18 February 2018 09:59

AR, MA & ARIMA

AR → Auto regressive process

Todays observation regressed on yesterdays observations on all times

AR Recursion → Today = Const + Slope * Yest + Noise

Mean centered version

$$(\text{Today} - \mu) = \text{slope} * (\text{Yest} - \mu) + \text{Noise}$$

$$\Rightarrow Y_t - \mu = \phi (Y_{t-1} - \mu) + \epsilon$$

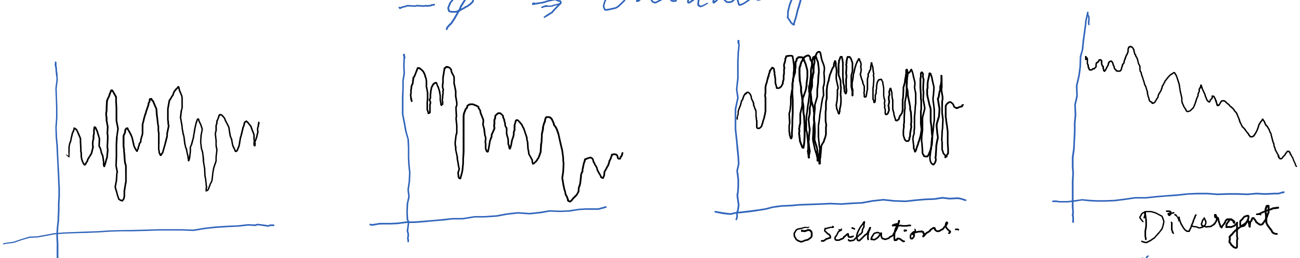
3 parameters → μ → Mean

ϕ → slope → If $\phi = 0$ → y is white Noise

→ If $\phi \neq 0$ → y depends of previous observation & current Noise & process is auto Corrected

$\phi \uparrow$ ⇒ great accuracy

$-\phi$ ⇒ oscillatory time series.



$\phi = 0.98$ $\phi = 1$ $\phi = -0.6$ $\phi = 1.01$

$\underbrace{\phi = 0.98 \quad \phi = 1}_{\text{Depends on Neighbors closely}}$

When $\mu = 0$ & $\phi = 1$ AR becomes RW & Not Stationary

Estimation & Forecast 1 step to h step process

Fitted graph ⇒ Estimation of today given yesterday

AR Fitted ⇒ $\widehat{\text{Today}} = \widehat{\text{Mean}} + \widehat{\text{Slope}} * (\text{Yest} - \widehat{\text{Mean}})$

AR Fitted = $\widehat{Today} = \widehat{Mean} + slope * (Yest - Mean)$

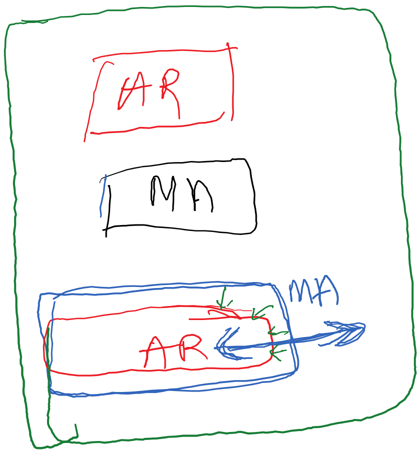
$$\Rightarrow \hat{y}_t = \hat{\mu} + \hat{\phi}(y_{t-1} - \hat{\mu})$$

Residual = $Today - \widehat{Today} \Rightarrow \hat{\epsilon}_t = Y_t - \hat{Y}_t$

SMA/SA → Moving Avg process.

Today = Mean + Noise + slope * (Yest Noise)

$$\Rightarrow y_t = \mu + \epsilon_t + \theta \epsilon_{t-1}$$

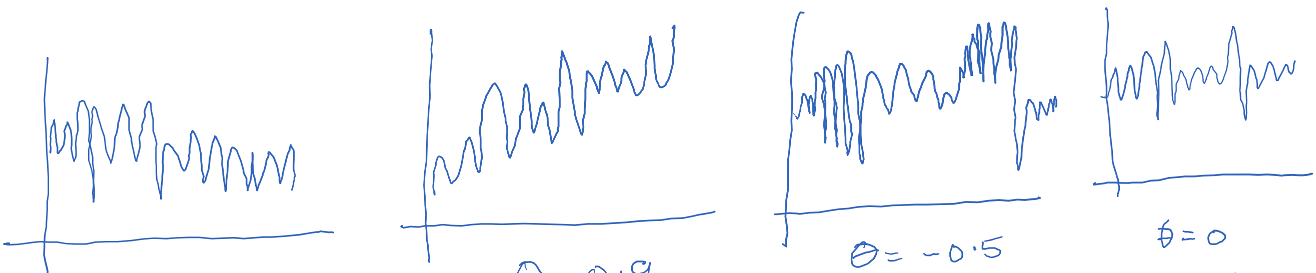


3 parameters → μ → Mean
ϕ → slope
σ_ϵ^2 → WN variance

If $\theta = 0$ → y is WN process.

If $\theta \neq 0$ → y depends on both ϵ_t & ϵ_{t-1} & auto correlated

$\theta \uparrow$ → great autocorrection, $-ve\ \theta$ → oscillatory time series.



