Business Report

Project – Time series Forecasting

Created by Amit Jain

Contents

List of Figure

Rose.csv

Introduction: This report explains the business requirements and provide the detailed solution based on the data provided for each problem statement. given in the assignment. Also, the purpose of this exercise is to execute various Timeseries forecasting learning techniques and building various models over Timeseries data, combine all predictions and find out the model with best prediction or accuracy. Timeseries data is unlikely normal data used in supervised/unsupervised learning, where we have dependent /independent variables.

In TS data, we need to use predictions on Same field, using Old data set in ordered Time manner.

1. Read the data as an appropriate Time Series data and plot the data.

Time Series is a sequence of observations recorded at regular time intervals.

	YearMonth	Rose
0	1980-01-01	112.0
1	1980-02-01	118.0
2	1980-03-01	129.0
3	1980-04-01	99.0
4	1980-05-01	116.0
...	...	...
182	1995-03-01	45.0
183	1995-04-01	52.0
184	1995-05-01	28.0
185	1995-06-01	40.0
186	1995-07-01	62.0

187 rows × 2 columns

Given data is not time. So, we parse the date range and create a timestamp.

Plot for Rose wine Sales data

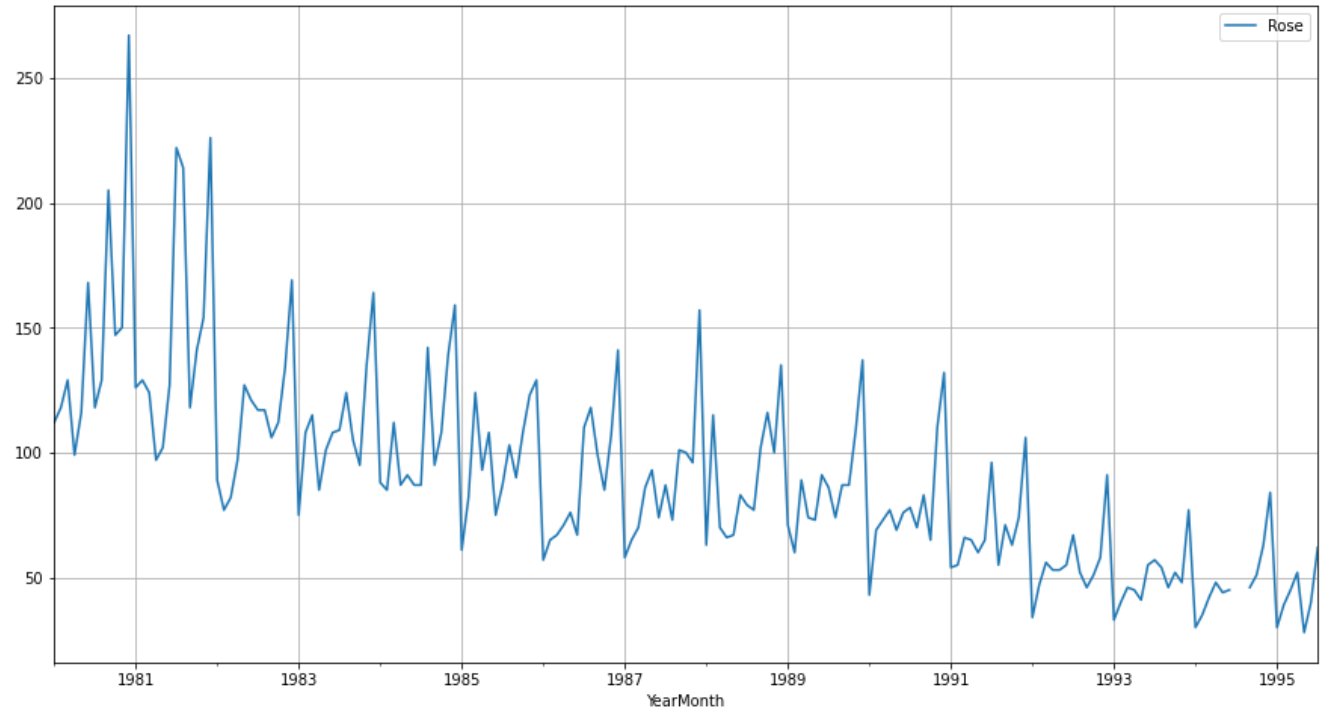


Figure 1 Time series Plot

```
Describer for Rose  Wine Sample data:
count  185.000000
mean    90.394595
std     39.175344
min     28.000000
25%     63.000000
50%     86.000000
75%    112.000000
max    267.000000
```

Insights:

1. Data consist of 187 data points
2. It seems to be contained seasonality
3. We also notice the fluctuations in the trend in the initial years and slowly decreasing the following years.
4. Minimum sales for the data in Any month are 28 and Max sales of Rose wines in any month is 267
5. Year 1981 has highest sales and 1995 has lowest sales

2. Perform appropriate Exploratory Data Analysis to understand the data and also perform decomposition.

Box Plot for Year on year sales data

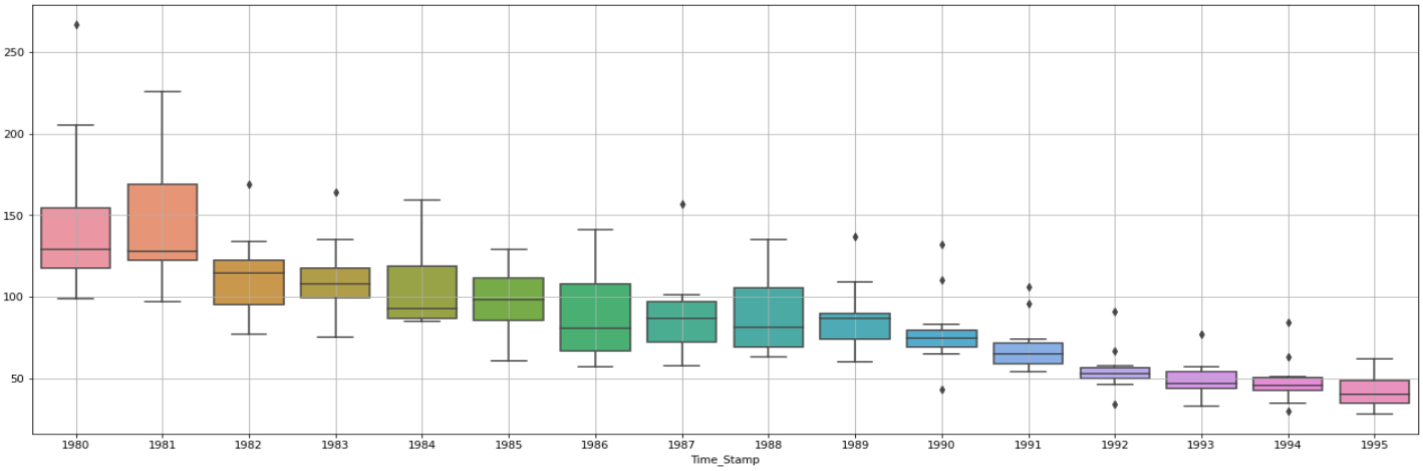


Figure 2 Box plot for year to year sales

Insights:

1. We see the Big increase in Sales for Rose wine in year 1981 and soon after that started decreasing Also there were years with Rise and fall equally. But gradually Sales goes down.
2. Boxplot helps to check the outliers in each year and month and we see there are outliers in almost all the year as per the box plot.
3. Average sales are lowest in the year of 1995

Monthly plot

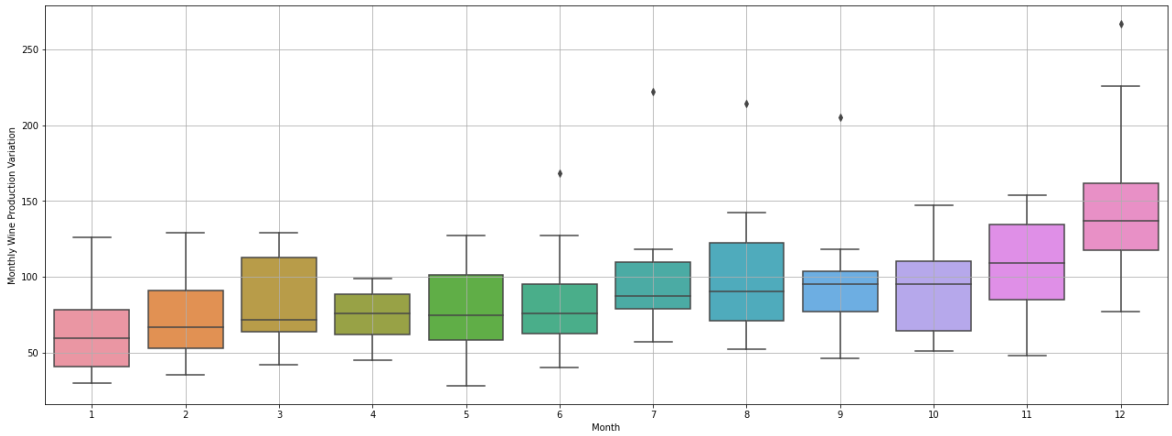


Figure 3 Time series Plot

Insights:

1. The box plot for various months is plotted
2. Monthly plot contains outliers in the month of June, July, August, September and December.
3. There are Highest sales in the Month of December followed by 2nd highest Sales in November Month, which indicates year end party and vacation celebration sales

Plot a month plot of the give Time Series:

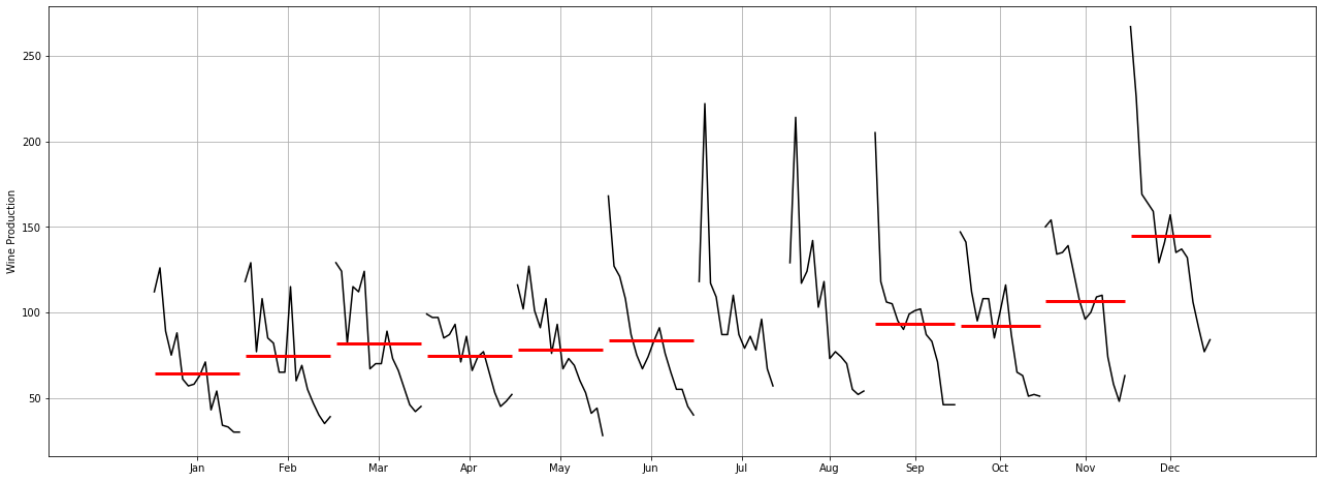


Figure 4 Month Plot of the Given time series

Insights:

1. We have accumulated all year's data for each month and plotted it against each month.
2. We see that there are High Sales in each month at the month Start and then Fall down till Mid of the month,
3. We also see small increase in Sales in every Mid of the Month and then again goes down till Month End.
4. It clearly indicates, when Anyone gets the Salary at the Start of the month and sometimes biweekly, then Sales for Month Start is always High and small Rise in Mid of the Month.

Plot the Time Series according to different months for different years

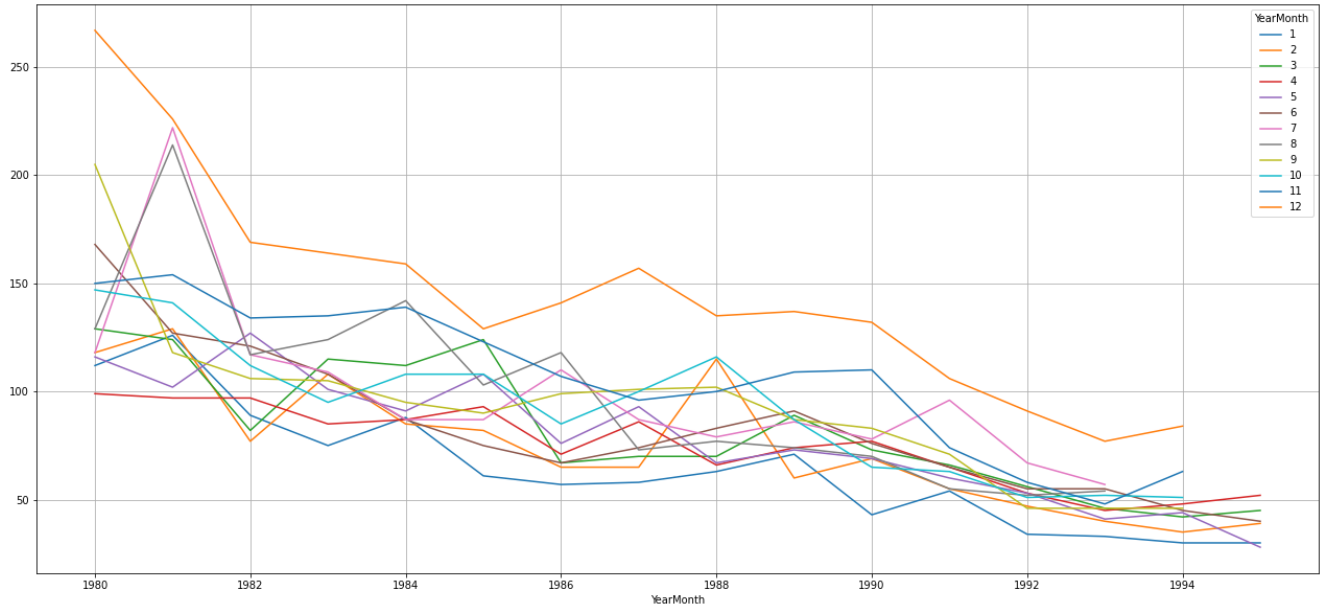


Figure 5 yearly sales across Month

YearMonth	1	2	3	4	5	6	7	8	9	10	11	12
YearMonth												
1980	112.0	118.0	129.0	99.0	116.0	168.0	118.0	129.0	205.0	147.0	150.0	267.0
1981	126.0	129.0	124.0	97.0	102.0	127.0	222.0	214.0	118.0	141.0	154.0	226.0
1982	89.0	77.0	82.0	97.0	127.0	121.0	117.0	117.0	106.0	112.0	134.0	169.0
1983	75.0	108.0	115.0	85.0	101.0	108.0	109.0	124.0	105.0	95.0	135.0	164.0
1984	88.0	85.0	112.0	87.0	91.0	87.0	87.0	142.0	95.0	108.0	139.0	159.0
1985	61.0	82.0	124.0	93.0	108.0	75.0	87.0	103.0	90.0	108.0	123.0	129.0
1986	57.0	65.0	67.0	71.0	76.0	67.0	110.0	118.0	99.0	85.0	107.0	141.0
1987	58.0	65.0	70.0	86.0	93.0	74.0	87.0	73.0	101.0	100.0	96.0	157.0
1988	63.0	115.0	70.0	66.0	67.0	83.0	79.0	77.0	102.0	116.0	100.0	135.0
1989	71.0	60.0	89.0	74.0	73.0	91.0	86.0	74.0	87.0	87.0	109.0	137.0
1990	43.0	69.0	73.0	77.0	69.0	76.0	78.0	70.0	83.0	65.0	110.0	132.0
1991	54.0	55.0	66.0	65.0	60.0	65.0	96.0	55.0	71.0	63.0	74.0	106.0
1992	34.0	47.0	56.0	53.0	53.0	55.0	67.0	52.0	46.0	51.0	58.0	91.0
1993	33.0	40.0	46.0	45.0	41.0	55.0	57.0	54.0	46.0	52.0	48.0	77.0
1994	30.0	35.0	42.0	48.0	44.0	45.0	NaN	NaN	46.0	51.0	63.0	84.0
1995	30.0	39.0	45.0	52.0	28.0	40.0	62.0	NaN	NaN	NaN	NaN	NaN

Insights:

1. December records have the high number of rose wine sales in each year.
2. May, January have low number of wine sales.
3. There are 2 Months data not available in July and August 1994 and then no data after July 1995.

Yearly Plot:

aggregate the time series from an annual perspective and summing up the observations

	YearMonth	Rose
0	1980-12-31	1758.0
1	1981-12-31	1780.0
2	1982-12-31	1348.0
3	1983-12-31	1324.0
4	1984-12-31	1280.0

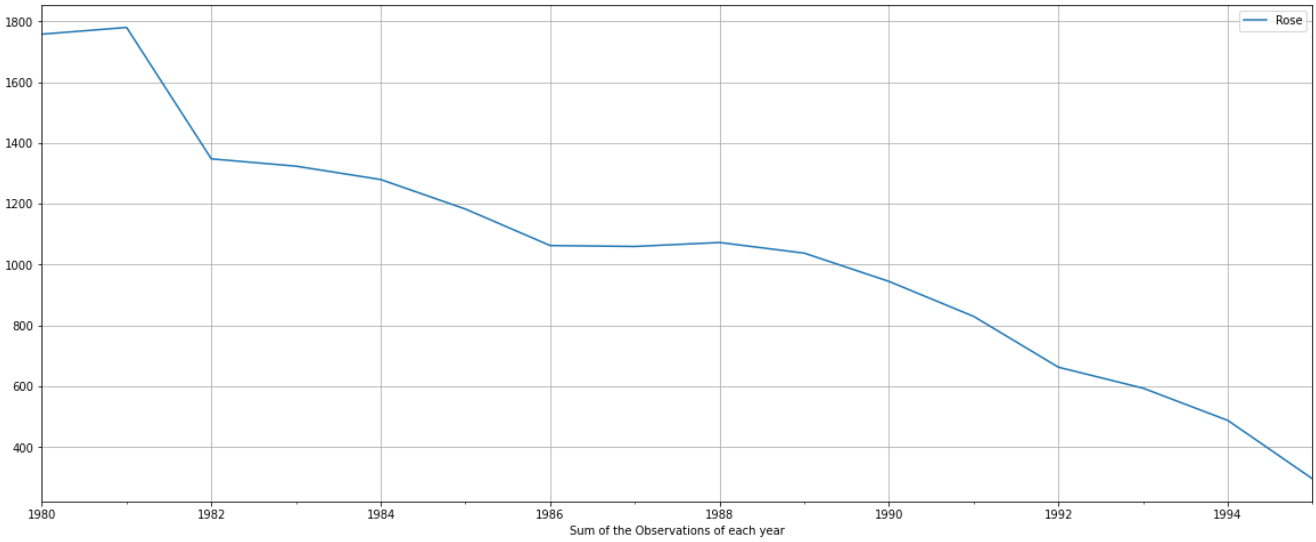


Figure 6 Sum of Sales Each year Plot

Insights:

1. The plot shows that in 1982 there is a fall in the wine sales and there is a steep downfall is observed.
2. The resampled yearly or annual series have smoothened out the seasonality and have only been able to capture the year on year trend where there was.

Quarterly plot –

aggregate the time series from a quarterly perspective and sum the observations of each quarter.

	YearMonth	Rose
0	1980-03-31	359.0
1	1980-06-30	383.0
2	1980-09-30	452.0
3	1980-12-31	564.0
4	1981-03-31	379.0

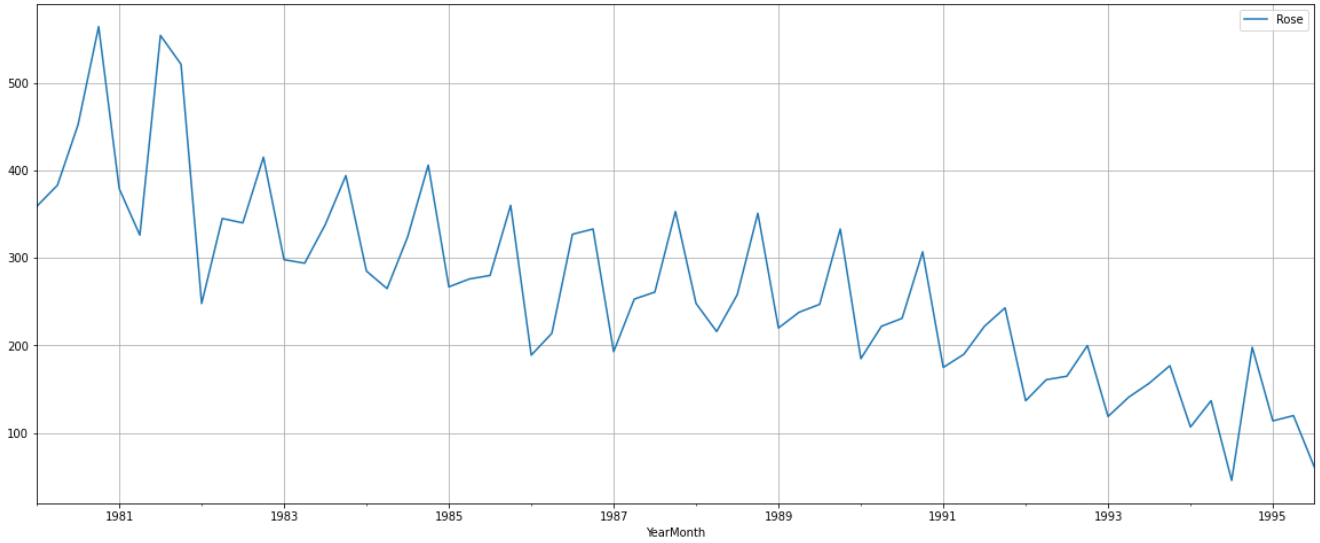


Figure 7 Quarterly sales for each year

Insights:

1. There is some rise is found in Year 1984 and 1988, but ultimately that also goes down.
2. We see that the year on year quarterly series represents the year on year monthly series. The quarterly series is able to catch the seasonality in the data.

Daily plot

aggregate the data from a daily perspective

YearMonth	Rose	
0	1980-01-01	112.0
1	1980-01-02	0.0
2	1980-01-03	0.0
3	1980-01-04	0.0
4	1980-01-05	0.0
...	...	...
5656	1995-06-27	0.0
5657	1995-06-28	0.0
5658	1995-06-29	0.0
5659	1995-06-30	0.0
5660	1995-07-01	62.0

5661 rows × 2 columns

The values which the original series cannot provide is taken as 0 by python if we try to resample the data on a daily basis.

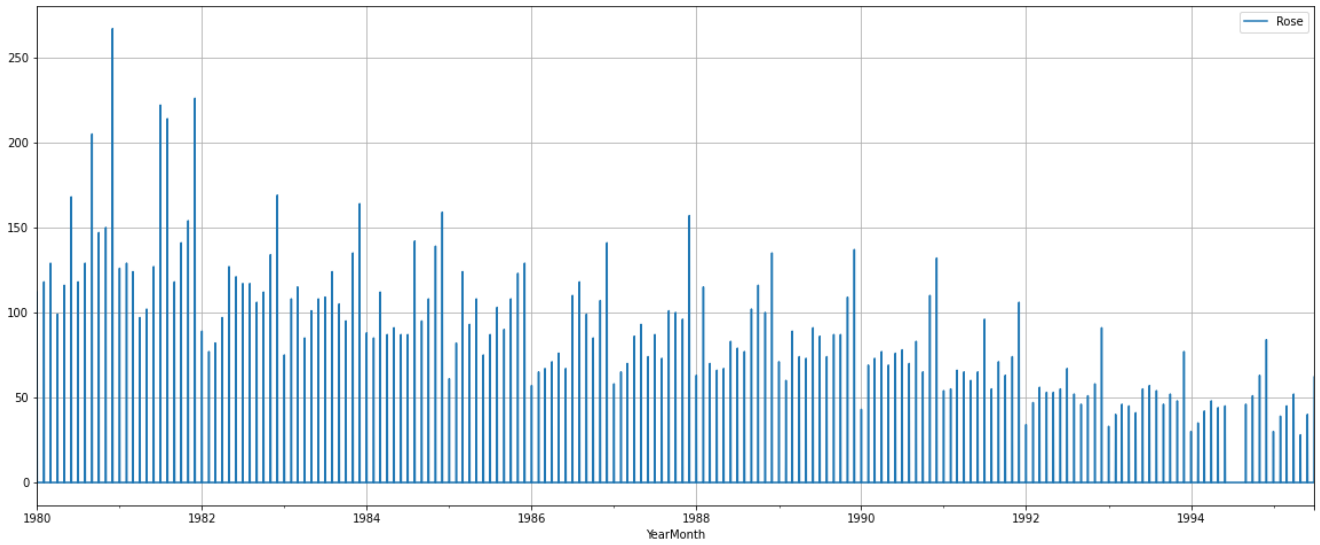


Figure 8 Daily Sales for all years

The above graph fails to give us a proper understanding of our data. Thus, resampling the data to intervals where a number of observations are 0 is not a good idea as that does not give us an understanding of the performance of the time series.

To get a very high-level overview of the trend of the Time Series Data (if Trend is present) can be understood by resampling the data keeping the intervals very large.

Decade Plot

	YearMonth	Rose
0	1980-12-31	1758.0
1	1990-12-31	12094.0
2	2000-12-31	2871.0

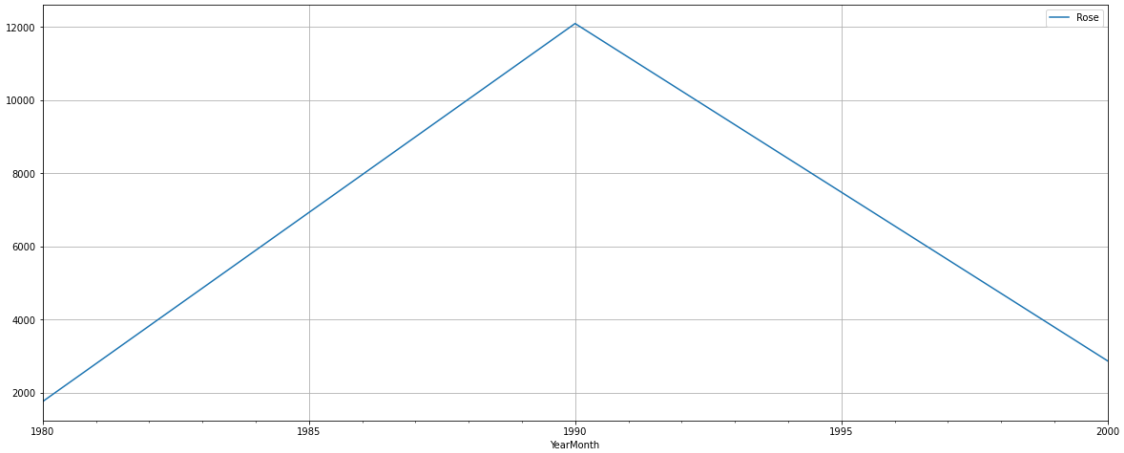


Figure 9 Decade Sales for all data

If we take the resampling period to be 10 years or a decade, we see that the seasonality present has been smoothed over and it is only giving an estimate of the trend.

