1. Compute $\int x^2(3-x)\,dx$

$$\int x^2(3-x)\,dx = \int 3x^2 - x^3\,dx = x^3 - \frac{x^4}{4} + C$$

2. Compute $\int 9\sqrt{x} - 3\sec(x)\tan(x)\,dx$

$$\int 9\sqrt{x} - 3\sec(x)\tan(x)\,dx = 9\int \sqrt{x}\,dx - 3\int \sec(x)\tan(x)\,dx$$

$$= 9\cdot\frac{2}{3}x^{3/2} - 3\sec(x) + C$$

$$= 6x^{3/2} - 3\sec(x) + C$$

3. Find an antiderivative of $f(x) = \frac{1}{x^2}$ that does not have the form $-1/x + C$.

$$f(x) = \begin{cases} -1/x + 5 & x > 0 \\ -1/x - 2 & x < 0 \end{cases}$$

4. Snow is falling on my garden at a rate of

$$A(t) = 10e^{-2t}$$

kilograms per hour for $0 \le t \le 2$, where t is measured in hours.

a. If $m(t)$ is the total mass of snow on my garden, how are $m(t)$ and $A(t)$ related to each other?

$m'(t) = A(t)$

b. What does $m(2) - m(0)$ represent?

Net accumulation of snow between $t=0$ and $t=2$.

c. Find an antiderivative of $A(t)$.

$-5e^{-2t}$

d. Compute the total amount of snow accumulation from $t = 0$ to $t = 1$.

$$m(1) - m(0) = \int_0^1 m'(t)\,dt = \int_0^1 A(t)\,dt = \int_0^1 10e^{-2t}\,dt = -5e^{-2t}\Big|_0^1 = 5(1-e^{-2})\text{ kg}$$

e. Compute the total amount of snow accumulation from $t = 0$ to $t = 2$.

$$m(2) - m(0) = \int_0^2 10e^{-2t}\,dt = -5e^{-2t}\Big|_0^2 = 5(1-e^{-4})\text{ kg}$$

f. From the information given so far, can you compute $m(2)$?

No.

g. Suppose $m(0) = 9$. Compute $m(1)$ and $m(2)$.

$m(1) = 9 + 5(1-e^{-2})$ kg

$m(2) = 9 + 5(1-e^{-4})$ kg

5. A airplane is descending. Its rate of change of height is $r(t) = -4t + \frac{t^2}{10}$ meters per second.

a. if $A(t)$ is the altitude of the airplane in meters, how are $A(t)$ and $r(t)$ related?

$A'(t) = r(t)$

b. What physical quantity does $\int_1^3 r(t)\,dt$ represent?

This is $A(3) - A(1)$, the net change in height from $t=1$ to $t=3$.

c. Compute $A(3) - A(1)$.

$$A(3) - A(1) = \int_1^3 -4t + \frac{t^2}{10}\,dt = -2t^2 + \frac{t^3}{30}\Big|_1^3$$

$$= -18 + \frac{9}{10} + 2 - \frac{1}{30}$$

$$= -16 + \frac{26}{30} \text{ m}$$

6. Gravel is being added to a pile at a rate of rate of $1+t^2$ tons per minute for $0 \le t \le 10$ minutes. If $G(t)$ is the amount of gravel (in tons) in the pile at time t, compute $G(10) - G(0)$.

$$G(10) - G(0) = \int_0^{10} G'(t)\,dt = \int_0^{10} 1 + t^2\,dt = t + \frac{t^3}{3}\Big|_0^{10}$$

$$= 10 + \frac{1000}{3}$$

$$\approx 343.3 \text{ tons}$$