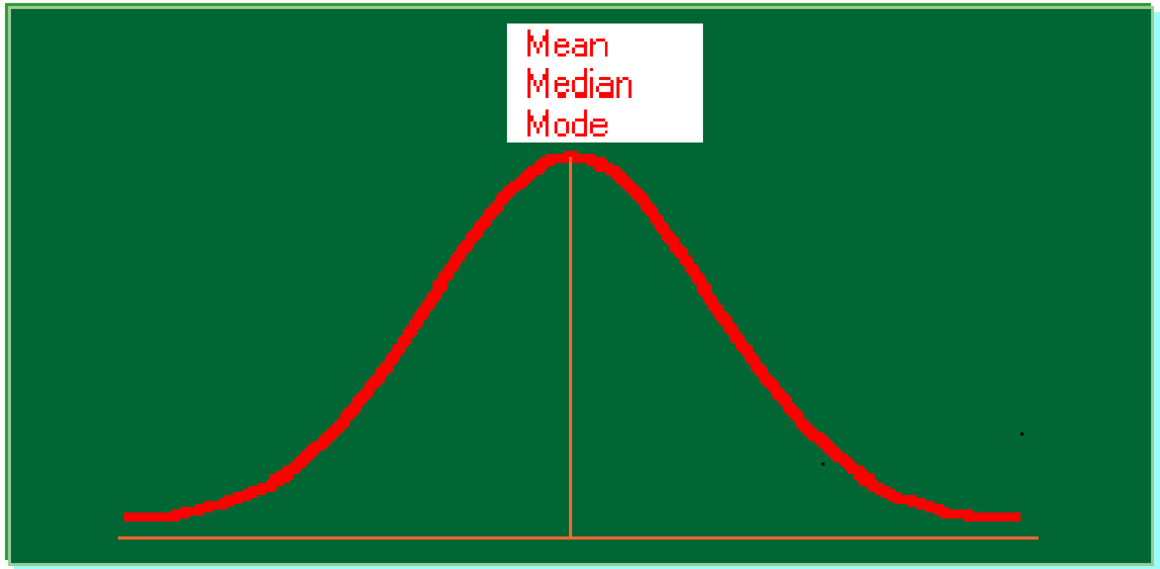
Statistical Methods for Decision Making

Probability Distributions

What is Probability Distribution?

- In precise terms, a probability distribution is a total listing of the various values the random variable can take along with the corresponding probability of each value. A real life example could be the pattern of distribution of the machine breakdowns in a manufacturing unit.
- The random variable in this example would be the various values the machine breakdowns could assume.
- The probability corresponding to each value of the breakdown is the relative frequency of occurrence of the breakdown.
- The probability distribution for this example is constructed by the actual breakdown pattern observed over a period of time. Statisticians use the term “observed distribution” of breakdowns.

Normal Distribution



- The Normal Distribution is the most widely used continuous distribution. It occupies a unique place in the field of statistics. In fact, the entire quality control function that employs the statistical techniques heavily will come to a grinding halt without the use of the normal distribution. The control charts for reducing and stabilizing variation relies on the normal distribution. Process capability studies to meet the customer specifications needs the normal distribution. The whole subject matter inferential statistics is based on the normal distribution. In all management functions including the human side, the observed frequency distributions encountered are all fairly close to the normal distribution when the sample size is reasonably large.

Properties of Normal Distribution

- The normal distribution is a continuous distribution looking like a bell. Statisticians use the expression “Bell Shaped Distribution”.
- It is a beautiful distribution in which the mean, the median, and the mode are all equal to one another.
- It is symmetrical about its mean.
- If the tails of the normal distribution are extended, they will run parallel to the horizontal axis without actually touching it. (asymptotic to the x-axis)
- The normal distribution has two parameters namely the mean μ and the standard deviation σ

Normal Probability Density Function

In the usual notation, the probability density function of the normal distribution is given below:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left[\frac{(x-\mu)^2}{2\sigma^2}\right]}$$

x is a continuous normal random variable with $-\infty < x < \infty$.

Standard Normal Distribution

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- The Standard Normal Variable is defined as follows:

$$z = \frac{x - \mu}{\sigma}$$

- Please note that Z is a pure number independent of the unit of measurement. The random variable Z follows a normal distribution with mean=0 and standard deviation =1.

$$f(Z) = \frac{1}{\sqrt{2\pi}} e^{-\left[\frac{z^2}{2}\right]}$$

Example Problem

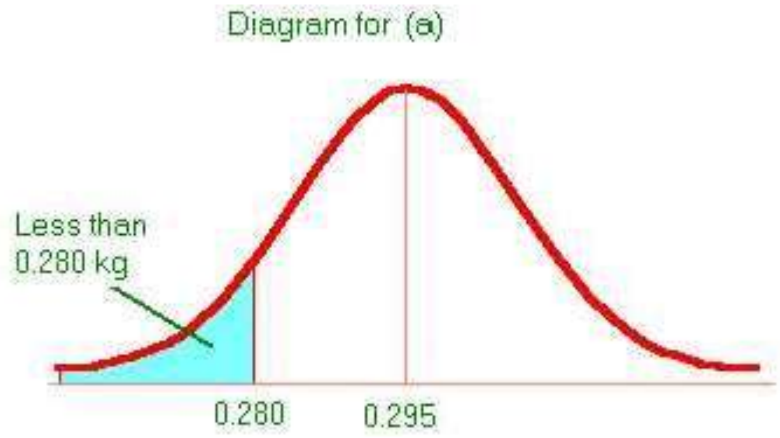
The mean weight of a morning breakfast cereal pack is 0.295 kg with a standard deviation of 0.025 kg. The random variable weight of the pack follows a normal distribution.

a)What is the probability that the pack weighs less than 0.280 kg?

b)What is the probability that the pack weighs more than 0.350 kg?

