

1. The volume of a snowball of radius  $r$  is  $V(r) = (4/3)\pi r^3$ , where  $r$  is measured in inches and  $V$  is measured in inches cubed. Explain what  $V'(2) \approx 50.265$  means in language your parents could understand.

When the snowball has a radius of 2 inches, increasing the radius increases the volume at a rate of 50.26 in<sup>3</sup>/in.

2. Compute  $\frac{d}{dx} \cot(x)$

$$\begin{aligned} \frac{d}{dx} \cot(x) &= \frac{d}{dx} \frac{\cos(x)}{\sin(x)} = \frac{\left(\frac{d}{dx} \cos(x)\right) \sin(x) - \cos(x) \frac{d}{dx} \sin(x)}{\sin^2(x)} \\ &= \frac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)} = \frac{-1}{\sin^2(x)} = -\csc^2(x) \end{aligned}$$

3. Compute  $\frac{d}{dx} \sec(x)$

$$\frac{d}{dx} \frac{1}{\cos(x)} = \frac{-\frac{d}{dx} \cos(x)}{\cos^2(x)} = \frac{\sin(x)}{\cos^2(x)} = \sec(x) \tan(x)$$

4. Compute the second derivative  $\frac{d^2}{dx^2} e^x \cos(x)$

$$\begin{aligned} \frac{d}{dx} e^x \cos(x) &= e^x \cos(x) - e^x \sin(x) \\ &= e^x (\cos(x) - \sin(x)) \end{aligned}$$

$$\begin{aligned} \frac{d^2}{dx^2} e^x \cos(x) &= e^x (\cos(x) - \sin(x)) + e^x (-\sin(x) - \cos(x)) \\ &= -2e^x \sin(x) \end{aligned}$$

5. A 12 foot ladder rests against a wall. Let  $\theta$  be the angle between the ladder and the wall and let  $x$  be the distance from the base of the ladder and the wall.

- a. Compute  $x$  as a function of  $\theta$ .

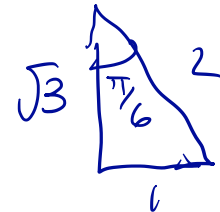


$$\frac{x}{12} = \sin(\theta)$$

$$x = 12 \sin(\theta)$$

- b. How fast does  $x$  change with respect to  $\theta$  when  $\theta = \pi/6$ ? Include units in your answer.

$$\frac{dx}{d\theta} = 12 \cos(\theta)$$



$$\begin{aligned} \text{at } \theta = \pi/6, \quad \frac{dx}{d\theta} &= 12 \cos(\pi/6) \\ &= 12 \frac{\sqrt{3}}{2} \\ &= 6\sqrt{3} \end{aligned}$$