## Hypothesis Testing → Tentative explanation of principles in Nature

→ Research Hypothesis

→ Statistical Hypothesis → Formal way → Evidence / Proof

→ Substantive Hypothesis

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## Statistical Hypothesis

NULL Hypothesis → $H_0$ → No change (old is gold)

Alternate Hypothesis → $H_A$ → Change has happened

$\underbrace{H_0 = 500\text{ gm}}$ $\underbrace{H_A \neq 500\text{ gm}}$

Board Two tail test → Directionless → $=$ or $\neq$ (Yes or No)

profits. One tail test → Directional → $>$, $<$ → Profit, height, lower.

ROI 28% 2016 → Rebrand Marketing Brand Ambassadors 2017 → Sales → ROI > 28% 2018

$\underbrace{H_0 = 28\%\,?}$ $\underbrace{H_A > 28\%\,?}$

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Hypothesis procedure → H.T.A.B (Assume → $H_0$ → TRUE)

H → Hypothesise → Develop → $H_0$ & $HA$

Select a test, $\alpha$ $\boxed{0.05}$, Decision Rule | Gather Data

T → Test →

T → Test → Select a test, $\alpha$ , Decision Rule

$\boxed{0.05}$
0.01
0.1
0.001

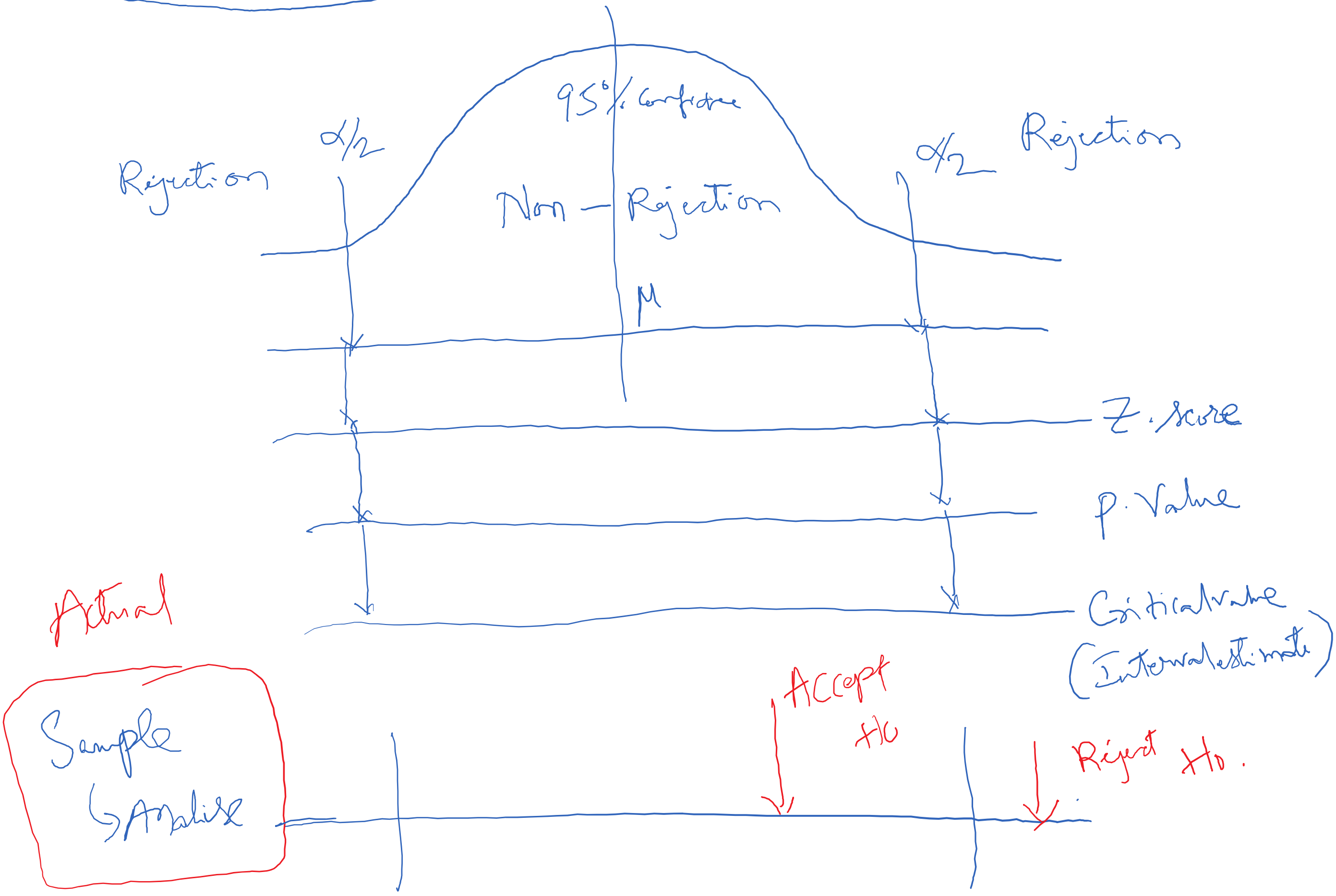
→ Critical value
→ p value
→ $Z / t / f^2$

Theory

y other Data
Analyse Data
Practical

A → Action → Decide to Accept / Reject → $H_0$

B → Business Implication → Apply / check → Business Implications.




$\mu = 74,914$  $n = 112$ , $\bar{x} = 78,695$ , $\sigma = 14,530$ (Assume $H_0$ → TRUE)

① → $H_0 = \mu = 74,914$ → Always have "="

$H_A = \mu \neq 74,914$ → Can have $>, <, \neq$

② → Z test → Random samples

→ $\delta$ not known

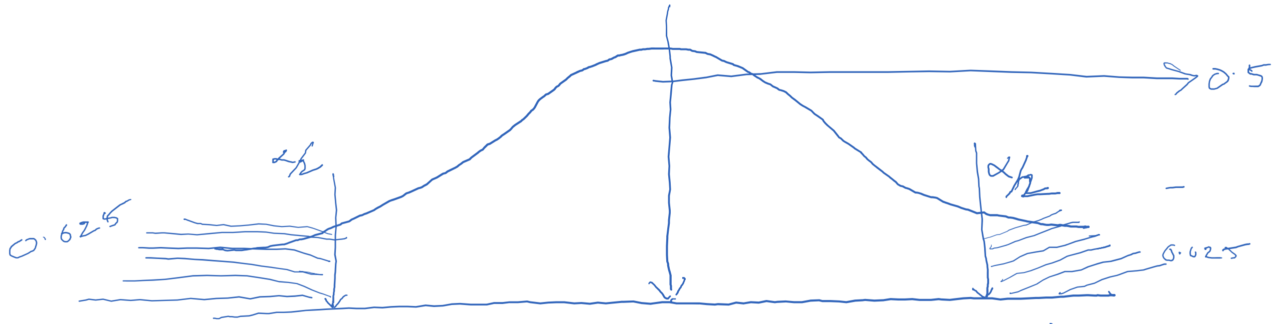
→ $n \geq 30$

→ Distri is normal

$$Z = \bar{x} - \mu / \sigma/\sqrt{n}$$

3 $\alpha = 0.05 \Rightarrow \alpha/2 = 0.025$

4 Critical value → $\alpha$ & Z stat




$Z(-0.475)$    74,914    $Z(0.475)$ → pvalue (T)

↓      ↓

$-1.96$      $+1.96$ → Z

$$\bar{x}_c = \mu \pm Z \cdot \frac{\sigma}{\sqrt{n}}$$

↓      ↓

74,917 − 2691      74917 + 2691

$\boxed{72,223}$      $\boxed{77,605}$ → Critical Value.

