

1. The cost of building wooden pencils is given by a function $C(n)$ where C is the cost in dollars and n is the number of pencils, measured in thousands. Explain what $C'(50) = 37.5$ means in language your parents could understand.

After 50 thousand pencils have been produced,
the cost of producing more pencils is \$37.5 / thousand
pencils.

Compute the derivatives of the following functions.

2. $f(x) = \sqrt{1+x^2}$

$$f'(x) = \frac{1}{2} \frac{1}{\sqrt{1+x^2}} \cdot 2x = \frac{x}{\sqrt{1+x^2}}$$

3. $f(\theta) = \tan(4\theta + 9)$

$$\begin{aligned} f'(\theta) &= \sec^2(4\theta + 9) \cdot \frac{d}{d\theta}(4\theta + 9) \\ &= 4 \sec^2(4\theta + 9) \end{aligned}$$

4. $f(t) = e^{t^2}(1 + \cos(t))$

$$\begin{aligned} f'(t) &= \frac{d}{dt} \left[e^{t^2} (1 + \cos(t)) \right] \\ &= \left(\frac{d}{dt} e^{t^2} \right) (1 + \cos(t)) + e^{t^2} \frac{d}{dt} (1 + \cos(t)) \\ &= e^{t^2} \left(\frac{d}{dt} t^2 \right) (1 + \cos(t)) + e^{t^2} (-\sin(t)) \\ &= e^{t^2} \left[2t (1 + \cos(t)) - \sin(t) \right] \end{aligned}$$

5. $f(v) = \sec\left(\frac{1}{1+v^2}\right)$

$$\begin{aligned} f'(v) &= \sec\left(\frac{1}{1+v^2}\right) \tan\left(\frac{1}{1+v^2}\right) \cdot \frac{d}{dv} \left(\frac{1}{1+v^2}\right) \\ &= \sec\left(\frac{1}{1+v^2}\right) \tan\left(\frac{1}{1+v^2}\right) \frac{-2v}{(1+v^2)^2} \end{aligned}$$

6. $f(x) = \cos(x^{1/3}e^x)$

$$\begin{aligned} f'(x) &= -\sin(x^{1/3}e^x) \cdot \frac{d}{dx} x^{1/3}e^x \\ &= -\sin(x^{1/3}e^x) \cdot \left[\frac{1}{3}x^{-2/3}e^x + x^{1/3}e^x \right] \end{aligned}$$

7. $f(x) = \sqrt{x + e^{x^2}}$

$$\begin{aligned} f'(x) &= \frac{1}{2\sqrt{x+e^{x^2}}} \cdot \frac{d}{dx} (x + e^{x^2}) \\ &= \frac{1}{2\sqrt{x+e^{x^2}}} (1 + 2xe^{x^2}) \end{aligned}$$