

Condensed Matter Physics Meets Python via SageMath

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Introduction

What is SageMath and SageMathCloud ?!

Simple Mathematics in SageMath

Introduction to Group Theory in SageMath

What are Groups ?!

Group Theory in SageMath

Quantum Double Lattice Models

Quantum Double Lattice Model construction

Ribbon operators, Excitations, Condensations

Quantum Double Lattice Model in SageMath

Let's try it out !

Z_2 , S_3 , D_4 and so on

Conclusion

SageMath and SageMathCloud features

Quantum Double Lattice Model explored

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- ▶ Bundled with many open source softwares including SageMath (installed on the cloud instance).

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- ▶ May be Integration and Differentiation

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- ▶ Group Properties and Operations (conjugacy classes and centralizers using symmetries)
- ▶ Other Examples (Z_2 , $SU(2)$,)

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- ▶ Playing around with groups, some checks like *is_subgroup*, *is_cyclic*, *is_abelian*
- ▶ Conjugacy classes, Centralizers, Character Tables

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- ▶ Face Operators restrict the multiplication of the elements on the edges to the identity in the group

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- ▶ The excitations are given by the irreducible representations of the centralizers of the conjugacy class and that which condense by looking at the inner product of the characters ! 

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- ▶ Notebook on SageMathCloud

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- ▶ It has a huge variety of software included required for scientific computing.
- ▶ Some aspects of Quantum Double Model have been focused upon ! These excitations are used to perform Quantum Computation !
- ▶ Thank you :)