Final project

Part 1 – Probability

.1

א. בערך 1/125 מהלידות זה תאומים לא זהים ו-1/300 מהלידות זה תאומים זהים. לאלביס היה את תאום שמת בלידה. מה ההסתברות שאלביס היה תאום זהה? (ניתן להניח שההסתברות להולדת בן ובת שווה ל-1/2).

ב. יש שתי קערות של עוגיות. בקערה 1 יש 10 עוגיות שקדים ו-30 עוגיות שוקולד. בקערה 2 יש 20 עוגיות שקדים ו-20 עוגיות שוקולד. אריק בחר קערה **באקראי** ובחר ממנה עוגיה **באקראי**. העוגיה שנבחרה היא שוקולד. מה ההסתברות שאריק בחר את קערה 1?

Exercise 1-

$\boldsymbol{a})$

$$P_{Birth\ of\ non-identical\ twins} = \frac{1}{125}$$

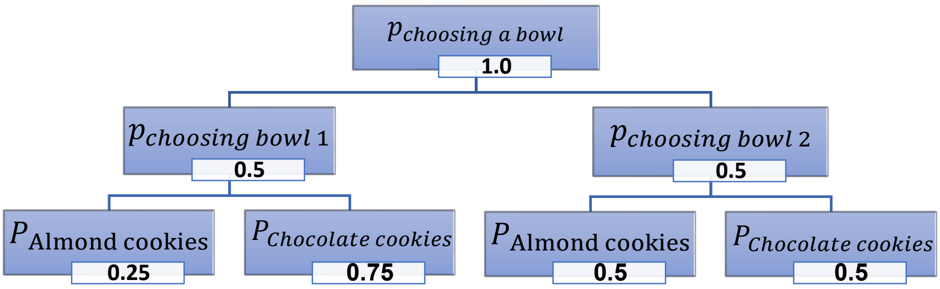
$$P_{Birth\ of\ identical\ twins} = \frac{1}{300}$$

$$P_{Birth\ of\ twins} = P_{Birth\ of\ identical\ twins} + P_{Birth\ of\ non-identical\ twins} = \frac{17}{1500}$$

$$P_{Elvis\ had\ an\ identical\ twin} = \frac{P_{Birth\ of\ identical\ twins}}{P_{Birth\ of\ twins}} = \frac{\frac{1}{300}}{\frac{17}{1500}} = \frac{5}{17}$$

(Because we know for sure that he had a twin brother.)

$\boldsymbol{b})$



$$P_{chocolate\ cookie} = P_{chocolate\ cookie(bowl\ 1)} + P_{chocolate\ cookie\ (bowl\ 2)} = \frac{1}{2} * \frac{1}{2} + \frac{1}{2} * \frac{3}{4} = \frac{5}{8}$$

$$P_{chocolate\ cookie\ (bowl\ 1)} = \frac{P_{chocolate\ cookie\ (bowl\ 1)}}{P_{chocolate\ cookie}} = \frac{\frac{3}{8}}{\frac{5}{8}} = \frac{3}{5}$$

.2

בשנת 1995 חברת M&M הוסיפה את הצבע כחול. לפני השנה הזו, התפלגות הצבעים
בשקית M&M נראית כך:

30% Brown, 20% Yellow, 20% Red, 10% Green, 10% Orange, 10% Tan

החל משנת 1995, ההתפלגות נראית כך:

24% Blue , 20% Green, 16% Orange, 14% Yellow, 13% Red, 13% Brown.

לחבר שלכם יש 2 שקיות M&M, אחת משנת 1994 ואחת משנת 1996 והוא לא מוכן לגלות לכם איזו שקית שייכת לאיזו שנה. אבל הוא נותן לכם סוכריה אחת מכל שקית. סוכריה אחת היא צהובה ואחת היא ירוקה. מה הסיכוי שהסוכריה הצהובה הגיעה מהשקית של 1994?