5 & 6 112 people → $\sigma = 14,530$, $n = 112$, $\boxed{\bar{x} = 78,695}$

practical

$$Z\text{ score} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{78,695 - 74,914}{14530/\sqrt{n}} = \boxed{2.75}$$

$Z_{(p)} > Z_{(T)} \Rightarrow 2.75 > 1.96 \Rightarrow$ Reject → $H_0$

Observe → $\boxed{\bar{x} = 78695} > \boxed{77605} \Rightarrow$ Reject → $H_0$

7

8 → The old Avg Salary ⇒ No longer Holds good.

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P-value (Practical) → Probability of getting sample means AS EXTREME or MORE EXTREME than observed

→ P-value is the smallest value of $\alpha$ for which we can Reject $H_0$

$\alpha_T = 0.025$, $\bar{x} = 78{,}695$

$$P\left(\bar{x} > 78{,}695 \;\middle|\; H_0 \to \text{True} \;\middle|\; \mu = 74914\right)$$

$$P\left[\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > \frac{78{,}695 - 74914}{14530/\sqrt{112}}\right] \Rightarrow P(z > 2.75)$$

$$= P(>(0.5 - 0.4970))$$

$\Rightarrow \underline{0.003}_{P} < 0.025_{T}$ → Reject $H_0$

↳ Reject $H_0$

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$\alpha + \beta = 1$

| | $H_0$ TRUE | $H_0$ FALSE |
|---|---|---|
| Accept Ho | Correct Judgment | Type 2 error $\beta$ |
| Reject Ho | Type 1 error $\alpha$ | Correct Judgement (Power) |

$T_1$ → Rejecting a True $H_0$

$T_2$ → Accepting a False $H_0$

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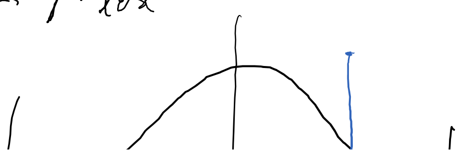
Hypothesis about $\mu$ & $\sigma$ (Unknown) → T-test

$H_0 \Rightarrow \mu = 25$



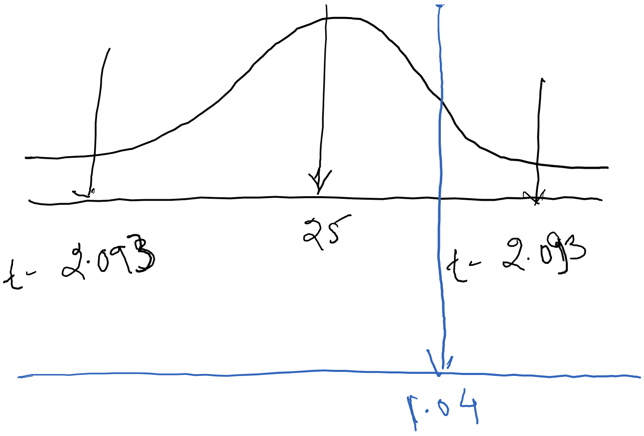
$H_0 \Rightarrow \mu = 25$

$H_A \Rightarrow \mu \neq 25$

$\alpha = 0.05 \Rightarrow \alpha/2 = 0.025$

$n = 20 \Rightarrow df = (n-1) = 19$

$\Rightarrow t_{(0.025)(19)} \rightarrow 2.093$




→ Select samples → $\bar{x} = 25.51$, $s = 2.193$, $n = 20$

→ $\boxed{\text{Reject } H_0 \text{ if } t < -2.093 \text{ or } > +2.093}$

$$\Rightarrow t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{25.51 - 25}{2.193/\sqrt{20}} = \underline{\underline{1.04}}$$

$t = 1.04 < 2.093 \rightarrow$ <u>$H_0$ is NOT Rejected</u>

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Testing for Proportion $(\hat{p})$

CLT $\quad Z = \dfrac{\hat{p} - p}{\sqrt{\dfrac{p \cdot q}{n}}}$

$n = 140$, $x = 48$

$\hat{p} = 0.34$

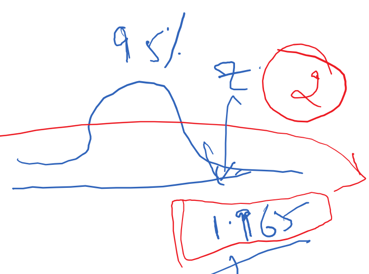
$$Z = \frac{0.34 - 0.26}{\sqrt{\dfrac{0.26 \times 0.74}{140}}}$$

$\alpha = 0.05 \Rightarrow \alpha/2 = 0.025$

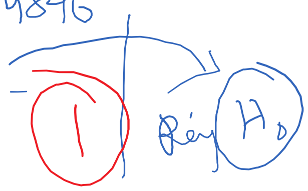
$H_0 = p = 26$

$H_A = p > 26$

$$\boxed{Z = 2.16}$$




$\alpha = 0.05 \Rightarrow \alpha/2 = 0.025 \rightarrow Z = 1.965$




Conclusion → Market Share is changed & $> 26\%$

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$\chi^2 \rightarrow$ Variance $\qquad$ ② → $\chi^2 = (n-1)s^2$

$\chi^2 \to$ Variance

Assume $H_0$ is TRUE

① $H_0 \to \sigma^2 = 25$

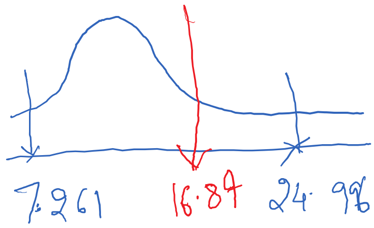
$H_A \to \sigma^2 \neq 25$

② $\to \chi^2 = \dfrac{(n-1)s^2}{\sigma^2}$

③ $\to \alpha = 0.10$

$\alpha/2 = 0.05$

④ $df = (n-1) = 15$




$\boxed{S^2 = 28.062}$

| | | | |
|---|---|---|---|
| 57 | 56 | 52 | 44 |
| 46 | 53 | 44 | 44 |
| 48 | 51 | 55 | 48 |
| 63 | 53 | 51 | 50 |

$\chi^2_{(1-0.05)(15)} = \chi^2_{(0.95)(15)} = \boxed{7.261}$

$\chi^2_{(0.05)(15)} = \boxed{24.996}$

Decision $\to$ Reject $H_0$ $\to$ observed value is $< 7.261$ & $> 24.996$

⑤ Table data $\to s^2 = 28.062$

observed $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{15(28.062)}{25} = \boxed{16.84} \to$ Accept $H_0$

Conclusion $\to$ There is no variance in my overtime value

How to solve for type 2 errors

$H_0 \to$ Correct

$H_0 \to$ Type 2 error $\to$ Business loss

$\to$ Miss opportunities

$\to$ Improvement can be stopped

Type 2 error $\to$ Probability

$\to$ Soft drink (Type ② $\to$ Fail to Reject a False $H_0$)

$H_0 \rightarrow \mu = 12$

$H_A \rightarrow \mu \neq 12$

If $\mu = 12$ is false

Various possibilities ⇒ $\mu = 11.9,\ 11.99,\ 12.5,\ 10$

% % % %

Diff possibility (%) of Commiting type 2 error

How?

