1. A rocket is launching, and its height h in meters is a function of t in seconds (so we are considering the function $h(t)$). Explain what $h'(10) = 1035$ means in language your mom could understand. You answer must include units.

At time $t = 10$ seconds the rocket is rising at a rate of 1035 feet per second.

Compute derivatives of the following functions using derivative rules.

2. $f(x) = \sqrt{t}e^t$

$$\frac{d}{dt}\sqrt{t}\,e^t = \left(\frac{d}{dt}\sqrt{t}\right)e^t + \sqrt{t}\,\frac{d}{dt}e^t$$

$$= \frac{1}{2}t^{-\frac{3}{2}}e^t + t^{1/2}e^t$$

3. $f(t) = e^{-t}$

$$\frac{d}{dt}e^{-t} = \frac{d}{dt}\frac{1}{e^t} = \frac{-1\frac{d}{dt}e^t}{(e^t)^2} = \frac{-e^t}{e^{2t}} = e^{-t}$$

4. $f(t) = e^{2t}$

$$\frac{d}{dt} e^{2t} = \frac{d}{dt} e^t e^t = \left(\frac{d}{dt} e^t\right) e^t + e^t \frac{d}{dt} e^t$$

$$= e^t e^t + e^t e^t$$

$$= 2e^{2t}$$

5. $f(v) = \left(1 + \frac{1}{v}\right)\left(2 - \frac{1}{v}\right)$

$$\frac{d}{dv}\left[\left(1+\frac{1}{v}\right)\left(2-\frac{1}{v}\right)\right] = \frac{-1}{v^2}\left(2-\frac{1}{v}\right) + \left(1+\frac{1}{v}\right)\left(\frac{1}{v^2}\right)$$

$$= \frac{-2}{v^2} + \frac{1}{v^3} + \frac{1}{v^2} + \frac{1}{v^3}$$

$$= -\frac{1}{v^2} + \frac{2}{v^3}$$

6. $f(x) = \dfrac{e^{2x}}{1 - e^x}$

$$\frac{d}{dx} \frac{e^{2x}}{1-e^x} = \frac{\left(\frac{d}{dx} e^{2x}\right)(1-e^x) - e^{2x} \frac{d}{dx}(1-e^x)}{(1-e^x)^2}$$

$$= \frac{2e^{2x}(1-e^x) - e^{2x}(-e^x)}{(1-e^x)^2}$$

$$= \frac{2e^{2x} - e^{3x}}{(1-e^x)^2}$$

7. $f(t) = \dfrac{\sin(x)}{\cos(x)}$

$$\frac{d}{dx}\frac{\sin(x)}{\cos(x)} = \frac{\left(\frac{d}{dx}\sin(x)\right)\cos(x) - \sin(x)\frac{d}{dx}\cos(x)}{\cos^2 x}$$

$$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)}$$

$$= \frac{1}{\cos^2(x)} = \sec^2(x)$$

8. $f(t) = e^{2x}\sin(x)$

$$\frac{d}{dx}e^{2x}\sin(x) = \left(\frac{d}{dx}e^{2x}\right)\sin(x) + e^{2x}\frac{d}{dx}\sin(x)$$

$$= 2e^{2x}\sin(x) + e^{2x}\cos(x)$$

$$= e^{2x}\left(2\sin(x) + \cos(x)\right)$$

9. $f(t) = (1+x^2)e^x\sin(x)$

$$\frac{d}{dx}\left[(1+x^2)e^x\sin(x)\right] = \left[\frac{d}{dx}(1+x^2)\right]e^x\sin(x) + (1+x^2)\left(\frac{d}{dx}e^x\right)\sin(x)$$

$$+ (1+x^2)e^x\frac{d}{dx}\sin(x)$$

$$= 2xe^x\sin(x) + (1+x^2)e^x\sin(x) + (1+x^2)e^x\cos(x)$$

$$= e^x\left[(1+x)^2\sin(x) + (1+x^2)\cos(x)\right]$$