1. Follow the guidelines from the previous worksheet to sketch the graph of

$$f(x) = \frac{2}{x} + \ln(x).$$

a. What is the function's domain?

$x > 0$

b. Does this function have any symmetry?

none

c. Find a few choice values of x to evaluate the function at.

$f(1) = 2$

d. What behaviour occurs for this function at ±∞?

$$\lim_{x\to\infty} \frac{2}{x} + \ln(x) = 0 + \infty = \infty$$

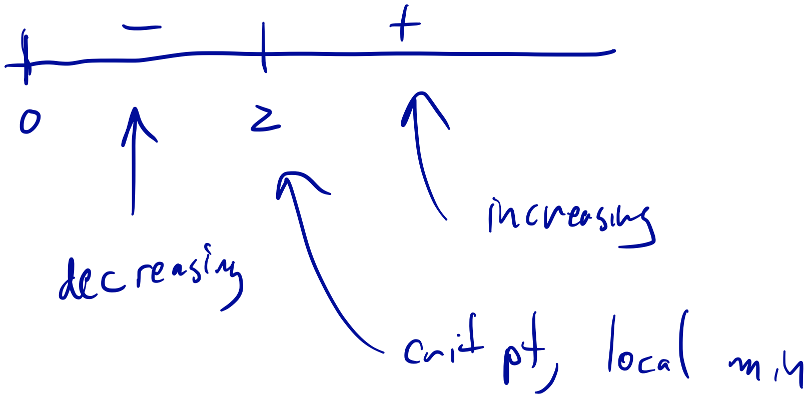
e. Does the function have any vertical asymptotes? Where?

$$\lim_{x\to0^+} \frac{2}{x} + \ln(x) = \lim_{x\to0^+} \frac{1}{x}\left[2 + x\ln(x)\right] = \infty\left[2 + 0\right] = \infty$$

$$\lim_{x\to0^+} x\ln(x) = \lim_{x\to0^+} \frac{\ln(x)}{1/x} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to0^+} \frac{\frac{1}{x}}{-1/x^2} = \lim_{x\to0^+} -x = 0.$$

f. Find intervals where f is increasing/decreasing and identify critical points.

$$f'(x) = \frac{-2}{x^2} + \frac{1}{x} = \frac{x-2}{x^2}$$

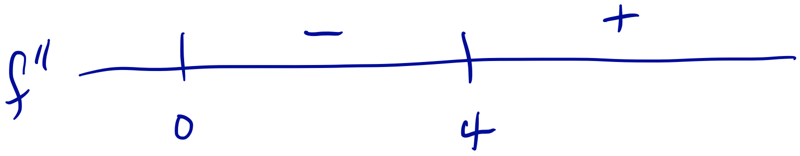


g. Classify each critical point as a local min/max/neither.

