1. Estimate

$$\lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h}$$

to 5 decimal digits.

$f(h) = \dfrac{\sqrt{2+h}-\sqrt{2}}{h}$

h	$f(h)$
0.1	0.349...
0.01	0.3531...
0.001	0.353509...
0.0001	0.353548...
0.00001	0.353552...
0.000001	0.35355533...

$\lim_{h\to 0} f(h) \approx 0.35355$

2. Estimate

$$\lim_{x\to 0} \frac{x^2}{\cos(x)-1}$$

to 5 decimal digits.

$f(x) = \dfrac{x^2}{\cos(x)-1}$

x	$f(x)$
0.1	-2.0016...
0.01	-2.000016...
0.001	-2.00000016...

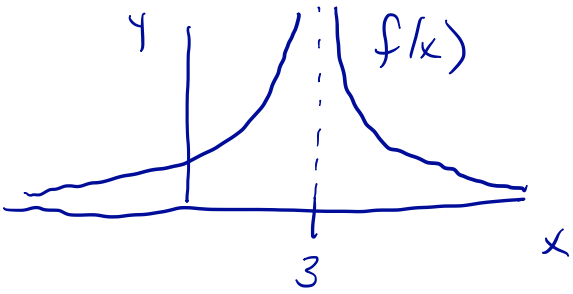
$\lim_{x\to 0} f(x) \approx -2$

3. Sketch the graph of

$$f(x) = \frac{1}{(3-x)^2}.$$

Then determine

$$\lim_{x\to 3} f(x).$$



$\lim_{x\to 3} f(x) = \infty$

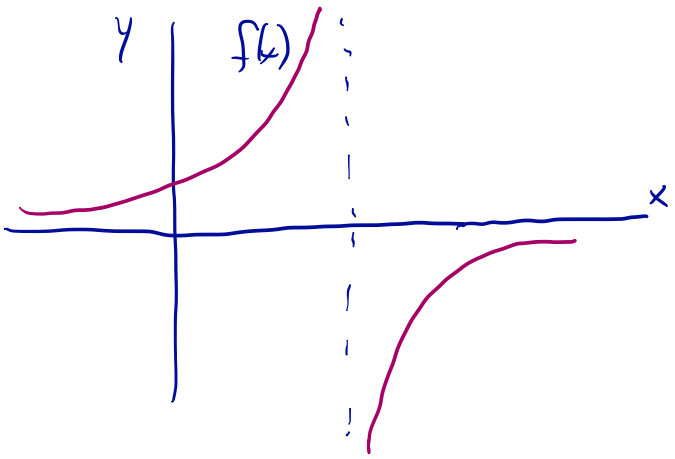
4. Determine

$$\lim_{x\to 3^+} \frac{1}{3-x}$$

and

$$\lim_{x\to 3^-} \frac{1}{3-x}.$$

A sketch of the graph might be helpful.



