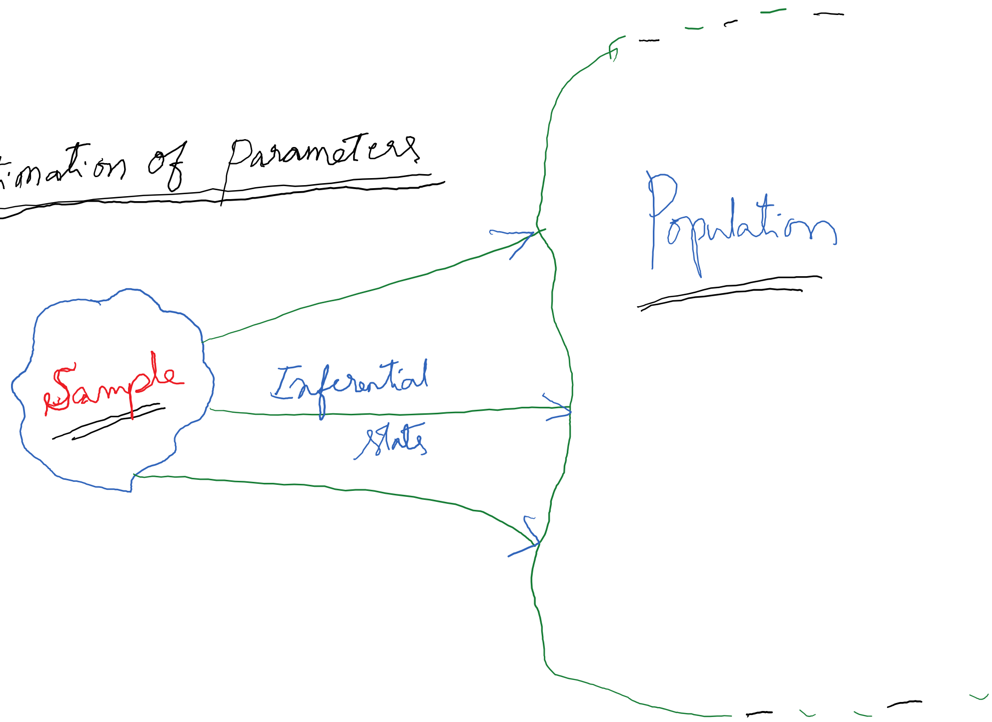
Estimation of Parameters



1 → Estimate μ (Population) From sample μ when σ (pop) is Known
μ $\bar{x}$ s

2 → ————— " ————— σ (Pop) is Unknown

3 Estimation of pop proportion using Z stat

4 Using $\boxed{\text{chi square dist}}$ → pop variance given sample var

5 Sample size needed? to estimate pop mean & pop proportion

→ Manager → Hrs → Illness

→ Random sample → Sample mean $\xrightarrow{\text{Infers}}$ Population μ

Sample mean ↓ $\boxed{\text{Statistic}}$ Population μ ↓ $\boxed{\text{Parameter}}$

