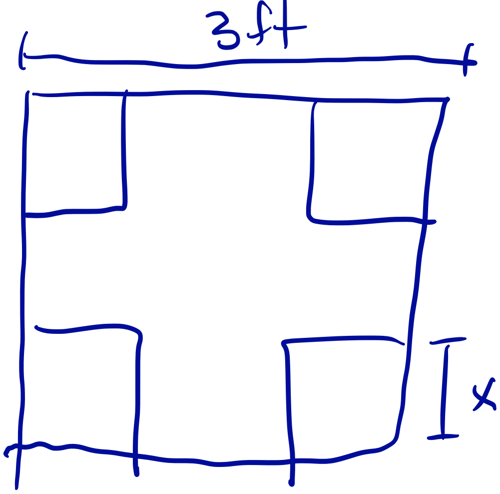
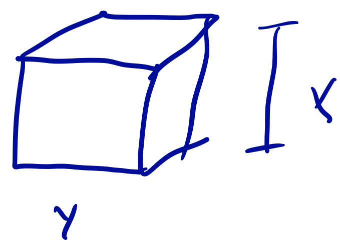
1. A box with an open top is to be constructed from a square piece of cardboard, 3 ft wide, by cutting a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.





$$V = y^2 x$$

$$y = 3 - 2x$$

$$V = (3-2x)^2 x \qquad 0 \le x \le \frac{3}{2}$$

$$\frac{dV}{dx} = -2(3-2x)x + (3-2x)^2$$

$$= (3-2x)\left[-2x + 3 - 2x\right]$$

$$= (3-2x)(3-4x)$$

$$V(0) = 0$$

$$V(3/2) = 0$$

$$\frac{dV}{dx} = 0 \Rightarrow x = \frac{3}{2},\ x = \frac{3}{4}$$

$$V(3/4) = \left(\frac{3}{2}\right)^2 \cdot \frac{3}{4} > 0$$

max value

2. The position of a mass on the x axis is given by $x(t) = t(e^t - 2)$ for $t \geq 0$. Find an equation involving a derivative to solve to determine the time when $x(t)$ is at a minimum. You will not be able to solve the equation by hand, so don't sweat it.

$$x'(t) = e^t - 2 + te^t$$

$$= (1+t)e^t - 2$$

Want: $(1+t)e^t - 2 = 0$

3. We can use Newton's method in the previous problem to find an approximate solution.

a. Explain why you expect the minimum to occur somewhere between $t = 0$ and $t = \ln(2) \approx 0.7$.

$x(0) = 0$, $x(\ln(2)) = 0$, $x \to \infty$ as $t \to \infty$

b. Apply one round of Newton's method to determine an approximate solution starting with $t = 1/2$.

$v(t) = x'(t) = (1+t)e^t - 2 = 0$. Want $v(t) = 0$

$$t_1 = t_0 - \frac{v(t_0)}{v'(t_0)}$$

$$v'(t) = (1+t)e^t + e^t = (2+t)e^t$$

$$t_1 = \frac{1}{2} - \frac{(1+\frac{1}{2})e^{1/2} - 2}{(2+\frac{1}{2})e^{1/2}} \approx 0.38522$$

4. Consider the function $G(x) = x^3 - x^2$.

(a) On what intervals is G increasing or decreasing?

$$G'(x) = 3x^2 - 2x$$
$$= x(3x-2)$$

G': $+$ | 0 | $-$ | $2/3$ | $+$

(b) Find the locations of any local maximum and minimum values of G.

