1 Euler Method

$$ih\frac{d\left|\psi\right\rangle}{dt} = H\left|\psi\right\rangle \tag{1}$$

$\Longrightarrow$

$$ih\frac{\left|\psi(t_{k+1})\right\rangle - \left|\psi(t_k)\right\rangle}{\triangle\ t} = H\left|\psi(t_k)\right\rangle \tag{2}$$

$\Longrightarrow$

$$\left|\psi(t_{k+1})\right\rangle - \left|\psi(t_k)\right\rangle = -iH\left|\psi(t_k)\right\rangle \triangle\ t \tag{3}$$

$\Longrightarrow$

$$\left|\psi(t_{k+1})\right\rangle = (\mathbb{I} - iH\ \triangle\ t)\left|\psi(t_k)\right\rangle \tag{4}$$

For code refer to Notebook

2 Crank Nicolson method

$$\left|\psi(t_{k+1})\right\rangle - \left|\psi(t_{k+1/2})\right\rangle = -iH\left|\psi(t_{k+1/2})\right\rangle \triangle\ t/2 \tag{5}$$

and

$$\left|\psi(t_{k+1/2})\right\rangle - \left|\psi(t_k)\right\rangle = -iH\left|\psi(t_{k+1/2})\right\rangle \triangle\ t/2 \tag{6}$$

From (5)

$$\left|\psi(t_{k+1})\right\rangle = (\mathbb{I} - iH\ \triangle\ t/2)\left|\psi(t_{k+1/2})\right\rangle \tag{7}$$

From (6)

$$\left|\psi(t_k)\right\rangle = (\mathbb{I} + iH\ \triangle\ t/2)\left|\psi(t_{k+1/2})\right\rangle \tag{8}$$

$\Longrightarrow$

$$[\mathbb{I} + \frac{iH\ \triangle\ t}{2}]\left|\psi(t_{k+1})\right\rangle = [\mathbb{I} - \frac{iH\ \triangle\ t}{2}]\left|\psi(t_k)\right\rangle \tag{9}$$

For code refer to Notebook