Compute derivatives of the following functions using derivative rules.

1. $f(x) = (x-2)(2x+3)$ $= 2x^2 - x - 6$

$\frac{d}{dx}\left(2x^2 - x - 6\right) = 4x - 1$

2. $f(t) = \sqrt{t} - e^t$

$\frac{d}{dx} \sqrt{t} - e^t = \frac{1}{2} t^{-\frac{1}{2}} - e^t$

3. $f(x) = \dfrac{x^2 + x - 1}{\sqrt{x}}$ $= x^{\frac{3}{2}} + x^{1/2} - x^{-1/2}$

$f'(x) = \frac{3}{2} x^{1/2} + \frac{1}{2} x^{-\frac{1}{2}} + \frac{1}{2} x^{-3/2}$

4. $V(r) = \dfrac{4}{3}\pi r^3$

$V'(r) = \frac{4}{3}\pi\, 3r^2 = 4\pi r^2$

5. $f(x) = e^{x-3}$ $= e^x e^{-3}$

$f'(x) = e^{-3} e^x = e^{x-3}$

6. Use the definition of the derivative to show $\frac{d}{dx}x^3 = 3x^2$.

$$\frac{d}{dx}x^3 = \lim_{h\to 0}\frac{(x+h)^3 - x^3}{h} = \lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$$

$$= \lim_{h\to 0}\frac{3x^2h+3xh^2+h^3}{h}$$

$$= \lim_{h\to 0} 3x^2+3xh+h^2$$

$$= 3x^2$$

7. Use the definition of the derivative to show $\frac{d}{dx}x^{-1} = (-1)x^{-2}$.

$$\frac{d}{dx}x^{-1} = \lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h} = \lim_{h\to 0}\frac{\frac{x-(x+h)}{x(x+h)}}{h}$$

$$= \lim_{h\to 0}\frac{-h}{h\,x(x+h)}$$

$$= \lim_{h\to 0}\frac{-1}{x(x+h)} = \frac{-1}{x^2}$$

8. Estimate $f'(0)$ to three decimal digits if $f(x) = 3^x$

$$f'(0) = \lim_{h\to 0}\frac{3^h-1}{h}$$

h	$\frac{3^h-1}{h}$
0.1	1.16 ....
0.01	1.1046 .....
0.001	1.0992 ....
0.0001	1.0986 .....
0.00001	1.0986 .....

$\hookrightarrow f'(0) \simeq 1.0986$