22 January 2018 09:50

Hypothesis Testing → Tentative explanation of principles in Nature

→ Research Hypothes

→ Statistical Hypothesis → Formal way → Evidence / Proof

→ Substantive Hypothesis

Statistical Hypothesis

NULL Hypothesis → H_0 → No change (old is gold)

Alternate Hypothesis → H_A → Change has happened

$\underbrace{H_0 = 500\ gm}$ $\underbrace{H_A \neq 500\ gm}$

Both **Two tail test** → Direction less → $=$ or $\neq$ (Yes or No)

profits. **One tail test** → Directional → $>$, $<$ → Profit, height, lower.

$\boxed{ROI\ 28\%}$ 2016 → $\boxed{\text{Rebrand, Marketing, Brand Ambassadors}}$ 2017 → Sales → $\boxed{ROI > 28\%}$ 2018

$\underbrace{H_0 = 28\%\ ?}$ $\underbrace{H_A > 28\%\ ?}$

Hypothesis procedure → H.T.A.B $\boxed{\text{Assume} \to H_0 \to \text{TRUE}}$

H → Hypothise → Develop → H_0 & HA

Select a test, α $\boxed{0.05}$, Decision Rule, Gather Data

T → Test →

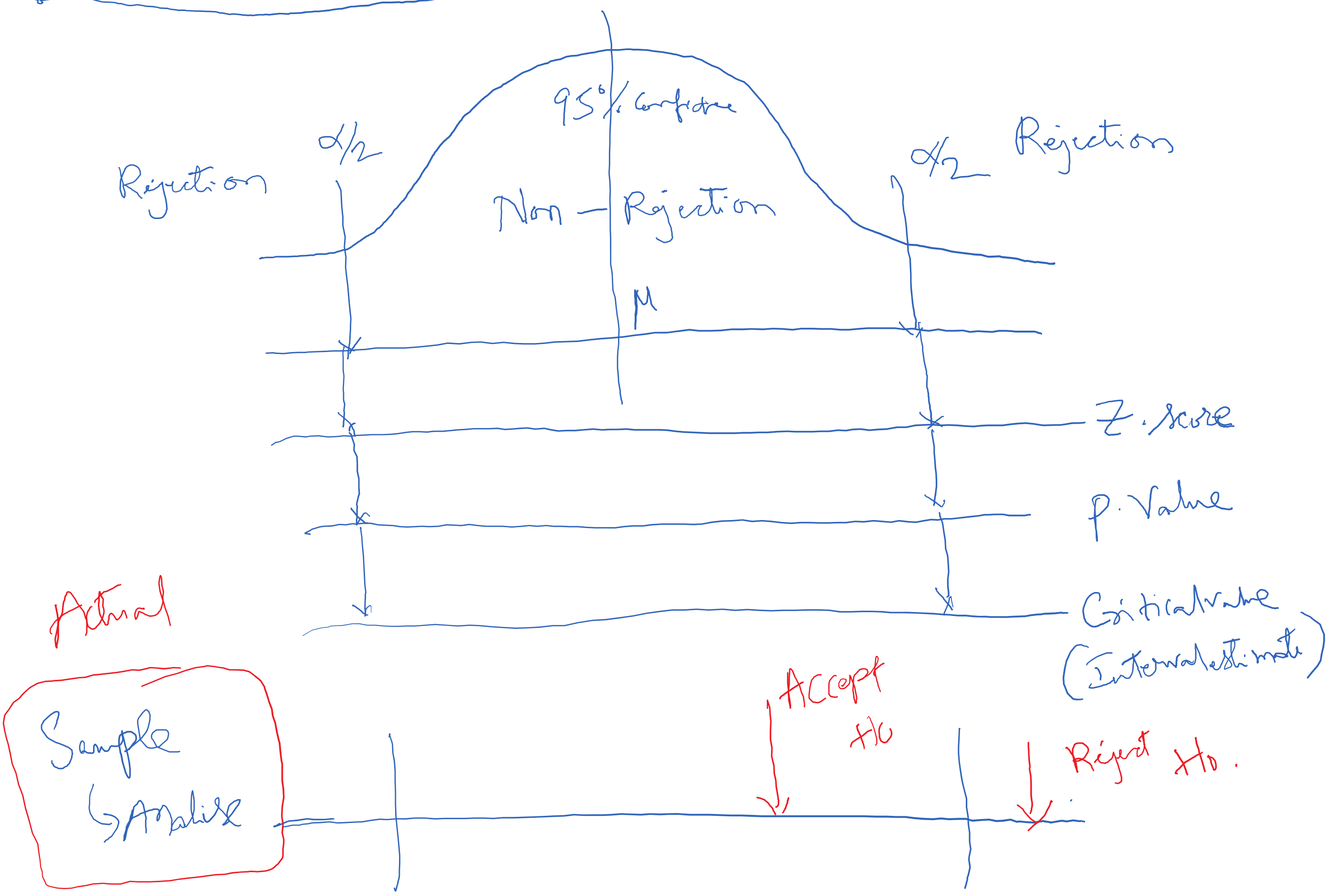
→ Critical value

T → Test → Select a test, α $\boxed{0.05}$ 0.01 0.1 0.001, Decision Rule → Critical value → p value → Z / t / f^2 | Gather Data / Analyse Data

Theory | Practical

A → Action → Decide to Accept / Reject → H_0

B → Business Implication → Apply / Check → Business Implications.