→ 60 Cans → $(\mu)_{\bar{x}} = 11.985$ → $\sigma = 0.10$, $\alpha = 0.05$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{11.985 - 12}{0.10/\sqrt{60}} = -1.16 \text{ (observed)}$$

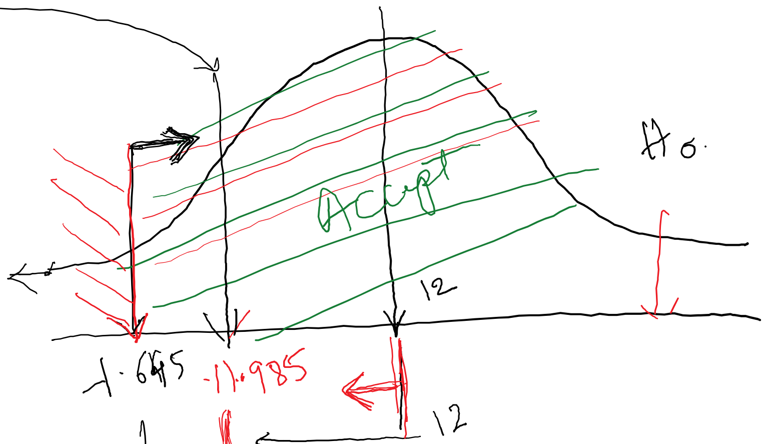
Observed

$\mu = 11.985 > 11.979$

→ Accept Ho

Correct

Type II



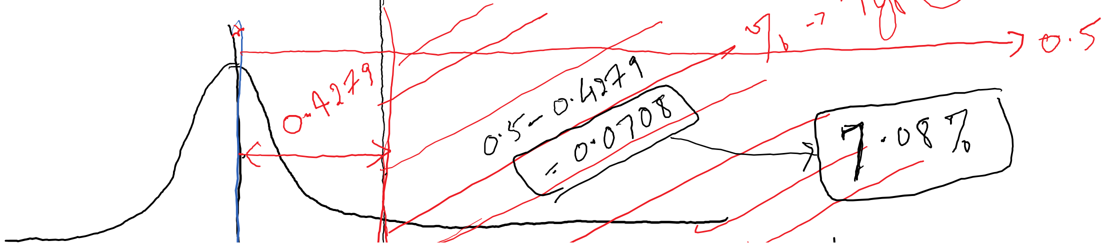

-11.979 → Theoritical $\bar{x}_c$

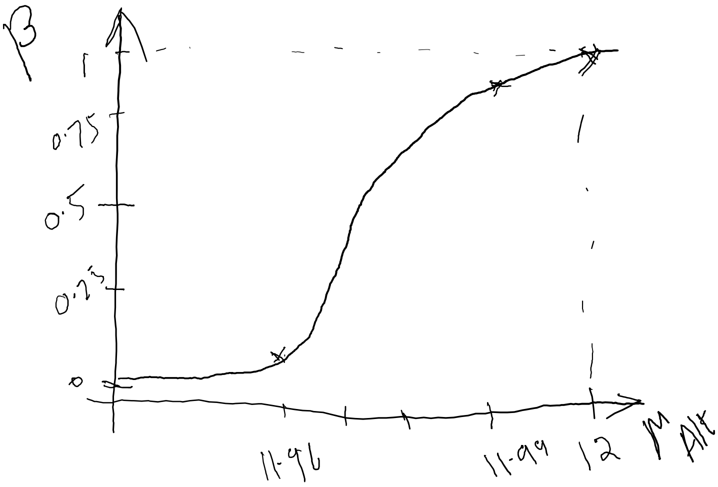
⇒ 0.5 + 0.32

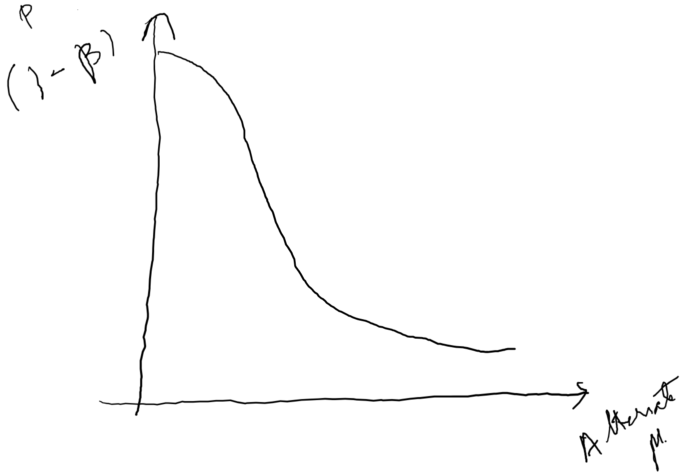
⇒ 0.802

⇒ 80%

Type 2 error

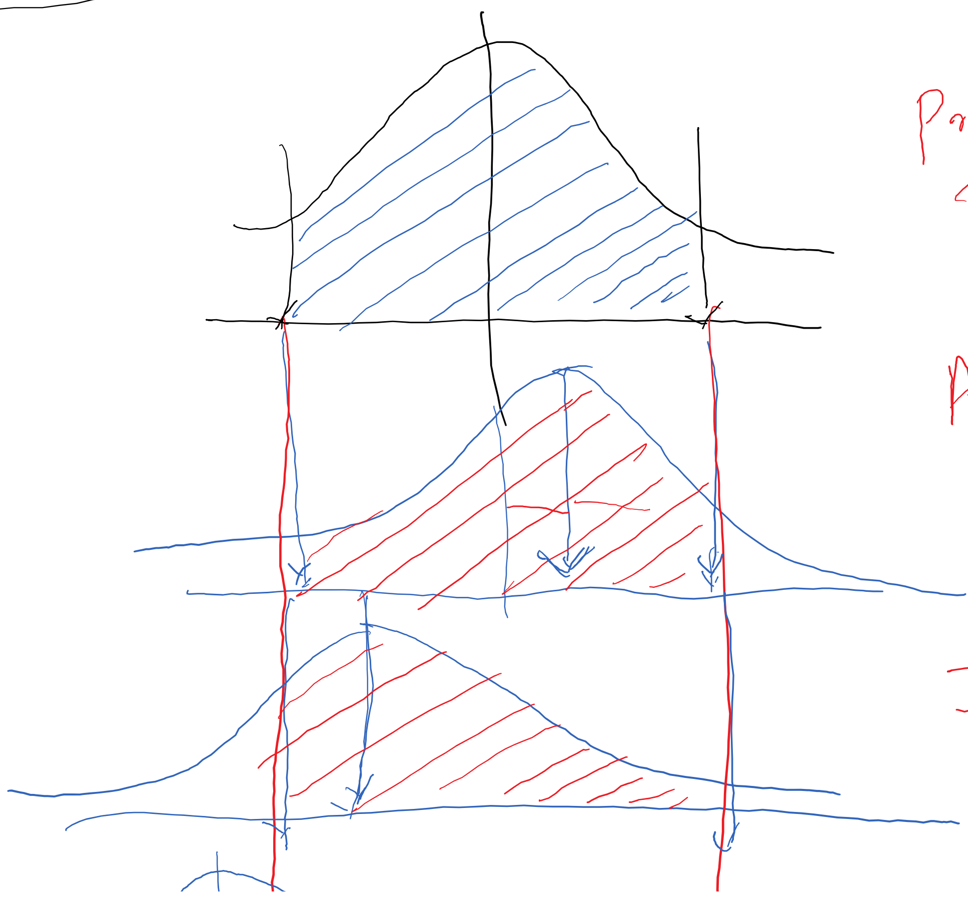



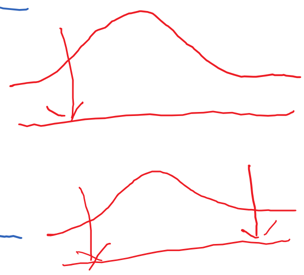

O-C curve

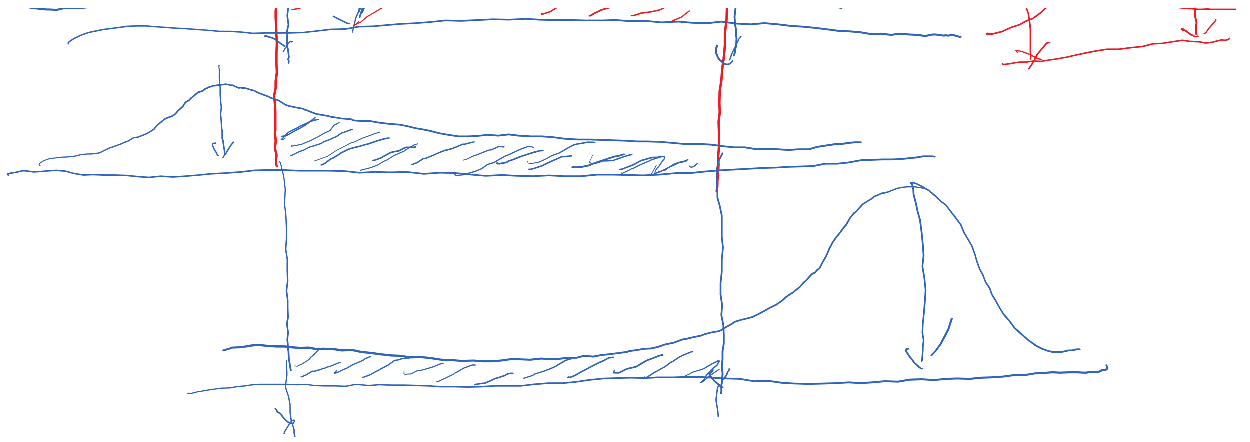
→ o/p character curve.

