For each limit in problems 1 through 5, verify that the expression is of the form 0/0 at the limit point. Then compute the limit using the ”Limits don’t care about one point” rule. For each limit computation, start by writing out the expression

$$\lim_{x\to a} f(x) =$$

for the specific values of f, a and x. Then carry on from here. Circle the equality in your computation where the “Limits don’t care about one point” rule gets used. See the example on the board for a template.

1. Compute $\lim_{h\to 0} \frac{(3+h)^2-9}{h}$.

$$\frac{(3+0)^2-9}{0} = \frac{0}{0} \checkmark$$

$$\lim_{h\to 0} \frac{(3+h)^2-9}{h} = \lim_{h\to 0} \frac{9+6h+h^2-9}{h}$$

$$= \lim_{h\to 0} \frac{(6+h)h}{h}$$

$$= \lim_{h\to 0} 6+h = 6+0 = 6$$

2. Compute $\lim_{h\to 0} \frac{\frac{1}{2+h}-\frac{1}{2}}{h}$.

$$\frac{\frac{1}{2+0}-\frac{1}{2}}{0} = \frac{0}{0} \checkmark$$

$$\lim_{h\to 0} \frac{\frac{1}{2+h}-\frac{1}{2}}{h} = \lim_{h\to 0} \left(\frac{1}{2+h}-\frac{1}{2}\right)\frac{1}{h}$$

$$= \lim_{h\to 0} \frac{2-(2+h)}{2(2+h)}\frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-h}{2(2+h)}\frac{1}{h} = \lim_{h\to 0} \frac{-1}{2(2+h)} = \frac{-1}{2(2+0)} = -\frac{1}{4}$$

3. Compute $\lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h}$.

$$\lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h} = \lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h} \, \frac{\sqrt{2+h}+\sqrt{2}}{\sqrt{2+h}+\sqrt{2}}$$

$$= \lim_{h\to 0} \frac{(2+h)-2}{h\left[\sqrt{2+h}+\sqrt{2}\right]}$$

$$= \lim_{h\to 0} \frac{h}{h\left(\sqrt{2+h}+\sqrt{2}\right)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{2+h}+\sqrt{2}} = \frac{1}{\sqrt{2+0}+\sqrt{2}} = \frac{1}{2\sqrt{2}}$$

4. Compute $\lim_{x\to 3} \frac{\frac{1}{x}-\frac{1}{3}}{x-3}$.

$$\lim_{x\to 3} \frac{\frac{1}{x}-\frac{1}{3}}{x-3} = \lim_{x\to 3} \left(\frac{1}{x}-\frac{1}{3}\right)\frac{1}{x-3}$$

$$= \lim_{x\to 3} \left(\frac{3-x}{3x}\right)\frac{1}{x-3}$$

$$= \lim_{x\to 3} \frac{-1}{3x}$$

$$= \frac{-1}{9}$$

5. Compute $\lim_{x\to 2} \dfrac{x^3 - 8}{x - 2}$.

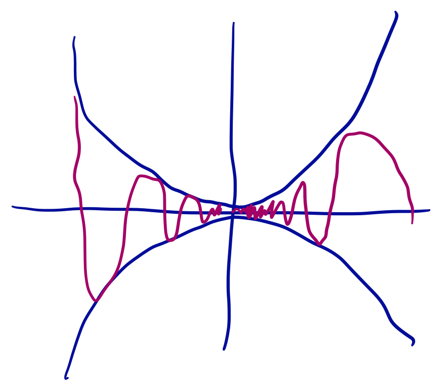
$$\lim_{x\to 2} \frac{x^3-8}{x-2} = \lim_{x\to 2} \frac{(x-2)(x^2+2x+4)}{x-2}$$

$$= \lim_{x\to 2} x^2+2x+4$$

$$= 2^2+2\cdot 2+4$$

$$= 12$$

6. Compute $\lim_{x\to 0} x^2 \sin(1/x)$.



$$-1 \le \sin\left(\tfrac{1}{x}\right) \le 1$$

$$-x^2 \le \sin\left(\tfrac{1}{x}\right) \le x^2$$

$$\left.\begin{array}{l} \lim_{x\to 0} x^2 = 0 \\ \lim_{x\to 0} -x^2 = 0 \end{array}\right] \Rightarrow$$ by Squeeze Theorem

$$\lim_{x\to 0} x^2 \sin\left(\tfrac{1}{x}\right) = 0$$

7. Compute $\displaystyle\lim_{x\to 6^+} \frac{6+|x|}{6-x}$.

For x near 6, $|x| = x$. "limits are nearsighted"

$$\lim_{x\to 6^+} \frac{6+|x|}{6-x} = \lim_{x\to 6^+} \frac{6+x}{6-x} = \frac{12}{0^-} = -\infty$$

8. Compute $\displaystyle\lim_{x\to 6^-} \frac{6+|x|}{6-x}$.

$$\lim_{x\to 6^-} \frac{6+|x|}{6-x} = \lim_{x\to 6^-} \frac{6+x}{6-x} = \frac{12}{0^+} = +\infty$$