Decompose the Time Series : Additive Model

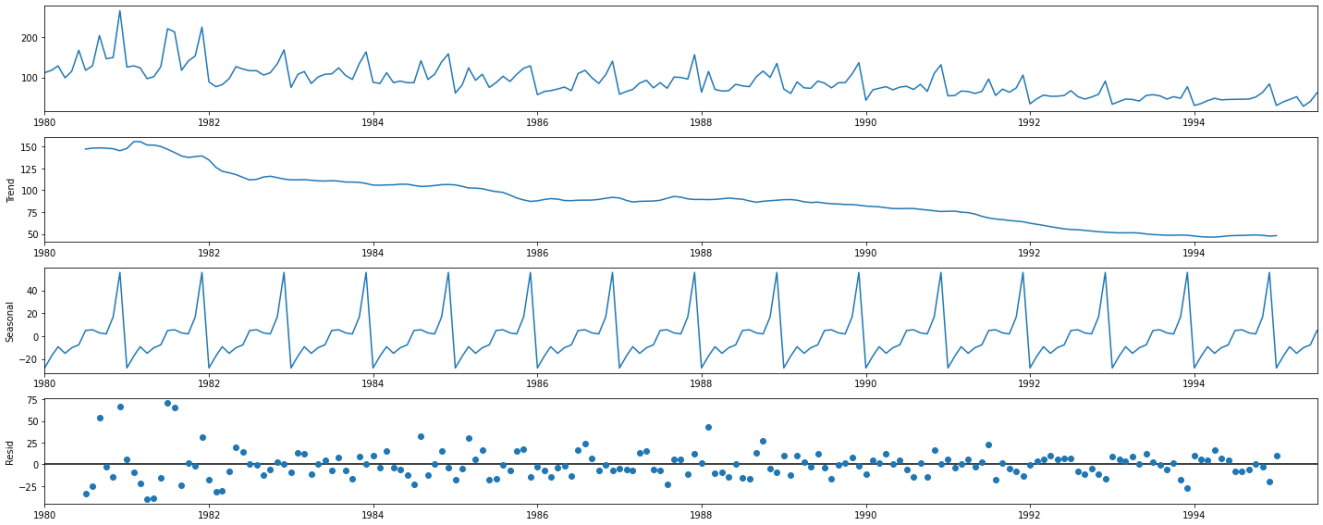


Figure 10 Additive Decomposition

Insights:

1. We have built 2 Models of the data Additive trend as well as Multiplicative Trend .
2. From the 'additive' decomposition, there is seasonality in the data. Which is At the Start of the year Sales goes down, and at the end of the year, sales goes High for that year.
3. Sales trend is going down year by year

```
Trend
 YearMonth
1980-01-01           NaN
1980-02-01           NaN
1980-03-01           NaN
1980-04-01           NaN
1980-05-01           NaN
1980-06-01           NaN
1980-07-01    147.083333
1980-08-01    148.125000
1980-09-01    148.375000
1980-10-01    148.083333
1980-11-01    147.416667
1980-12-01    145.125000
Name: trend, dtype: float64

Seasonality
 YearMonth
1980-01-01   -27.908647
1980-02-01   -17.435632
1980-03-01    -9.285830
1980-04-01   -15.098330
1980-05-01   -10.196544
1980-06-01    -7.678687
1980-07-01     4.896908
```

```
1980-08-01     5.499686
1980-09-01     2.774686
1980-10-01     1.871908
1980-11-01    16.846908
1980-12-01    55.713575
Name: seasonal, dtype: float64

Residual
 YearMonth
1980-01-01          NaN
1980-02-01          NaN
1980-03-01          NaN
1980-04-01          NaN
1980-05-01          NaN
1980-06-01          NaN
1980-07-01   -33.980241
1980-08-01   -24.624686
1980-09-01    53.850314
1980-10-01    -2.955241
1980-11-01   -14.263575
1980-12-01    66.161425
Name: resid, dtype: float64
```

Sales data without Seasonality component:

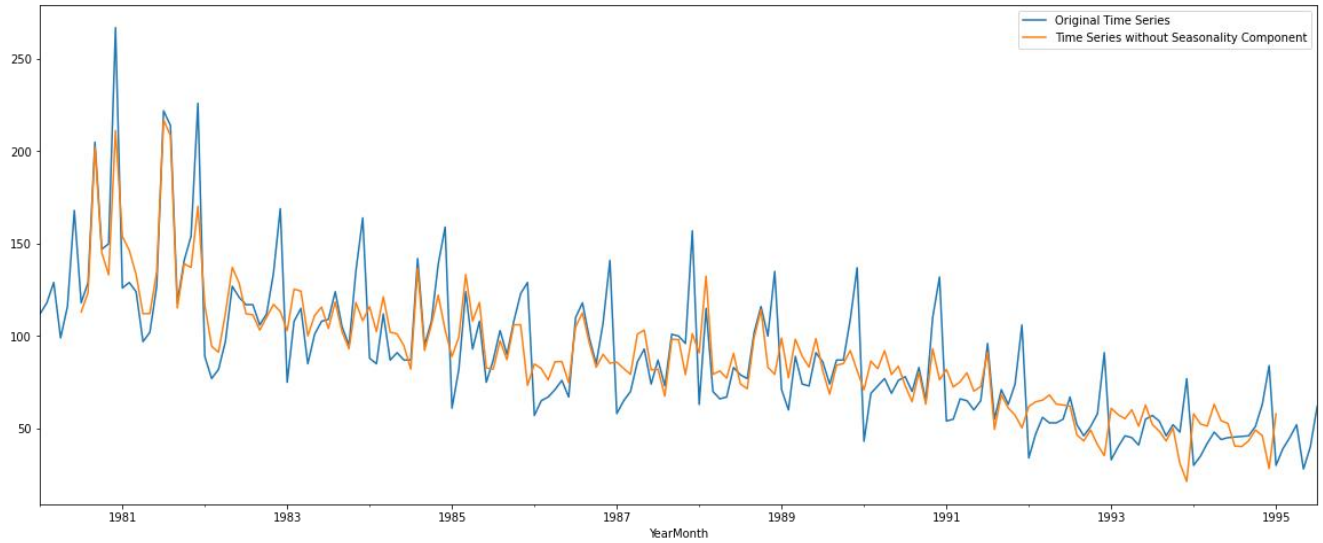


Figure 11 Sales data without Seasonality

```
YearMonth
1980-01-01          NaN
1980-02-01          NaN
1980-03-01          NaN
1980-04-01          NaN
1980-05-01          NaN
1980-06-01          NaN
1980-07-01   113.103092
```

```
1980-08-01    123.500314
1980-09-01    202.225314
1980-10-01    145.128092
1980-11-01    133.153092
1980-12-01    211.286425
dtype: float64
```

Multiplicative Model for Rose data problem

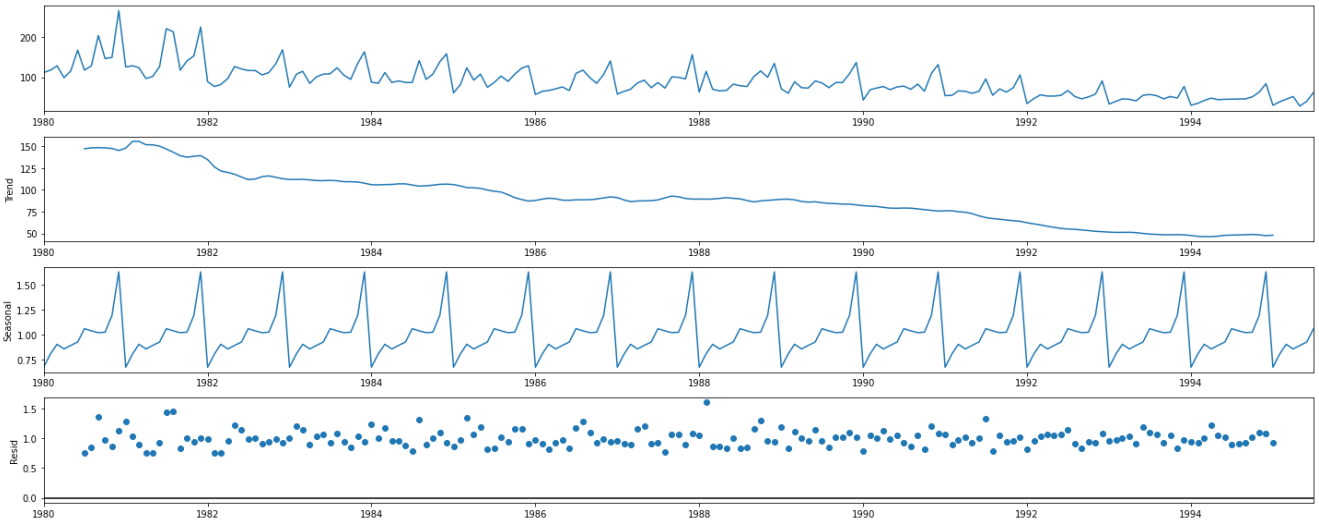


Figure 12 Multiplicative Sales Decomposition

Since Residual is more close to a single line, we would choose Additive Decomposition over Multiplicative decomposing.

3. Split the data into training and test. The test data should start in 1991.

We have split the data in train and test data set,

Data before 1991 is considered as train set and data after 1991 is considered as test data set:

After splitting this is the final shape of the Train and Test data

```
(132, 1)
(55, 1)
```

First few rows of Training Data

YearMonth	Rose
1980-01-01	112.0
1980-02-01	118.0
1980-03-01	129.0
1980-04-01	99.0
1980-05-01	116.0

Last few rows of Training Data

YearMonth	Rose
1990-08-01	70.0
1990-09-01	83.0
1990-10-01	65.0
1990-11-01	110.0
1990-12-01	132.0

First few rows of Test Data

YearMonth	Rose
1991-01-01	54.0
1991-02-01	55.0
1991-03-01	66.0
1991-04-01	65.0
1991-05-01	60.0

Last few rows of Test Data

YearMonth	Rose
1995-03-01	45.0
1995-04-01	52.0
1995-05-01	28.0
1995-06-01	40.0
1995-07-01	62.0

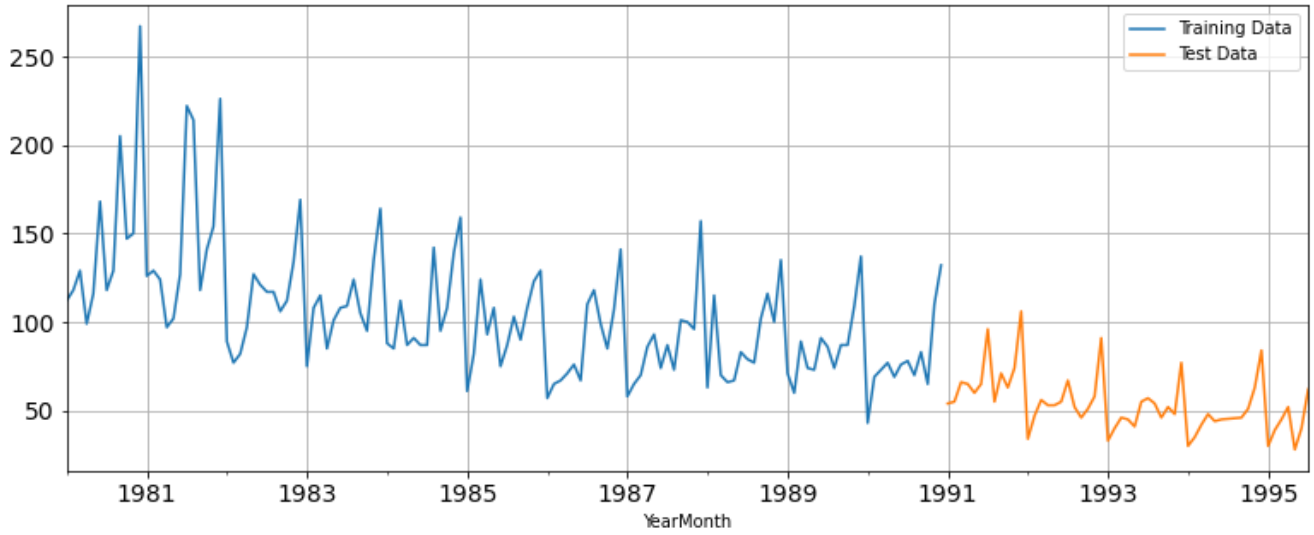


Figure 13 Plot Train and test data

It is difficult to predict the future if the past is not happened. From the above split, we are predicting similar to the past data.

4. Build all the exponential smoothing models on the training data and evaluate the model using RMSE on the test data. Other additional models such as regression, naïve forecast models, simple average models, moving average models should also be built on the training data and check the performance on the test data using RMSE

Model1: Linear Regression

For this particular linear regression, we are going to regress the 'Rose' variable against the order of the occurrence. For this we need to modify our training data before fitting it into a linear regression.
Regress the “Rose” variable against the order of occurrence. We have also generated the numerical instance order for both training and test set . Linear Regression is built on the training and test dataset

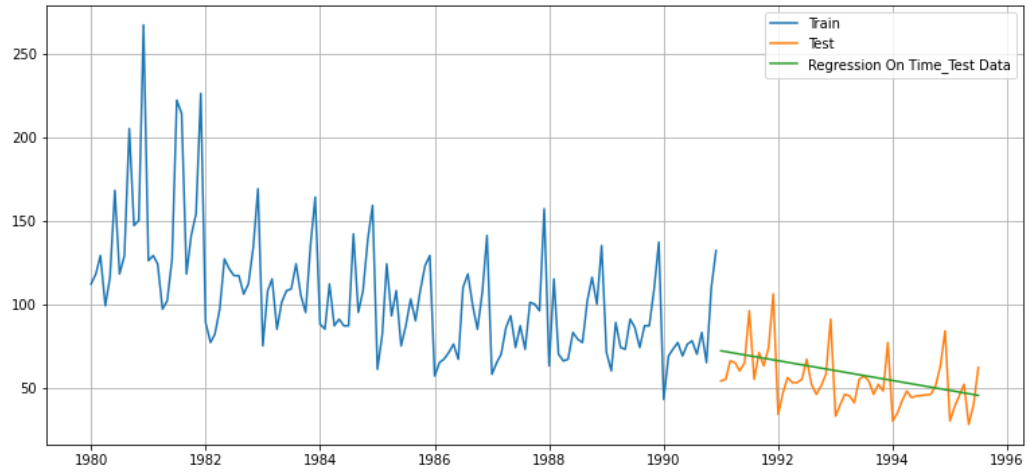


Figure 14 Linear regression Prediction Plot

We have evaluated the Model based on RMSE parameter and put this in a Data frame, which we would use later for Comparing multiple models
Model evaluation :

	index	Test RMSE
0	RegressionOnTime	15.268955

Model2 – Naïve model

For this particular naive model, we say that the prediction for tomorrow is the same as today and the prediction for day after tomorrow is tomorrow and since the prediction of tomorrow is same as today, therefore the prediction for day after tomorrow is also today.

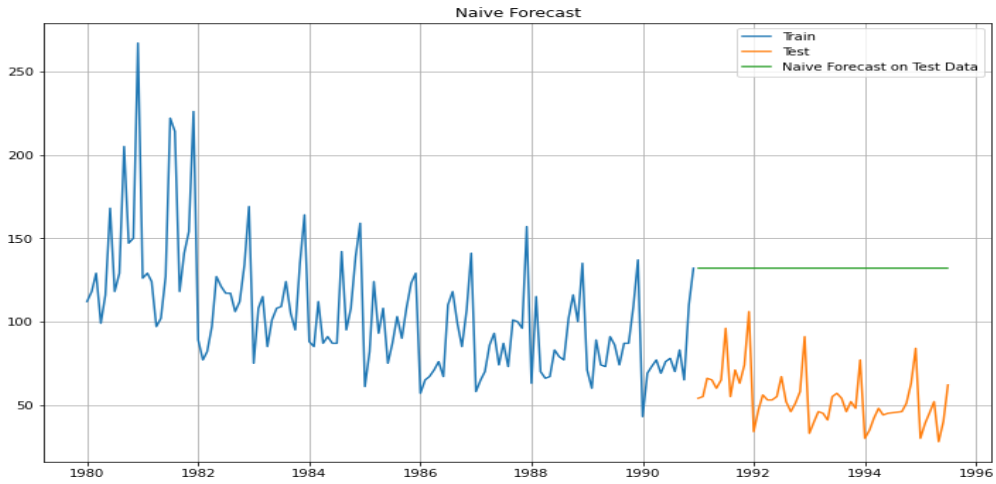


Figure 15 Prediction Plot for Naïve based Model

Model Evaluation

	index	Test RMSE
0	RegressionOnTime	15.268955
1	NaiveModel	79.718773

Model 3 – Simple Average

For this particular simple average method, we will forecast by using the average of the training values

YearMonth	Rose	mean_forecast
1991-01-01	54.0	104.939394
1991-02-01	55.0	104.939394
1991-03-01	66.0	104.939394
1991-04-01	65.0	104.939394
1991-05-01	60.0	104.939394

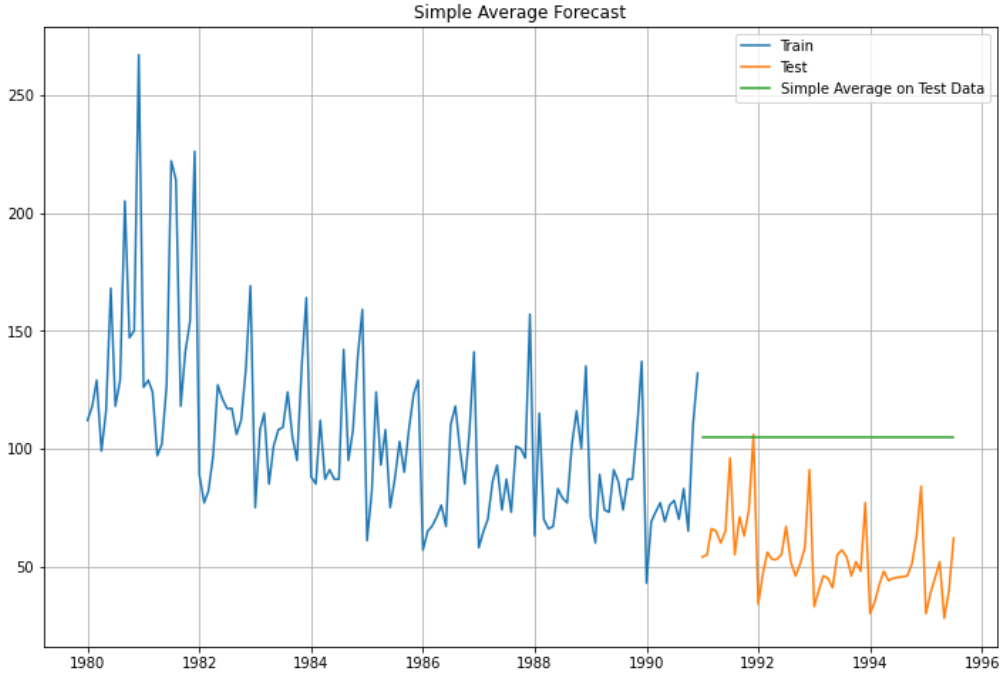


Figure 16 Prediction Plot for Simple Average

Model Evaluation:

	Test RMSE
RegressionOnTime	15.268955
NaiveModel	79.718773
SimpleAverageModel	53.460570

Model4- Moving Average –

For the moving average model, we are going to calculate rolling means (or moving averages) for different intervals. The best interval can be determined by the maximum accuracy (or the minimum error) over here. Let's take moving average of previous 2, 3, 4 and 6 data elements:

YearMonth	Rose	Trailing_2	Trailing_3	Trailing_4	Trailing_6
1980-01-01	112.0	NaN	NaN	NaN	NaN
1980-02-01	118.0	115.0	NaN	NaN	NaN
1980-03-01	129.0	123.5	119.666667	NaN	NaN
1980-04-01	99.0	114.0	115.333333	114.5	NaN
1980-05-01	116.0	107.5	114.666667	115.5	NaN

Plot the predictions based on Moving averages:

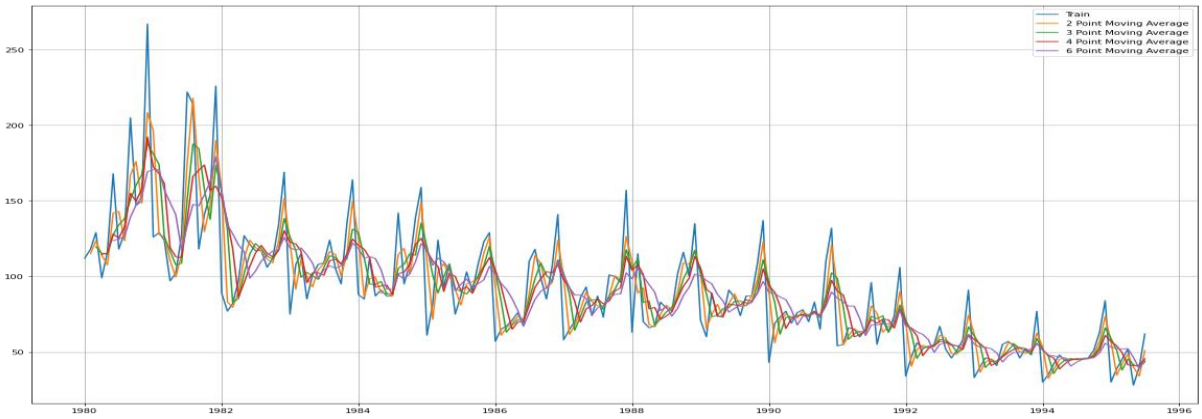


Figure 17 Moving Average for 2,3,4 and 6 MA

Let us split the data into train and test and plot this Time Series. The window of the moving average is need to be carefully selected as too big a window will result in not having any test set as the whole series might get averaged over.

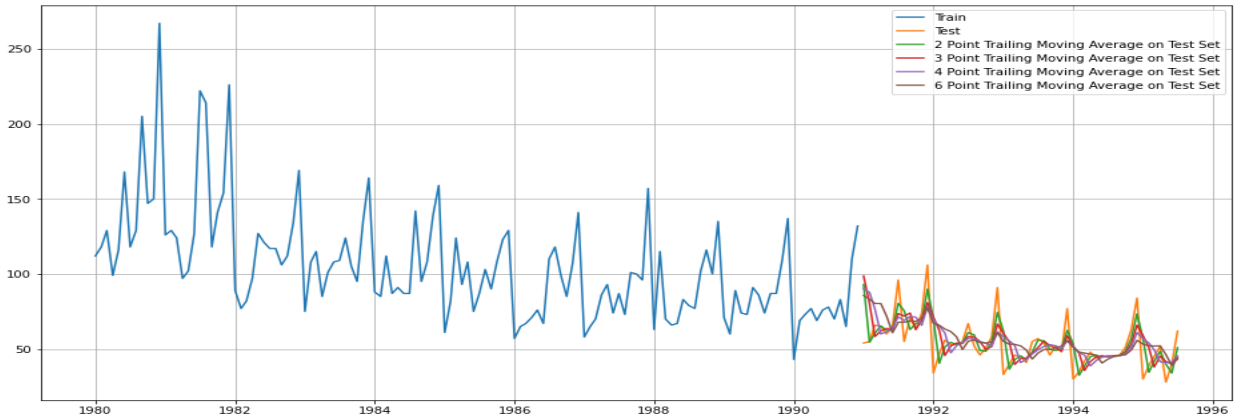


Figure 18 Plot MA on test data

Model Evaluation:

```
For 2 point Moving Average Model forecast on the Training Data,  RMSE is 11.529

For 3 point Moving Average Model forecast on the Training Data,  RMSE is 14.127

For 4 point Moving Average Model forecast on the Training Data,  RMSE is 14.451

For 6 point Moving Average Model forecast on the Training Data,  RMSE is 14.566
```

	Test RMSE
RegressionOnTime	15.268955
NaiveModel	79.718773
SimpleAverageModel	53.460570
2pointTrailingMovingAverage	11.529278
3pointTrailingMovingAverage	14.126525
4pointTrailingMovingAverage	14.451403
6pointTrailingMovingAverage	14.566327

The Best Model for Moving average is 2 points trailing moving average with lowest RMSE of 11.52, We would choose this model for predicting the Test model, if we are to choose the best one from above all models:

Before we go on to build the various Exponential Smoothing models, let us plot all the models and compare the Time Series plots

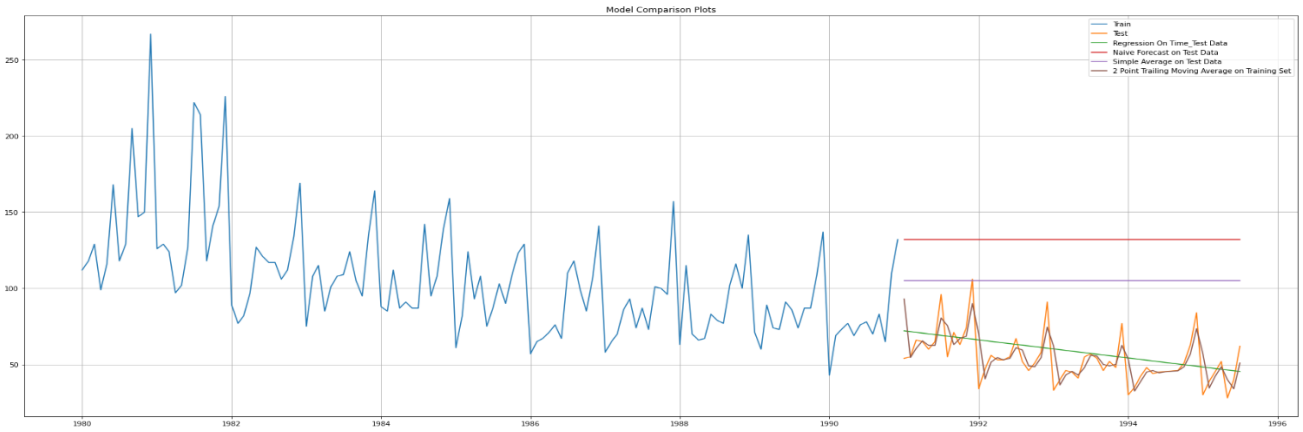


Figure 19 Building all Models on Test data

Model -5- Exponential Smoothing

Exponential smoothing methods consist of flattening time series data. Exponential smoothing averages or exponentially weighted moving averages consist of forecast based on previous periods data with exponentially declining influence on the older observations.

Exponential smoothing methods consist of special case exponential moving with notation ETS (Error, Trend, Seasonality) where each can be none(N), additive (N), additive damped (Ad), Multiplicative (M) or multiplicative damped (Md). One or more parameters control how fast the weights decay. These parameters have values between 0 and 1

First Model in Exponential smoothing is SES (Simple Exponential Smoothing) .

SES - ETS(A, N, N) - Simple Exponential Smoothing with additive errors

The simplest of the exponentially smoothing methods is naturally called simple exponential smoothing (SES). This method is suitable for forecasting data with no clear trend or seasonal pattern.

In Single ES, the forecast at time (t + 1) is given by Winters,1960

Ft+1=αYt+(1−α)Ft

Parameter α is called the smoothing constant and its value lies between 0 and 1. Since the model uses only one smoothing constant, it is called Single Exponential Smoothing.

Note: Here, there is both trend and seasonality in the data. So, we should have directly gone for the Triple Exponential Smoothing but Simple Exponential Smoothing and the Double Exponential Smoothing models are built over here to get an idea of how the three types of models compare in this case.

SimpleExpSmoothing class must be instantiated and passed the training data.

The fit() function is then called providing the fit configuration, the alpha value, smoothing_level. If this is omitted or set to None, the model will automatically optimize the value

Auto parameters:

```
{'smoothing_level': 0.09874989743650385,
 'smoothing_trend': nan,
 'smoothing_seasonal': nan,
 'damping_trend': nan,
 'initial_level': 134.38699692184085,
 'initial_trend': nan,
 'initial_seasons': array([], dtype=float64),
 'use_boxcox': False,
 'lamda': None,
 'remove_bias': False}
```

Perform Prediction on test data set and these are the result look like:

YearMonth	Rose	predict
1991-01-01	54.0	87.104999
1991-02-01	55.0	87.104999
1991-03-01	66.0	87.104999
1991-04-01	65.0	87.104999
1991-05-01	60.0	87.104999

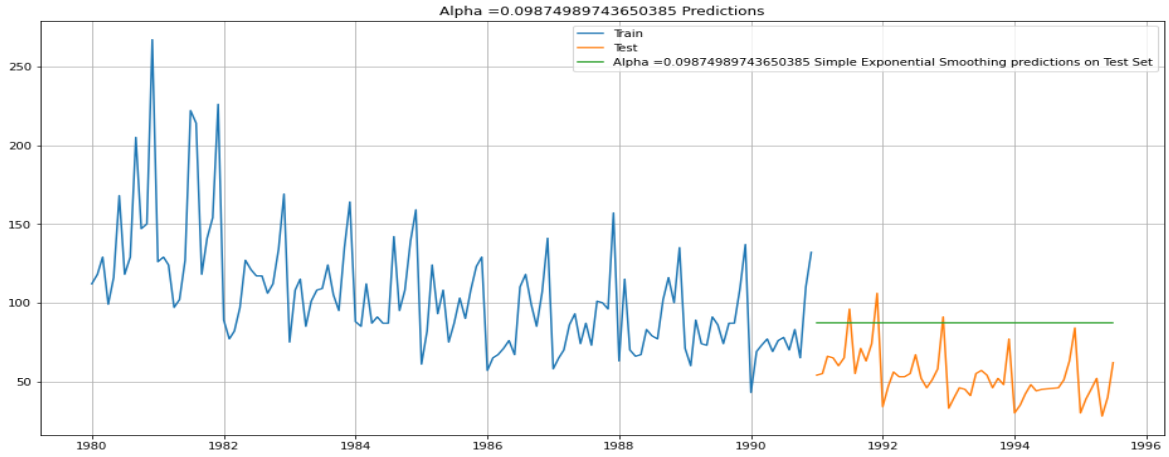


Figure 20 SES Model on Auto parameters

Model Evaluation for α = 0.09874989743650385 : Simple Exponential Smoothing For Alpha =0.09874989743650385 Simple Exponential Smoothing Model forecast on the Test Data, RMSE is 36.796

	Test RMSE
RegressionOnTime	15.268955
NaiveModel	79.718773
SimpleAverageModel	53.460570
2pointTrailingMovingAverage	11.529278
3pointTrailingMovingAverage	14.126525
4pointTrailingMovingAverage	14.451403
6pointTrailingMovingAverage	14.566327
Alpha=0.098749,SimpleExponentialSmoothing	36.796242

Setting different alpha values. The higher the alpha value more weightage is given to the more recent observation. That means, what happened recently will happen again.

We will run a loop with different alpha values to understand which particular value works best for alpha on the test set. We have given a range of values between 0.01 to 1 with the interval of 0.01 so , Alpha values like 0.01, 0.02, 0.03 … upto 1.

We tried all 100 possible values and calculated the RMSE value for that, and chosen best Alpha value for Lowest RMSE value

	Alpha Values	Train RMSE	Test RMSE
6	0.07	32.046904	36.435772
7	0.08	31.936243	36.462965
5	0.06	32.209657	36.580469
8	0.09	31.862435	36.604118
9	0.10	31.815610	36.828033
...	...	...	...
94	0.95	38.112735	78.532696
95	0.96	38.243543	78.786884
96	0.97	38.376021	79.032686
97	0.98	38.510198	79.270003
98	0.99	38.646108	79.498734

99 rows × 3 columns

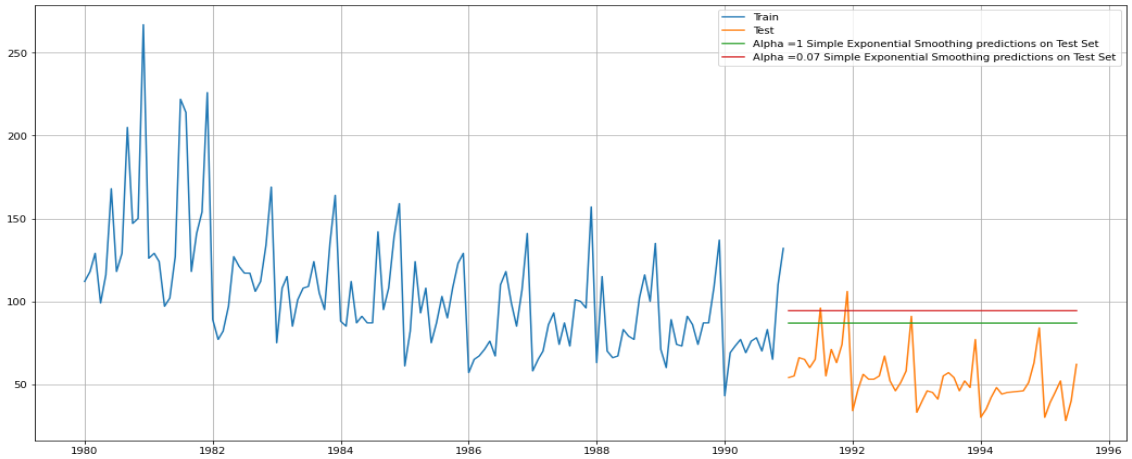


Figure 21 SES Model on Corrected parameters

Model evaluation:

	Test RMSE
RegressionOnTime	15.268955
NaiveModel	79.718773
SimpleAverageModel	53.460570
2pointTrailingMovingAverage	11.529278
3pointTrailingMovingAverage	14.126525
4pointTrailingMovingAverage	14.451403
6pointTrailingMovingAverage	14.566327
Alpha=0.098749,SimpleExponentialSmoothing	36.796242
Alpha=0.07,SimpleExponentialSmoothing	36.435772

Method 6: Double Exponential Smoothing (Holt's Model)

Holt - ETS(A, A, N) - Holt's linear method with additive errors , Double Exponential Smoothing.

One of the drawbacks of the simple exponential smoothing is that the model does not do well in the presence of the trend. This model is an extension of SES known as Double Exponential model which estimates two smoothing parameters. This is applicable when data has Trend but no seasonality. In this model two separate components are considered: **Level** and **Trend**. **Level** is the local mean.

One smoothing parameter α corresponds to the level series.

A second smoothing parameter β corresponds to the trend series.

Double Exponential Smoothing uses two equations to forecast future values of the time series, one for forecasting the short-term average value or level and the other for capturing the trend.

Intercept or Level equation, Lt is given by: $Lt=\alpha Yt+(1-\alpha)Ft$

Trend equation is given by $Tt=\beta(Lt-Lt-1)+(1-\beta)Tt-1$

Here, α and β are the smoothing constants for level and trend, respectively,

$0 < \alpha < 1$ and $0 < \beta < 1$.

The forecast at time t + 1 is given by

$Ft+1=Lt+Tt$

$Ft+n=Lt+nTt$

Two parameters α and β are estimated in this model. Level and Trend are accounted for in this model

Similar to Above SES model, we will calculate Alpha and beta values by using the for loops and calculate the RMSE value. Then we would choose Alpha and beta value for the Lowest RMSE.

