \dots)$ in R_+, $\lim_{p \to \infty} V_\pi^{t_p} = v$. By possibly taking a subsequence of $(t_0, t_1, \dots)$, one may assume that $\lim_{p \to \infty} \bar{x}^{t_p}$ and $\lim_{p \to \infty} \bar{y}^{t_p}$ exist. Let $\bar{x}$ and $\bar{y}$ be the corresponding limits. Then $v = \lim_{p \to \infty} V_\pi^{t_p}(b\bar{x}^{t_p} + c\bar{y}^{t_p}) = b\bar{x} + c\bar{y}$, establishing (4.2). Next, (4.1a) follows directly from (2.1a), and (4.1b) follows from (2.1b) and the fact that $\operatorname{conv} X$ is compact, as X is finite. Finally, (2.1c) and (2.1d) imply that $A\bar{y}^{t_p} = t^{-1}\int_0^t Ay(\tau)\,d\tau = t^{-1}[z(t) - z(0)] \geqslant t^{-1}[Ly(t) - z(0)]$, and therefore $A\bar{y} = \lim_{p \to \infty} A\bar{y}^{t_p} \geqslant \lim_{p \to \infty} t_p^{-1}[Ly(t_p) - z(0)] = 0$, establishing (4.1c). ∎

Lemma 1 is a continuous-time analogue of Orlin (1981, Chapter 2, Theorem 1).

Lemma 1 implies that any optimal solution (x^*, y^*) of

Program I: minimize $bx + cy$

subject to $0 \leqslant y \leqslant Bx, \quad x \in \operatorname{conv} X, \quad Ay \geqslant 0.$

has

$$bx^* + cy^* \leqslant \liminf_{t \to \infty} V_\pi^t \tag{4.3}$$

for every policy π. In particular, we get the following immediate corollary of Lemma 1.

COROLLARY 2. *If π is a policy for which $\lim_{t \to \infty} V_\pi^t$ equals the optimal objective value of Program* I, *then π is average-optimal.*

Call a policy $\pi = (x(\cdot), y(\cdot), z(\cdot))$ *stationary* if $x(\cdot)$ and $y(\cdot)$ are constant, that is, invariant in t. Evidently for every given $x \in R^k$ and $y \in R^n$, there exists a stationary policy $\pi = (x(\cdot), y(\cdot), z(\cdot))$ with $x(t) = x$ and $y(t) = y$ for all $t \in R_+$ if and only if $0 \leqslant y \leqslant Bx$, $x \in X$, and $Ay \geqslant 0$, in which case $z(t) = z(0) + tAy$ for arbitrarily selected $z(0) \geqslant Ly$ and $V_\pi^t = bx + cy$ for all $t \in R_+$. Thus, a stationary policy π satisfying (2.3) for every stationary policy ρ can be obtained from any optimal solution, say (x^*, y^*), of

Program I′: minimize $bx + cy$

subject to $0 \leqslant y \leqslant Bx, \quad x \in X, \quad Ay \geqslant 0.$

However, an optimal solution to Program I′ need not be optimal for Program I; hence, the best stationary policy need not be optimal. In the following we consider a broader class of policies than the stationary ones and demonstrate that average-optimal policies exist in that class. We also show how such policies can be computed.

Call a policy $\pi = (x(\cdot), y(\cdot), z(\cdot))$ *periodic* if $x(t + 1) = x(t)$ and $y(t + 1) = y(t)$ for all $t \in R_+$; we emphasize that this definition does not require that $z(t + 1) = z(t)$ for $t \in R_+$.

The next lemma states the observation that for periodic policies the long-term average reward converges as the horizon grows.

LEMMA 3. *Suppose* $\pi = (x(\cdot), y(\cdot), z(\cdot))$ *is a periodic policy. Then* $\lim_{t \to \infty} V_\pi^t$ *exists and*

$$\lim_{t \to \infty} V_\pi^t = \int_0^1 [bx(\tau) + cy(\tau)]\, d\tau. \tag{4.4}$$

A periodic policy $\pi = (x(\cdot), y(\cdot), z(\cdot))$ is called *piecewise constant* if for some real numbers $t_0 = 0 < t_1 < \cdots < t_q = 1$, $x(\cdot)$ and $y(\cdot)$ are constant on each interval $[t_r, t_{r+1})$, $r = 0, \ldots, q-1$. The following theorem establishes existence of a periodic, piecewise constant, average-optimal policy.

