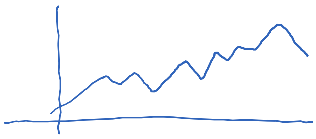
Time series → It is a sequence of data in a chronological order which is recorded sequentially or over time (dated / Time stamped)

Ex Temp reports, stock price movement etc

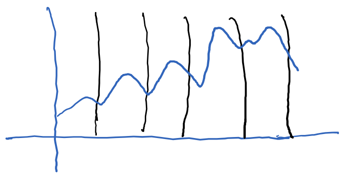


Sampling frequency →

Series is evenly spread → (Temp / hr)

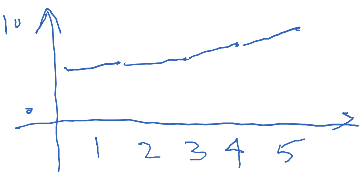
Series can be approx evenly spread → (Temp / email)

Series is even with Missing values → (stock price)

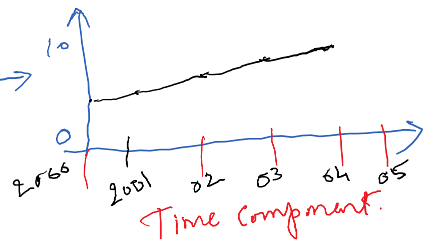


Basic time series objects : ts() ⇒ ts(data, start = 2001, frequency = 4)

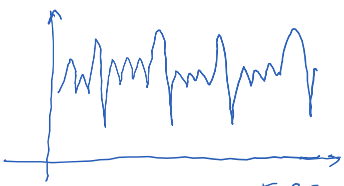
Vector → 4 5 6 7 8 + time component → Time series object



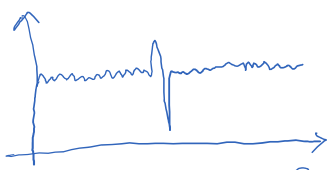
+ (start + freq) →



Trend spotting → Some t.s will not have trend.

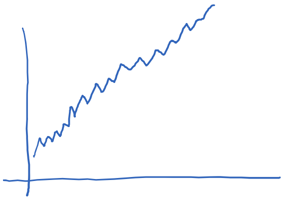


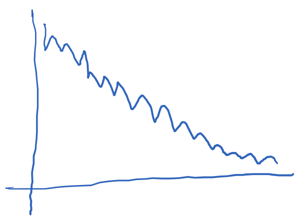
ECG

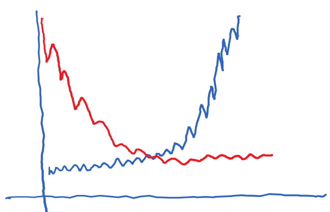


Seismograph

Linear Trend

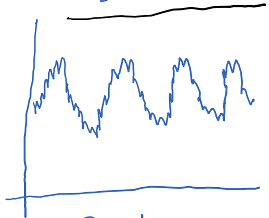






Exponential growth / Decay

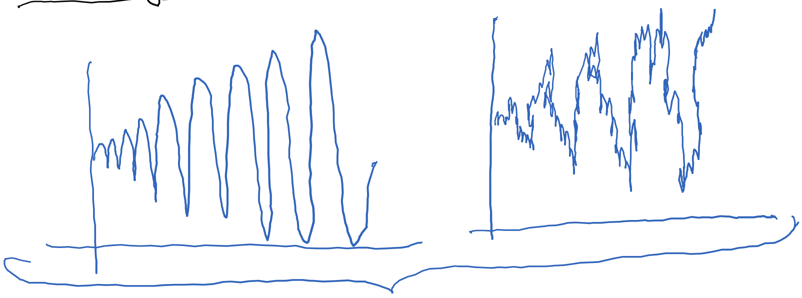
Oscillations.