	Alpha Values	Beta Values	Train RMSE	Test RMSE
0	0.1	0.1	34.439111	36.923416
1	0.1	0.2	33.450729	48.688648
10	0.2	0.1	33.097427	65.731702
2	0.1	0.3	33.145789	78.156641
20	0.3	0.1	33.611269	98.653317
...	...	...	...	...
78	0.8	0.9	51.756649	1052.464792
68	0.7	0.9	48.539838	1061.847942
79	0.8	1.0	53.844112	1095.112285
59	0.6	1.0	47.190957	1102.085828
69	0.7	1.0	50.266943	1125.186726

100 rows × 4 columns

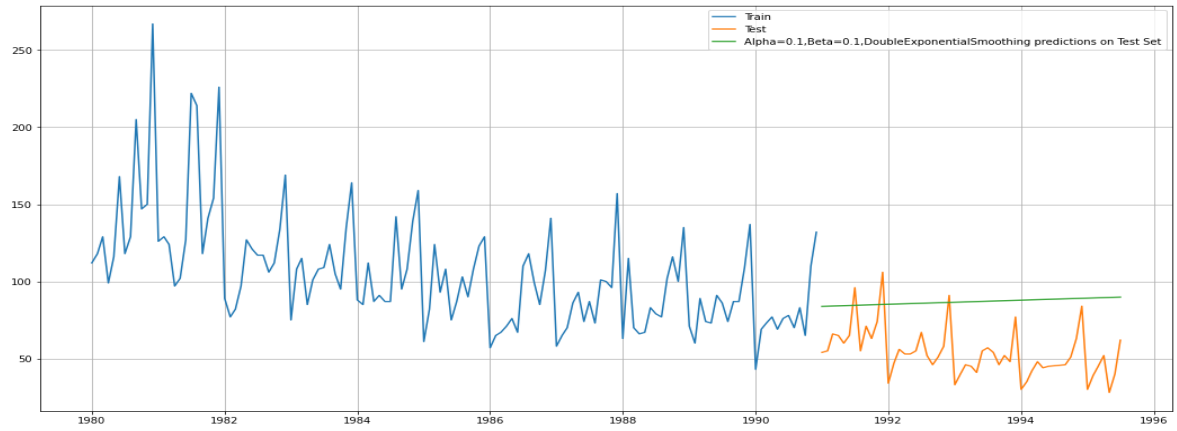


Figure 22 DES model

We see that the double exponential smoothing is picking up the trend component along with the level component as well.

	Test RMSE
RegressionOnTime	15.268955
NaiveModel	79.718773
SimpleAverageModel	53.460570
2pointTrailingMovingAverage	11.529278
3pointTrailingMovingAverage	14.126525
4pointTrailingMovingAverage	14.451403
6pointTrailingMovingAverage	14.566327
Alpha=0.098749,SimpleExponentialSmoothing	36.796242
Alpha=0.07,SimpleExponentialSmoothing	36.435772
Alpha=0.1,Beta=0.1,DoubleExponentialSmoothing	36.923416

Inference

Here, we see that the Double Exponential Smoothing has actually done well when compared to the Simple Exponential Smoothing. This is because of the fact that the Double Exponential Smoothing model has picked up the trend component as well.

The Holt's model in Python has certain other options of exponential trends or whether the smoothing parameters should be damped. You can try these out later to check whether you get a better forecast.

Method 7: Triple Exponential Smoothing (Holt - Winter's Model)

Three parameters α , β and γ are estimated in this model. Level, Trend and Seasonality are accounted for in this model. First of all build the model with default parameters:

```
{'smoothing_level': 0.07736040004765096,
 'smoothing_trend': 0.03936496779735522,
 'smoothing_seasonal': 0.0008375039104357999,
 'damping_trend': nan,
 'initial_level': 156.90674503596637,
 'initial_trend': -0.9061396720042346,
 'initial_seasons': array([0.7142168 , 0.80982439, 0.88543128, 0.77363782, 0.87046319
,
        0.94699283, 1.04196135, 1.11012703, 1.04835489, 1.0276963 ,
        1.19783562, 1.6514144 ]),
 'use_boxcox': False,
 'lamda': None,
 'remove_bias': False}
```

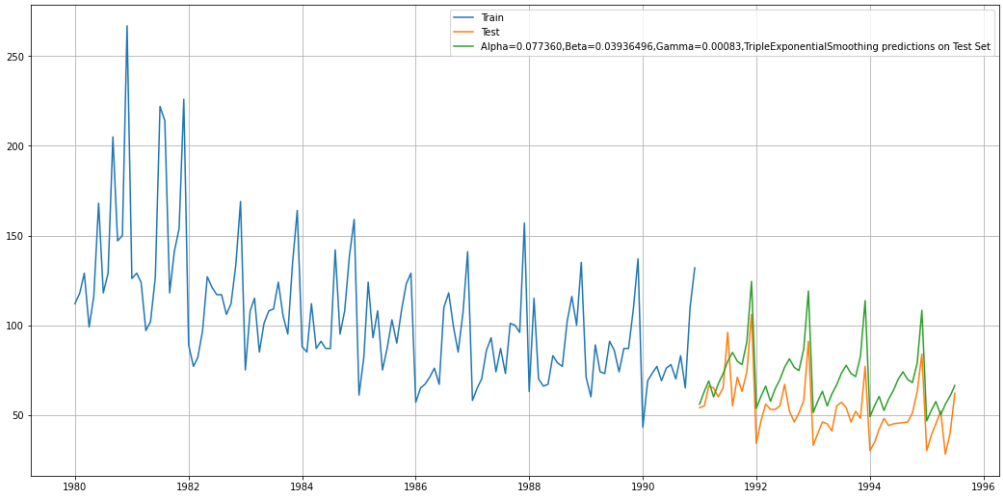


Figure 23 TES Model on Auto Parameters

We see that the Triple Exponential Smoothing is picking up the seasonal component as well.

Inference

Triple Exponential Smoothing has performed the best on the test as expected since the data had both trend and seasonality.

But we see that our triple exponential smoothing is under forecasting. Let us try to tweak some of the parameters in order to get a better forecast on the test set.

	Test RMSE
RegressionOnTime	15.268955
NaiveModel	79.718773
SimpleAverageModel	53.460570
2pointTrailingMovingAverage	11.529278
3pointTrailingMovingAverage	14.126525
4pointTrailingMovingAverage	14.451403
6pointTrailingMovingAverage	14.566327
Alpha=0.098749,SimpleExponentialSmoothing	36.796242
Alpha=0.07,SimpleExponentialSmoothing	36.435772
Alpha=0.1,Beta=0.1,DoubleExponentialSmoothing	36.923416
Alpha=0.077360,Beta=0.03936496,Gamma=0.00083,TripleExponentialSmoothing	19.113110

Auto generated Alpha , beta and game values and decide best based on lowest RMSE values

	Alpha Values	Beta Values	Gamma Values	Train RMSE	Test RMSE
9	0.1	0.2	0.1	19.77	9.22
10	0.1	0.2	0.2	20.25	9.50
127	0.2	0.6	0.2	23.13	9.57
119	0.2	0.5	0.3	23.66	9.89
11	0.1	0.2	0.3	20.87	9.89

Final values are chosen based as 0.1, 0.2 and0.1 on lowest RMSE value of 9.22 .

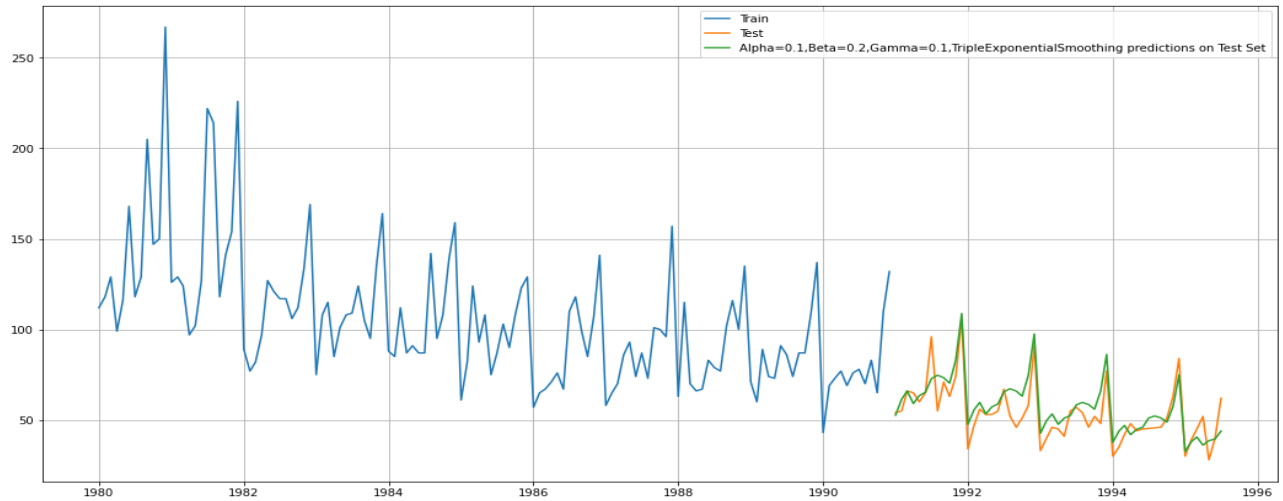


Figure 24 TES Model on Corrected parameters

Compare the Model generated so far:

	Test RMSE
Alpha=0.1,Beta=0.2,Gamma=0.1,TripleExponentialSmoothing	9.220000
2pointTrailingMovingAverage	11.529278
3pointTrailingMovingAverage	14.126525
4pointTrailingMovingAverage	14.451403
6pointTrailingMovingAverage	14.566327
RegressionOnTime	15.268955
Alpha=0.077360,Beta=0.03936496,Gamma=0.00083,TripleExponentialSmoothing	19.113110
Alpha=0.07,SimpleExponentialSmoothing	36.435772
Alpha=0.098749,SimpleExponentialSmoothing	36.796242
Alpha=0.1,Beta=0.1,DoubleExponentialSmoothing	36.923416
SimpleAverageModel	53.460570
NaiveModel	79.718773

We see that the best model is the Triple Exponential Smoothing with multiplicative seasonality with the parameters α = 0.1 , β = 0.2 and γ = 0.1.

For this data, we had both trend and seasonality so by definition Triple Exponential Smoothing is supposed to work better than the Simple Exponential Smoothing as well as the Double Exponential Smoothing.

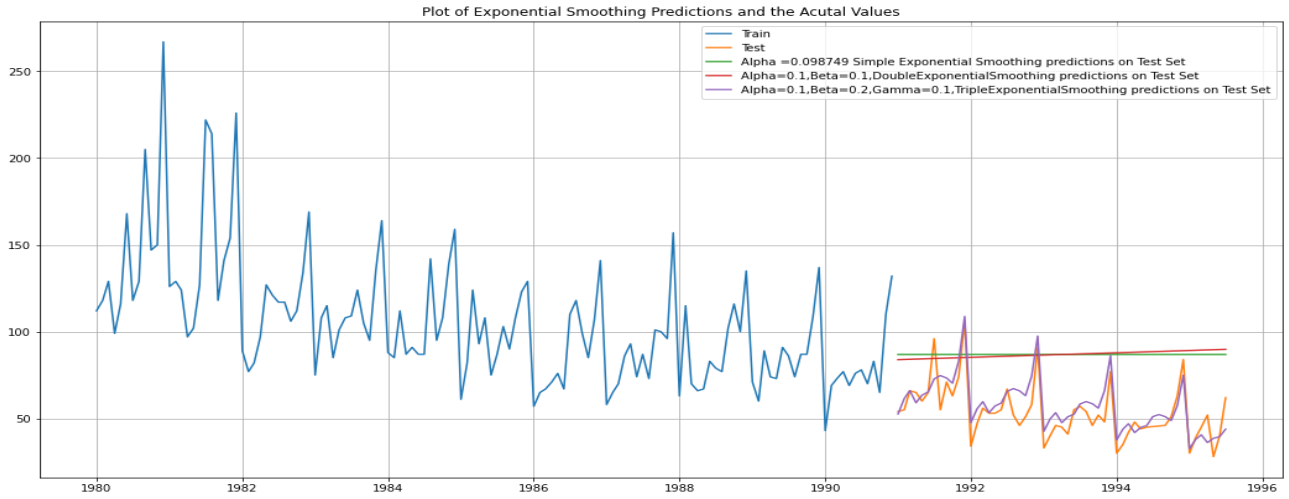


Figure 25 SES, DES and TES Model together comparison on Test data

In this particular we have built several models and went through a model building exercise. This particular exercise has given us an idea as to which particular model gives us the least error on our test set for this data. But in Time Series Forecasting, we need to be very vigil about the fact that after we have done this exercise we need to build the model on the whole data. Remember, the training data that we have used to build the model stops much before the data ends. In order to forecast using any of the models built, we need to build the models again (this time on the complete data) with the same parameters. For this particular mentored learning session, we will go ahead and build only the top 1 model which gave us the best accuracy (least RMSE). The two models to be built on the whole data are the following:

Alpha=0.4,Beta=0.1,Gamma=0.3,TripleExponentialSmoothing

Alpha=0.111,Beta=0.049,Gamma=0.362,TripleExponentialSmoothing

5. Check for the stationarity of the data on which the model is being built on using appropriate statistical tests and also mention the hypothesis for the statistical test. If the data is found to be non-stationary, take appropriate steps to make it stationary. Check the new data for stationarity and comment.

Note: Stationarity should be checked at alpha = 0.05

A time series has stationarity when the observations are not dependent on the time. Statistical properties of these time series will not change with time thus they will have constant mean, variance, and covariance.

The time series which have trends or with seasonality, are not stationary. Because trends will have a change in the movement of data concerning time which will cause the change in mean over time. Whereas seasonality occurs when the pattern in time series shows a variation for a regular time interval which will cause the variance to change over time.

Stationarity of time series can be detected by: Visually Plotting the time series and check for trend or seasonality. By Splitting time series into the different partitions and compare the statistical inference.

We can also perform Augmented Dickey-Fuller test to check the stationarity.

The Augmented Dickey-Fuller test is a unit root test which determines whether there is a unit root and subsequently whether the series is non-stationary.

The hypothesis in a simple form for the ADF test is:

H0H0 : The Time Series has a unit root and is thus non-stationary.

H1H1 : The Time Series does not have a unit root and is thus stationary.

We would want the series to be stationary for building ARIMA models and thus we would want the p-value of this test to be less than the $\alpha\alpha$ value. We have performed ADF test and following are the results for the same:

DF test statistic is -2.240

DF test p-value is **0.4671371627793189**

Number of lags used 13

Since P value for above test is greater than 0.05 , which is 0.4671371627793189, so we are fail to reject the Null hypothesis and we accept that it is a Non stationary Time series. We see that at 5% significant level the Time Series is non-stationary.

There are various ways that Python allows us to select the appropriate number of lags at which we check whether the Time Series is stationary. To know more about the how to select the various ways, please refer to the link over $here$.

Let us take one level of differencing to see whether the series becomes stationary.

DF test statistic is -8.162

DF test p-value is 3.015976115827045e-11

Number of lags used 12

Since P value is way less than 0.05 so going back by one level of differencing will make time series as Stationary. Now, let us go ahead and plot the stationary series.

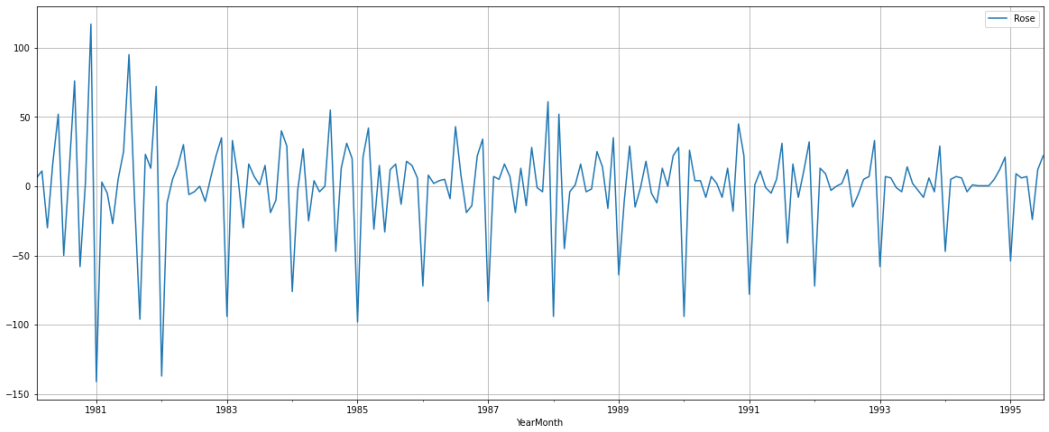


Figure 26 Stationary Time series

Also, if the series is non-stationary, stationaries the Time Series by taking a difference of the Time Series. Then we can use this particular differenced series to train the ARIMA models. We do not need to worry about stationarity for the Test Data because we are not building any models on the Test Data, we are evaluating our models over there. You can look at other kinds of transformations as part of making the time series stationary like taking logarithms

6. Build an automated version of the ARIMA/SARIMA model in which the parameters are selected using the lowest Akaike Information Criteria (AIC) on the training data and evaluate this model on the test data using RMSE

ARIMA Model:

An ARIMA and SARIMA models are class of statistical models for analyzing and forecasting time series data. Lets break it down :

- AR: Autoregression. A model that uses the dependent relationship between an observation and some number of lagged observations.
- I: Integrated. The use of differencing of raw observations in order to make the time series stationary.
- MA: Moving Average. A model that uses the dependency between an observation and a residual error from a moving average model applied to lagged observations.

The parameters of the ARIMA model are defined as follows:

- p: The number of lag observations included in the model, also called the lag order.
- d: The number of times that the raw observations are differenced, also called the degree of differencing.
- q: The size of the moving average window, also called the order of moving average.

The main assumption of AR model is that the time series data is stationary.

A stationary time series is one whose statistical properties such as mean, variance, autocorrelation, etc. are all constant over time.

When the time series data is not stationary, then we convert the non-stationary data before applying AR models. Method we used for making timeseries as Stationary is : Taking the difference between consecutive observations, we also call it a lag-1 difference. For time series with a seasonal component, the lag may be expected to be the period (width) of the seasonality.

White noise of the residuals:

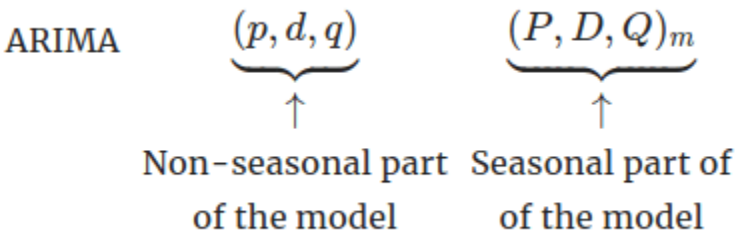
White noise is a process of residuals ϵt that are uncorrelated and follow normal distribution with mean 0 and constant standard deviation. In AR models, one of the main assumptions is the errors follow a white noise.

SARIMA Model:

The difference between ARIMA and SARIMA (SARIMAX) is about the seasonality of the dataset. if your data is seasonal, like it happens after a certain period of time. then we will use SARIMA.

SARIMA stands for Seasonal-ARIMA and it includes seasonality contribution to the forecast. The importance of seasonality is quite evident and ARIMA fails to encapsulate that information implicitly.

The Autoregressive (AR), Integrated (I), and Moving Average (MA) parts of the model remain as that of ARIMA. The addition of Seasonality adds robustness to the SARIMA model. It's represented as:



Source

where m is the number of observations per year. We use the uppercase notation for the seasonal parts of the model, and lowercase notation for the non-seasonal parts of the model.

Similar to ARIMA, the P,D,Q values for seasonal parts of the model can be deduced from the ACF and PACF plots of the data. Let's implement SARIMA for the same Catfish sales model.

Both ARIMA and SARIMA can be build using Automated way of generating values p,d and q value or manual way of building ACF / PACF graph and observe the numbers for p , d and q.

Auto ARIMA Model:

Since we have already seen that Stationarity can be achieved with Lag of 1 . So d value is 1. And We have taken chosen default value for p and q between 0 and 4, And then generated all possible combination for p,d and q. There are the parameters we have chosen for testing our ARIMA Model and gathered AIC value for all the models:

```
Examples of the parameter combinations for the Model
Model: (0, 1, 0)
Model: (0, 1, 1)
Model: (0, 1, 2)
Model: (0, 1, 3)
Model: (1, 1, 0)
Model: (1, 1, 1)
Model: (1, 1, 2)
Model: (1, 1, 3)
Model: (2, 1, 0)
Model: (2, 1, 1)
Model: (2, 1, 2)
Model: (2, 1, 3)
Model: (3, 1, 0)
Model: (3, 1, 1)
Model: (3, 1, 2)
Model: (3, 1, 3)
```

Following are the AIC values for above listed all parameters , when we fit Timeseries data into ARIMA model :

```
ARIMA(0, 1, 0) - AIC:1333.1546729124348
ARIMA(0, 1, 1) - AIC:1282.3098319748315
ARIMA(0, 1, 2) - AIC:1279.6715288535752
ARIMA(0, 1, 3) - AIC:1280.5453761734652
ARIMA(1, 1, 0) - AIC:1317.3503105381526
ARIMA(1, 1, 1) - AIC:1280.574229538006
ARIMA(1, 1, 2) - AIC:1279.870723423191
ARIMA(1, 1, 3) - AIC:1281.8707223309964
ARIMA(2, 1, 0) - AIC:1298.6110341604958
ARIMA(2, 1, 1) - AIC:1281.5078621868606
ARIMA(2, 1, 2) - AIC:1281.8707222264304
ARIMA(2, 1, 3) - AIC:1274.6951493753345
ARIMA(3, 1, 0) - AIC:1297.481091727174
ARIMA(3, 1, 1) - AIC:1282.4192776271934
ARIMA(3, 1, 2) - AIC:1283.720740597716
ARIMA(3, 1, 3) - AIC:1278.6699617388035
```

Then we have sported the data, based on AIC value and least value of AIC have following records:

	param	AIC
11	(2, 1, 3)	1274.695149
15	(3, 1, 3)	1278.669962
2	(0, 1, 2)	1279.671529
6	(1, 1, 2)	1279.870723
3	(0, 1, 3)	1280.545376

Lets build the SARIMAX report for the Best parameter (2,1,3)

```
                               SARIMAX Results
==========================================================================================
Dep. Variable:                           Rose   No. Observations:                  132
Model:                 ARIMA(2, 1, 3)   Log Likelihood                -631.348
Date:                Thu, 25 Aug 2022   AIC                           1274.695
Time:                        22:11:02   BIC                           1291.946
Sample:                    01-01-1980   HQIC                  1281.705
- 12-01-1990        Covariance Type:                        opg
==========================================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.L1         -1.6777      0.084    -20.037      0.000      -1.842      -1.514
ar.L2         -0.7285      0.084     -8.701      0.000      -0.893      -0.564
ma.L1          1.0445      0.650      1.606      0.108      -0.230       2.319
ma.L2         -0.7724      0.134     -5.772      0.000      -1.035      -0.510
ma.L3         -0.9049      0.590     -1.533      0.125      -2.061       0.252
sigma2       858.9672    547.433      1.569      0.117    -213.981    1931.916
==========================================================================================
Ljung-Box (L1) (Q):                   0.02   Jarque-Bera (JB): 24.48
Prob(Q):                              0.88   Prob(JB):  0.00
Heteroskedasticity (H):               0.40   Skew:    0.71
Prob(H) (two-sided):                  0.00   Kurtosis:  4.57
==========================================================================================
Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
```

Lets Build the Diagnostic Plot:

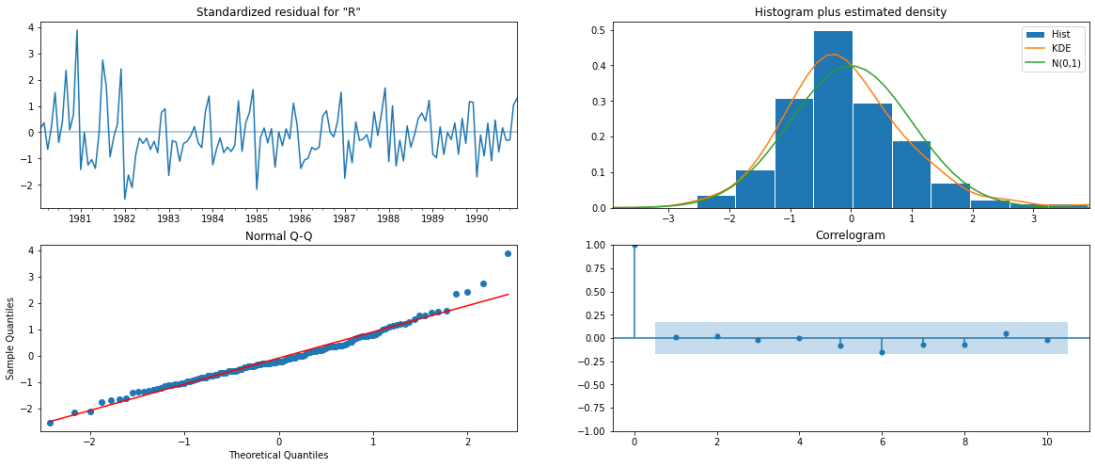


Figure 27 Diagnostic Plot for Auto ARIMA

Lets find the RMSE Value for Auto ARIMA with p, d, q value of 2,1,3:

	RMSE
Auto_ARIMA(2,1,3)	36.809324

Auto SARIMA:

Similar to Auto Arima Model, We have taken all possible combination of p,d,q and P,D,Q values for SARIMA Model, along with the Seasonality parameter. We have taken these combinations:

```
Examples of the parameter combinations for the Model are
Model: (0, 1, 1)(0, 0, 1, 6)
Model: (0, 1, 2)(0, 0, 2, 6)
Model: (0, 1, 3)(0, 0, 3, 6)
Model: (1, 1, 0)(1, 0, 0, 6)
Model: (1, 1, 1)(1, 0, 1, 6)
Model: (1, 1, 2)(1, 0, 2, 6)
Model: (1, 1, 3)(1, 0, 3, 6)
Model: (2, 1, 0)(2, 0, 0, 6)
Model: (2, 1, 1)(2, 0, 1, 6)
Model: (2, 1, 2)(2, 0, 2, 6)
Model: (2, 1, 3)(2, 0, 3, 6)
Model: (3, 1, 0)(3, 0, 0, 6)
Model: (3, 1, 1)(3, 0, 1, 6)
Model: (3, 1, 2)(3, 0, 2, 6)
Model: (3, 1, 3)(3, 0, 3, 6)
```

Then we fit our TS data into SARIMA model to calculate the AIC value and sorted AIC values. Following is Sorted AIC value achieved with p,d,q values:

	param	seasonal	AIC
187	(2, 1, 3)	(2, 0, 3, 6)	951.744297
59	(0, 1, 3)	(2, 0, 3, 6)	952.073632
251	(3, 1, 3)	(2, 0, 3, 6)	952.582102
191	(2, 1, 3)	(3, 0, 3, 6)	953.205616
123	(1, 1, 3)	(2, 0, 3, 6)	953.684951

So the Best parameter for SARIMA Model will be (2,1,3) (2,0,3,6) . RMSE and MEP values are:

```
RMSE: 27.124535166501488

MAPE: 55.23994932700531
```

SARIMAX Results

Dep. Variable:	Rose	No. Observations:	132
Model:	SARIMAX(2, 1, 3)x(2, 0, 3, 6)	Log Likelihood	-464.872
Date:	Thu, 25 Aug 2022	AIC	951.744
Time:	22:13:31	BIC	981.349
Sample:	01-01-1980 - 12-01-1990	HQIC	963.750
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
ar.L1	-0.5028	0.083	-6.082	0.000	-0.665	-0.341
ar.L2	-0.6628	0.084	-7.919	0.000	-0.827	-0.499
ma.L1	-0.3714	1869.343	-0.000	1.000	-3664.216	3663.474
ma.L2	0.2033	1175.137	0.000	1.000	-2303.023	2303.430
ma.L3	-0.8320	1555.238	-0.001	1.000	-3049.043	3047.379
ar.S.L6	-0.0838	0.049	-1.720	0.085	-0.179	0.012
ar.S.L12	0.8099	0.052	15.465	0.000	0.707	0.913
ma.S.L6	0.1701	0.248	0.687	0.492	-0.315	0.656
ma.S.L12	-0.5646	0.199	-2.837	0.005	-0.955	-0.174
ma.S.L18	0.1710	0.143	1.198	0.231	-0.109	0.451
sigma2	260.7967	4.88e+05	0.001	1.000	-9.55e+05	9.56e+05

Ljung-Box (L1) (Q):	0.72	Jarque-Bera (JB):	4.77
Prob(Q):	0.40	Prob(JB):	0.09
Heteroskedasticity (H):	0.54	Skew:	-0.36
Prob(H) (two-sided):	0.06	Kurtosis:	3.73

Warnings: [1] Covariance matrix calculated using the outer product of gradients (complex-step). [2] Covariance matrix is singular or near-singular, with condition number 5.98e+14. Standard errors may be unstable.

Diagnostics Plot:

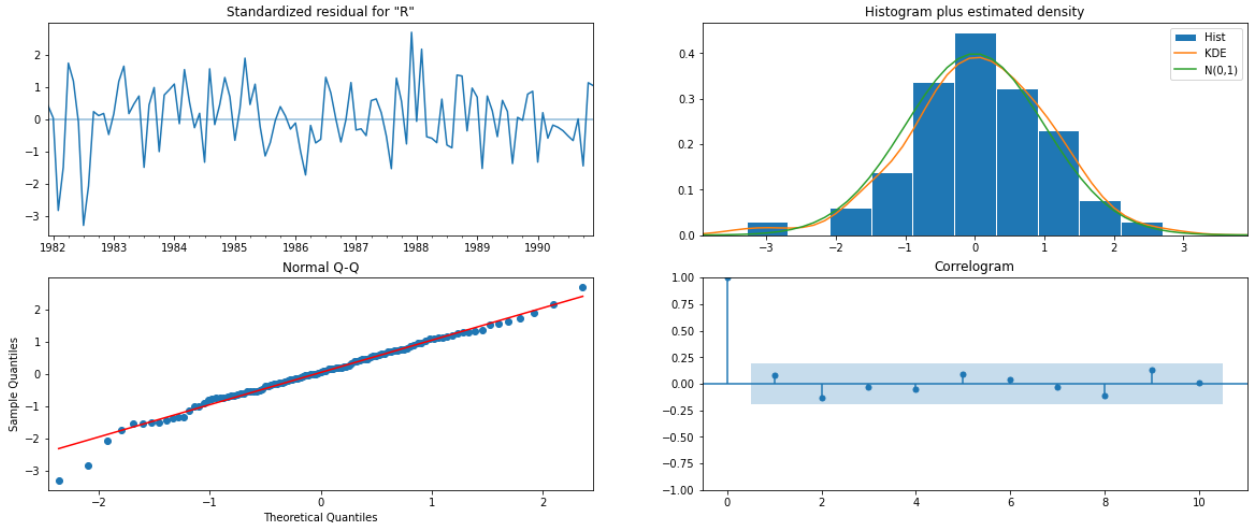


Figure 28 Auto SARIMA Model Diagnostic Plot

7. Build ARIMA/SARIMA models based on the cut-off points of ACF and PACF on the training data and evaluate this model on the test data using RMSE.

In the above question we build Automated version of ARIMA/SARIMA model, in that we have used all possible combinations of p,d and q values and generated Chosen AIC, RMSE and MEP parameters to identify, which combination of pdq is good for the timeseries data.

One problem with Auto version is, it needs lot of computations and we should have good memory and processing power in the system for iterating all possible values. So to avoid that scenario , we can use building ACF/PACF graph for generating Manual version of ARIMA/SARIMA Model.

Important Component of Automated version of ARIMA Model :

Auto-Correlation Function (ACF) or correlogram : A plot of auto-correlation of different lags is called ACF. The plot summarizes the correlation of an observation with lag values. The x-axis shows the lag and the y-axis shows the correlation coefficient between -1 and 1 for negative and positive correlation.

Partial Auto-Correlation Function (PACF) Autocorrelation Function (ACF) : A plot of partial auto-correlation for different values of lags is called PACF.The plot summarizes the correlations for an observation with lag values that is not accounted for by prior lagged observations.

Both plots are drawn as bar charts showing the 95% and 99% confidence intervals as horizontal lines. Bars that cross these confidence intervals are therefore more significant and worth noting.

