1. Compute $\int \frac{\sec^2(x)}{\tan(x)}\,dx$ $= \int \frac{1}{u}\,du = \ln(|u|) + C$

$= \ln(|\tan(x)|) + C$

$u = \tan(x)$

$du = \sec^2(x)\,dx$

2. Compute $\int \sec^2(x)\tan(x)\,dx$ $= \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}\sec^2(x) + C$

$u = \sec(x)$

$du = \sec(x)\tan(x)\,dx$

Note: $\tan^2(x) = \sec^2(x) + 1$

Alt: $u = \tan(x)$

$du = \sec^2(x)\,dx$

$\int \sec^2(x)\tan(x)\,dx = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}\tan^2(x) + C$

3. Compute $\int \frac{\sin(\theta)}{1+\cos(\theta)}\,d\theta$

$u = 1 + \cos(x)$

$du = -\sin(x)\,dx$

$\int \frac{\sin(\theta)}{1+\cos\theta}\,d\theta = \int \frac{-du}{u} = -\ln(|u|) + C$

$= -\ln(|1+\cos\theta|) + C$

$= -\ln(1+\cos\theta) + C$

$= \ln\left(\frac{1}{1+\cos\theta}\right) + C$

4. Compute $\int \frac{1}{x\ln(x)}\,dx$ $= \int \frac{1}{u}\,du = \ln(|u|) + C$

$u = \ln(x)$ $= \ln(|\ln(x)|) + C$

$du = \frac{1}{x}\,dx$

5. Compute $\int \frac{\sin(4/x)}{x^2}\,dx$ $= \int -\frac{1}{4}\sin(u)\,du = \frac{1}{4}\cos(u) + C$

$u = 4/x$ $= \frac{1}{4}\cos(4/x) + C$

$du = -4/x^2\,dx$

6. Compute $\int \frac{e^x}{e^x - 3}\,dx$ $= \int \frac{1}{u}\,du = \ln(|u|) + C$

$u = e^x - 3$ $= \ln(|e^x - 3|) + C$

$du = e^x\,dx$

7. Compute $\int \frac{1}{9+x^2}\,dx$ $= \int \frac{\frac{1}{9}}{1+(\frac{x}{3})^2}\,dx = \frac{1}{3}\int \frac{1}{1+u^2}\,du$

$u = \frac{x}{3}$

$du = \frac{1}{3}dx$

$= \frac{1}{3}\arctan(u) + C$

$= \frac{1}{3}\arctan(\frac{x}{3}) + C$

8. Compute $\int \sqrt{x}(x^4+x)\,dx$ $= \int x^{9/2} + x^{3/2}\,dx$

$= \frac{2}{11}x^{11/2} + \frac{2}{5}x^{5/2} + C$

9. Compute $\int \cos(x)\sin(\sin(x))\,dx$ $= \int \sin(u)\,du = -\cos(u) + C$

$u = \sin(x)$

$du = \cos(x)dx$

$= -\cos(\sin(x)) + C$

10. Compute $\frac{d}{dx}[x\ln(x) - x]$. Then compute $\int s^2 \ln(s^3)\, ds$

$\frac{d}{dx}[x\ln(x) - x] = \ln(x) + x\cdot\frac{1}{x} - 1 = \ln(x)$

i.e. $\int \ln(x)\,dx = x\ln(x) - x$

$u = s^3$
$du = 3s^2\,ds$

$\int s^2\ln(s^3)\,ds = \frac{1}{3}\int \ln(u)\,du = \frac{1}{3}[u\ln(u) - u] + C$

$= \frac{1}{3}[s^3\ln(s^3) - s^3] + C$

11. Compute $\int x\sqrt{x-1}\,dx$

$u = x - 1 \Rightarrow x = u + 1$

$du = dx$

$\int x\sqrt{x-1}\,dx = \int (u+1)\sqrt{u}\,du = \int u^{3/2} + u^{1/2}\,du$

$= \frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C$

$= \frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C$

12. Compute $\int_1^3 \frac{(\ln(x))^3}{x}\,dx$

$u = \ln(x)$

$du = \frac{1}{x}\,dx$

$x = 1 \Rightarrow u = 0$

$x = 3 \Rightarrow u = \ln(3)$

$\int_1^3 \frac{(\ln(x))^3}{x}\,dx = \int_0^{\ln(3)} u^3\,du = \frac{u^4}{4}\Big|_0^{\ln(3)} = \frac{(\ln(3))^4}{4}$