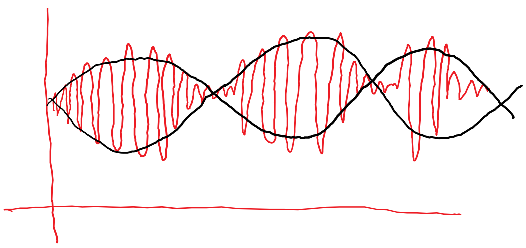
Periodic Sinusoidal

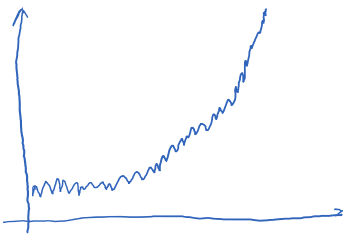
Change in Variance

Change in Variance



Increase in Variance

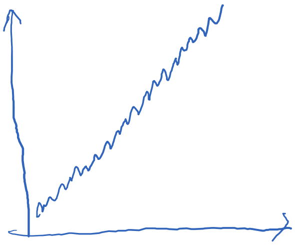




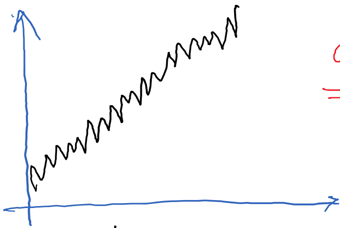
Exponential

log()

only +ve Trends

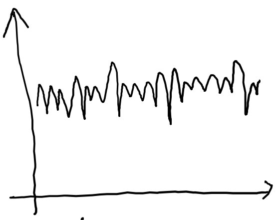


Linear

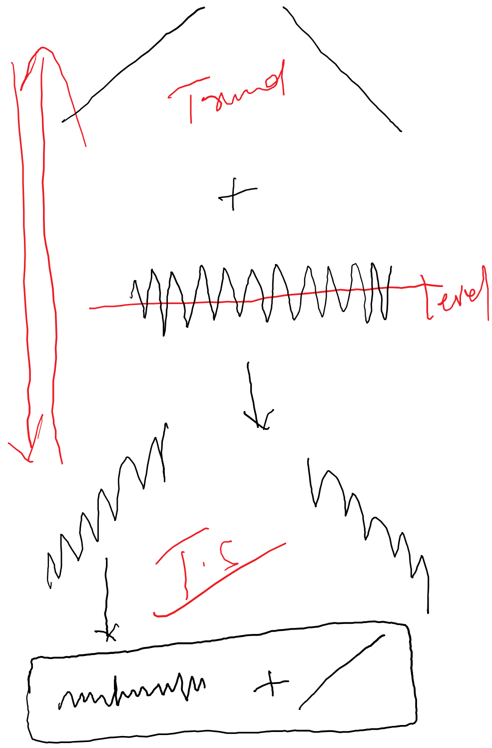


Linear.

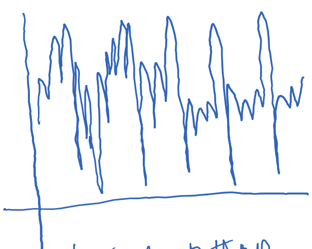
diff()



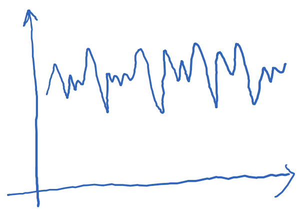
diff / change series.



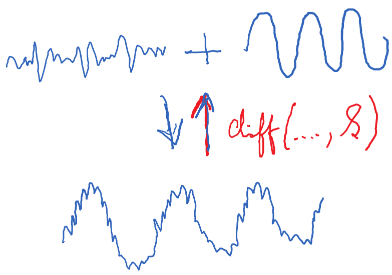
Seasonal Difference Transformation - diff(..., S)



