1. Compute $\int_0^{\pi/2} \cos^3(x)\sin(x)\,dx$

$$= \int_1^0 u^3(-du) = \int_0^1 u^3\,du = \frac{u^4}{4}\Big|_0^1 = \frac{1}{4}$$

$u = \cos(x)$

$du = -\sin(x)dx$

2. Compute $\int \cos(x)\sin(\sin(x))\,dx = \int \sin(u)\,du$

$u = \sin(x)$

$du = \cos(x)dx$

$$= -\cos(u) + C$$

$$= -\cos(\sin(x)) + C$$

3. Compute $\int \frac{1}{9+x^2}\,dx = \int \frac{1}{9}\frac{1}{1+(\frac{x}{3})^2}\,dx$

$u = \frac{x}{3}$

$du = \frac{dx}{3}$

$$= \int \frac{1}{3}\frac{1}{1+u^2}\,du$$

$$= \frac{1}{3}\arctan(u) + C$$

$$= \frac{1}{3}\arctan\left(\frac{x}{3}\right) + C$$

4. Compute $\int \sqrt{x}(x^4 + x)\, dx$ $= \int x^{9/2} + x^{3/2}\, dx$

$= \frac{2}{11} x^{11/2} + \frac{2}{5} x^{5/2} + C$

5. Compute $\int x\sqrt{x-1}\, dx$

$u = x - 1$

$du = dx$

$= \int (u+1)\sqrt{u}\, du = \int u^{3/2} + u^{1/2}\, du$

$= \frac{2}{5} u^{5/2} + \frac{2}{3} u^{3/2} + C$

$= \frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C$

6. Compute $\int_1^3 \frac{(\ln(x))^3}{x}\, dx$

$u = \ln(x)$

$du = \frac{1}{x}\, dx$

$\hookrightarrow = \int_{\ln(1)}^{\ln(3)} u^3\, du = u^4 \Big|_0^{\ln(3)}$

$= [\ln(3)]^4$

7. Compute $\frac{d}{dx}[x\ln(x) - x]$. Then compute $\int s^2\ln(s^3)\,ds$

$$\frac{d}{dx}\left[x\ln(x)-x\right] = \ln(x) + \frac{x}{x} - 1 = \ln(x)$$

$$\int s^2\ln(s^3)ds = \frac{1}{3}\int \ln(u)\,du = \frac{1}{3}\left[u\ln(u) - u\right]$$

$$= \frac{1}{3}\left[s^3\ln(s^3) - s^3\right]$$

$u = s^3$

$du = 3s^2 ds$

8. Compute $\int \cot(\theta)\,d\theta$

$$\int \frac{\cos(\theta)}{\sin(\theta)}d\theta = \int \frac{1}{u}du = \ln(|u|) + C$$

$$= \ln(|\sin\theta|) + C$$

$u = \sin\theta$

$du = \cos\theta\, d\theta$

9. Compute $\int x(x+1)^{1/4}dx$

$u = x+1$

$du = dx$

$$\int (u-1)u^{1/4}du = \int u^{5/4} - u^{1/4}du$$

$$= \frac{4}{9}u^{9/4} - \frac{4}{5}u^{5/4}$$

$$= \frac{4}{9}(x+1)^{9/4} - \frac{4}{5}(x+1)^{5/4}$$

10. Challenge! Compute

$$\frac{d}{dx}\int_5^{x^3} \cos(\sqrt{s})\ ds.$$

Hint: Let $H(x) = \int_5^x \cos(\sqrt{s})\ ds$. You're interested in $H(x^3)$. Apply the Chain Rule!

$$H'(x) = \cos(\sqrt{x})$$

$$\frac{d}{dx}\int_5^{x^3} \cos(\sqrt{s})\,ds = \frac{d}{dx} H(x^3)$$

$$= H'(x^3)\,3x^2$$

$$= \cos(\sqrt{x^3})\cdot 3x^2$$

11. Challenge! Compute

$$\frac{d}{dx}\int_x^{x+1} \sqrt{s^2+1}\, ds.$$

$$\frac{d}{dx}\int_x^{x+1} \sqrt{s^2+1}\, ds = \frac{d}{dx}\int_x^0 \sqrt{s^2+1}\,ds + \frac{d}{dx}\int_0^{x+1} \sqrt{s^2+1}\,ds$$

$$= -\frac{d}{dx}\int_0^x \sqrt{s^2+1}\,ds + \frac{d}{dx}\int_0^{x+1} \sqrt{s^2+1}\,ds$$

$$= -\sqrt{x^2+1} + \sqrt{(x+1)^2+1}$$