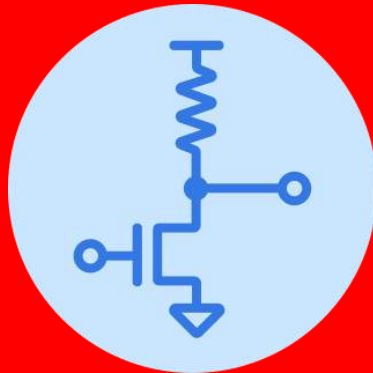


Analog Electronics Notes



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MOS DEVICE PHYSICS.

→ The level of abstraction of knowledge required to study analog is

Quantum physics → Solid state physics → Semiconductor physics
→ device Modelling → design of circuits.

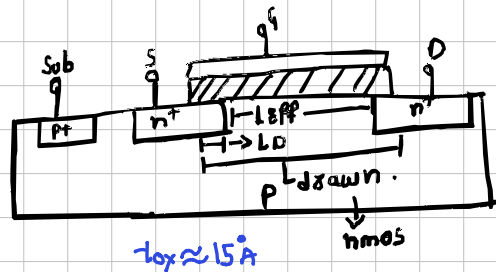
Mosfet as switch.

* To use mosfet as a switch. the drain to source Resistance should be very low. when gate voltage of mosfet (nmos) is very high, in ideal case it should be zero. & the resistance should be very high for low gate voltage. in ideal case it should be infinity.

Mosfet structure.

* The effective length of mosfet is given by $L_{\text{drawn}} - 2L_D = L_{\text{eff}}$

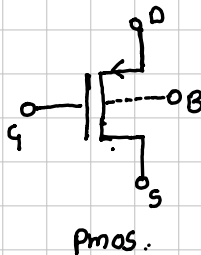
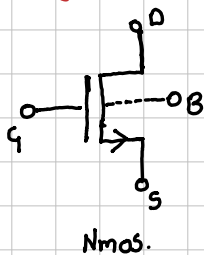
* mosfet is a symmetrical device i.e the drain & source of the device can be interchanged.



* The substrate of the mosfet must be reverse bias with respect to source. or else the current will directly flow from S/D to the substrate. if it is forward bias. this is called latch up.

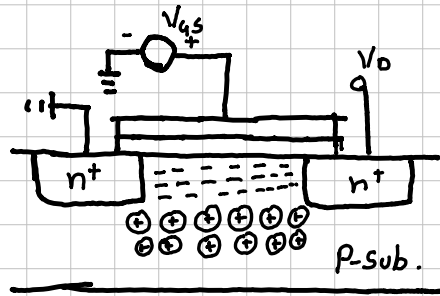
* Pmos is negation of nmos. & it is fabricated in an N-well & substrate. is connected to highest potential.

Mosfet Symbol



Mosfet V-I characteristics.

* When we apply voltage to the gate, the holes move away from gate and electrons move towards the gate. This will create a channel. If we increase V_g then the depletion region will increase. This region can be modelled as two cap i.e. gate oxide cap & depletion cap connected in series. If we increase V_g the current will start to flow. Then we can say device is inverted. The V_g required for this inverted channel is called V_{TH} . If V_g is increased further the change in depletion region will be same but charge density i.e. current will increase.



$$V_{TH} = \phi_{MS} + 2\phi_F + \frac{\phi_{dep}}{C_{ox}}$$

ϕ_{MS} :- Polysilicon & silicon workfunction.

$$\phi_F = V_T \ln\left(\frac{N_{sub}}{N_i}\right)$$

$N_{sub} \rightarrow$ charge density. $N_i \rightarrow$ intrinsic carrier
 $V_T =$ thermal voltage.

$$\phi_{dep} = \sqrt{4q\epsilon_{si}|\phi_F|N_{sub}}$$

$$C_{ox} = \frac{\epsilon_{si}}{t_{ox}} \approx 17.25 \text{ fF}/\mu\text{m}^2$$

* For Pmos theory is same but the polarity is reversed.

Derivation of I-V characteristics.

* Current is the amount of charge flowed per unit time i.e. it is equal to

$$I_D = Q_d \cdot V \quad (1)$$

$Q_d \rightarrow$ charge density
 $V \rightarrow$ velocity of charge.



* for $V_{GS} > V_{th}$ the charge density is proportional to $V_{GS} - V_{th}$. For $V_{GS} \leq V_{th}$, $Q_d = 0$.

$$Q_d = W C_{ox} (V_{GS} - V_{th}) \quad (2)$$

* When we increase $V_{DS} > 0$ the channel potential varies from $V_G(x) = 0$ to $V_D(x) = V_{DD}$. \therefore the charge density can be written as.

$$Q_d = W C_{ox} (V_{GS} - V_{th} - V(x)) \quad (3) \quad V(x) \text{ varies from } 0 \text{ to } V_D.$$

* Substitute equation 3 into 1.

$$I_D = -W C_{ox} (V_{GS} - V_{th} - V(x)) \cdot V \quad (4) \quad V \rightarrow \text{velocity of electron.}$$

* Negative sign is due to negative charge we know that.

$$V = \mu_n \vec{E} \Rightarrow V = \mu_n \left(-\frac{dV}{dx} \right) \quad (5) \quad V \Rightarrow \text{potential.}$$

Substituting equation 5 in 4.

$$I_D = W C_{ox} (V_{GS} - V_{th} - V(x)) \times \mu_n \times \frac{dV}{dx}.$$

Integrating on both side.

$$\int_0^L I_{D0} dx = \int_{V_{DS}=0}^{V_{DS}=V_{DD}} W C_{ox} (V_{GS} - V_{th} - V(x)) \times \mu_n \times dV$$

\therefore the current in channel is given by

$$I_D = \frac{W C_{ox} \mu_n}{L} \left[(V_{GS} - V_{th}) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

* for $V_{GS} - V_{th} \geq V_{DS}$ \therefore the device act as Voltage controlled resistor.

$$I_D \approx \frac{W C_{ox} \mu_n}{L} (V_{GS} - V_{th}) V_{DS}$$

$$\therefore R_{on} = \frac{1}{\frac{W}{L} C_{ox} \mu_n (V_{GS} - V_{th})}$$

* for $V_{GS} - V_{th} < V_{DS}$ the current in the mosfet become constant due to the pinch off. the effective length of transistor decrease. there is current even after the pinch off because there is large electric field in the channel. which will shoot electron to the drain of mosfet.

$$I_D = \frac{W C_{ox} \mu_n}{2 \times L'} (V_{GS} - V_{th})^2$$

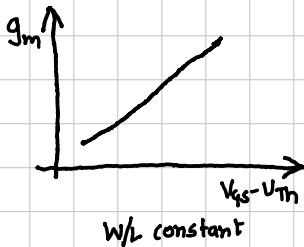
* In saturation mosfet act as the current source when L is constant. Nmos current source inject current into ground. Where as pmos current source draw current from the V_{DD} .

MOS Transconductance.

→ The change in drain current with change in gate-source voltage is called transconductance. it is denoted by g_m .

$$g_m = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{V_{DS} \text{ const}} \Rightarrow \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}).$$

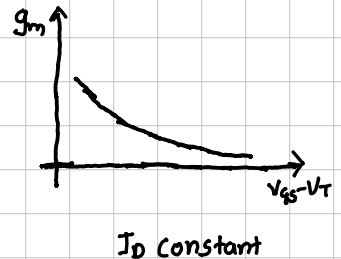
$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$



$$g_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$



$$g_m = \frac{2 I_D}{V_{GS} - V_{TH}}$$



* g_m increase with overdrive voltage when W/L kept constant whereas g_m decrease with overdrive voltage when I_D is kept constant.

