In the first part of this worksheet we will get to know a method for computing an approximation of  $\sqrt{2}$  to many digits of accuracy using only addition, subtraction, multiplication and division, and indeed using only a few such operations.

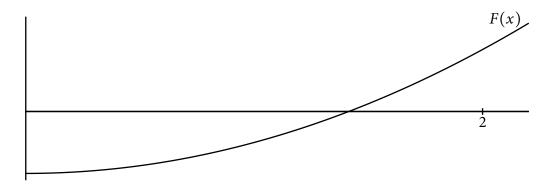
**1.** Consider the function

$$F(x) = x^2 - 2.$$

If we solve F(a) = 0 for some  $a \ge 0$ , what is the value of a?

**2.** Find the linearization L(x) of F(x) at x = 2. Leave your answer in point-slope form.

**3.** I've graphed F(x) for you below. Add to this diagram the graph of L(x).



**4.** Find the number  $x_1$  such that  $L(x_1) = 0$ .

- **5.** What good is the number  $x_1$ ? Keep in mind that you want to solve F(x) = 0. You solved L(x) = 0 instead.
- **6.** In the diagram above, label the point  $x_1$  on the x-axis.

7. Let's do it again! Find the linearization L(x) of F(x) at  $x = x_1$ .

- **8.** Add the graph of this new linearization to your diagram on the first page.
- **9.** Find the number  $x_2$  such that  $L(x_2) = 0$ . Then label the point  $x = x_2$  in the diagram.

- **10.** To how many digits does  $x_2$  agree with  $\sqrt{2}$
- 11. Let's be a little more systematic. Suppose we have an estimate  $x_k$  for  $\sqrt{2}$ .
  - Compute  $F(x_k)$ .
  - Compute  $F'(x_k)$ .
  - Compute the linearization of F(x) at  $x = x_k$ .

$$L(x) =$$

• Find the number  $x_{k+1}$  such that  $L(x_{k+1}) = 0$ . You should try to find as simple an expression as you can.

12. Starting with  $x_0 = 2$ , compute  $x_1$  and  $x_2$  with your shiny new formula. Verify that they agree with your earlier expressions for  $x_1$  and  $x_2$ .

**13.** Compute  $x_4$ . To how many digits does it agree with  $\sqrt{2}$ ?

## Newton's Method In General

We wish to solve F(x) = 0 for a differentiable function F(x). We have an initial estimate  $x_0$  for the solution.

**14.** Try to solve

$$e^{-x} - x = 0$$

by hand.

**15.** Explain why there is a solution between x = 0 and x = 1.

**16.** Starting with  $x_0 = 1$ , find an approximation of the solution of  $e^{-x} - x = 0$  to 6 decimal places. During your computation, keep track of each  $x_k$  to at least 10 decimal places of accuracy.