1. Compute

$$\lim_{x\to-\infty} \frac{\sqrt{1+2x^4}}{2-x^2}.$$

$$\lim_{x\to-\infty} \frac{\sqrt{1+2x^4}}{2-x^2} = \lim_{x\to-\infty} \frac{\sqrt{x^4}\sqrt{1/x^4+2}}{2-x^2}$$

$$= \lim_{x\to-\infty} \frac{|x^2|\sqrt{1/x^4+2}}{2-x^2}$$

$$= \lim_{x\to-\infty} \frac{\sqrt{1/x^4+2}}{\frac{2}{x^2}-1}$$

$$= \frac{\sqrt{0+2}}{0-1} = \sqrt{2}$$

2. Compute

$$\lim_{x\to\infty} \sqrt{9x^2+1}-3x.$$

Hint: Multiply by $1 = \dfrac{\sqrt{9x^2+1}+3x}{\sqrt{9x^2+1}+3x}$.

$$\lim_{x\to\infty} \sqrt{9x^2+1}-3x = \lim_{x\to\infty} \frac{(\sqrt{9x^2+1}-3x)}{1}\frac{(\sqrt{9x^2+1}+3x)}{\sqrt{9x^2+1}+3x}$$

$$= \lim_{x\to\infty} \frac{1}{\sqrt{9x^2+1}+3x}$$

$$= \lim_{x\to\infty} \frac{1}{x}\left[\frac{1}{\sqrt{9+1/x}+3}\right]$$

$$= 0\cdot\frac{1}{\sqrt{9+0}+3} = 0$$

3. Compute
$$\lim_{x\to\infty} \frac{2+e^x}{1-e^x}.$$

$$\lim_{x\to\infty} \frac{2+e^x}{1-e^x} = \lim_{x\to\infty} \frac{2+e^x}{1-e^x} \cdot \frac{e^{-x}}{e^{-x}}$$

$$= \lim_{x\to\infty} \frac{2e^{-x}+1}{e^{-x}-1}$$

$$= \frac{2\cdot 0+1}{0-1} = -1$$

4. Compute
$$\lim_{x\to-\infty} \frac{2+e^x}{1-e^x}.$$

$$\lim_{x\to-\infty} \frac{2+e^x}{1-e^x} = \frac{2+0}{1-0} = 2$$

5. Compute
$$\lim_{x\to\infty} \ln(3+x) - \ln(1+x)$$

$$\lim_{x\to\infty} \ln(3+x) - \ln(1+x) = \lim_{x\to\infty} \ln\left(\frac{3+x}{1+x}\right)$$

$$= \ln\left(\lim_{x\to\infty} \frac{3+x}{1+x}\right)$$

$$= \ln\left(\lim_{x\to\infty} \frac{3/x+1}{1/x+1}\right)$$

$$= \ln\left(\frac{0+1}{0+1}\right) = \ln(1) = 0$$

6. Compute
$$\lim_{x\to\infty} \arctan(2^{-x})$$

Let $y = 2^{-x}$. Since $\lim_{x\to\infty} y = \lim_{x\to\infty} 2^{-x} = 0$,

$$\lim_{x\to\infty} \arctan(2^{-x}) = \lim_{y\to 0} \arctan(y) = \arctan(0) = 0.$$

7. Compute

$$\lim_{x\to\infty} \frac{x^3 - 12x + 1}{x^4 + 7}$$

$$\lim_{x\to\infty} \frac{x^3 - 12x + 1}{x^4 + 7} = \lim_{x\to\infty} \frac{x^{-1} - 12x^{-3} + x^{-4}}{1 + 7x^{-4}}$$

$$= \frac{0 - 0 + 0}{1 + 0} = 0$$

8. Compute

$$\lim_{x\to\infty} \frac{x^4 + 7}{x^3 - 12x + 1}$$

$$\lim_{x\to\infty} \frac{x^4 + 7}{x^3 - 12x + 1} = \lim_{x\to\infty} x \left[\frac{1 + 7x^{-3}}{1 - 12x^{-2} + x^{-3}} \right]$$

$$= \infty \cdot \frac{1}{1} = \infty$$