$\mu = 74,914$ $n = 112$, $\bar{x} = 78,695$, $\sigma = 14,530$ (Assume H_0 → TRUE)

① → $H_0 = \mu = 74,914$ → Always have "="

$H_A = \mu \neq 74,914$ → Can have $>, <, \neq$

② → Z test → Random samples

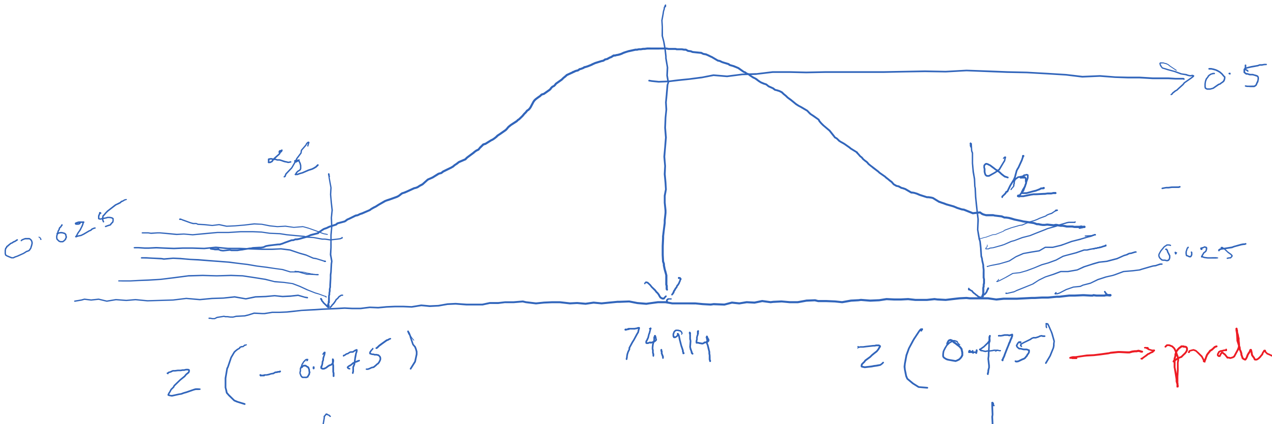
→ δ not Known

$\rightarrow n \geq 30$
$\rightarrow$ Distr is normal

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

3 $\alpha = 0.05 \Rightarrow \alpha/2 = 0.025$

4 Critical value $\rightarrow \alpha$ & Z stat



-1.96 $\qquad$ $+1.96 \rightarrow Z$

$$\bar{x}_c = \mu \pm Z \cdot \frac{\sigma}{\sqrt{n}}$$

$74,917 - \underline{2691}$ $\qquad$ $74917 + \underline{2691}$

$\boxed{72,223}$ $\qquad$ $\boxed{77,605} \rightarrow$ Critical Value.

5 & 6 112 people $\rightarrow \sigma = 14,530$, $n = 112$, $\boxed{\bar{x} = 78,695}$

practical

$$Z\ \text{score} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{78,695 - 74,914}{14530/\sqrt{n}} = \boxed{2.75}$$

$Z_{(p)} > Z_{(T)} \Rightarrow 2.75 > 1.96 \Rightarrow$ Reject $\rightarrow H_0$

Observe $\rightarrow \boxed{\bar{x} = 78695} > \boxed{77605} \Rightarrow$ Reject $\rightarrow H_0$

7

8 → The old Avg Salary ⇒ No longer Holds good.

P value (practical) → Probability of getting sample means AS EXTREME or MORE EXTREME than observed

→ P-value is the smallest value of α for which we can Reject H_0

$\alpha_T = 0.025$, $\bar{x} = 78{,}695$

$$P\left(\bar{x} > 78{,}695 \;\middle|\; H_0 \to \text{True} \;\middle|\; \mu = 74914\right)$$

$$P\left[\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > \frac{78{,}695 - 74914}{14530/\sqrt{112}}\right] \Rightarrow P(z > 2.75)$$

$$= P(>(0.5 - 0.4970))$$

$$\Rightarrow \underbrace{0.003}_{P} < \underbrace{0.025}_{T} \to \text{Reject } H_0$$

↳ Reject H_0

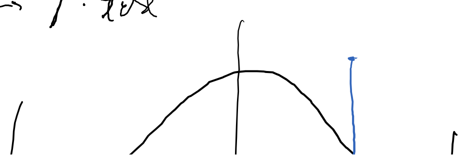
$\alpha + \beta = 1$	H_0 TRUE	H_0 FALSE
Accept H_0	Correct Judgment	Type 2 error β
Reject H_0	Type 1 error α	Correct Judgement (Power)

