Vocabulary

Suppose $f(x)$ is a real-valued function with domain D and suppose c is a point in D.

1. $f(c)$ is an **absolute maximum value** for f if $f(c) \geq f(x)$ for each x in D.
2. $f(c)$ is a **(absolute) minimum value** for f if $f(c) \leq f(x)$ for each x in D.
3. $f(c)$ is a **local maximum value** for f if $f(c) \geq f(x)$ for each x in D *near* c.
4. $f(c)$ is a **local minimum value** for f if $f(c) \leq f(x)$ for each x in D *near* c.
5. We say c is a **critical point** for f if either $f'(c) = 0$ or $f'(c)$ does not exist.

Key Tools

1. [Fermat's Theorem] If $f(c)$ is a (local or absolute) maximum/minimum value, and if f is defined on both sides of c, and if $f'(c)$ exists, then $f'(c) = 0$.
2. [Extreme Value Theorem] If the domain of f is a closed, bounded interval, and if f is continuous, then f is guaranteed to have both a maximum and a minimum value.

So, to find a maximum or minimum value for a function defined on an closed, bounded interval $[a, b]$, look in all of the following locations:

1. The end points.
2. The critical points.

1. Find the maximum and minimum values of $f(x) = x - x^{1/3}$ on the interval [-1,4]. Determine where those maximum and minimum values occur.

① Find critical points:

$f'(x) = 1 - \frac{1}{3}x^{-2/3}$

$f'(c) = 0: \quad 1 - \frac{1}{3}c^{-2/3} = 0$

$c^{-2/3} = 3$

$c = 3^{-3/2} \approx 0.19$

$f'(c)$ does not exist: $c = 0$

Critical points: $c = 0, 3^{-3/2}$

End points: $c = -1, 4$

② Substitute:

$f(0) = 0$

$f(3^{-3/2}) = 3^{-3/2} - 3^{-1/2}$

≈ -0.38

$f(-1) = 0$

$f(4) = 4 - 4^{1/3}$

≈ 2.4

3) Max value $4 - 4^{1/3}$ at $x = 4$

Min value $3^{-3/2} - 3^{1/2}$ at $x = 3^{-3/2}$

2. Find the maximum and minimum values of $f(x) = x + \frac{1}{x}$ on the interval [1/5,4]. Determine where those maximum and minimum values occur.

3. Find the maximum and minimum values of $f(x) = x^{2/3}$ on the interval [-8,8]. Determine where those maximum and minimum values occur.