1. Estimate

$$\lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h}$$

to 5 decimal digits.

$$f(h) = \frac{\sqrt{2+h}-\sqrt{2}}{h}$$

h	$f(h)$
0.1	0.349....
0.01	0.35311.....
0.001	0.353509....
0.0001	0.35354895....
0.00001	0.3535529....
0.000001	0.3535533....

$$\lim_{h\to 0} f(h) \approx 0.35355$$

2. Estimate

$$\lim_{x\to 0} \frac{x^2}{\cos(x)-1}$$

to 5 decimal digits.

$$f(x) = \frac{x^2}{\cos^2(x)-1}$$

x	$f(x)$
0.1	-2.0016....
0.01	-2.000016....
0.001	-2.000000016.....
0.0001	-2.000000000016.....

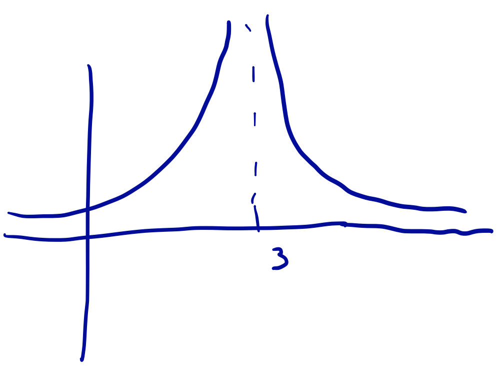
$$\lim_{x\to 0} f(x) \approx -2$$

3. Sketch the graph of

$$f(x) = \frac{1}{(3-x)^2}.$$

Then determine

$$\lim_{x\to 3} f(x).$$



$$\lim_{x\to 3} f(x) = \infty$$

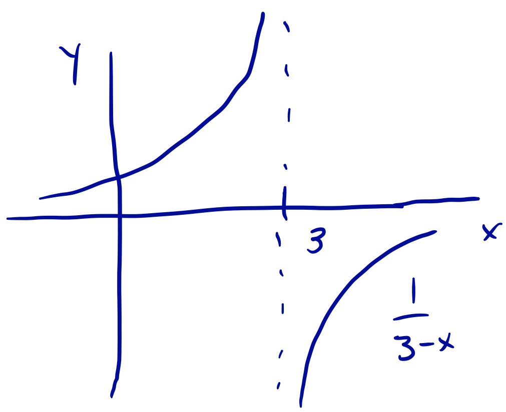
4. Determine

$$\lim_{x\to 3^+} \frac{1}{3-x}$$

and

$$\lim_{x\to 3^-} \frac{1}{3-x}.$$

A sketch of the graph might be helpful.



$$\lim_{x\to 3^+} \frac{1}{3-x} = \frac{1}{0^-} = -\infty$$

$$\lim_{x\to 3^-} \frac{1}{3-x} = \frac{1}{0^+} = +\infty$$

5. Determine exactly

$$\lim_{x\to 2} \frac{x^2-7x+10}{x-2}$$

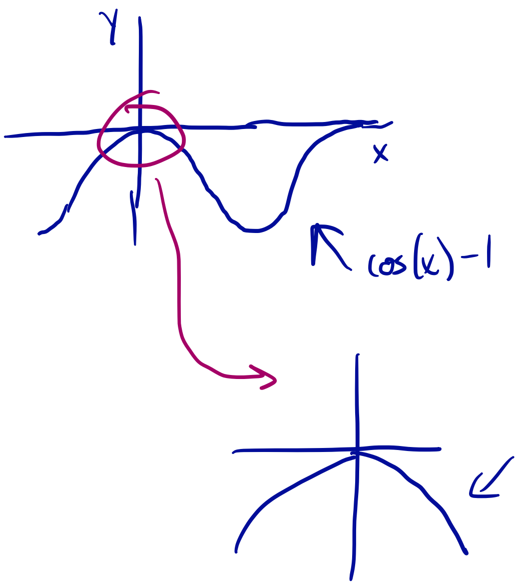
$$\lim_{x\to 2} \frac{x^2-7x+10}{x-2} = \lim_{x\to 2} \frac{(x-5)(x-2)}{x-2}$$

$$= \lim_{x\to 2} x-5 = -3.$$

6. Determine if

$$\lim_{x\to 0} \frac{1}{\cos(x)-1}$$

exists. If not, determine if the left- and right-hand limits exist.



$$\lim_{x\to 0^+} \frac{1}{\cos(x)-1} = \frac{1}{0^-} = -\infty$$

$$\lim_{x\to 0^-} \frac{1}{\cos(x)-1} = \frac{1}{0^-} = -\infty$$

$$\Rightarrow \lim_{x\to 0} \frac{1}{\cos(x)-1} = -\infty$$

$\cos(x)-1 < 0$ for x near 0, $x \neq 0$.

also.

7. Determine the left- and right-hand limits at 0 of $f(x) = x/|x|$.

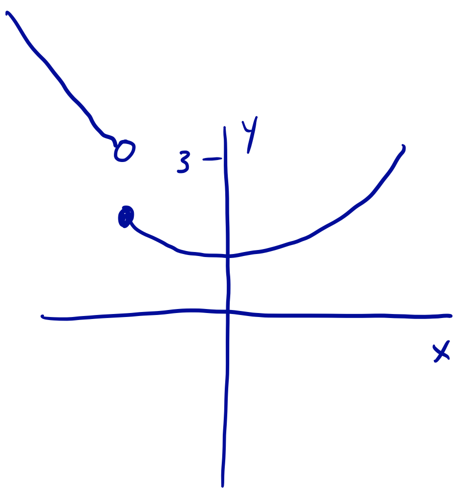
$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{x}{|x|} = \lim_{x\to 0^+} 1 = 1$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{x}{|x|} = \lim_{x\to 0^-} \frac{x}{-x} = \lim_{x\to 0^-} -1 = -1.$$

8. Suppose

$$g(x) = \begin{cases} x^2+1 & x \geq -1 \\ 2-x & x < -1. \end{cases}$$

Sketch the graph. Then determine if $\lim_{x\to -1} g(x)$ exists. If not, determine if the left- and right-hand limits exist.



$$\lim_{x\to -1^-} g(x) = \lim_{x\to -1^-} 2-x = 3$$

$$\lim_{x\to -1^+} g(x) = \lim_{x\to -1^+} x^2+1 = 1+1 = 2$$

9. Determine

$$\lim_{x\to 0^+} 10^{-\frac{1}{x}}$$

and

$$\lim_{x\to 0^-} 10^{-\frac{1}{x}}.$$

As $x \to 0^+$, $-\frac{1}{x} \to \frac{-1}{0^+} = -\infty$ and $10^{-\frac{1}{x}} \to 0$.

As $x \to 0^-$ $-\frac{1}{x} \to \frac{-1}{0^-} = +\infty$ and $10^{-1/x} \to \infty$.