L'Hôpital's Rule

If f and g are differentiable and $g'(x) \neq 0$ on an interval containing a (except possibly at $x = a$). If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$ then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

so long as the right-hand limit exists, or is $\pm\infty$. Moreover, the same technique can be used

- if $\lim_{x\to a} f(x) = \pm\infty$ and $\lim_{x\to a} g(x) = \pm\infty$,
- for one-sided limits,
- for limits at infinity.

Compute the following limits.

1. $\displaystyle \lim_{x\to 0} \frac{\sin(5x)}{\sin(3x)}$

2. $\displaystyle \lim_{x\to 0} \frac{\cos(x) - 1}{x}$

3. $\lim_{x \to 0} \dfrac{\cos(x) - 1}{x^2}$

4. $\lim_{x \to -\infty} x e^x$.

5. $\lim_{x \to 0} \dfrac{\arcsin(x)}{x}$

6. $\displaystyle\lim_{x\to 0} \frac{e^x}{x+3}$. Careful!!

7. $\displaystyle\lim_{x\to 0^+} \frac{e^{1/x}}{\ln x}$.

8. $\displaystyle\lim_{x\to\infty} \left(1+\frac{5}{x}\right)^x$.