THEOREM 4. *A policy exists if and only if Program I is feasible. Moreover, in this case there exists a periodic, piecewise constant, average-optimal policy* π *such that* $\lim_{t \to \infty} V_\pi^t$ *equals the optimal objective value of Program I.*

Proof. Lemma 1 shows that Program I is feasible whenever a policy exists. Next assume that Program I is feasible. We will construct a piecewise-constant, periodic, average-optimal policy π such that $\lim_{t \to \infty} V_\pi^t$ equals the optimal objective value of Program I. In particular, this construction will demonstrate the existence of a policy.

Compactness arguments show that Program I has an optimal solution, say (x^*, y^*). Now, as $x^* \in \operatorname{conv} X$, there exist a positive integer q, vectors $f^1, \ldots, f^q$ in X, and positive numbers $\lambda_1, \ldots, \lambda_q$ with $\sum_{r=1}^q \lambda_r = 1$ and $\sum_{r=1}^q \lambda_r f^r = x^*$. Then $0 \leqslant y^* \leqslant Bx^* = \sum_{r=1}^q \lambda_r Bf^r$ and $\lambda_r Bf^r \geqslant 0$ for $r = 1, \ldots, q$. Let D be the $m \times m$ diagonal matrix with $D_{jj} = y_j^*/(Bx^*)_j$ for $j = 1, \ldots, m$, and let $g^r = DBf^r$ for $r = 1, \ldots, q$. As $0 \leqslant D \leqslant I$ and each Bf^r is nonnegative, we have that

$$0 \leqslant DBf^r = g^r \leqslant Bf^r, \qquad r = 1, \ldots, q. \tag{4.5}$$

Also,

$$\sum_{r=1}^q \lambda_r g^r = D\left(\sum_{r=1}^q \lambda_r Bf^r \right) = DBx^* = y^*. \tag{4.6}$$

It is easy to verify the existence of a vector z^0 in R^n with

$$z^0 + \sum_{j=1}^{u-1} \lambda_j A g^j + \xi A g^u \geqslant L g^u \qquad \text{for all} \quad u = 1, \ldots, q \text{ and } 0 \leqslant \xi \leqslant \lambda_u. \tag{4.7}$$

Now, for $t \in R_+$ there exists a unique integer $r \in \{1, \ldots, q)$ with $\sum_{j=1}^{r-1} \lambda_j \leqslant t - \lfloor t \rfloor < \sum_{j=1}^{r} \lambda_j$, where $\lfloor t \rfloor$ is the largest integer smaller than or equal to t. For such t and corresponding r, define $\pi = (x(t), y(t), z(t))$ by letting $x(t) = f^r$, $y(t) = g^r$, and $z(t) = z^0 + \lfloor t \rfloor A y^* + \sum_{j=1}^{r-1} \lambda_j A g^j + (t - \lfloor t \rfloor - \sum_{j=1}^{r-1} \lambda_j)^{-1} A g^r$. We next argue that π is a periodic, piecewise-constant, average-optimal policy.

We first establish that π is a policy. For every integer p, the explicit definition of $y(\cdot)$ and (4.6) imply that

$$\int_0^p A y(\tau)\, d\tau = \sum_{s=0}^{p-1} \left(\int_s^{s+1} A y(\tau)\, d\tau \right) = \sum_{s=0}^{p-1} \left(\sum_{r=1}^{q} \lambda_r A g^r \right)$$

$$= p \left(\sum_{r=1}^{q} \lambda_r A g^r \right) = p A y^* \geqslant 0,$$

Hence, for $t \in R_+$ and $r \in \{1, \ldots, q\}$ satisfying $\sum_{j=1}^{r-1} \lambda_j \leqslant t - \lfloor t \rfloor < \sum_{j=1}^{r} \lambda_j$,

$$\int_0^t A y(\tau)\, d\tau = \int_0^{\lfloor t \rfloor} A y(\tau)\, d\tau + \int_{\lfloor t \rfloor}^t A y(\tau)\, d\tau$$

$$= \lfloor t \rfloor A y^* + \sum_{j=1}^{r-1} \lambda_j A g^j + \left(t - \lfloor t \rfloor - \sum_{j=1}^{r-1} \lambda_j \right) A g^r$$

$$= z(t) - z^0 = z(t) - z(0),$$

establishing (2.1c). Further, the fact that $A y^* \geqslant 0$, (4.7), and the definition of $y(t)$ imply that

$$z(t) = z(0) + \lfloor t \rfloor A y^* + \sum_{j=1}^{r-1} \lambda_j A g^i + \left(t - \lfloor t \rfloor - \sum_{j=1}^{r-1} \lambda_j \right) A g^r$$

$$\geqslant L g^r = L y(t),$$

establishing (2.1d). Next, (4.5) shows that for $r = 1, \ldots, q$, $0 \leqslant g^r \leqslant Bf^r$, immediately implying (2.1a). Also, (2.1b) is immediate from the fact that the f^r's are all in X. So indeed π is a policy.

