1. Compute $\int e^{4x-9}\,dx$

$u = 4x-9$

$du = 4dx$

$\frac{1}{4}du = dx$

$\int e^{4x-9}dx = \int e^u \frac{1}{4}du$

$= \frac{1}{4}e^u = \frac{1}{4}e^{4x-9}$

2. Compute $\int x\sin(x^2+1)\,dx$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\int x\sin(x^2+1)dx = \int \sin(u)\frac{1}{2}du$

$= -\frac{1}{2}\cos(u)$

$= -\frac{1}{2}\cos(x^2+1)$

3. Compute $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$.

$u = \sqrt{x}$

$du = \frac{1}{2}\frac{1}{\sqrt{x}}dx$

$2du = \frac{1}{\sqrt{x}}dx$

$\int e^{\sqrt{x}}\frac{1}{\sqrt{x}}dx = \int e^u \cdot 2 \cdot du$

$= 2e^u = 2e^{\sqrt{x}}$

4. Compute $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$.

$\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = 2e^{\sqrt{x}}\Big|_1^4 = 2e^2 - 2e^1$

$= 2(e^2 - e)$

5. Compute $\int \frac{\arctan(x)}{1+x^2}\,dx$

$u = \arctan(x)$

$du = \frac{1}{1+x^2}$

$\int \frac{\arctan(x)}{1+x^2}\,dx = \int u\,du = \frac{1}{2}u^2$

$= \frac{1}{2}\left(\arctan(x)\right)^2$

6. Compute $\int \frac{x^3}{\sqrt{1-x^4}}\,dx$

$u = 1-x^4$

$du = -4x^3\,dx$

$-\frac{1}{4}du = x^3\,dx$

$\int \frac{x^3}{\sqrt{1-x^4}}\,dx = \int -\frac{1}{4}\frac{1}{\sqrt{u}}\,du$

$= -\frac{1}{4}\,2u^{1/2}$

$= -\frac{1}{2}(1-x^4)^{1/2}$

$= -\frac{1}{2}\sqrt{1-x^4}$

7. Compute $\int \frac{x}{\sqrt{1-x^4}}\,dx$.

$u = x^2$

$du = 2x\,dx$

$\int \frac{x}{\sqrt{1-x^4}}\,dx = \int \frac{1}{2}\frac{1}{\sqrt{1-u^2}}\,du$

$= \frac{1}{2}\arcsin(u)$

8. Compute $\int \frac{\sec^2(x)}{\tan(x)}\,dx$

$u = \tan(x)$

$du = \sec^2(x)\,dx$

$\int \frac{\sec^2(x)}{\tan(x)}\,dx = \int \frac{1}{u}\,du$

$= \ln(|u|)$

$= \ln(|\tan(x)|)$

9. Compute $\int \sec^2(x)\tan(x)\,dx$

$u = \sec(x)$

$du = \sec(x)\tan(x)$

$\int \sec^2(x)\tan(x)\,dx = \int u\,du = \frac{1}{2}u^2$

$= \frac{1}{2}\sec^2(x)$

Alt: $v = \tan(x)$

$dv = \sec^2(x)\,dx$

$\int u\,du = \frac{1}{2}u^2 = \frac{1}{2}\tan^2(x)$

Why is this ok?

$+C$

10. Compute $\int \frac{\sin(\theta)}{1+\cos(\theta)}\,d\theta$

$u = 1+\cos\theta$

$du = -\sin(\theta)\,d\theta$

$\int \frac{\sin\theta}{1+\cos\theta}\,d\theta = \int -\frac{du}{u}$

$= -\ln(|u|)$

$= \ln(|1+\cos\theta|^{-1})$

$= \ln\left(\frac{1}{1+\cos\theta}\right)$

11. Compute $\int \frac{1}{x\ln(x)}\,dx$

$u = \ln(x)$

$du = \frac{1}{x}dx$

$$\int \frac{1}{x\ln(x)}dx = \int \frac{1}{u}\,du$$

$$= \ln(|u|)$$

$$= \ln(|\ln(x)|)$$

12. Compute $\int \frac{\sin(4/x)}{x^2}\,dx$

$u = 4/x$

$du = -4/x^2$

$$\int \frac{\sin(4/x)}{x^2} = \int -\frac{1}{4}\sin(u)\,du$$

$$= \frac{1}{4}\cos(u) + C$$

$$= \frac{1}{4}\cos\left(\frac{4}{x}\right) + C$$

13. Compute $\int \frac{e^x}{e^x - 3}\,dx$

$u = e^x - 3$

$du = e^x dx$

$$\int \frac{e^x}{e^x-3}dx = \int \frac{du}{u}$$

$$= \ln(|u|) + C$$

$$= \ln(|e^x - 3|) + C$$