<u>Exercise 2-</u>

This table shows the distributions of the color in MNM before 1994 and after this year:

	Yellow	*Red*	*Green*	*Orange*	*Tan*	*Blue*	*Brown*
1994	0.2	0.2	0.1	0.1	0.1	0	0.3
1996	0.14	0.13	0.2	0.16	0	0.24	0.13

$$P_{\text{choosing a yellow candy before }1994} = \frac{20}{100} * \frac{1}{2} = \frac{1}{10}$$

$$P_{\text{choosing a yellow candy}} = P_{\text{choosing a yellow candy before }1994} + P_{\text{choosing a yellow candy after }1994}$$
$$= \frac{1}{10} + \left(\frac{14}{100} * \frac{1}{2}\right) = \frac{1}{10} + \frac{14}{200} = \frac{17}{100}$$

$$P_{\text{choosing a green candy after }1994(1996)} = \frac{20}{100} * \frac{1}{2} = \frac{1}{10}$$

$$P_{\text{choosing a green candy}} = P_{\text{choosing a green candy after }1994(1996)} + P_{\text{choosing a green candy before }1994}$$
$$= \frac{1}{10} + \left(\frac{1}{2} * \frac{1}{10}\right) = \frac{1}{10} + \frac{1}{20} = \frac{15}{100}$$

$$P_{\text{yellow candy from }1994} = P_{\text{choosing a green candy after }1994(1996)} * P_{\text{choosing a yellow candy before }1994}$$
$$= \frac{P_{\text{choosing a green candy after }1994(1996)}}{P_{\text{choosing a green candy}}} * \frac{P_{\text{choosing a yellow candy before }1994}}{P_{\text{choosing a yellow candy}}}$$
$$= \frac{\frac{1}{10}}{\frac{15}{100}} * \frac{\frac{1}{10}}{\frac{17}{100}} = \frac{20}{51}$$

3. הלכת לדוקטור בעקבות ציפורן חודרנית. הדוקטור בחר בך **<u>באקראי</u>** לבצע בדיקת דם הבודקת שפעת חזירים. ידוע סטטיסטית ששפעת זו פוגעת ב-1 מתוך 10,000 אנשים באוכלוסייה. הבדיקה מדויקת ב-99 אחוז במובן שההסתברות ל false positive היא 1%. הווה אומר שהבדיקה סיווגה בטעות אדם בריא כאדם חולה היא 1 אחוז. ההסתברות ל- false negative היא 0 – אין סיכוי שהבדיקה תגיד על אדם החולה בשפעת חזירים שהוא בריא. בבדיקה יצאת חיובי (יש לך שפעת).

א. מה ההסתברות שיש לך שפעת חזירים?

ב. נניח שחזרת מתאילנד לאחרונה ואתה יודע ש-1 מתוך 200 אנשים שחזרו לאחרונה מתאילנד, חזרו עם שפעת חזירים. בהינתן אותה סיטואציה כמו בשאלה א, מה ההסתברות (המתוקנת) שיש לך שפעת חזירים?

<u>Exercise 3</u>

First, we need to organize all the data we got, in order to make our way to the answer more easily.

	Wrong	Right
Sick	0%	100%
Healthy	0.01%	0.99%

$$P_{Sick} = \frac{1}{10,000}$$

$$P_{healthy} = 1 - P_{Sick} = 1 - \frac{1}{10,000} = \frac{9,999}{10,000}$$

$$P_{True\ Positive} = \frac{1}{10,000}$$

$$P_{False\ Positive} = \frac{9,999}{10,000} * \frac{1}{100} = 0.009999$$

$$P_{Positive} = P_{False\ Positive} + P_{True\ Positive} = \frac{1}{10,000} + 0.009999 = 0.010099$$

$\boldsymbol{a})$

$$P_{I\ rlly\ sick} = \frac{P_{True\ Positive}}{P_{Positive}} = \frac{\frac{1}{10000}}{0.010099} = \frac{100}{10,099} = 0.00990197$$

$\boldsymbol{b})$

$$P_{Sick} = \frac{1}{200}$$

$$P_{healthy} = 1 - P_{Sick} = 1 - \frac{1}{200} = \frac{199}{200}$$

$$P_{True\ Positive} = \frac{1}{200}$$

$$P_{False\ Positive} = \frac{199}{200} * \frac{1}{100} = \frac{199}{20,000} = 0.00995$$

$$P_{Positive} = P_{False\ Positive} + P_{True\ Positive} = \frac{1}{200} + \frac{199}{20,000} = \frac{299}{20,000} = 0.01495$$

$$P_{I\ rlly\ sick} = \frac{P_{True\ Positive}}{P_{Positive}} = \frac{\frac{1}{200}}{\frac{299}{20,000}} = \frac{100}{299} = 0.3344481605$$

Exercise 4-

1. Roi is playing a dice game with Yael.

Roi will roll 2 six-sided dice, and if the sum of the dice is divisible by 3, he will win 6$. If the sum is not divisible by 3, he will lose 3$.

What is Roi's expected value of playing this game?

First, we will check which sum of the numbers that Roi got are divisible by 3.

The number that dividing by 3 are: 3, 6, 9, 12.

