1. Compute $\int_1^2 \frac{t^3 - 3t^2}{t^4}\, dt$.

$$\int_1^2 \frac{t^3 - 3t^2}{t^4} dt = \int_1^2 \frac{1}{t} - \frac{3}{t^2} dt = \ln(|t|) + \frac{3}{t} \Big|_1^2$$

$$= \ln(2) - \ln(1) + \frac{3}{2} - 3$$

$$= \ln(2) - \frac{3}{2}$$

2. Compute $\frac{d}{dx} \int_5^x \cos(\sqrt{s})\, ds$.

$\cos(\sqrt{x})$ (!)

3. Compute $\int x^2(3 - x)\, dx$

$$\int 3x^2 - x^3 dx = x^3 - \frac{x^4}{4} + C$$

4. Compute $\int 9\sqrt{x} - 3\sec(x)\tan(x)\, dx$

$$\int 9\sqrt{x} - 3\sec(x)\tan(x) dx = 9\int \sqrt{x} dx - 3\int \sec(x)\tan(x) dx$$

$$= 9 \frac{2}{3} x^{3/2} - 3\sec(x) + C$$

$$= 6x^{3/2} - 3\sec(x) + C$$

AFES

5. Snow is falling on my garden at a rate of

$$A(t) = 10e^{-2t}$$

kilograms per hour for $0 \le t \le 2$, where t is measured in hours.

a. If $m(t)$ is the total mass of snow on my garden, how are $m(t)$ and $A(t)$ related to each other?

$m'(t) = A(t)$

b. What does $m(2) - m(0)$ represent?

Net change of mass of snow on garden from $t=0$ to $t=2$

c. Find an antiderivative of $A(t)$.

$\int A(t)\,dt = -5e^{-2t}$

d. Compute the total amount of snow accumulation from $t = 0$ to $t = 1$.

$m(1) - m(0) = \int_0^1 m'(t)\,dt = \int_0^1 10e^{-2t}\,dt = -5e^{-2t}\Big|_0^1$

$= -5e^{-2} + 5$

$= 5(1-e^{-2})$

e. Compute the total amount of snow accumulation from $t = 0$ to $t = 2$.

Same as above, except $1 \to 2$:

$m(2) - m(0) = 5(1-e^{-4})$

f. From the information given so far, can you compute $m(2)$?

No

g. Suppose $m(0) = 9$. Compute $m(1)$ and $m(2)$.

$m(1) = m(0) + m(1) - m(0)$

$= 9 + 5(1-e^{-2})$

$m(2) = m(0) + (m(2) - m(0))$

$= 9 + 5(1-e^{-4})$

6. A airplane is descending. Its rate of change of height is $r(t) = -4t + \frac{t^2}{10}$ meters per second.

a. if $A(t)$ is the altitude of the airplane in meters, how are $A(t)$ and $r(t)$ related?

$A'(t) = r(t)$

b. What physical quantity does $\int_1^3 r(t)\,dt$ represent?

Net change in height from $t=1$ to $t=3$.

c. Compute $A(3) - A(1)$.

$$A(3) - A(1) = \int_1^3 A'(t)\,dt = \int_1^3 r(t)\,dt$$

$$= \int_1^3 -4t + \frac{t^2}{10}\,dt$$

$$= -2t + \frac{t^3}{30}\Big|_1^3 = -6 + \frac{9}{30} - \left(-2 + \frac{1}{30}\right)$$

$$= -4 + \frac{2}{10} = -3.8 \text{ m}$$

7. Gravel is being added to a pile at a rate of rate of $1 + t^2$ tons per minute for $0 \le t \le 10$ minutes. If $G(t)$ is the amount of gravel (in tons) in the pile at time t, compute $G(10) - G(0)$.

$$G(10) - G(0) = \int_0^{10} G'(t)\,dt = \int_0^{10} 1 + t^2\,dt$$

$$= t + \frac{t^3}{3}\Big|_0^{10} = 10 + \frac{1000}{3}$$

$$= 343.\overline{3} \text{ tons}$$

8. Challenge! Compute

$$\frac{d}{dx}\int_5^{x^3} \cos(\sqrt{s})\, ds.$$

Hint: Let $H(x) = \int_5^x \cos(\sqrt{s})\, ds$. You're interested in $H(x^3)$. Apply the Chain Rule!

$$H'(x) = \cos(\sqrt{x})$$

$$\frac{d}{dx}\int_5^{x^3} \cos(\sqrt{s})\,ds = \frac{d}{dx} H(x^3)$$

$$= H'(x^3)\, 3x^2$$

$$= \cos(\sqrt{x^3}) \cdot 3x^2$$

9. Challenge! Compute

$$\frac{d}{dx}\int_x^{x+1} \sqrt{s^2+1}\, ds.$$

$$\frac{d}{dx}\int_x^{x+1} \sqrt{s^2+1}\, ds = \frac{d}{dx}\int_x^0 \sqrt{s^2+1}\,ds + \frac{d}{dx}\int_0^{x+1}\sqrt{s^2+1}\,ds$$

$$= -\frac{d}{dx}\int_0^x \sqrt{s^2+1}\,ds + \frac{d}{dx}\int_0^{x+1}\sqrt{s^2+1}\,ds$$

$$= -\sqrt{x^2+1} + \sqrt{(x+1)^2+1}$$