Periodic pattern with a cycle length

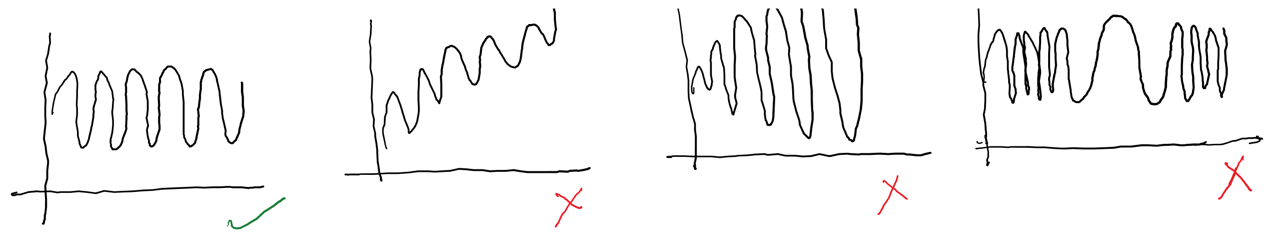


Periodic Trend Removed



(WN) → White Noise → Simple ex. of a stationary process

Stationary process



White noise process has -

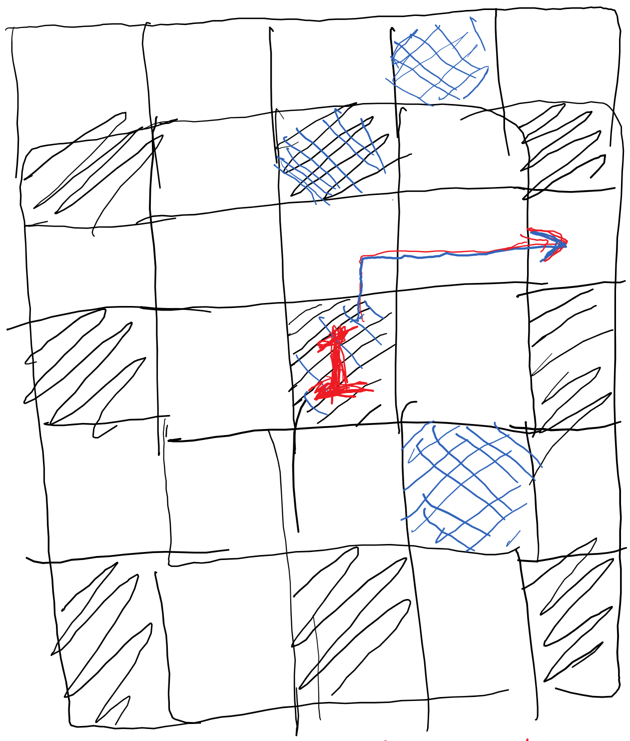
→ Fixed Constant Mean & Variance & No Correlation (Pattern) over time

arima.sim( ) to simulate a white Noise.

ARIMA → Auto Regressive Integrated Moving Avg. → default $\mu = 0$ & S.D = 1

arima( ) to estimate a WN in a time series

Random Walk → (RW)

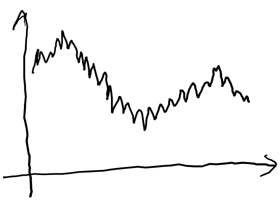


→ Ex of a Non-stationary process.

→ No specific μ or Variance & Shows very strong dependency over time

→ Each observation is closely related to its Neighbours.

→ However → change or increment follows a white Noise process.



Random Walk Recursion

Next step depends on Previous step.

Today = Yesterday + Noise

RW process is Recursive.

$$\boxed{Y_t = Y_{t-1} + \epsilon}$$

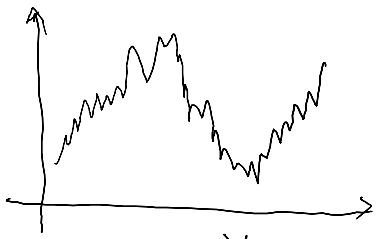
→ Error is $\mu = 0$ → white Noise

→ Simulation requires an initial point

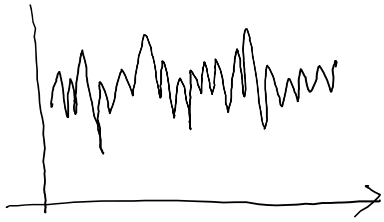
... parameter → WN variance = σ^2

→ Only one parameter → WN variance $= \sigma_\epsilon^2$

$$\Rightarrow \boxed{Y_t - Y_{t-1} = \epsilon} \Rightarrow \text{diff}(Y) \text{ is WN}$$