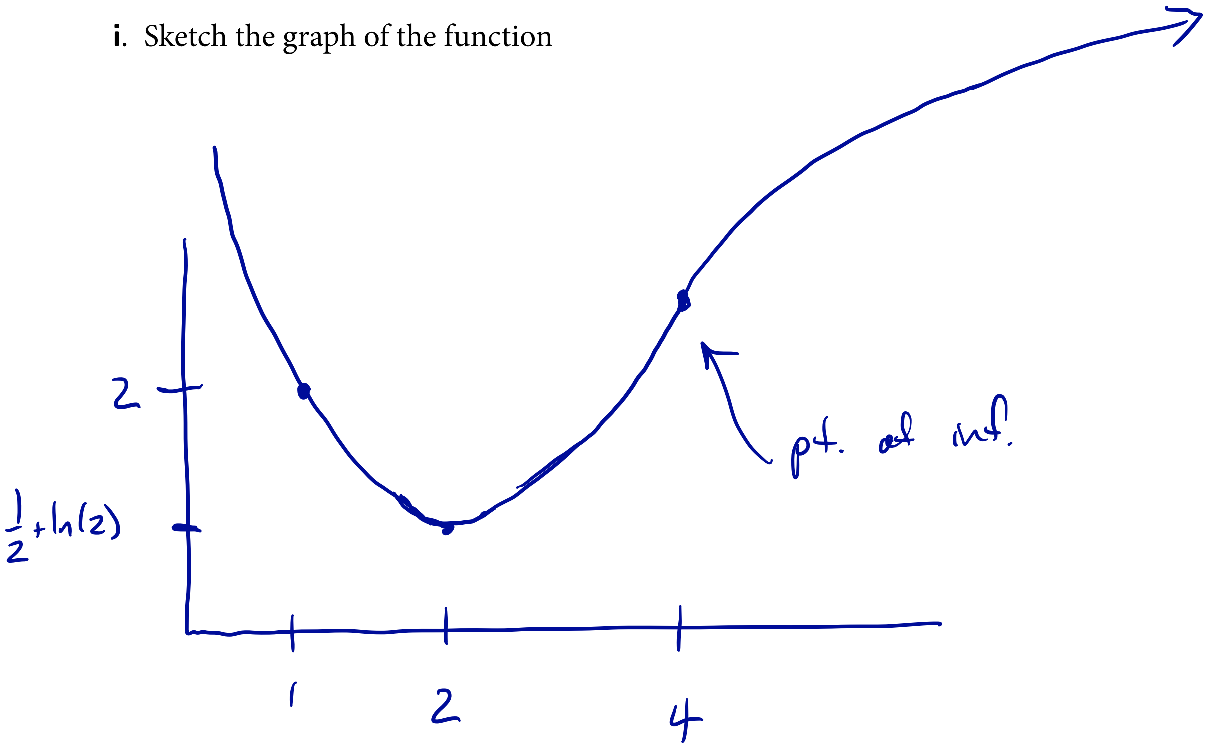
$x=2$ is the location of a local min

h. Find intervals where f is concave up/concave down and identify points of inflection

$$f''(x) = \frac{x^2 - 2x(x-2)}{x^4} = \frac{x - 2(x-2)}{x^3} = \frac{4-x}{x^3}$$



i. Sketch the graph of the function



2. Follow the guidelines from the previous worksheet to sketch the graph of

$$f(x) = x\sqrt{4 - x^2}.$$

a. What is the function's domain?

$-2 \leq x \leq 2$

b. Does this function have any symmetry?

odd symmetry

c. Find a few choice values of x to evaluate the function at.

$f(0) = 0, \quad f(\pm 2) = 0$

d. What behaviour occurs for this function at $\pm\infty$?

not defined near $\pm\infty$

e. Does the function have any vertical asymptotes? Where?

none

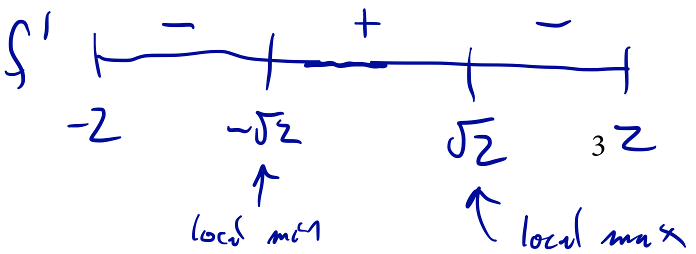
f. Find intervals where f is increasing/decreasing and identify critical points.

$$f'(x) = \sqrt{4-x^2} + \frac{x(-2x)}{2\sqrt{4-x^2}}$$

$$= \frac{4-x^2-x^2}{\sqrt{4-x^2}} = \frac{2(2-x^2)}{\sqrt{4-x^2}}$$

← controls sign.

← ≥ 0



g. Classify each critical point as a local min/max/neither.

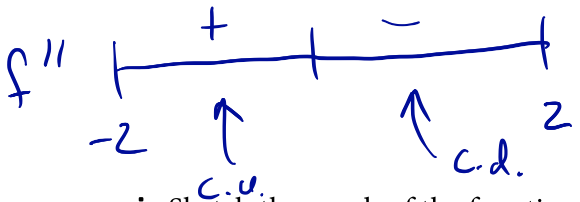
see previous q.

h. Find intervals where f is concave up/concave down and identify points of inflection

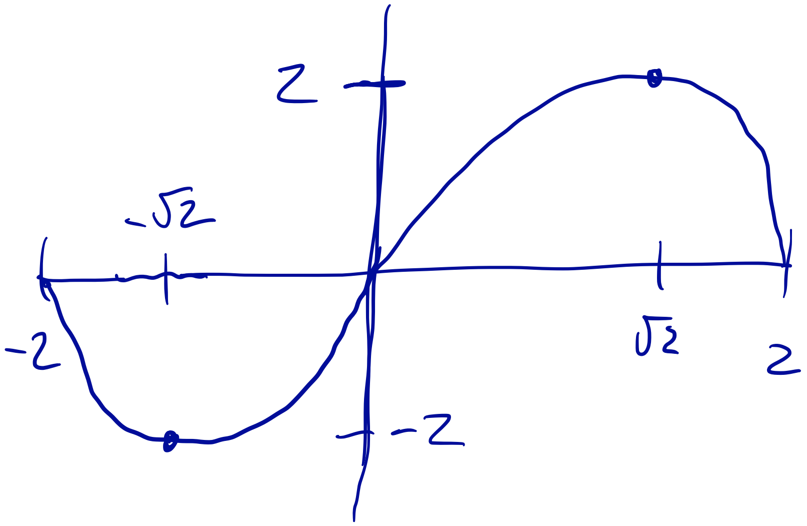
$$f'(x) = \frac{2(2-x^2)}{\sqrt{4-x^2}}, \quad f''(x) = 2\left[\frac{-2x\sqrt{4-x^2} - (2-x^2)\frac{-x}{\sqrt{4-x^2}}}{(4-x^2)}\right]$$