* the pinch off occurs when Voltage of drain is at $V_{GS} - V_{Th}$ if voltage of drain is increased further the effective length of mosfet is decreased and current is given by

$$I_D = \frac{\mu_n W}{2 L'} C_{ox} (V_{GS} - V_{Th})^2 \quad L' \rightarrow \text{effective length after pinch off}$$

* mobility of electron is twice the mobility of holes

> Transconductance of mosfet

$$* \quad g_m = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{\text{constant } V_{DS}} \Rightarrow \frac{\mu_n W}{L} C_{ox} (V_{GS} - V_{Th}) \Rightarrow \sqrt{2 \frac{\mu_n W}{L} C_{ox} I_D}$$

Body effect.

→ when Negative Voltage is applied to the body, more positive charge hole move towards the body which will increase the channel width. in order to balance this charge we need to add more positive charge to the gate. this will increase the V_{Th} of the transistor.

$$V_{Th} = V_{Th0} + \gamma \left[\sqrt{2\phi_f + \phi_{SB}} - \sqrt{2\phi_f} \right] \quad \gamma = \frac{\sqrt{2q\epsilon_s N_{sub}}}{C_{ox}}$$

→ γ range is from 0.3 to 0.4

channel length modulation.

→ The length of the channel will decrease when the voltage of the drain. increase. in other words the current i_d is function of V_{DS} . the effective change in length is given by.

$$L' = L - \Delta L. \quad \therefore \frac{L'}{L} = 1 - \frac{\Delta L}{L}$$

$$I_D = \frac{kW}{L \times \frac{L'}{L}} (V_{GS} - V_T)^2 \Rightarrow \frac{kW}{L \times \left[1 - \frac{\Delta L}{L}\right]} (V_{GS} - V_T)^2 \quad k \Rightarrow \frac{C_{ox} \mu_n}{2}$$

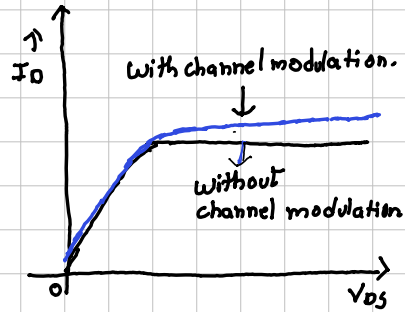
$$I_D \approx \frac{kW}{L} (V_{GS} - V_T)^2 \left[1 + \frac{\Delta L}{L}\right] \quad \frac{\Delta L}{L} \propto V_{DS} \Rightarrow \frac{\Delta L}{L} = \lambda V_{DS}$$

$$I_D \approx \frac{kW}{L} (V_{GS} - V_T)^2 [1 + \lambda V_{DS}]$$

→ λ is called channel modulation coefficient

→ the gm of the transistor is.

$$g_m = 2 \frac{kW}{L} (V_{GS} - V_T) [1 + \lambda V_{DS}] \quad k = \frac{C_{ox} \mu_n}{2}$$



→ This formula is less accurate for small channel mosfet. In short channel mosfet I_D is also dependent on V_{DS} .

Subthreshold conductance.

→ we have assumed that the current in mosfet is zero for $V_{GS} < V_T$ but this assumption is not right. There is current in the channel. This current is due to the weak inversion. & the current will rise exponentially with respect to V_{GS} . When subthreshold current reaches 100nA the gate voltage is taken as V_T of transistor.

$$I_D = I_0 e^{\frac{V_{GS}}{nV_T}} \quad I_0 \propto \frac{W}{L} \quad n \rightarrow \text{ideality factor} \quad V_T \rightarrow \text{thermal voltage.}$$

→ for voltage below V_T the current will drop 10 times per 80mV of voltage drop.

Mos device model :-

→ Parasitic caps of mosfet

- gate to channel $\rightarrow C_1$ (oxide).
- channel to substrate $\rightarrow C_2$ (depletion)
- gate to source & drain $\rightarrow C_3 \& C_4 \rightarrow$ (overlap).
- Substrate to source & drain $\rightarrow C_5 \& C_6 \rightarrow$ (junction)

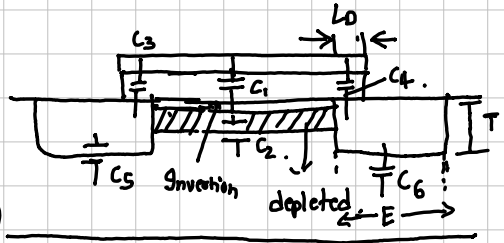


fig 1

$$C_1 = WL C_{ox} \quad C_{ox} = \epsilon / t_{ox}$$

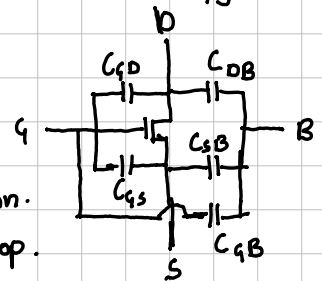
$$C_2 = WL \sqrt{q \epsilon_s N_{sub} / (4 \phi_F)}$$

$N_{sub} \rightarrow$ donor concentration.

$\phi_F \rightarrow$ voltage drop.

$$C_3 \& C_4 \Rightarrow C_{ox} L_0 \times W$$

$$\Rightarrow C_{ov} W \rightarrow C_{ov} \rightarrow \text{overlap cap per unit width.}$$



$C_5 \& C_6$

- $C_j \times A_{area} \rightarrow$ due to bottom plate. $\Rightarrow A_{area} = W \times L$ from fig 1.
- $C_{jsw} \times \text{Perimeter} \rightarrow$ due to side wall (perimeter) $\Rightarrow \text{perimeter} = 2(W+L)T$

$$C_j = C_{j0} / \left[1 + \frac{V_R}{\phi_B} \right]^m$$

$\rightarrow V_R \rightarrow$ reverse bias voltage.
 $\rightarrow \phi_B \rightarrow$ built in potential.
 $m \rightarrow 0.3 \text{ to } 0.4$

→ Similar for C_{jsw}

→ C_{jsw} for region marked in yellow from figure 2 may be different compared to region marked in pink. but we assume it is equal to other regions

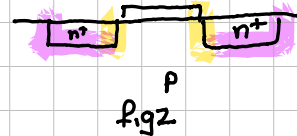


fig 2

Case I \Rightarrow when device is off $V_{GS} < V_{Th}$.

1) $C_{GD} = C_{GS} = C_{ov}W$

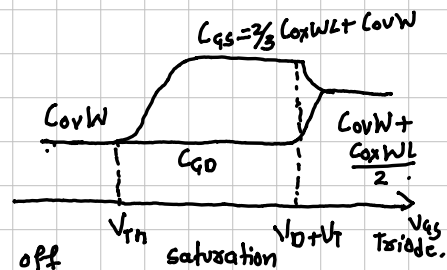
2) gate to bulk $\Rightarrow C_1 \parallel C_2$

3) $C_{DB}, C_{SB} \Rightarrow$ depends on bias voltage with substrate.

Case II :- when device is in triode, $V_{GS} > V_{Th}$ $V_S = V_D$

1) the gate to channel capacitance is divide between source & drain.

1) $C_{GD} = C_{GS} = C_{ov}W + \frac{C_{ox}WL}{2}$



Case III :- when device is in saturation.
 $V_{GS} > V_{Th}$ $V_{GS} - V_{Th} < V_{DS}$.

$$C_{GS} = \frac{2}{3}C_{ox}WL + C_{ov}W$$

$$C_{GD} = C_{ov}W$$

* the capacitance doesn't provide smooth transition from one region to other. this will create difficult to converge in simulation.

* C_{GB} is neglected in saturation & triode region. because of formation of channel which act as a shield between gate & bulk.

Mos small signal model

→ Small signal analysis is applied when perturbation is small & non-linear effect are not concerned.

