

1. The volume of a snowball of radius r is $V(r) = (4/3)\pi r^3$, where r is measured in inches and V is measured in inches cubed. Explain what $V'(2) \approx 50.265$ means in language your parents could understand.

Starting with a snowball with a 2 inch radius,
as we increase the radius the volume grows at a rate
of 50.265 cubic inches per inch.

2. If you increase the radius of a snowball from 2 inches to 2.02 inches, estimate the change in volume of the snowball.

$$\Delta r = 2.02 - 2 = 0.02.$$

$$\Delta V \approx V'(2) \cdot \Delta r = 50.265 \cdot (0.02) = 1.0053 \text{ cubic inches}$$

3. Compute $\frac{d}{dx} \tan(x)$

$$\frac{d}{dx} \tan(x) = \frac{d}{dx} \frac{\sin(x)}{\cos(x)} = \sec^2(x) \text{ by last worksheet}$$

4. Compute $\frac{d}{dx} \sec(x)$

$$\frac{d}{dx} \frac{1}{\cos(x)} = \frac{-1 \frac{d}{dx} \cos(x)}{\cos^2(x)} = + \frac{\sin(x)}{\cos^2(x)} = \frac{\sin(x)}{\cos(x)} \frac{1}{\cos(x)} = \tan(x) \sec(x)$$

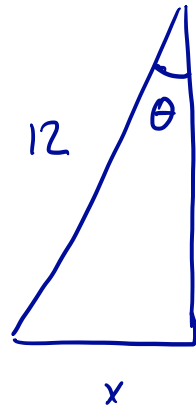
5. Compute the second derivative $\frac{d^2}{dx^2} e^x \cos(x)$

$$\frac{d}{dx} e^x \cos x = e^x \cos(x) - e^x \sin(x) = e^x [\cos(x) - \sin(x)]$$

$$\begin{aligned} \frac{d^2}{dx^2} e^x \cos(x) &= \frac{d}{dx} e^x [\cos(x) - \sin(x)] = e^x [\cos(x) - \sin(x)] \\ &\quad + e^x [-\sin(x) - \cos(x)] \\ &= -2e^x \sin x \end{aligned}$$

6. A 12 foot ladder rests against a wall. Let θ be the angle between the ladder and the wall and let x be the distance from the base of the ladder and the wall.

- a. Compute x as a function of θ .



$$\sin \theta = \frac{x}{12}$$

$$x = 12 \sin \theta$$

- b. How fast does x change with respect to θ when $\theta = \pi/6$?

$$x(\theta) = 12 \sin \theta$$

$$x'(\theta) = 12 \cos \theta$$

$$x'\left(\frac{\pi}{6}\right) = 12 \cos \frac{\pi}{6}$$

$$= 12 \cdot \frac{1}{2} = 6.$$

