1. Compute
$$\lim_{x\to\infty} \frac{x^2-1}{x^2+1}$$

$$\lim_{x\to\infty} \frac{x^2-1}{x^2+1} = \lim_{x\to\infty} \frac{x^2-1}{x^2+1}\,\frac{1/x^2}{1/x^2}$$

$$= \lim_{x\to\infty} \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^2}}$$

$$= \frac{1-0}{1+0} = \frac{1}{1} = \boxed{1}$$

2. Compute
$$\lim_{x\to-\infty} \frac{x-1}{|x+2|}$$

As $x \to -\infty$, $x+2$ is negative so $|x+2| = -(x+2)$.

Thus
$$\lim_{x\to-\infty} \frac{x-1}{|x+2|} = \lim_{x\to-\infty} \frac{x-1}{-(x+2)}$$

$$= \lim_{x\to-\infty} \frac{(x-1)\frac{1}{x}}{-(x+2)\frac{1}{x}}$$

$$= \lim_{x\to-\infty} \frac{1-\frac{1}{x}}{-1-2/x} = \frac{1-0}{-1-0} = \boxed{-1}$$

3. Compute

$$\lim_{x\to-\infty} \frac{\sqrt{1+2x^4}}{2-x^2}.$$

$$\lim_{x\to-\infty} \frac{\sqrt{1+2x^4}}{2-x^2} = \lim_{x\to-\infty} \frac{\frac{1}{x^2}\sqrt{1+2x^4}}{\frac{1}{x^2}(2-x^2)}$$

$$= \lim_{x\to-\infty} \frac{\sqrt{1/x^4+2}}{\frac{2}{x^2}-1}$$

$$= \frac{\sqrt{0+2}}{0-1} = \sqrt{2}$$

4. Compute

$$\lim_{x\to\infty} \sqrt{9x^2+1} - 3x.$$

Hint: Multiply by $1 = \dfrac{\sqrt{9x^2+1}+3x}{\sqrt{9x^2+1}+3x}$.

$$\lim_{x\to\infty} \sqrt{9x^2+1} - 3x = \lim_{x\to\infty} \sqrt{9x^2+1} - 3x \left(\frac{\sqrt{9x^2+1}+3x}{\sqrt{9x^2+1}+3x}\right)$$

$$= \lim_{x\to\infty} \frac{9x^2+1-9x^2}{\sqrt{9x^2+1}+3x}$$

$$= \lim_{x\to\infty} \frac{1}{\sqrt{9x^2+1}+3x}$$

$= 0$ since $\sqrt{9x^2+1}+3x \to \infty$

5. Compute
$$\lim_{x\to\infty} \frac{2+e^x}{1-e^x}.$$

$$\lim_{x\to\infty} \frac{2+e^x}{1-e^x} = \lim_{x\to\infty} \frac{2+e^x}{1-e^x}\,\frac{e^{-x}}{e^{-x}}$$

$$= \lim_{x\to\infty} \frac{2e^{-x}+1}{e^{-x}-1}$$

$$= \frac{0+1}{0-1} = \boxed{-1}$$

6. Compute
$$\lim_{x\to-\infty} \frac{2+e^x}{1-e^x}.$$

$$\lim_{x\to-\infty} \frac{2+e^x}{1-e^x} = \lim_{x\to-\infty} \frac{2+0}{1-0} = 2$$

7. Compute
$$\lim_{x\to\infty} \ln(3+x) - \ln(1+x)$$

$$\lim_{x\to\infty} \ln(3+x) - \ln(1+x) = \lim_{x\to\infty} \ln\left[\frac{3+x}{1+x}\right]$$

$$= \lim_{x\to\infty} \ln\left(\frac{\frac{3}{x}+1}{\frac{1}{x}+1}\right)$$

$$= \ln\left(\frac{0+1}{0+1}\right) = \ln(1) = \boxed{0}$$

8. Compute
$$\lim_{x\to\infty} \arctan(2^{-x})$$

$$\lim_{x\to\infty} 2^{-x} = 0.$$

So

$$\lim_{x\to\infty} \arctan(2^{-x}) = \arctan(0) = \boxed{0.}$$