→ drain current is the function of drain voltage which is modelled by

$$\delta_o = \frac{\partial V_{DS}}{\partial I_D} = \frac{1}{\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 \lambda} = \frac{1}{I_D \lambda}$$

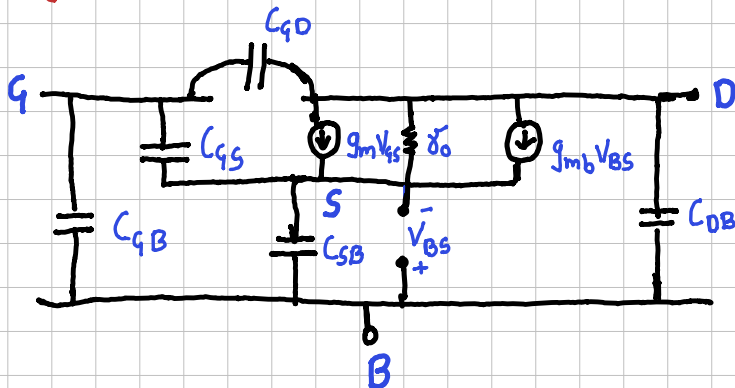
→ the bulk potential effect the threshold voltage of mosfet. It effect the current I_D . it behave as the second gate.

$$g_{mb} = \frac{\partial I_D}{\partial V_{BS}} = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T) \frac{\partial V_{TH}}{\partial V_{BS}}$$

$$\frac{\partial V_{TH}}{\partial V_{BS}} = -\frac{\partial V_{TH}}{\partial V_{SB}} = -\frac{\gamma}{2} (2\phi_F + V_{SB})^{-1/2}$$

$$g_{mb} = g_m \eta \quad \text{where } \eta = -\frac{\gamma}{2} (2\phi_F + V_{SB})^{-1/2}$$

Small signal model.



→ For pmos the small signal is similar to that of nmos.

level one Spice parameters:-

- * V_{TO} :- threshold voltage with zero $V_{SB} \rightarrow V$
- * γ :- body-effect coefficient $\rightarrow V^{1/2}$.
- * ϕ :- $2\phi_F \rightarrow V$
- * t_{ox} :- gate oxide thickness $\rightarrow m$.
- * N_{sub} :- Substrate doping $\rightarrow cm^{-3}$
- * L_D :- source/drain side diffusion $\rightarrow m$.
- * μ_0 :- channel mobility $\rightarrow cm^2/V$.
- * λ :- channel-length modulation coefficient $\rightarrow V^{-1}$
- * C_{j0} :- source/drain bottom-plate junction capacitance per unit area $\rightarrow F/m^2$
- * C_{jsw} :- sidewall junction capacitance per unit length $\rightarrow F/m$.
- * ϕ_B :- source/drain junction built-in potential $\rightarrow V$
- * m_j :- exponent in C_j equation \rightarrow unitless
- * m_{jsw} :- exponent in C_{jsw} equation \rightarrow unitless.
- * C_{gd0} :- gate-drain overlap capacitance per unit width $\rightarrow F/m$.
- * C_{gs0} :- gate-source overlap capacitance per unit width $\rightarrow F/m$.
- * J_s :- source/drain leakage current per unit area $\rightarrow A/m^2$

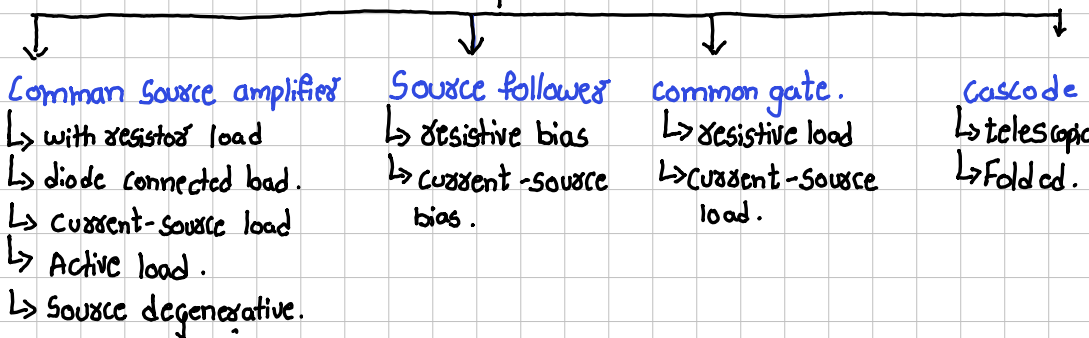
SINGLE STAGE AMPLIFIERS.

* The ideal input and output relation of amplifier $y(t) = a_0 + a_1 x(t)$.

* the real input & output relation of amplifier due to distortion is given by $y(t) = a_0 + a_1 x(t) + \underbrace{a_2 x^2(t) + a_3 x^3(t) + \dots}_{\text{higher order harmonics.}}$

* Parameters that used to evaluate the performance of amplifier are
1) speed 2) gain 3) maximum swing 4) power consumption
5) supply Voltage 6) linearity 7) input output impedance.

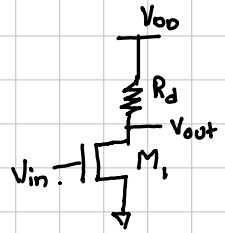
Amplifier categories



Common Source Stage. [CS].

* CS with Resistance.

$$\therefore V_{out} = V_{DD} - R_d \times \left[\frac{1}{2} \frac{C_{ox} W}{L} \mu_n (V_{GS} - V_{TH})^2 \right]$$



triode region $V_{out} < V_{in} - V_{TH}$.

$$A_v = \frac{\partial V_{out}}{\partial V_{in}} = -R_o \mu_n \frac{C_{ox} W}{L} (V_{in} - V_{TH}) = -g_m R_o.$$

- * Since g_m is function of V_{in} the swing of signal should be small to prevent non linear behaviour.

$$A_v = - \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D} \frac{V_{R_D}}{I_D} \Rightarrow - \sqrt{2 \mu_n C_{ox} \frac{W}{L}} \frac{V_{R_D}}{\sqrt{I_D}}$$

- * A_v can be increased by increasing W/L which in lead to increase in cap & high V_{DD} which will reduce the swing
- * if V_{DD} is kept constant and I_D is reduced which will increase R_D & the time constant
- * for large R_D the channel length modulation become significant.

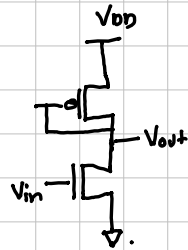
$$\frac{\partial V_{out}}{\partial V_{in}} = -R_D \frac{C_{ox} W}{L} \mu_n (V_{in} - V_{TH}) (1 + \lambda V_{DS}) - \frac{R_D C_{ox} W}{2L} (V_{in} - V_{TH})^2 \lambda \frac{\partial V_{out}}{\partial V_{in}}$$

$$A_v = -g_m R_D \frac{R_D}{\delta_o} A_v = A_v = -g_m [R_D \parallel \delta_o].$$

- * Maximum gain is achieved when $R_D = \infty$ ie $= -g_m \delta_o$ which is almost equal to 10.

CS stage with diode connected load.

It is difficult to fabricate resistor on chip so we use diode connected resistor to get resistor. the resistance of diode connected resistor is equal to.



$$r = \frac{1}{g_{mp} + g_{mbp}} \parallel r_{op} \approx \frac{1}{g_{mp} + g_{mbp}}$$

$$\therefore \text{gain } A_v = -g_{mn} r \parallel r_{on} = -g_{mn} \left(\frac{1}{g_{mp} + g_{mbp}} \parallel r_{on} \right) \approx \frac{-g_{mn}}{g_{mp} + g_{mbp}}$$

$$\therefore A_v = \frac{-g_{mn}}{g_{mp}(1 + \eta)} = \frac{\sqrt{\frac{W}{L} \times u_n}}{\sqrt{\frac{W}{L} \times u_p}} \times \frac{1}{(1 + \eta)} \quad \eta = \frac{g_{mbp}}{g_{mp}}$$

* the gain is independent of bias current or voltage it is more linear compared to previous amplifiers.

