1. Compute the linearization of $f(x) = 1/x$ at $x = 2$.

$f(2) = \frac{1}{2}$

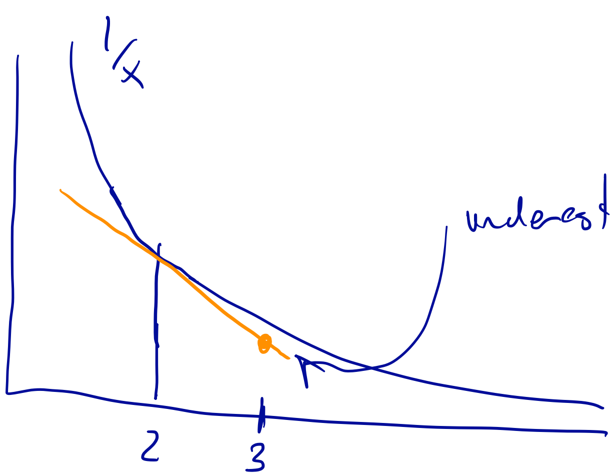
$f'(2) = \frac{-1}{4}$

$L(x) = \frac{1}{2} - \frac{1}{4}(x-2)$

2. Use your linearization to estimate $1/3$.

$\frac{1}{3} = f(3) \approx L(3) = \frac{1}{2} - \frac{1}{4}(3-2) = \frac{1}{4}$

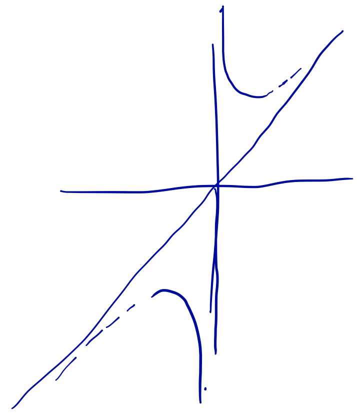
3. Draw a graph that illustrates the computation you just did. Then use the graph to determine if your estimate for $1/3$ is an underestimate or an overestimate.



underestimate

The problems on this page refer to the function $f(x) = \frac{1}{x} + x$.

4. On what intervals is the function increasing? Decreasing?



$$f'(x) = -\frac{1}{x^2} + 1 = \frac{x^2 - 1}{x^2}$$

$+ \quad - \quad - \quad +$

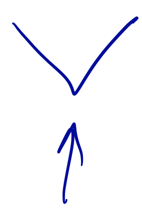
inc $\quad -1 \quad$ dec $\quad 0 \quad$ dec $\quad 1 \quad$ inc

5. Find the critical points of $f(x)$.

$x = \pm 1$

6. Use the first derivative test to classify the only positive critical point as a local min/max/neither.

$- \quad 1 \quad +$



local min

7. Use the second derivative test to classify the only positive critical point as a local min/max if this is possible

$$f''(x) = \frac{2}{x^3}$$

$$f''(1) = 2$$ local min

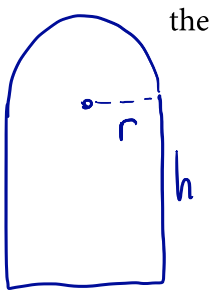
8. A circular metal plate is being heated in an oven. The radius of the plate is increasing at a rate of 0.01 cm/min when the radius is 50cm. How fast is the area of the plate increasing?

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$= 2\pi \cdot 50 \cdot \frac{1}{100} = \pi \text{ cm}^2/\text{min}$$

9. A Norman window is has a rectangular base and a semi-circle on top. What dimensions of the window minimize the perimeter if the area of the window is to be 4 ft².



$$A = 2rh + \frac{\pi r^2}{2} \qquad A = 4 \Rightarrow h = \frac{4 - \frac{\pi r^2}{2}}{2r} = \frac{8 - \pi r^2}{4r}$$

$$P = 2h + 2r + \pi r$$

$$P = \frac{8 - \pi r^2}{2r} + 2r + \pi r$$

$$= \frac{4}{r} + \frac{\pi}{2} r + 2r$$

$$P' = -\frac{4}{r^2} + \left(2 + \frac{\pi}{2}\right)$$

$$P' = 0 \Rightarrow r = \frac{4}{2 + \pi/2} = \frac{8}{2 + \pi}$$

$$P'' = \frac{8}{r^3} > 0$$

so a global min at

$$r = \frac{8}{2+\pi}$$

10. The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$ where r is the radius of the base of the cone and h is the height of the cone. Use a differential to estimate the change in volume of the cone if the height is fixed at 9 feet and the radius changes from 5 feet to 5.5 feet.

$$dV = \frac{2}{3}\pi h r \, dr$$

$$dV = \frac{2}{3}\pi \cdot 9 \cdot 5 \cdot \frac{1}{2} = 15\pi$$

11. Compute $\lim_{x\to 0} \frac{\sec(x) - 1}{x^2}$

$$\lim_{x\to 0} \frac{\sec(x)-1}{x^2} \overset{\frac{0}{0}}{=} \lim_{x\to 0} \frac{\sec(x)\tan(x)}{2x}$$

$$\overset{\frac{0}{0}}{=} \lim_{x\to 0} \frac{\sec(x)\tan^2(x) + \sec^3(x)}{2}$$

$$= \frac{0+1}{2}$$

$$= \frac{1}{2}$$

12. Consider the curve defined implicitly by

$$x^4 + y^4 = 2.$$

a. Show that the point $(1, 1)$ lies on this curve.

$1^4 + 1^4 = 2$

b. Find the slope of the tangent line to the curve at this point.

$4x^3 + 4y^3 y' = 0$

$y' = -\frac{4x^3}{4y^3} = -\frac{x^3}{y^3}$

at $(1,1)$ $y' = -\frac{1}{1} = -1$