The trunk of a tree is growing. The radius r of the trunk, in centimeters, is given by

$$r(t) = 2\sqrt{t}$$

where t is measured in years.

1. Find the average rate of change from $t = 1$ to $t = 2$ years.

$$\frac{r(2) - r(1)}{2 - 1} = \frac{2\sqrt{2} - 2\sqrt{1}}{1}$$

$$= 2(\sqrt{2} - 1)$$

$$\approx 0.83 \text{ cm/year}$$

2. Use the h-version of the limit definition of the derivative to find the instantaneous rate of change at $t = 1$ year.

$$r'(1) = \lim_{h \to 0} \frac{r(1+h) - r(1)}{h}$$

$$= \lim_{h \to 0} \frac{2\sqrt{1+h} - 2}{h}$$

$$= \lim_{h \to 0} \frac{2(\sqrt{1+h} - 1)}{h} \frac{(\sqrt{1+h} + 1)}{\sqrt{1+h} + 1}$$

$$= \lim_{h \to 0} \frac{2(1 + h - 1)}{h(\sqrt{1+h} + 1)} = \lim_{h \to 0} \frac{2}{\sqrt{1+h} + 1} = \frac{2}{\sqrt{1} + 1} = 1 \text{ cm/year}$$

limits don't care!

3. Use the a, b-version of the limit definition of the derivative to find the instantaneous rate of change of radius at $t = 1$ year.

$$r'(1) = \lim_{b\to 1} \frac{r(b)-r(1)}{b-1} = \lim_{b\to 1} \frac{2\sqrt{b}-2\sqrt{1}}{b-1}$$

$$= \lim_{b\to 1} \frac{2(\sqrt{b}-1)(\sqrt{b}+1)}{(b-1)(\sqrt{b}+1)}$$

$$= \lim_{b\to 1} \frac{2(b-1)}{(b-1)(\sqrt{b}+1}$$

$$= \lim_{b\to 1} \frac{2}{\sqrt{b}+1} = \frac{2}{\sqrt{1}+1} = 1 \text{ cm/year}$$

4. I promise you that $r(4) = 4\text{cm}$ and $r'(4) = 1/2$ cm/year. From this data alone, approximate the radius at 4 years and one month. Then compare your approximation with the true value.

4 years, one month: $4 + \frac{1}{12} = \frac{49}{12}$

$$r\left(\frac{49}{12}\right) = r(4) + \Delta r$$

$$= r(4) + \frac{\Delta r}{\Delta t}\Delta t$$

$$\approx r(4) + r'(4)\Delta t$$

$$= 4 + \frac{1}{2}\cdot\frac{1}{12} = \frac{97}{24} \approx 4.04167$$

$$r\left(\frac{49}{12}\right) = 2\sqrt{49/12} = 4.041451\ldots$$

first error here