Math F251: Section 4.3 Worksheet October 29, 2018

First Derivative Test
Suppose f is a function with a derivative on (a, b), and if c is a point in the interval with $f'(c) = 0$.

- If $f'(x) > 0$ for x just to the left of c and $f'(x) < 0$ for x just to the right of c, then f has a local maximum at c.
- If $f'(x) < 0$ for x just to the left of c and $f'(x) > 0$ for x just to the right of c, then f has a local minimum at c.
- If $f'(c) = 0$ and $f'(x) < 0$ on both sides of c or $f'(x) > 0$ on both sides of c, then there is neither a local min nor a local max at c.

Second Derivative Test
Suppose f is a function with a continuous second derivative on (a, b), and that c is a point in the interval with $f'(c) = 0$.

- If $f''(c) > 0$ then f has a local minimum at c.
- If $f''(c) < 0$ then f has a local maximum at c.

Concave Up: $f'(x)$ increasing; $f''(x) > 0$

Concave Down: $f'(x)$ decreasing; $f''(x) < 0$

Point of Inflection: Value x where concavity changes; often $f''(x) = 0$

This worksheet considers the function

$$g(x) = x^2 e^x$$

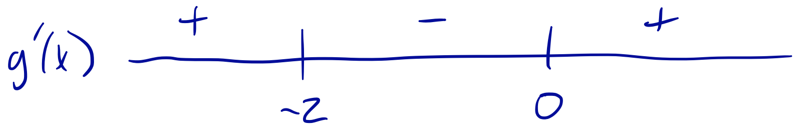
1. Find all critical points of g.

$g'(x) = 2xe^x + x^2e^x = (2x+x^2)e^x$

$x = 0$

$x = -2$

2. Determine the intervals where g is increasing and where g is decreasing.



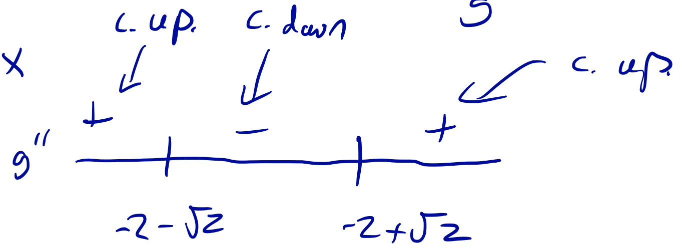
(same as for $(2x+x^2)$ since $e^x > 0$). So increasing on $(-\infty, -2)$ and on $(0, \infty)$

Dec. on $(-2, 0)$.

3. Determine the intervals where g is concave up and where ~~f~~ g is concave down.

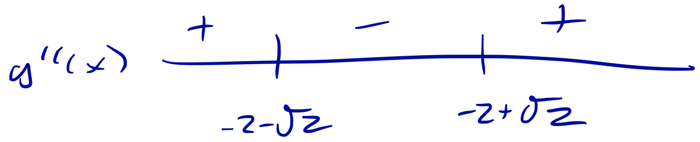
$g''(x) = (2+4x+x^2)e^x$

$g'(x) = 0$ at $-2 \pm \sqrt{2}$



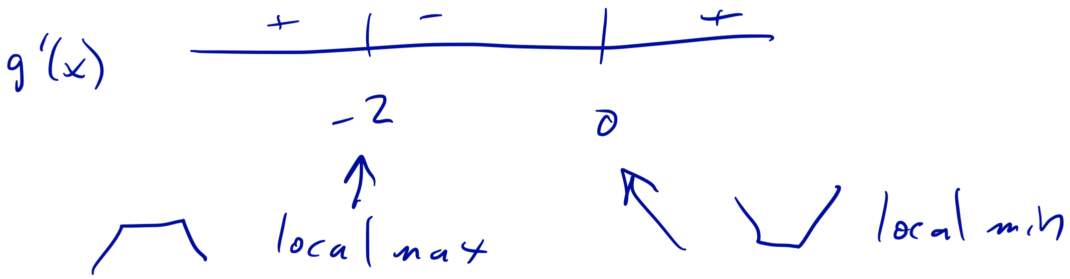
4. Find all points of inflection if g.

From previous:

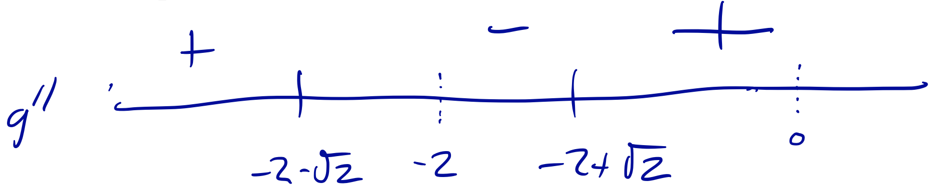


POI: $-2-\sqrt{2}$, $-2+\sqrt{2}$

5. Use the First Derivative Test to classify each critical point as a local min/local max.



6. Use the Second Derivative Test to classify each critical point as a local min/local max (if possible).



$g''(-2) < 0 \Rightarrow$ local max $\qquad g''(0) > 0 \Rightarrow$ local min

7. Determine the value of g at each of its critical points.

$g(0) = 0^2 e^0 = 1$

$g(-2) = (-2)^2 e^{-2} = 4e^{-2} = 0.54....$

8. Use the information determined thus far to sketch the graph of $g(x)$. You may used the fact, which we will justify next class, that $\lim_{x\to-\infty} f(x) = 0$.