Some useful patterns you may observe on these plots are:

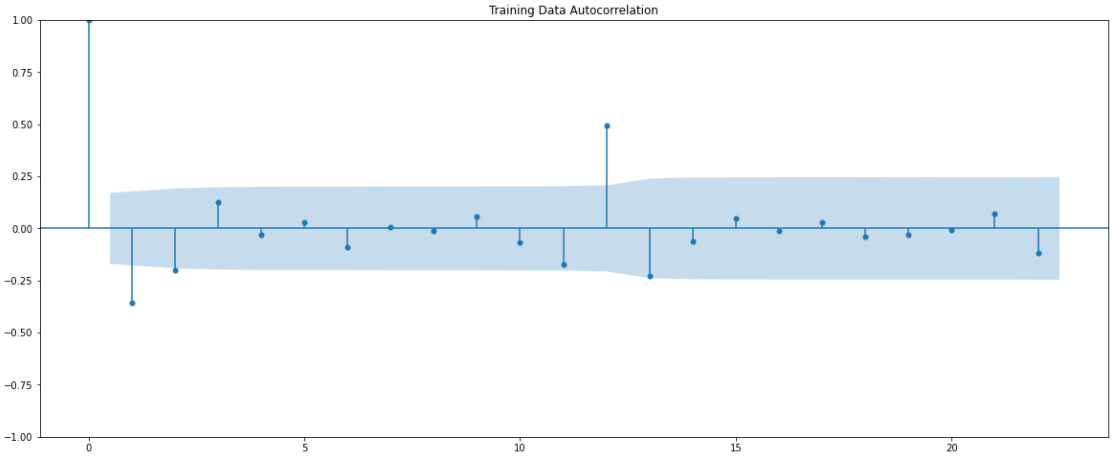
The number of lags is p when:

- The partial auto-correlation, | ppk | > 1.96 / n−−√ for first p values and cuts off to zero. The auto-correlation function, ρk decreases exponentially.
- The model is AR of order p when the PACF cuts-off after a lag p.
- The model is MA of order p when the ACF cuts-off after a lag q.
- The model is a mix of AR and MA if both the PACF and ACF trail off and cuts-off at p and q respectively.

For an ARIMA (p,d,q) process, it becomes non-stationary to stationary after differencing it for d times

Build Manual ARIMA Model

ACF graph for ARIMA Model:



PACF Graph for Arima Model

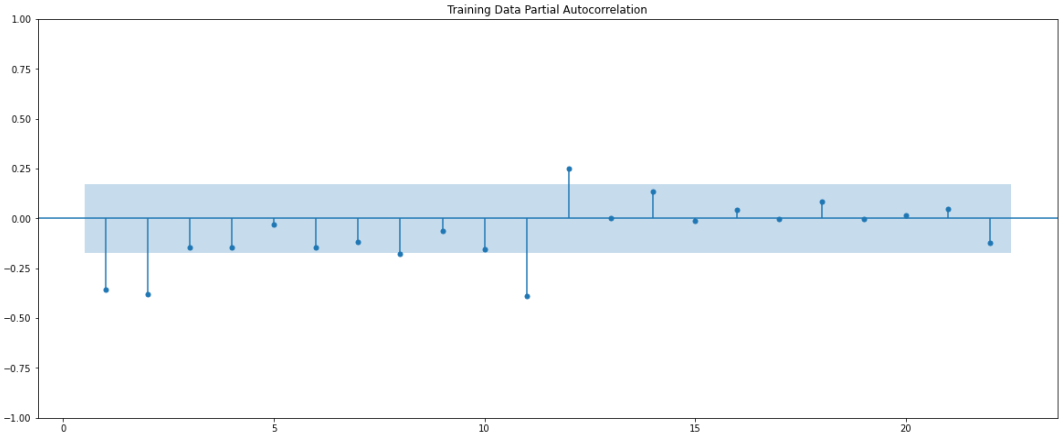


Figure 29 Manual ARIMA ACF/PACF Plot

Here, we have taken alpha=0.05.

- The Auto-Regressive parameter in an ARIMA model is 'p' which comes from the significant lag before which the PACF plot cuts-off to 2.
- The Moving-Average parameter in an ARIMA model is 'q' which comes from the significant lag before the ACF plot cuts-off to 2.

By looking at the above plots, we will take the value of p and q to be 2 and 2 respectively.

SARIMAX Results

Dep. Variable:	Rose	No. Observations:	132
Model:	ARIMA(2, 1, 2)	Log Likelihood	-635.935
Date:	Thu, 25 Aug 2022	AIC	1281.871
Time:	22:11:03	BIC	1296.247
Sample:	01-01-1980	HQIC	1287.712
	- 12-01-1990	Covariance Type:	opg

	coef	std err	z	P>|z|	[0.025	0.975]
ar.L1	-0.4540	0.469	-0.969	0.333	-1.372	0.464
ar.L2	0.0001	0.170	0.001	0.999	-0.334	0.334
ma.L1	-0.2541	0.459	-0.554	0.580	-1.154	0.646
ma.L2	-0.5984	0.430	-1.390	0.164	-1.442	0.245
sigma2	952.1601	91.424	10.415	0.000	772.973	1131.347

Ljung-Box (L1) (Q):	0.02	Jarque-Bera (JB):	34.16
Prob(Q):	0.88	Prob(JB):	0.00
Heteroskedasticity (H):	0.37	Skew:	0.79
Prob(H) (two-sided):	0.00	Kurtosis:	4.94

Warnings: [1] Covariance matrix calculated using the outer product of gradients (complex-step)

Diagnostic Plot:

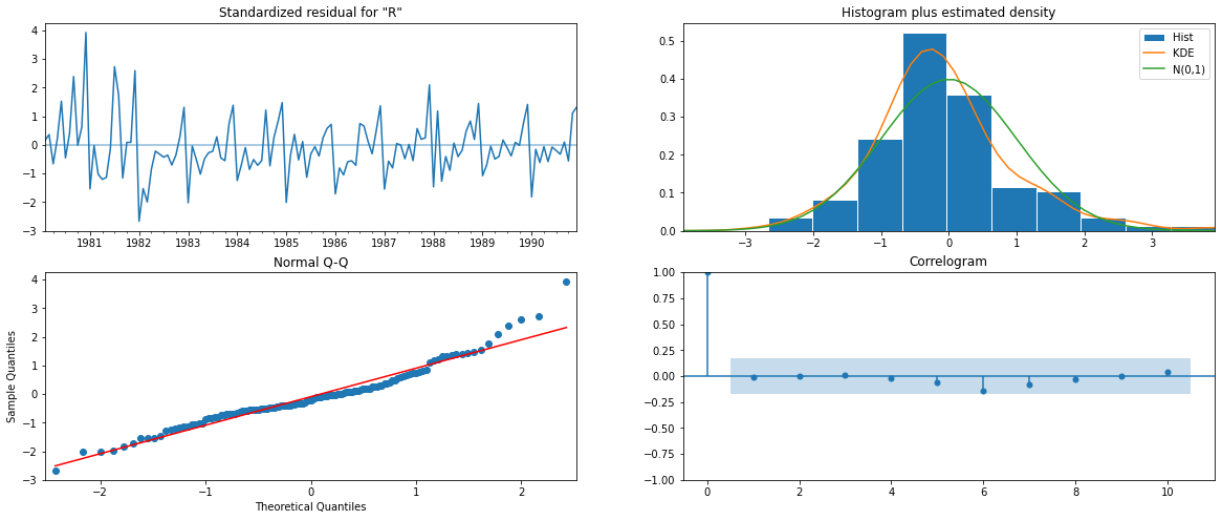


Figure 30 Manual ARIMA Diagnostic Plot

Predict on the Test Set using this model and evaluate the model

```
RMSE: 36.87119662176807
MAPE: 76.05621272229534
```

Build Manual SARIMA Model:

We will star from where we left in ARIMA Model, from the above manual version of ARMIA model, we had chosen value for p,d,q as (2,1,2). So we will choose same for SARIMA Model and then add the parameter for Seasonality . Lets plot ACF and PACF plot one more time:

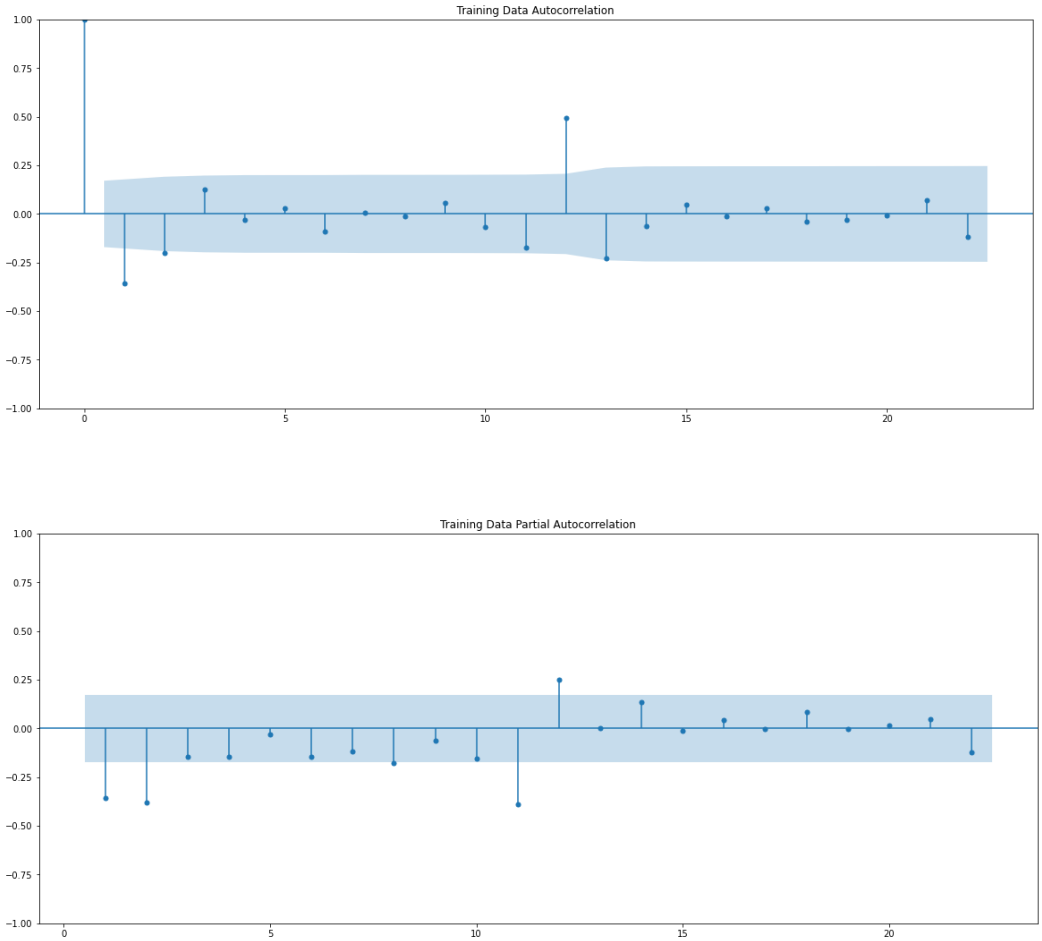


Figure 31 Manual SARIMA ACF/PACF Plot

Here, we have taken alpha=0.05. We can not see that there is a seasonality. and P value and Q value would be 0 from above graphs.

We are going to take the seasonal period as 3 or its multiple e.g. 6. We are taking the p value to be 3 and the q value also to be 3 as the parameters same as the ARIMA model.

The Auto-Regressive parameter in an SARIMA model is 'P' which comes from the significant lag after which the PACF plot cuts-off to 0.

The Moving-Average parameter in an SARIMA model is 'Q' which comes from the significant lag after which the ACF plot cuts-off to 0.

So final values are as follows: (2,1,2)(0,1,0,6) . Lets build the Sarimax result:

```
                      SARIMAX Results
==========================================================================================
Dep. Variable:                  Rose   No. Observations:                  132
Model:             SARIMAX(2, 1, 2)x(0, 1, [], 6)   Log Likelihood                -634.370
Date:                                Thu, 25 Aug 2022   AIC                        1278.739
Time:                                        22:11:04   BIC                        1292.759
Sample:                                  01-01-1980   HQIC                       1284.434
                                       - 12-01-1990
Covariance Type:                                 opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.L1          1.1123      0.055     20.150      0.000       1.004       1.220
ar.L2         -0.3756      0.029    -13.012      0.000      -0.432      -0.319
ma.L1         -1.9944      0.114    -17.561      0.000      -2.217      -1.772
ma.L2          1.0000      0.114      8.786      0.000       0.777       1.223
sigma2      1774.6506      0.000   1.39e+07      0.000    1774.650    1774.651
===================================================================================
Ljung-Box (L1) (Q):                   0.32   Jarque-Bera (JB):                 4.04
Prob(Q):                              0.57   Prob(JB):                         0.13
Heteroskedasticity (H):               0.37   Skew:                            -0.24
Prob(H) (two-sided):                  0.00   Kurtosis:                         3.76
===================================================================
Warnings: [1] Covariance matrix calculated using the outer product of gradients (comp
lex-step). [2] Covariance matrix is singular or near-singular, with condition number
1.8e+22. Standard errors may be unstable.
```

Diagnostics Plot:

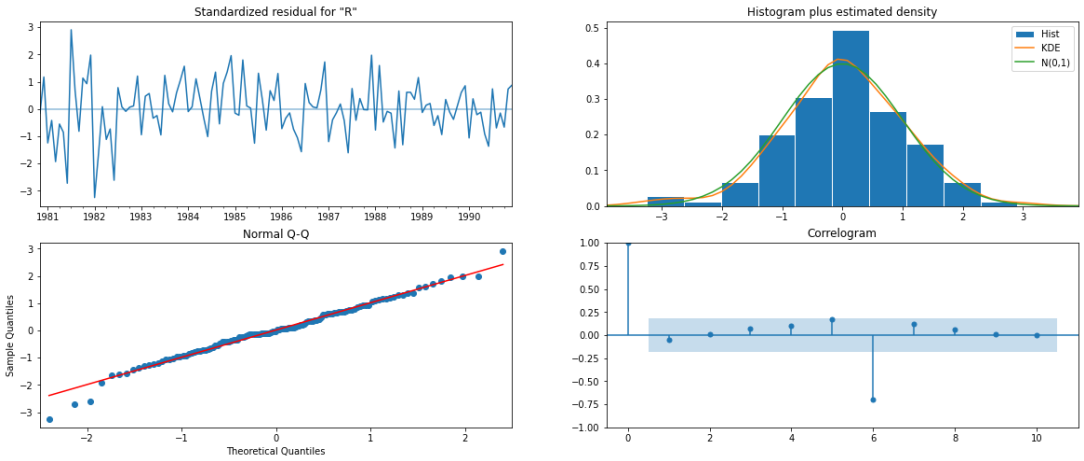


Figure 32 Manual SAIRMA Diagnostic Plot

Calculate the RMSE and MAPE value for test data for Manual SARIMA Model:

```
RMSE: 31.65138712114397

MAPE: 51.711250860674
```

8. Build a table with all the models built along with their corresponding parameters and the respective RMSE values on the test data

We have built several Models, with different parameters, and calculated the RMSE values for all the parameters . After combining all RMSE values for all the models, following is the table for that:

Model name with Parameters	**Test RMSE**
Alpha=0.1,Beta=0.2,Gamma=0.1,TripleExponentialSmoothing	9.22
2pointTrailingMovingAverage	11.529278
3pointTrailingMovingAverage	14.126525
4pointTrailingMovingAverage	14.451403
6pointTrailingMovingAverage	14.566327
RegressionOnTime	15.268955
Alpha=0.077360,Beta=0.03936496,Gamma=0.00083,TripleExponentialSmoothing	19.11311
Auto_SARIMA(2,1,3)(2, 0, 3, 6)	27.124535
manual_SARIMA(2,1,2)(0,1,0,6)	31.651387
Alpha=0.07,SimpleExponentialSmoothing	36.435772
Alpha=0.098749,SimpleExponentialSmoothing	36.796242
Auto_ARIMA(2,1,3)	36.809324
Manual_ARIMA(2,1,2)	36.871197
Alpha=0.1,Beta=0.1,DoubleExponentialSmoothing	36.923416
SimpleAverageModel	53.46057
NaiveModel	79.718773

9. Based on the model-building exercise, build the most optimum model(s) on the complete data and predict 12 months into the future with appropriate confidence intervals/bands.

Based on the table above, we have see that best model so far given least value of RMSE is Triple exponential Model with parameters of Alpha=0.1,Beta=0.2,Gamma=0.1. It's RMSE values is only 9.22 as compared to the worst RMSE value of 79.718773 of Naïve based Model .

Lets build the Triple exponential Smoothing model based on above parameters and fit full data set , we were using only test data for the predictions earlier. We have also calculated the RMSE value for full data set .

```
RMSE of the Full Model 17.023705516791587
```

Also as required, we have predicted nest 12 months of data for using above model:

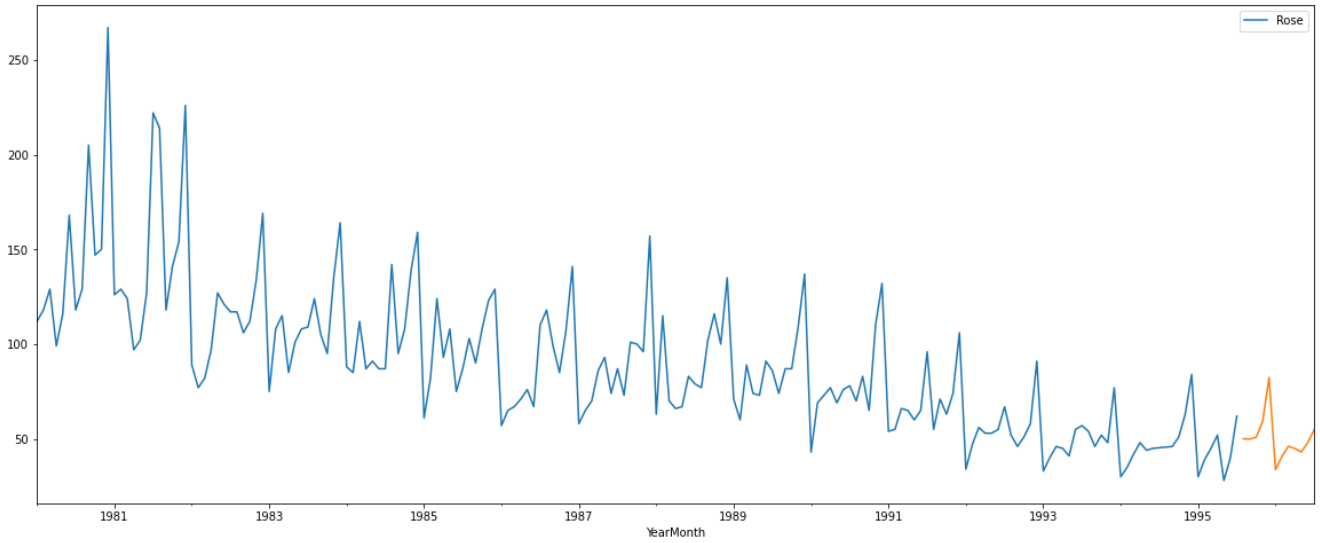


Figure 33 Best Model TES plot for next 12 Months predictions

Plotting the forecast with the confidence band.

Confidence band for forecasting: A confidence band is used in statistical analysis to represent the uncertainty in an estimate of a curve or function based on limited or noisy data. The 95% confidence bands enclose the area that you can be 95% sure contains the true curve. It gives you a visual sense of how well your data define the best-fit curve. It is closely related to the 95% prediction bands , which enclose the area that you expect to enclose 95% of future data points. This includes both the uncertainty in the true position of the curve (enclosed by the confidence bands), and also accounts for scatter of data around the curve. Therefore, prediction bands are always wider than confidence bands

For building the Confidence band we need to find lower and upper values of actual predictions with 95% confidentiality . In our case

Following are the Table for Predictions of upper and lower values along with Predicted values:

	lower_CI	prediction	upper_ci
1995-08-01	1192.471126	1955.959170	2719.447214
1995-09-01	1833.607745	2597.095788	3360.583832
1995-10-01	2622.596318	3386.084362	4149.572406
1995-11-01	3450.819866	4214.307910	4977.795954
1995-12-01	5899.849951	6663.337995	7426.826039
1996-01-01	727.675695	1491.163739	2254.651783
1996-02-01	1147.379748	1910.867791	2674.355835
1996-03-01	1402.958117	2166.446161	2929.934205
1996-04-01	1307.848108	2071.336151	2834.824195
1996-05-01	1089.007967	1852.496011	2615.984055
1996-06-01	987.056069	1750.544113	2514.032156
1996-07-01	1385.839853	2149.327896	2912.815940

Plot the time series with next 12 months of unseen data :

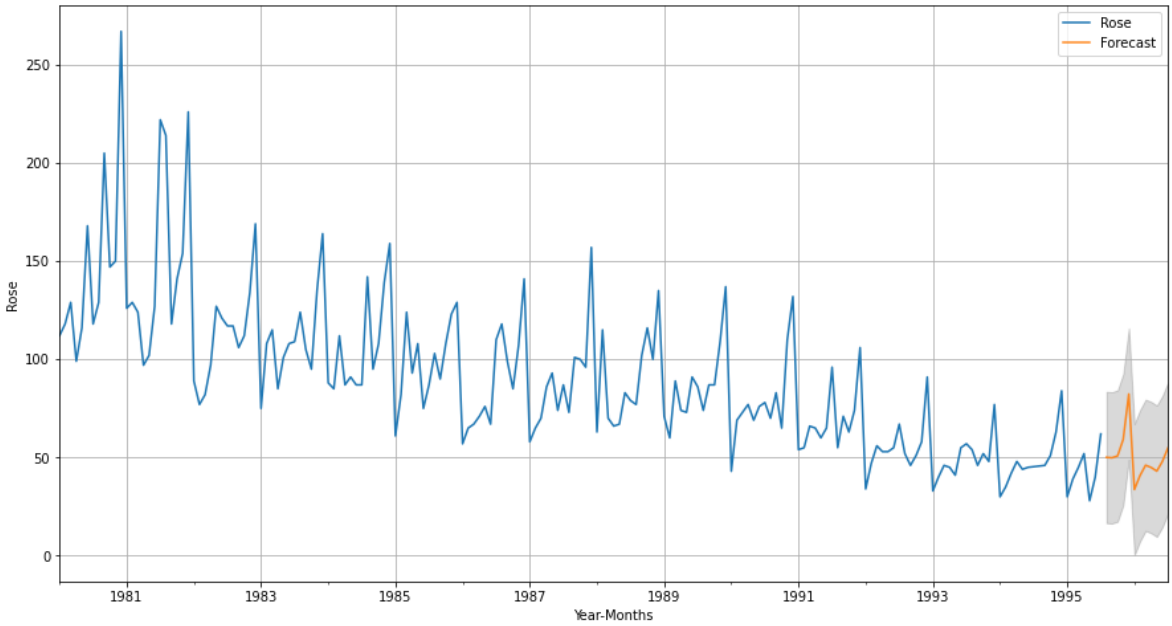


Figure 34 TES Model next 12 Month Predictions with Confidence Band

10. Comment on the model thus built and report your findings and suggest the measures that the company should be taking for future sales.

Analysis Rose wine productions Sales data . These are the observations about the data:

1. Data consist of 187 data points
2. It seems to be contained seasonality
3. We also notice the fluctuations in the trend in the initial years and slowly decreasing the following years.
4. Minimum sales for the data in any month is 28 and Max sales of Rose wines in any month is 267
5. Year 1981 has highest sales and 1995 has lowest sales
6. We see the Big increase in Sales for Rose wine in year 1981 and soon after that started decreasing Also there were years with Rise and fall equally. But gradually Sales goes down.
7. We see there are outliers in almost all the year as per the box plot.
8. Average sales is lowest in the year of 1995
9. Monthly plot contains outliers in the month of June, July, August, September and December.
10. There are Highest sales in the Month of December followed by 2nd highest Sales in November Month, which indicates year end party and vacation celebration sales
11. We see that there are High Sales in each month at the month Start and then Fall down till Mid of the month,
12. We also see small increase in Sales in every Mid of the Month and then again goes down till Month End.
13. It clearly indicates, when Anyone gets the Salary at the Start of the month and sometimes biweekly, then Sales for Month Start is always High and small Rise in Mid of the Month.
14. There are 2 Months data not available in July and August 1994 and then no data after July 1995.
15. The plot shows that in 1982 there is a fall in the wine sales and there is a steep downfall is observed.
16. The resampled yearly or annual series have smoothened out the seasonality and have only been able to capture the year on year trend where there was.
17. There is some rise is found in Year 1984 and 1988, but ultimately that also goes down.
18. We see that the year on year quarterly series represents the year on year monthly series. The quarterly series is able to catch the seasonality in the data.
19. If we take the resampling period to be 10 years or a decade, we see that the seasonality present has been smoothed over and it is only giving an estimate of the trend.
20. Sales trend is going down year by year
21. We have done up sampling of the data for Quarterly, yearly and a Decade sale, there are seasonality in the data, and it gets flatter out when we Up sample the data
22. We have also done Down sampling of the data for analyzing Daily Sales.
23. Correlogram, histogram, residual and quartiles were plotted.

Forecasting analysis:

We have built 17 Models on our data and Compared all of them based on RMSE value for each Model: Results with the parameters used for comparisons are as follows:

	Test RMSE
Alpha=0.1,Beta=0.2,Gamma=0.1,TripleExponentialSmoothing	9.220000
2pointTrailingMovingAverage	11.529278
3pointTrailingMovingAverage	14.126525
4pointTrailingMovingAverage	14.451403
6pointTrailingMovingAverage	14.566327
RegressionOnTime	15.268955
Alpha=0.077360,Beta=0.03936496,Gamma=0.00083,TripleExponentialSmoothing	19.113110
Auto_SARIMA(2,1,3)(2, 0, 3, 6)	27.124535
manual_SARIMA(2,1,2)(0,1,0,6)	31.651387
Alpha=0.07,SimpleExponentialSmoothing	36.435772
Alpha=0.098749,SimpleExponentialSmoothing	36.796242
Auto_ARIMA(2,1,3)	36.809324
Manual_ARIMA(2,1,2)	36.871197
Alpha=0.1,Beta=0.1,DoubleExponentialSmoothing	36.923416
SimpleAverageModel	53.460570
NaiveModel	79.718773

It is clear that Alpha=0.1,Beta=0.2,Gamma=0.1,TripleExponentialSmoothing has the lower RMSE of 9.22 and Naive Model has the highest RMSE value of 79.718

Based on above listed table, we have found that triple exponential Model doing the best prediction , based on Lowest RMSE value of 9.22 as compared to other models.

Recommendations:

From the forecast it is being predicted that the sales will increase in the next 12 months and efforts should be made to keep inventory as per the predicted forecast. Also as observed before sales will increase till December and then there is a likely chance of sharp drop in January followed by a very gradual increase till July 1996. This is similar observations during Monthly and yearly plot with seasonality .

Also, wine producing company should take efforts or run marketing campaign for promoting wine productions at the start of the year and especially in the period of Jan-July 1996 Months.

Company should also analyze further various business methods like marketing, optimization of raw materials and for better profitability.

Sparkling.csv

Introduction: This report explains the business requirements and provide the detailed solution based on the data provided for each problem statement. given in the assignment. Also, the purpose of this exercise is to execute various Timeseries forecasting learning techniques and building various models over Timeseries data, combine all predictions and find out the model with best prediction or accuracy. Timeseries data is unlikely normal data used in supervised/unsupervised learning, where we have dependent /independent variables.

In TS data, we need to use predictions on Same field, using Old data set in ordered Time manner.

1. Read the data as an appropriate Time Series data and plot the data.

Time Series is a sequence of observations recorded at regular time intervals.

YearMonth	Sparkling	
0	1980-01-01	1686
1	1980-02-01	1591
2	1980-03-01	2304
3	1980-04-01	1712
4	1980-05-01	1471
...	...	...
182	1995-03-01	1897
183	1995-04-01	1862
184	1995-05-01	1670
185	1995-06-01	1688
186	1995-07-01	2031

187 rows × 2 columns

Given data is not time. So, we parse the date range and create a timestamp.

