1. Justify

$$\lim_{x\to 5} \frac{x^2 - 6x + 5}{x - 5} = 4$$

using the "Limits don't care about one point" rule.

$$\lim_{x\to 5} \frac{x^2-6x+5}{x-5} = \lim_{x\to 5} \frac{(x-5)(x-1)}{x-5}$$

limits don't care about one point

$$= \lim_{x\to 5} x-1$$

$$= 5-1$$

$$= 4$$

2. Compute

$$\lim_{h\to 0} \frac{\sqrt{4+h} - 2}{h}$$

using the "Limits don't care about one point" rule. Hint: Multiply top and bottom by $\sqrt{4+h} + 2$ early in the computation.

$$\lim_{h\to 0} \frac{\sqrt{4+h}-2}{h} = \lim_{h\to 0} \frac{(\sqrt{4+h}-2)(\sqrt{4+h}+2)}{h(\sqrt{4+h}+2)}$$

$$= \lim_{h\to 0} \frac{h}{h(\sqrt{4+h}+2)}$$

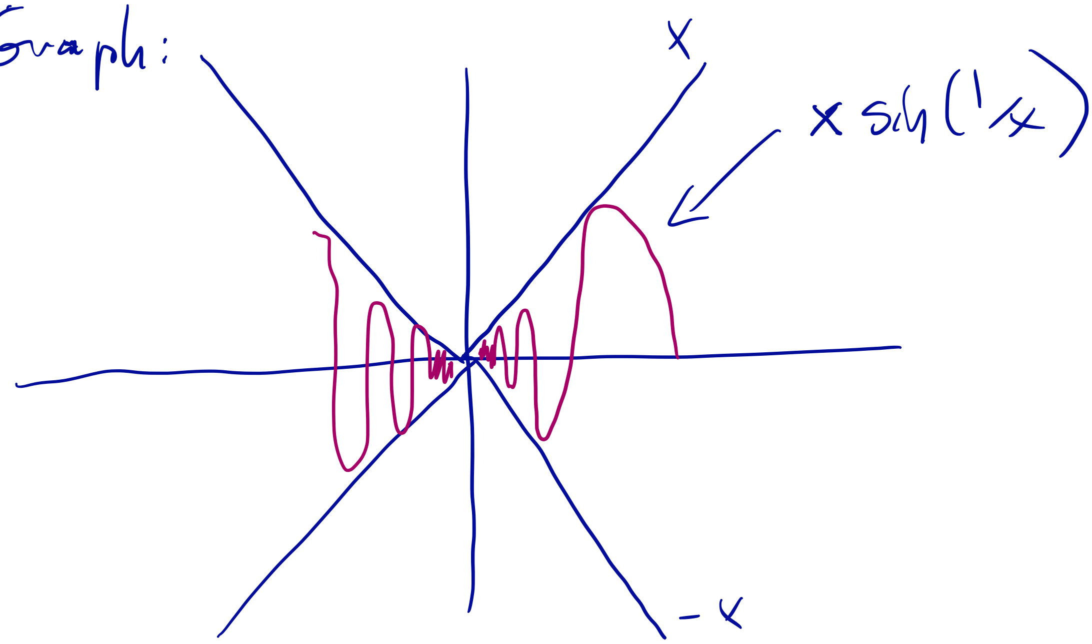
$$= \lim_{h\to 0} \frac{1}{\sqrt{4+h}+2} = \frac{1}{\sqrt{4+0}+2} = \frac{1}{4}$$

3. Use the squeeze theorem to show

$$\lim_{x\to 0} x \sin\left(\frac{1}{x}\right) = 0.$$

See next page:

Graph:



Since $-1 \le \sin(1/x) \le 1$, for $x > 0$,

$$-|x| = -x \le x\sin(1/x) \le x = |x|.$$

Since $\lim_{x\to 0^+} x = 0$ and $\lim_{x\to 0^+} -x = 0$, $\lim_{x\to 0^+} x\sin\left(\frac{1}{x}\right) = 0$

by the squeeze theorem.

For $x < 0$, it's the same argument now using

$$x \le x\sin(1/x) \le -x.$$

So $\lim_{x\to 0^-} x\sin(1/x) = 0$ and

$$\lim_{x\to 0} x\sin(1/x) = 0.$$