

1. The cost of building wooden pencils is given by a function  $C(n)$  where  $C$  is the cost in dollars and  $n$  is the number of pencils, measured in thousands. Explain what  $C'(50) = 37.5$  means in language your parents could understand.

Once 50000 pencils have been constructed,  
the cost of making additional pencils increases at  
a rate of 37.5 dollars per 1000 pencils.

Compute the derivatives of the following functions.

2.  $f(x) = \sqrt{1+x^2}$

$$\frac{d}{dx} \sqrt{1+x^2} = \frac{1}{2\sqrt{1+x^2}} \cdot 2x = \frac{x}{\sqrt{1+x^2}}$$

3.  $f(\theta) = \tan(4\theta + 9)$

$$\begin{aligned} \frac{d}{d\theta} \tan(4\theta + 9) &= \sec^2(4\theta + 9) \cdot \frac{d}{d\theta} (4\theta + 9) \\ &= 4 \sec^2(4\theta + 9) \end{aligned}$$

4.  $f(t) = e^{t^2}(1 + \cos(t))$

$$\begin{aligned}\frac{d}{dt} [e^{t^2}(1 + \cos(t))] &= \left( \frac{d}{dt} e^{t^2} \right) (1 + \cos t) + e^{t^2} \frac{d}{dt} (1 + \cos t) \\&= \left( e^{t^2} \frac{d}{dt} t^2 \right) (1 + \cos t) - e^{t^2} \sin t \\&= e^{t^2} [2t(1 + \cos t) - \sin t]\end{aligned}$$

5.  $f(v) = \sec\left(\frac{1}{1+v^2}\right)$

$$\begin{aligned}f'(v) &= \sec\left(\frac{1}{1+v^2}\right) \tan\left(\frac{1}{1+v^2}\right) \cdot \frac{d}{dv} \left( \frac{1}{1+v^2} \right) \\&= \sec\left(\frac{1}{1+v^2}\right) \tan\left(\frac{1}{1+v^2}\right) \frac{-2v}{(1+v^2)^2} \\&= -2 \sec\left(\frac{1}{1+v^2}\right) \tan\left(\frac{1}{1+v^2}\right) \frac{v}{(1+v^2)^2}\end{aligned}$$

6.  $f(x) = \cos(x^{1/3}e^x)$

$$\begin{aligned}\frac{d}{dx} \cos(x^{1/3}e^x) &= -\sin(x^{1/3}e^x) \frac{d}{dx} x^{1/3}e^x \\ &= -\sin(x^{1/3}e^x) \left[ \frac{1}{3} x^{-2/3} e^x + x^{1/3} e^x \right] \\ &= -x^{-2/3} e^x \sin(x^{1/3}e^x) \left[ \frac{1}{3} + x \right]\end{aligned}$$

7.  $f(x) = \sqrt{x + e^{x^2}}$

$$\begin{aligned}f'(x) &= \frac{1}{2\sqrt{x+e^{x^2}}} \cdot \frac{d}{dx} (x + e^{x^2}) \\ &= \frac{1}{2\sqrt{x+e^{x^2}}} \cdot (1 + 2xe^{x^2})\end{aligned}$$