Analog and Digital Electronics (EC13103)
IT-B
Lecture-4
(Boolean Algebra)

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Principal of Duality

- In Boolean algebra, principal of duality is obtained by interchanging AND & OR operators as well as replacing 0’s by 1’s and 1’s by 0’s

Example:

$X+1=1$ then $X.0=0$

Basic Theorem of Boolean Algebra

Theorem-1: Properties of 0:

(a) $0+X = X$

(b) $0.X = 0$

Theorem-2: Properties of 1:

(a) $1+X = 1$

(b) $1.X = X$

Theorem-3: Commutative Law

(a) $X+Y=Y+X$

(b) $X.Y = Y.X$

Cont….

Theorem-4: Associate Law

(a) $(X+Y)+Z=X+(Y+Z)$

(b) $(X.Y).Z = X.(Y.Z)$

Theorem-5: Distributive Law

(a) $X.(Y+Z)=X.Y+X.Z$

(b) $X+YZ= (X+Y)(X+Z)$

(c) $X + \overline{X}Y = X + Y$

Theorem-6: Identity Law

(a) $X+X= X$

(b) $X.X= X$

Theorem-7: Absorption Law (Redundance Law)

(a) $A+AB= A$

(b) $A(A+B)=A$

Theorem-8: Complementary Law:

(a) $X + \overline{X} = 1$

(b) $X.\overline{X} = 0$

Cont….

Theorem-9: Involution

$$\bar{\bar{X}} = \mathrm{X}$$

Theorem-10: De Morgan's Theorem

$$\text{(a) } \overline{\mathrm{X} + \mathrm{Y}} = \overline{\mathrm{X}}.\overline{\mathrm{Y}}$$

$$\text{(b) } \overline{\mathrm{X}.\mathrm{Y}} = \overline{\mathrm{X}} + \overline{\mathrm{Y}}$$

Theorem-11: Consensus Theorem:

$$\text{(a) } \mathrm{A}.\mathrm{B} + \overline{\mathrm{A}}.\mathrm{C} + \mathrm{BC} = \mathrm{A}.\mathrm{B} + \overline{\mathrm{A}}.\mathrm{C}$$

$$\text{(b) } (\mathrm{A} + \mathrm{B}).(\overline{\mathrm{A}} + \mathrm{C}).(\mathrm{B} + \mathrm{C}) = (\mathrm{A} + \mathrm{B}).(\overline{\mathrm{A}} + \mathrm{C})$$

Theorem-12: Transposition Theorem:

$$\text{(a) } \mathrm{A}.\mathrm{B} + \overline{\mathrm{A}}.\mathrm{C} = (\mathrm{A} + \mathrm{C}).(\bar{A} + B)$$

Cont….

Ex.1: Reduce the Boolean Expression $AB + A\overline{B}C + B\overline{C}$

Solution: $AB + A\overline{B}C + B\overline{C}$

$=A(B + \overline{B}C) + B\overline{C}$

$=A(B + \overline{B})(B + C) + B\overline{C}$

$=AB+AC+B\overline{C}$

$=AB(C + \overline{C})+AC+B\overline{C}$

$=ABC + AB\overline{C} + AC + B\overline{C}$

$=AC(1+B)+ B\overline{C}\,(1+A)$

$=AC+ B\overline{C}$

Ex.2: Apply the Demorgan′s Theorem to the expression

$$\overline{(A + \overline{B})(C + \overline{D})}$$

Solution: After applying Demorgan's theorem,

$$\left(\overline{A + \overline{B}}\right) + \left(\overline{C + \overline{D}}\right)$$

$$= \bar{A}.\bar{\bar{B}} + \bar{C}.\bar{\bar{D}}$$

$$=\bar{A}.B \; + \bar{C}.D$$

For Practice:

- Reduce the Boolean Expression

$$A + B[AC + (B + \overline{C})D]$$

- Reduce the Boolean Expression

$$(B + BC)(B + \overline{B}C)(B + D)$$

- Reduce the Boolean Expression

$$A\overline{B}C + B + \overline{D}B + AB\overline{D} + \overline{A}C$$

Representation of Boolean Expression

- Boolean Expression can be represented by two ways:
  - Sum of Product form (SOP)
  - Product of Sum form (POS)

  Example: AB+AC - SOP form

  (A+B).(A+C) - POS form

Canonical Form of Boolean Expression (Standard Form)

- In standard SOP and POS, each term of Boolean expression must contain all the literals that has been used in the Boolean expression.
- If above condition is satisfied, then expression is said to be canonical form of Boolean expression.

Cont….

- In Boolean expression, AB+AC the literal C is missing in 1st term and B is missing from 2nd term. That's why; AB+AC is not the Canonical form

- In order to convert it in canonical SOP, following procedure has to be followed:

$$AB + AC = AB(C + \overline{C}) + AC(B + \overline{B})$$
$$= ABC + AB\overline{C} + ABC + A\overline{B}C$$
$$= ABC + AB\overline{C} + A\overline{B}C$$

- Convert (A+B).(A+C) into canonical POS

$$(A+B).(A+C) = (A + B + C\overline{C}).(A + C + B.\overline{B})$$
$$= (A + B + C).(A + B + \overline{C}).(A + B + C).(A + \overline{B} + C)$$
$$= (A + B + C).(A + B + \overline{C}).(A + \overline{B} + C)$$