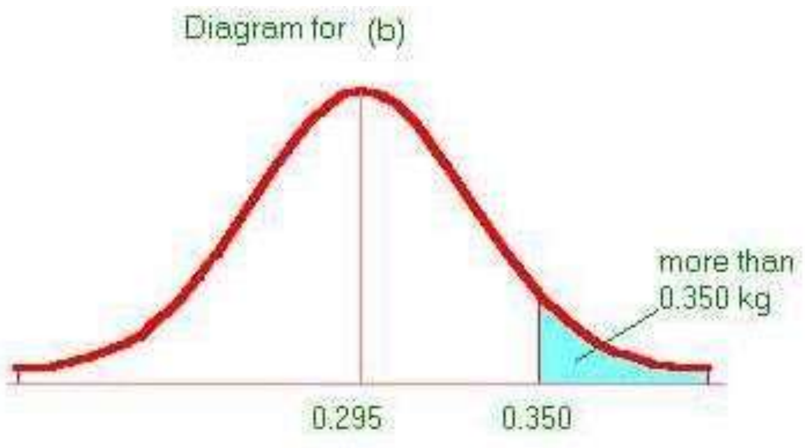
c)What is the probability that the pack weighs between 0.260 kg to 0.340 kg?

Solution-a)



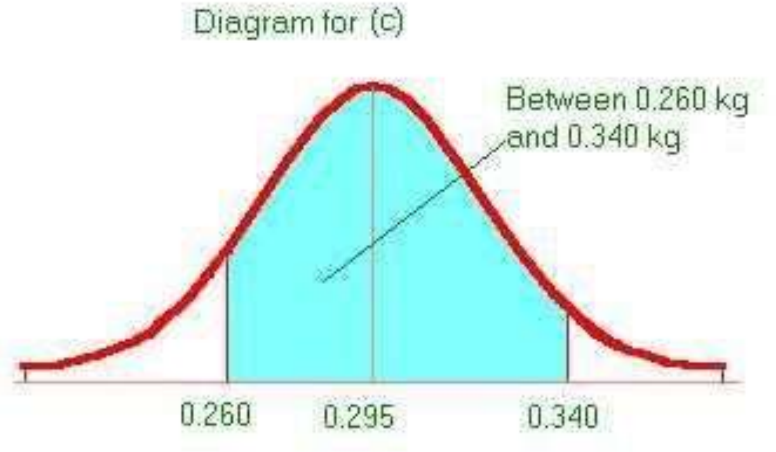
z = $\dfrac{x - \mu}{\sigma}$ = (0.280-0.295)/0.025 = -0.6. Click "Paste Function" of Microsoft Excel, then click the "statistical" option, then click the standard normal distribution option and enter the z value. You get the answer. Excel accepts directly both the negative and positive values of z. Excel always returns the cumulative probability value. When z is negative, the answer is direct. When z is positive, the answer is =1- the probability value returned by Excel. The answer for part a) of the question = 0.2743(Direct from Excel since z is negative). This says that 27.43 % of the packs weigh less than 0.280 kg.

Solution-b)



z = $\frac{x-\mu}{\sigma}$ = (0.350-0.295)/0.025 =2.2. Excel returns a value of 0.9861. Since z is positive, the required probability is = 1-0.9861 =0.0139. This means that 1.39% of the packs weigh more than 0.350 kg.

Solution-c)



For this part, you have to first get the cumulative probability up to 0.340 kg and then subtract the cumulative probability up to 0.260. $z = \frac{x-\mu}{\sigma}$ = (0.340-0.295)/0.025 =1.8(up to 0.340)

$z = \frac{x-\mu}{\sigma}$ = (0.260-0.295)/0.025 = -1.4(up to 0.260). These two probabilities from Excel are 0.9641 and 0.0808 respectively. The answer is = 0.9641 - 0.0808 = 0.8833. This means that 88.33% of the packs weigh between 0.260 kg and 0.340 kg.

Example 2

A company produces a bolt of length 10mm for its customers   The bolts produces are normally distributed with average length of 10.01mm & standard deviation 0.06mm

(a) What is the probability that the bolt produced will be longer than 10.2 mm

(b)The sales team is negotiating with a new customer who has more stringent quality requirements. The new customer requires bolts shall be between 9.9 and 10.15 mm. What is the probability that a bolt produces by the current process will be acceptable to the new customer

(c) What is the length for which 99% of bolts produced will be less that the length?

Solution a)

The value required is greater than 10.2 (i.e. right tail)

The answer is 0.00077 = 0.077% probability

Solution b)

Since the probability between 10.15 and 9.90 is needed, we need to subtract the cumulative probability of 9.90 from the cumulative probability of 10.15

Answer: 0.9568 i.e. 95.68% chance of meeting customer criteria.

Solution c)

In this question the probability is given and we are required to calculate the value below which (to left of which) 99% of area lies.

Answer: 99% of bolts will be smaller than 10.14958 mm