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1) Using Cramer's rule to obtain the solutions to the following equation.

$2x_1 + x_2 - x_3 = 0$
$x_1 + x_3 = 4$
$x_1 + x_2 + x_3 = 0.$

solution:

Writing the above equations in matrix form.

$$\begin{pmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}$$

Now,

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} + (-1) \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$

$= 2 \times (-1) - 1 \times 0 - 1 \times 1.$
$= -2 - 1$
$= -3$

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Then,

$$\Delta x_1 = \begin{vmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= 0\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 4 & 1 \\ 0 & 1 \end{vmatrix} + (-1)\begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= 0(-1) - 1\times(4) - 4\times 1$$
$$= 0 - 4 - 4$$
$$= -8.$$

$$\Delta x_2 = \begin{vmatrix} 2 & 0 & -1 \\ 1 & 4 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

$$= 2\begin{vmatrix} 4 & 1 \\ 0 & 1 \end{vmatrix} - 0\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 4 \\ 1 & 0 \end{vmatrix}$$

$$= 2\times(4) - 0\times 0 - 1(-4)$$
$$= 8 - 0 + 4$$
$$= 12$$

$$\Delta x_3 = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 0 & 4 \\ 1 & 1 & 0 \end{vmatrix}$$

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$$= 2\begin{vmatrix} 0 & 4 \\ 1 & 0 \end{vmatrix} - 1\begin{vmatrix} 1 & 4 \\ 1 & 0 \end{vmatrix} + 0\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$

$= 2(-4) - (-4) + 0.$

$= -8 + 4 + 0$

$= -4.$

$x_1 = \dfrac{\Delta x_1}{\Delta}$ $= \dfrac{+8}{+3}$ $= \dfrac{8}{3}$

$x_2 = \dfrac{\Delta x_2}{\Delta}$ $= \dfrac{12}{-3}$ $= -4.$

$x_3 = \dfrac{\Delta x_3}{\Delta}$ $= \dfrac{-4}{-3}$ $= \dfrac{4}{3}.$

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2) a) solve the following using Gauss elimination:

$x_1 + x_2 + x_3 = 2$
$2x_1 + 3x_2 + 4x_3 = 3$
$x_1 - 2x_2 - x_3 = 1$

solution

$x_1 + x_2 + x_3 = 2$ ——— (i)
$2x_1 + 3x_2 + 4x_3 = 3$ ——— (ii)
$x_1 - 2x_2 - x_3 = 1$ ——— (iii)

Now,

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & -2 & -1 & 1 \end{array}\right]$$

or, $$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 1 & -2 & -1 & 1 \end{array}\right] \longrightarrow R_2 \rightarrow R_2 - 2R_1$$

or, $$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & -3 & -2 & -1 \end{array}\right] \quad R_3 \rightarrow R_3 - R_1$$

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or, $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 4 & -4 \end{array}\right]$ $R_3 \rightarrow R_3 + 3R_2$

Writing in equation form,

$4x_3 = -4$ —— (iv)
$x_2 + 2x_3 = -1$ —— (v)
$x_1 + x_2 + x_3 = 2$ —— (vi)

From eqn (iv).

$x_3 = \frac{-4}{4} = -1.$

Putting the value of x_3 in eqn (v)

$x_2 + 2x_3 = -1$
or, $x_2 + 2 \times (-1) = -1.$
or, $x_2 - 2 = -1$
or, $x_2 = -1 + 2.$
$\therefore$ $x_2 = 1.$

Putting the value of x_2 and x_3 in eqn (vi).

$x_1 + x_2 + x_3 = 2.$
or, $x_1 + 1 - 1 = 2.$
$\therefore$ $x_1 = 2.$

$\therefore$ $x_1 = 2, x_2 = 1$ and $x_3 = -1.$

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b). Find the inverse of the matrix from (a) using elimination.

solution,

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & -2 & -1 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Augmenting matrix A with I.

$$[A \quad I] = \left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 3 & 4 & 0 & 1 & 0 \\ 1 & -2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 1 & -2 & -1 & 0 & 0 & 1 \end{array}\right] \quad R_2 \to R_2 - 2R_1$$

$$= \left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & -3 & -2 & -1 & 0 & 1 \end{array}\right] \quad R_3 \to R_3 - R_1$$

$$= \left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 0 & 4 & -7 & 3 & 1 \end{array}\right] \quad R_3 \to R_3 + 3R_2$$

