1. I injest a 100mg aspirin at noon. Asperin in the body, at this dosage, has a half life of 3 hours. How much asperin is in my body at:

a) 6pm

$100 \cdot \frac{1}{2} \cdot \frac{1}{2} = 25\,mg$ (two half lives!)

b) 3pm

$100 \cdot \frac{1}{2} = 50\,mg$ (one half life)

c) 1pm

$100 \cdot 2^{-1/3} \approx 79.4\,mg$

d) 4:45pm

$100 \cdot 2^{-4.75/3} \approx 33.37\,mg$

2. You start with a 100g lump of a radioactive isotope. A year later the lump has a mass of 97.7g. What is the half life of the isotope?

$m(t) = 100 \cdot 2^{-t/b}$

$m(1) = 97.7$

$100 \cdot 2^{-1/b} = 97.7$

$2^{-1/b} = 0.977$

$-\frac{1}{b} \log_{10} 2 = \log_{10} 0.977$

$$\boxed{b = \frac{-\log_{10} 2}{\log_{10} 0.977} \approx 29.8 \text{ years}}$$

3. At time $t = 0$ minutes, a colony of E. coli has 10000 cells. The population is growing exponentially, and after 60 minutes it has 90000 members. Find a function of the form

$$p(t) = C\,10^{at}$$

that describes the population size.

$$p(0) = C\,10^{a\cdot 0} = C\cdot 1 = C$$
$$p(0) = 10000$$
$$C = 10000$$

$$p(60) = 10000\,10^{a60}$$
$$p(60) = 90000$$

$$10000\cdot 10^{60a} = 90000$$
$$10^{60a} = 9$$
$$60a \log_{10} 10 = \log_{10} 9$$
$$60a = \log_{10} 9$$

$$\boxed{a = \frac{1}{60}\log_{10} 9 \approx 0.0159}$$

4. The function $f(x) = 2^{-3x}$ can be written in the form $f(x) = 10^{-ax}$ for a certain constant a. Determine the value of a.

$$2^{-3x} = 10^{-ax}$$

$$\log_{10} 2^{-3x} = \log_{10} 10^{-ax}$$

$$-3x \log_{10} 2 = -ax \log_{10} 10$$

$$-3\log_{10} 2 = -a$$

$$a = 3\log_{10} 2 \approx 0.903$$

5. Use the change of base formula to rewrite $\log_{10}(7)$ in terms of the natural logarithm, ln.

$$\log_{10}(x) = \frac{\ln(x)}{\ln(10)} \Rightarrow \boxed{\log_{10}(7) = \frac{\ln(7)}{\ln(10)}}$$

6. Solve the following equation for x:

$$\ln(x) + \ln(x-1) = 2.$$

$$\ln(x(x-1)) = 2$$

$$x(x-1) = e^2$$

$$x^2 - x - e^2 = 0$$

quadratic formula

$$\boxed{x = \frac{1 \pm \sqrt{1+4e^2}}{2}}$$

7. Find the inverse function of $f(x) = 1 + \sqrt{2-3x}$. Remember:

a) Write $y = f(x)$.

b) Solve for x.

c) The resulting expression in terms of y is $f^{-1}(y)$.

$$y = 1 + \sqrt{2-3x}$$

$$y - 1 = \sqrt{2-3x}$$

$$(y-1)^2 = 2 - 3x$$

$$x = \boxed{\frac{1}{3}\left(2 - (y-1)^2\right) = f^{-1}(y)}$$