Vocabulary

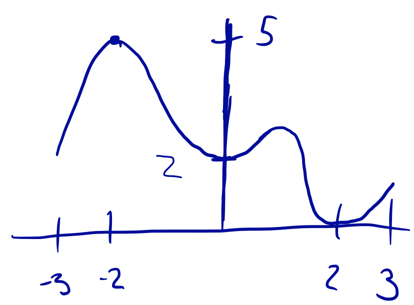
Suppose $f(x)$ is a real-valued function with domain D and suppose c is a point in D.

1. $f(c)$ is an **absolute maximum value** for f if $f(c) \geq f(x)$ for each x in D.
2. $f(c)$ is a **(absolute) minimum value** for f if $f(c) \leq f(x)$ for each x in D.
3. $f(c)$ is a **local maximum value** for f if $f(c) \geq f(x)$ for each x in D *near* c.
4. $f(c)$ is a **local minimum value** for f if $f(c) \leq f(x)$ for each x in D *near* c.
5. We say c is a **critical point** for f if either $f'(c) = 0$ or $f'(c)$ does not exist.

Key Tools

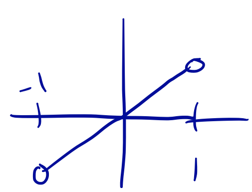
1. [Fermat's Theorem] If $f(c)$ is a (local or absolute) maximum/minimum value, and if f is defined on both sides of c, and if $f'(c)$ exists, then $f'(c) = 0$.
2. [Extreme Value Theorem] If the domain of f is a closed, bounded interval, and if f is continuous, then f is guaranteed to have both a maximum and a minimum value.

1. Sketch the graph of a function with domain $[-3, 3]$ that has an absolute maximum of 5 at $x = -2$, an absolute minimum of 0 at $x = 2$ and a local minimum of 2 at $x = 0$ that is not an absolute minimum.



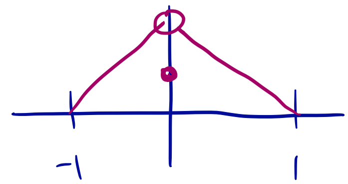
2. Give an example of a function with domain $(-1, 1)$ that does not attain either an absolute minimum or an absolute maximum value. Why doesn't your example violate the Extreme Value Theorem?

$f(x) = x$



no violation: interval not closed.

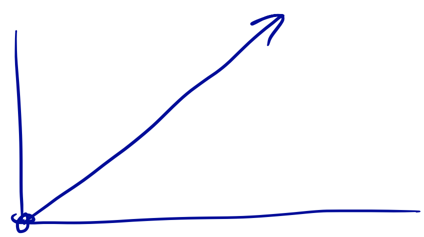
3. Sketch a discontinuous function with domain $[-1, 1]$ that attains a minimum but does not attain a maximum value. Why doesn't your example violate the Extreme Value Theorem?



No violation: function not continuous

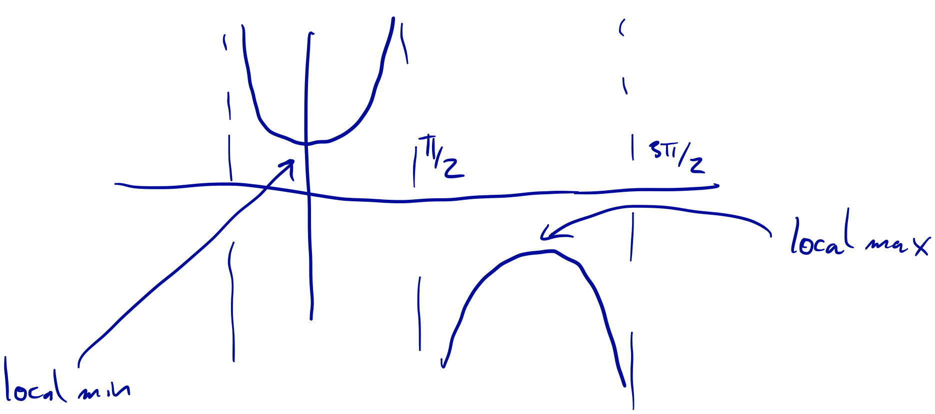
4. Give an example of a continuous function with domain $[0, \infty)$ that attains a minimum but does not attain a maximum value. Why doesn't your example violate the Extreme Value Theorem?