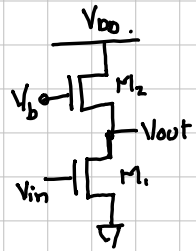
* gain is weak function of W/L if $u_n = 2u_p$ then $(W/L)_n = 12.5(W/L)_p$ to get gain of 5.

* In today technology channel length modulation can't be ignored \therefore gain is given by

$$A_v = -g_{mn} \left(\frac{1}{g_{mp} + g_{mbp}} \parallel r_{on} \parallel r_{op} \right)$$

CS Stage with Current Source.

- * to achieve higher gain we need to have larger R_o in order to achieve that we use current source.



$$A_v = -g_m (\delta_{o1} || \delta_{o2})$$

- to achieve larger swing V_{dsat} of M_2 should be less. this can be achieved by increasing width, but this will lead to reduction of δ_{o2} . then we have to change λ by increasing length to increase δ_{o2} . but increasing in width will also increase the capacitance at output node.

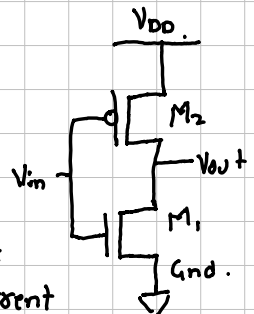
$$g_{m1} \delta_{o1} = \sqrt{2 \left(\frac{W}{L} \right) \mu_n C_{ox} I_0} \frac{1}{I_0 \lambda}$$

- * gain increase. with increasing L as λ is function of L .
- * gain increase with decrease in I_0
- * increase in length of M_2 will increase δ_{o2} but it will also increase the V_{dsat} of M_2 .

CS Stage with active load

- * the gain of the following circuit is

$$A_v = -(g_{m1} + g_{m2}) (\delta_{o1} || \delta_{o2})$$



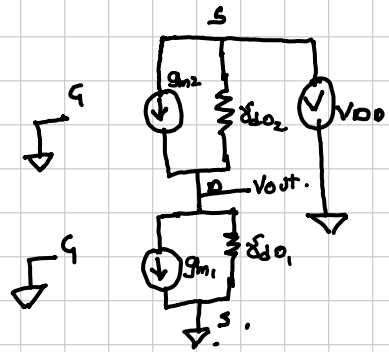
- * the following circuit has same impedance as current source circuit but it has higher gain compared to current source.
- * Since $V_{GS1} + |V_{GS2}| = V_{DD}$. the variation in V_{DD} or V_{TH} will translate.

to current at output. the bias point of the circuit depends upon the PVT. this is the drawback of the circuit.

* the output gain given by variation in supply

$$V_{out} = (g_{m2} V_{DD} \delta o_2 + V_{DD}) \frac{\delta d o_r}{\delta d o_2 + \delta d o_1}$$

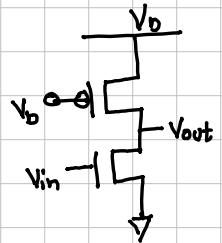
$$\therefore \frac{V_{out}}{V_{DD}} = \frac{(g_{m2} \delta o_2 + 1) \delta d o_r}{\delta d o_2 + \delta d o_1}$$



- CS with triode load.

* here mosfet is operating in deep triode.

$$R_{on} = \frac{1}{\frac{W}{L_2} \mu_n C_{ox} (V_{DD} - V_b - V_T)}$$



* the drawback of the following circuit is the R_{on} depend on V_b, V_T, W, L, V_{DD} which change with PVT. and it is difficult to produce constant bias voltage V_b .

CS with source degenerated.

* this configuration will make output more linear compared to other configuration. it make gain less dependent on g_m

* the output of this circuit is linearized but the gain is reduced due to the feedback.

$$G_m = \frac{\partial I_o}{\partial V_{in}} = \frac{\partial I_o}{\partial V_{GS}} \times \frac{\partial V_{GS}}{\partial V_{in}} \quad (1)$$

We know that

$$V_{GS} = V_{in} - I_o R_s$$

$$\therefore \frac{\partial V_{GS}}{\partial V_{in}} = \left(1 - \frac{\partial I_o}{\partial V_{in}}\right) R_s \Rightarrow (1 - G_m) R_s \quad (2)$$

Substitute 2 in (1).

$$G_m = g_m (1 - G_m) R_s \Rightarrow G_m = \frac{g_m}{1 + g_m R_s}$$

$$\therefore \text{Small signal gain} = -G_m R_o = \frac{-g_m R_o}{1 + g_m R_s} \approx \frac{-R_o}{R_s} \text{ if } g_m R_s \gg 1.$$

* Gain of source degenerative circuit with body effect and channel length modulation is given below.

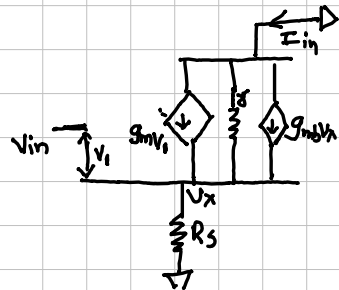
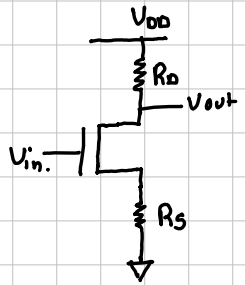
$$\therefore I_{out} = g_m V_i - g_{mb} V_x - \frac{I_{out} R_s}{\delta_o}$$

$$I_{out} = g_m (V_{in} - I_{out} R_s) + g_{mb} (-I_{out} R_s) - \frac{I_{out} R_s}{\delta_o}$$

$$G_m = \frac{g_m \delta_o}{R_s + [1 + (g_m + g_{mb}) R_s] \delta_o}$$

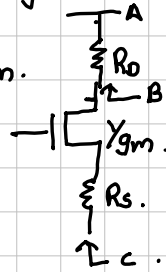
$$A_v = \frac{-g_m \delta_o \parallel R_{eq}}{R_s + [1 + (g_m + g_{mb}) R_s] \delta_o}$$

$$R_{eq} = R_o \parallel [R_s + \delta_o + (g_m + g_{mb}) \delta_o R_s]$$



finding gain by inspection for source degeneration

gain is equal to impedance looking into drain.
divide by the impedance looking into the source $\Rightarrow c$.

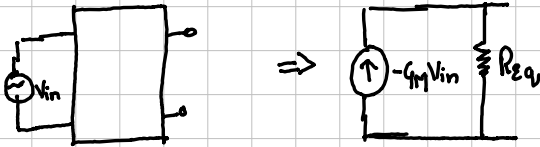


$$A_V = \frac{-R_D}{\frac{1}{g_m} + R_D} \Rightarrow \frac{R_{AB}}{R_{BC}} = \frac{-R_D}{\frac{1}{g_m} + R_S}$$

* after source degeneration output resistance is increase therefore output resistance is

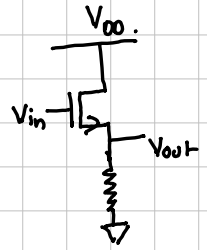
$$R_{out} = \left[\left(1 + (g_m + g_{mb}) R_D \right) r_o + R_S \right] \parallel R_D$$

* any circuit can be represented by noxtan equivalent circuit.
by short circuit current parallel with open circuit resistance.
and gain is given by $-g_m R_{eq}$.



Source follower \rightarrow Common drain.

