1. R & Y 3.21

2. R & Y 3.22

3. R & Y 3.23

4. R & Y 4.3

5. Consider the map $I : C[0,1] \to C[0,1]$ given by

$$(I(f))(x) = \int_0^x f(s)\, ds.$$

Find a sequence of functions f_n such that $||f_n||_\infty = 1$ and such that $||If_n||_\infty \to 0$. Then tell me what this has to do with Example 4.10.

6. For each $p > 1$, find a $w \in \ell^p$ such that the map $Z \mapsto Z$:

$$T(z) = (w_1 z_1, w_2 z_2, \ldots)$$

is discontinuous if Z is given the ℓ^1 norm.

7. Consider Z with the ℓ^2 norm. Fix $w \in \ell^p$ for some $1 \le p \le \infty$ and define

$$T(z) = \sum_{k=1}^{\infty} w_k z_k.$$

Determine the values of p such that T is necessarily continuous.