1. A ball is tossed straight up into the air. It has a velocity at time $t = 0$ seconds of 5 meters per second. It undergoes a constant acceleration due to gravity of -9.8 meters per second per second, m/s^2. The height of the ball can be written in the form

$$h(t) = at + bt^2$$

where h is measured in meters, time is measured in seconds, and a and b are certain constants.

   1. Determine the values for the constants.

$h'(t) = a + 2bt$ $h'(0) = a$ $a = 5 \ m/s$

$h''(t) = 2b$ $2b = -9.8 \ m/s^2 \rightarrow b = -4.9 \ m/s^2$

   2. What is the height of the ball at time $t = 0$? At $t = 1$?

$h(0) = a \cdot 0 + b 0^2 = 0$

$h(1) = 5 \cdot 1 - 4.9 \cdot 1^2 = 0.1 \ m$

   3. At what times is the ball at height 0?

$h(t) = 0$ $at + bt^2 = 0$ $t = 0$ or

$t(a + bt) = 0$ $a + bt = 0 \rightarrow t = \frac{-a}{b} = \frac{5}{4.9} = 1.02 s$

   4. What is the average velocity of the ball over the time interval $[0.2, 0.21]$?

$\dfrac{h(0.21) - h(0.2)}{0.01} = 2.991 \ m/s$

   5. What is the average velocity of the ball over the time interval $[0.2, 0.201]$?

$\dfrac{h(0.201) - h(0.2)}{0.001} = 3.0351 \ m/s$

   6. What is the instantaneous velocity of the ball at time $t = 0.2$??

$h'(t) = 5 - 9.8t$; $h'(0.2) = 3.04 \ m/s$

   7. At what time t is the ball motionless?

$h'(t) = 0$ when $5 - 9.8t = 0$ so $t = 0.5102 s$

   8. What is the velocity of the ball at time $t = 0$? At $t = 0.1$? At $t = 1$?

$h'(0) = 5 \ m/s$

$h'(0.1) = 4.02 \ m/s$

$h'(1) = -4.8 \ m/s$

2. A stone is thrown in a pond and a circular ripple travels outward at a speed of 60 cm/s. Determine the rate of change of area inside the ripple at time $t = 1$ second and at time $t = 2$ seconds.

3. A current is passsing through a wire. The amount of charge that has passed by a measuring point on the wire at time t is

$$Q(t) = te^{-t}$$

for $t > 0$. Here, the charge Q is measured in Coulombs (which is a count of the number of electrons) and time t is measured in seconds.

Determine the current in the wire at time $t = 0$ and $t = 2$ seconds. Current is measured in Coulombs per second, and one Coulomb per second is known as an Ampere (an amp).

4. A population of bacteria starts at 500 cells and doubles every 30 minutes. Find a function $P(t)$ that describes this situation. Then compute the rate of change of the bacteria population at time $t = 60$ minutes.

5. A one-meter rod has non uniform mass. The mass of the rod from one end to distance x along it is

$$m(x) = x + \frac{1}{3}\sqrt{x}$$

where mass is measured in grams and x is in centimeters.

  1. What is the total mass of the rod?

  2. What is the mass of the first half of the rod? The second half?

  3. What is the average density (in grams/centimeter) of the first half of the rod?

  4. What is the density of the rod at $x = 30$ centimeters?

6. A population of caribou is growing, and its population is

$$P(t) = 4000\frac{3e^{t/5}}{1 + 2e^{t/5}}.$$

1. What is the population at time $t = 0$?

2. Determine the rate of change of the population at any time t.

3. Determine the rate of change of the population at time $t = 0$ years.

4. Determine the long term population.