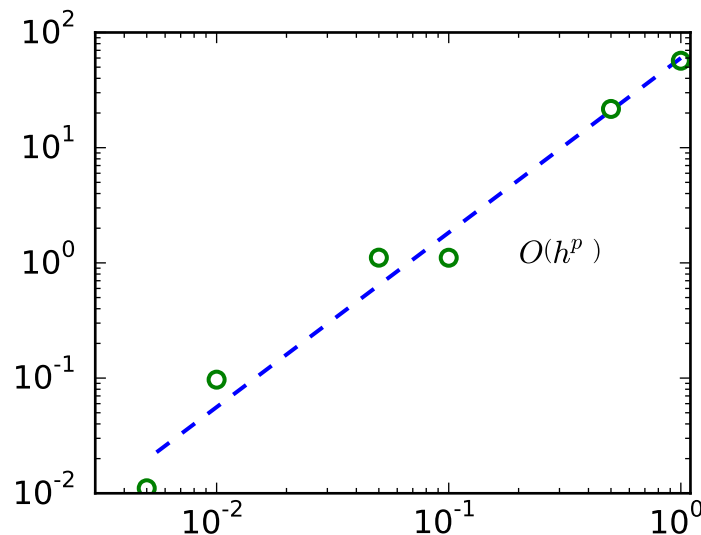


1. Suppose this table of “data” is samples of an $O(h^p)$ function:

h	1.0	0.5	0.1	0.05	0.01	0.005
Z	56.859	21.694	1.1081	1.1101	0.096909	0.011051

This data may be fitted (linear regression) by a function $f(h) = Mh^p$ for some values M and p , as in the following figure. Find p by fitting a straight line to the data, and reproduce the figure. Your version of the figure should have the value of p filled in.



2. Use Taylor's Theorem to verify the truncation term for the “Centered” row of Table 1.1 of your text. Hint: center all Taylor expansions at the same point.

Substantial partial credit will be awarded for showing the truncation term is $O(h^2)$, but try to get the exact expression with its constant. Hint: The average of two numbers lies in between the two numbers.

3. Implement the following schemes for a scalar ODE:

1. Forward Euler
2. Backwards Euler
3. Trapezoidal

Each method should be implemented with a function that takes the following arguments:

1. The right-hand side function $f(t, u)$.
2. The initial time t_0 .
3. The initial value u_0 .
4. The final time T .

5. The number M of time steps.

It should return a vector of sample times t_k , and a vector of solution values u_k .

Test your methods against $u' = -u$ and $u' = -\sin(t)$ with initial condition $u(0) = 1$ and confirm (using the technique of problem 1) that the order of convergence is the theoretically expected order for each method.

4. Consider the linear multistep method

$$u_{n+2} + 4u_{n+1} - 5u_n = h(4f_{n+1} + 2f_n)$$

where $f_k = f(t_k, u_k)$.

- Show that this method is consistent.
- In the case $f = 0$, the method reduces to a linear recurrence relation

$$u_{n+2} + 4u_{n+1} - 5u_n = 0.$$

The characteristic polynomial of this relation is $\sigma(\rho) = \rho^2 + 4\rho - 5$. Show that if ρ is a root of the characteristic polynomial, then $u_n = C\rho^n$ is a solution of the recurrence relation for any constant C . Moreover, if ρ_1 and ρ_2 are roots of the characteristic polynomial, then $u_n = C_1\rho_1^n + C_2\rho_2^n$ is a solution of the recurrence relation for any constants C_1 and C_2 .

- Compute the roots of the characteristic polynomial.
- Implement this method (using Euler's method to compute u_1) and apply it to the IVP

$$\begin{aligned} u' &= -u \\ u(0) &= 1 \end{aligned}$$

on the t -interval $[0, 1]$ with $M = 10, 50$ and 100 .

- Compute the global error in each of these three cases. Why is the error growing? Can you give an rough explanation for the rate of growth you observed?