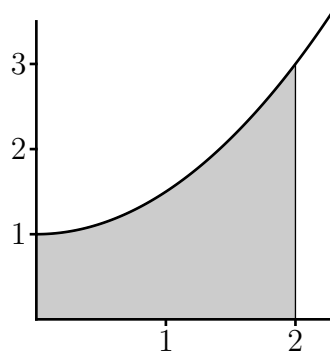
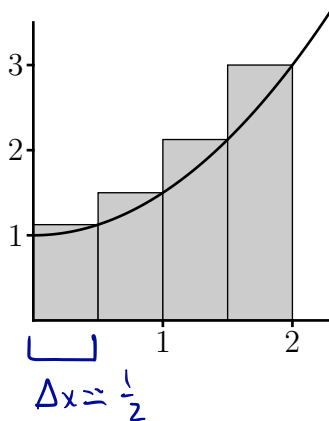


The goal of the first part is to estimate the area under the curve $y = \frac{1}{2}x^2 + 1$ and above the x -axis on the interval $[0, 2]$.



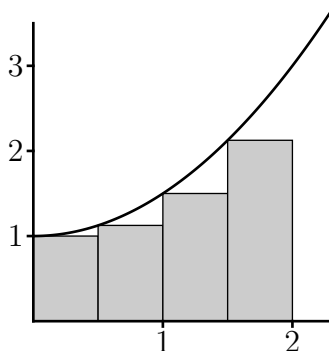
1. Use $n = 4$ rectangles with right-hand endpoints. Overestimate or underestimate?



$$\begin{array}{l|l} x_0 = 0 & y_0 = f(x_0) = 1 \\ x_1 = \frac{1}{2} & y_1 = f(x_1) = 1 + \frac{1}{2}\left(\frac{1}{2}\right)^2 \\ x_2 = 1 & y_2 = f(x_2) = 1 + \frac{1}{2}(1)^2 \\ x_3 = \frac{3}{2} & y_3 = f(x_3) = 1 + \frac{1}{2}\left(\frac{3}{2}\right)^2 \\ x_4 = 2 & y_4 = f(x_4) = 1 + \frac{1}{2}(2)^2 \end{array}$$

$$\begin{aligned} A &= y_1 \Delta x + y_2 \Delta x + y_3 \Delta x + y_4 \Delta x \\ &= \Delta x (y_1 + y_2 + y_3 + y_4) = 3.875, \text{ overestimate} \end{aligned}$$

2. Use $n = 4$ rectangles with left-hand endpoints. Overestimate or underestimate?

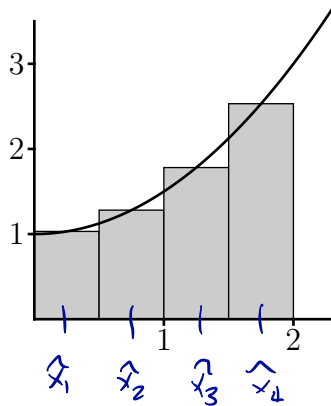


$$\begin{aligned} A &= y_0 \Delta x + y_1 \Delta x + y_2 \Delta x + y_3 \Delta x \\ &= \Delta x (y_0 + y_1 + y_2 + y_3) \\ &= 2.875, \text{ underestimate} \end{aligned}$$

3. From your last two answers, give your best estimate for the area.

$$\frac{1}{2} (3.875 + 2.875) = 3.375$$

4. Use $n = 4$ rectangles with midpoints. Overestimate or underestimate?



$$\hat{x}_1 = \frac{1}{4}$$

$$\hat{x}_2 = \frac{3}{4}$$

$$\hat{x}_3 = \frac{5}{4}$$

$$\hat{x}_4 = \frac{7}{4}$$

$$y_1 = f(\hat{x}_1) = 1 + \frac{1}{2} \left(\frac{1}{4}\right)^2$$

$$y_2 = 1 + \frac{1}{2} \left(\frac{3}{4}\right)^2$$

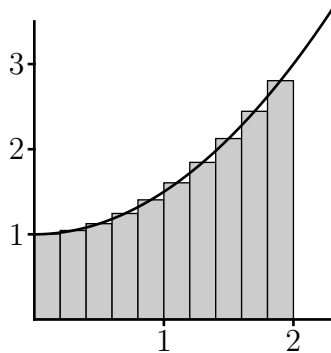
$$y_3 = 1 + \frac{1}{2} \left(\frac{5}{4}\right)^2$$

$$y_4 = 1 + \frac{1}{2} \left(\frac{7}{4}\right)^2$$

$$A = \Delta x (y_1 + y_2 + y_3 + y_4)$$

$$= 3.3125$$

5. Use $n = 10$ rectangles with midpoints.



$$\Delta x = \frac{1}{10}$$

$$x_1 = \frac{1}{20}$$

$$x_2 = \frac{1}{20} + \frac{1}{10}$$

$$x_3 = \frac{1}{20} + \frac{2}{10}$$

⋮

$$x_{10} = \frac{1}{20} + \frac{9}{10}$$

$$h_1 = f(x_1) = 1 + \frac{1}{2} \left(\frac{1}{20}\right)^2$$

,

,

⋮

$$h_{10} = f(x_{10}) = 1 + \frac{1}{2} \left(\frac{1}{20} + \frac{9}{10}\right)^2$$

$$\text{Area: } \Delta x (h_1 + h_2 + \dots + h_{10})$$

$$= 3.33$$

6. Suppose the odometer on our car is broken and we want to estimate the distance driven over a 1.5 hour time period. We take speedometer readings every 15 minutes and then record them in the table below. Estimate the distance traveled by the car. What method are you using?

Time (minutes)	0	15	30	45	60	75	90
Velocity (mi/h)	17	21	24	29	32	31	28

$$\begin{array}{l}
 \Delta t = \frac{1}{4} \text{ hour} \\
 \Delta x_1 = v_1 \cdot \Delta t = 17 \cdot \frac{1}{4} \\
 \Delta x_2 = v_2 \cdot \Delta t = 21 \cdot \frac{1}{4} \\
 \vdots \\
 \Delta x_6 = v_6 \cdot \Delta t = 31 \cdot \frac{1}{4}
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} \rightarrow \text{using left endpoints} \\ \\ \\ \\ \\ \end{array}$$

$$\frac{1}{4} (17 + 21 + 24 + 29 + 32 + 31) = 38.5 \text{ miles}$$

$$\text{using right endpoints:}$$

$$\frac{1}{4} (21 + 24 + 29 + 32 + 31 + 28) = 41.25 \text{ miles}$$

7. Oil leaked out of a tank at a rate of $r(t)$ liters per hour. The rate decreased as time passed and values of the rate at 2 hour time intervals are shown in the table. Estimate how much oil leaked out. What method are you using? Is it an overestimate or an underestimate.

t (h)	0	2	4	6	8	10
r(t) (L/h)	8.7	7.6	6.8	6.2	5.7	5.3

$$\Delta t = 2 \text{ hr}$$

$$\text{using right endpoints: } (7.6 + 6.8 + 6.2 + 5.7 + 5.3) \cdot 2 = 63.2 \text{ L}$$

$$\text{using left endpoints: } (8.7 + 7.6 + 6.8 + 6.2 + 5.7) \cdot 2 = 70 \text{ L}$$