$\theta = 0.5$ | $\theta = 0.9$ | $\theta = -0.5$ Oscillations | $\theta = 0$ White Noise.

Estimation & forecast

MA fitted $\widehat{Today} = \widehat{Mean} + \widehat{slope} * (Yest \widehat{Noise})$

$$Y_t = \mu + \theta_{\epsilon\, t-1}$$

Residual = $Today - \widehat{Today} = \hat{\epsilon}_t = Y_t - \hat{Y}_t$

Residual = Today − $\hat{\text{Today}}$ = $\hat{\epsilon}_t = Y_t - \hat{Y}_t$

Both give almost same fit (AR & MA) → we need selection process for best fit

Goodness of fit → Information Criterion

AIC → Alkaline IC
BIC → Baysean IC

} Lower AIC/BIC better is model.

ARIMA → Auto Regressive inclusive Moving Avg →

Also Known as Box-Jenkins process ⇒ Combination of AR & MA

Non Seasonal ARIMA is given by ARIMA(p, d, q)

p → no of regression terms → Order of AR Model

d → no of Non Seasonal diff needed for stationarity

q → no of lagged forecast errors in the prediction equation.

AR → linear regression of current value with one or more prior values

MA → ~ linear regression of current value with WN of one or more prior value series.

Steps → given in Code → ARIMA

→ Load data.

→ MA(q) → Visualise Moving Avg

→ STL () → Decompose Moving Avg

→ Test Stationality & Non-Stationality

→ Analyse Auto-Correlation ACF & find q in MA(q)

q → no of lagged forecasts error in prediction eqn

→ Analyse Partial ACF (PACF) & find P in AR(P)

p → no of autoregressive terms.

→ Analyse ACF & PACF & start with d = 1.

⇒ Forecast eqn → First cut fit → auto defined order (P, Q, d)

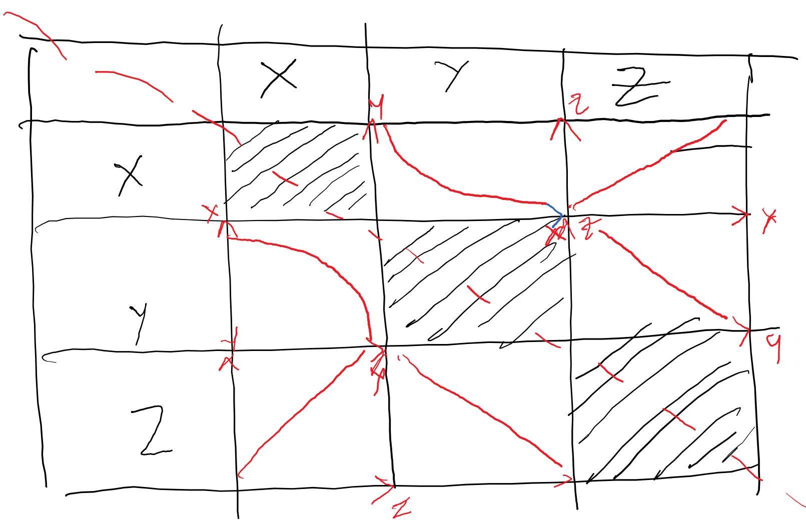
→ user ——— (p, q, d)

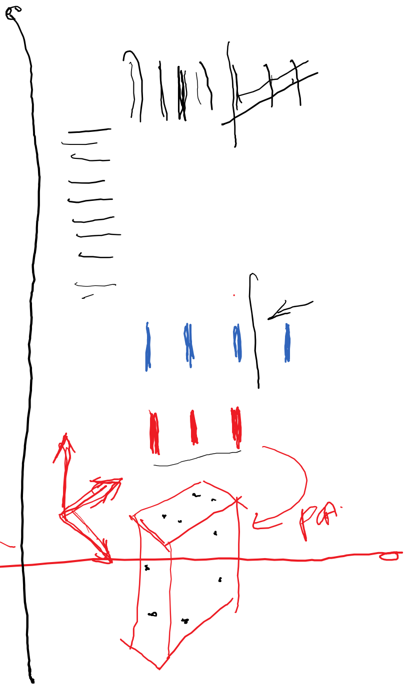
⇒ Forecast from fit model

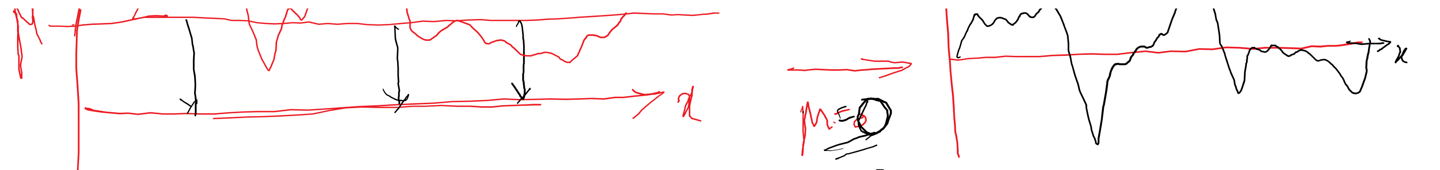
→ Forecast without seasonal component

→ Forecast with seasonal component

Correlation Matrix







Mean Centered Normalisation