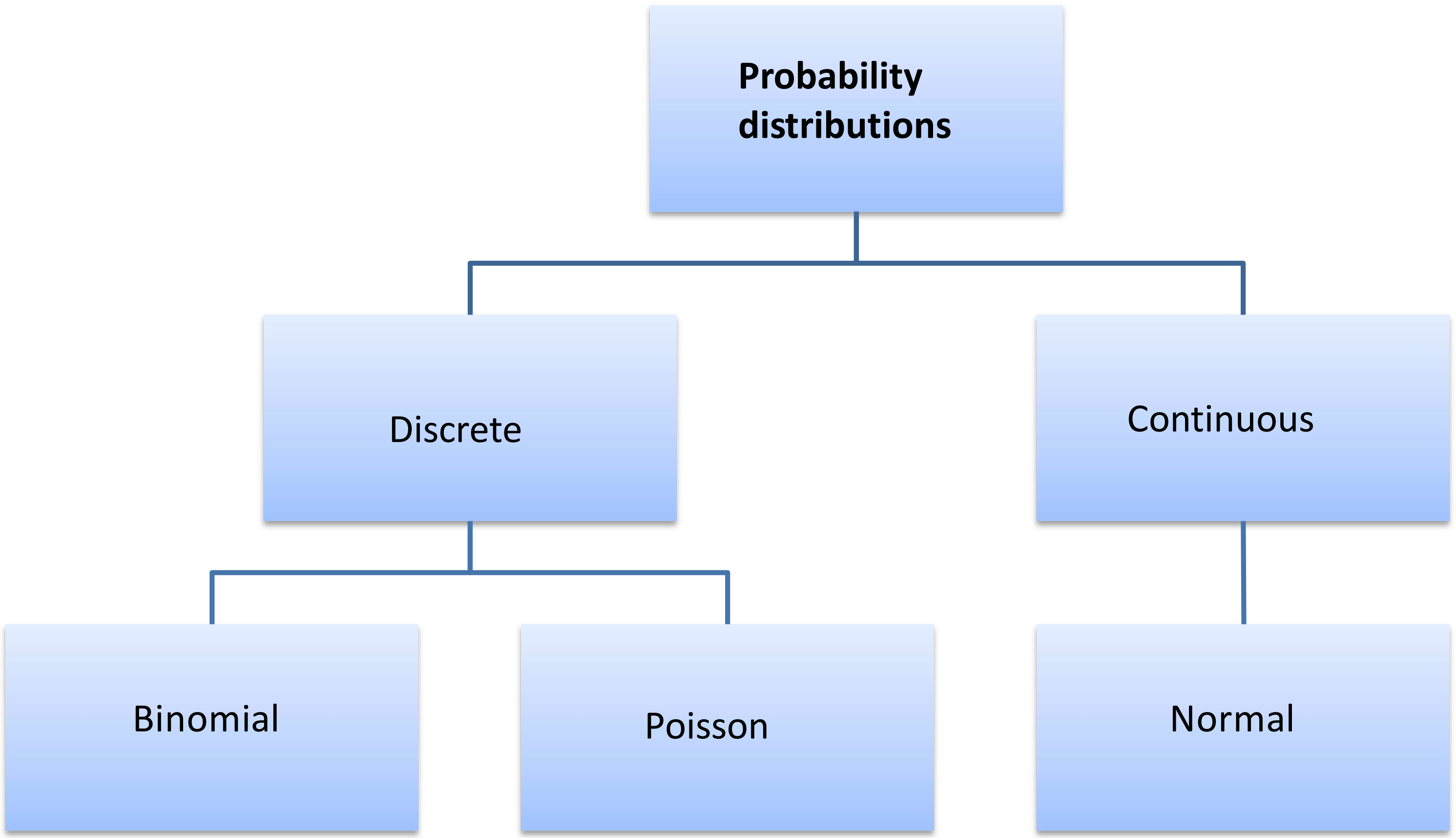
Statistical Methods for Decision Making

Probability Distributions

Binomial & Poisson

What is Probability Distribution?

- In precise terms, a probability distribution is a total listing of the various values the random variable can take along with the corresponding probability of each value. A real life example could be the pattern of distribution of the machine breakdowns in a manufacturing unit.
- The random variable in this example would be the various values the machine breakdowns could assume.
- The probability corresponding to each value of the breakdown is the relative frequency of occurrence of the breakdown.
- The probability distribution for this example is constructed by the actual breakdown pattern observed over a period of time. Statisticians use the term “observed distribution” of breakdowns.


Probability distributions
Discrete
Continuous
Binomial
Poisson
Normal

Binomial Distribution

- The Binomial Distribution is a widely used probability distribution of a discrete random variable.

- It plays a major role in **quality control** and **quality assurance** function. Manufacturing units do use the binomial distribution for **defective** analysis.

- Reducing the number of defectives using the proportion defective control chart (p chart) is an accepted practice in manufacturing organizations.

- Binomial distribution is also being used in **service organizations** like banks, and insurance corporations to get an idea of the proportion customers who are satisfied with the service quality.