T_1 → Rejecting a True H_0

T_2 → Accepting a False H_0

Hypothesis about μ, σ (Unknown) → T-test

$H_0 \Rightarrow \mu = 25$



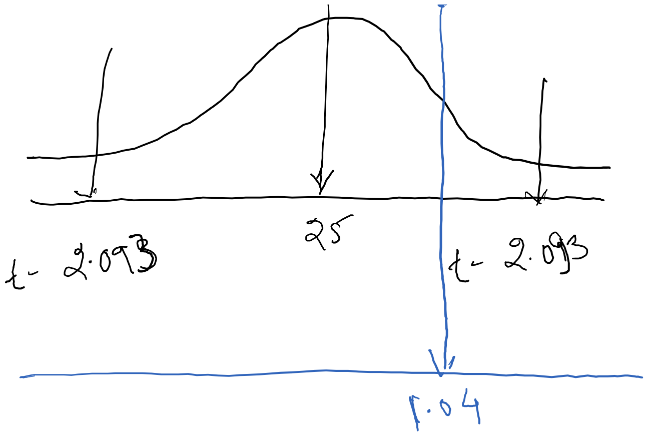
$H_0 \Rightarrow \mu = 25$

$H_A \Rightarrow \mu \neq 25$

$\alpha = 0.05 \Rightarrow \alpha/2 = 0.025$

$n = 20 \Rightarrow df = (n-1) = 19$

$\Rightarrow t_{(0.025)(19)} \rightarrow 2.093$



Select samples → $\bar{x} = 25.51$, $s = 2.193$, $n = 20$

$\boxed{\text{Reject } H_0 \text{ if } t < -2.093 \text{ or } > +2.093}$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{25.51 - 25}{2.193/\sqrt{20}} = \underline{1.04}$$

$t = 1.04 < 2.093 \rightarrow$ H_0 is NOT Rejected

Testing for Proportion $(\hat{p})$

CLT $$Z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}}$$

$n = 140$, $x = 48$

$\hat{p} = 0.34$

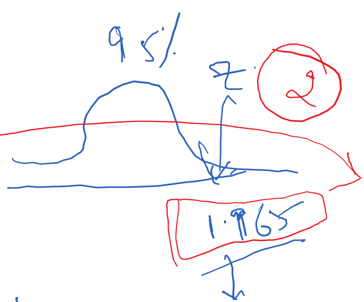
$$Z = \frac{0.34 - 0.26}{\sqrt{\frac{0.26 \times 0.74}{140}}}$$

$\alpha = 0.05 \Rightarrow \alpha/2 = 0.025$

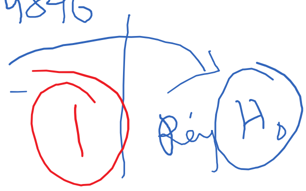
$H_0 = p = 26$

$H_A = p > 26$

$\boxed{Z = 2.16}$



0.4846



$\alpha = 0.05 \Rightarrow \alpha/2 = 0.025 \rightarrow Z = 1.965$

Conclusion → Market share is changed & > 26%

$\chi^2 \rightarrow$ Variance

(2) → $\chi^2 = (n-1)s^2$

$\chi^2 \rightarrow$ Variance

Assume H_0 is TRUE

① $H_0 \rightarrow \sigma^2 = 25$

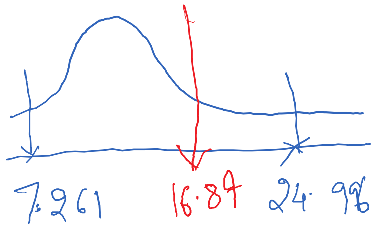
$H_A \rightarrow \sigma^2 \neq 25$

② $\rightarrow \chi^2 = \dfrac{(n-1)s^2}{\sigma^2}$

③ $\rightarrow \alpha = 0.10$

$\alpha/2 = 0.05$

④ $df = (n-1) = 15$



$\boxed{s^2 = 28.062}$

57	56	52	44
46	53	44	44
48	51	55	48
63	53	51	50

$\chi^2_{(1-0.05)(15)} = \chi^2_{(0.95)(15)} = \boxed{7.261}$

$\chi^2_{(0.05)(15)} = \boxed{24.996}$

Decision $\rightarrow$ Reject H_0 $\rightarrow$ observed value is < 7.261 & > 24.996

⑤ Table data $\rightarrow$ $s^2 = 28.062$

observed $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{15(28.062)}{25} = \boxed{16.84} \rightarrow$ Accept H_0

Conclusion $\rightarrow$ There is no variance in my overtime value

How to solve for type 2 errors

$H_0 \rightarrow$ Correct

$H_0 \rightarrow$ Type 2 error $\rightarrow$ Business loss

$\rightarrow$ Miss opportunities

$\rightarrow$ Improvement can be stopped

$\rightarrow$ Probability

$\rightarrow$ Soft drink (Type ② $\rightarrow$ Fail to Reject a False H_0)

$H_0 \to \mu = 12$

$H_A \to \mu \neq 12$

If $\mu = 12$ is false

Various possibilities ⇒ $\mu = 11.9,\ 11.99,\ 12.5,\ 10$

↓ % ↓ % ↓ % ↓ %

Diff possibility (%) of Committing type 2 error ← %

How?

