1. SR 1.1

2. SR 1.2 (i) - (ii)

3. The general linear group $GL(\mathbb{R},3)$ is the set of 3×3 invertible matrics. In this exercise, we show that the Euclidean group $E(\mathbb{R}^2)$ can be seen as a subgroup of $GL(\mathbb{R},3)$. (More formally, for the cognoscenti, we construct a group homomorphism from $E(\mathbb{R}^2)$ to $GL(\mathbb{R},3)$, i.e. a group representation of $E(\mathbb{R}^2)$.)

If

$$i(x,y) = \begin{pmatrix} c & \mp s \\ s & \pm c \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix} \tag{1}$$

we define

$$M_i = \begin{pmatrix} c & \mp s & t_x \\ s & \pm c & t_y \\ 0 & 0 & 1 \end{pmatrix}. \tag{2}$$

a) Suppose $x, y \in \mathbb{R}$. Show that

$$M_i \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \tag{3}$$

has the form

$$\begin{pmatrix} a \\ b \\ 1 \end{pmatrix} \tag{4}$$

and that $i(x,y) = (a,b)$.

b) Show that if i_1 and i_2 belong to $E(\mathbb{R}^2)$ then

$$M_{i_2 \circ i_1} = M_{i_2} M_{i_1}. \tag{5}$$

Note that on the right-hand side of this equation we are multiplying matrics.

c) Conclude that if $i \in E(\mathbb{R}^2)$, then

$$M_{i^{-1}} = (M_i)^{-1}. \tag{6}$$

4. [Challenge, not due!] SR 1.2 (iii)