1. The average BAC of eight male subjects was measured after consumption of 15 mL of ethanol. The resulting data were modeled by the concentration function

$$C(t) = 0.0225te^{-0.0467t}$$

where t is measured in minutes after consumption and C is measured in mg/mL.

(a) How rapidly was BAC increasing after 10 minutes?

$a = 0.0225$

$b = 0.0467$

$$C(t) = ate^{-bt}$$

$$C'(t) = ae^{-bt} - abte^{-bt}$$

$$= ae^{-bt}(1-bt)$$

$$C'(10) = 0.00752 \frac{mg}{mL} \cdot \frac{1}{minute}$$

(b) How rapidly was BAC decreasing half an hour later?

$$C'(30) = -0.0022 \frac{mg}{mL} \cdot \frac{1}{minute} = -0.133 \frac{mg}{mL} \cdot \frac{1}{hour}$$

2. The brightness of a star in units of m_V (apparent magnitude) is given by

$$B(t) = 4.0 + 0.35\sin\left(\frac{2\pi t}{5.4}\right)$$

where t is measured in days. Find the rate of change of brightness after one day and interpret your answer. Include units.

$$B'(t) = 0.35 \frac{2\pi}{5.4} \cos\left(\frac{2\pi t}{5.4}\right)$$

$$B'(1) = 0.35 \frac{2\pi}{5.4} \cos\left(\frac{2\pi}{5.4}\right)$$

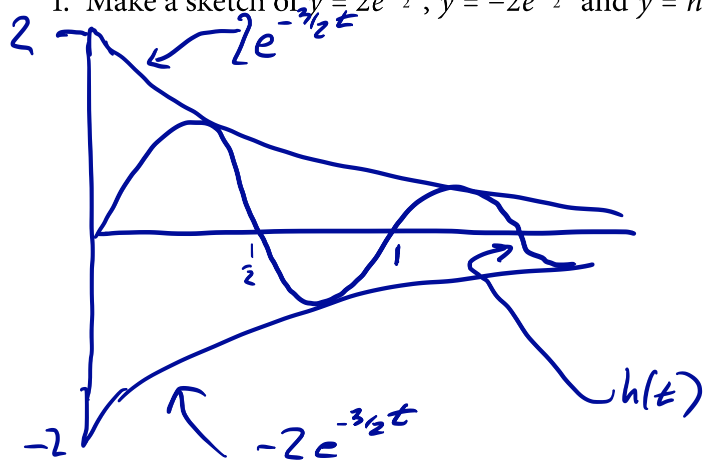
$$= 0.16 \; m_V / day$$

3. A mass on a spring is oscillating. Its height at time t is

$$h(t) = 2e^{-\frac{3}{2}t}\sin(2\pi t)$$

where t is measured in seconds and h is measured in centimeters.

1. Make a sketch of $y = 2e^{-\frac{3}{2}t}$, $y = -2e^{-\frac{3}{2}t}$ and $y = h(t)$.



2. Find the velocity of the mass at time t in general and at time $t = 1$ second in particular.

$$h'(t) = -3e^{-3/2 t}\sin(2\pi t) + 4\pi e^{-3/2 t}\cos(2\pi t)$$

$$h'(1) = 0 + 4\pi e^{-3/2}$$

$$= 2.80\ldots \text{ cm/s}$$

3. Compute $\lim_{t\to\infty} h(t)$ and interpret what this means.

Since $\lim_{t\to\infty} \pm 2e^{-3/2 t} = 0$

and since $-2e^{-3/2 t} \le h(t) \le 2e^{-3/2 t}$,

$\lim_{t\to\infty} h(t) = 0$. As $t \to \infty$, the oscillations decay and the mass approaches a constant height $h = 0$.

4. Find all the locations where the tangent to the curve $y = 2\cos(x) + \cos^2(x)$ is horizontal.

$y' = 2\sin x + 2\sin x \cos x$

$y' = 0$ if $\sin x(1+\cos x) = 0$ so

$\sin(x) = 0$ $(x = \pi k,\ k \in \mathbb{Z})$ or

$\cos(x) = -1$ $(x = \pi + 2\pi k\ \ k \in \mathbb{Z})$.

Combining these conditions, the tangent is horizontal at $x = \pi k$, k any integer

5. Compute $f'(t)$ if $f(t) = e^{at}\sin(bt)$, where a and b are constants.

$$f'(t) = \frac{d}{dt} e^{at}\sin(bt) = \left(\frac{d}{dt}e^{at}\right)\sin(bt) + e^{at}\frac{d}{dt}\sin bt$$

$$= a e^{at}\sin(bt) + b e^{at}\cos bt$$

$$= e^{at}\left[a\sin(bt) + b\cos bt\right]$$

6. Find y'' if $y = \cos(\sin(3x))$.

$$y' = -\sin(\sin(3x))\frac{d}{dx}\sin(3x)$$

$$= -\sin(\sin(3x))\,3\cos(3x)$$

$$y'' = -\cos(\sin(3x))\,(3\cos(3x))^2 + \sin(\sin(3x))\,9\sin(3x)$$