→ 60 cans → $(\mu)_{\bar{x}} = 11.985$ → $\sigma = 0.10$, $\alpha = 0.05$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{11.985 - 12}{0.10/\sqrt{60}} = -1.16 \text{ (observed)}$$

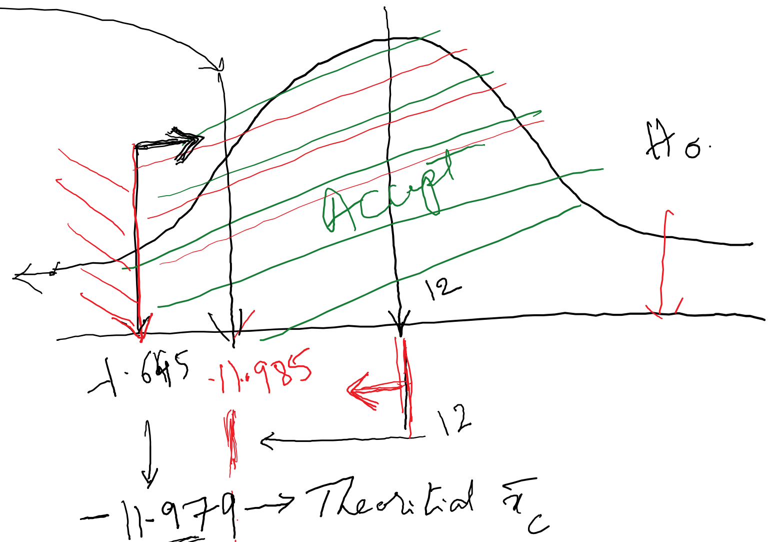
Observed

$\mu = 11.985 > 11.979$

↳ Accept Ho

Correct

Type II