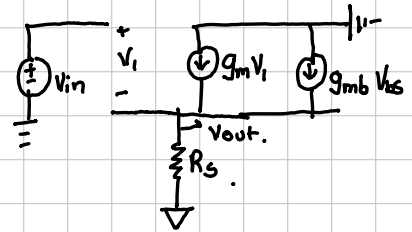
* To achieve large gain for common source we need to have large output resistance. When this is connected to next stage with low impedance, the effective impedance decreases, so the voltage gain. We can use source follower as a buffer between two stages. The gain of source follower is almost equal to 1.



$$V_{gs} = -V_{out}$$

$$V_{out} = R_s [g_m V_i - g_{mb} V_{out}]$$

$$V_{out} = R_s [g_m [V_{in} - R_s V_{out}] - g_{mb} V_{out}]$$



$$\frac{V_{out}}{V_{in}} = \frac{g_m R_s}{1 + [g_m + g_{mb}] R_s} \quad \text{if } (g_m + g_{mb}) R_s \gg 1 \approx \frac{g_m}{g_m + g_{mb}} \approx \frac{1}{1 + \eta} =$$

- * Even $R_s = \infty$ the output gain will not be equal to 1.
- * If V_{in} increases by $\sqrt{2}$, I_{out} & V_{out} will increase by 2. This will create non-linearity in gain. To resolve this issue, output resistance is replaced by a constant current source.
- * It has high input impedance & moderate output impedance.
- * It has low output headroom for swing.

Common gate.

$$* V_{out} = V_{DD} - \frac{1}{2} \mu_0 C_{ox} \frac{W}{L} (V_b - V_{in} - V_{Th})^2 R_D.$$

\therefore

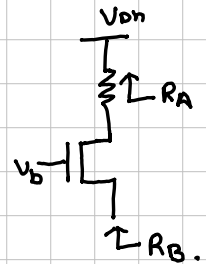
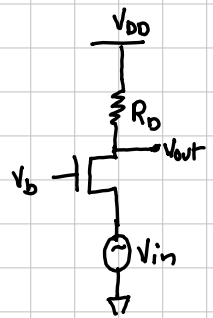
$$\frac{\partial V_{out}}{\partial V_{in}} = -\frac{1}{2} \mu_0 C_{ox} \frac{W}{L} (V_b - V_{in} - V_{Th}) \left(-1 - \frac{\partial V_{Th}}{\partial V_{in}} \right) R_D.$$

$$\frac{\partial V_{Th}}{\partial V_{in}} = \frac{\partial V_{Th}}{\partial V_{SB}} \frac{\partial V_{SB}}{\partial V_{in}} = \eta \times \frac{\partial V_{in}}{\partial V_{in}} = \eta$$

$$A_v = +\frac{1}{2} \mu_0 C_{ox} \frac{W}{L} (V_b - V_{in} - V_{Th}) (1 + \eta) R_D.$$

$$A_v = g_m (1 + \eta) R_D.$$

$$\text{gain by inspection} = \frac{\text{Impedence at drain } (R_D) - R_D}{\text{Impedence at source } (R_D) \times (g_m + g_{mb})} = (g_m + g_{mb}) R_D$$



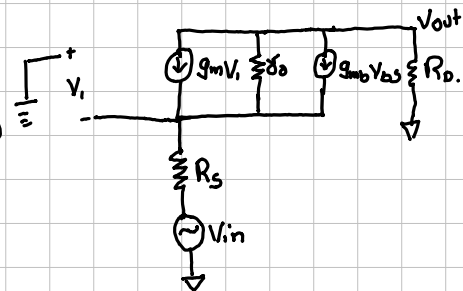
* gain considering δ_o & R_S .

$$V_i = +\frac{R_S V_{out}}{R_D} - V_{in}. \quad (1)$$

$$\delta_o \left[\frac{-V_{out}}{R_D} - g_m V_i - g_{mb} V_i \right] - \frac{V_{out} R_S + V_{in}}{R_D} = V_{out} \quad (2)$$

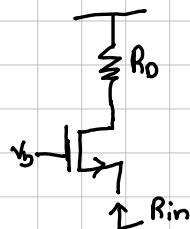
from 1 & 2.

$$A_v = \frac{(g_m + g_{mb}) \delta_o + 1}{\delta_o + (g_m + g_{mb}) \delta_o R_S + R_S + R_D} R_D$$



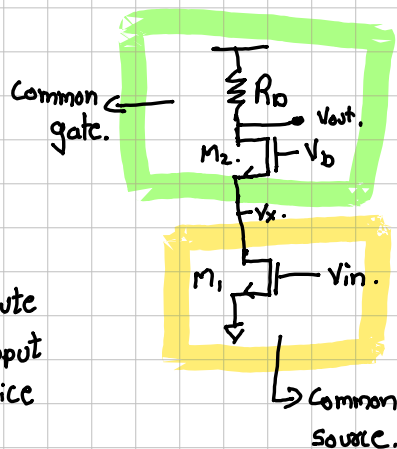
* Input impedance at the source is

$$R_{in} = \frac{R_D + r_o}{1 + (g_m + g_{mb})r_o}$$



Cascode stage.

* the input from common source is converted into current & this current is feed to common gate this type of configuration is called cascode stage



* M_1 provide the gain to small signal V_{in} & M_2 route the current to the R_D . device M_1 is called input device & device M_2 is called cascode. the device configuration is called telescopic cascode.

* let us consider if we apply ΔV voltage to V_{in} . and $r_o = \lambda = 0$. $r_{o2} = \infty$ then output current is $g_{m1}\Delta V$. the voltage at V_x is $(\frac{1}{g_{m2}})g_{m1}\Delta V$ & the voltage at the output is $R_D g_{m1}\Delta V$.

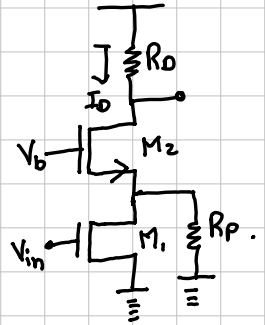
* let us consider if we keep V_{in} constant & vary V_b . the Mosfet M_1 act as a current source. the node X act as source follower which has gain Equal to 1. the node output doesn't change with. the input due to constant current is flowing in the circuit.

* To get both the device into saturation we need to have $V_{out} \geq V_{dsat1} + V_{dsat2}$.

$$A_v = -I_D R_D$$

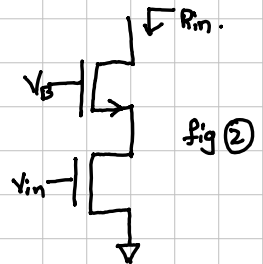
$$I_D = \frac{R_p}{R_p + \frac{1}{g_m + g_{mb}}} g_m V_{in}. \quad \text{Assume } \delta_{o2} = \infty$$

→ current divider between resistance at M_2 & R_p .



$$A_v = \frac{-R_p \times R_D}{R_p + \frac{1}{g_m + g_{mb}}} g_m V_{in}.$$

$$R_{in} \text{ of figure 2} = \delta_{o2} + \delta_{o1} + (g_{m2} + g_{mb}) \delta_{o2} \delta_{o1} \\ \approx (g_{m2} + g_{mb}) \delta_{o2} \delta_{o1}$$

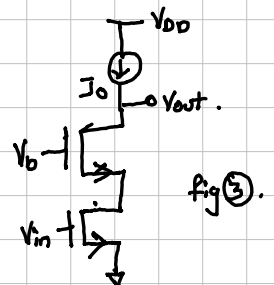


gain for figure 3 is

$$A_v = G_m R_{out}$$

$$R_{out} = \delta_{o1} + \delta_{o2} + (g_{m2} + g_{mb2}) \delta_{o2} \delta_{o1}.$$

$$I_{out} = \frac{\delta_{o2} g_m V_{in}}{\delta_{o2} + \left(\frac{1}{g_{m1} + g_{mb1}} \right) \parallel \delta_{o2}}.$$

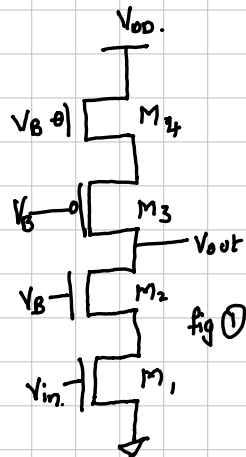


$$A_v = G_m R_{out} \Rightarrow g_{m1} \delta_{o1} \left[(g_{m2} + g_{mb2}) \delta_{o2} + 1 \right].$$

The output impedance of figure 1 is .

$$\left[\delta_{o4} + \delta_{o3} + (g_{m3} + g_{m_{b3}}) \delta_{o3} \delta_{o4} \right] \parallel \left[\delta_{o2} + \delta_{o1} + (g_{m2} + g_{m_{b2}}) \delta_{o1} \delta_{o2} \right]$$

$$\text{gain } A_v \approx .g_{m1} \left[(g_{m3} + g_{m_{b3}}) \delta_{o3} \delta_{o4} \parallel (g_{m2} + g_{m_{b2}}) \delta_{o1} \delta_{o2} \right]$$



* Since cascode has high impedance it can be used as shield for voltage variations. in the circuit.

