Last class: worksheet

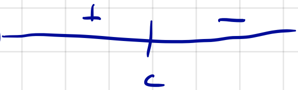
Increasing/Decreasing Test:

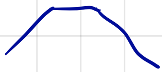
$f'(x) > 0$ on (a,b) $\Rightarrow$ f is increasing on (a,b)

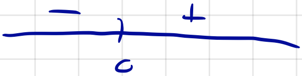
$f'(x) < 0$ on (a,b) $\Rightarrow$ f is decreasing on (a,b)

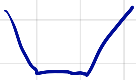
First Derivative Test

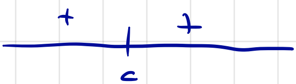
At a point where $f'(c) = 0$:

$f'(x)$ 

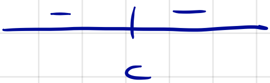
$\Rightarrow$  $\Rightarrow$ local max

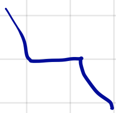
$f'(x)$ 

$\Rightarrow$  $\Rightarrow$ local min

$f'(x)$ 

$\Rightarrow$  $\Rightarrow$ neither

$f'(x)$ 

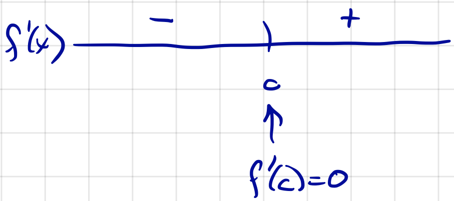
$\Rightarrow$  $\Rightarrow$ neither

2^{nd} Derivative test

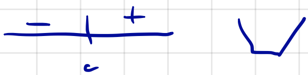
E.g.: $f'(c) = 0$

$f''(c) > 0$ (and $f''(x)$ continuous near c, so $f''(x) > 0$ near c)

Since $f''(x) > 0$ near c, $f'(x)$ is <u>increasing</u> near c.



$\leftarrow$ only possibility if f' is increasing and $f'(c) = 0$.

So: 

$\Rightarrow$ local min.

Full test: Suppose $f(x)$ has a continuous 2^{nd} derivative near c and $f'(c) = 0$.

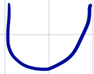
a) If $f''(c) > 0$, f achieves a local min at c.

b) If $f''(c) < 0$, f achieves a local max at c.

(c) if $f''(c) = 0$, the test is inconclusive. could be min/max/neither).

How to remember:

$f(x)=x^2$: $f'(0)=0$, $f''(0)=2$



$\hookrightarrow f''(0)>0 \to$ local min

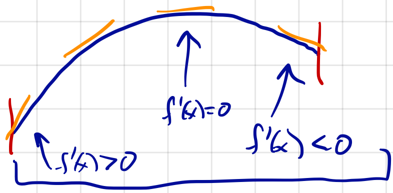
$f(x)=-x^2$ $f'(0)=0$ $f''(0)=-2$

$f''(0)<0 \to$ local max

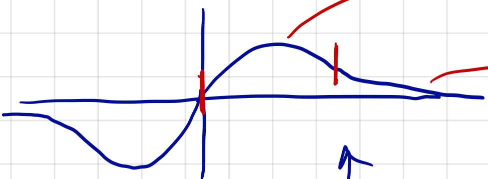
Concavity:

Regions where $f'(x)$ is increasing/decreasing are easy to spot geometrically and are useful mathematically.

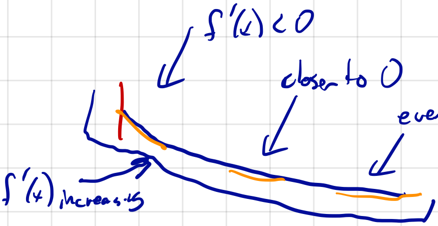
e.g. $f(x)=\dfrac{x}{1+x^2}$



$\hookrightarrow f'(x)$ decreasing



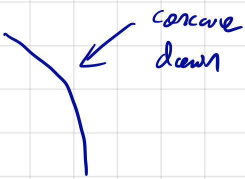
rough sketch.

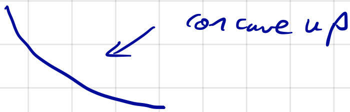


Def: We say a function is <u>concave up</u> on an interval if $f'(x)$ is <u>increasing</u> on the interval. [It is concave up, in particular, if $f''(x)$ exists and $f''(x)>0$ on the interval.

We say a function is concave down on an interval if $f'(x)$ is decreasing on the interval. [It is concave down if $f''(x)<0$].

Rule of Thumb: Up "U" $\Rightarrow$ concave up
Down "U" $\Rightarrow$ concave down



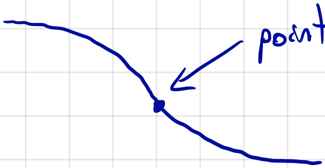


A spot where concavity changes from up to down or vice-versa, is called a **point of** inflection.

Look for $f''(a)=0$ but you need to see a sign change:

$f''(x)$ $+ \underset{a}{|} -$ e.g.   not: f'' $+ \underset{a}{|} +$.

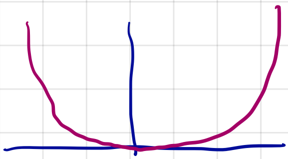
(no change in concavity.



point of inflection.

For $f(x)=x^4$
$f''(x)=12x^2$

$f''(x)$ $+ \; | \; +$



$f''(x)=0$ but not a point of inflection