It remains to show that π is periodic, piecewise constant, and average-optimal. The periodicity of $x(\cdot)$ and $y(\cdot)$ is immediate from their definition and the fact that for all $t \in R_+$, $t + 1 - \lfloor t + 1 \rfloor = t - \lfloor t \rfloor$. Also, the assertion that π is piecewise constant follows directly from the definition of $x(\cdot)$ and $y(\cdot)$. Finally, Lemma 3 implies that $\lim_{t \to \infty} V_\pi^t$ exists, and that

$$\lim_{t \to \infty} V_\pi^t = \int_0^1 [bx(\tau) + cy(\tau)]\, d\tau = \sum_{r=1}^{q} \lambda_r (bf^r + cg^r) = bx^* + cy^*.$$

So $\lim_{t \to \infty} V_\pi^t$ equals the optimal objective value of Program I, and Corollary 2 implies that π is average-optimal. ∎

With regard to the proof just given, the naive approach for constructing an optimal policy for MICP from an optimal solution (x^*, y^*) of Program I is to first express x^* as a convex combination of elements of X, then match each x in X with an optimal y and use the corresponding pairs over time at the right proportions. This procedure will potentially result in a policy whose long term average cost is lower than the optimal objective value of Program I. But the proof of Theorem 4 shows that the procedure can be modified to produce an optimal policy whose long-term average cost equals the optimal objective value of Program I. This is achieved by relaxing stationarity and carrying inventory.

The proof of Theorem 4 is constructive and suggests the use of the following steps to compute a piecewise-constant, optimal policy:

(1) compute an optimal solution (x^*, y^*) to Program I;

(2) find $f^1, \ldots, f^q$ in X and positive real numbers $\lambda_1, \ldots, \lambda_q$, where q is a positive integer such that $\Sigma_{r=1}^q \lambda_r = 1$ and $\Sigma_{r=1}^q \lambda_r f^r = x^*$;

(3) for $r = 1, \ldots, q$, compute the vectors $g^1, \ldots, g^q$ for which (4.5) and (4.6) hold; and

(4) compute $z^0 \in R^n$ that satisfies (4.7).

The discussion in Section 5 of Eaves and Rothblum (1987) shows how to execute the above steps efficiently. In particular, if A, B, c and d have rational elements one can require that the f^r's, g^r's, and λ_r's do also. Also, by the Carathéodory theorem, one can require that the number of break points per unit time in the average-optimal policy constructed in the proof of Theorem 4 not exceed $k + 1$. Also, if free disposal is allowed, one can obtain an optimal policy with uniformly bounded inventory levels over time.

We note that the rational periodic optimal policy π constructed in the proof of Theorem 4 has the strong property that V_π^t attains $\inf_\rho(\liminf_{t\to\infty} V_\rho^t)$ at each period k for every positive integer k.

Our analysis applies to several extensions of the basic continuous-time FMOS presented here; see Appendix C of Eaves and Rothblum (1987), where such extensions are considered for discrete-time FMOS. In particular, if inventory levels are required to be bounded from above, we get that $Ay \geqslant 0$ in Program I is replaced by $Ay = 0$.

REFERENCES

Aumann, R. J. 1981. Survey of repeated games, in *Essays in Game Theory and Mathematical Economics* (R. J. Aumann and J. C. Harsanyi, Eds.), Birkhäuser, pp. 11–42.

Brown, D., Holmquist, K., Muir, D., Porter, K., and Prusak, K. 1986. Hewlett-Packard Assembly Line Scheduling, a field case study, Operations Research Dept., Stanford Univ., Stanford, Calif. 94305.

Eaves, B. C. and Rothblum, U. G. 1987. A flexible manufacturing and operator scheduling model solved by deconvexification over time-discrete-time model, manuscript.

Orlin, J. 1981. Dynamic Convex Programming, Ph.D. Dissertation, Dept. of Operations Research, Stanford Univ.

Orlin, James B. 1984. Minimum convex cost dynamic network flows, *Math. Oper. Res.* 9:190–207.

Stecke, K. E. and Suri, R. (Eds.). 1986. *Proceedings of the Second ORSA/TIMS Conference on Flexible Manufacturing Systems: Operations Research Models and Applications.*

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