$$= 2\left[\frac{-2x + x[2-x^2]}{(4-x^2)^{3/2}}\right]$$

$$= 2\left[\frac{-x^3}{(4-x^2)^{3/2}}\right]$$



i. Sketch the graph of the function



3. Follow the guidelines from the previous worksheet to sketch the graph of

$$f(x) = \frac{x}{\sqrt{9+x^2}}.$$

a. What is the function's domain?

$\mathbb{R}$

b. Does this function have any symmetry?

odd

c. Find a few choice values of x to evaluate the function at.

$f(0) = 0$

d. What behaviour occurs for this function at ±∞?

$$\lim_{x\to\infty} \frac{x}{\sqrt{9+x^2}} = \lim_{x\to\infty} \frac{1}{\sqrt{9/x^2+1}} = 1$$

$$\lim_{x\to-\infty} \frac{x}{\sqrt{9+x^2}} = -1$$

e. Does the function have any vertical asymptotes? Where?

None

f. Find intervals where f is increasing/decreasing and identify critical points.

$$f'(x) = \frac{1}{\sqrt{9+x^2}} + \frac{x}{(9+x^2)^{3/2}}\left(\frac{-1}{2}\right)\cdot 2x$$

No crit pts.

Always increasing.

$$= \frac{9+x^2-x^2}{(9+x^2)^{3/2}} = \frac{9}{(9+x^2)^{3/2}} > 0$$

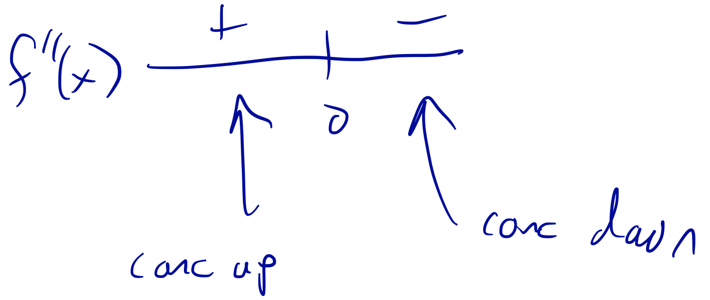
g. Classify each critical point as a local min/max/neither.

None to classify

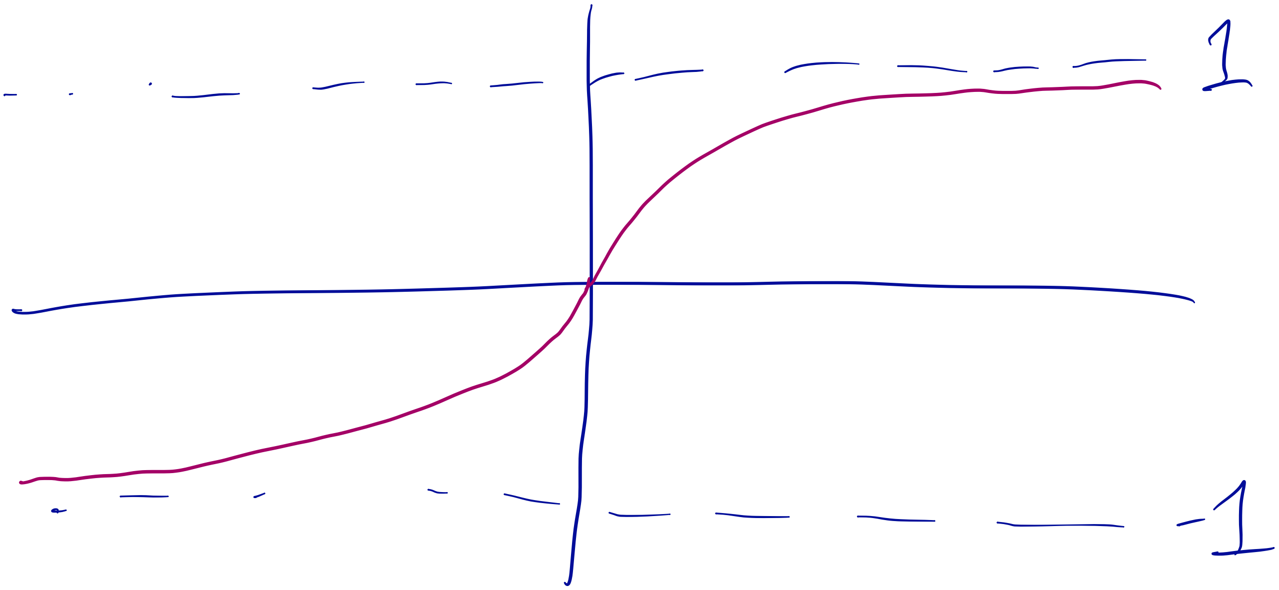
h. Find intervals where f is concave up/concave down and identify points of inflection

