

1. [Mason] Suppose  $f \in \mathcal{R}[a, b]$  and  $\alpha \in \mathbb{R}$ . Show that  $\alpha f \in \mathcal{R}[a, b]$  and

$$\int_a^b \alpha f = \alpha \int_a^b f$$

2. [Sakti] Show that the uniform limit of Riemann integrable functions is Riemann integrable. Conclude that  $\mathcal{R}[a, b]$  is a closed subset of  $B[a, b]$ .
3. [Max] Find (with proof) an element of  $\mathcal{R}[a, b]$  that is not a uniform limit of step functions.
4. [Jody] Show that integration on  $\mathcal{R}[a, b]$  (as a closed subset of  $B[a, b]$ ) is continuous by showing that the map

$$f \mapsto \int_a^b f$$

is a bounded linear map.

5. [Lander] Determine if  $\chi_\Delta \in \mathcal{R}[0, 1]$ , where  $\Delta$  is the Cantor set.
6. [Jody] Suppose  $l : \mathcal{P}(\mathbb{R}) \rightarrow [0, \infty]$  and that it is either additive or that it is countably additive. Show that either  $l(\emptyset) = 0$  or  $l(A) = \infty$  for all  $A \in \mathcal{P}(\mathbb{R})$ . Regardless, show that  $l$  is monotone.
7. [Mason] Suppose  $l : \mathcal{P}(\mathbb{R}) \rightarrow [0, \infty]$ . Show that  $l$  is countably additive if and only if  $l$  is finitely additive and countably subadditive.