

1. A rocket is launching, and its height h in meters is a function of t in seconds (so we are considering the function $h(t)$). Explain what $h'(10) = 1035$ means in language your parents could understand. Your answer must include units.

The rocket is rising at a rate of 1035 m/s at time $t = 10$ seconds.

Compute derivatives of the following functions using derivative rules.

2. $f(t) = \sqrt{t}e^t$

$$f'(t) = \frac{1}{2}t^{-1/2}e^t + \sqrt{t}e^t$$

3. $f(t) = e^{-t}$

$$\frac{d}{dt} e^{-t} = \frac{d}{dt} \frac{1}{e^t} = -\frac{\frac{d}{dt} e^t}{(e^t)^2} = \frac{-e^t}{e^{2t}} = -e^{-t}$$

4. $f(t) = e^{2t}$

$$\begin{aligned}\frac{d}{dt} e^{2t} &= \frac{d}{dt} (e^t e^t) = \left(\frac{d}{dt} e^t \right) e^t + e^t \frac{d}{dt} e^t \\ &= e^t e^t + e^t e^t \\ &= 2e^{2t}\end{aligned}$$

5. $f(v) = \left(1 + \frac{1}{v}\right) \left(2 - \frac{1}{v}\right)$

$$f(v) = 2 + \frac{1}{v} - \frac{1}{v^2}$$

$$f'(v) = -\frac{1}{v^2} + \frac{2}{v^3}$$

6. $f(x) = \frac{e^{2x}}{1 - e^x}$

$$\begin{aligned}f'(x) &= \frac{\left(\frac{d}{dx} e^{2x} \right) (1 - e^x) + e^{2x} \frac{d}{dx} (1 - e^x)}{(1 - e^x)^2} \\ &= \frac{2e^{2x} (1 - e^x) + e^{2x} (-e^x)}{(1 - e^x)^2} \\ &= \frac{2e^{2x} - 3e^{3x}}{(1 - e^x)^2}\end{aligned}$$

7. $f(x) = \frac{\sin(x)}{\cos(x)}$

$$\begin{aligned} f'(x) &= \frac{d}{dx} \frac{\sin(x)}{\cos(x)} = \frac{\left(\frac{d}{dx} \sin(x) \right) \cos(x) - \sin(x) \frac{d}{dx} \cos(x)}{\cos^2(x)} \\ &= \frac{\cos^2(x) - \sin(x)(-\sin(x))}{\cos^2(x)} \\ &= \frac{1}{\cos^2(x)} = \sec^2(x) \end{aligned}$$

8. $f(x) = e^{2x} \sin(x)$

$$f'(x) = 2e^{2x} \sin(x) + e^{2x} \cos(x)$$

9. $f(x) = (1+x^2)e^x \sin(x)$

$$f'(x) = 2x e^x \sin(x) + (1+x^2)e^x \sin(x) + (1+x^2)e^x \cos(x)$$