

## 1 Euler Method

$$i\hbar \frac{d|\psi\rangle}{dt} = H |\psi\rangle \quad (1)$$

$\Rightarrow$

$$i\hbar \frac{|\psi(t_{k+1})\rangle - |\psi(t_k)\rangle}{\Delta t} = H |\psi(t_k)\rangle \quad (2)$$

$\Rightarrow$

$$|\psi(t_{k+1})\rangle - |\psi(t_k)\rangle = -iH |\psi(t_k)\rangle \Delta t \quad (3)$$

$\Rightarrow$

$$|\psi(t_{k+1})\rangle = (\mathbb{I} - iH \Delta t) |\psi(t_k)\rangle \quad (4)$$

For code refer to Notebook

## 2 Crank Nicolson method

$$|\psi(t_{k+1})\rangle - |\psi(t_{k+1/2})\rangle = -iH |\psi(t_{k+1/2})\rangle \Delta t/2 \quad (5)$$

and

$$|\psi(t_{k+1/2})\rangle - |\psi(t_k)\rangle = -iH |\psi(t_{k+1/2})\rangle \Delta t/2 \quad (6)$$

From (5)

$$|\psi(t_{k+1})\rangle = (\mathbb{I} - iH \Delta t/2) |\psi(t_{k+1/2})\rangle \quad (7)$$

From (6)

$$|\psi(t_k)\rangle = (\mathbb{I} + iH \Delta t/2) |\psi(t_{k+1/2})\rangle \quad (8)$$

$\Rightarrow$

$$[\mathbb{I} + \frac{iH \Delta t}{2}] |\psi(t_{k+1})\rangle = [\mathbb{I} - \frac{iH \Delta t}{2}] |\psi(t_k)\rangle \quad (9)$$

For code refer to Notebook