1. (Young's Inequality) Let $p \in (1, \infty)$ and define q by $\frac{1}{p} + \frac{1}{q} = 1$. Suppose $a, b \geq 0$. Show

$$ab \leq \frac{a^p}{p} + \frac{b^q}{q}$$

and that the inequality is strict unless either $a^p = ab$ or $b^q = ab$ (in which case both of these equalities hold!).

Hint: Fix $b \geq 0$ and consider $f(a) = a^p/p + b^q/q - ab$ on $[0, \infty)$. Look at the first and second derivatives of f.

Remark: You proof should clearly note the place where $p > 1$ is used.

2. Carothers 3.34

3. Carothers 3.36

4. Carothers 3.39

5. Carothers 3.44

6. Carothers 3.46

7. Carothers 4.3

8. Carothers 4.11

9. Carothers 4.19

10. Carothers 4.14

11. Carothers 5.17

12. Carothers 5.24

13. Carothers 5.25