5 hrs → Population

$$\bar{x} = \frac{\sum x}{n} \rightarrow \text{Point estimate}$$

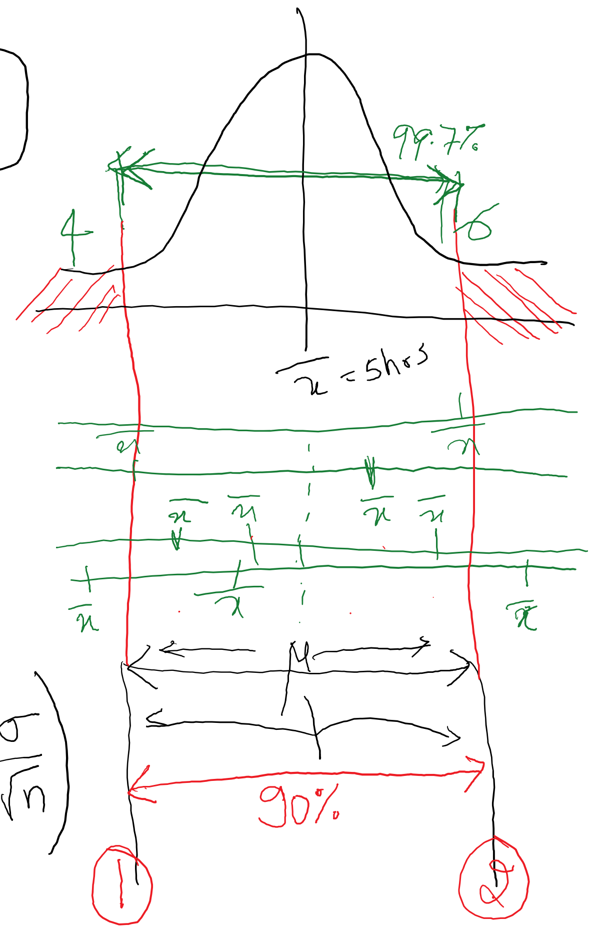
Interval Estimate

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$\Rightarrow \mu = \bar{x} \pm Z \cdot \frac{\sigma}{\sqrt{n}}$$

$$\left(\bar{x} - z \cdot \frac{\sigma}{\sqrt{n}}\right) \le \mu \le \left(\bar{x} + z \cdot \frac{\sigma}{\sqrt{n}}\right)$$

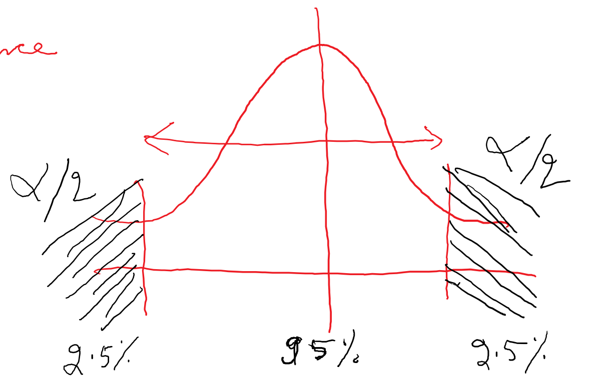
① ②

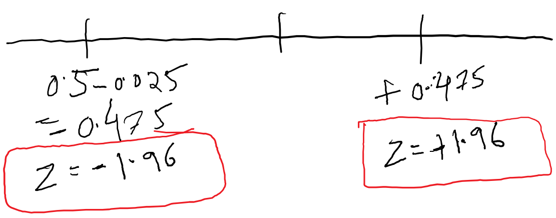


$\alpha \rightarrow$ Area outside the Confidence interval.

$\bar{x} = 1300$
$\sigma = 160$
$n = 85$
$Z = 1.96$

95% Confident
& Normal





Confidence Interval

$$\bar{x} - z \cdot \frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z \cdot \frac{\sigma}{\sqrt{n}}$$

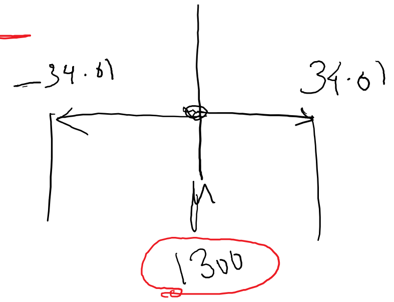
$$= 1300 - 1.96 \cdot \frac{160}{\sqrt{85}} \le \mu \le 1300 + 1.96 \cdot \frac{160}{\sqrt{85}}$$

$$1300 - 34.01 \le \mu \le 1300 + 34.01$$

34.01 | 34.01

$$1300 - 34.01 \le \mu \le 1300 + 34.01$$

$$\boxed{1265.9 \le \mu \le 1334.01}$$

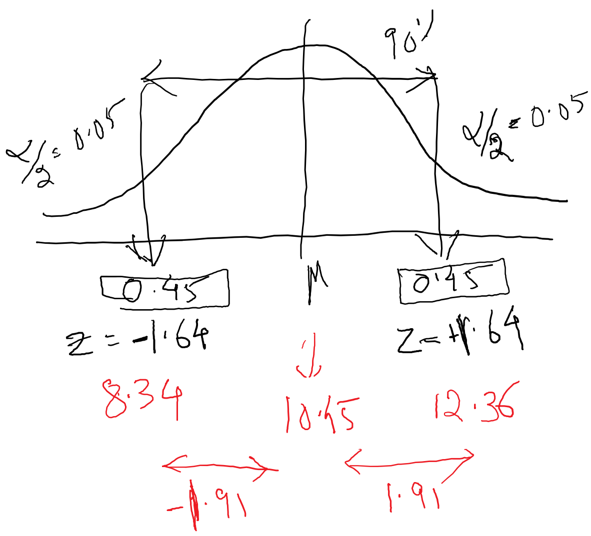


$\bar{x} = 10.455$, $\sigma = 7.7$, $n = 44$

$$\bar{x} - z \cdot \frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z \cdot \frac{\sigma}{\sqrt{n}}$$

