UNIVERSITY PARTNER

UNIVERSITY OF WOLVERHAMPTON

HERALD COLLEGE KATHMANDU

4MM013 - Computational Mathematics

Mathematics Assignment-1

Full Marks: 10

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1. State the definition of a function and composite function.
let f and g be functions defined as follow:

$f: R \to R,\ f(x) = \frac{x-3}{x+1}\ \ f(2) = 9$ and

$g: R \to R,\ g(x) = \frac{1}{x},\ x \neq 0$

calculate $(f o g)(x)$ and $(g o f)(x)$.

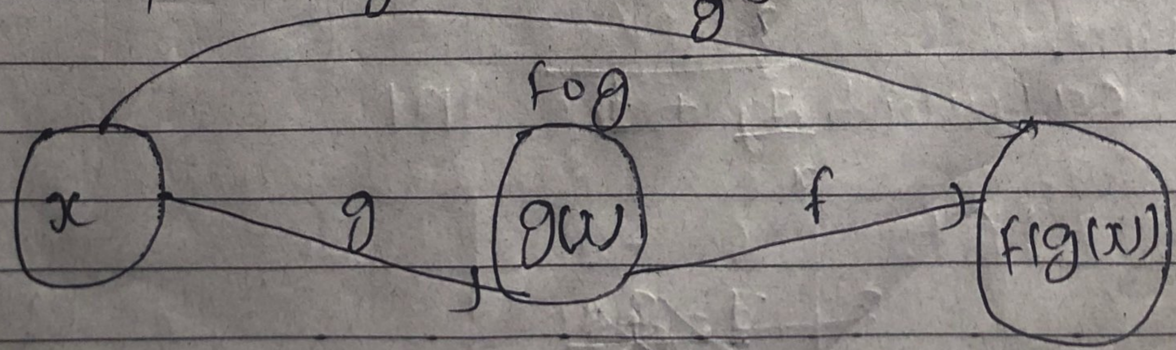
Soln

⇒ function is the rule that operates on an input and produce single output from that input.

i.e $y = f(x)$

where, y is the result, f is the function and x is the input.

composite function is an operation that takes two functions f and g and produces a function h such as $h(x) = g$. For example $fog(x)$ and $gof(x)$



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Given,

$$f(x) = \frac{x-3}{x+1}$$

for f(2) we can have as,

$$f(2) = \frac{2-3}{2+1}$$

$$= \frac{-1}{3}$$

$$\therefore f(2) = \frac{-1}{3}$$

again, $g(x) = \frac{1}{x}$, $x \neq 0$

for fog(x)

$$fog(x) = fog(x)$$

$$= f\left(\frac{1}{x}\right)$$

$$= \frac{\frac{1}{x} - 3}{\frac{1}{x} + 1}$$

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$$= \frac{\frac{1-3x}{x}}{\frac{1+x}{x}}$$

$$= \frac{1-3x}{1+x}$$

now,

for gof(x)

$$gof(x) = g\left(\frac{x-3}{x+1}\right)$$

$$= \frac{1}{\frac{x-3}{x+1}}$$

$$= \frac{x+1}{x-3}$$

$$\therefore fog(x) = \frac{1-3x}{1+x}$$

$$gof(x) = \frac{x+1}{x-3}$$ ans

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2. Solve the following using inverse matrix method:

$$6x - y = 0$$
$$2x - 4y = 1$$

=> Soln

Given Eqn can be written in the form of matrix as,

$$\begin{bmatrix} 6 & -1 \\ 2 & -4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

from above Eqn can be written as,

$$Ax = B$$
$$x = A^{-1} B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \text{--- (i)}$$

now, $A^{-1} = ?$

First,

calculating the $|A|$

$$|A| = \begin{vmatrix} 6 & -1 \\ 2 & -4 \end{vmatrix}$$

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$= -4 \times 6 - (2 \times (-1))$
$= -24 + 2$
$= -22$, $|A| \neq 0$ so, A^{-1} exist.

now,
$$A^{-1} = \frac{1}{\det \cdot A} \times Adj \cdot A$$

$$= \frac{1}{-22} \begin{bmatrix} -4 & 1 \\ -2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{22} & -\frac{1}{22} \\ \frac{2}{22} & -\frac{6}{22} \end{bmatrix}$$

now,
Eqn (1) become as,

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{4}{22} & -\frac{1}{22} \\ \frac{2}{22} & -\frac{6}{22} \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{22} \times 0 - \frac{1}{22} \times 1 \\ \frac{2}{22} \times 0 - \frac{6}{22} \times 1 \end{bmatrix}$$