Power curve



Probability of Type 2 error is Area within Critical Values

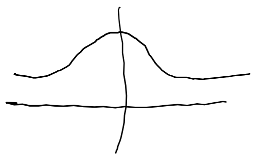




Testing for difference

CLT $\rightarrow$ $Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$n \geq 30$



$$\boxed{Z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}}$$

Find the difference in Sample mean $\rightarrow$ $\sigma$ Known

Managers Avg Annual wages. ($H_0 \rightarrow$ TRUE)

$H_0: \mu_1 = \mu_2$

$H_A: \mu_1 \neq \mu_2$

$\alpha = 0.05 \Rightarrow \dfrac{\alpha}{2} = 0.025$

$z = \pm 1.96$


$\longrightarrow 2.35$

$$Z = \dfrac{(70.70 - 62.18) - (0)}{\sqrt{\dfrac{264.160}{32} + \dfrac{166.411}{34}}}$$

$Z = 2.35 > 1.96 \Rightarrow$ Reject $H_0$

$\rightarrow$ Reject $H_0$

$\Rightarrow$ There is difference in Avg salary

... interval. (Actual)

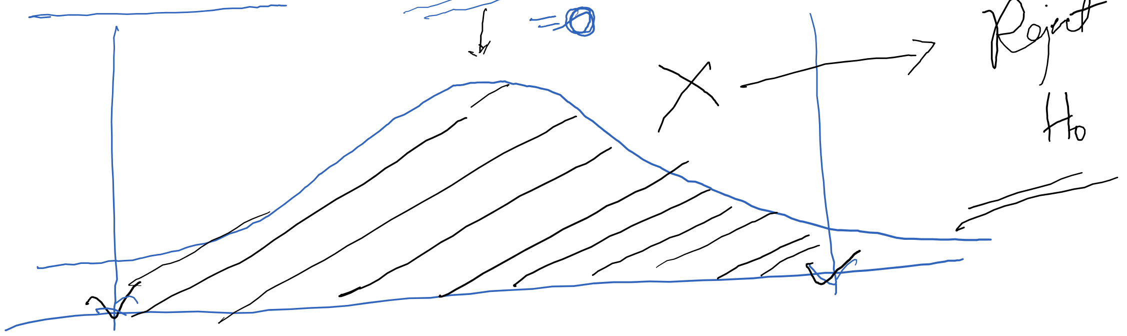
Confidence interval. (Actual)

$$(\bar{x}_1 - \bar{x}_2) - Z\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + Z\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$8.513 - 7.1074 \le \mu_1 - \mu_2 \le 8.513 + 7.1074$$

$$1.405 \le \mu_1 - \mu_2 \le 15.620$$

$\mu_1 - \mu_2 = 0$




$\sigma$ Unknown $\rightarrow$ T Test $\Rightarrow$ T test $\rightarrow$ 2 sample equal variance

Excel.

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} \Rightarrow \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2(n-1) + s_2^2(n-1)}{n_1 + n_2 - 2}}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$s_p^2 \rightarrow$ Weighted Avg $s_1^2$ & $s_2^2$


$(n-1)$ df $\quad \boxed{n_1 + n_2 - 2} \quad (n-1)$

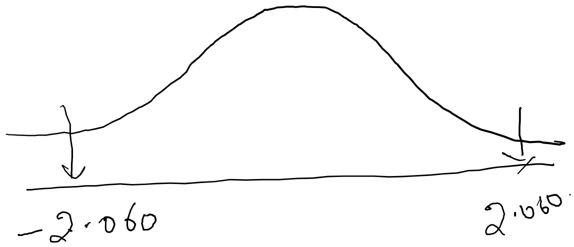
$H_0: \mu_1 = \mu_2$

$H_A: \mu_1 \neq \mu_2$

$\alpha : 0.05$

$\alpha/2 = 0.025$

| | $\mu_1$ | $\mu_2$ |
|---|---|---|
| | $n_1 = 15$ | $n_2 = 12$ |
| | $\bar{x}_1 = 47.73$ | $\bar{x}_2 = 56.56$ |
| | $S_1^2 = 19.49$ | $S_2^2 = 18.273$ |




$t_{(0.025)(25)} = 2.060$

$$t = \frac{(47.73 - 56.56) - (0)}{\sqrt{\frac{(19.47)(14) + (18.27)(11)}{25}} \sqrt{\frac{1}{15} + \frac{1}{12}}} = -5.20$$

## Confidence Interval

$$(x_1 - x_2) \pm t \sqrt{\frac{S_1^2(n-1) + S_2^2(n-1)}{n_1 + n_2 - 2}} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \leq \mu_1 - \mu_2$$

## Statistical inference for 2 Related Populations (Matched, Dependent, Correlated)

Excel → T. test, Paired sample for means

Ex. Before → After, Twins.

$n$ = no of sample in pairs.

$d$ = sample diff in pairs

$D$ = $\mu$ population diff

$S_d$ = Sd of sample diff

$\bar{d}$ = $\mu$ of sample diff

$$t = \frac{\bar{d} - D}{S_d / \sqrt{n}}$$

$$\bar{d} = \frac{\Sigma d}{n} \qquad S_d = \sqrt{\frac{\Sigma (d - \bar{d})^2}{n-1}} \Rightarrow \sqrt{\frac{\Sigma d^2 - (\Sigma d)^2 / n}{n-1}}$$

## Statistical inference for Proportions $(\hat{p}_1 - \hat{p}_2)$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{p_1 q_1}{\;} \quad \frac{p_2 q_2}{\;}}} \longrightarrow z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{(\;)(\frac{1}{\;} + \frac{1}{\;})}}$$

$$z = \frac{(p_1 - p_2) - (P_1 - P_2)}{\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}} \longrightarrow z = \frac{\ldots}{\sqrt{(\bar{p} \cdot \bar{q})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where $\boxed{\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{n_1 \hat{p} + n_2 \hat{p}}{n_1 + n_2}}$

$\boxed{\bar{q} = 1 - \bar{p}}$

Confidence Interval:

$\boxed{\phantom{xx}} \leq \qquad \leq \boxed{\phantom{xx}}$

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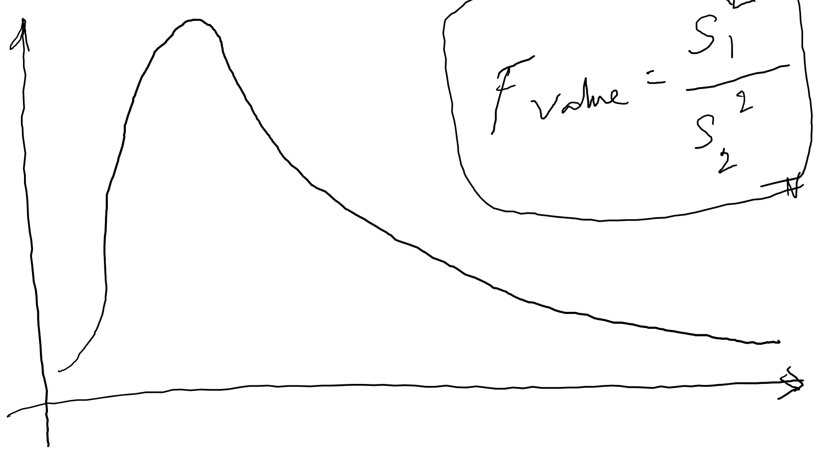
## Testing hypothesis for 2 population Variance