$$= 10.455 - 1.91 \le \mu \le 10.455 - 1.91$$

$$\boxed{8.54 \le \mu \le 12.36}$$



$$\bar{x} - Z \frac{\sigma}{\sqrt{n}} \underbrace{\sqrt{\frac{N-n}{N-1}}} \le \mu \le \bar{x} - Z \frac{\sigma}{\sqrt{n}} \underbrace{\sqrt{\frac{N-n}{N-1}}}$$

Sample $\lesssim$ 5% of pop size

Finite Corrections → Accuracy

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② σ (Pop) Unknown → Estimate population μ

NY – LA → flight time (avg)

① Random sample

② Sample μ → $\boxed{S.D = S}$

③ Pop mean → $Z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}} =$

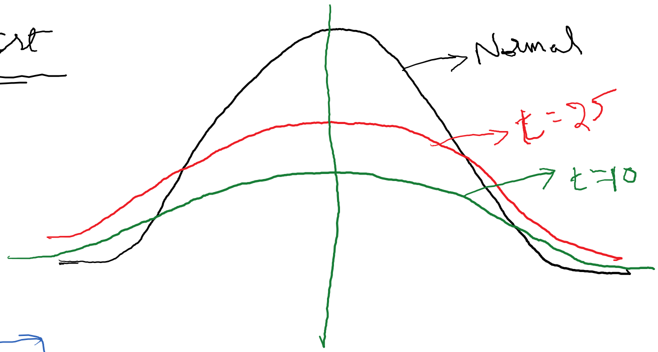
Zscore ✗

William S. Gosset → T. dist

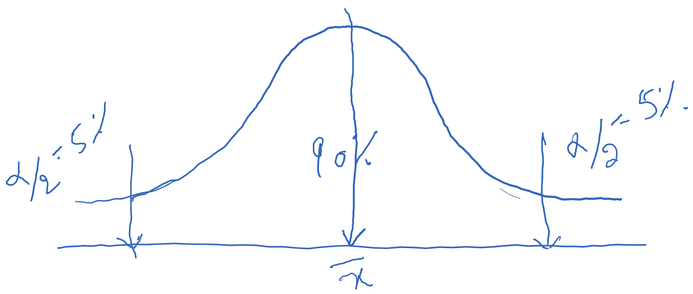
William. S. Gosset $\rightarrow$ T. dist

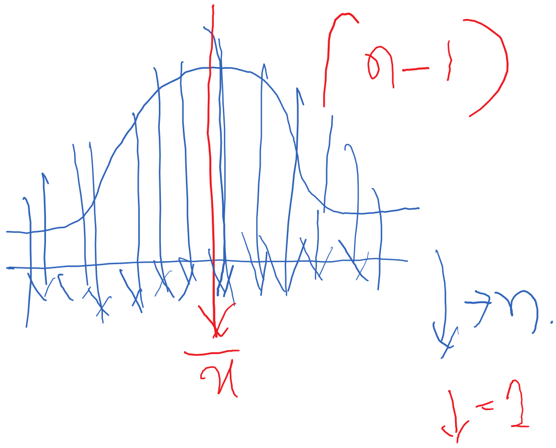
$$T = \frac{\bar{x} - \mu_{\bar{x}}}{s/\sqrt{n}}$$

$\boxed{t \Rightarrow n >>> \simeq \text{Normal}}$



Flatter in the middle





$df = (n-1)$

1. $\bar{x}$ $\rightarrow$ point estimate
2. Interval estimate $\Rightarrow$ $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$ $\Rightarrow$ $\boxed{\mu = \bar{x} \pm t_{(\alpha/2)(n-1)} \cdot s/\sqrt{n}}$