Conditions for Applying Binomial Distribution (Bernoulli Process)

- Trials are independent and random.
- There are fixed number of trials (n trials).
- There are only two outcomes of the trial designated as success or failure.
- The probability of success is uniform through out the n trials

Binomial Probability Function

- The probability of getting x successes out of n trials is indeed the definition of a Binomial Distribution. The Binomial Probability Function is given by the following expression

$$P(x) = \binom{n}{x} p^{x} (1-p)^{n-x}$$

x can take values 0, 1, 2, ………, n

- Where P(x) is the probability of getting x successes in n trials

$\binom{n}{x}$ is the number of ways in which x successes can take place out of n trials $= \dfrac{n!}{x!\,(n-x)!}$

- p is the probability of success, which is the same through out the n trials.
- p is the parameter of the Binomial distribution

Example for Binomial Distribution

- A bank issues credit cards to customers under the scheme of Master Card. Based on the past data, the bank has found out that 60% of all accounts pay on time following the bill. If a sample of 7 accounts is selected at random from the current database, construct the Binomial Probability Distribution of accounts paying on time.

Solution

- This problem can be structured as a Bernoulli Process where an account paying on time is taken as success and an account not paying on time is taken as failure. The random variable x represents here an account paying on time, which can take values 0,1,2,3,4,5,6,7. You need to prepare a table containing x and P(x) for all the values of x. Performing calculations using Binomial Probability Function

$$P(x) = \binom{n}{x} p^{x} (1-p)^{n-x}$$

For x = 0, 1, 2, 3, 4, 5, 6, 7 is very tedious

- The best option is to use the Microsoft Excel to calculate the Binomial Probabilities both for individual values and for the cumulative position. This facility is available under the option "Paste Function". The form of the function is: n, p, O or 1) where x is the number of successes, n is the number of trials, and p is the probability of success in each trial. The last term 0 or I performs a logical operation. If you enter 0, the computer returns the individual probability value; if 1 is entered, the computer gives the cumulative probability value

Spreadsheet Showing the Solution

A	B	C	D	
1		Example Problem-Master Card		
2				
3	x	P(x)	Cumulative	
4			Probability	
5	0	0.0016384	0.0016384	
6	1	0.0172032	0.0188416	
7	2	0.0774144	0.0962560	
8	3	0.1935360	0.2897920	
9	4	0.2903040	0.5800960	
10	5	0.2612736	0.8413696	
11	6	0.1306368	0.9720064	
12	7	0.0279936	1.0000000	

Example problem 2 – binomial distribution

- Jones makes an average of 10 calls per day and has a success rate of 75%. Kate makes and average of 16 calls per day but has a success rate of 45%.
  - What is the probability of the salespersons making 6 sales on any given day?
  - What is the probability of the salespersons making upto 6 sales on any given day?
  - What is the probability of the salespersons making atleast 6 sales on any given day?

Mean and Standard Deviation of the Binomial Distribution

- The mean μ of the Binomial Distribution is
- given by $\mu = E(x) = np$
- The Standard Deviation σ is given by
- $\sigma = \sqrt{np\,(1-p)}$
- For the example problem in the previous two slides,
- Mean $\mu = 7 * 0.6 = 4.2$
- Standard Deviation = $\sqrt{4.2\,(1-0.60)} = 1.30$

Poisson Distribution

- Poisson Distribution is another discrete distribution which also plays a major role in quality control in the context of reducing the number of defects per standard unit.

- Examples include number of defects per item, number of defects per transformer produced, number of defects per 100 m2 of cloth, etc.

- Other real life examples would include 1) The number of cars arriving at a highway check post per hour; 2) The number of customers visiting a bank per hour during peak business period.

Poisson Process

- The probability of getting an event in a continuous interval such as length, area, time and the like is constant.
- The probability of an event occurs in any one interval is independent of the probability of event occurring in any other interval.
- The probability of getting more than one event in an interval approaches 0 as the interval becomes smaller.

Poisson Probability Function

Poisson Distribution Formula

$$P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

where

P(x) = Probability of x events in an interval given an idea of λ

λ = Average number of events per unit

e = 2.71828(based on natural logarithm)

x = events per unit which can take values 0, 1, 2, 3,…………….∞

λ is the Parameter of the Poisson Distribution.

Example

If on an average, 6 customers arrive every two minutes at a bank during the busy hours of working, a) what is the probability that exactly four customers arrive in a given minute? b) What is the probability that more than three customers will arrive in a given minute?

6 customers arrive every two minutes. Therefore , 3 customers arrive every minute. That implies my lambda=3

P(X=4)=?

P(X>3)=? Implies 1-P(X< =3)?

Spreadsheet showing Solution

Poisson Distribution	
Question	**Solution (formula)**
Find P(X=4)	=POISSON.DIST(4,3,0)
Find P(X > 3) i.e. Find 1 - P(X <= 3)	=1-POISSON.DIST(3,3,1)

Poisson Distribution	
Question	**Solution**
Find P(X=4)	0.168031356
Find P(X > 3) i.e. Find 1 - P(X <= 3)	0.352768111

The above spreadsheet shows the formula and solution.