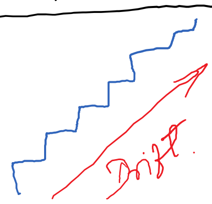


Y



diff(Y)

Random walk with Drift



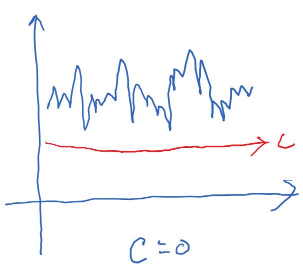
Todays = Constant + Yest + Noise

Intercept

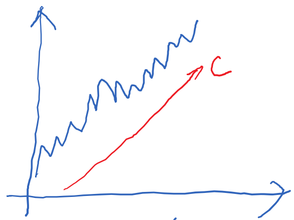
$$\Rightarrow Y_t = c + Y_{t-1} + \epsilon$$

There are 2 parameters → Constant 'c' & WN variance (σ_ϵ^2)

Ex.

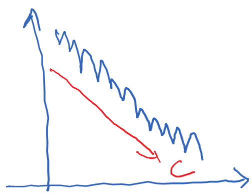


$c = 0$



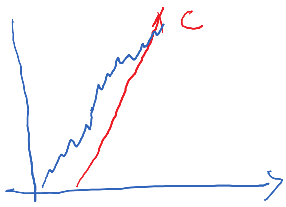
$c = +ve$

Uptrend.



$c = -ve$

Downtrend



c is >> +ve

Steep uptrend.

Holt winters forecasting → Tripple exponential Smoothing

→ Holts linear method → Allows forecasting of data with trend.

→ Holts winter method → Adds seasonality to Holts linear method.

→ Exponential smoothing → Dates back to (1700/1800) → Poisson dist

→ Charls C. Holt → Established this forecasting method.

... applied in forecasting

Brief History

→ Charls C. Holt → Established this forecasting ...
⇒ 2000's → It became famous & applied in forecasting } History

Steps: **Time series** → sequence of data with timestamp

Observed value vs Expected value (Forecasting) → Time series is estimating the values that we dont know from the values that we know

Time series may vary

(+1) (Fibonacci)

Ex. Series = 1, 2, 3 | 4, 5 (Forecast) (or) 5, 8 (Forecast)

Errors → Errors can be +ve/−ve → SSE or MSE = $\sqrt{SSE}$

Methods of forecasting

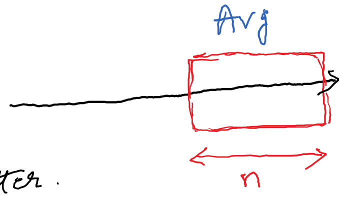
Naive method → Expected pt = previous observed pt.

$$\hat{y}_{n+1} = y_n$$

Simple Avg → Arithmetic Avg = Mean = μ.

$$\hat{y}_{n+1} = \frac{1}{n} \sum_{i=1}^{n} y_i$$

Moving Avg → Sliding window →



Only the recent values matter.
n → window size

Weighted Moving Avg → More recent points matter more {Weights MUST Add to 1}

Instead of window size we select weights.