3 1 3 2 5 1 2 1 4 2 1 3 1 1

$\boxed{\bar{x} = 2.14}$ $\rightarrow$ pt

99% Confidence $\Rightarrow$ $\alpha/2 = 0.005$

$n = 14$

$df = n-1 = 13$

$t_{(0.005)(13)} = 3.012$

$\boxed{\pm 1.038}$

$s = 1.29$

$$IE \Rightarrow \bar{x} - t\, s/\sqrt{n} \leq \mu \leq \bar{x} + t \cdot s/\sqrt{n}$$

$$= 2.14 - 3.012 \cdot \frac{1.29}{\sqrt{14}} \leq \mu \leq 2.14 + 3.012 \cdot \frac{1.29}{\sqrt{14}}$$

$$= 2.14 - \underline{1.038} \leq \mu \leq 2.14 + \underline{1.038}$$

$= 2.14 - 1.058 \le \mu \le 2.14 + \ldots$

$$= \boxed{1.10 \le \mu \le 3.18} \rightarrow 99\%$$

18 - staff = n   90%

6 21 17 20 7 0 8 16 29

3 8 12 11 9 21 25 15 16

df = 17 . 90% → $\alpha/2 = 0.05$

$$\boxed{t_{(0.05)(17)} = 1.740}$$ $sd = s = 7.80$

$$\therefore \boxed{\mu_{\bar{x}} = 13.5}$$

$$= \bar{x} - t_{(\alpha/2)(n-1)} \cdot s/\sqrt{n} \le \mu_{\bar{x}} \le \bar{x} + t \cdot s/\sqrt{n}$$

$$= 13.56 - 1.740 \cdot \frac{7.80}{\sqrt{18}} \le \mu_{\bar{x}} \le 13.56 + 1.740 \cdot \frac{7.80}{\sqrt{18}}$$

$$= \boxed{10.36 \le \mu \le 16.76} \rightarrow 90\%$$

Estimate population proportion " p " $\hat{p} \rightarrow$ sample proportion

$$CLT = \quad Z = \hat{p} - p \Big/ \sqrt{\frac{p \cdot q}{n}}$$

$$\boxed{p + q = 1}$$

$n \cdot p > 5$

$n \cdot q > 5$

⇒ For Interval estimate

$$Z = \hat{p} - p \Big/ \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}} \quad = p = \hat{p} \pm Z \cdot \sqrt{\frac{\hat{p}\,\hat{q}}{n}}$$

$$\Rightarrow \boxed{\hat{p} - Z_{(\alpha/2)} \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}} \le p \le \hat{p} + Z_{(\alpha/2)} \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}}}$$

$n = 423$

$x = 72$

$$\hat{p} = \frac{x}{n} = \frac{72}{423} = 0.17$$

Point

$90\% \Rightarrow \alpha/2 = 0.05$

$\Rightarrow z = 1.642$

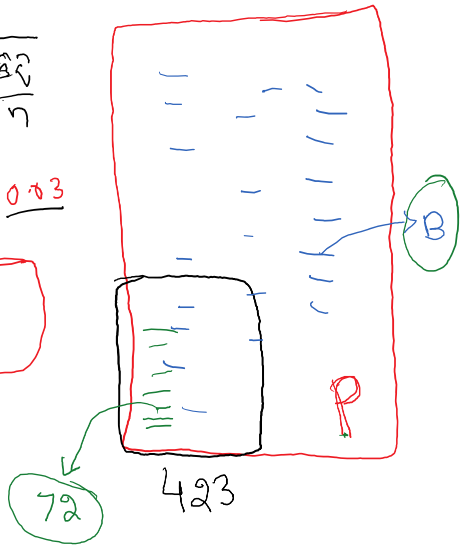
$\hat{q} = 0.83$

$$\hat{p} - z\sqrt{\frac{\hat{p}\hat{q}}{n}} \le p \le \hat{p} + z\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$= 0.17 - 0.03 \le p \le 0.17 + 0.03$$

$$= 0.14 \le p \le 0.20$$

Interval



Estimate Population Variance

→ Variance, estimator → $(n-1)$ denominator = df

$\boxed{\text{(Chi-square dist) } \chi^2 \quad \text{→ Relation b/n sample variance \& population variance}}$

$$\boxed{\chi^2 = (n-1)\, s^2 / \sigma^2}$$

$n-1$ times ratio of sample var & pop variance

$\boxed{\text{WARNING}}$ χ^2 → Find "population var"

→ Extremely sensitive to violations of Assumption that "Population is NORMAL"