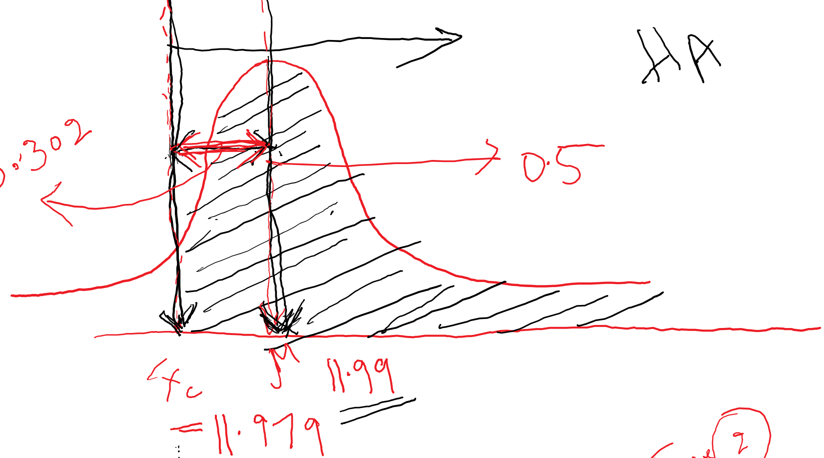


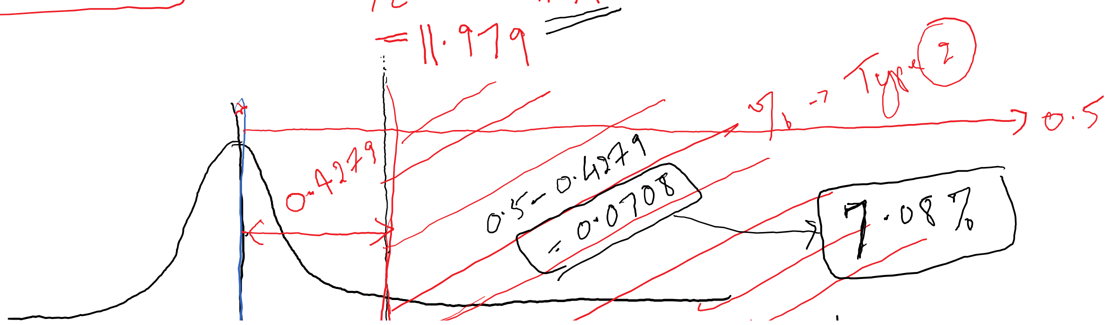
⇒ 0.5 + 0.32

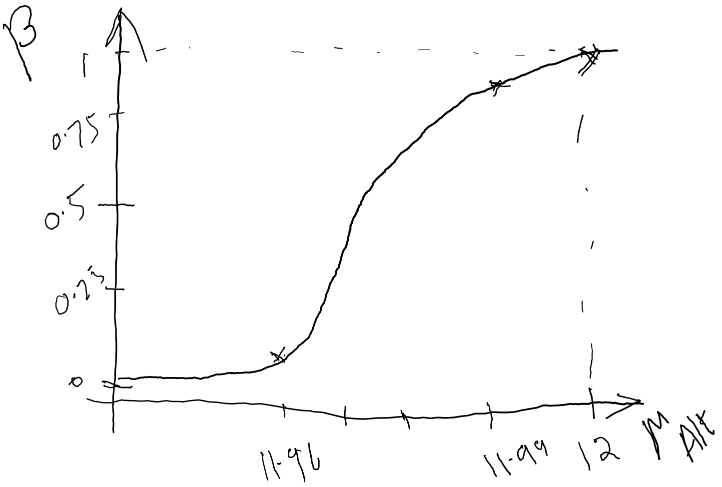
⇒ 0.802

⇒ 80%

Type 2 error



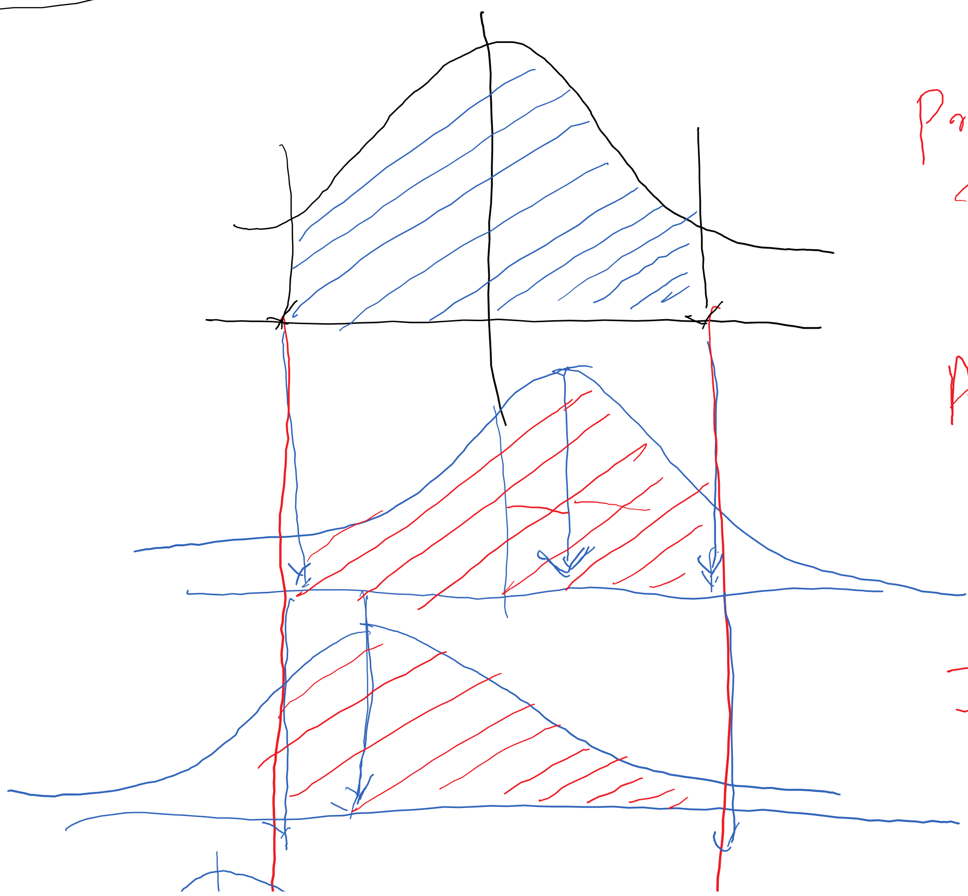




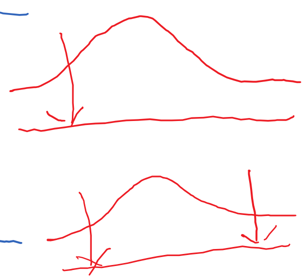
O-C curve

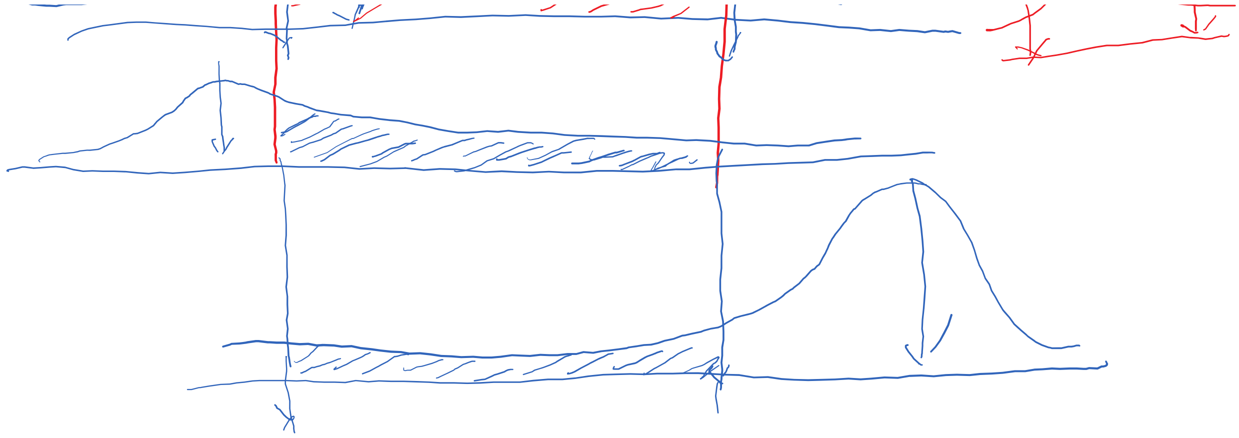
→ o/p character curve.