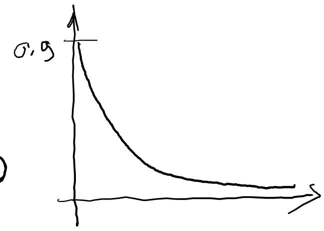
Ex. If we pick weights = 0.1, 0.2, 0.3, 0.4

Ex If we pick weights = '0.1, 0.2, 0.3, 0.4'

Weight will be given 10%, 20%, 30%, 40% to last 4 pts. Respectively

Single exponential Smoothing → Imagin weighted average considered for all points in a series → We assign exponentially smaller weights when we go back in time

Ex $0.9^1, 0.9^2, 0.9^3 \ldots\ldots\ \underbrace{0.9^n}_{\simeq 0}$



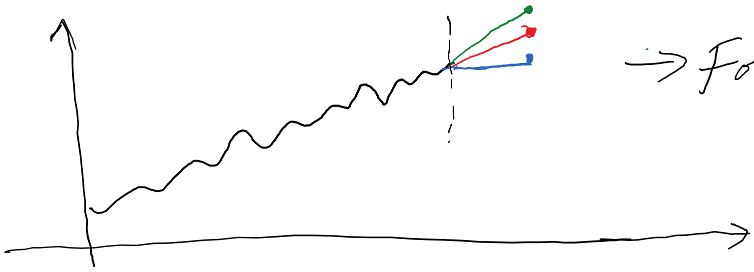
$$\boxed{\hat{y}_x = \alpha \cdot y_x + (1-\alpha)\,\hat{y}_{x-1}} \rightarrow \text{Level}$$

α → something like a starting weight

⇒ Weighted moving avg with 2 weights, α & $\alpha - 1$.

⇝ $(1-\alpha)$ is multiplied by the previous expected $\hat{y}_{x-1}$ → (This was also the result of same formula)

⇒ α → Memory decay rate → More the α → Faster the method forgets.



→ Forecasts happen using different methods

(a) → level → Expected Values (baseline, intercept etc)

→ level is the predicted point in exponential smoothing

(b) → Trend or Slope → $m = \Delta y / \Delta x$ → Algebra Δx → anything

In series $\Delta x = 1$ ⇒ $\Delta y \Rightarrow b = y_x - y_{x-1}$

We can also divide y_{x-1} from y_x & take ratio.

DE-S → Double Exponential Smoothing

→ We have 2 components in this series → Level & Trend

→ DES is exponential smoothing applied to both level & trend

$$l_x = \alpha y_x + (1-\alpha)(l_{x-1} + b_{x-1}) \rightarrow \text{level eqn}$$

$$b_x = \beta(l_x - l_{x-1}) + (1-\beta) b_{x-1} \rightarrow \text{trend}$$

β - Trend factor

$$\hat{y}_{x+1} = l_x + b_x \rightarrow \text{Forecast}$$

Season - Series appears to be repetitive over certain interval

Holt winters is applied only to seasonal trends

(L) Seasonal length → No of data pts after which new season begins.

(S) Seasonal component — It is an additional deviation from levels + trend that repeats itself at the same offset in to the season.

→ There is a seasonal component for every point in the series

→ If 12 is the season length we have 12 Seasonal components.

Applying exponential smoothing to : level, trend & seasonal comp

Tripple exponential smoothing → Holt winters method

→ 3rd pt from S_1 will be exponentially smoothed with 3rd point of S_2 & so on...

$$l_x = \alpha(y_x - s_{x-L}) + (1-\alpha)(l_{x-1} + b_{x-1}) \rightarrow \text{level}$$

$$\boxed{l_x = \alpha(y_x - s_{x-L}) + (1-\alpha)(l_{x-1} + b_{x-1})} \rightarrow \text{level}$$

$$\boxed{b_x = \beta(l_x - l_{x-1}) + (1-\beta) b_{x-1} \rightarrow \text{trend}}$$

$$\boxed{s_x = \gamma(y_x - l_x) + (1+\gamma) s_{x-L} \rightarrow \text{Seasonal}}$$

$\gamma \rightarrow$ Smoothing factor (Gamma)

$$\hat{y}_{x+m} = \underline{l_x} + m\underline{b_x} + \underline{s_{x-L+1+(m-1) \bmod L}} \rightarrow \text{Forecast}$$