Folded Cascode.

* Cascode circuit can also be made by using PMOS - NMOS or NMOS - PMOS configuration.

* the current $I_x = I_1 + I_2$

* In fig 2 V_{in} become more positive. I_1 decrease. forcing I_2 to increase. hence V_{out} to drop.

* if V_{in} (figure 2) decrease from V_{DD} to zero M_2 carries all current & $V_{in} < V_{DD} - V_{th}$. then $I_2 = I_x - I_1$, $V_{out} = V_{DD} - I_2 R_o$.

* if V_{in} is decreased further. $I_1 = I_x$ & $I_2 = 0$.

* if V_{in} falls below this level ie $I_1 > I_x$ then M_1 enters into triode region.

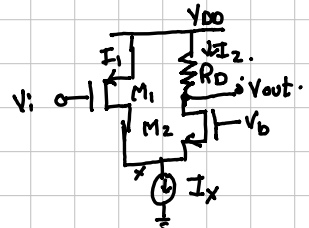


Fig 2

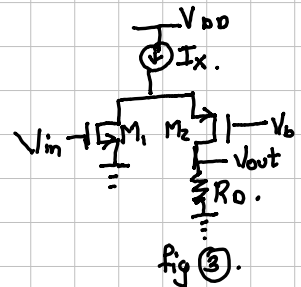


Fig 3

DIFFERENTIAL AMPLIFIERS

- * Single Ended :- output is taken between ground & the signal.
- * differential signal :- the signal is measured between two nodes. & the two nodes should have equal impedance to the potential. if single end peak to peak is V_o . the differential peak to peak is $2V_o$. the center point of differential signal is called common mode signal.
- * the advantage of differential over single end is it is immune to the noise. the noise coupling will be more in high frequency circuit. this noise can be eliminated by the differential signal.
- * differential will also reduce power coupling from V_{DD} to output. consider an example of common source where V_{DD} change. ΔV this will also lead to change at V_{out} . Since we are taking differential output at output this change can be cancelled.
- * differential signalling also increase the output swing. & make circuit highly linear. Some of the drawback of differential circuit is it consumes more area.

Basic differential pair

- * by our intuition. we can build differential amplifier as shown in figure 1 and V_{in1} & V_{in2} to V_{cm} . and applying differential input at V_{in1} & V_{in2} . this will give differential output at V_{out1} & V_{out2} . the problem with this circuit if there is any mismatch in. the bias point output will not be differential. this problem is resolved by using differential pair.

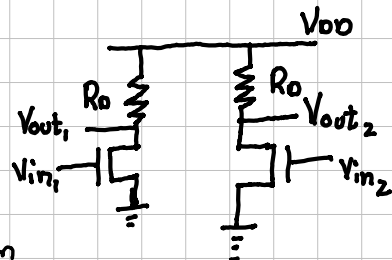
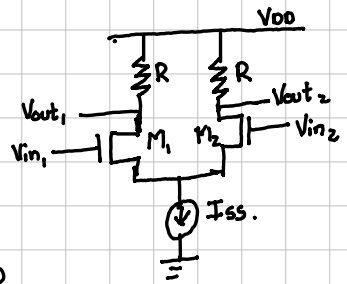


fig 1

* In differential pair bias is done by current source I_{SS} where $I_{SS} = I_1 + I_2$ independent of V_{in1} & V_{in2} : if $V_{in1} = V_{in2}$ then $I_1 = I_2 = I_{SS}/2$ and $V_{outcm} = V_{DD} - \frac{I_{SS}}{2} \times R$



* for large signal analysis we can neglect body & channel length modulation effects i.e. $\gamma = 0$ & $\lambda = 0$

Qualitative Analysis

→ let us assume $V_{in1} - V_{in2}$ varies $-\infty$ to ∞

Case I:- V_{in1} is more negative than V_{in2} .
then M_1 is off M_2 is ON, $V_{out1} = V_{DD}$.
 $V_{out2} = V_{DD} - I_{SS} R_D$

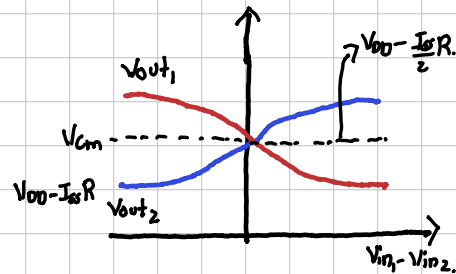


Fig - 1

* as we increase V_{in1} , M_1 starts conducting current in M_2 starts decreasing since $I_1 + I_2 = I_{SS}$ then V_{out1} will reduce & V_{out2} will increase.

Case II:- $V_{in1} = V_{in2}$
then $V_{out1} = V_{out2}$ i.e. $V_{DD} - \frac{I_{SS}}{2} R_D$.

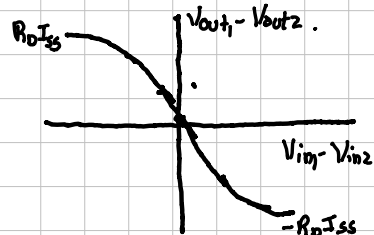


Fig - 2.

Case III:- V_{in1} is more positive than V_{in2} .
then M_1 carry more current than M_2 \therefore the voltage at V_{out1} is less than V_{out2} . if we further increase V_{in1} then M_2 will be off then $V_{out1} = V_{DD} - I_{SS} R_D$ & $V_{out2} = V_{DD}$.

* The maximum & minimum level are V_{DD} & $V_{DD} - I_{SS} R_D$ which are independent of input common mode voltage.

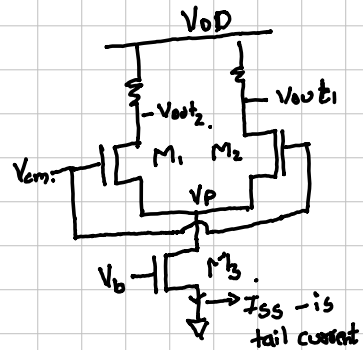
* gain will be maximum and linear when $V_{in_1} = V_{in_2}$ & become more nonlinear as $|V_{in_2} - V_{in_1}| \gg 0$. which can be seen from figure - 2 of previous page.

Common mode analysis of differential Amplifier.

* let us set $V_{in_1} = V_{in_2} = V_{cm}$ & vary V_{cm} from 0 to V_{DD} and Analysis the above circuit.

Case I:- $V_{cm} = 0$

When $V_{cm} = 0$ M_1 & M_2 are turned off V_{out_1} & $V_{out_2} = V_{DD}$ & transistor M_3 is in triode region. & $V_p = 0$ & circuit can't be used for amplification.



Case II:- V_{cm} is increased.

* When $V_p < V_b - V_{th}$: Transistor M_3 act as a resistor the circuit will behave has a source follower where $V_p = V_{cm} - V_{th}$. & the current I_1 & I_2 will increase with increase with V_{cm} .

