1. I'm tired of doing all the work around here. It's your turn. You're going to show that

$$\frac{d}{dx}\ln(x) = \frac{1}{x}.$$

Start with the equation $y = \ln(x)$.

1. Solve this equation for x.

$$e^y = x$$

2. Take an implicit derivative with respect to x, and solve for dy/dx.

$$e^y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{e^y}$$

3. Now convert dy/dx into an expression that only involves x. (Tah dah!)

$$\frac{dy}{dx} = \frac{1}{x}$$

2. Compute $\dfrac{d}{dx}\ln\left(x + e^{3x}\right)$.

$$\frac{1}{x+e^{3x}} \cdot \left(1 + 3e^{3x}\right)$$

3. Compute $\dfrac{d}{dx}\ln(\cos(x))$ and simplify your expression.

$$\frac{1}{\cos(x)} \cdot \sin(x) = \tan(x)$$

4. How can we compute $\dfrac{d}{dx}5^x$?

1. Rewrite $5^x = e^{ax}$ for a certain constant a. Your job is to find a!

$$\ln(5^x) = \ln(e^{ax}) = ax$$

$$x\ln(5) = ax \Rightarrow a = \ln(5)$$

2. Now compute $\dfrac{d}{dx}5^x$ by taking the derivative of e^{ax} instead.

$$\frac{d}{dx}5^x = \frac{d}{dx}e^{\ln(5)x} = \ln(5)e^{\ln(5)x}$$

3. Rewrite your previous answer so that the letter e does not appear.

$$e^{\ln(5)x} = 5^x \text{ so } \frac{d}{dx}5^x = \ln(5)5^x$$

5. Derive a formula for $\dfrac{d}{dx}\log_5(x)$. You can either use a change of base formula, or you can repeat the technique used to find the derivative of $\ln(x)$. Heck, do it both ways.

$$\log_5(x) = \frac{\ln(x)}{\ln(5)} \Rightarrow \frac{d}{dx}\log_5(x) = \frac{1}{\ln(5)}\frac{d}{dx}\ln(x)$$

$$= \frac{1}{x\ln(5)}$$

$$y = \log_5(x)$$

$$5^y = x$$

$$\ln(5)5^y\frac{dy}{dx} = 1 \longrightarrow \frac{dy}{dx} = \frac{1}{\ln(5)5^y} = \frac{1}{x\ln(5)}$$

6. Suppose you wish to differentiate

$$f(x) = x^x.$$

The tool to use is called logarithmic differentiation.

Start with the equation $y = x^x$.

1. Apply the natural logarithm to both sides of the equation and simplify.

$$\ln(y) = x\ln(x)$$

2. Take an implicit derivative with respect to x, and solve for dy/dx.

$$\frac{1}{y}\frac{dy}{dx} = \ln(x) + 1$$

$$\frac{dy}{dx} = y\left(\ln(x)+1\right)$$

3. Now convert dy/dx into an expression that only involves x. (Tah dah!)

$$\frac{dy}{dx} = x^x\left(\ln(x)+1\right)$$

7. Differentiate $f(x) = x^{\sin(x)}$.

$$y = x^{\sin(x)}$$

$$\ln(y) = \sin(x)\ln(x)$$

$$\frac{1}{y}y' = \cos(x)\ln(x) + \sin(x)/x$$

$$y' = x^{\sin(x)}\left[\cos(x)\ln(x) + \frac{\sin(x)}{x}\right]$$

8. We wish, for whatever bizarre reason, to compute dy/dx if

$$y = \frac{(x^2+1)(x+3)^{1/2}}{x-1}.$$

One can use the product and quotient rules. But logarithmic differentiation can be a useful tool instead. known as logarithmic differentiation.

1. Take the natural logarithm of both sides of the equation.

$$\ln(y) = \ln\left(\frac{(x^2+1)(x+3)^{1/2}}{x-1}\right)$$

2. Use log rules such as $\ln(AB) = \ln(A) + \ln(B)$ to expand the right-hand side of this equation

$$\ln(y) = \ln(x^2+1) + \frac{1}{2}\ln(x+3) - \ln(x-1)$$

3. Compute (implicitly) dy/dx and solve for dy/dx.

$$\frac{dy}{dx} = y\left[\frac{2x}{x^2+1} + \frac{1}{2}\frac{1}{x+3} - \frac{1}{x-1}\right]$$

4. Convert the expression for dy/dx so that it only involves x, and there are no appearances of y.

$$\frac{dy}{dx} = \frac{(x^2+1)(x+3)^{1/2}}{x-1}\left[\frac{2x}{x^2+1} + \frac{1}{2}\frac{1}{x+3} - \frac{1}{x-1}\right]$$