Plot for Rose wine Sales data

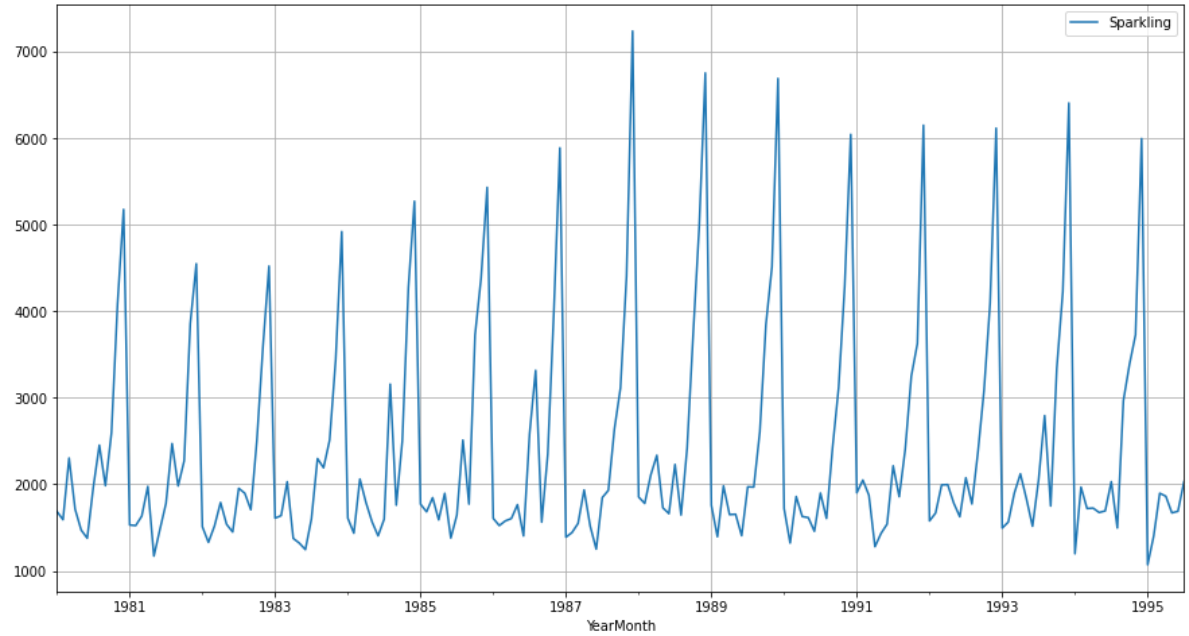


Figure 35 Time series Plot

```
Describer for Sparkling Wine Sample data :            Sparkling
count   187.000000
mean   2402.417112
std    1295.111540
min    1070.000000
25%    1605.000000
50%    1874.000000
75%    2549.000000
max    7242.000000
```

Insights:

1. Data consist of 187 data points
2. It seems to be contained seasonality
3. We also notice that there is small portion of the trend in data but altogether its not visible very clearly by plotting it in years.
4. Minimum sales for the data in Any month are 1070 and Max sales of Rose wines in any month is 7242
5. Year 1988 has highest sales
6. We also notice that 1995 has lowest sales , but this is not correct because we don't have full data point for this year

2. Perform appropriate Exploratory Data Analysis to understand the data and also perform decomposition.

Box Plot for Year on year sales data

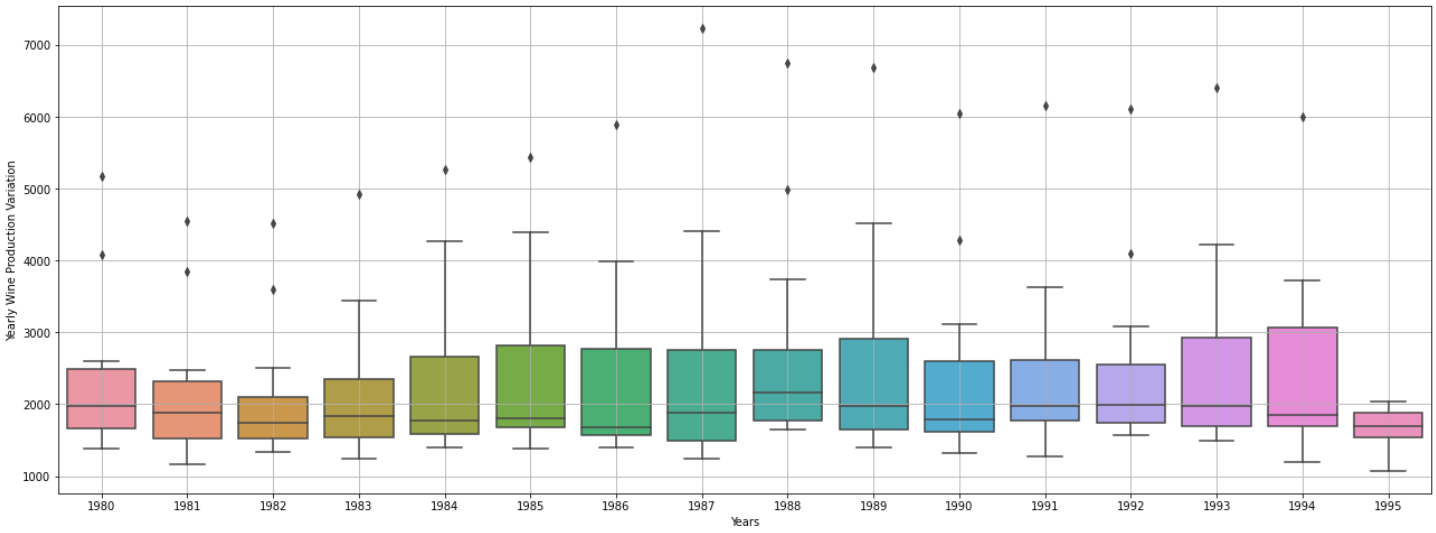


Figure 36 Box plot for year to year sales

Insights:

1. We see the decrease in Sales for Sparkling wine in initial first 3-4 years , then again Sales increased , which remain almost constant and no big increase in sales .
2. Boxplot helps to check the outliers in each year and month and we see there are outliers in almost all the year as per the box plot.
3. Average sales are lowest in the year of 1995

Monthly plot

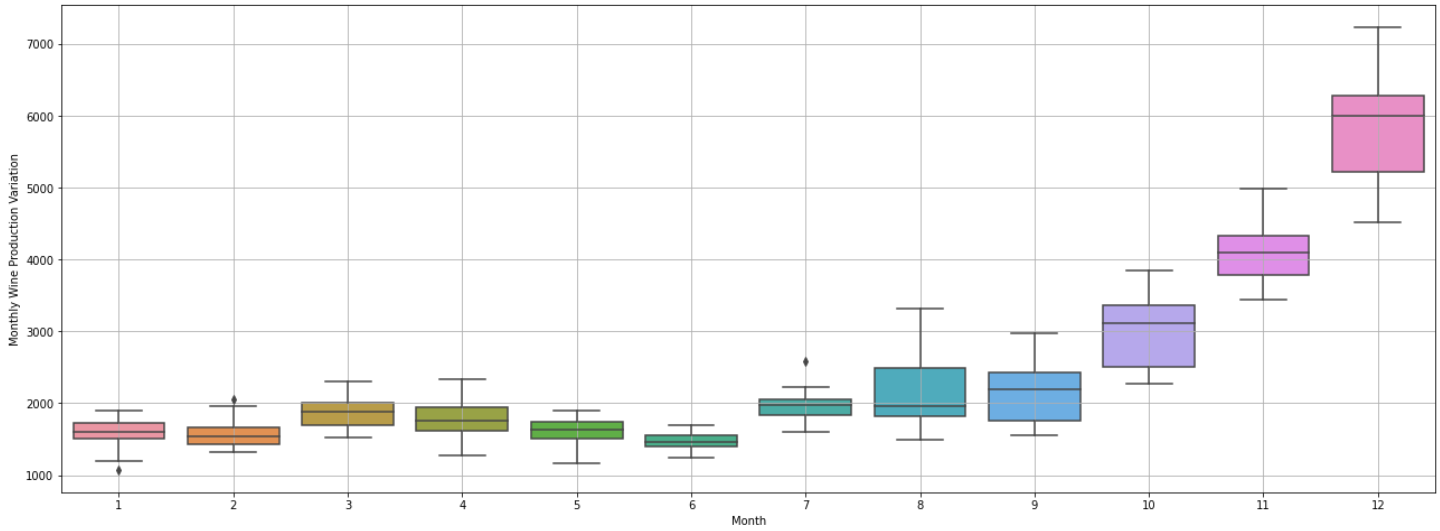


Figure 37 Time series Plot

Insights:

1. The box plot for various months is plotted
2. Monthly plot contains outliers in the month of February and July month Sales
3. There are Highest sales in the Month of December followed by 2nd highest Sales in November Month, which indicates year end party and vacation celebration sales
4. As year end comes, sales start growing and it is about similar sales , which is very less for the first 6 months of any year

Plot a month plot of the give Time Series:

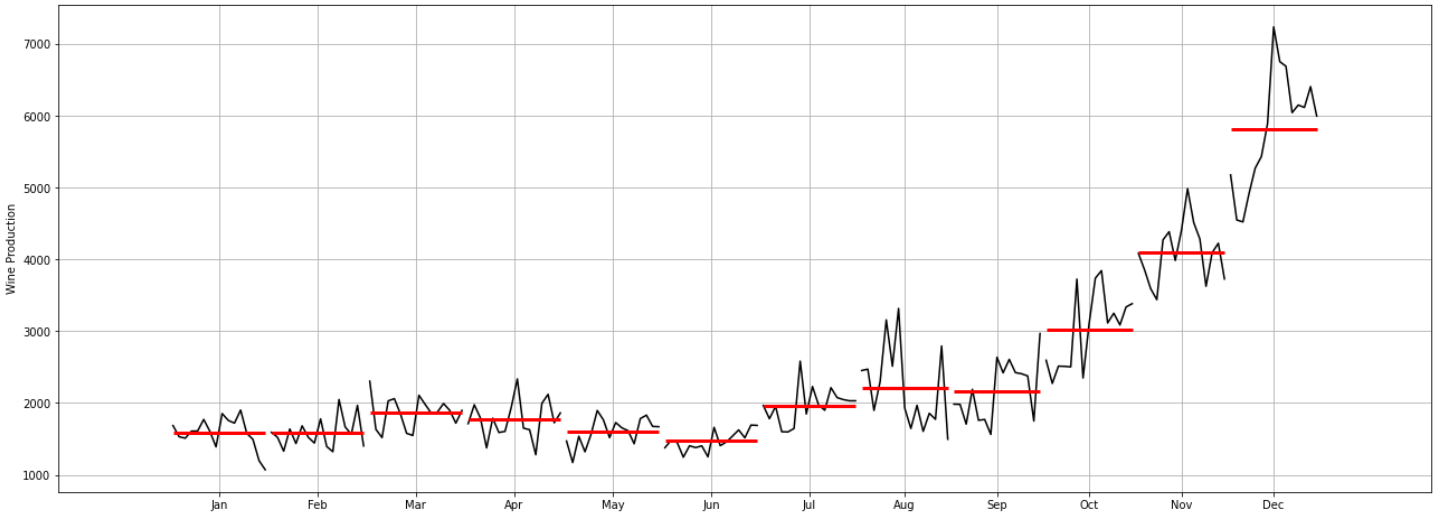


Figure 38 Month Plot of the Given time series

Insights:

1. We have accumulated all year's data for each month and plotted it against each month.
2. We see that there are High Sales in each month at the Mid of the month and then Fall down till end of the month.
3. We also see decrease and lowest Sales in every Start of the month and then slightly cover up , goes highest in Mid of the month.-

Plot the Time Series according to different months for different years

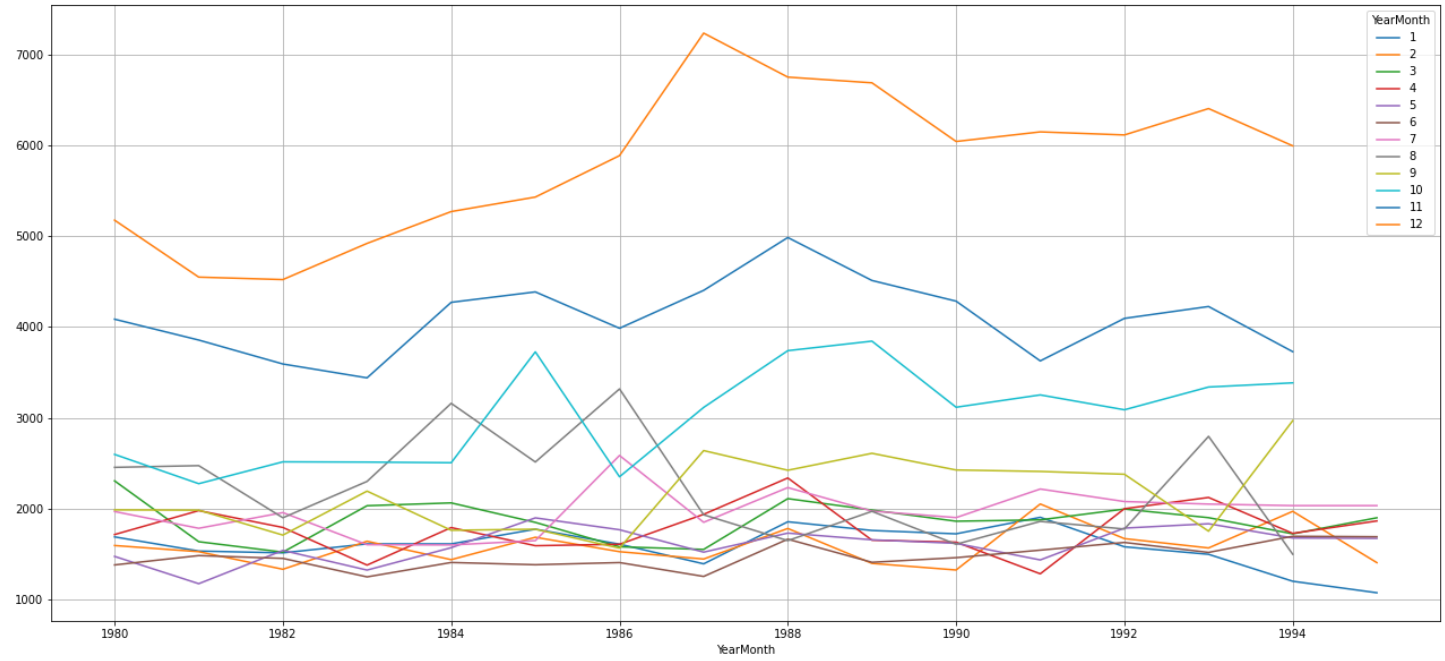


Figure 39 yearly sales across Month

YearMonth	1	2	3	4	5	6	7	8	9	10	11	12
YearMonth												
1980	1686.0	1591.0	2304.0	1712.0	1471.0	1377.0	1966.0	2453.0	1984.0	2596.0	4087.0	5179.0
1981	1530.0	1523.0	1633.0	1976.0	1170.0	1480.0	1781.0	2472.0	1981.0	2273.0	3857.0	4551.0
1982	1510.0	1329.0	1518.0	1790.0	1537.0	1449.0	1954.0	1897.0	1706.0	2514.0	3593.0	4524.0
1983	1609.0	1638.0	2030.0	1375.0	1320.0	1245.0	1600.0	2298.0	2191.0	2511.0	3440.0	4923.0
1984	1609.0	1435.0	2061.0	1789.0	1567.0	1404.0	1597.0	3159.0	1759.0	2504.0	4273.0	5274.0
1985	1771.0	1682.0	1846.0	1589.0	1896.0	1379.0	1645.0	2512.0	1771.0	3727.0	4388.0	5434.0
1986	1606.0	1523.0	1577.0	1605.0	1765.0	1403.0	2584.0	3318.0	1562.0	2349.0	3987.0	5891.0
1987	1389.0	1442.0	1548.0	1935.0	1518.0	1250.0	1847.0	1930.0	2638.0	3114.0	4405.0	7242.0
1988	1853.0	1779.0	2108.0	2336.0	1728.0	1661.0	2230.0	1645.0	2421.0	3740.0	4988.0	6757.0
1989	1757.0	1394.0	1982.0	1650.0	1654.0	1406.0	1971.0	1968.0	2608.0	3845.0	4514.0	6694.0
1990	1720.0	1321.0	1859.0	1628.0	1615.0	1457.0	1899.0	1605.0	2424.0	3116.0	4286.0	6047.0
1991	1902.0	2049.0	1874.0	1279.0	1432.0	1540.0	2214.0	1857.0	2408.0	3252.0	3627.0	6153.0
1992	1577.0	1667.0	1993.0	1997.0	1783.0	1625.0	2076.0	1773.0	2377.0	3088.0	4096.0	6119.0
1993	1494.0	1564.0	1898.0	2121.0	1831.0	1515.0	2048.0	2795.0	1749.0	3339.0	4227.0	6410.0
1994	1197.0	1968.0	1720.0	1725.0	1674.0	1693.0	2031.0	1495.0	2968.0	3385.0	3729.0	5999.0
1995	1070.0	1402.0	1897.0	1862.0	1670.0	1688.0	2031.0	NaN	NaN	NaN	NaN	NaN

Insights:

1. December records have the high number of Sparkling wine sales in each year.
2. January have low number of wine sales.
3. There are 5 Months data not available for year 1995. So we have all data point available from Start of Year 1980 till July 1995.

Yearly Plot:

aggregate the time series from an annual perspective and summing up the observations

	YearMonth	Sparkling
0	1980-12-31	28406
1	1981-12-31	26227
2	1982-12-31	25321
3	1983-12-31	26180
4	1984-12-31	28431

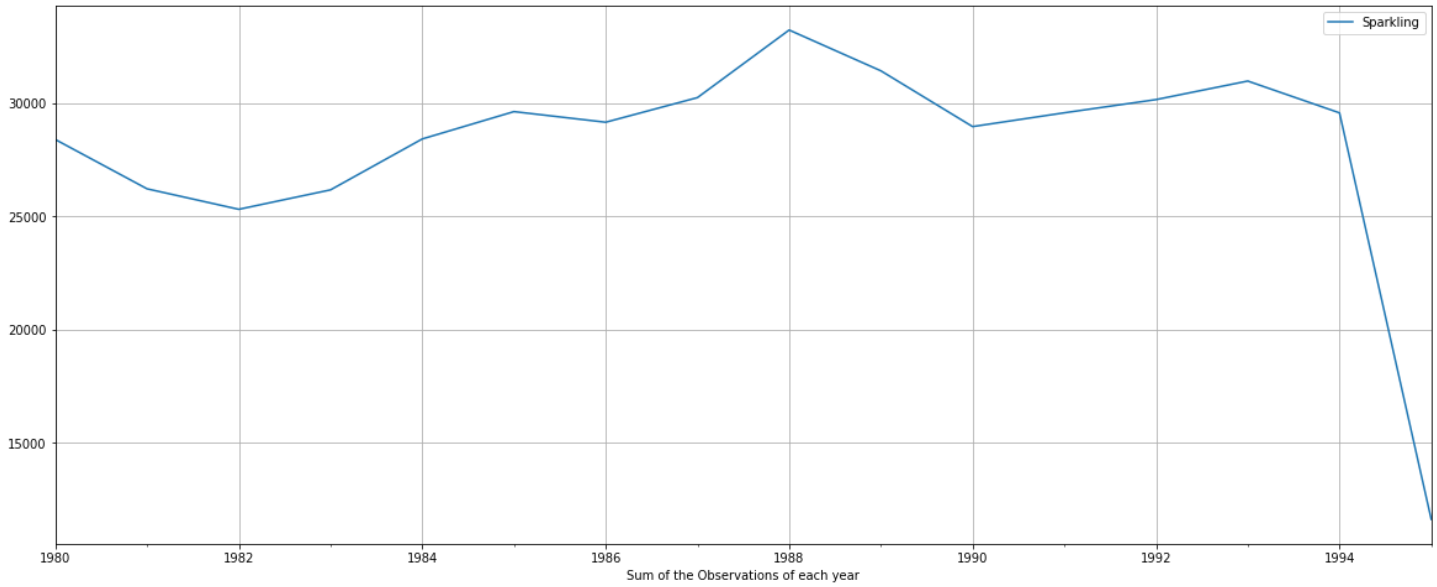


Figure 40 Sum of Sales Each year Plot

Insights:

1. The plot shows that in 1982 there is a fall in the wine sales
2. Then sales goes steady will 1987
3. Highest sales observed in year 1988 , then goes down again for 89 and 90s.
4. After 1994, Sales goes down rapidly, but this is all because we don't have data for full year 1995.
5. The resampled yearly or annual series have smoothened out the seasonality and have only been able to capture the year on year trend where there was.

Quarterly plot –

aggregate the time series from a quarterly perspective and sum the observations of each quarter.

	Sparkling
YearMonth	
1980-03-31	5581
1980-06-30	4560
1980-09-30	6403
1980-12-31	11862
1981-03-31	4686

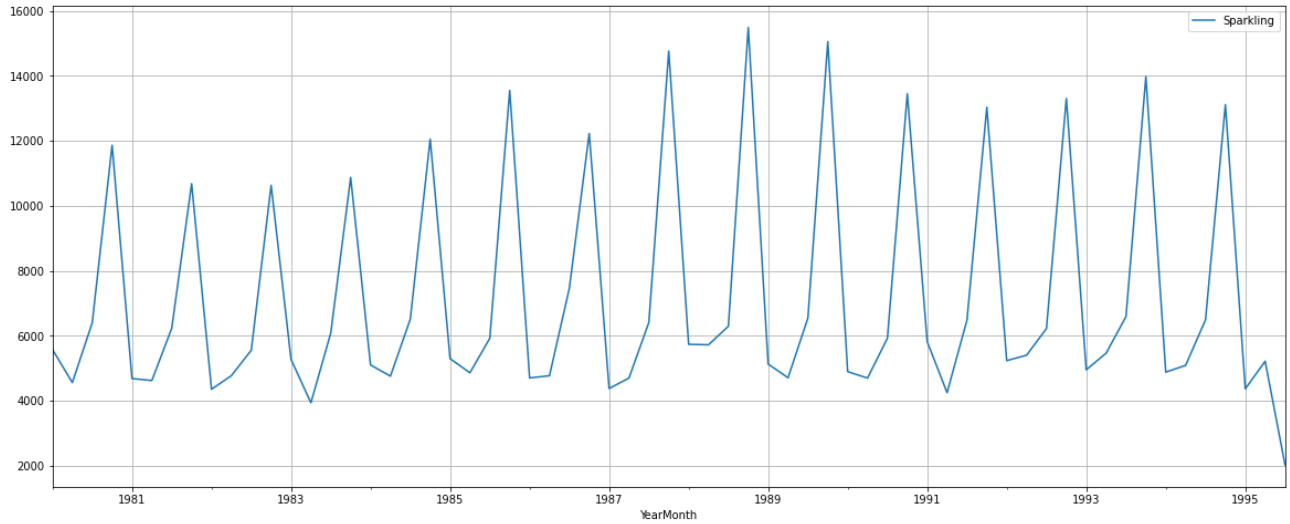


Figure 41 Quarterly sales for each year

Insights:

1. We see that the year on year quarterly series represents the year on year monthly series. The quarterly series is able to catch the seasonality in the data.

Daily plot

aggregate the data from a daily perspective

YearMonth	Sparkling
1980-01-01	1686
1980-01-02	0
1980-01-03	0
1980-01-04	0
1980-01-05	0
...	...
1995-06-27	0
1995-06-28	0
1995-06-29	0
1995-06-30	0
1995-07-01	2031

5661 rows × 1 columns

The values which the original series cannot provide is taken as 0 by python if we try to resample the data on a daily basis.

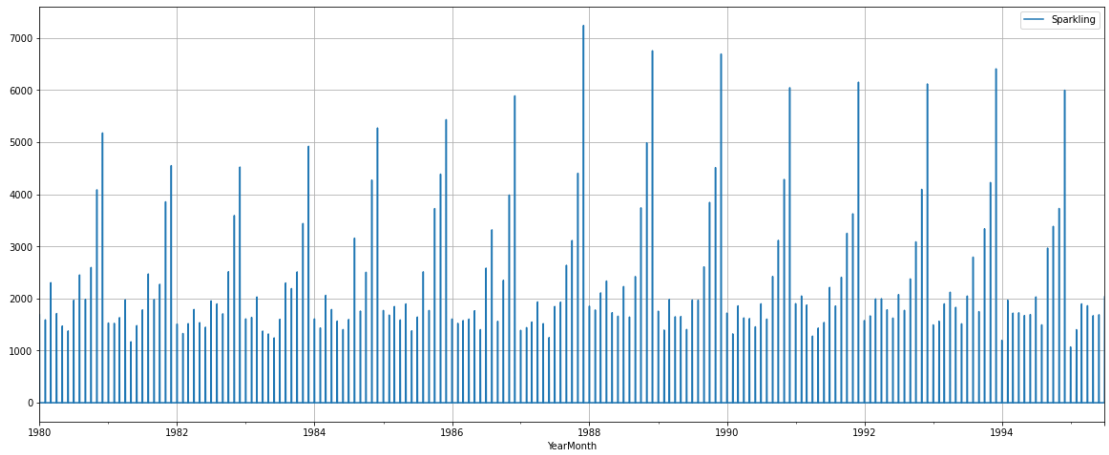


Figure 42 Daily Sales for all years

The above graph fails to give us a proper understanding of our data. Thus, resampling the data to intervals where a number of observations are 0 is not a good idea as that does not give us an understanding of the performance of the time series.

To get a very high-level overview of the trend of the Time Series Data (if Trend is present) can be understood by resampling the data keeping the intervals very large.

Decade Plot

	Sparkling
YearMonth	
1980-12-31	28406
1990-12-31	288893
2000-12-31	131953

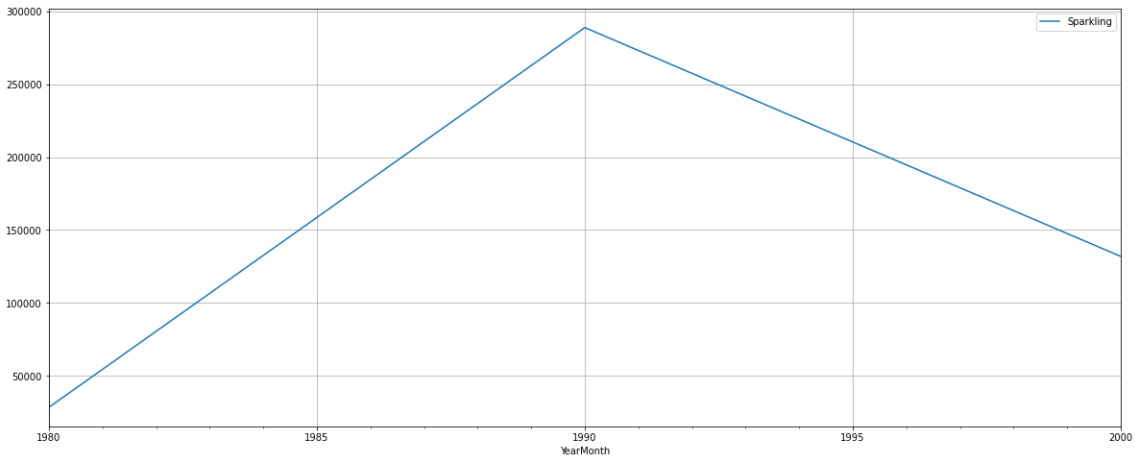


Figure 43 Decade Sales for all data

If we take the resampling period to be 10 years or a decade, we see that the seasonality present has been smoothed over and it is only giving an estimate of the trend.

There are always Increase in decade data, which gets flattened out in future decades

Decompose the Time Series : Additive Model

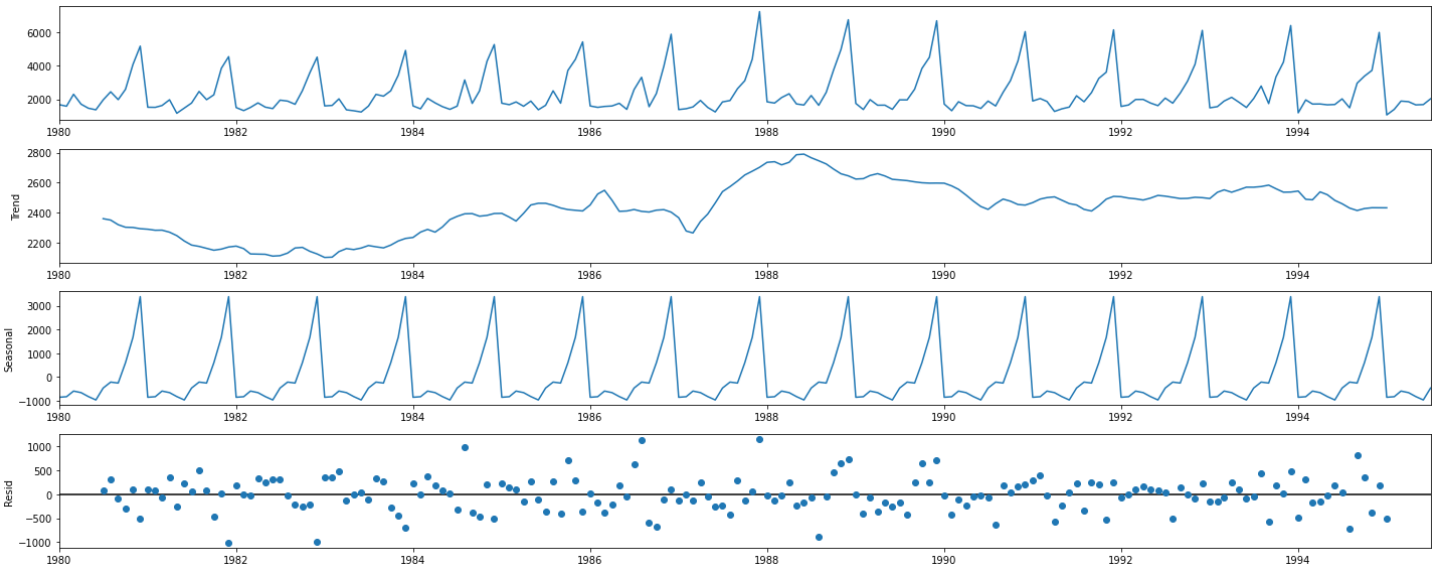


Figure 44 Additive Decomposition

Insights:

1. We have built 2 Models of the data Additive trend as well as Multiplicative Trend .
2. From the 'additive' decomposition, there is seasonality in the data. Which is at the End of the year when Sales goes High, and at the Start of the year, sales goes down for following years.
3. Sales trend is almost constant for 2nd half of the data and don't see , a big difference in total Sales

```
Trend
 YearMonth
1980-01-01            NaN
1980-02-01            NaN
1980-03-01            NaN
1980-04-01            NaN
1980-05-01            NaN
1980-06-01            NaN
1980-07-01    2360.666667
1980-08-01    2351.333333
1980-09-01    2320.541667
1980-10-01    2303.583333
1980-11-01    2302.041667
1980-12-01    2293.791667
Name: trend, dtype: float64

Seasonality
 YearMonth
1980-01-01    -854.260599
1980-02-01    -830.350678
1980-03-01    -592.356630
1980-04-01    -658.490559
1980-05-01    -824.416154
1980-06-01    -967.434011
```

```
1980-07-01    -465.502265
1980-08-01    -214.332821
1980-09-01    -254.677265
1980-10-01     599.769957
1980-11-01    1675.067179
1980-12-01    3386.983846
Name: seasonal, dtype: float64

Residual
 YearMonth
1980-01-01           NaN
1980-02-01           NaN
1980-03-01           NaN
1980-04-01           NaN
1980-05-01           NaN
1980-06-01           NaN
1980-07-01     70.835599
1980-08-01    315.999487
1980-09-01    -81.864401
1980-10-01   -307.353290
1980-11-01    109.891154
1980-12-01   -501.775513
Name: resid, dtype: float64
```

Sales data without Seasonality component:

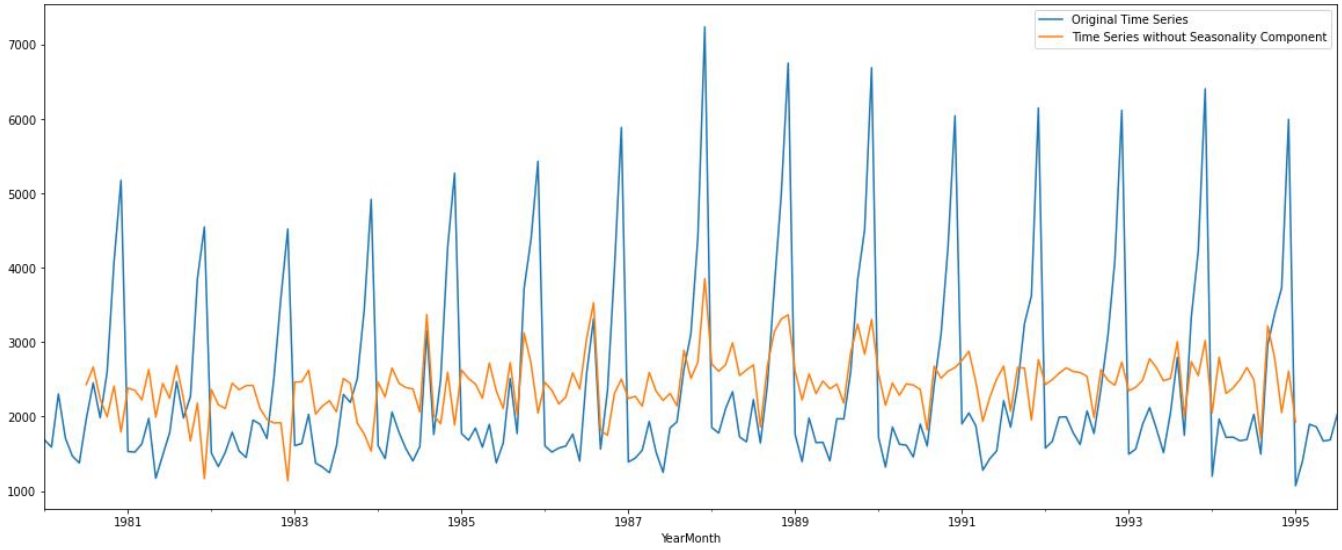


Figure 45 Sales data without Seasonality

Multiplicative Model for Rose data problem

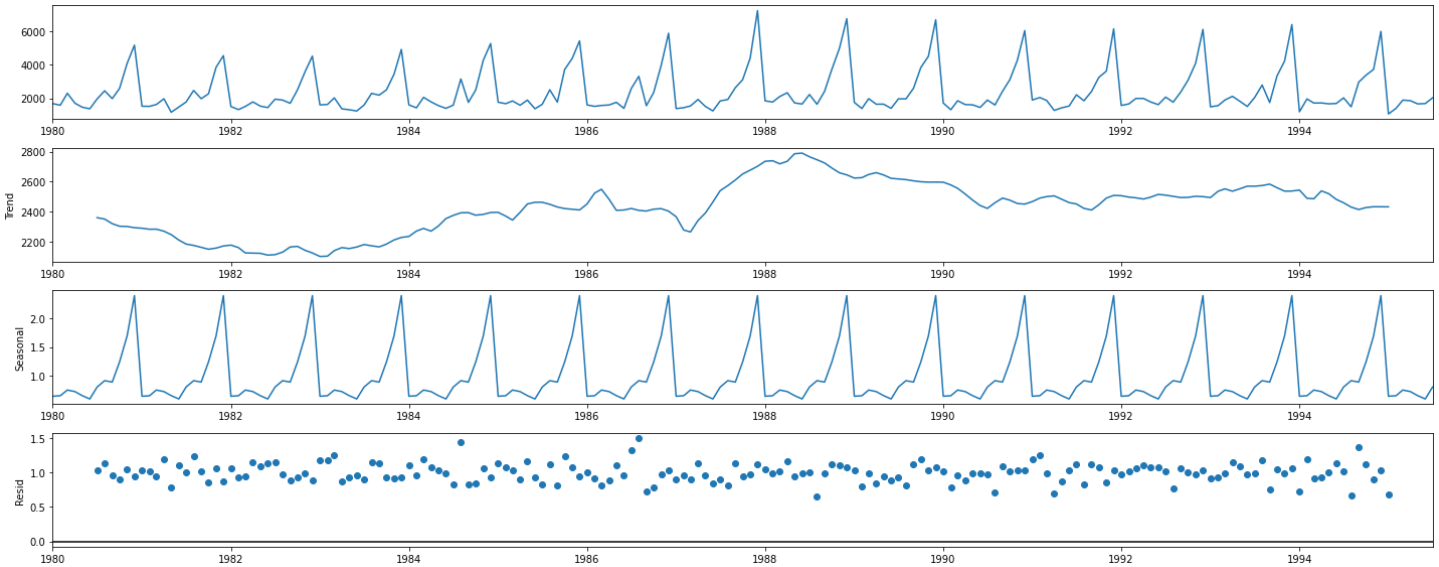


Figure 46 Multiplicative Sales Decomposition

Since Residual is more close to a single line, we would choose Multiplicative Decomposition over Additive decomposing.