Excel ⇒ F test - 2 sample for variance

1 population: $\chi^2$ ⇒ Analysis; $\chi^2$ ⇒ Hypothesis test



$$F\ \text{value} = \frac{S_1^2}{S_2^2}$$

F Dist → 2 population
↓
ANOVA

→ Varies with 'n'

→ Extremely sensitive to violation of normal dist assumption

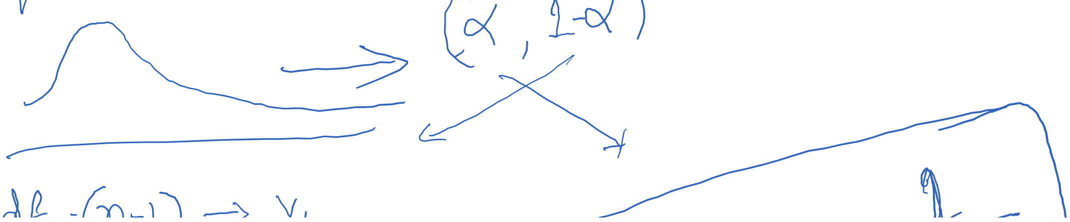
→ Non-Symmetric

→ Varies with $\alpha$ → Different plots for diff $\alpha$

$\alpha = 0.10$, $0.05$, $0.025$
↓
Different F-tables.

→ No mean = 0.

→ F table gives only values of upper table $(\alpha, 1-\alpha)$



$n \to df\ (n-1) \to v_1$

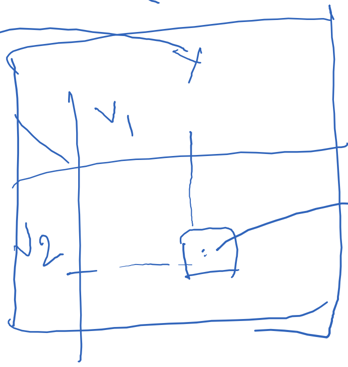
$n_1 \Rightarrow df = (n_1 - 1) \rightarrow v_1$

$n_2 \rightarrow df = (n_2 - 1) \rightarrow v_2$

$$F_{(1-\alpha/2), v_1, v_2} = \frac{1}{F_{(\alpha/2), v_1, v_2}}$$

$F_{(\alpha/2)(v_1)(v_2)} =$

df df




→ Right tail value

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$$F = \frac{S_1^2}{S_2^2}$$

$\Rightarrow 5.62$

**$M_1$**

| | |
|---|---|
| 22.3 | 21.9 |
| 21.8 | 22.4 |
| 22.3 | 22.5 |
| 21.6 | 22.2 |
| 21.8 | 21.6 |

$S_1^2 = 0.1137$

$n = 10$

$df = 9$

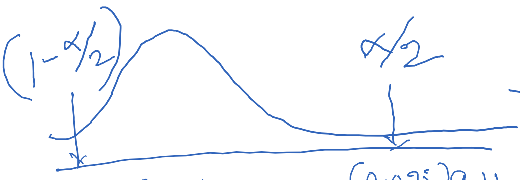
**$M_2$**

| | |
|---|---|
| 22.0 | 21.7 |
| 22.1 | 21.9 |
| 21.8 | 22.0 |
| 21.9 | 22.1 |
| 22.2 | 22.9 |
| 22.0 | 22.1 |

$S_2^2 = 0.0202$

$n = 12$

$df = 11$




$(0.975)_{9,11}$ → 0.28

$(0.025)_{9,11}$ → $\boxed{3.59}$ → Reject $H_0$ ⇒

$$F_{(1-\alpha/2)} = \frac{1}{F(\alpha/2)}$$

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Anova → Analysis of variance

Experiment Design → Plan / Design for Hypothesis Test: Researcher Can Control / Modify the variables.

Independent variables :→ Controlled / Modified.

| → levels of treatment

1p

$\chi^2$ → 1 population Variance

$\chi^2$ → Hypothesis Variance

F → Dist → 2 pop variance

⇓ → 3 sets of samples

Anova.

Independent variable ...

↳ levels of treatment

Anova.

Dependant variables : Response of Independent variables.

Tyre Quality : → Independent variable

| Low | Med | High |
|---|---|---|
| 1 k | 1.5 k | 2.1 K |
| 1.5 K | 2 K | 2.2 K |
| 1.25 | 2.1 K | 2.3 K |
| 1.7 | 2.3 | 3.1 |

→ Levels / Treatment

→ Dependant variable

METRO

56%

| | E | W | N | S |
|---|---|---|---|---|
| 1 | x K | y k | 2 k | A k |
| 2 | $x_2$ k | y k | 2 k | A k |
| 3 | $x_3$ k | y k | 2 k | A k |
| 4 | $x_4$ k | y k | 2 k | A k |
| 5 | | | | |

Sales Amount

| Valve Opening By Operator | | | |
|---|---|---|---|
| Opr 1 | Op 2 | Op 3 | Op 4 |
| 6.33 | 6.26 | 6.44 | 6.29 |
| 6.26 | 6.36 | 6.38 | 6.23 |
| 6.31 | 6.23 | 6.58 | 6.19 |
| 6.29 | 6.27 | 6.54 | 6.21 |
| 6.4 | 6.19 | 6.56 | |
| | 6.5 | 6.34 | |
| | 6.19 | 6.58 | |
| | 6.22 | | |

→ Independent

→ Levels

Dependant

Unexplained

$${}^nC_r = \frac{n!}{(n-r)!\,r!} = \frac{24}{4} = 6$$

→ $\alpha$ → Error of estimation

$\alpha \times \alpha \times \alpha$ Exponentially

Anova in general → K samples.

(1) $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \dots \mu_K$

$H_a$ : Atleast one $\mu$ is not same

Ind ← Phones

Levels ←

| Nokia | Samsung | Iphone |
|---|---|---|
| 2D | 14h | 12h |
| 3D | 15h | 9h |
| 2.5D | 18h | 16h |
| | 20h | 30h |

Dep

Expl

Unexp

(2) Total variance $\boxed{SST}$

(+)

$SST = SSC + SSE$

$\boxed{+}$

Variance from Treatment of Columns (Levels)

$\boxed{\text{or}}$

Explained Variance

$\boxed{\text{or}}$

Between group Variance

$\boxed{\text{or}}$

SSC

Error Variance Within Columns.

