

1. Compute

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{1+2x^4}}{2-x^2}.$$

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{\sqrt{1+2x^4}}{2-x^2} &= \lim_{x \rightarrow -\infty} \frac{\sqrt{x^4} \sqrt{1/x^4 + 2}}{2-x^2} \\ &= \lim_{x \rightarrow -\infty} \frac{|x^2| \sqrt{1/x^4 + 2}}{2-x^2} \\ &= \lim_{x \rightarrow -\infty} \frac{\sqrt{1/x^4 + 2}}{\frac{2}{x^2} - 1} \\ &= \frac{\sqrt{0+2}}{0-1} = -\sqrt{2} \end{aligned}$$

2. Compute

$$\lim_{x \rightarrow \infty} \sqrt{9x^2+1} - 3x.$$

Hint: Multiply by $1 = \frac{\sqrt{9x^2+1} + 3x}{\sqrt{9x^2+1} + 3x}.$

$$\begin{aligned} \lim_{x \rightarrow \infty} \sqrt{9x^2+1} - 3x &= \lim_{x \rightarrow \infty} \frac{(\sqrt{9x^2+1} - 3x)(\sqrt{9x^2+1} + 3x)}{\sqrt{9x^2+1} + 3x} \\ &= \lim_{x \rightarrow \infty} \frac{1}{\sqrt{9x^2+1} + 3x} \\ &= \lim_{x \rightarrow \infty} \frac{1}{x} \left[\frac{1}{\sqrt{9+1/x} + 3} \right] \\ &= 0 \cdot \frac{1}{\sqrt{9+0} + 3} = 0 \end{aligned}$$

3. Compute

$$\lim_{x \rightarrow \infty} \frac{2 + e^x}{1 - e^x}.$$

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{2 + e^x}{1 - e^x} &= \lim_{x \rightarrow \infty} \frac{2 + e^x}{1 - e^x} \cdot \frac{e^{-x}}{e^{-x}} \\ &= \lim_{x \rightarrow \infty} \frac{2e^{-x} + 1}{e^{-x} - 1} \\ &= \frac{2 \cdot 0 + 1}{0 - 1} = -1 \end{aligned}$$

4. Compute

$$\lim_{x \rightarrow -\infty} \frac{2 + e^x}{1 - e^x}.$$

$$\lim_{x \rightarrow -\infty} \frac{2 + e^x}{1 - e^x} = \frac{2 + 0}{1 - 0} = 2$$

5. Compute

$$\lim_{x \rightarrow \infty} \ln(3+x) - \ln(1+x)$$

$$\begin{aligned} \lim_{x \rightarrow \infty} \ln(3+x) - \ln(1+x) &= \lim_{x \rightarrow \infty} \ln\left(\frac{3+x}{1+x}\right) \\ &= \ln\left(\lim_{x \rightarrow \infty} \frac{3+x}{1+x}\right) \\ &= \ln\left(\lim_{x \rightarrow \infty} \frac{3/x + 1}{1/x + 1}\right) \\ &= \ln\left(\frac{0+1}{0+1}\right) = \ln(1) = 0 \end{aligned}$$

6. Compute

$$\lim_{x \rightarrow \infty} \arctan(2^{-x})$$

Let $y = 2^{-x}$. Since $\lim_{x \rightarrow \infty} y = \lim_{x \rightarrow \infty} 2^{-x} = 0$,

$$\lim_{x \rightarrow \infty} \arctan(2^{-x}) = \lim_{y \rightarrow 0} \arctan(y) = \arctan(0) = 0.$$

7. Compute

$$\lim_{x \rightarrow \infty} \frac{x^3 - 12x + 1}{x^4 + 7}$$

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x^3 - 12x + 1}{x^4 + 7} &= \lim_{x \rightarrow \infty} \frac{x^{-1} - 12x^{-3} + x^{-4}}{1 + 7x^{-4}} \\ &= \frac{0 - 0 + 0}{1 + 0} = 0 \end{aligned}$$

8. Compute

$$\lim_{x \rightarrow \infty} \frac{x^4 + 7}{x^3 - 12x + 1}$$

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x^4 + 7}{x^3 - 12x + 1} &= \lim_{x \rightarrow \infty} x \left[\frac{1 + 7x^{-3}}{1 - 12x^{-2} + x^{-3}} \right] \\ &= \infty \cdot \frac{1}{1} = \infty \end{aligned}$$