3. Split the data into training and test. The test data should start in 1991.

We have split the data in train and test data set,

Data before 1991 is considered as train set and data after 1991 is considered as test data set:

After splitting this is the final shape of the Train and Test data

```
(132, 1)
(55, 1)


First few rows of Training Data
            Sparkling
YearMonth
1980-01-01       1686
1980-02-01       1591
1980-03-01       2304
1980-04-01       1712
1980-05-01       1471

Last few rows of Training Data
            Sparkling
YearMonth
1990-08-01       1605
1990-09-01       2424
1990-10-01       3116
1990-11-01       4286
1990-12-01       6047

First few rows of Test Data
            Sparkling
YearMonth
1991-01-01       1902
1991-02-01       2049
1991-03-01       1874
1991-04-01       1279
1991-05-01       1432

Last few rows of Test Data
            Sparkling
YearMonth
1995-03-01       1897
1995-04-01       1862
1995-05-01       1670
1995-06-01       1688
1995-07-01       2031
```

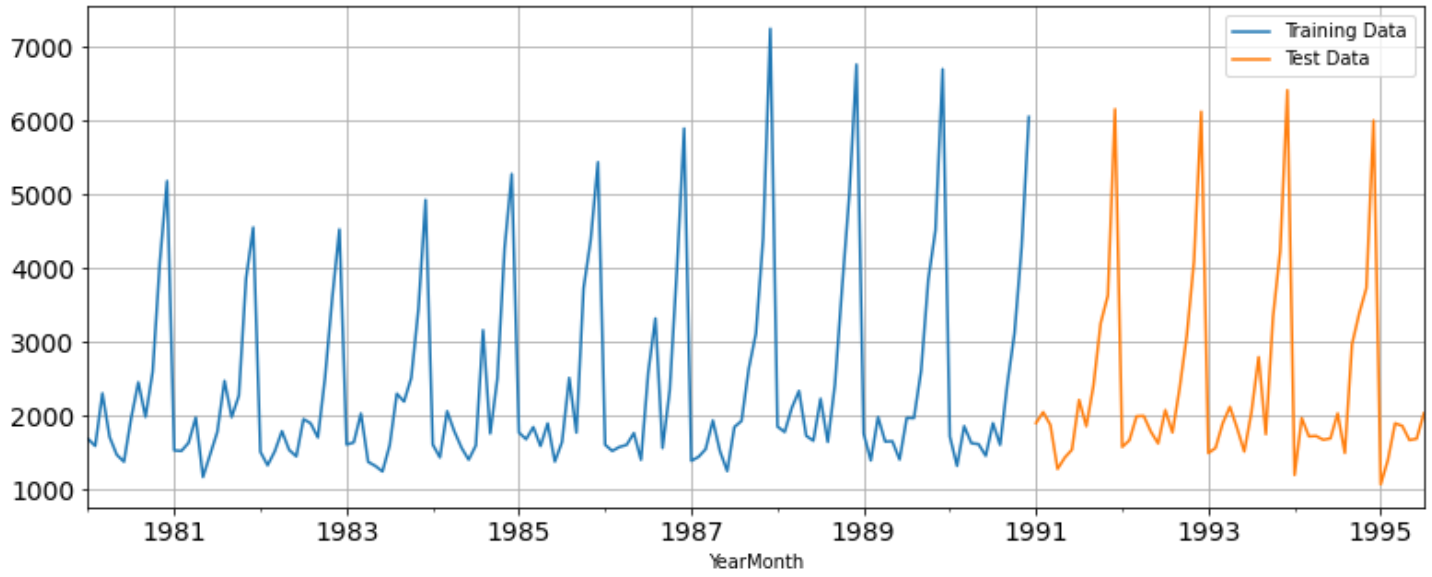


Figure 47 Plot Train and test data

It is difficult to predict the future if the past is not happened. From the above split, we are predicting similar to the past data.

4. Build all the exponential smoothing models on the training data and evaluate the model using RMSE on the test data. Other additional models such as regression, naïve forecast models, simple average models, moving average models should also be built on the training data and check the performance on the test data using RMSE

Model1: Linear Regression

For this particular linear regression, we are going to regress the 'Sparkling' variable against the order of the occurrence. For this we need to modify our training data before fitting it into a linear regression.
Regress the "Sparkling'" variable against the order of occurrence. We have also generated the numerical instance order for both training and test set . Linear Regression is built on the training and test dataset

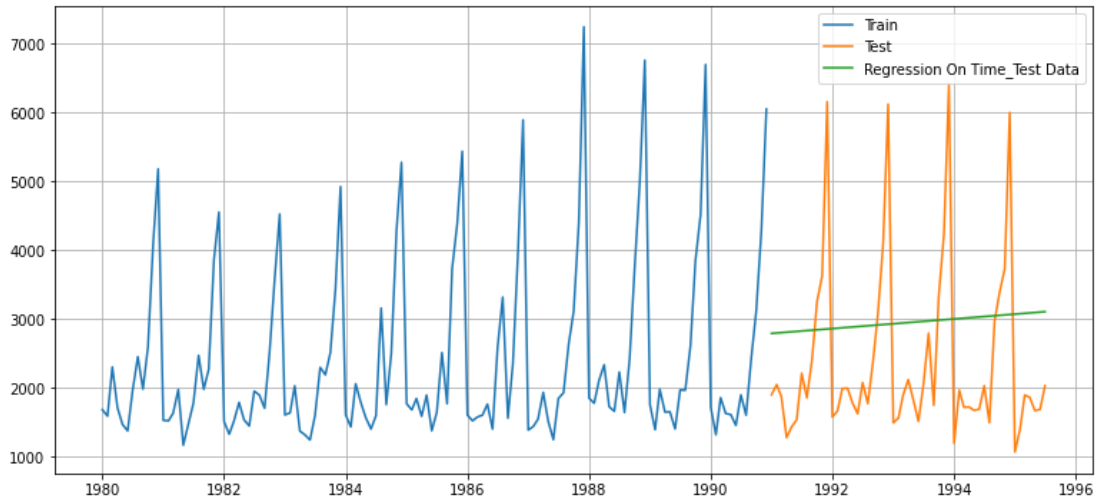


Figure 48 Linear regression Prediction Plot

We have evaluated the Model based on RMSE parameter and put this in a Data frame, which we would use later for Comparing multiple models
Model evaluation :

	Test RMSE
RegressionOnTime	1389.135175

Model2 – Naïve model

For this particular naive model, we say that the prediction for tomorrow is the same as today and the prediction for day after tomorrow is tomorrow and since the prediction of tomorrow is same as today, therefore the prediction for day after tomorrow is also today.

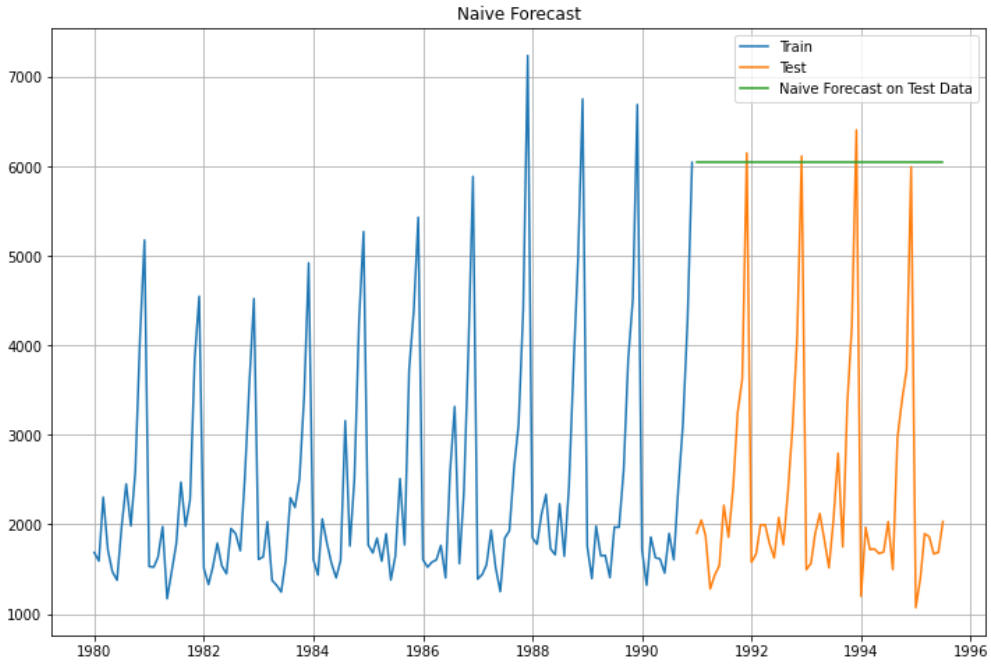


Figure 49 Prediction Plot for Naïve based Model

Model Evaluation

	Test RMSE
RegressionOnTime	1389.135175
NaiveModel	3864.279352

Model 3 – Simple Average

For this particular simple average method, we will forecast by using the average of the training values.

	Sparkling	mean_forecast
YearMonth		
1991-01-01	1902	2403.780303
1991-02-01	2049	2403.780303
1991-03-01	1874	2403.780303
1991-04-01	1279	2403.780303
1991-05-01	1432	2403.780303

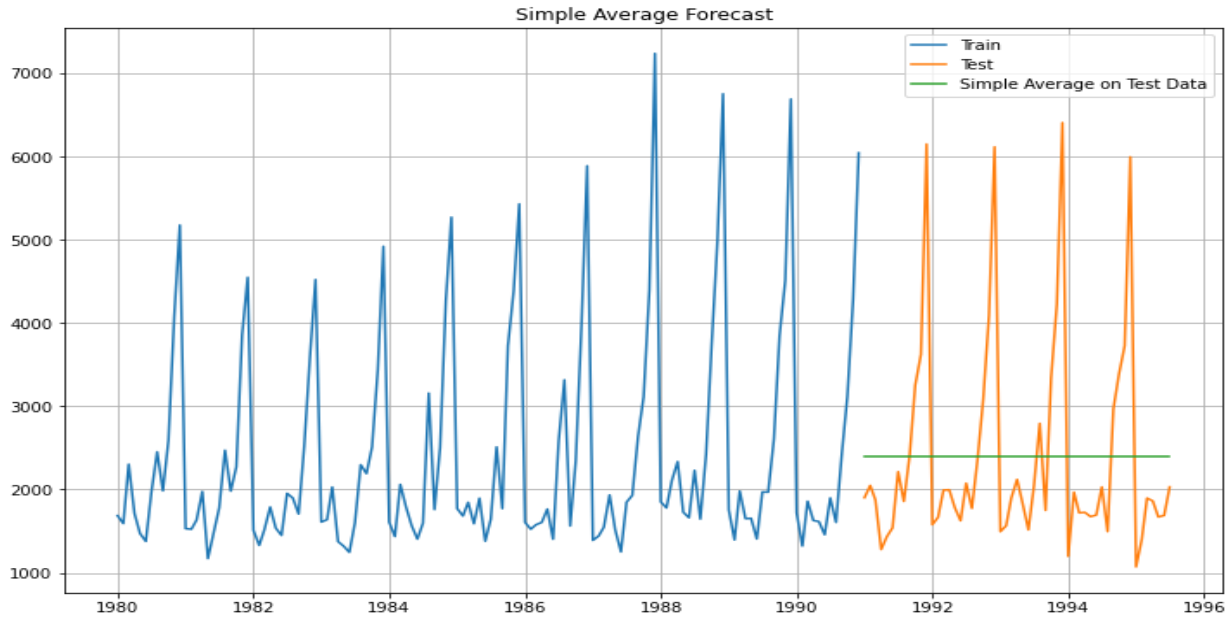


Figure 50 Prediction Plot for Simple Average

Model Evaluation:	Test RMSE
RegressionOnTime	1389.135175
NaiveModel	3864.279352
SimpleAverageModel	1275.081804

Model4- Moving Average –

For the moving average model, we are going to calculate rolling means (or moving averages) for different intervals. The best interval can be determined by the maximum accuracy (or the minimum error) over here. Let's take moving average of previous 2, 3, 4 and 6 data elements:

YearMonth	Sparkling	Trailing_2	Trailing_3	Trailing_4	Trailing_6
1980-01-01	1686	NaN	NaN	NaN	NaN
1980-02-01	1591	1638.5	NaN	NaN	NaN
1980-03-01	2304	1947.5	1860.333333	NaN	NaN
1980-04-01	1712	2008.0	1869.000000	1823.25	NaN
1980-05-01	1471	1591.5	1829.000000	1769.50	NaN

Plot the predictions based on Moving averages:

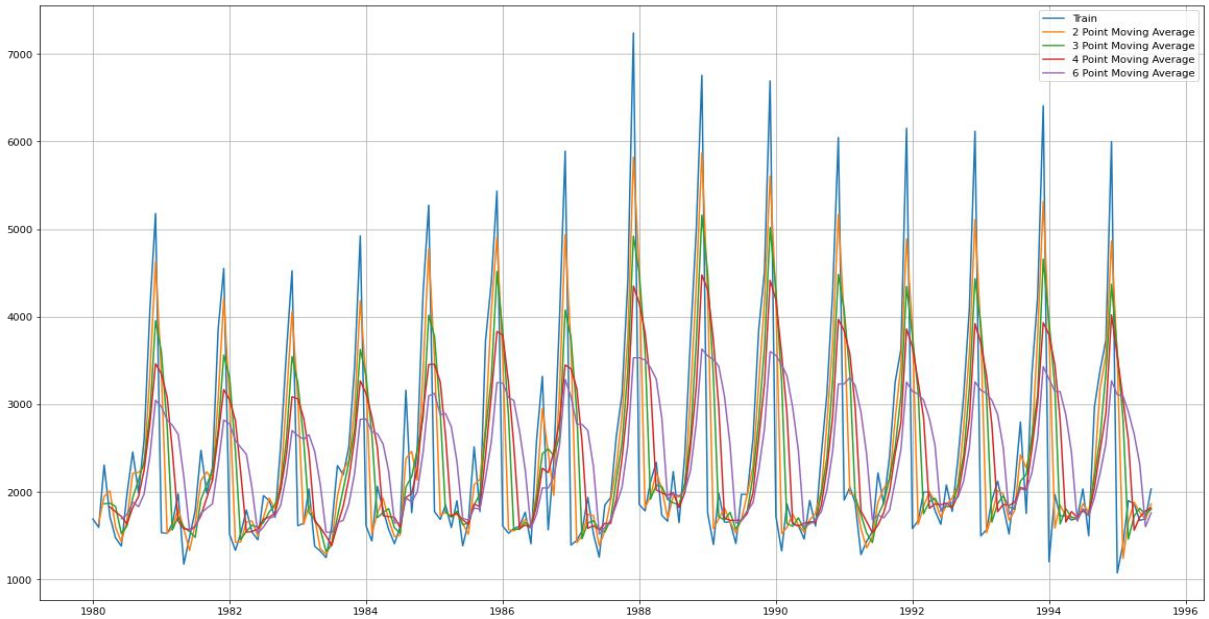


Figure 51 Moving Average for 2,3,4 and 6 MA

Let us split the data into train and test and plot this Time Series. The window of the moving average is need to be carefully selected as too big a window will result in not having any test set as the whole series might get averaged over.

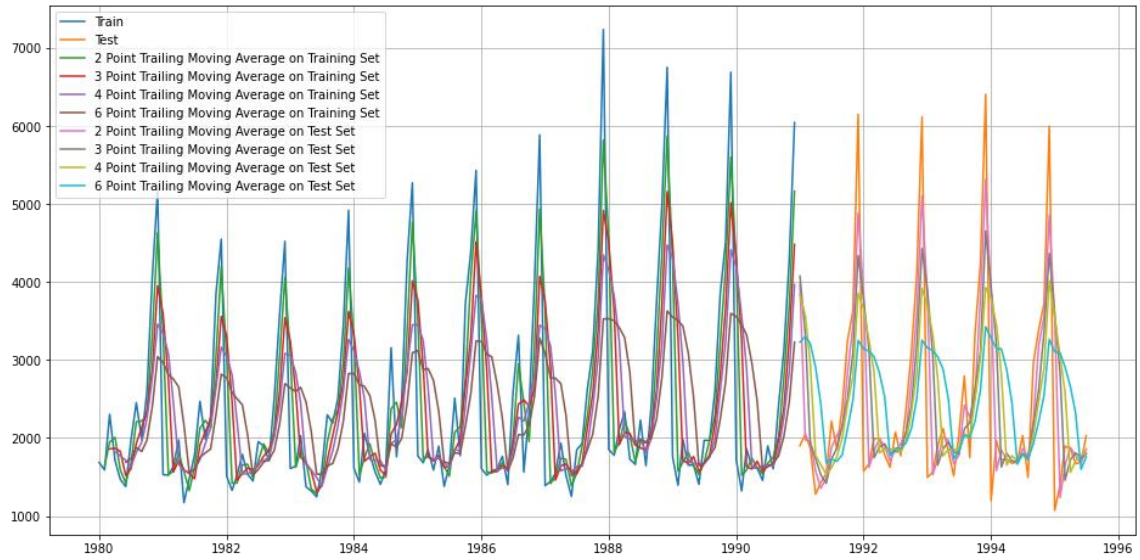


Figure 52 Plot MA on Full data

Model Evaluation:

```
For 2 point Moving Average Model forecast on the Training Data,  RMSE is 813.401
For 3 point Moving Average Model forecast on the Training Data,  RMSE is 1028.606
For 4 point Moving Average Model forecast on the Training Data,  RMSE is 1156.590
For 6 point Moving Average Model forecast on the Training Data,  RMSE is 1283.927
```

	Test RMSE
RegressionOnTime	1389.135175
NaiveModel	3864.279352
SimpleAverageModel	1275.081804
2pointTrailingMovingAverage	813.400684
3pointTrailingMovingAverage	1028.605756
4pointTrailingMovingAverage	1156.589694
6pointTrailingMovingAverage	1283.927428

The Best Model for Moving average is 2 points trailing moving average with lowest RMSE of 813.401, We would choose this model for predicting the Test model, if we are to choose the best one from above all models:

Before we go on to build the various Exponential Smoothing models, let us plot all the models and compare the Time Series plots

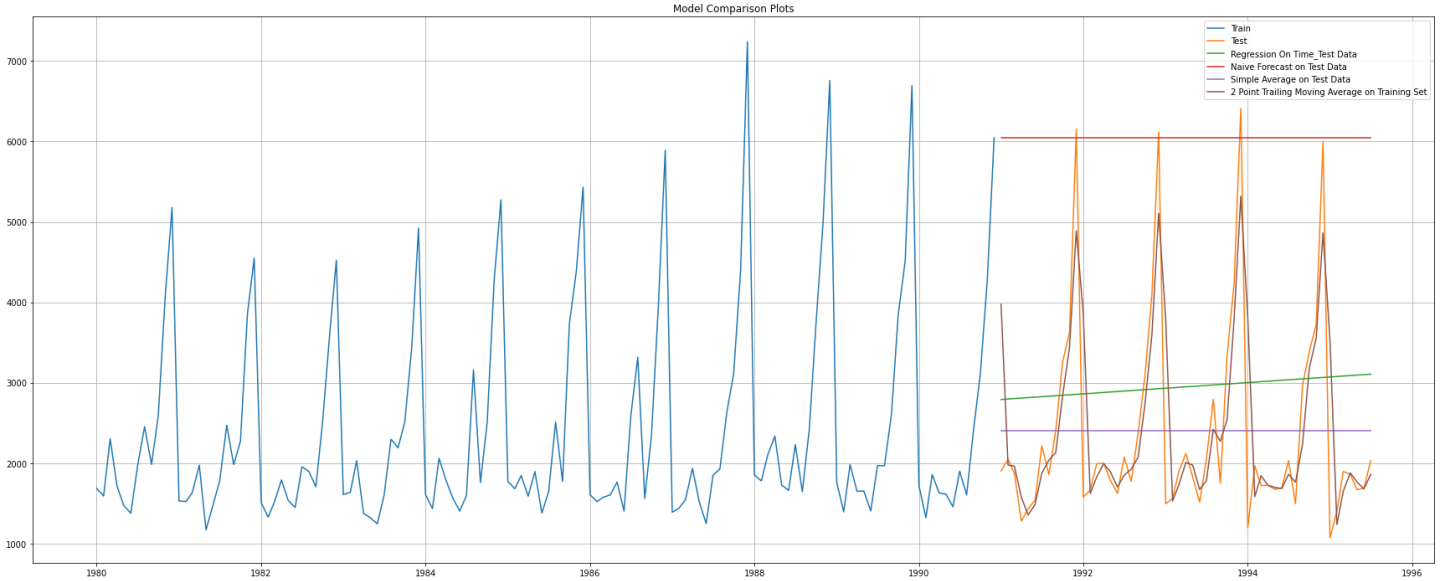


Figure 53 Building all Models on Test data

Model -5- Exponential Smoothing

Exponential smoothing methods consist of flattening time series data. Exponential smoothing averages or exponentially weighted moving averages consist of forecast based on previous periods data with exponentially declining influence on the older observations.

Exponential smoothing methods consist of special case exponential moving with notation ETS (Error, Trend, Seasonality) where each can be none(N), additive (N), additive damped (Ad), Multiplicative (M) or multiplicative damped (Md). One or more parameters control how fast the weights decay. These parameters have values between 0 and 1

First Model in Exponential smoothing is SES (Simple Exponential Smoothing) .

SES - ETS(A, N, N) - Simple Exponential Smoothing with additive errors

The simplest of the exponentially smoothing methods is naturally called simple exponential smoothing (SES). This method is suitable for forecasting data with no clear trend or seasonal pattern.

In Single ES, the forecast at time (t + 1) is given by Winters,1960

Ft+1=αYt+(1−α)Ft

Parameter α is called the smoothing constant and its value lies between 0 and 1. Since the model uses only one smoothing constant, it is called Single Exponential Smoothing.

Note: Here, there is both trend and seasonality in the data. So, we should have directly gone for the Triple Exponential Smoothing but Simple Exponential Smoothing and the Double Exponential Smoothing models are built over here to get an idea of how the three types of models compare in this case.

SimpleExpSmoothing class must be instantiated and passed the training data.

The fit() function is then called providing the fit configuration, the alpha value, smoothing_level. If this is omitted or set to None, the model will automatically optimize the value

Auto parameters:

```
{'smoothing_level': 0.04960659880745982,
 'smoothing_trend': nan,
 'smoothing_seasonal': nan,
 'damping_trend': nan,
 'initial_level': 1818.5047538435374,
 'initial_trend': nan,
 'initial_seasons': array([], dtype=float64),
 'use_boxcox': False,
 'lamda': None,
 'remove_bias': False}
```

Perform Prediction on test data set and these are the result look like:

YearMonth	Sparkling	predict
1991-01-01	1902	2724.929339
1991-02-01	2049	2724.929339
1991-03-01	1874	2724.929339
1991-04-01	1279	2724.929339
1991-05-01	1432	2724.929339

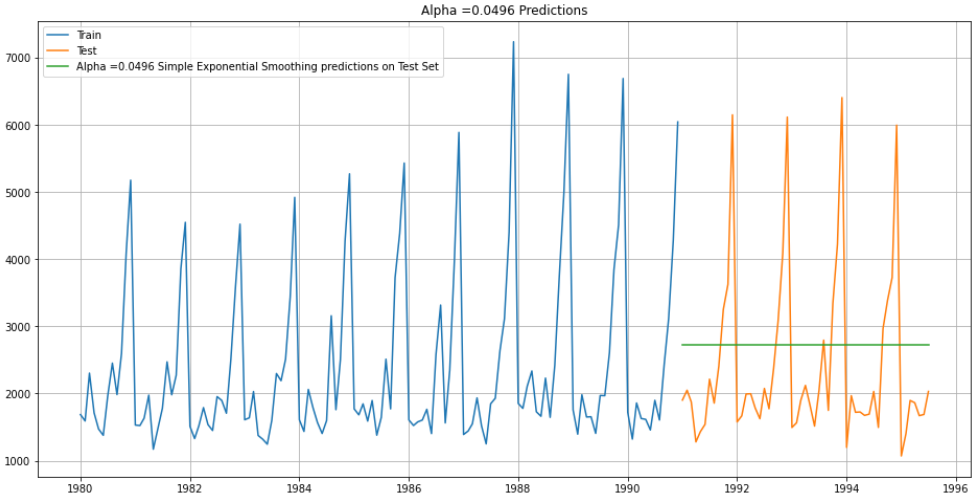


Figure 54 SES Model on Auto parameters

Model Evaluation for α = 0.09874989743650385 : Simple Exponential Smoothing For Alpha =0.09874989743650385 Simple Exponential Smoothing Model forecast on the Test Data, RMSE is 36.796

	Test RMSE
RegressionOnTime	1389.135175
NaiveModel	3864.279352
SimpleAverageModel	1275.081804
2pointTrailingMovingAverage	813.400684
3pointTrailingMovingAverage	1028.605756
4pointTrailingMovingAverage	1156.589694
6pointTrailingMovingAverage	1283.927428
Alpha=0.0496,SimpleExponentialSmoothing	1316.034674

Setting different alpha values. The higher the alpha value more weightage is given to the more recent observation. That means, what happened recently will happen again.

We will run a loop with different alpha values to understand which particular value works best for alpha on the test set. We have given a range of values between 0.01 to 1 with the interval of 0.01 so , Alpha values like 0.01, 0.02, 0.03 … up to 1.

We tried all 100 possible values and calculated the RMSE value for that, and chosen best Alpha value for Lowest RMSE value

	Alpha Values	Train RMSE	Test RMSE
1	0.02	1328.406554	1279.495201
0	0.01	1361.997529	1281.032699
2	0.03	1318.846031	1293.110073
3	0.04	1317.138929	1305.462953
4	0.05	1318.429335	1316.411742
...	...	...	...
94	0.95	1363.586057	3778.432623
95	0.96	1365.349793	3796.048620
96	0.97	1367.179935	3813.437370
97	0.98	1369.077807	3830.602869
98	0.99	1371.044831	3847.548965

99 rows × 3 columns

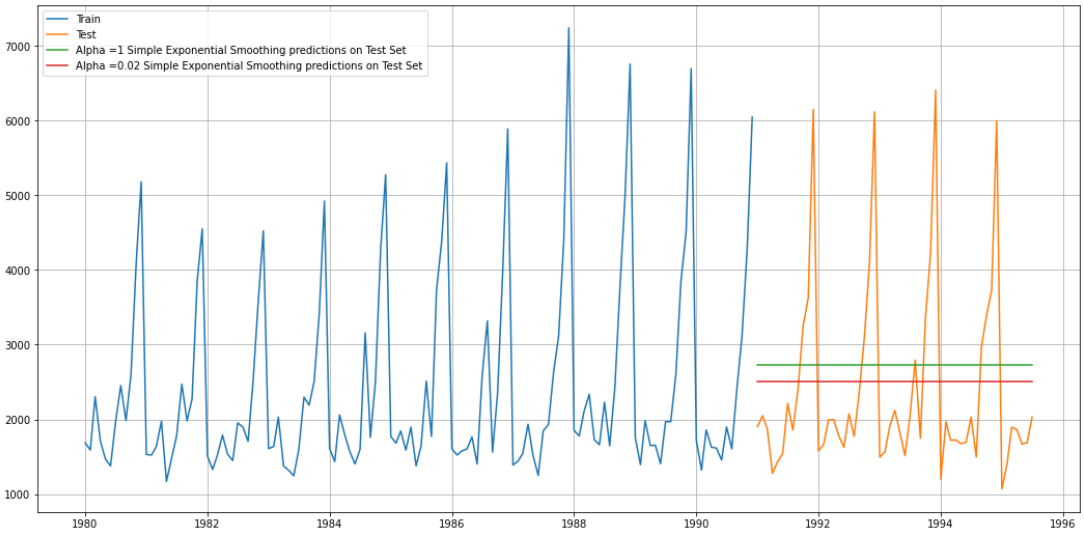


Figure 55 SES Model on Corrected parameters

Model evaluation:

	Test RMSE
RegressionOnTime	1389.135175
NaiveModel	3864.279352
SimpleAverageModel	1275.081804
2pointTrailingMovingAverage	813.400684
3pointTrailingMovingAverage	1028.605756
4pointTrailingMovingAverage	1156.589694
6pointTrailingMovingAverage	1283.927428
Alpha=0.0496,SimpleExponentialSmoothing	1316.034674
Alpha=0.02,SimpleExponentialSmoothing	1279.495201

Method 6: Double Exponential Smoothing (Holt's Model)

Holt - ETS(A, A, N) - Holt's linear method with additive errors , Double Exponential Smoothing.

One of the drawbacks of the simple exponential smoothing is that the model does not do well in the presence of the trend. This model is an extension of SES known as Double Exponential model which estimates two smoothing parameters. This is applicable when data has Trend but no seasonality. In this model two separate components are considered: **Level** and **Trend**. **Level** is the local mean.

One smoothing parameter α corresponds to the level series.

A second smoothing parameter β corresponds to the trend series.

Double Exponential Smoothing uses two equations to forecast future values of the time series, one for forecasting the short-term average value or level and the other for capturing the trend.

Intercept or Level equation, Lt is given by: $Lt=\alpha Yt+(1-\alpha)Ft$

Trend equation is given by $Tt=\beta(Lt-Lt-1)+(1-\beta)Tt-1$

Here, α and β are the smoothing constants for level and trend, respectively,

$0 < \alpha < 1$ and $0 < \beta < 1$.

The forecast at time t + 1 is given by

$Ft+1=Lt+Tt$

$Ft+n=Lt+nTt$

Two parameters α and β are estimated in this model. Level and Trend are accounted for in this model

Similar to Above SES model, we will calculate Alpha and beta values by using the for loops and calculate the RMSE value. Then we would choose Alpha and beta value for the Lowest RMSE.