$\boxed{\text{or}}$

Unexplained Variance

$\boxed{\text{or}}$

Within group Variance

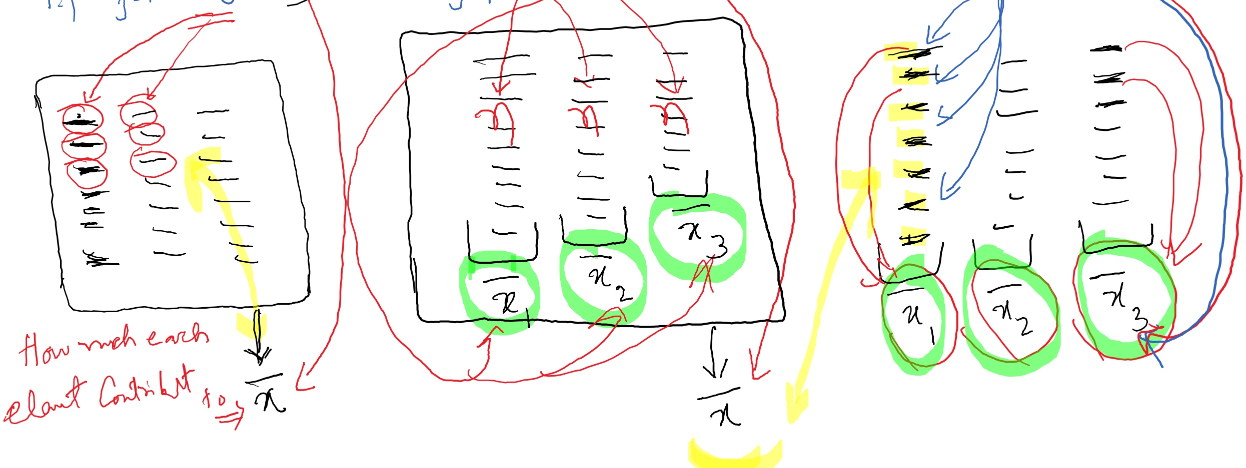
$\boxed{\text{or}}$

SSE

---

SST = SSC + SSE

$$\sum_{i=1}^{n}\sum_{j=1}^{c}\left(x_{ij}-\bar{\bar{x}}\right)^2 = \sum_{j=1}^{c} n_j\left(\bar{x}_j-\bar{\bar{x}}\right)^2 + \sum_{i=1}^{n_j}\sum_{j=1}^{c}\left(x_{ij}-\bar{x}_j\right)^2$$




SST → Total Sum of Squares

SSC → Column Sum of Squares

SS E → Error Sum of Squares

$i$ = Element no in a level

$j$ = Treatement label / Column no

$C$ = no of Columns / Levels

$n$ = no of observations in a level

$\bar{\bar{x}}$ = grand mean.

$\bar{x}_j$ = Column mean

$df_c$ = C − 1 (Numerator df)

$df_T$ = N − 1 (Denominator df)

$df_E$ = N − C

$$MSC = \frac{SSC}{df_c}$$

$$MSE = \frac{SSE}{df_E}$$

$F = \frac{MSC}{\ldots}$ → Practical Value of → Compare

$\bar{x}_{j}$ : Column mean

$x_{ij}$ = individual value

$$\boxed{F_{value} = \frac{MSC}{MSE}} \rightarrow \text{Practical Value}$$

$F \rightarrow$ Compare with Theory value

⊛ Problem in Excel Sheet ⊛

---

Anova → test the hypothesis about the difference in means.

Advantage → probability of error ↓

4 groups 2 → 6 tests

$T_1 \quad T_2 \quad T_3 \quad T_4 \quad T_5 \quad T_6$

error % → $e_1\% \times e_2\% \times e_3\% \times e_4\% \times e_5\% \times e_6\%$ ↑

Anova →

$C_1 \quad C_2 \quad C_3 \quad C_4$

Var 1 Var 2 Var 3 Var 4

Total

→ Test for difference → More to total

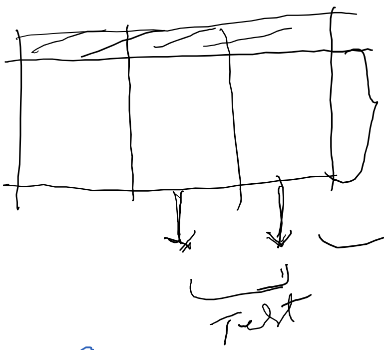
→ Priori → Test Differe

→ Posteriori → Actual test → Test diff

---

Tukey's HSD (Same sample size)

$$T_{HSD} = q_{\alpha, C, N-C} \sqrt{\frac{MSE}{n}}$$

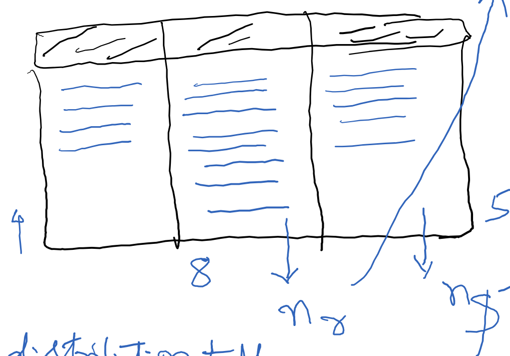


equal = n

Test

Tukey-Kramer's test (Diff Sample size)

$$T_{TK} = q_{\alpha, C, N-C} \sqrt{\frac{MSE}{2}\left(\frac{1}{n_r} + \frac{1}{n_s}\right)}$$




q table → Studentized range distribution table

---

$\chi^2$ = 1 Sample, $\chi^2$ → hypothesis, F = 2 Samples, > 2 Samples ← Anova

Z = 1 Sample, Z → hypothesis, T = 2 Samples, > 2 → Anova Samples.

## Anova

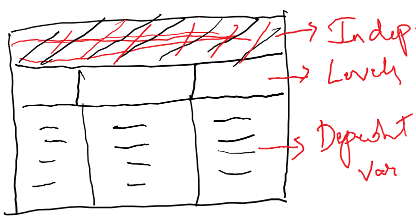
**Completely Randomized design**

⇓

1 Independent var

↓

ONE WAY ANOVA




Excel → Anova: Single factor

SST = SSE + SSC

1 F value

**Randomized Block Design**

⇓

1 Independent var + Blocking var

Anova - 2 factor - without Replication

$SST = SSC + \underbrace{SSR + SSE'}_{SSE}$

2 F values.

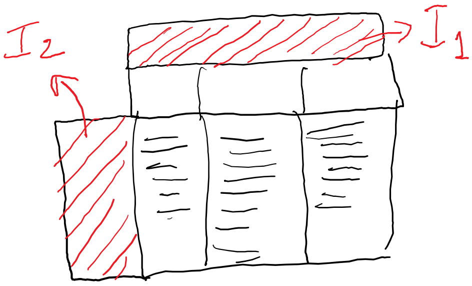
**Factorial Design**

⇓

2 Independent var

↓

TWO WAY ANOVA



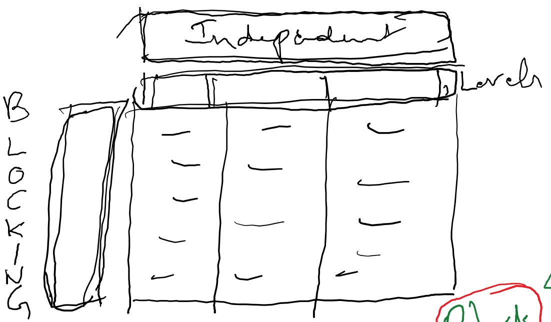

Anova - 2 factor With Replication

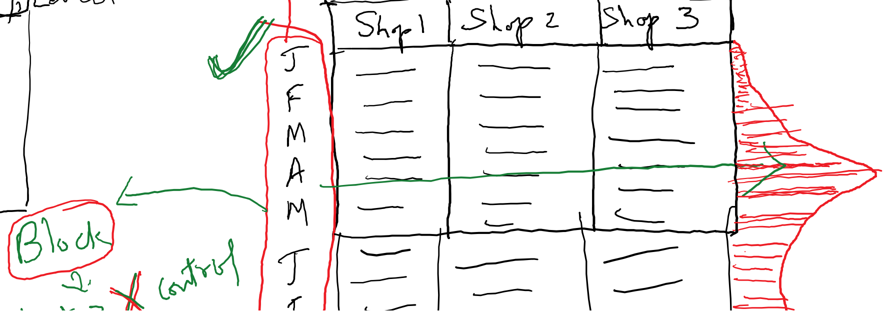
SST = SSC + SSR + SSE' + SSI

3 F values

---

Random block design → Blocking Var, Confounding var, Concommitent var.