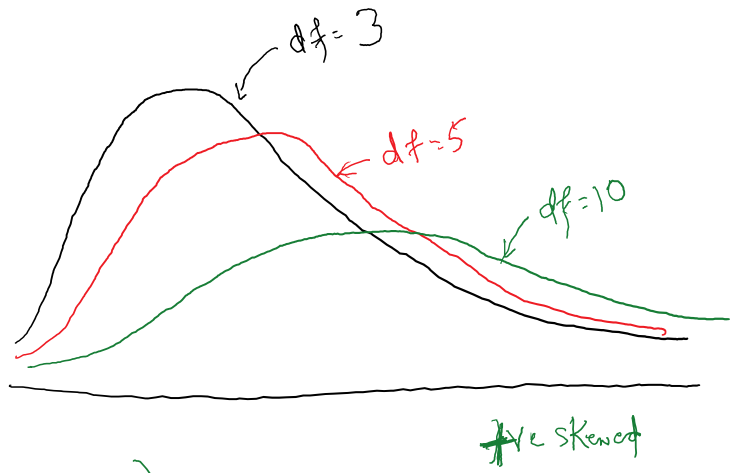
O/P

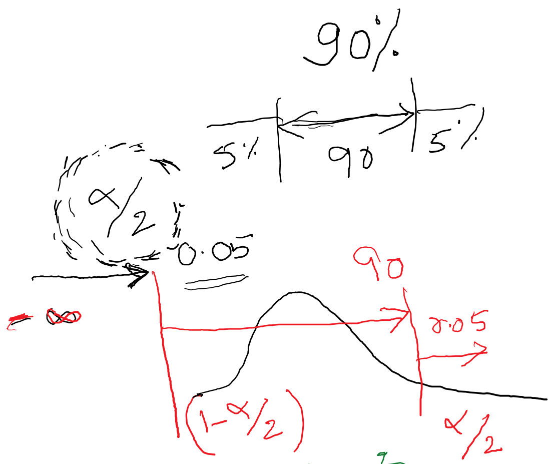
→ Apply χ^2 → ONLY, Population is NORMAL

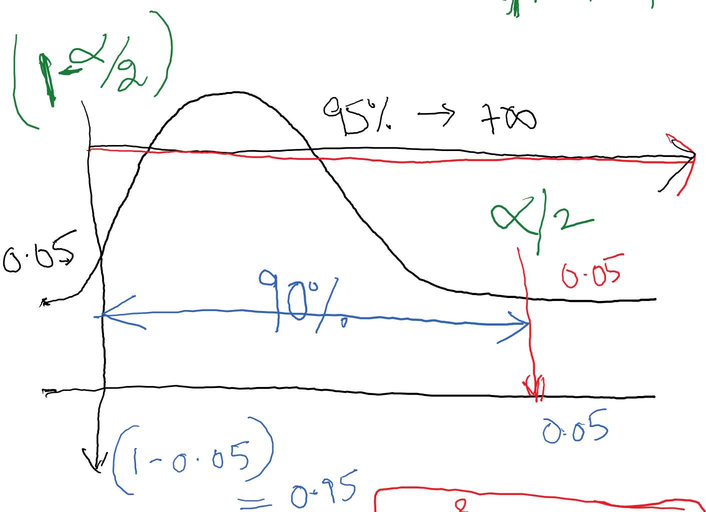
→ Chi-square → $\boxed{\text{ASSYMETRIC}}$ → shape varies with $\boxed{\text{df}}$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

$$\boxed{\sigma^2 = \frac{(n-1)s^2}{\chi^2}}$$







$$\sigma^2 = \frac{(n-1)\cdot s^2}{\chi^2} \rightarrow \text{Interval estimate}$$

χ^2 table
→ values of Right tail

$$\frac{(n-1)s^2}{\chi^2_{(1-\alpha/2),(df)}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{(\alpha/2),(df)}}$$

$\sigma = 1.12$

$\boxed{s^2 = \sigma^2 = 1.25}$

$n = 25$

$df = 24$

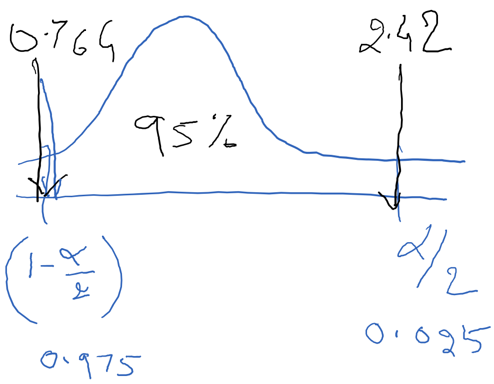
95%

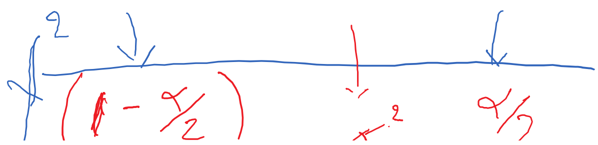
$1 - \alpha = 0.05$

$\alpha/2 = 0.025$

$$\chi^2_{(0.025)(24)} = 39.364$$

$$\chi^2_{(0.975)(24)} = 12.401$$





$\alpha/2 = 0.025$

$\chi^2\left(1-\alpha/2\right)$ χ^2 $\alpha/2$

Interval estimate

$$\frac{(n-1)s^2}{\chi^2\left(1-\alpha/2\right)} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2\left(\alpha/2\right)}$$