Power curve



Probability of Type 2 error is Area within Critical Values

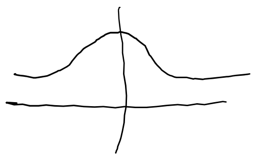




Testing for difference

CLT $\rightarrow$ $Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$n \geq 30$



$$\boxed{Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}}$$

Find the difference in Sample mean $\rightarrow$ σ Known

Managers Avg Annual wages. ($H_0 \rightarrow$ TRUE)

$H_0: \mu_1 = \mu_2$

$H_A: \mu_1 \neq \mu_2$

$\alpha = 0.05 \Rightarrow \dfrac{\alpha}{2} = 0.025$

$z = \pm 1.96$

$\longrightarrow$ 2.35

$$Z = \frac{(70.70 - 62.18) - (0)}{\sqrt{\dfrac{264.160}{32} + \dfrac{166.411}{34}}}$$

$Z = 2.35 > 1.96 \Rightarrow$ Reject H_0

$\rightarrow$ Reject H_0

$\Rightarrow$ There is difference in Avg salary

... interval. (Actual)

Confidence interval (Actual)

$$\left(\bar{x}_1-\bar{x}_2\right) - Z\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}} \le \mu_1-\mu_2 \le \left(\bar{x}_1-\bar{x}_2\right) + Z\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$$

$$8.513 - 7.1074 \le \mu_1-\mu_2 \le 8.513 + 7.1074$$

$$1.405 \le \mu_1-\mu_2 \le 15.620$$

$= 0$

σ Unknown $\rightarrow$ T Test $\Rightarrow$ T test $\rightarrow$ 2 Sample equal variance

Excel

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n}} \Rightarrow \frac{\left(\bar{x}_1-\bar{x}_2\right)-\left(\mu_1-\mu_2\right)}{\boxed{\sqrt{\frac{S_1^2(n-1)+S_2^2(n-1)}{n_1+n_2-2}}}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

$S_p^2 \rightarrow$ Weighted Avg S_1^2 & S_2^2

$(n-1)$ df $\quad \boxed{n_1+n_2-2} \quad (n-1)$

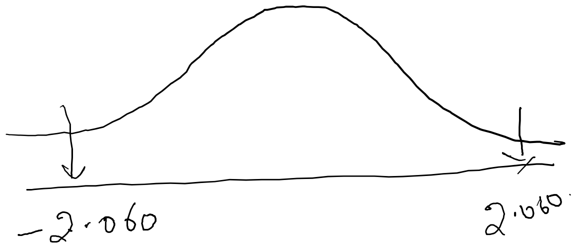
$H_0: \mu_1 = \mu_2$

$H_A: \mu_1 \neq \mu_2$

$\alpha = 0.05$

$\alpha/2 = 0.025$

μ_1	μ_2
$n_1 = 15$	$n_2 = 12$
$\bar{x}_1 = 47.73$	$\bar{x}_2 = 56.56$
$s_1^2 = 19.49$	$s_2^2 = 18.273$



$t_{(0.025)(25)} = 2.060$

$$t = \frac{(47.73 - 56.56) - (0)}{\sqrt{\frac{(19.47)(14) + (18.27)(11)}{25}} \sqrt{\frac{1}{15} + \frac{1}{12}}} = \underline{\underline{-5.20}}$$

Confidence Interval

$$(x_1 - x_2) \pm t \sqrt{\frac{S_1^2(n-1) + S_2^2(n-1)}{n_1 + n_2 - 2}} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \leq \mu_1 - \mu_2$$

Statistical inference for 2 Related Populations (Matched, Dependant, co-related)

Excel → T. test, paired sample for means

Ex Before → After, Twins.

n = no of sample in pairs.

d = sample diff in pairs

$$t = \frac{\bar{d} - D}{S_d / \sqrt{n}}$$

D = μ population diff

S_d = SD of sample diff

$\bar{d}$ = μ of sample diff

$$\bar{d} = \frac{\Sigma d}{n} \qquad S_d = \sqrt{\frac{\Sigma(d - \bar{d})^2}{n-1}} \Rightarrow \sqrt{\frac{\Sigma d^2 - (\Sigma d)^2/n}{n-1}}$$