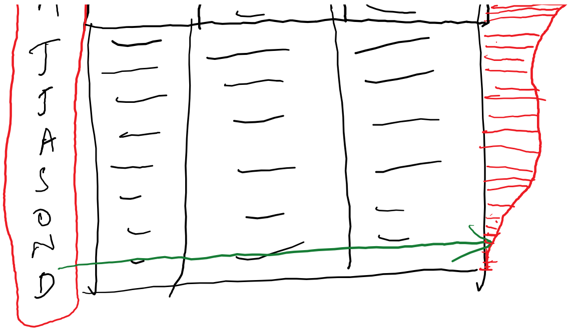



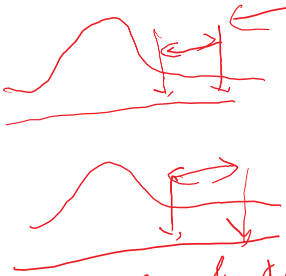
Block

Analyst → ✗ control

Independent

Analy → ✓ control

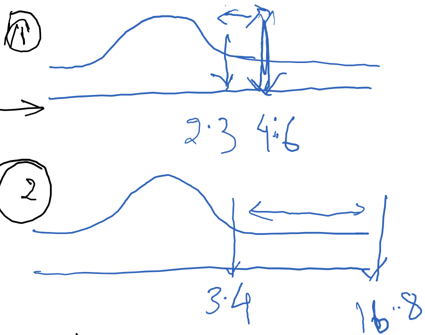


| | F value | | F crit |
|---|---|---|---|
| Without Block → | 4.631 | 4.613 | 2.336 |
| With block → | 16.818 | 5.13 | 3.418 |

→ Effect of blocking var.

No significant effect of blocking




1 → $H_0$ → TRUE

2 → $H_0 = \mu_1 = \mu_2 = \mu_3 = \cdots \mu_C$ $\qquad$ $H_0 = \mu_1 = \mu_2 = \mu_3 = \cdots \mu_R$

$H_A$ = Atleast one $\mu$ is $\neq$ $\qquad$ $H_A$ = Atleast one $\mu$ is $\neq$

3 $\alpha = 0.05$ $\qquad$ 4 Select F test → $\boxed{F_{1} \quad F_{2}}$ → Theory / Critical

$$SST = SSC + SSR + SSE$$

$$\sum^{n}\sum^{c}(x_i - \bar{x})^2 = n\sum^{c}(\bar{x}_j - \bar{x})^2 + c\sum^{n}(\bar{x}_i - \bar{x})^2 + \sum^{n}\sum^{c}(x_{ij} - \bar{x}_{i\cdot} - \bar{x}_{\cdot j} + \bar{x})^2$$

$df_c = c-1$ $\qquad$ $MSC = \dfrac{SSC}{c-1}$ $\qquad$ $MSE = \dfrac{SSE}{N-n-C+1}$

$df_R = n-1$ $\qquad$ → $H_0$ → TRUE / FALSE

$df_E = (c-1)(n-1)$ $\qquad$ $MSR = \dfrac{SSR}{n-1}$

$F_{\alpha,(c-1)((c-1)(n-1))} \rightarrow F_1$

$F_{\alpha,(n-1)((c-1)(n-1))} \rightarrow F_2$

Theory

$F = \dfrac{MSC}{MSE}$ → Treatment → level → Colums $\qquad$ $F = \dfrac{MSR}{MSE}$ block

practical / Actual

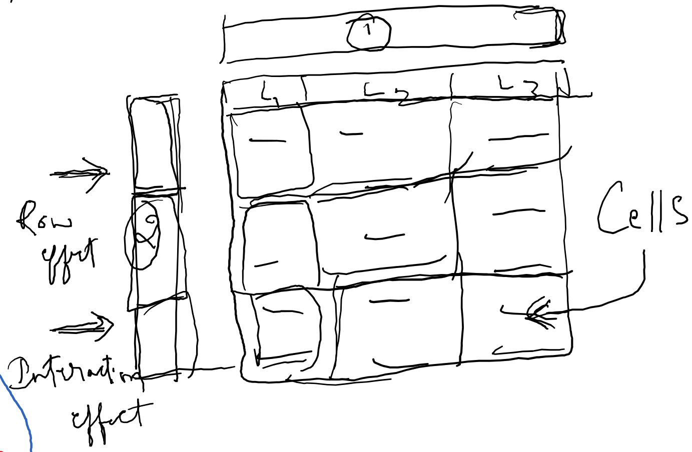
Factorial design → 2 Way Anova.

Factorial design → 2 way ...

Row effect
- $H_0$ - Row $\mu$ are equal
- $H_A$ - $\neq$

Column effect
- $H_0$ - Col means equal
- $H_A$ - $\neq$

Interaction effect
- $H_0$ - Interaction Not Present $= 0$
- $H_A$ - Present




SST = SSC + SSR + SSE + SSI

$$\sum_{i}^{R}\sum_{j}^{C}\sum_{k}^{n}\left(x_{ijk}-\bar{x}\right)^2 = nR\sum^{C}\left(\bar{x}_{i}-\bar{x}\right)^2 + nC\sum^{R}\left(\bar{x}_{j}-\bar{x}\right)^2 + \sum^{R}\sum^{C}\sum^{n}\left(x_{ijk}-\bar{x}_{ij}\right)^2 + n\sum^{R}\sum^{C}\left(\bar{x}_{ij}-\bar{x}_{i}-\bar{x}_{j}+\bar{x}\right)^2$$

$df_R = R-1$

$df_C = C-1$

$df_I = (R-1)(C-1)$

$df_E = RC(n-1)$

$df_T = N-1$

$MSR = \dfrac{SSR}{R-1}$ $\quad MSC = \dfrac{SSC}{C-1}$ $\quad MSI = \dfrac{SSI}{(R-1)(C-1)}$ $\quad MSE = \dfrac{SSE}{RC(n-1)}$

$$\boxed{F_R = \frac{MSR}{MSE} \qquad F_C = \frac{MSC}{MSE} \qquad F_I = \frac{MSI}{MSE}}$$

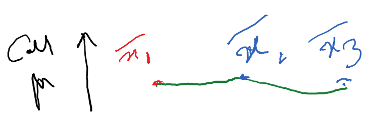
Practical/Actual

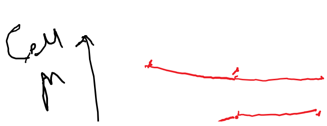
Row effect $\qquad$ Col effect $\qquad$ Interaction effect

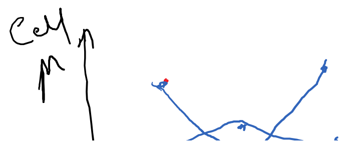
$$\boxed{F_{\alpha,\,R-1,\,RC(n-1)} \qquad F_{\alpha,\,C-1,\,RC(n-1)} \qquad F_{\alpha,\,(R-1)(C-1),\,RC(n-1)}}$$

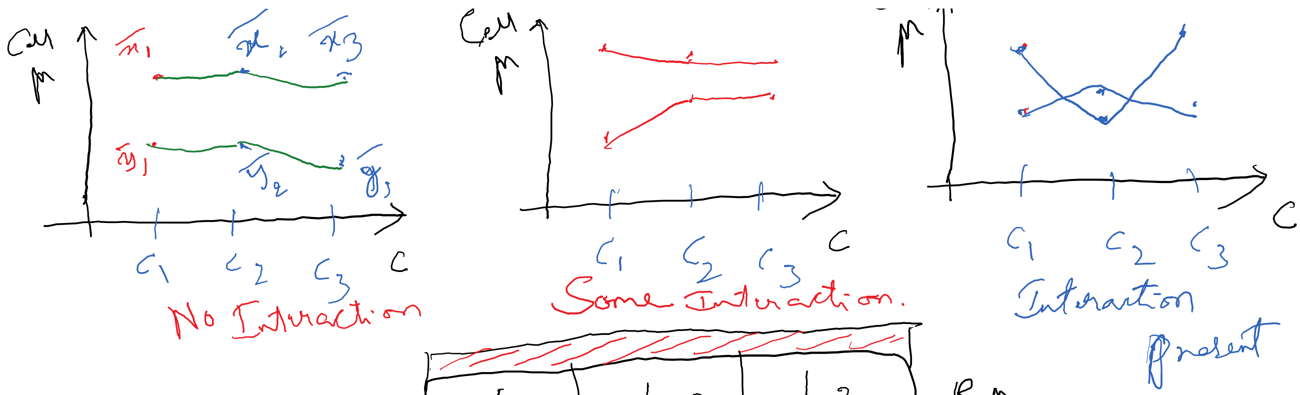
Theory

Interaction → Effect of one treatment varies accordingly to the level of treatment of other levels.

