1. Compute

$$\frac{d}{dt}\left[2\pi\frac{t-80}{365}\right].$$

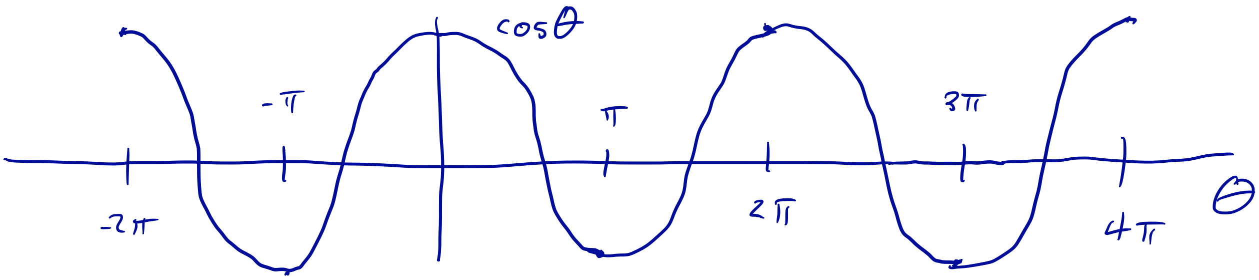
Don't you dare use the quotient rule.

$$2\pi\left(\frac{t-80}{365}\right) = \frac{2\pi}{365}t - \frac{2\pi\cdot 80}{365}$$

$$\frac{d}{dt}(at+b) = a\frac{d}{dt}t + \frac{d}{dt}b = a\cdot 1 + 0 = a.$$

So $$\frac{d}{dt}\left[2\pi\frac{t-80}{365}\right] = \frac{2\pi}{365}$$

2. Find **all** values θ such that $\cos(\theta) = 1$.



integers

$\cos\theta = 1$ when $\theta = 2\pi k,\ k\in\mathbb{Z}$

($\mathbb{Z}$ is the set of numbers: $0, 1, 2, 3, \ldots$ $-1, -2, -3, \ldots$)

3. Find all values x such that $\cos(3x) = 1$.

$$3x = 2\pi k$$

$$x = \tfrac{2}{3}\pi k \qquad k\in\mathbb{Z}$$

4. I'm tired of doing all the work around here. It's your turn. You're going to show that

$$\frac{d}{dx}\ln(x) = \frac{1}{x}.$$

Start with the equation $y = \ln(x)$.

1. Solve this equation for x.

$$e^y = x$$

2. Take an implicit derivative with respect to x, and solve for dy/dx.

$$e^y \frac{dy}{dx} = 1, \qquad \frac{dy}{dx} = \frac{1}{e^y}$$

3. Now convert dy/dx into an expression that only involves x. (Tah dah!)

$$\frac{dy}{dx} = \frac{1}{x}$$

5. Compute $\frac{d}{dx}\ln\left(x + e^{3x}\right)$.

$$\frac{d}{dx}\ln\left(x+e^{3x}\right) = \frac{1}{x+e^{3x}} \cdot \frac{d}{dx}\left(x+e^{3x}\right) = \frac{1+3e^{3x}}{x+e^{3x}}$$

6. Compute $\frac{d}{dx}\ln(\cos(x))$ and simplify your expression.

$$\frac{d}{dx}\ln(\cos(x)) = \frac{1}{\cos(x)} \cdot \frac{d}{dx}\cos(x) = \frac{-\sin(x)}{\cos(x)} = -\tan(x)$$

7. How can we compute $\frac{d}{dx}5^x$?

1. Rewrite $5^x = e^{ax}$ for a certain constant a. Your job is to find a!

$5 = e^{\ln 5}$ so $5^x = (e^{\ln 5})^x = e^{(\ln 5)x}$

2. Now compute $\frac{d}{dx}5^x$ by taking the derivative of e^{ax} instead.

$\frac{d}{dx}e^{ax} = ae^{ax}$ so $\frac{d}{dx}5^x = \frac{d}{dx}e^{(\ln 5)x} = \ln 5 e^{(\ln 5)x}$

3. Rewrite your previous answer so that the letter e does not appear.

$\frac{d}{dx}5^x = \ln(5)e^{\ln 5 x}$

8. Derive a formula for $\frac{d}{dx}\log_5(x)$. You can either use a change of base formula, or you can repeat the technique used to find the derivative of $\ln(x)$. Heck, do it both ways.

$y = \log_5 x$

$5^y = x$

$\frac{d}{dx}5^y = \frac{d}{dx}x$

$\ln(5)\,5^y\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\ln(5)5^y} = \frac{1}{\ln(5)x}$

$\log_5(x) = \frac{\ln(x)}{\ln(5)}$

$\frac{d}{dx}\log_5(x) = \frac{1}{\ln(5)x}$

9. We wish, for whatever bizarre reason, to compute dy/dx if

$$y = \frac{(x^2+1)(x+3)^{1/2}}{x-1}.$$

One can use the product and quotient rules. Here's an alternative technique known as logarithmic differentiation.

1. Take the natural logarithm of both sides of the equation.

$$\ln(y) = \ln\left(\frac{(x^2+1)(x+3)^{1/2}}{x-1}\right)$$

2. Use log rules such as $\ln(AB) = \ln(A) + \ln(B)$ to expand the right-hand side of this equation

$$\ln(y) = \ln(x^2+1) + \frac{1}{2}\ln(x+3) - \ln(x-1)$$

3. Compute (implicitly) dy/dx and solve for dy/dx.

$$\frac{1}{y}\frac{dy}{dx} = \frac{2x}{x^2+1} + \frac{1}{2}\frac{1}{x+3} - \frac{1}{x-1}$$

$$\frac{dy}{dx} = y\left[\frac{2x}{x^2+1} + \frac{1}{2}\frac{1}{x+3} - \frac{1}{x-1}\right]$$

4. Convert the expression for dy/dx so that it only involves x, and there are no appearances of y.

$$\frac{dy}{dx} = \frac{(x^2+1)(x+3)^{1/3}}{x-1} \cdot \left[\frac{2x}{x^2+1} + \frac{1}{2}\frac{1}{x+3} - \frac{1}{x-1}\right]$$