Statistical inference for Proportions $(\hat{p}_1 - \hat{p}_2)$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}} \longrightarrow z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{(\bar{p}\,\bar{q})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$z = \frac{(p_1 - p_2) - (P_1 - P_2)}{\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}} \longrightarrow z = \frac{\quad}{\sqrt{(\bar{p}\cdot\bar{q})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

where $\boxed{\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{n_1\hat{p} + n_2\hat{p}}{n_1 + n_2}}$

$\boxed{\bar{q} = 1 - \bar{p}}$

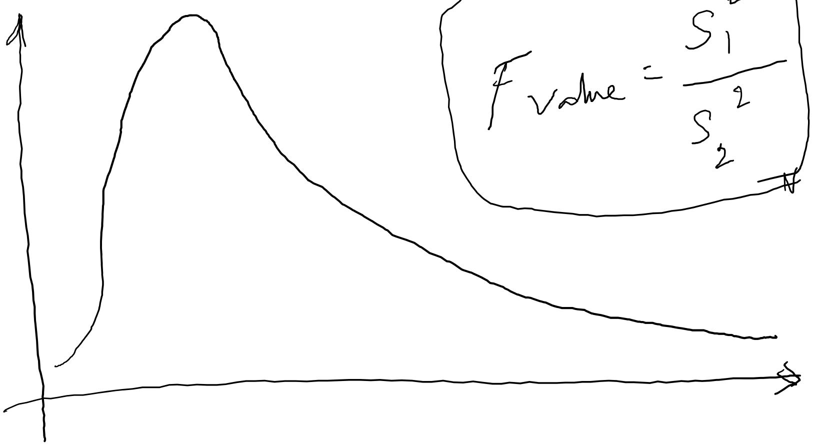
Confidence Interval:

$\boxed{\quad} \leq \quad \leq \boxed{\quad}$

Testing hypothesis for 2 population Variance

Excel ⇒ F test - 2 samples for variance

1 population: χ^2 ⇒ Analysis; χ^2 ⇒ Hypothesis test



$$F \text{ value} = \frac{S_1^2}{S_2^2}$$

F Dist → 2 population

↓

ANOVA

→ Varies with 'n'

→ Extremely sensitive to violation of normal dist assumption

→ Non-Symmetric

→ Varies with α → Different plots for diff α

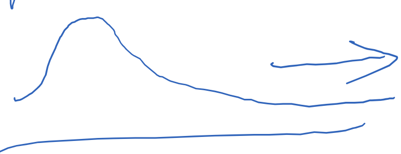
$\alpha = 0.10$, 0.05 , 0.025

↓ ↓ ↓

Different F-tables.

→ No mean $= 0$.

→ F table gives only values of upper table



$(\alpha, 1-\alpha)$

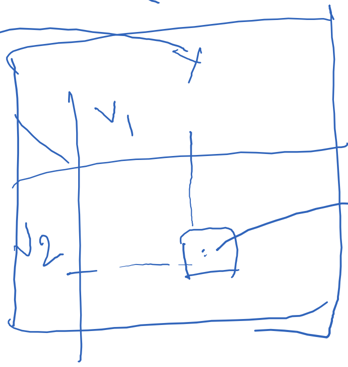
$n \rightarrow df \cdot (n-1) \rightarrow V_1$

$n_1 \to df : (n_1 - 1) \to v_1$

$n_2 \to df : (n_2 - 1) \to v_2$

$F_{(\alpha/2),(v_1),(v_2)} =$

df df



→ Right tail value

$$F_{(1-\alpha/2), v_1, v_2} = \frac{1}{F_{(\alpha/2), v_1, v_2}}$$

$$F = \frac{S_1^2}{S_2^2}$$

$\boxed{\Rightarrow 5.62}$

M_1	
22.3	21.9
21.8	22.4
22.3	22.5
21.6	22.2
21.8	21.6

$S_1^2 = 0.1137$

$n = 10$

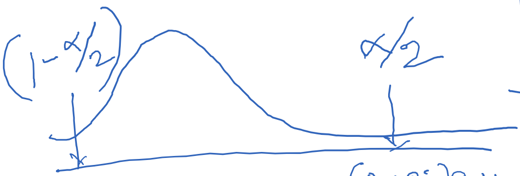
$df = 9$

M_2	
22.0	21.7
22.1	21.9
21.8	22.0
21.9	22.1
22.2	22.9
22.0	22.1

$S_2^2 = 0.0202$

$n = 12$

$df = 11$



$(0.975)9,11$ → 0.28

$(0.025)9,11$ → $\boxed{3.59}$

$$F_{(1-\alpha/2)} = \frac{1}{F(\alpha/2)}$$

→ Reject H_0

Anova → Analysis of Variance

Experiment Design → Plan / Design for Hypothesis Test → Researcher Can Control / Modify the variables.

Independent variables → Controlled / Modified.