2 $\boxed{6}$ 4

$$\frac{6}{2} \le \sigma \le \frac{6}{4}$$

$$\boxed{3 \le \sigma^2 \le 1.5}$$

$$\boxed{\times\ 3 \le 1.5}$$

$$\frac{(n-1)s^2}{\chi^2\left(\alpha/2\right)} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2\left(1-\alpha/2\right)}$$

$$\frac{(24)(1.254)}{39.36} \le \sigma^2 \le \frac{(24)(1.25)}{12.401}$$

$$\boxed{0.764 \le \sigma^2 \le 2.428}$$

Estimating Sample Size

Sample size → linked → Sampling error

→ Level of Confidence

→ Width of estimation

1 → Sample size estimating μ

2 → —— 11 —— p

$(\bar{x} - \mu)$ → Error

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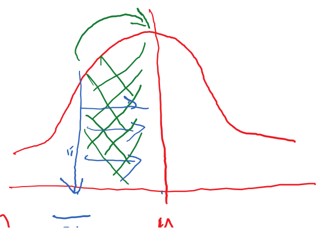
1 $\quad Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$\boxed{\begin{array}{c} \text{Diff} \\ (\bar{x} - \mu) \\ \downarrow \quad \downarrow \\ \text{sm} \quad \text{pm} \end{array}}$

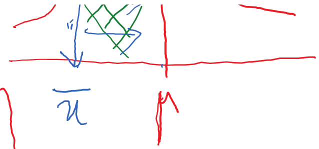


$\sigma/\sqrt{n}$

SM pm

Error

E Estimation



$$Z = \frac{E}{\sigma/\sqrt{n}} \quad \text{SBS}$$

$$Z^2 = \frac{E^2}{\sigma^2/n} \Rightarrow n = \frac{Z_{(\alpha/2)}^2\,\sigma^2}{E^2} = \left(\frac{Z\sigma}{E}\right)^2$$

$$\boxed{n = \left(\frac{Z\sigma}{E}\right)^2}$$

σ unknown

approximate $\sigma = \frac{1}{4}(\text{Range})$

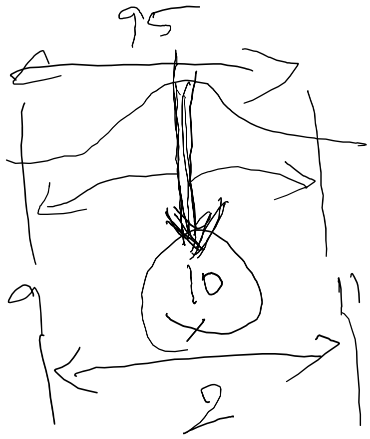
Range → 20

$E = 2$

95% ⇒ $Z = 1.96$

$\sigma = 5$

$$n = \frac{Z^2\sigma^2}{E^2} =$$



$$\frac{(1.96)^2\,5^2}{2^2}$$

$$\simeq 24.01 \simeq \boxed{25}$$

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② Sample Size when estimating (p)

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}}$$

$\hat{p} \neq p$

$E = \text{diff}(\hat{p} \,\&\, p)$

$$Z = \frac{E}{\sqrt{\frac{p \cdot q}{n}}} \quad \text{SBS} \quad Z^2 = \frac{E^2}{\left(\frac{p \cdot q}{n}\right)}$$

$$\Rightarrow n = \left(Z^2\,p\,q\right)/E^2$$

Rare Case

Approximate $p = 0.5$

$E = 0.03$

$p = 0.40$

$\Rightarrow q = 0.60$

98%

$z = 2.33$

$$n = (z^2 \cdot p \cdot q) / E^2$$

$$= \frac{(2.33)^2 (0.4)(0.6)}{(0.03)^2}$$

$$\boxed{n = 1447.70 \simeq 1448}$$