1. Compute $\int_2^4 t^3\, dt$.

$$\int_2^4 t^3 dt = \frac{t^4}{4}\Big|_2^4 = \frac{4^4}{4} - \frac{2^4}{4}$$

$$= 4^3 - 4 = 60$$

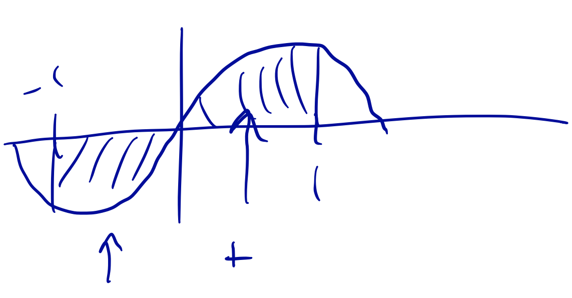
2. Compute $\int_2^4 e^{-t}\, dt$.

$$\int_2^4 e^{-t} dt = -e^{-t}\Big|_2^4 = -e^{-4} - (-e^{-2}) = e^{-2} - e^{-4}$$

3. Compute $\int_0^1 \frac{1}{1+s^2}\, ds$

$$\int_0^1 \frac{1}{1+s^2} ds = \arctan(s)\Big|_0^1 = \arctan(1) = \frac{\pi}{4}$$

4. Compute $\int_{-1}^1 \sin(x)\, dx$. Then give a geometric answer to justify your result.



net is 0

$$\int_{-1}^1 \sin(x) dx = -\cos(x)\Big|_{-1}^1$$

$$= -\cos(1) - (-\cos(-1))$$

$$= -\cos(1) + \cos(-1)$$

$$= -\cos(1) + \cos(1)$$

$$= 0$$

5. Compute $\int_0^{\frac{\pi}{2}} \cos(5x)\,dx$. You'll need to play around to find an antiderivative.

$$\int_0^{\pi/2} \cos(5x)\,dx = \frac{1}{5}\sin(5x)\Big|_0^{\pi/2} = \frac{1}{5}\left[\sin\left(\frac{5\pi}{2}\right) - \sin(0)\right]$$

$$= \frac{1}{5}\sin\left(\frac{5\pi}{2}\right) = \frac{1}{5}\sin\left(\pi/2\right) = \frac{1}{5}$$

6. Compute $\int_1^2 \frac{t^3 - 3t^2}{t^4}\,dt$

$$\int_1^2 \frac{t^3-3t^2}{t^4}\,dt = \int_1^2 \frac{1}{t} - \frac{3}{t^2}\,dt = \ln(|t|) + \frac{3}{t}\Big|_1^2$$

$$= \ln(2) - \ln(1) + \frac{3}{2} - 3$$

$$= \ln(2) - \frac{3}{2}$$

7. Can the Fundamental Theorem of Calculus help you compute $\int_0^{\pi} \tan(x)\,dx$?

No. The integrand is not cts on $[0,\pi]$

8. ~~Can the Fundamental Theorem of Calculus help you compute $\int_0^{\pi} \tan(x)\,dx$?~~

Compute $\frac{d}{dx}\arctan(x^3)$.

$$\frac{d}{dx}\arctan(x^3) = \frac{1}{1+(x^3)^2}\cdot 3x^2$$

9. Compute

$$\frac{d}{dx}\int_5^x \tan(\sqrt{s})\,ds$$

$$\frac{d}{dx}\int_5^x \tan(\sqrt{s})\,ds = \tan(\sqrt{x})$$

10. Compute

$$\frac{d}{dx}\int_5^{x^3} \tan(\sqrt{s})\,ds.$$

Hint: Let $H(x) = \int_5^x \tan(\sqrt{s})\,ds$. You're interested in $H(x^3)$. Apply the Chain Rule!

$$\frac{d}{dx}\int_5^{x^3} \tan(\sqrt{s})\,ds = \tan(\sqrt{x^3})\cdot 3x^2$$

11. Challenge! Compute

$$\frac{d}{dx}\int_x^{x+1} \sqrt{s^2+1}\,ds.$$

$$\int_x^{x+1}\sqrt{s^2+1}\,ds = \int_0^{x+1}\sqrt{s^2+1}\,ds + \int_x^0\sqrt{s^2+1}\,ds$$

$$= \int_0^{x+1}\sqrt{s^2+1} - \int_0^x\sqrt{s^2+1}\,ds$$

$$\frac{d}{dx}\int_x^{x+1}\sqrt{s^2+1}\,ds = \sqrt{(x+1)^2+1} - \sqrt{x^2+1}$$