1. Refresh your memory about the statement of Taylor's Theorem with remainder. (Appendix A.2 in your text will be helpful). Then use it to estimate the number of terms needed for the Taylor polynomial $p(x)$ for $\sin(x)$ such that

$$|p(x) - \sin(x)| < 10^{-4}$$

for all x in $[0, \pi]$. This will require estimating the size of the remainder term, and you are welcome to use a computer or calculator to assist in this computation. Then generate a graph showing the difference between $p(x)$ and $\sin(x)$ on the interval. What is the maximum error you actually observe?

2. Solve the initial value problem

$$y'' - y' - 6y = 0$$

for $y(t)$ subject to the initial condition $y(0) = 0$ and $y'(0) = 1$.

Then find the general solution of

$$y'' - y' - 6y = 1 + t.$$

3. Implement the bisection algorithm for finding roots. Your code should take as its arguments:

1. A function f. You will be solving $f(x) = 0$.
2. Two numbers a and b. The desired root should be in $[a, b]$.
3. A tolerance ϵ. The approximate root should be within distance ϵ of the true root.

Hand in both your code and a session showing its results in approximating $\sqrt{2}$ to within 10^{-8}.

4. Implement Newton's method. Your code should take as its arguments:

1. A function f. You will be solving $f(x) = 0$.
2. A number x_0, which is the initial approximate root.
3. A tolerance ϵ. The approximate root should be be returned when two subsequent iterations of Newton's method yield approximations within ϵ of each other.

Newton's method can be finicky. Your code should handle gracefully error conditions that can occur.

Hand in both your code, and a session showing its results approximating $\sqrt{2}$ to within 10^{-12} starting from an initial approximation $x_0 = 2$.