

# Condensed Matter Physics Meets Python via Sage

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## Introduction

What is SageMath and SageMathCloud ?!

Simple Mathematics in SageMath

## Introduction to Group Theory in SageMath

What are Groups ?!

Group Theory in SageMath

## Quantum Double Lattice Models

Quantum Double Lattice Model construction

Ribbon operators, Excitations, Condensations

## Quantum Double Lattice Model in SageMath

Let's try it out !

$Z_2$ ,  $S_3$ ,  $D_4$  and so on

## Conclusion

SageMath and SageMathCloud features

Quantum Double Lattice Model explored

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- ▶ Bundled with many open source softwares including SageMath (installed on the cloud instance).



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- ▶ May be Integration and Differentiation ....

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- ▶ Symmetries and Groups
- ▶ Group Properties and Operations (conjugacy classes and centralizers using symmetries)
- ▶ Other Examples ( $Z_2$ ,  $SU(2)$ , ....)

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- ▶ Conjugacy classes, Centralizers, Character Tables

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- ▶ Face Operators restrict the multiplication of the elements on the edges to the identity in the group

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- ▶ The excitations are given by the irreducible representations of the centralizers of the conjugacy class and that which condense by looking at the inner product of the characters ! 

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- ▶ Notebook on SageMathCloud

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- ▶ It has a huge variety of software included required for scientific computing.
- ▶ Some aspects of Quantum Double Model have been focused upon ! These excitations are used to perform Quantum Computation !
- ▶ Thank you :)