↓ → levels of treatment

S^2 → 1 population Variance

σ^2 → Hypothesis Variance

1p

F → Dist → 2 pop variance

↓ → 3 sets of samples

Anova.

Independent variable

↳ levels of treatment

Anova.

Dependant variables : Response of Independent variables.

Tyre Quality : → Independent variable		
Low	Med	High
1 k	1.5 k	2.1 K
1.5 K	2 K	2.2 K
1.25	2.1 K	2.3 K

→ Levels / Treatment

→ Dependent variable

→ METRO

56%

	E	W	N	S
1	x k	y k	z k	A k
2	x_2 k	y k	2 k	A k
3	x_3 k	y k	z k	A k
4	x_4 k	y k	z k	A k
5				

Sales Amount

Valve Opening By Operator → Independent

→ Levels

Oper 1	Op 2	Op 3	Op 4
6.33	6.26	6.44	6.29
6.26	6.36	6.38	6.23
6.31	6.23	6.58	6.19
6.29	6.27	6.54	6.21
6.4	6.19	6.56	
	6.5	6.34	
	6.19	6.58	
	6.22		

Dependant

Unexplained

$$^nC_r = \frac{n!}{(n-r)!\,r!} = \frac{24}{4} = 6$$

→ α → Error of estimation

$\boxed{\alpha \times \alpha \times \alpha \quad \text{Exponentially}}$

Anova in general → K samples.

(1) $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \dots \mu_K$

H_0 : Atleast one μ is not same

(2) Total variance $\boxed{SST}$

(+)

$SST = SSC + SSE$

+

Variance from Treatment of Columns (Levels)

or

Explained Variance

or

Between group Variance

or

SSC

Error Variance within Columns.

or

Unexplained Variance

or

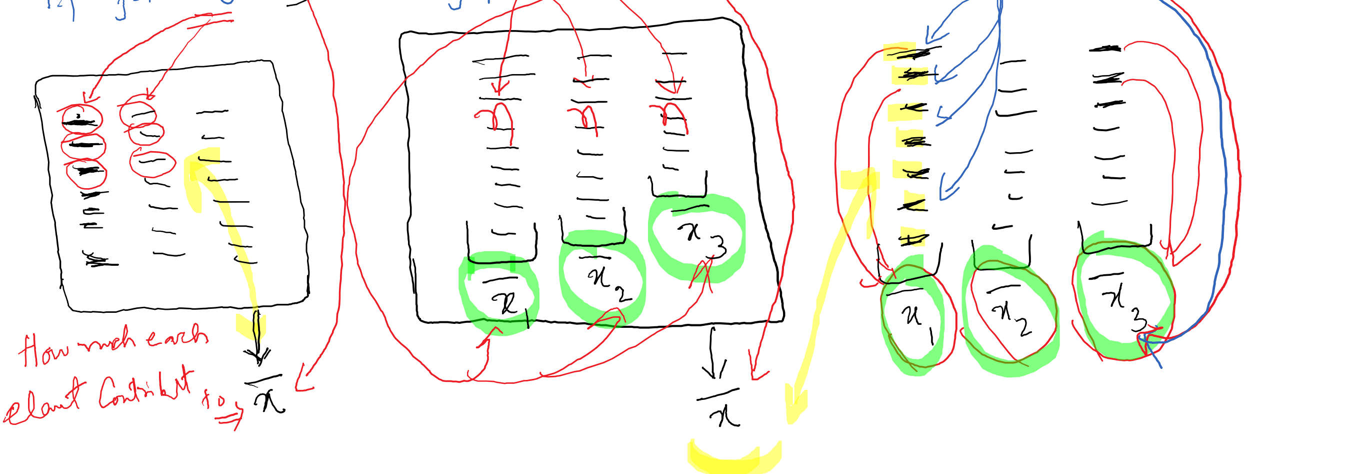
Within group Variance

or

SSE

SST = SSC + SSE

$$\sum_{i=1}^{n}\sum_{j=1}^{c}\left(x_{ij}-\bar{\bar{x}}\right)^2 = \sum_{j=1}^{c} n_j\left(\bar{x}_j-\bar{\bar{x}}\right)^2 + \sum_{i=1}^{n_j}\sum_{j=1}^{c}\left(x_{ij}-\bar{x}_j\right)^2$$



SST → Total Sum of Squares

SSC → Column Sum of Squares

SS E → Error Sum of Squares

i = Element no in a level

j = Treatment label / Column no

C = no of Columns / Levels

n = no of observations in a level

$\bar{\bar{x}}$ = grand mean

$\bar{x}_j$ = Column mean

df_c = $C-1$ (Numerator df)

df_T = $N-1$ (Denominator df)

df_E = $N-C$

$$MSC = \frac{SSC}{df_c}$$

$$MSE = \frac{SSE}{df_E}$$

$$F = \frac{MSC}{\ldots}$$ → Practical Value of → Compare table

$\bar{x}_j$ = Column mean

x_{ij} = individual value

$$\boxed{F_{value} = \frac{MSC}{MSE}}$$ → Practical value

F → Compare with Theory value