$f(x) = x$



No violation: interval not bounded

5. Consider the function $\sec(x)$. Sketch this function. From the sketch answer the following. Does this function have any absolute maximums? Absolute minimums? Local maximums? Local minimums?



no absolute max/mins

6. Find all critical points of the function $f(x) = \sin(x)^{1/3}$.

$$f'(x) = \frac{1}{3}(\sin(x))^{-2/3} \cdot \cos(x)$$

$f'(x)$ does not exist if $\sin(x) = 0$ $(x = k\pi \quad k \in \mathbb{Z})$

$f'(x) = 0$ if $\cos(x) = 0$ $(x = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z})$

all critical points

7. Key Tool: Closed Interval Method

To find a maximum or minimum value for a continuous function defined on an closed, bounded interval $[a, b]$, look in all of the following locations:

1. The end points.
2. The critical points.

Find the absolute maximum and minimum values of $f(x) = x - x^{1/3}$ on the interval $[-1, 4]$, and the locations where those values are attained.

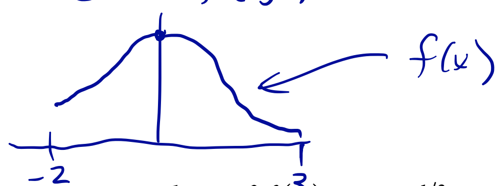
8. Find the absolute maximum and minimum values of $f(x) = e^{-x^2}$ on the interval $[-2, 3]$, and the locations where those values are attained.

$f'(x) = -2xe^{-x^2}$

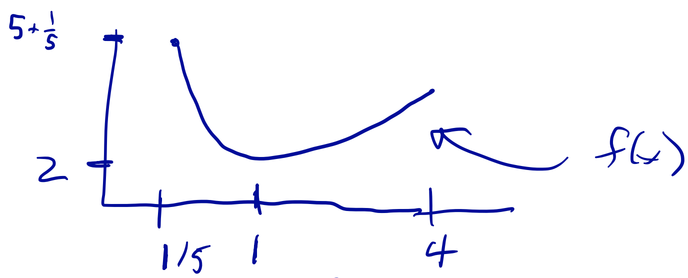
critical point: $x = 0$ $\quad f(0) = 1$ ← maximum value at $x = 0$

end point $x = -2$ $\quad f(-2) = e^{-4}$

endpoint $x = 3$ $\quad f(3) = e^{-9}$ ← minimum value at $x = 3$



9. ~~Find the maximum and minimum values of $f(x) = x - x^{1/3}$ on the interval [-1,4]. Determine where those maximum and minimum values occur.~~



10. Find the maximum and minimum values of $f(x) = x + \dfrac{1}{x}$ on the interval [1/5,4]. Determine where those maximum and minimum values occur.

$f'(x) = 1 - \frac{1}{x^2}$

critical point: $x = 1$ $\quad f(1) = 2$ ← min value at $x = 1$

endpoint $x = \frac{1}{5}$ $\quad f(1/5) = 5 + \frac{1}{5}$

end point $x = 4$ $\quad f(4) = 4 + \frac{1}{4}$ ← max value at $x = 4$

11. Find the maximum and minimum values of $f(x) = x^{2/3}$ on the interval [-8,8]. Determine where those maximum and minimum values occur.

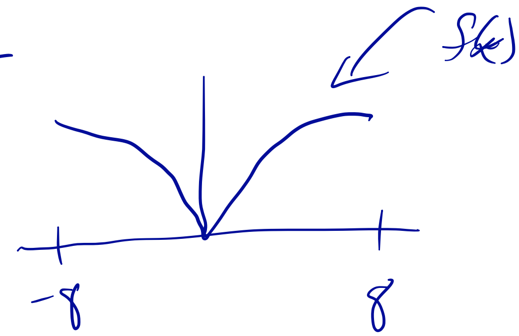
$f'(x) = \frac{2}{3} x^{-1/3}$

$f'(x)$ does not exist at $x = 0$

critical point: $x = 0$ $\quad f(0) = 0$ $\quad$ minimum value at $x = 0$

end point $x = -8$ $\quad f(-8) = 4$ $\quad$ max value at $x = 8, -8$

end point $x = 8$ $\quad f(8) = 4$



12. A ball thrown in the air at time $t = 0$ has a height given by

$$h(t) = h_0 + v_0 t - \frac{1}{2} g_0 t^2$$

meters where t is measured in seconds, h_0 is the height at time 0, v_0 is the velocity (in meters per second) at time 0 and g_0 is the constant acceleration due to gravity (in m/s^2). Assuming $v_0 > 0$, find the time that the ball attains its maximum height. Then find the maximum hight.

$h'(t) = v_0 - g_0 t$

$h'(t) = 0 \Rightarrow t = \frac{v_0}{g_0}$ ← time of max height

$h\left(\frac{v_0}{g_0}\right) = h_0 + v_0\left(\frac{v_0}{g_0}\right) - \frac{1}{2} g_0 \left(\frac{v_0^2}{g_0^2}\right)$

$= h_0 + \frac{1}{2}\frac{v_0^2}{g_0}$ ← max height