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$$= \begin{bmatrix} 1 & 1 & 1 & : & 1 & 0 & 0 \\ 0 & 2 & 0 & : & 3 & -1 & -1 \\ 0 & 0 & 4 & : & -7 & 3 & 1 \end{bmatrix} \quad R_2 \to 2R_2 - R_3$$

$$= \begin{bmatrix} 4 & 4 & 0 & : & 11 & -3 & -1 \\ 0 & 2 & 0 & : & 3 & -1 & -1 \\ 0 & 0 & 4 & : & -7 & 3 & 1 \end{bmatrix} \quad R_1 \to 4R_1 - R_3$$

$$= \begin{bmatrix} 4 & 0 & 0 & : & 5 & -1 & 1 \\ 0 & 2 & 0 & : & 3 & -1 & -1 \\ 0 & 0 & 4 & : & -7 & 3 & 1 \end{bmatrix} \quad R_1 \to R_1 - 2R_2$$

$$= \begin{bmatrix} 1 & 0 & 0 & : & 5/4 & -1/4 & 1/4 \\ 0 & 1 & 0 & : & 3/2 & -1/2 & -1/2 \\ 0 & 0 & 1 & : & -7/4 & 3/4 & 1/4 \end{bmatrix} \quad \begin{matrix} R_1 \to 1/4\, R_1 \\ R_2 \to 1/2\, R_2 \\ R_3 \to 1/4\, R_3 \end{matrix}$$

We know,

$$[A \; I] = [I \; A^{-1}]$$

$\therefore$ Hence, Inverse of matrix $A = A^{-1}$

$$= \begin{bmatrix} 5/4 & -1/4 & 1/4 \\ 3/2 & -1/2 & -1/2 \\ -7/4 & 3/4 & 1/4 \end{bmatrix}$$

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3. Determine whether the following sequence converges or diverges.

$$t_n = (-1)^{n+1} \frac{n+1}{n^2+3}$$

solution

The sequence is $t_n = (-1)^{n+1} \frac{n+1}{n^2+3}$.

Now,

$$a_n = \frac{n+1}{n^2+3}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n+1}{n^2+3}.$$

$$= \lim_{n\to\infty} \frac{(n+1)/n^2}{(n^2+3)/n^2}$$ [dividing numerator and denominator by n^2].

$$= \lim_{n\to\infty} \frac{1/n + 1/n^2}{1 + 3/n^2}.$$

$$= \frac{1/\infty + 1/\infty^2}{1 + 3/\infty^2}$$

$$= \frac{0+0}{1+0}$$

$$= 0$$

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∴ We know, if $\lim_{n \to \infty} a_n = k$ (real no.), the sequence is convergent and since 0 is a real number, this sequence is convergent.

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4) Find the Maclaurin series expansion of $\sin x$, also calculate the radius of convergence.

Solution,

Let $f(x) = \sin x$.

$f'(x) = \cos x$

$f''(x) = -\sin x$

$f'''(x) = -\cos x$.

For maclaurin series, $x = 0$.

$f(0) = \sin 0 = 0$

$f'(0) = \cos 0 = 1$

$f''(0) = -\sin 0 = 0$.

$f'''(0) = -\cos 0 = -1$.

The maclaurin series is,

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \ldots$$

$$= 0 + x.1 + \frac{x^2}{2!} + \frac{x^3}{3!} -1 + \ldots$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$$

From the above series,

1st term $(t_1) = \frac{1}{1!}$

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3rd term(t_3) = $\frac{1}{5!}$, nth term (t_n) = $\frac{1}{(2n+1)!}$

$$t_{n+1} = \frac{1}{(2n+3)!}$$

We know,

Radius of convergence (R) = $\lim_{n \to \infty} \frac{t_n}{t_{n+1}}$

$$= \lim_{n \to \infty} \frac{1}{(2n+1)!} \times (2n+3)!$$

$$= \lim_{n \to \infty} \frac{(2n+3)(2n+2)\cancel{(2n+1)!}}{\cancel{(2n+1)!}}$$

$$= (2\infty+3)(2\infty+2)$$

$$= \infty.$$

∴ The radius of convergence is $-\infty \leq x \leq \infty$.