* when $V_p > V_b - V_{th}$: if we further increase V_{cm} $V_p > V_b - V_{th}$ the transistor M_3 will be in saturation. this is the ideal condition for amplification. where current $I_1 + I_2 = I_{SS}$ & $V_{cm} \geq V_{GS_1} + V_{GS_2} - V_{th_3}$.

Case III:- V_{cm} is increased further

as we increase $V_{cm} > V_{out} + V_{th_1}$, the transistor M_1 & M_2 will come out of saturation. this set upper limit for V_{cm}

$$V_{GS} + V_{GS} - V_{th_3} \leq V_{cm} \leq \min \left[V_{DD} - \frac{I_{SS} R}{2} + V_{th_1}, V_{DD} \right]$$

* V_{cm} input is always less than V_{cm} output. if the gain of the circuit is higher input swing will be lower. output can go as high as V_{DD} & low as $V_{cm} - V_{th}$ for transistor to remain in saturation. the maximum peak to peak output swing is $V_{DD} - (V_{cm} - V_{th})$ i.e. $V_{DD} - (V_{GS1} + V_{GS3} - V_{th3} - V_{th1})$. = $V_{DD} - (V_{sat1} + V_{sat3})$.

Large Signal Analysis

* The output of the system is $V_{out1} - V_{out2}$. which is equal to $V_{DD} - I_1 R - (V_{DD} - I_2 R) \Rightarrow R(I_2 - I_1)$. & the voltage at node p [previous figure] is $V_{in1} - V_{GS1} = V_{in2} - V_{GS2}$ i.e. $V_{in1} - V_{in2} = V_{GS1} - V_{GS2}$.

$$V_{GS} = \sqrt{\frac{2I_D}{\mu_n C_{ox} W/L}} + V_{th}$$

$$V_{in1} - V_{in2} = \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} W/L}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox} W/L}}$$

Squaring on both side.

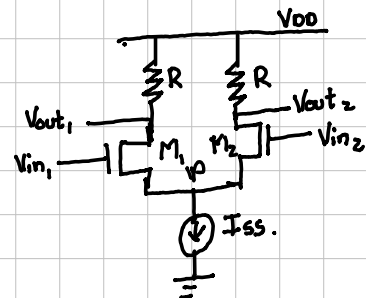
$$(V_{in1} - V_{in2})^2 = \frac{2}{\mu_n C_{ox} W/L} [I_{SS} - 2\sqrt{I_1 I_2}]$$

Squaring on both side. & applying $4I_1 I_2 = (I_1 + I_2)^2 - (I_1 - I_2)^2$:

$$\left[\frac{\mu_n C_{ox} W/L}{2} (V_{in1} - V_{in2})^2 - I_{SS} \right]^2 = (I_1 + I_2)^2 - (I_1 - I_2)^2$$

$$\Rightarrow \left(\frac{\mu_n C_{ox} W/L}{2} \right)^2 (V_{in1} - V_{in2})^4 + I_{SS}^2 - \mu_n C_{ox} W/L (V_{in1} - V_{in2})^2 I_{SS} = (I_{SS})^2 - (I_1 - I_2)^2$$

$$I_{D1} - I_{D2} = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS} (V_{in1} - V_{in2})} \sqrt{1 - \frac{\mu_n C_{ox} (W/L)}{4 I_{SS}} (V_{in1} - V_{in2})^2}$$



differentiating on both side.

$$\frac{\partial \Delta I_0}{\partial \Delta V_{in}} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[\frac{\frac{4I_{ss}}{\mu_n C_{ox} W/L} - 2\Delta V_{in}^2}{\sqrt{\frac{4I_{ss}}{\mu_n C_{ox} W/L} - \Delta V_{in}^2}} \right]$$

G_m is maximum when $\Delta V_{in} = 0$ i.e. $= \sqrt{\mu_n C_{ox} \frac{W}{L} I_{ss}}$

$$\Delta V_{out} = R \Delta I = R G_m \Delta V_{in}.$$

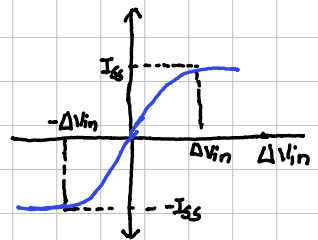
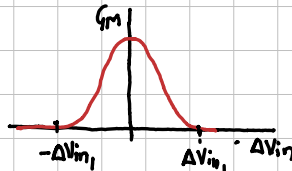
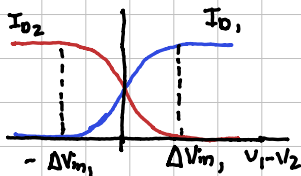
$$\frac{\Delta V_{out}}{\Delta V_{in}} = R G_m = R \sqrt{\mu_n C_{ox} (W/L) I_{ss}}$$

* Gain is same as gain of single device i.e. $R \sqrt{2 \times \mu_n C_{ox} \frac{W}{L} \frac{I_{ss}}{2}} = R g_m$.

* Gain is zero when $\Delta V_{in} = \sqrt{\frac{2I_{ss}}{\mu_n C_{ox} W/L}}$

* As we increase $|V_{in1} - V_{in2}|$ one of the transistor will go off other will draw all the current I_{ss} . \therefore the maximum ΔV_{in} change is

$$\Delta V_{in} = \sqrt{\frac{2I_{ss}}{\mu_n C_{ox} W/L}}.$$

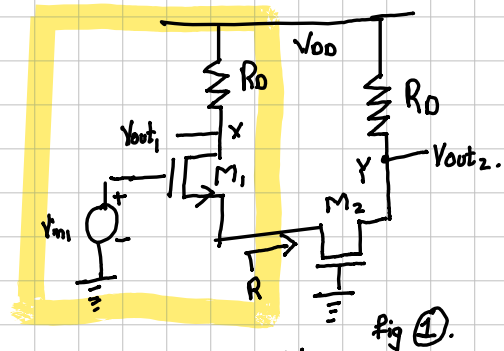


Small signal Analysis.

let us ground V_{in2} & find the effect of V_{in1} on V_{out1} & V_{out2} .

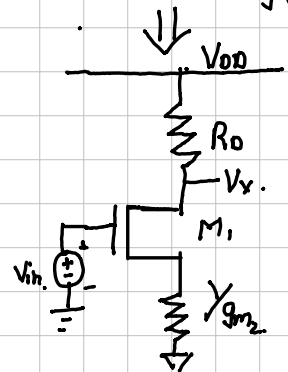
* from fig 2 we can see the output V_x can be found by considering it as common source amplifier

$$\frac{V_x}{V_{in}} = \frac{-R_D}{Y_{gm2} + Y_{gm1}} \quad (1)$$



* The circuit in the yellow box in fig 1 is an source follower circuit which can be modelled into Thevenin voltage of V_{in} with series with resistor of Y_{gm1} , as shown in figure 3 yellow box. and the circuit will behave as the common gate amplifier

$$\therefore \frac{V_y}{V_{in1}} = \frac{+R_D}{Y_{gm1} + Y_{gm2}} \quad (2)$$

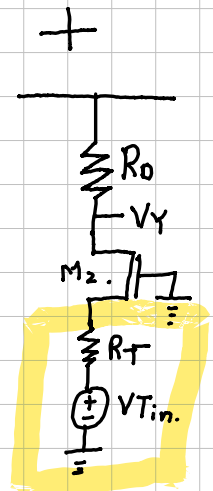


subtracting 2 from 1.

$$\frac{V_x - V_y}{V_{in1}} \Big|_{V_{in2}=0} = \frac{-2R_D}{Y_{gm1} + Y_{gm2}}$$

due to symmetry V_{in2} when $V_{in1}=0$ is same as above but put is superposition of both.