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or, $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1/22 \\ -6/22 \end{bmatrix}$

or, $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{1}{22} \\ -\frac{3}{11} \end{bmatrix}$

Equating crossponding elements we get,

$x = -\frac{1}{22}$

$y = -\frac{3}{11}$ ans

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3. Calculate the inverse of the following functions.

a. $f(x) = \frac{6+x}{7}$

Soln

let $f(x) = y$

$y = \frac{6+x}{7}$ — (1)

Interchanging value of x and y in Eqn (1)

$x = \frac{6+y}{7}$

or, $7x = 6+y$

or, $7x-6 = y$

$\therefore f^{-1}(x) = 7x-6$

$\therefore$ The inverse function of $f^{-1}(x) = 7x-6$

ans

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b. $f(x) = \frac{3}{2-x}$

Soln

let $y = f(x)$

$$y = \frac{3}{2-x} \quad \text{--- (i)}$$

now,
Interchanging the value of x and y we get,

$$x = \frac{3}{2-y}$$

or, $2x - xy = 3$

or, $2x - 3 = xy$

or, $\frac{2x-3}{x} = y$

or, $\frac{2x}{x} - \frac{3}{x} = y$

or, $2 - \frac{3}{x} = y$

$$\therefore f^{-1}(x) = 2 - \frac{3}{x}$$

$\therefore$ The inverse function $f^{-1}(x) = 2 - \frac{3}{x}$ ans

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4. Sketch the graph of following functions $f(x) = 2(x-1)^2+3$, $-2<x<2$; In the interval of $x = 0.5$ and state domain and range of function.

Soln
Given Eqn

$f(x) = 2(x-1)^2+3$, $-2<x<2$
Interval of $x = 0.5$

$x=-1.5$, $y = 2(-1.5-1)^2+3 = 15.5$, $(-1.5, 15.5)$

$x=-1$, $y = 2(-1-1)^2+3 = 11$, $(-1, 11)$

$x=-0.5$, $y = 2(-0.5-1)^2+3 = 7.5$, $(-0.5, 7.5)$

$x=0$, $y = 2(0-1)^2+3 = 5$, $(0, 5)$

$x=1$, $y = 2(1-1)^2+3 = 3$, $(1, 3)$

$x=1.5$, $y = 2(1.5-1)^2+3 = 3.5$, $(1.5, 3.5)$

$x=0.5$, $y = 2(0.5-1)^2+3 = 3.5$, $(0.5, 3.5$

now, ploting into the graph

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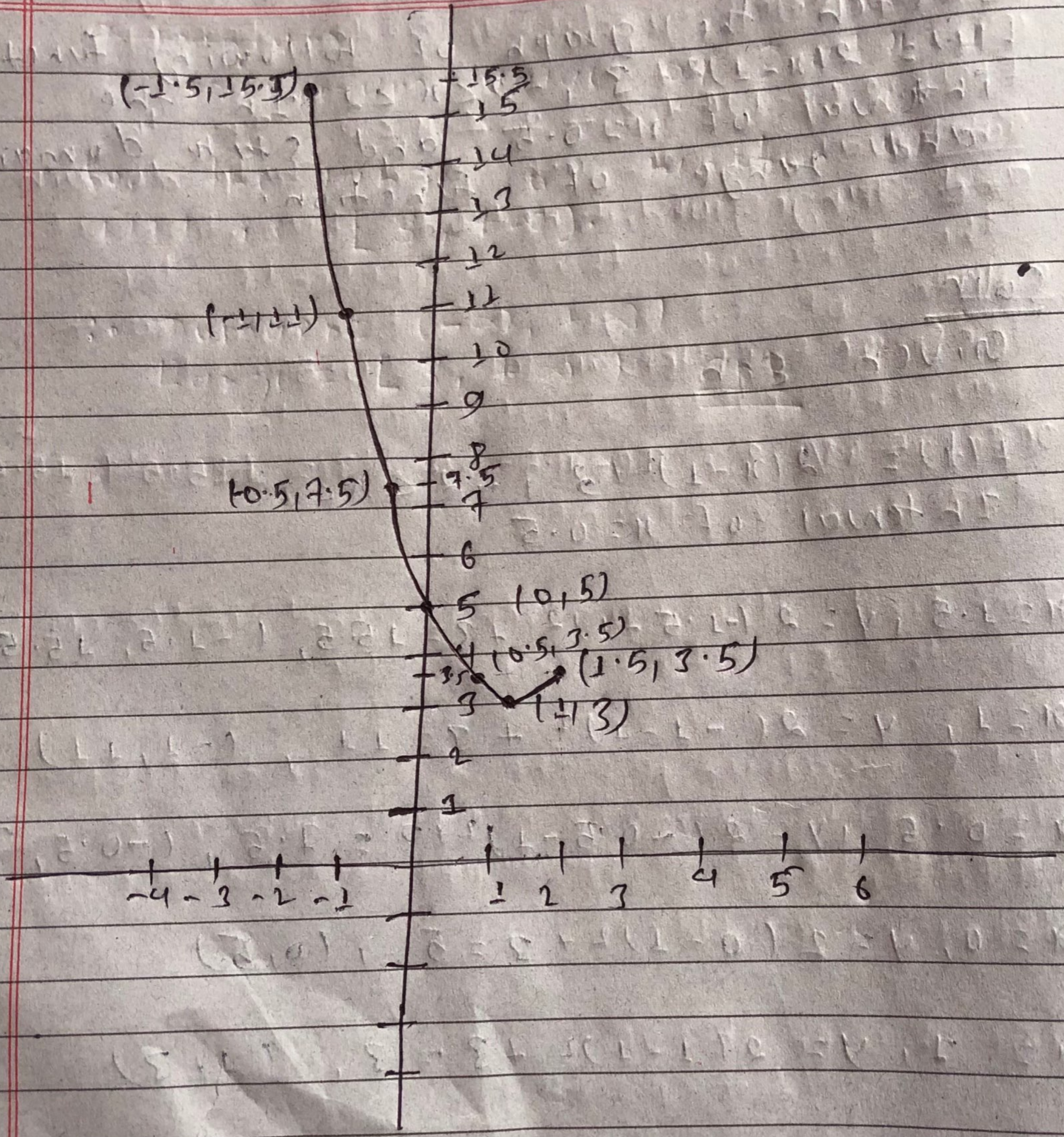

