Compute derivatives of the following functions using derivative rules.

1. $f(x) = (x-2)(2x+3)$

$f(x) = 2x^2 - x - 6$

$f'(x) = 4x - 1$

2. $f(t) = \sqrt{t} - e^t$

$f'(t) = \frac{1}{2}t^{-1/2} - e^t$

3. $f(x) = \dfrac{x^2 + x - 1}{\sqrt{x}}$

$f(x) = x^{3/2} + x^{1/2} - x^{-1/2}$

$f'(x) = \frac{3}{2}x^{1/2} + \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{-3/2}$

4. $V(r) = \frac{4}{3}\pi r^3$

$V'(r) = \frac{4\pi}{3} 3r^2 = 4\pi r^2$

5. $f(x) = e^{x-3}$

$f(x) = e^x e^{-3}$

$f'(x) = e^x e^{-3} = e^{x-3}$

6. Use the definition of the derivative to show $\frac{d}{dx}x^3 = 3x^2$.

$$\lim_{h\to 0}\frac{(x+h)^3 - x^3}{h} = \lim_{h\to 0}\frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$

$$= \lim_{h\to 0}\frac{(3x^2 + 3xh + h^2)h}{h}$$

$$= \lim_{h\to 0} 3x^2 + 3xh + h^2 = 3x^2 + 0 + 0 = 3x^2$$

7. Use the definition of the derivative to show $\frac{d}{dx}x^{-1} = (-1)x^{-2}$.

$$\frac{d}{dx}x^{-1} = \lim_{h\to 0}\frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h\to 0}\frac{\frac{x-(x+h)}{(x+h)x}}{h}$$

$$= \lim_{h\to 0}\frac{-h}{x(x+h)h} = \lim_{h\to 0}\frac{-1}{x(x+h)} = \frac{-1}{x^2}$$

8. Estimate $f'(0)$ to three decimal digits if $f(x) = 3^x$

$$f'(0) = \lim_{h\to 0}\frac{3^h - 3^0}{h} = \lim_{h\to 0}\frac{3^h - 1}{h}$$

h	$(3^h - 1)/h$
0.01	1.1047
0.001	1.09921
0.0001	1.09867
0.00001	$\underbrace{1.098}_{\text{trusted}}622$