	Alpha Values	Beta Values	Train RMSE	Test RMSE
0	0.1	0.1	1382.520870	1778.564670
1	0.1	0.2	1413.598835	2599.439986
10	0.2	0.1	1418.041591	3611.763322
2	0.1	0.3	1445.762015	4293.084674
20	0.3	0.1	1431.169601	5908.185554
...	...	...	...	...
98	1.0	0.9	1985.368445	57823.177011
79	0.8	1.0	1872.711054	57990.117908
89	0.9	1.0	1948.020916	59008.254331
99	1.0	1.0	2077.672157	59877.076519
19	0.2	1.0	2325.013004	60749.773505

100 rows × 4 columns

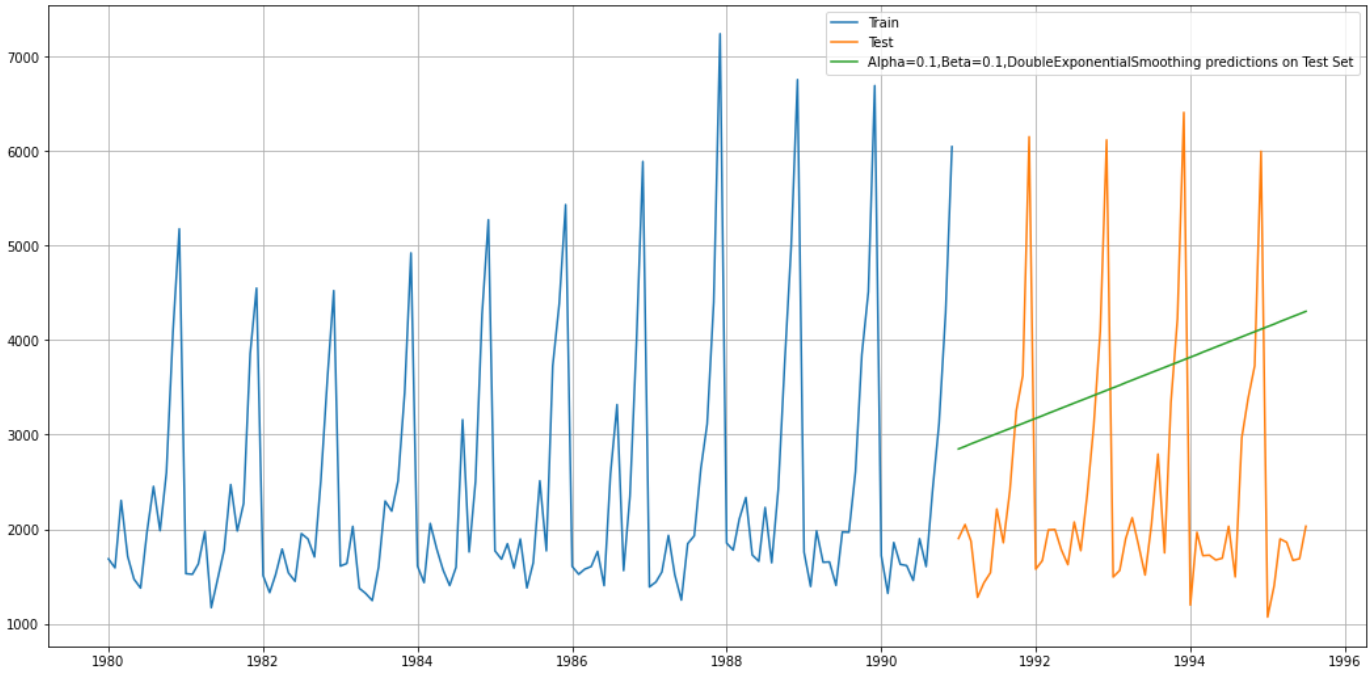


Figure 56 DES model

We see that the double exponential smoothing is picking up the trend component along with the level component as well.

	Test RMSE
RegressionOnTime	1389.135175
NaiveModel	3864.279352
SimpleAverageModel	1275.081804
2pointTrailingMovingAverage	813.400684
3pointTrailingMovingAverage	1028.605756
4pointTrailingMovingAverage	1156.589694
6pointTrailingMovingAverage	1283.927428
Alpha=0.0496,SimpleExponentialSmoothing	1316.034674
Alpha=0.02,SimpleExponentialSmoothing	1279.495201
Alpha=0.1,Beta=0.1,DoubleExponentialSmoothing	1778.564670

Inference

Here, we see that the Double Exponential Smoothing has actually done well when compared to the Simple Exponential Smoothing. This is because of the fact that the Double Exponential Smoothing model has picked up the trend component as well.

The Holt's model in Python has certain other options of exponential trends or whether the smoothing parameters should be damped. You can try these out later to check whether you get a better forecast.

Method 7: Triple Exponential Smoothing (Holt - Winter's Model)

Three parameters α , β and γ are estimated in this model. Level, Trend and Seasonality are accounted for in this model. First of all build the model with default parameters:

```
{'smoothing_level': 0.11119949831569428,
 'smoothing_trend': 0.049430920023313805,
 'smoothing_seasonal': 0.3620525701498937,
 'damping_trend': nan,
 'initial_level': 2356.5264391986907,
 'initial_trend': -9.443690175376352,
 'initial_seasons': array([0.71325627, 0.68332509, 0.90537798, 0.80561841, 0.65639659
,
        0.65451508, 0.88690241, 1.13423953, 0.91927727, 1.21396745,
        1.86941738, 2.3734461 ]),
 'use_boxcox': False,
 'lamda': None,
 'remove_bias': False}
```

And predicted values look like this, as compare to Actual values

YearMonth	Sparkling	auto_predict
1991-01-01	1902	1587.685845
1991-02-01	2049	1356.590237
1991-03-01	1874	1763.121866
1991-04-01	1279	1656.379813
1991-05-01	1432	1542.186697

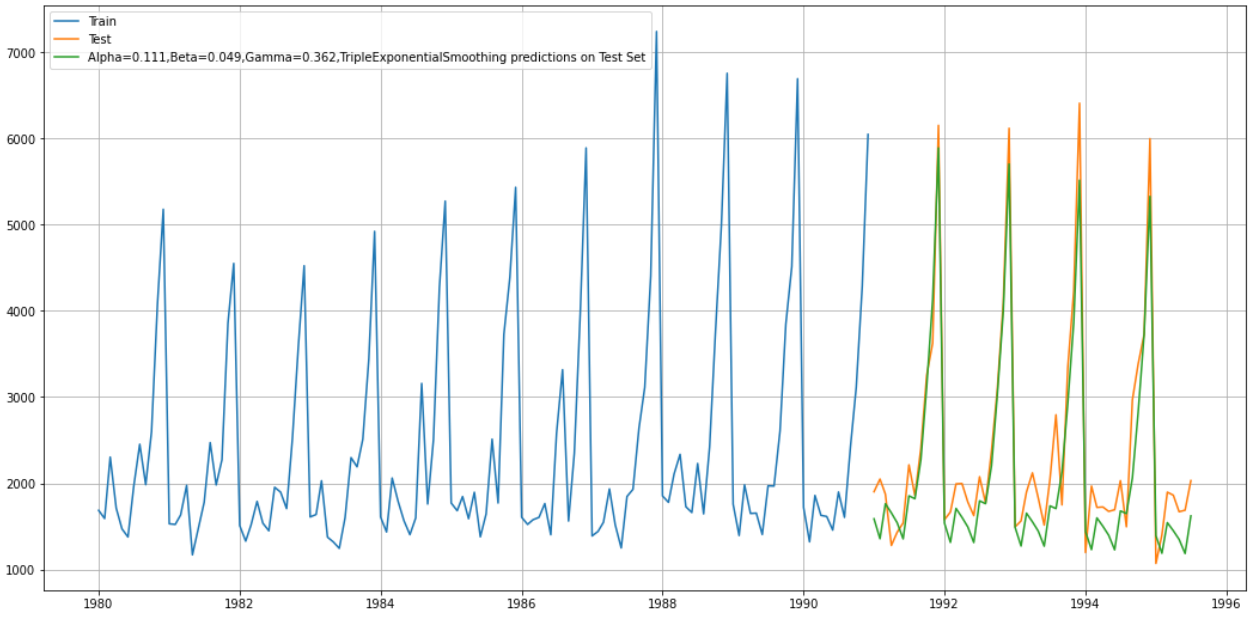


Figure 57 TES Model on Auto Parameters

We see that the Triple Exponential Smoothing is picking up the seasonal component as well.

Inference

Triple Exponential Smoothing has performed the best on the test as expected since the data had both trend and seasonality. But we see that our triple exponential smoothing is under forecasting. Let us try to tweak some of the parameters in order to get a better forecast on the test set.

	Test RMSE
RegressionOnTime	1389.135175
NaiveModel	3864.279352
SimpleAverageModel	1275.081804
2pointTrailingMovingAverage	813.400684
3pointTrailingMovingAverage	1028.605756
4pointTrailingMovingAverage	1156.589694
6pointTrailingMovingAverage	1283.927428
Alpha=0.0496,SimpleExponentialSmoothing	1316.034674
Alpha=0.02,SimpleExponentialSmoothing	1279.495201
Alpha=0.1,Beta=0.1,DoubleExponentialSmoothing	1778.564670
Alpha=0.111,Beta=0.049,Gamma=0.362,TripleExponentialSmoothing	403.706228

Auto generated Alpha , beta and game values and decide best based on lowest RMSE values

	Alpha Values	Beta Values	Gamma Values	Train RMSE	Test RMSE
244	0.4	0.1	0.2	384.47	317.43
172	0.3	0.2	0.2	388.54	329.04
162	0.3	0.1	0.1	388.22	337.08
90	0.2	0.2	0.1	398.48	340.19
326	0.5	0.1	0.3	396.60	345.91

Final values are chosen based as 0.4, 0.1 and 0.2 on lowest RMSE value of 317.43.

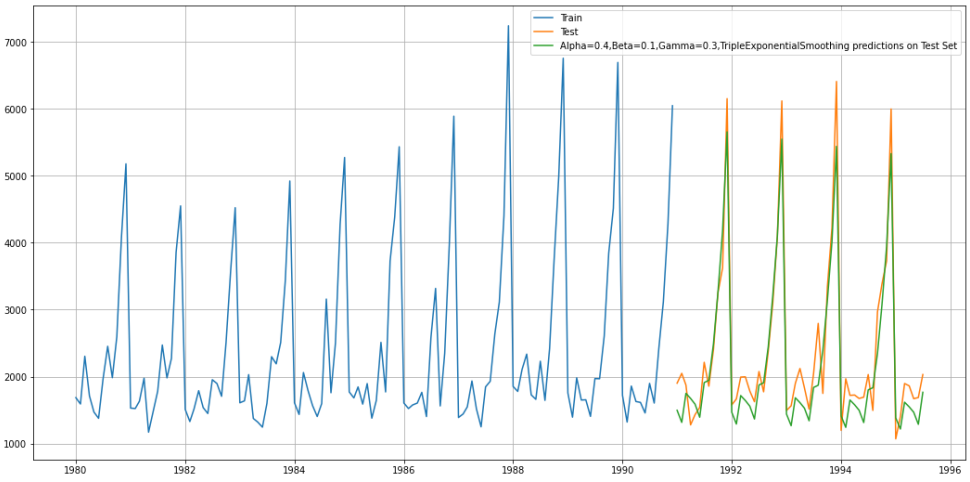


Figure 58 TES Model on Corrected parameters

Compare the Model generated so far:

	Test RMSE
RegressionOnTime	1389.135175
NaiveModel	3864.279352
SimpleAverageModel	1275.081804
2pointTrailingMovingAverage	813.400684
3pointTrailingMovingAverage	1028.605756
4pointTrailingMovingAverage	1156.589694
6pointTrailingMovingAverage	1283.927428
Alpha=0.0496,SimpleExponentialSmoothing	1316.034674
Alpha=0.02,SimpleExponentialSmoothing	1279.495201
Alpha=0.1,Beta=0.1,DoubleExponentialSmoothing	1778.564670
Alpha=0.111,Beta=0.049,Gamma=0.362,TripleExponentialSmoothing	403.706228
Alpha=0.4,Beta=0.1,Gamma=0.3,TripleExponentialSmoothing	317.430000

We see that the best model is the Triple Exponential Smoothing with multiplicative seasonality with the parameters α = 0.4 , β = 0.1 and γ = 0.3.

For this data, we had both trend and seasonality so by definition Triple Exponential Smoothing is supposed to work better than the Simple Exponential Smoothing as well as the Double Exponential Smoothing.

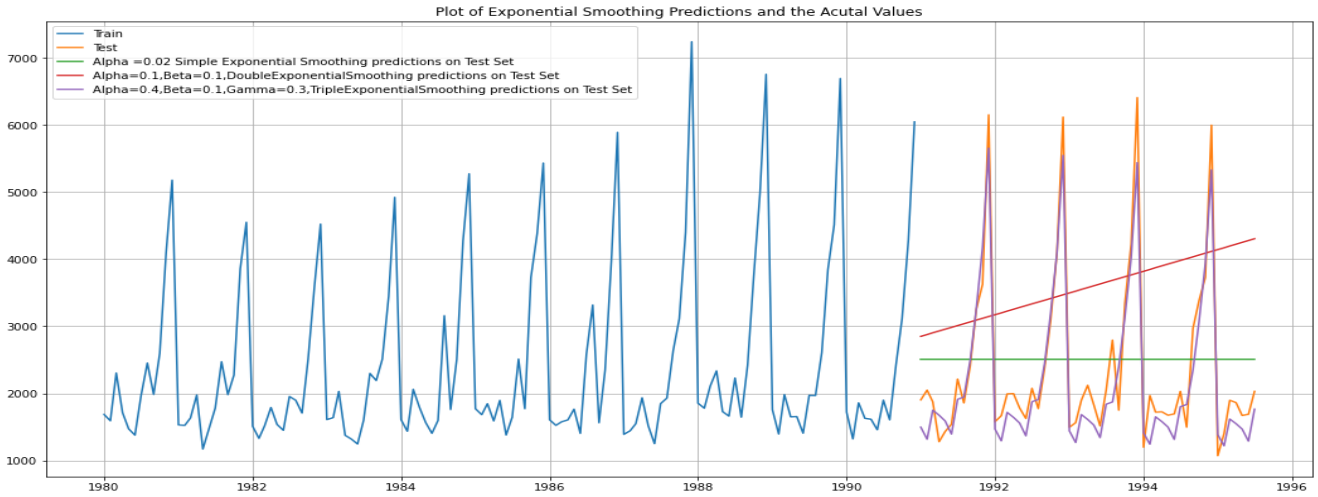


Figure 59 SES, DES and TES Model together comparison on Test data

In this particular we have built several models and went through a model building exercise. This particular exercise has given us an idea as to which particular model gives us the least error on our test set for this data. But in Time Series Forecasting, we need to be very vigil about the fact that after we have done this exercise we need to build the model on the whole data. Remember, the training data that we have used to build the model stops much before the data ends. In order to forecast using any of the models built, we need to build the models again (this time on the complete data) with

the same parameters. For this particular mentored learning session, we will go ahead and build only the top 1 model which gave us the best accuracy (least RMSE).

The two models to be built on the whole data are the following:

Alpha=0.4,Beta=0.1,Gamma=0.3,TripleExponentialSmoothing

Alpha=0.111,Beta=0.049,Gamma=0.362,TripleExponentialSmoothing

5. Check for the stationarity of the data on which the model is being built on using appropriate statistical tests and also mention the hypothesis for the statistical test. If the data is found to be non-stationary, take appropriate steps to make it stationary. Check the new data for stationarity and comment.

Note: Stationarity should be checked at alpha = 0.05

A time series has stationarity when the observations are not dependent on the time. Statistical properties of these time series will not change with time thus they will have constant mean, variance, and covariance.

The time series which have trends or with seasonality, are not stationary. Because trends will have a change in the movement of data concerning time which will cause the change in mean over time. Whereas seasonality occurs when the pattern in time series shows a variation for a regular time interval which will cause the variance to change over time.

Stationarity of time series can be detected by: Visually Plotting the time series and check for trend or seasonality. By Splitting time series into the different partitions and compare the statistical inference.

We can also perform Augmented Dickey-Fuller test to check the stationarity.

The Augmented Dickey-Fuller test is a unit root test which determines whether there is a unit root and subsequently whether the series is non-stationary.

The hypothesis in a simple form for the ADF test is:

***H*0H0 : The Time Series has a unit root and is thus non-stationary.**

***H*1H1 : The Time Series does not have a unit root and is thus stationary.**

We would want the series to be stationary for building ARIMA models and thus we would want the p-value of this test to be less than the $\alpha\alpha$ value. We have performed ADF test and following are the results for the same:

```
DF test statistic is -1.798
DF test p-value is 0.7055958459932058
Number of lags used 12
```

Since P value for above test is greater than 0.05 , which is 0.4671371627793189, so we are fail to reject the Null hypothesis and we accept that it is a Non stationary Time series. We see that at 5% significant level the Time Series is non-stationary.

There are various ways that Python allows us to select the appropriate number of lags at which we check whether the Time Series is stationary. To know more about the how to select the various ways, please refer to the link over *here*.

Let us take one level of differencing to see whether the series becomes stationary.

```
DF test statistic is -44.912
DF test p-value is 0.0
Number of lags used 10
```

Since P value is way less than 0.05 so going back by one level of differencing will make time series as Stationary. Now, let us go ahead and plot the stationary series.

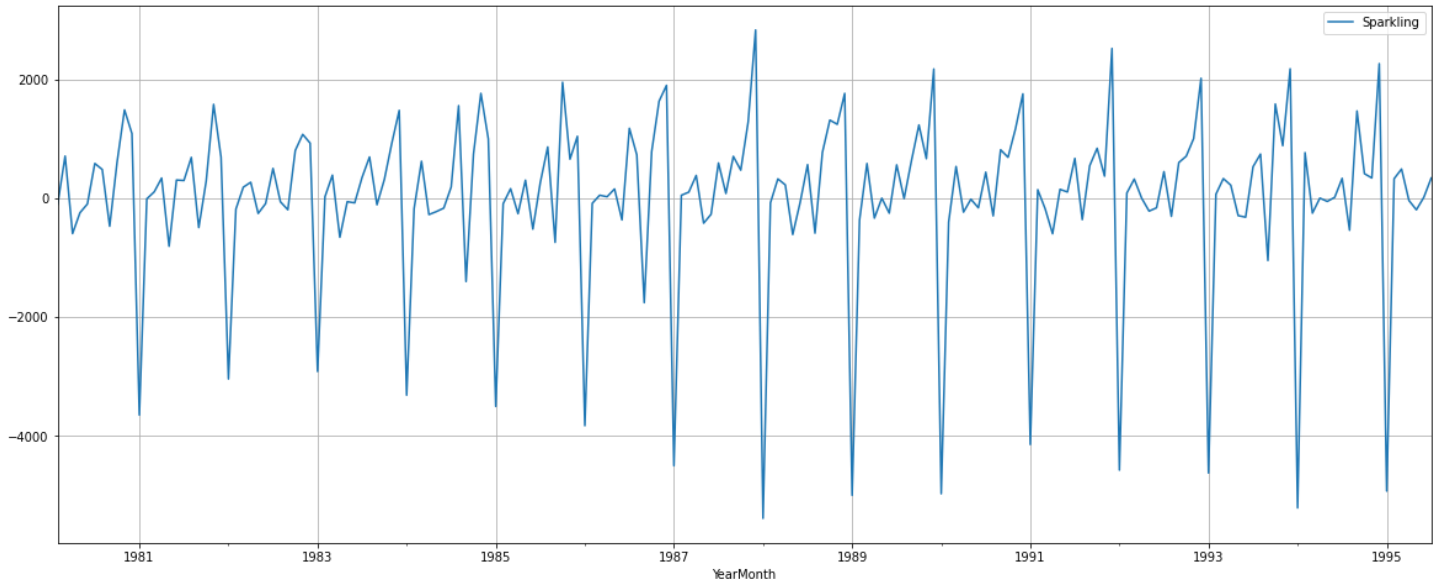


Figure 60 Stationary Time series

Also, if the series is non-stationary, stationaries the Time Series by taking a difference of the Time Series. Then we can use this particular differenced series to train the ARIMA models. We do not need to worry about stationarity for the Test Data because we are not building any models on the Test Data, we are evaluating our models over there. You can look at other kinds of transformations as part of making the time series stationary like taking logarithms

6. Build an automated version of the ARIMA/SARIMA model in which the parameters are selected using the lowest Akaike Information Criteria (AIC) on the training data and evaluate this model on the test data using RMSE

ARIMA Model:

An ARIMA and SARIMA models are class of statistical models for analyzing and forecasting time series data. Lets break it down :

- AR: Autoregression. A model that uses the dependent relationship between an observation and some number of lagged observations.
- I: Integrated. The use of differencing of raw observations in order to make the time series stationary.
- MA: Moving Average. A model that uses the dependency between an observation and a residual error from a moving average model applied to lagged observations.

The parameters of the ARIMA model are defined as follows:

- p: The number of lag observations included in the model, also called the lag order.
- d: The number of times that the raw observations are differenced, also called the degree of differencing.
- q: The size of the moving average window, also called the order of moving average.

The main assumption of AR model is that the time series data is stationary.

A stationary time series is one whose statistical properties such as mean, variance, autocorrelation, etc. are all constant over time.

When the time series data is not stationary, then we convert the non-stationary data before applying AR models. Method we used for making timeseries as Stationary is : Taking the difference between consecutive observations, we also call it a lag-1 difference. For time series with a seasonal component, the lag may be expected to be the period (width) of the seasonality.

White noise of the residuals:

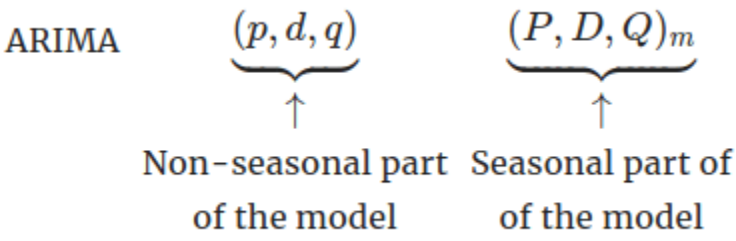
White noise is a process of residuals ϵt that are uncorrelated and follow normal distribution with mean 0 and constant standard deviation. In AR models, one of the main assumptions is the errors follow a white noise.

SARIMA Model:

The difference between ARIMA and SARIMA (SARIMAX) is about the seasonality of the dataset. if your data is seasonal, like it happens after a certain period of time. then we will use SARIMA.

SARIMA stands for Seasonal-ARIMA and it includes seasonality contribution to the forecast. The importance of seasonality is quite evident and ARIMA fails to encapsulate that information implicitly.

The Autoregressive (AR), Integrated (I), and Moving Average (MA) parts of the model remain as that of ARIMA. The addition of Seasonality adds robustness to the SARIMA model. It's represented as:



Source

where m is the number of observations per year. We use the uppercase notation for the seasonal parts of the model, and lowercase notation for the non-seasonal parts of the model.

Similar to ARIMA, the P,D,Q values for seasonal parts of the model can be deduced from the ACF and PACF plots of the data. Let's implement SARIMA for the same Catfish sales model.

Both ARIMA and SARIMA can be build using Automated way of generating values p,d and q value or manual way of building ACF / PACF graph and observe the numbers for p , d and q.

Auto ARIMA Model:

Since we have already seen that Stationarity can be achieved with Lag of 1 . So d value is 1. And We have taken chosen default value for p and q between 0 and 4, And then generated all possible combination for p,d and q. There are the parameters we have chosen for testing our ARIMA Model and gathered AIC value for all the models:

```
Examples of the parameter combinations for the Model
Model: (0, 1, 0)
Model: (0, 1, 1)
Model: (0, 1, 2)
Model: (0, 1, 3)
Model: (1, 1, 0)
Model: (1, 1, 1)
Model: (1, 1, 2)
Model: (1, 1, 3)
Model: (2, 1, 0)
Model: (2, 1, 1)
Model: (2, 1, 2)
Model: (2, 1, 3)
Model: (3, 1, 0)
Model: (3, 1, 1)
Model: (3, 1, 2)
Model: (3, 1, 3)
```

Following are the AIC values for above listed all parameters , when we fit Timeseries data into ARIMA model :

```
ARIMA(0, 1, 0) - AIC:2267.6630357855465
ARIMA(0, 1, 1) - AIC:2263.060015591336
ARIMA(0, 1, 2) - AIC:2234.4083231280847
ARIMA(0, 1, 3) - AIC:2233.994857754891
ARIMA(1, 1, 0) - AIC:2266.6085393190087
ARIMA(1, 1, 1) - AIC:2235.7550946815218
ARIMA(1, 1, 2) - AIC:2234.5272004521566
ARIMA(1, 1, 3) - AIC:2235.6078038022133
ARIMA(2, 1, 0) - AIC:2260.36574396809
ARIMA(2, 1, 1) - AIC:2233.77762634562
ARIMA(2, 1, 2) - AIC:2213.509212685558
ARIMA(2, 1, 3) - AIC:2232.8363546880437
ARIMA(3, 1, 0) - AIC:2257.72337899794
ARIMA(3, 1, 1) - AIC:2235.4986052031873
ARIMA(3, 1, 2) - AIC:2230.8080878060878
ARIMA(3, 1, 3) - AIC:2221.459263352943
```

Then we have sported the data, based on AIC value and least value of AIC have following records:

	param	AIC
10	(2, 1, 2)	2213.509213
15	(3, 1, 3)	2221.459263
14	(3, 1, 2)	2230.808088
11	(2, 1, 3)	2232.836355
9	(2, 1, 1)	2233.777626

Lets build the SARIMAX report for the Best parameter (2,1,3)

SARIMAX Results

Dep. Variable:	Sparkling	No. Observations:	132
Model:	ARIMA(2, 1, 2)	Log Likelihood	-1101.755
Date:	Thu, 25 Aug 2022	AIC	2213.509
Time:	22:13:59	BIC	2227.885
Sample:	01-01-1980	HQIC	2219.351
	- 12-01-1990		
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
ar.L1	1.3121	0.046	28.781	0.000	1.223	1.401
ar.L2	-0.5593	0.072	-7.740	0.000	-0.701	-0.418
ma.L1	-1.9917	0.109	-18.217	0.000	-2.206	-1.777
ma.L2	0.9999	0.110	9.109	0.000	0.785	1.215
sigma2	1.099e+06	1.99e-07	5.51e+12	0.000	1.1e+06	1.1e+06

Ljung-Box (L1) (Q):	0.19	Jarque-Bera (JB):	14.46
Prob(Q):	0.67	Prob(JB):	0.00
Heteroskedasticity (H):	2.43	Skew:	0.61
Prob(H) (two-sided):	0.00	Kurtosis:	4.08

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
[2] Covariance matrix is singular or near-singular, with condition number 6.98e+27. Standard errors may be unstable.

Lets Build the Diagnostic Plot:

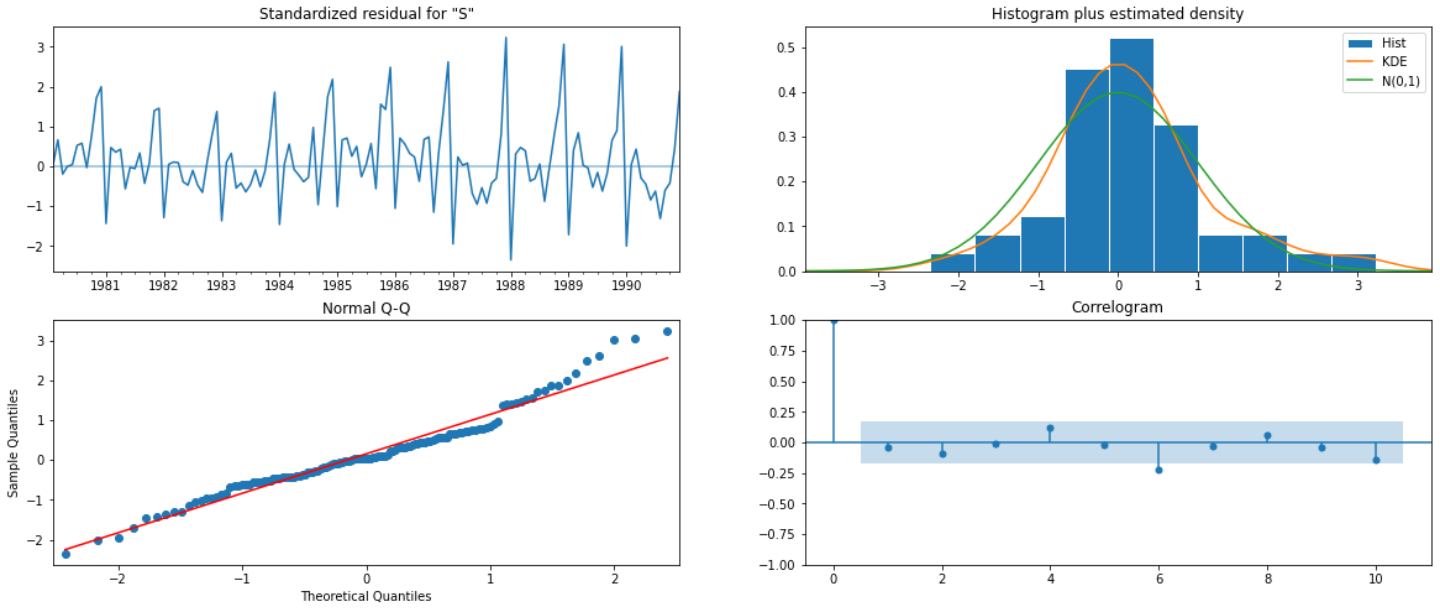


Figure 61 Diagnostic Plot for Auto ARIMA

Lets find the RMSE Value for Auto ARIMA with p, d, q value of 2,1,2:

```
RMSE: 1299.980240900083
MAPE: 47.099874793454504
```

Auto SARIMA:

Similar to Auto Arima Model, We have taken all possible combination of p,d,q and P,D,Q values for SARIMA Model, along with the Seasonality parameter. We have taken these combinations:

```
Examples of the parameter combinations for the Model are
Model: (0, 1, 1)(0, 0, 1, 6)
Model: (0, 1, 2)(0, 0, 2, 6)
Model: (0, 1, 3)(0, 0, 3, 6)
Model: (1, 1, 0)(1, 0, 0, 6)
Model: (1, 1, 1)(1, 0, 1, 6)
Model: (1, 1, 2)(1, 0, 2, 6)
Model: (1, 1, 3)(1, 0, 3, 6)
Model: (2, 1, 0)(2, 0, 0, 6)
Model: (2, 1, 1)(2, 0, 1, 6)
Model: (2, 1, 2)(2, 0, 2, 6)
Model: (2, 1, 3)(2, 0, 3, 6)
Model: (3, 1, 0)(3, 0, 0, 6)
Model: (3, 1, 1)(3, 0, 1, 6)
Model: (3, 1, 2)(3, 0, 2, 6)
Model: (3, 1, 3)(3, 0, 3, 6)
```

Then we fit our TS data into SARIMA model to calculate the AIC value and sorted AIC values. Following is Sorted AIC value achieved with p,d,q values:

	param	seasonal	AIC
187	(2, 1, 3)	(2, 0, 3, 6)	1632.621824
59	(0, 1, 3)	(2, 0, 3, 6)	1633.327862
251	(3, 1, 3)	(2, 0, 3, 6)	1634.474632
63	(0, 1, 3)	(3, 0, 3, 6)	1635.054387
123	(1, 1, 3)	(2, 0, 3, 6)	1635.424423

So the Best parameter for SARIMA Model will be (2,1,3) (2,0,3,6) . RMSE and MEP values are:

```
RMSE: 743.2229288149066
MAPE: 32.480506654342875
```

SARIMAX Results

Dep. Variable:	Sparkling	No. Observations:	132
Model:	SARIMAX(2, 1, 3)x(2, 0, 3, 6)	Log Likelihood	-805.311
Date:	Thu, 25 Aug 2022	AIC	1632.622
Time:	22:17:47	BIC	1662.227
Sample:	01-01-1980	HQIC	1644.628
	- 12-01-1990		
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
ar.L1	-1.1886	0.032	-37.062	0.000	-1.251	-1.126
ar.L2	-0.9414	0.045	-20.726	0.000	-1.030	-0.852
ma.L1	0.4320	0.488	0.885	0.376	-0.525	1.388
ma.L2	-0.1071	0.340	-0.315	0.753	-0.774	0.560
ma.L3	-0.8582	0.569	-1.509	0.131	-1.973	0.256
ar.S.L6	-0.0007	0.028	-0.026	0.979	-0.056	0.055
ar.S.L12	1.0464	0.020	52.773	0.000	1.008	1.085
ma.S.L6	-0.7316	0.667	-1.097	0.273	-2.039	0.576
ma.S.L12	-1.1091	0.328	-3.384	0.001	-1.751	-0.467
ma.S.L18	0.2487	0.495	0.502	0.616	-0.722	1.219
sigma2	7.147e+04	1.32e-05	5.42e+09	0.000	7.15e+04	7.15e+04

Ljung-Box (L1) (Q):	0.01	Jarque-Bera (JB):	21.89
Prob(Q):	0.91	Prob(JB):	0.00
Heteroskedasticity (H):	1.42	Skew:	0.34
Prob(H) (two-sided):	0.30	Kurtosis:	5.09

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
[2] Covariance matrix is singular or near-singular, with condition number 1.26e+26. Standard errors may be unstable.