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For domain and range:

Domain:
The given expression $2(x-1)^2+3$ is a polynomial function and there are no restrictions on the values of x that can be plugged into a polynomial. So, the domain of the function is all real numbers.
$\therefore$ Domain $= (-\infty, +\infty)$.

Range
We analyze the behavior of the function. The given function is a quadratic function in form of $f(x) = a(x-h)^2+k$ where a, h and k are constants.
So, comparing Eqn, ~~a=0~~ $a=2$, $h=1$, $k=3$
Since vertex of parabola defined by function occurs at point (h, k) which in this case is $(1, 3)$

Since the cofficient a is positive the parabola opens upwards.

The lowest point of parabola occurs at the vertex, so the function has

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~~a minimum~~

a minimum value of 3 at $x = 1$

So, the range of function of $f(x)$ is $[3, +\infty)$.

$\therefore$ Domain $= (-\infty, +\infty)$
Range $= [3, +\infty)$ ans

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5a. Define ~~gar~~ gradient of a function. State the gradient and intercept of:

$2y+8=6x$

⇒ Soln

~~The~~ The gradient of function is the function that represents the rate of change of function with respect to ~~it's~~ it's independent variable (s). It indicates the steepness or slope of the function at any give point.

To find the gradient and intercept of the equation $2y+8=6x$.

let's rearrange the eqn into slope-intercept form ($y=mx+c$) where m represents the gradient and c represents the y-intercept.

we ~~haver~~ have,

$2y+8=6x$

let's isolate the y term as:

$2y=6x-8$

now,

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$2y = 6x - 8$

Dividing both sides by 2 we get:

$\frac{2y}{2} = \frac{6x}{2} - \frac{8}{2}$

or, $y = 3x - 4$

now,
comparing this eqn with $y = mx + c$
we get,

$m = 3$
y - intercept $(c) = -4$

$\therefore$ Gradient $(m) = 3$
y - intercept $(c) = -4$ ans

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5b. Solve the following equations.

$$x^2 + \frac{17}{6}x + 2 = 0$$

⇒ Soln

$$x^2 + \frac{17}{6}x + 2 = 0$$

or, $\frac{6x^2 + 17x + 12}{6} = 0$

or, $6x^2 + 17x + 12 = 0$

or, $6x^2 + 8x + 9x + 12 = 0$

or, $2x(3x+4) + 3(3x+4) = 0$

or, $(2x+3)(3x+4) = 0$

now,

OR, $2x+3 = 0$

$2x = -3$

$x = -\frac{3}{2}$

Either, $3x+4 = 0$

$3x = -4$

$x = -\frac{4}{3}$

$\therefore x = -\frac{3}{2}, -\frac{4}{3}$ ans