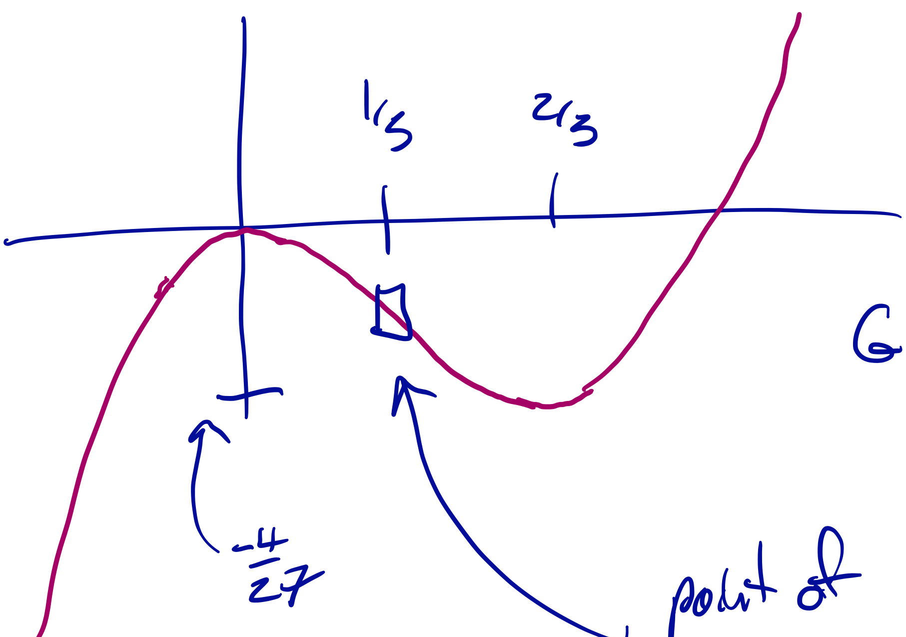
0: local max

$2/3$: local min

(c) Find the intervals of concavity and the inflection points.

$G''(x) = 6x - 2$ $\quad G''$: $-$ | $1/3$ | $+$

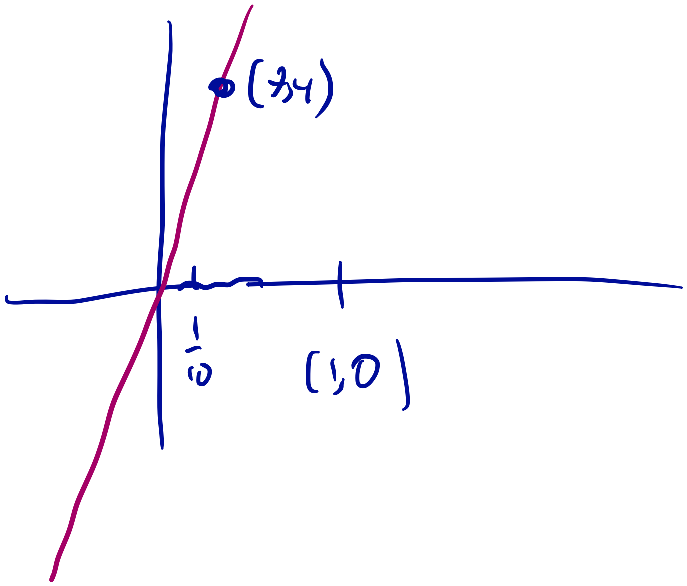
(d) Sketch the graph of the function including the data already determined.



point of inflection

$$G\left(\tfrac{2}{3}\right) = \left(\tfrac{2}{3}\right)^2 \cdot \left(\tfrac{2}{3} - 1\right)$$
$$= \tfrac{4}{9} \cdot \left(\tfrac{-1}{3}\right)$$
$$= \tfrac{-4}{27}$$

5. Find the point on the line $y = 3x$ that is closest to the point $(1, 0)$.



$$D = (1-x)^2 + (0-y)^2$$

dist squared

$$= (x-1)^2 + y^2$$

But $y = 3x$

$$D = (x-1)^2 + (3x)^2$$

$$D' = 2\left[(x-1) + 9x\right]$$

$$= 2\left[10x - 1\right]$$

min at $x = \frac{1}{10}$

$y = \frac{3}{10}$

6. Find the linearization of $f(x) = \sqrt{x}$ at $a = 4$ and use it to estimate $\sqrt{4.1}$.

$$L(x) = \sqrt{4} + f'(4)(x-4)$$

$$f'(x) = \frac{1}{2}\frac{1}{\sqrt{x}} \qquad f'(4) = \frac{1}{4}$$

$$L(x) = 2 + \frac{1}{4}(x-4)$$

$$\sqrt{4.1} \approx L(4.1) = 2 + \frac{1}{4}(4.1-4) = 2 + \frac{1}{40}$$

$$= 2.025$$