1. A ball is tossed straight up into the air. It has a velocity at time $t = 0$ seconds of 5 meters per second. It undergoes a constant acceleration due to gravity of -9.8 meters per second per second, m/s^2. The height of the ball can be written in the form

$$h(t) = at + bt^2$$

where h is measured in meters, time is measured in seconds, and a and b are certain constants.

1. Determine the values for the constants.

$h'(t) = a + 2bt$ $\quad h'(0) = a$ $\quad a = 5\ m/s$

$h''(t) = 2b$ $\quad 2b = -9.8\ m/s^2 \to b = -4.9\ m/s^2$

2. What is the height of the ball at time $t = 0$? At $t = 1$?

$h(0) = a \cdot 0 + b0^2 = 0$

$h(1) = 5 \cdot 1 - 4.9 \cdot 1^2 = 0.1\ m$

3. At what times is the ball at height 0?

$h(t) = 0$ $\quad at + bt^2 = 0$ $\quad t(a+bt) = 0$ $\to$ $t = 0$ or $a + bt = 0 \to t = \frac{-a}{b} = \frac{5}{4.9} = 1.02s$

4. What is the average velocity of the ball over the time interval $[0.2, 0.21]$?

$$\frac{h(0.21) - h(0.2)}{0.01} = 2.991\ m/s$$

5. What is the average velocity of the ball over the time interval $[0.2, 0.201]$?

$$\frac{h(0.201) - h(0.2)}{0.001} = 3.0351\ m/s$$

6. What is the instantaneous velocity of the ball at time $t = 0.2$??

$h'(t) = 5 - 9.8t$; $\quad h'(0.2) = 3.04\ m/s$

7. At what time t is the ball motionless?

$h'(t) = 0$ when $5 - 9.8t = 0$ so $t = 0.5102s$

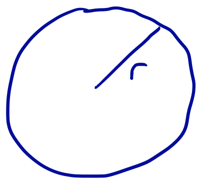
8. What is the velocity of the ball at time $t = 0$? At $t = 0.1$? At $t = 1$?

$h'(0) = 5\ m/s$

$h'(0.1) = 4.02\ m/s$

$h'(1) = -4.8\ m/s$

2. A stone is thrown in a pond and a circular ripple travels outward at a speed of 60 cm/s. Determine the rate of change of area inside the ripple at time $t = 1$ second and at time $t = 2$ seconds.



$r(t) = 60t$

Area: πr^2

$A(t) = \pi (60t)^2$

$A'(t) = \pi \cdot 2(60t) \cdot 60$

$A'(1) = 2\pi 60^2 = 22619 \text{ cm}^2/\text{s} = 2.26 \text{ m}^2/\text{s}$

$A'(2) = 2\pi 60^2 \cdot 2^2 = 90477 \text{ cm}^2/\text{s} = 9.04 \text{ m}^2/\text{s}$

3. A current is passsing through a wire. The amount of charge that has passed by a measuring point on the wire at time t is

$$Q(t) = te^{-t}$$

for $t > 0$. Here, the charge Q is measured in Coulombs (which is a count of the number of electrons) and time t is measured in seconds.

Determine the current in the wire at time $t = 0$ and $t = 2$ seconds. Current is measured in Coulombs per second, and one Coulomb per second is known as an Ampere (an amp).

$Q'(t) = e^{-t} - te^{-t}$

$= (1-t)e^{-t}$

$Q'(0) = 1$ Coulomb/second = 1 amp

$Q'(2) = -e^{-2} = -0.135$ amp

current is running in opposite direction

4. A population of bacteria starts at 500 cells and doubles every 30 minutes. Find a function $P(t)$ that describes this situation. Then compute the rate of change of the bacteria population at time $t = 60$ minutes.

$P(t) = 500 \cdot 2^{t/30}$

$P'(t) = 500 \cdot \frac{1}{30} \ln(2)\, 2^{t/30}$

$P'(60) = 500 \cdot \frac{1}{30} \ln(2)\, 2^{2} = 46$ bacteria/minute

5. A one-meter rod has non uniform mass. The mass of the rod from one end to distance x along it is

$$m(x) = x + \frac{1}{3}\sqrt{x}$$

where mass is measured in grams and x is in centimeters.

1. What is the total mass of the rod?

$m(100) = 103.3$ g

2. What is the mass of the first half of the rod? The second half?

$\underbrace{m(50) = 52.35\text{g}}_{\text{first half}}$ $\underbrace{m(100) - m(50)}_{\text{second half}} = 50.97$ g

3. What is the average density (in grams/centimeter) of the first half of the rod?

$\frac{52.35\text{g}}{50\text{ cm}} = 1.047$ g/cm

4. What is the density of the rod at $x = 30$ centimeters?

$m'(x) = 1 + \frac{1}{6\sqrt{x}}$ $m'(30) = 1.0304$ g/cm

6. A population of caribou is growing, and its population is

$$P(t) = 4000\frac{3e^{t/5}}{1+2e^{t/5}}.$$

t in years

1. What is the population at time $t = 0$?

$$P(0) = 4000\,\frac{3}{1+2} = 4000$$

2. Determine the rate of change of the population at any time t.

$$P'(t) = 4000\left[\tfrac{3}{5}e^{t/5}\left(1+2e^{t/5}\right) - 3e^{t/5}\left(2\cdot\tfrac{1}{5}e^{t/5}\right)\right]\left(1+2e^{t/5}\right)^{-2}$$

$$= 800e^{t/5}\left[3\left(1+2e^{t/5}\right) - 3\cdot 2e^{t/5}\right]\left(1+2e^{t/5}\right)^{-2}$$

$$= 2400\,e^{t/5}\left(1+2e^{t/5}\right)^{-2}$$

3. Determine the rate of change of the population at time $t = 0$ years.

$$P'(0) = \frac{2400}{3^2} = \frac{2400}{9} = 266 \text{ caribou/year}$$

4. Determine the long term population.

$$\lim_{t\to\infty} P(t) = \lim_{t\to\infty} 4000\,\frac{3e^{t/5}}{1+2e^{t/5}} = \lim_{t\to\infty} 4000\,\frac{3}{e^{-t/5}+2} = 6000 \text{ caribou}$$