$f(x) = \frac{1}{3-x}$

$\lim_{x\to 3^+} f(x) = -\infty$

$\lim_{x\to 3^-} f(x) = \infty$

5. Determine exactly

$$\lim_{x\to 2} \frac{x^2-7x+10}{x-2}$$

$$\lim_{x\to 2} \frac{x^2-7x+10}{x-2} = \lim_{x\to 2} \frac{(x-2)(x-5)}{(x-2)}$$

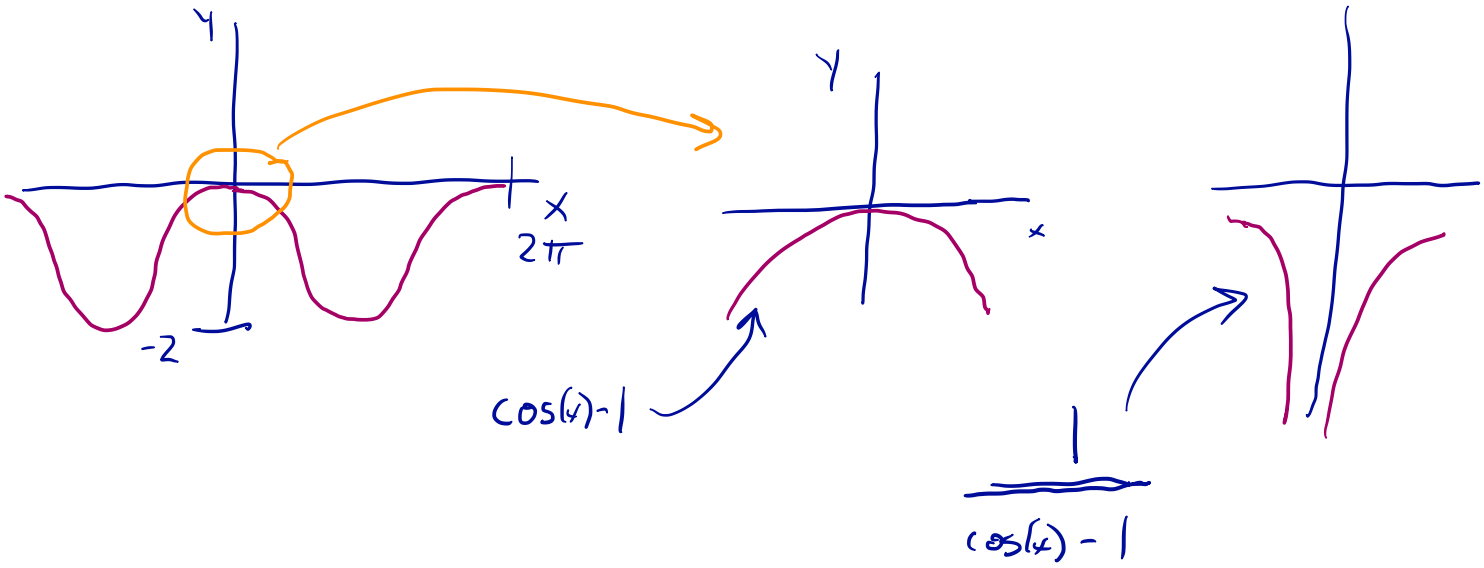
$$= \lim_{x\to 2} x-5$$

$$= -3$$

6. Determine if

$$\lim_{x\to 0} \frac{1}{\cos(x)-1}$$

exists. If not, determine if the left- and right-hand limits exist.



$$\lim_{x\to 0} \frac{1}{\cos(x)-1} = -\infty$$

7. Determine the left- and right-hand limits at 0 of $f(x) = x/|x|$.

$$f(x) = \frac{x}{|x|} = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \\ \text{undefined} & x = 0 \end{cases}$$

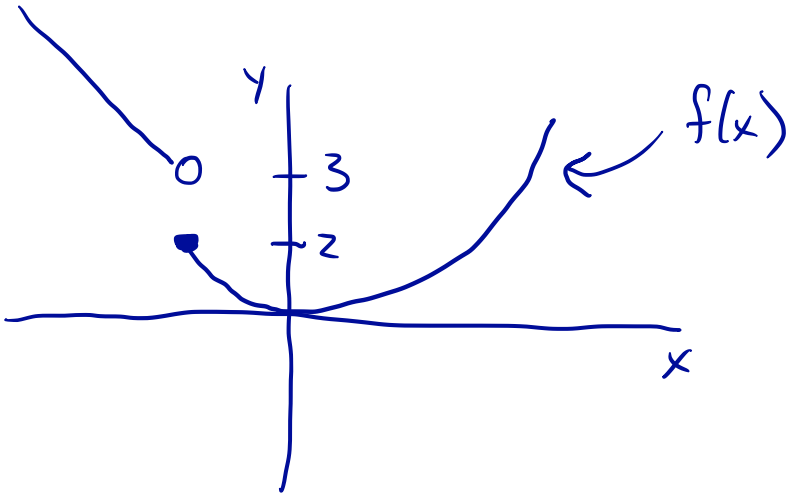
$$\lim_{x\to 0^+} f(x) = 1$$

$$\lim_{x\to 0^-} f(x) = -1$$

8. Suppose

$$g(x) = \begin{cases} x^2 + 1 & x \geq -1 \\ 2 - x & x < -1. \end{cases}$$

Sketch the graph. Then determine if $\lim_{x \to -1} g(x)$ exists. If not, determine if the left- and right-hand limits exist.



$\lim_{x \to -1^+} f(x) = (-1)^2 + 1 = 2$

$\lim_{x \to -1^-} f(x) = 2 - (-1) = 3$

9. Determine

$$\lim_{x \to 0^+} 10^{-\frac{1}{x}}$$

and

$$\lim_{x \to 0^-} 10^{-\frac{1}{x}}.$$

As $x \to 0^+$, $\frac{-1}{x} \to -\infty$ and $10^{-\frac{1}{x}} \to 0$

As $x \to 0^-$, $-\frac{1}{x} \to +\infty$ and $10^{-\frac{1}{x}} \to \infty$