$$f''(x) = -\frac{3}{2}\cdot 9\cdot (9+x^2)^{-5/2}\cdot 2x$$

$$= -27(9+x^2)^{-5/2}\cdot x$$



i. Sketch the graph of the function



4. Follow the guidelines from the previous worksheet to sketch the graph of

$$f(x) = xe^{-1/x}.$$

a. What is the function's domain?

$x \neq 0$

b. Does this function have any symmetry?

None

c. Find a few choice values of x to evaluate the function at.

$f(1) = e^{-1}$

$f(-1) = -e$

d. What behaviour occurs for this function at ±∞?

$$\lim_{x \to \infty} xe^{-1/x} = \infty \cdot 1 = \infty$$

$$\lim_{x \to -\infty} xe^{-1/x} = -\infty \cdot 1 = -\infty$$

e. Does the function have any vertical asymptotes? Where?

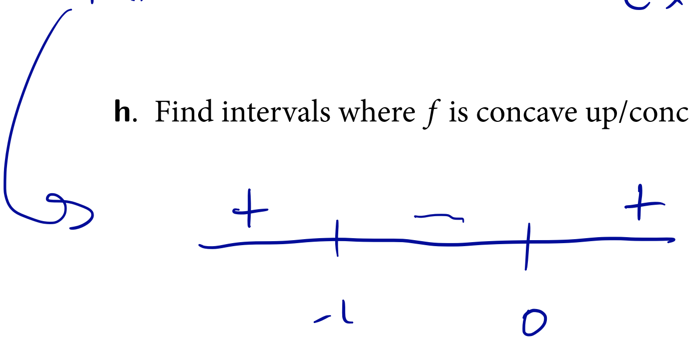
At $x = 0$:

$$\lim_{x \to 0^-} xe^{-1/x} = \lim_{x \to 0^-} \frac{x}{e^{1/x}} \overset{\frac{0}{0}}{=} \lim_{x \to 0^-} \frac{1}{e^{1/x}\left(\frac{-1}{x^2}\right)} = -\infty$$

f. Find intervals where f is increasing/decreasing and identify critical points.

g. Classify each critical point as a local min/max/neither.

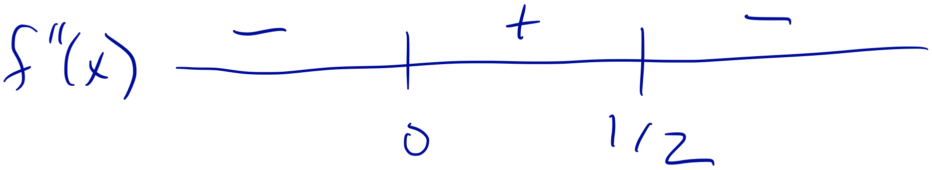
$$f'(x) = e^{-1/x} + x e^{-1/x} \cdot \left(+\frac{1}{x^2}\right) = e^{-1/x}\left[1+\frac{1}{x}\right]$$



h. Find intervals where f is concave up/concave down and identify points of inflection

$$f''(x) = e^{-1/x}\left(\frac{-1}{x^2}\right) + \left[1+\frac{1}{x}\right]e^{-1/x}\left(\frac{-1}{x^2}\right)$$

$$= \left(\frac{-1}{x^2}\right)e^{-1/x}\left[2+\frac{1}{x}\right]$$



i. Sketch the graph of the function