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Which means that the probability of Roi to win this game stand on $\frac{4}{12} = \frac{1}{3}$.

However, the probability of Roi to lose stand on $\frac{8}{12} = \frac{2}{3}$

Therefore, we can conclude that if Roi win a game he getting 6$ but while he winning a game he loosing 2 games, which means he loosing 6$.

Then, Roi's expected value of plating this game stand on 0$.

Exercise 5-

2. Sharon has challenged Alex to a round of Marker Mixup. Marker Mixup is a game where there is a bag of 5 red markers numbered 1 through 5, and another bag with 5 green markers numbered 6 through 10.

Alex will grab 1 marker from each bag, and if the 2 markers add up to more than 12, he will win 5$, 5. If the sum is exactly 12, he will break even, and If the sum is less than 12, he will lose 6$.

What is Alex's expected value of playing Marker Mixup?

	6	7	8	9	10
1	7	8	9	10	11
2	8	9	10	13	12
3	9	10	11	12	13
4	10	11	12	13	14
5	11	12	13	14	15

The probability of Alex to win 5$ standing on: $\frac{6}{25}$

The probability of Alex to not earn nothing standing on: $\frac{4}{25}$

The probability of Alex to lose 6$ standing on: $\frac{15}{25} = \frac{3}{5}$

From this data we can conclude that Alex will lose more then he will gain. More than that, he will be losing 15 games, win 6 games and didn't lose and didn't win 4 games every 25 rounds.

From this data he will:

$$(5 * 6) + (0 * 4) - (15 * 6) = -60$$

Which means, that every 25 games he will lose 60$.

3. A division of a company has 200 employees, 40%, percent of which are male. Each month, the company randomly selects 8 of these employees to have lunch with the CEO.

What are the mean and standard deviation of the number of males selected each month?

$$P_{male} = \frac{Total\ employees * \ Percentage\ of\ men}{Total\ employees} = \frac{200 * 0.4}{200} = \frac{80}{200} = \frac{2}{5}$$

$$P_{female} = 1 - \ P_{male} = 1 - \frac{2}{5} = \frac{3}{5}$$

$$mean = 8 * \frac{2}{5} = 3.2$$

I will use the following formula:

$$\sigma = \sqrt{\frac{\sum_{i=0}^{n}(x_i - \mu)^2}{n}}$$

First I'm going to calculate the sigma:

$$\sum_{i=0}^{n}(x_i - \mu)^2 = \sum_{i=0}^{8}(x_i - 3.2)^2 = 65.76$$

Calculating the Std:

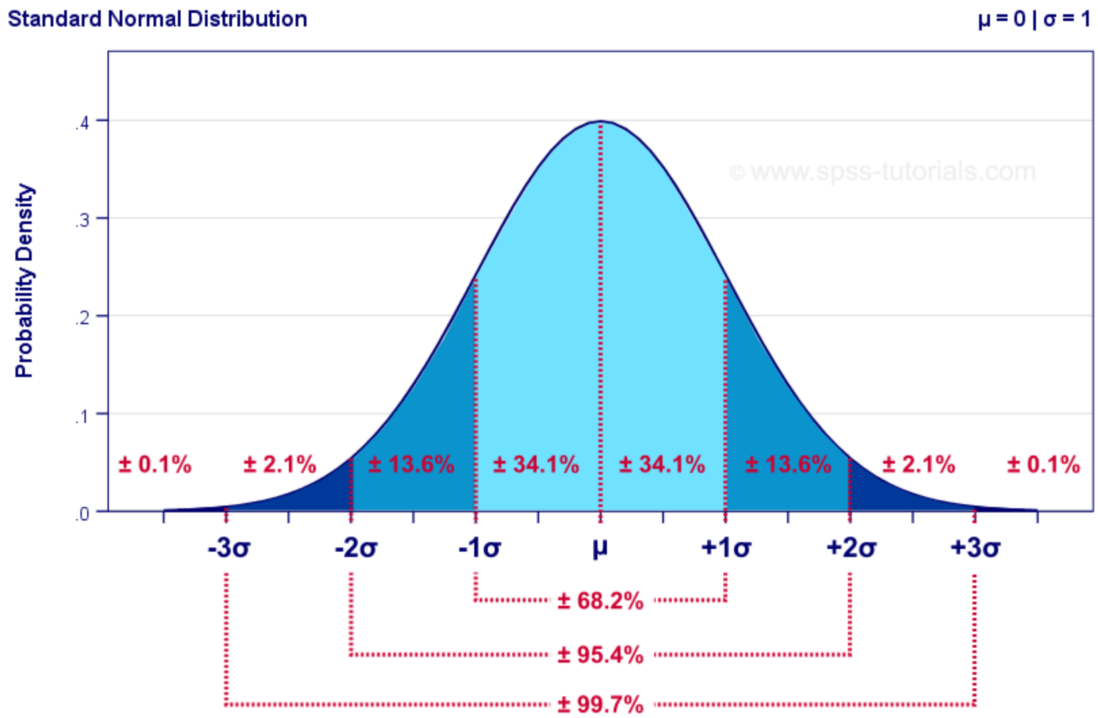
$$std = \sqrt{\frac{65.76}{8}} = \sqrt{8}.22 = 2.867$$