Diagnostics Plot:

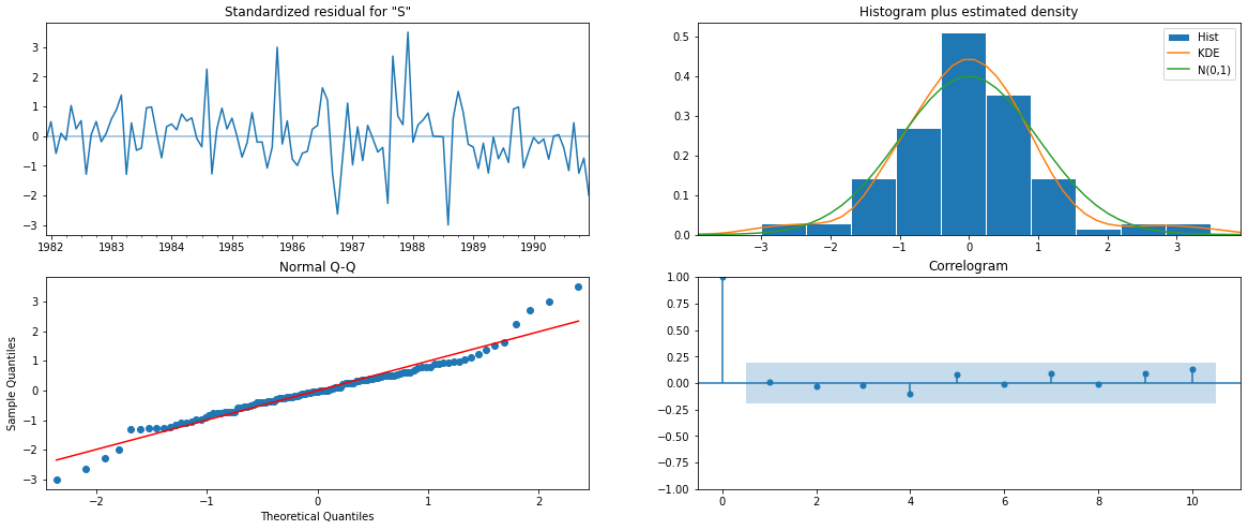


Figure 62 Auto SARIMA Model Diagnostic Plot

7. Build ARIMA/SARIMA models based on the cut-off points of ACF and PACF on the training data and evaluate this model on the test data using RMSE.

In the above question we build Automated version of ARIMA/SARIMA model, in that we have used all possible combinations of p,d and q values and generated Chosen AIC, RMSE and MEP parameters to identify, which combination of pdq is good for the timeseries data.

One problem with Auto version is, it needs lot of computations and we should have good memory and processing power in the system for iterating all possible values. So to avoid that scenario , we can use building ACF/PACF graph for generating Manual version of ARIMA/SARIMA Model.

Important Component of Automated version of ARIMA Model :

Auto-Correlation Function (ACF) or correlogram : A plot of auto-correlation of different lags is called ACF. The plot summarizes the correlation of an observation with lag values. The x-axis shows the lag and the y-axis shows the correlation coefficient between -1 and 1 for negative and positive correlation.

Partial Auto-Correlation Function (PACF) Autocorrelation Function (ACF) : A plot of partial auto-correlation for different values of lags is called PACF.The plot summarizes the correlations for an observation with lag values that is not accounted for by prior lagged observations.

Both plots are drawn as bar charts showing the 95% and 99% confidence intervals as horizontal lines. Bars that cross these confidence intervals are therefore more significant and worth noting.

Some useful patterns you may observe on these plots are:

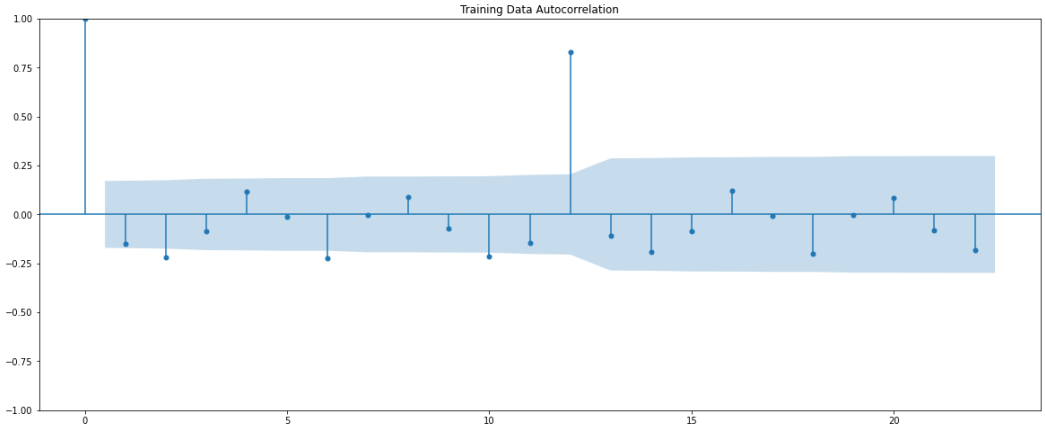
The number of lags is p when:

- The partial auto-correlation, | ppk | > 1.96 / n−−√ for first p values and cuts off to zero. The auto-correlation function, ρk decreases exponentially.
- The model is AR of order p when the PACF cuts-off after a lag p.
- The model is MA of order p when the ACF cuts-off after a lag q.
- The model is a mix of AR and MA if both the PACF and ACF trail off and cuts-off at p and q respectively.

For an ARIMA (p,d,q) process, it becomes non-stationary to stationary after differencing it for d times

Build Manual ARIMA Model

ACF graph for ARIMA Model:



PACF Graph for Arima Model

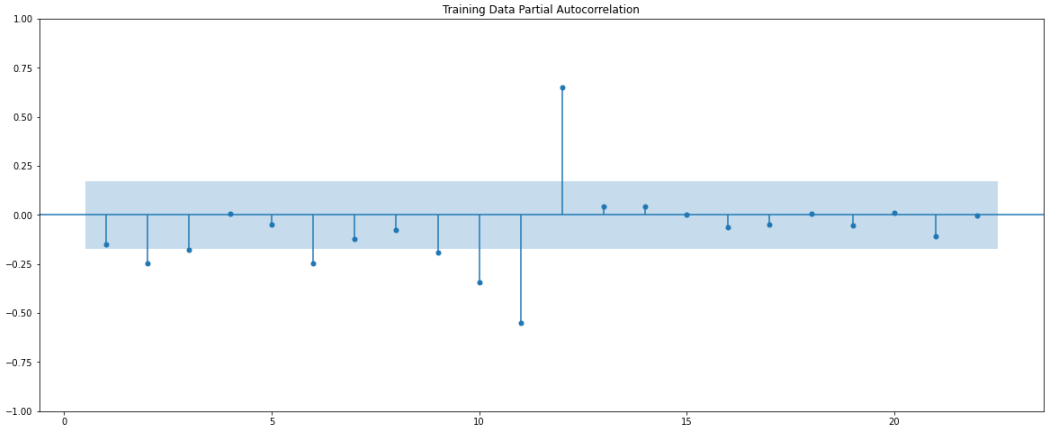


Figure 63 Manual ARIMA ACF/PACF Plot

Here, we have taken alpha=0.05.

- The Auto-Regressive parameter in an ARIMA model is 'p' which comes from the significant lag before which the PACF plot cuts-off to 0.
- The Moving-Average parameter in an ARIMA model is 'q' which comes from the significant lag before the ACF plot cuts-off to 0.

By looking at the above plots, we will take the value of p and q to be 0 and 0 respectively.

```
SARIMAX Results
==========================================================================================
Dep. Variable:                  Sparkling   No. Observations:                  132
Model:                 ARIMA(0, 1, 0)   Log Likelihood               -1132.832
Date:                Thu, 25 Aug 2022   AIC                           2267.663
Time:                        22:14:01   BIC                           2270.538
Sample:                    01-01-1980   HQIC                          2268.831
                         - 12-01-1990
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
sigma2      1.885e+06   1.29e+05     14.658      0.000    1.63e+06    2.14e+06
===================================================================================
Ljung-Box (L1) (Q):                   3.07   Jarque-Bera (JB):               198.83
Prob(Q):                              0.08   Prob(JB):                         0.00
Heteroskedasticity (H):               2.46   Skew:                            -1.92
Prob(H) (two-sided):                  0.00   Kurtosis:                         7.65
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
```

Diagnostic Plot:

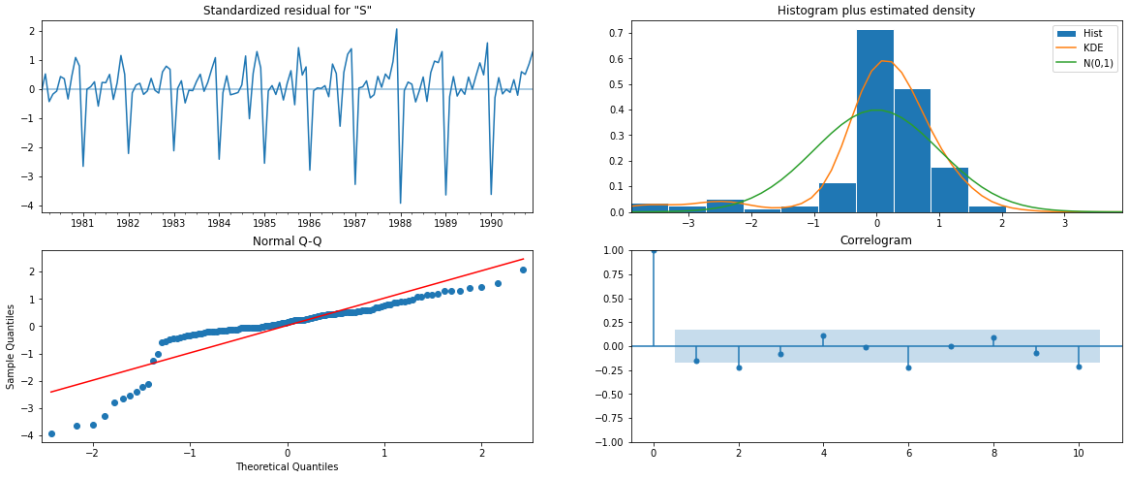


Figure 64 Manual ARIMA Diagnostic Plot

Predict on the Test Set using this model and evaluate the model

```
RMSE: 3864.2793518443914
MAPE: 201.32764950352743
```

Build Manual SARIMA Model:

We will star from where we left in ARIMA Model, from the above manual version of ARMIA model, we had chosen value for p,d,q as (0,1,0). So we will choose same for SARIMA Model and then add the parameter for Seasonality . Lets plot ACF and PACF plot one more time:

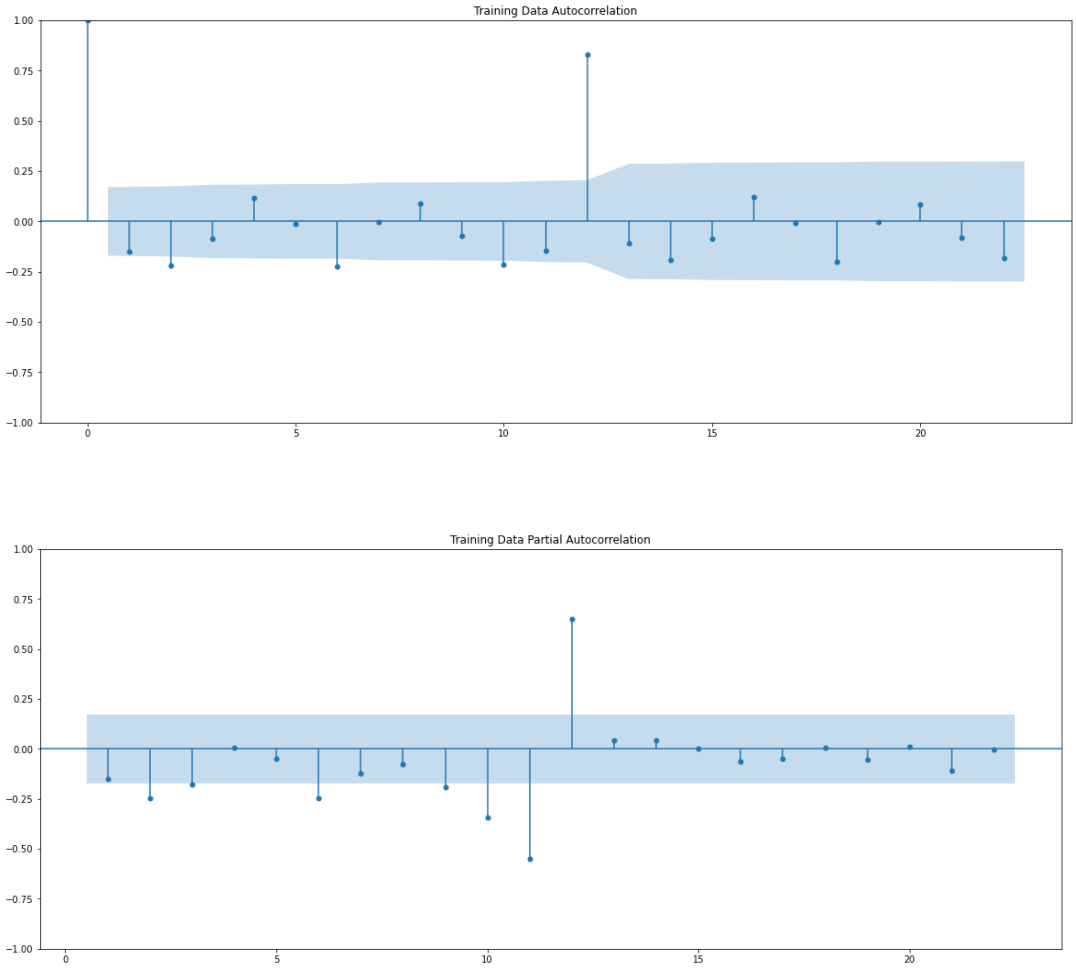


Figure 65 Manual SARIMA ACF/PACF Plot

Here, we have taken alpha=0.05. We are going to take the seasonal period as 3 or its multiple e.g. 6. We are taking the p value to be 3 and the q value also to be 3 as the parameters same as the ARIMA model.

The Auto-Regressive parameter in an SARIMA model is 'P' which comes from the significant lag after which the PACF plot cuts-off to 2.

The Moving-Average parameter in an SARIMA model is 'Q' which comes from the significant lag after which the ACF plot cuts-off to 2.

The Moving-Average parameter in an SARIMA model is 'Q' which comes from the significant lag after which the ACF plot cuts-off to 0.

So final values are as follows: (0,1,0) (2, 1, 2, 6). Lets build the Sarimax result:

```
                SARIMAX Results
================================================
Dep. Variable:                          Sparkling   No. Observations: 132
Model:             SARIMAX(0, 1, 0)x(2, 1, [1, 2], 6)   Log Likelihood   -854.548
Date:                                Thu, 25 Aug 2022   AIC               1719.096
Time:                                        22:14:02   BIC               1732.689
Sample:                                    01-01-1980   HQIC              1724.611
                                         - 12-01-1990
Covariance Type:                                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.S.L6       -1.1528      0.267     -4.322      0.000      -1.676      -0.630
ar.S.L12      -0.1397      0.271     -0.515      0.607      -0.672       0.392
ma.S.L6        0.1665      0.290      0.573      0.566      -0.403       0.736
ma.S.L12      -0.4794      0.077     -6.211      0.000      -0.631      -0.328
sigma2      2.47e+05   2.44e+04     10.103      0.000    1.99e+05    2.95e+05
===================================================================================
Ljung-Box (L1) (Q):                   15.56   Jarque-Bera (JB):                41.05
Prob(Q):                              0.00   Prob(JB):                         0.00
Heteroskedasticity (H):               1.33   Skew:                             0.71
Prob(H) (two-sided):                  0.39   Kurtosis:                         5.61
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
```

Diagnostics Plot:

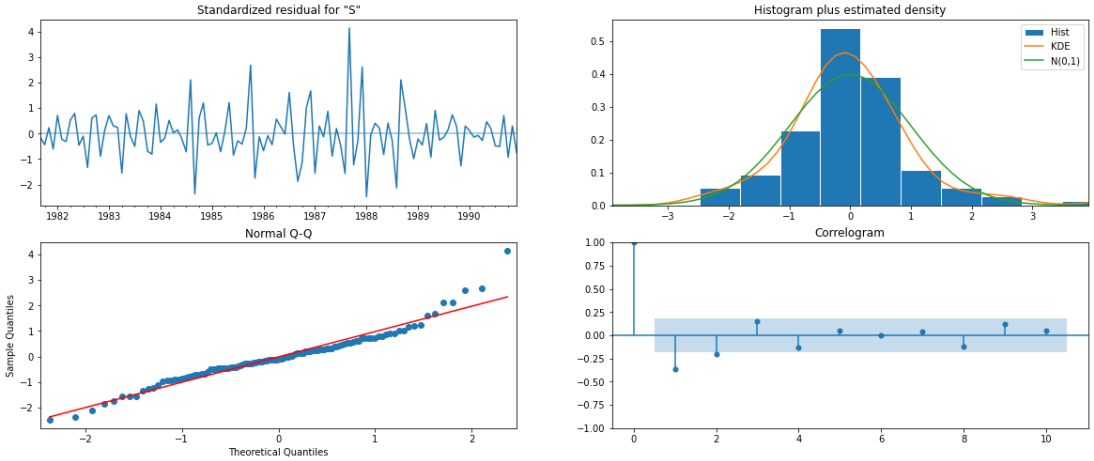


Figure 66 Manual SAIRMA Diagnostic Plot

Calculate the RMSE and MAPE value for test data for Manual SARIMA Model:

```
RMSE: 1750.2510595324502
MAPE: 82.17368719075145
```

8. Build a table with all the models built along with their corresponding parameters and the respective RMSE values on the test data

We have built several Models, with different parameters, and calculated the RMSE values for all the parameters . After combining all RMSE values for all the models, following is the table for that:

	Test RMSE
Alpha=0.4,Beta=0.1,Gamma=0.3,TripleExponentialSmoothing	317.430000
Alpha=0.111,Beta=0.049,Gamma=0.362,TripleExponentialSmoothing	403.706228
Auto_SARIMA(2,1,3)(2, 0, 3, 6)	743.222929
2pointTrailingMovingAverage	813.400684
3pointTrailingMovingAverage	1028.605756
4pointTrailingMovingAverage	1156.589694
SimpleAverageModel	1275.081804
Alpha=0.02,SimpleExponentialSmoothing	1279.495201
6pointTrailingMovingAverage	1283.927428
Auto_ARIMA(2,1,2)	1299.980241
Alpha=0.0496,SimpleExponentialSmoothing	1316.034674
RegressionOnTime	1389.135175
manual_SARIMA(0,1,0)(2,0,2,6)	1750.251060
Alpha=0.1,Beta=0.1,DoubleExponentialSmoothing	1778.564670
Manual_ARIMA(0,1,0)	3864.279352
NaiveModel	3864.279352

9. Based on the model-building exercise, build the most optimum model(s) on the complete data and predict 12 months into the future with appropriate confidence intervals/bands.

Based on the table above, we have see that best model so far given least value of RMSE is Triple exponential Model with parameters of Alpha=0.4,Beta=0.1,Gamma=0.3. It's RMSE values is only 317.4300 as compared to the worst RMSE value of 3864.279 of Naïve based Model .

Lets build the Triple exponential Smoothing model based on above parameters and fit full data set , we were using only test data for the predictions earlier. We have also calculated the RMSE value for full data set .

```
RMSE of the Full Model 388.5276378831874
```

Also as required, we have predicted nest 12 months of data for using above model:

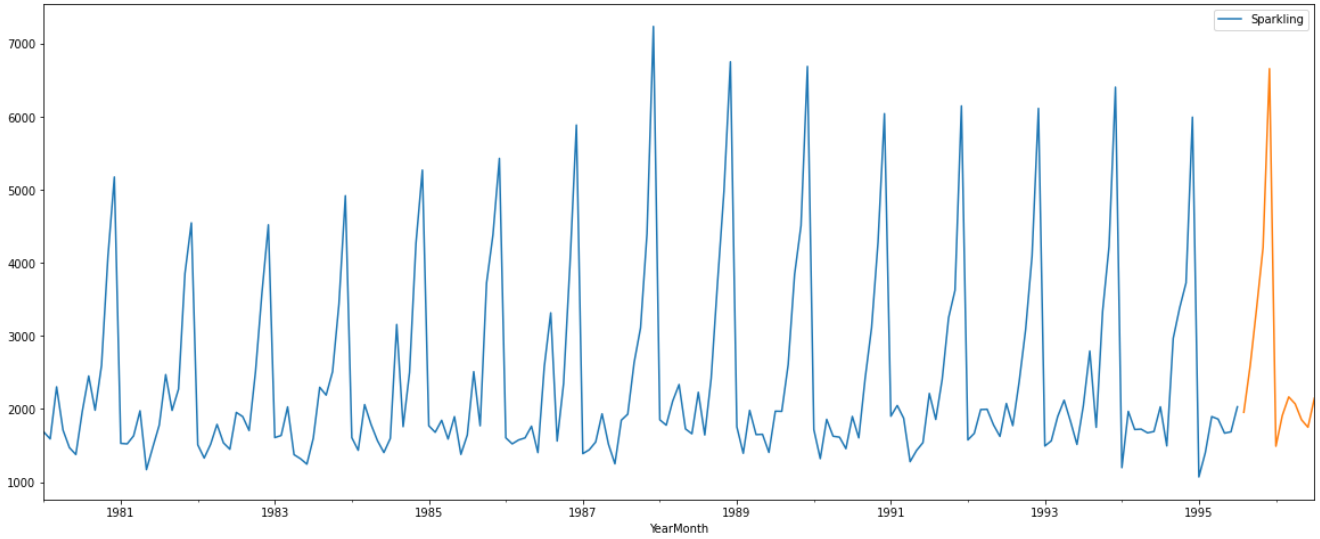


Figure 67 Best Model TES plot for next 12 Months predictions

Plotting the forecast with the confidence band.

Confidence band for forecasting: A confidence band is used in statistical analysis to represent the uncertainty in an estimate of a curve or function based on limited or noisy data. The 95% confidence bands enclose the area that you can be 95% sure contains the true curve. It gives you a visual sense of how well your data define the best-fit curve. It is closely related to the 95% prediction bands , which enclose the area that you expect to enclose 95% of future data points. This includes both the uncertainty in the true position of the curve (enclosed by the confidence bands), and also accounts for scatter of data around the curve. Therefore, prediction bands are always wider than confidence bands

For building the Confidence band we need to find lower and upper values of actual predictions with 95% confidentiality . In our case

Following are the Table for Predictions of upper and lower values along with Predicted values:

	lower_CI	prediction	upper_ci
1995-08-01	1192.471126	1955.959170	2719.447214
1995-09-01	1833.607745	2597.095788	3360.583832
1995-10-01	2622.596318	3386.084362	4149.572406
1995-11-01	3450.819866	4214.307910	4977.795954
1995-12-01	5899.849951	6663.337995	7426.826039

Plot the time series with next 12 months of unseen data :

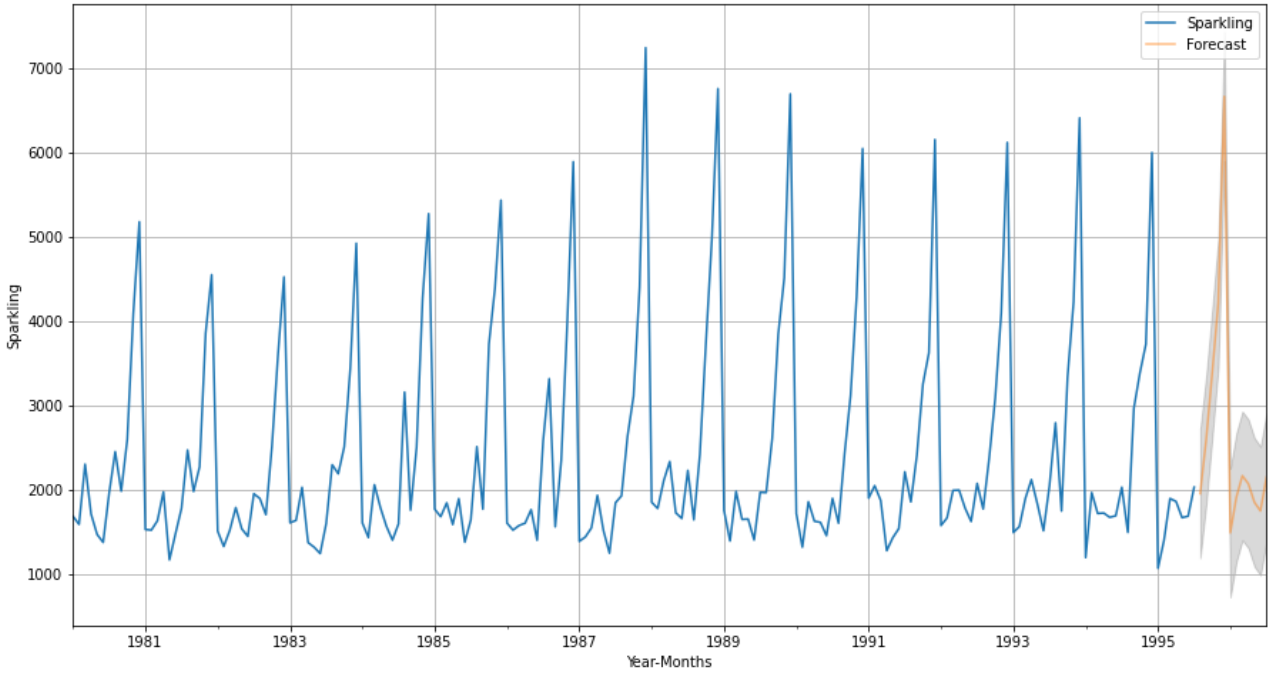


Figure 68 TES Model next 12 Month Predictions with Confidence Band

10. Comment on the model thus built and report your findings and suggest the measures that the company should be taking for future sales.

Analysis about the data:

1. This is a Sales data for Sparkling wine product of Wine manufacturing company , with the mean sales of 2302.417 in per Month and Minimum Sales of 1070 . whereas max Sales of 7242 in any Month.
2. Data consist of 187 data points
3. It seems to be contained seasonality
4. We also notice that there is small portion of the trend in data but altogether its not visible very clearly by plotting it in years.
5. Minimum sales for the data in Any month are 1070 and Max sales of Rose wines in any month is 7242
6. Year 1988 has highest sales
7. We also notice that 1995 has lowest sales , but this is not correct because we don't have full data point for this year
8. We have also seen that all year sales have outlier Sales in a few months of each year.
9. We have done upsampling of the data for Quarterly, yearly and a Decade sales, There are seasonality in the data, and it gets flatternout when we Upsample the data
10. We have also done Downsampling of the data for analyzing Daily Sales.
11. We see the decrease in Sales for Sparkling wine in initial first 3-4 years , then again Sales increased , which remain almost constant and no big increase in sales .
12. Boxplot helps to check the outliers in each year and month and we see there are outliers in almost all the year as per the box plot.
13. Average sales are lowest in the year of 1995
14. Monthly plot contains outliers in the month of February and July month Sales
15. There are Highest sales in the Month of December followed by 2nd highest Sales in November Month, which indicates year end party and vacation celebration sales
16. As yearend comes, sales start growing and it is about similar sales , which is very less for the first 6 months of any year
17. We have accumulated all year's data for each month and plotted it against each month.
18. We see that there are High Sales in each month at the Mid of the month and then Fall down till end of the month.
19. We also see decrease and lowest Sales in every Start of the month and then slightly cover up , goes highest in Mid of the month.-
20. January have low number of wine sales.
21. Highest sales observed in year 1988 , then goes down again for 89 and 90s.
22. After 1994, Sales goes down rapidly, but this is all because we don't have data for full year 1995.
23. The resampled yearly or annual series have smoothened out the seasonality and have only been able to capture the year on year trend where there was.

24. The quarterly series is able to catch the seasonality in the data.
25. If we take the resampling period to be 10 years or a decade, we see that the seasonality present has been smoothed over and it is only giving an estimate of the trend.
26. There are always Increase in decade data, which gets flattened out in future decades
27. We have built 2 Models of the data Additive trend as well as Multiplicative Trend .
28. From the 'additive' decomposition, there is seasonality in the data. Which is at the End of the year when Sales goes High, and at the Start of the year, sales goes down for following years.
29. Sales trend is almost constant for 2nd half of the data and don't see , a big difference in total Sales

Forecasting analysis:

We have built 17 Models on our data and Compared all of them based on RMSE value for each Model: Results with the parameters used for comparisons are as follows:

	Test RMSE
Alpha=0.4,Beta=0.1,Gamma=0.3,TripleExponentialSmoothing	317.430000
Alpha=0.111,Beta=0.049,Gamma=0.362,TripleExponentialSmoothing	403.706228
Auto_SARIMA(2,1,3)(2, 0, 3, 6)	743.222929
2pointTrailingMovingAverage	813.400684
3pointTrailingMovingAverage	1028.605756
4pointTrailingMovingAverage	1156.589694
SimpleAverageModel	1275.081804
Alpha=0.02,SimpleExponentialSmoothing	1279.495201
6pointTrailingMovingAverage	1283.927428
Auto_ARIMA(2,1,2)	1299.980241
Alpha=0.0496,SimpleExponentialSmoothing	1316.034674
RegressionOnTime	1389.135175
manual_SARIMA(0,1,0)(2,0,2,6)	1750.251060
Alpha=0.1,Beta=0.1,DoubleExponentialSmoothing	1778.564670
Manual_ARIMA(0,1,0)	3864.279352
NaiveModel	3864.279352

It is clear that Alpha=0.4,Beta=0.1,Gamma=0.3,TripleExponentialSmoothing has the lower RMSE and NaiveModel has the hieghest RMSE value of 3864.27935

Based on above listed table, we have found that Alpha=0.1,Beta=0.2,Gamma=0.1,TripleExponentialSmoothing has the lower RMSE of 9.22 and NaiveModel has the highest RMSE value of 79.718

To find the most optimum model, we run the model on the full data . We predict for the next 12 months for next years. RMSE of the Full Model 388.5276378831874

Recommendations:

From the forecast it is being predicted that the sales will remain steady and it will follow same pattern, as it has been doing for previous years, where Year end will be good and Start of new year will be bad for the sales.

Company should start promotions and give more deals, to increase sales for all the months, and not only for the end of the year.

Also as observed before sales will increase till December and then there is a likely chance of sharp drop in January followed by a very gradual increase in the middle of 1996. This is similar observations during Monthly and yearly plot with seasonality as well.

Also, wine producing company should take efforts or run marketing campaign for promoting wine productions at the start of the year and especially in the period of Jan-July 1996 Months.

Company should also analyze further various business methods like marketing, optimization of raw materials and for better profitability.