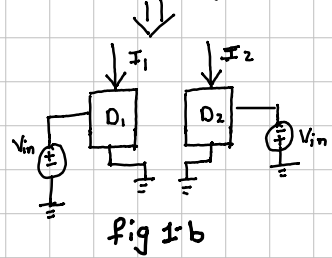
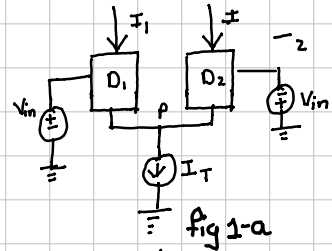
$$V_x - V_y = \frac{-2R_D}{Y_{gm1} + Y_{gm2}} [V_{in1} - V_{in2}]$$



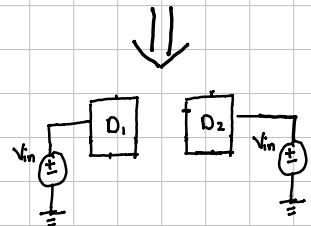
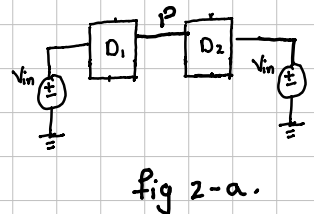
We know that $g_{m1} = g_{m2} = g_m$. when both transistor are matched

$$V_x - V_y = -g_m R_D [V_{in1} - V_{in2}]$$

Lemma I :- Consider a symmetric circuit where D_1 is symmetric to D_2 and both are linear if we apply differential signal to both the input the node p voltage is constant. for ac Analysis we can assume the node is at virtual ground. as shown in figure 1-b.



Lemma II :- Consider a symmetric circuit where D_1 is symmetric to D_2 and both are linear if we apply common mode signal to both the input. there is no current in Node p we can consider the circuit is open at p. which is shown in figure 2-b.

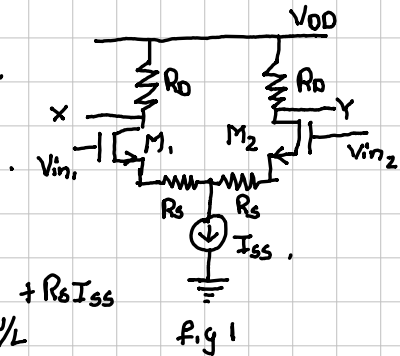


Degenerated Differential pair.

* this circuit make output more linearize.

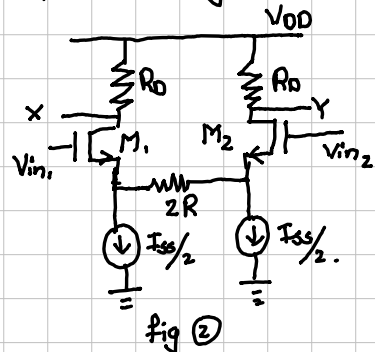
$$* \text{ gain} = \frac{R_o}{\frac{1}{g_m} + R_s}$$

$$* \text{ Maximum input range} = V_{in1} - V_{in2} = \frac{\sqrt{2I_{SS}}}{\sqrt{\mu_n C_{ox} W/L}} + R_s I_{SS}$$



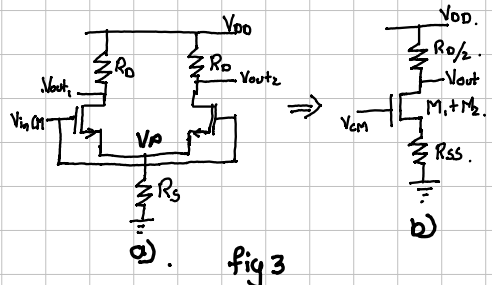
* The gain of the circuit is reduced. It has less headroom since there is drop $(I_{SS}/2)R_s$ in resistor. and gain is independent of g_m .

* The headroom can be increased. in the figure 2 where there is no current $2R$ when V_{in1} & V_{in2} are equal so the drop across $2R$ is zero. there will be drop only due to differential signal.



Common mode response

* Ideally common mode response should not effect the output response. but due difference in g_m of both the transistor or finite resistance of current mirror. fraction of CM Voltage will appear at the output.



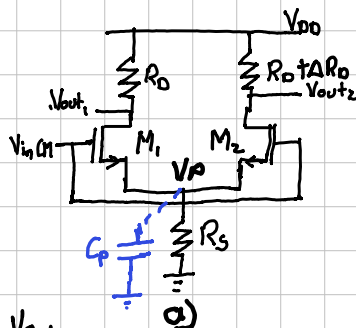
* let us assume the circuit is symmetric & has finite R_s as shown in the figure. 3-b. if V_{cm} is increased V_x & V_y decrease due to increase in current. which will change the small signal gain and the output swing the common signal gain is given by.

$$A_{cm} = \frac{V_{out_{cm}}}{V_{in_{cm}}} = \frac{R_o/2}{1/2g_m + R_s}$$

* let us study the mismatch of R_o on the output of the circuit.

V_p act as a source follower the change in V_p is given by

$$\Delta V_p = \frac{2R_s}{1/g_m + 2R_s} V_{in_{cm}}.$$



M_1 & M_2 act as Common source with source degenerative when output is taken at V_{out1} & V_{out2} .

$$\Delta V_{out1} = -\Delta V_{cm} \frac{R_o}{1/g_m + 2R_s}$$

$$\Delta V_{out2} = -\Delta V_{cm} \frac{R_o + \Delta R_o}{1/g_m + 2R_s}$$

* If the common mode noise for higher frequency input. due to the parasitic capacitance $[C_p]$ of current mirrors will be high at output is high due to the low impedance path created by cap

The Effect of Mismatch of transconductance on the output.

* The mismatch is mainly due the variation in Length, width, $\propto V_T$ of the mosfet the current in the each Mos fet is given by

$$I_{D1} = g_{m1}(V_{in_{cm}} - V_P) \quad \& \quad I_{D2} = g_{m2}(V_{in_{cm}} - V_P), \quad (I_{D1} + I_{D2})R_S = V_P$$

$$\therefore (g_{m1} + g_{m2})(V_{in_{cm}} - V_P)R_{SS} = V_P$$

$$V_P = \frac{(g_{m1} + g_{m2})R_{SS}}{(g_{m1} + g_{m2})R_{SS} + 1} V_{in_{cm}}.$$

Output Voltage is given by

$$V_X = -g_{m1}(V_{cm} - V_P)R_D$$

$$V_X = \frac{-g_{m1} \times R_D V_{in_{cm}}}{(g_{m1} + g_{m2})R_{SS} + 1}, \quad \& \quad V_Y = \frac{-g_{m2} R_D V_{in_{cm}}}{(g_{m1} + g_{m2})R_{SS} + 1}$$

$$V_X - V_Y = \frac{-[g_{m1} - g_{m2}]R_D V_{in_{cm}}}{(g_{m1} + g_{m2})R_{SS} + 1} = \frac{-\Delta g_m R_D V_{in_{cm}}}{(g_{m1} + g_{m2})R_{SS} + 1}.$$

* Common Mode rejection ratio :- It is the ratio of differential gain with the common mode gain. for ideal op-amp CMRR will be ∞

$$CMRR = \left| \frac{A_{DM}}{A_{CM}} \right|$$

* differential gain. of Amplifier with different g_m is

$$|A_{DM}| = \frac{R_D}{2} \frac{g_{m1} + g_{m2} + 4g_{m1}g_{m2}R_{SS}}{1 + (g_{m1} + g_{m2})R_{SS}}.$$

$$\therefore CMRR = \frac{g_{m1} + g_{m2} + 4g_{m1}g_{m2}R_{ss}}{2\Delta g_m}$$

$$\approx \frac{g_m(1 + 2g_m R_{ss})}{\Delta g_m} \quad \text{where } g_m = \frac{g_{m1} + g_{m2}}{2}$$

Differential pair with mos load.

* The gain of the following circuit is

$$A_v = -g_{m1} \left[\frac{1}{g_{m3}} \parallel r_{o3} \parallel r_{o1} \right]$$

$$\approx \frac{-g_{m1}}{g_{m3}}$$