Which means, the std is 2.867.

4. Different dealers may sell the same car for different prices. The sale prices for a particular car are normally distributed with a mean and standard deviation of 26,000$ and 2,000$, respectively. Suppose we select one of these cars at random. Let X = the sale price (in thousands of dollars) for the selected car.

Find P(26<X<30),

I'm going to use the normal distribution's graph below:



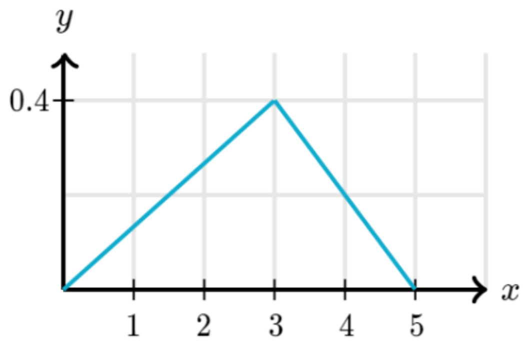
At first lets calculate:

- mean = 26,000
- standard deviation = 2

in addition when x in range $\mathbf{26,000 < x < 30,000 = \mu < x < \mu + 2\sigma} = 34.1\% + 13.6\% = 47.7\%$

which means that the $P_{26<x<30} = 0.477$.

5. Given the following distribution, what is P(x>3)?



$i\ am\ going\ to\ calculate\ the\ distribution\ by\ triangle's\ area\ formula$:
$height = 0.4$
$triangle\ side = 5 - 3 = 2$

$$S_{triangle} = \frac{2 * 0.4}{2} = 0.4$$

Which means that $P_{x>3} = 0.4$

But first we have to check if the big-triangle's area equal to 1:

$height = 0.4$
$triangle\ side = 5 - 0 = 1$

$$S_{big\_triangle} = \frac{5 * 0.4}{2} = 1$$

Now we can say for sure that $P_{x>3} = 0.4$.

6. A company has 500 employees, and 60% of them have children. Suppose that we randomly select 4 of these employees.

   What is the probability that exactly 3 of the 4 employees selected have children?

$P_{employees\ with\ children} = \frac{3}{5}$

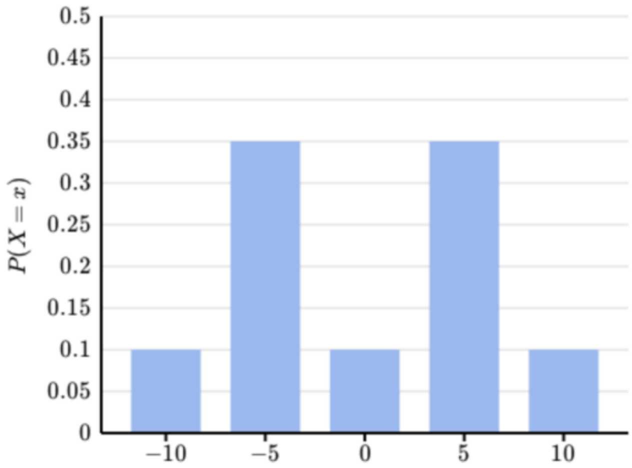
I am going to use 'Bernoli's formula':

$$P_{A_k} = \binom{n}{k} * P^k * (1-P)^{n-k}$$

Calculate:

$$P_{exacly\ 3} = \binom{4}{3} * \left(\frac{3}{5}\right)^3 * \left(1-\frac{3}{5}\right)^{4-3} = 4 * 0.216 * 0.4 = 0.3456$$

7. Look at the next Graph. What is the expected value of X?



Lets calculate x's value:

$$x_{expected} = -10 * 0.1 + (-5) * 0.35 + 0 * 0.1 + 5 * 0.35 + 10 * 0.1 = 0$$

Which means that the expected value of x is 0.