1. The conversion formula $E = mc^2$ for rest mass allows one to specify mass as energy. In particle physics, a standard measure of energy is a "gigaelectronvolt". Recall that the volt is the unit of electric potential: the energy required to move a coulomb of charge from a point with voltage V_1 to a point with voltage V_2 is $V_2 - V_1$. An electron volt is the energy in Joules required to move an electron accross a single volt of potential, and a gigaelectronvolt (GeV) is the energy needed to move 10^9 electrons over a volt of potential.

a) Recalling that the charge of an electron is 1.6×10^{-19} coulombs, show that a mass of 1GeV is equivalent to 1.8×10^{-27}kg.

b) A muon has a mass of 0.106 GeV and a rest frame half life of 2.19×10^{-6} seconds. It is moving in a circular particle accelerator, 1 km in diameter, with energy 1000 GeV. How far around the circle to you expect it will travel?

2. When we first introduced the equivalence principle, we observed that it would predict a change in wavelength of a photon travelling up a building of height z as

$$\Delta\lambda = a\frac{z}{c^2} \tag{1}$$

where a is gravitational acceleration at the surface of the earth.

In class, we saw that the formula for gravitational redshift in Schwarzschild is given by

$$\frac{\omega_2}{\omega_1} = \sqrt{\frac{1 - \frac{2GM}{r_1}}{1 - \frac{2GM}{r_2}}}. \tag{2}$$

Use equation (1) to derive equation (2).

3. The rules for computing Cristoffel symbols, parallel transport, covariant derivatives, and so forth work equally well for Riemannian metrics (i.e. metrics g_{ab} with signature $(+, +, \ldots, +)$) and in any dimension. Consider the Riemannian metric $d\phi^2 + \sin^2\phi d\theta^2$, which is the metric for the sphere in polar coordinates. Here, $\phi \in (0, \pi)$ and $\theta \in (-\pi, \pi)$.

1. Show that curves of constant θ are geodesics, and that the only line of constant ϕ that is a geodesic is the curve $\phi = 0$.

2. In this coordinate system, take the vector $X = [1, 0]$ and parallel transport it around a line of constant ϕ. What is the resulting vector? Your answer should depend on ϕ.

4. GR: 5.7

5. Consider a metric of the form

$$ds^2 = dt^2 - t^{2a_1}dx_1^2 - t^{2a_2}dx_2^2 - t^{2a_3}dx_3^2. \tag{3}$$

Find necessary and sufficient conditions on the numbers a_1, a_2 and a_3 such that the metric is a solution of the vacuum Einstein equations.

6. Consider Einstein's vaccum equation with a cosmological constant Λ:

$$G_{ab} = -\Lambda g_{ab}.$$

Find the analog of the Schwarzschild solution for $\Lambda \neq 0$. The equation of motion for geodesics can be written in the form $p^2 = f(u)$ as at the bottom of page 112. What is $f(u)$ in this case?

7. GR 8.4

8. The equation

$$u_{tt} - c^2 \Delta u + m^2 u = 0 \tag{4}$$

with the constant $m > 0$ is known as the Klein-Gordon equation. Find an energy for it and show that the causality principle holds for this equation as well.