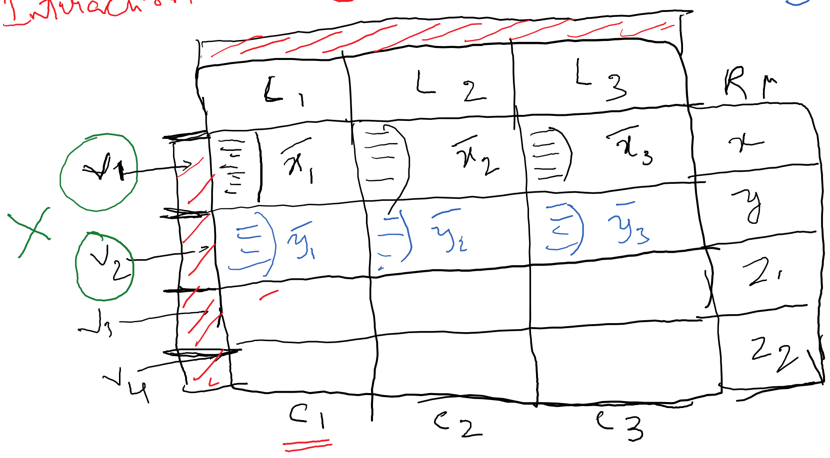


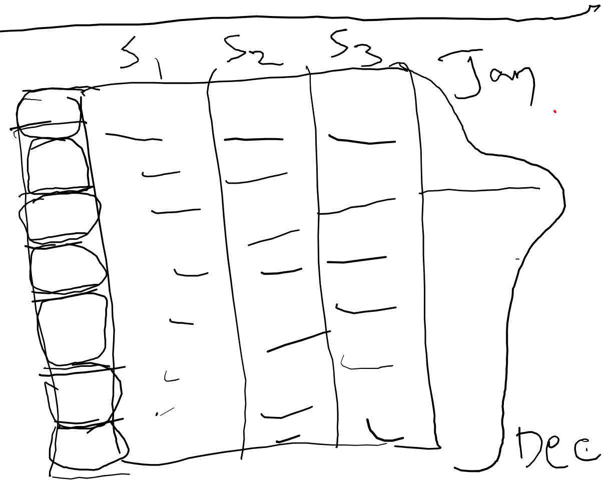


Cell
μ
$\bar{x}_1$ $\bar{x}_2$ $\bar{x}_3$
$\bar{y}_1$ $\bar{y}_2$ $\bar{y}_3$
$c_1$ $c_2$ $c_3$ C
No Interaction
Cell
μ
$c_1$ $c_2$ $c_3$ C
Some Interaction.
μ
$c_1$ $c_2$ $c_3$ C
Interaction Present

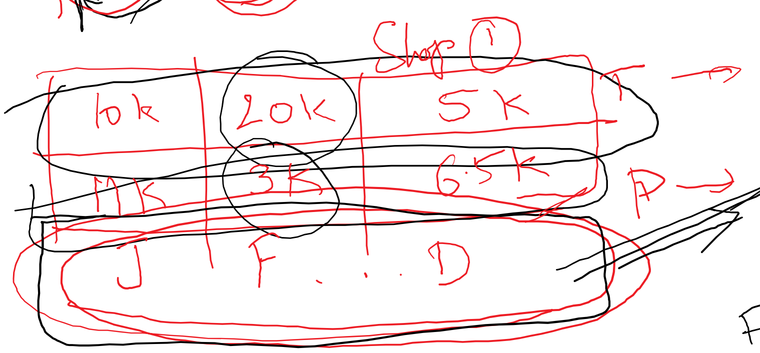



$L_1$ $L_2$ $L_3$
Rm
$\bar{x}_1$ $\bar{x}_2$ $\bar{x}_3$ x
$\bar{y}_1$ $\bar{y}_2$ $\bar{y}_3$ y
$z_1$
$z_2$
$v_1$ $v_2$ $v_3$ $v_4$
$c_1$ $c_2$ $c_3$

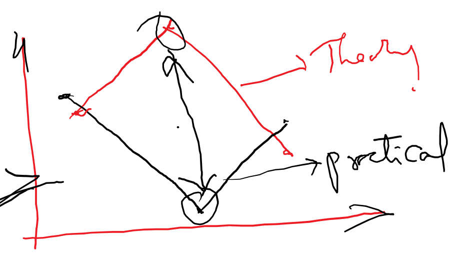

Sales
$S_3$
$S_1$
$S_2$
J F F A M D.

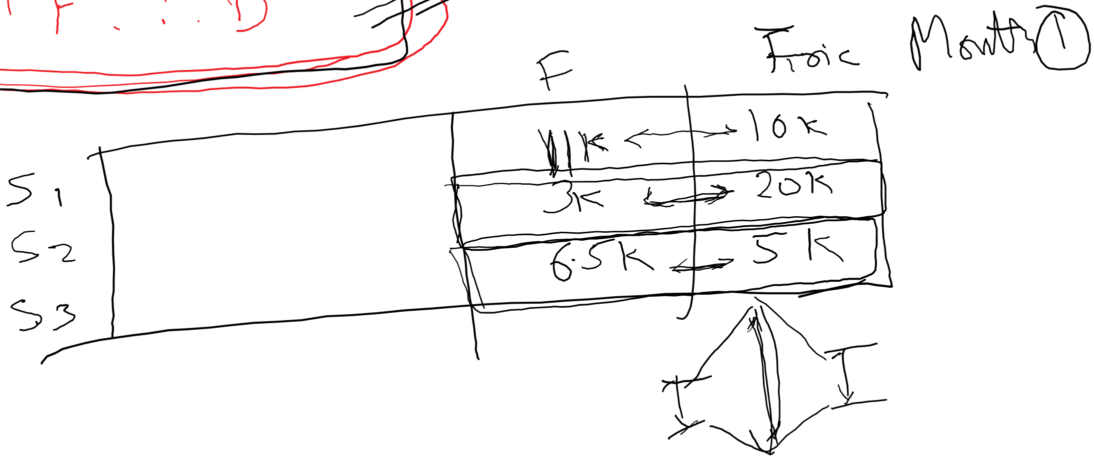
$S_1$ $S_2$ $S_3$
Jan
Dec

Shop 1
10k 20K 5K
11K 3K 6.5K
J F . . . D
T
P

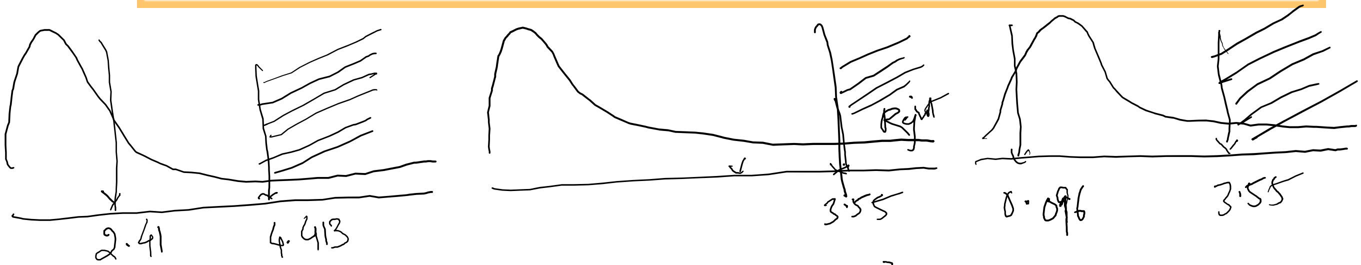



y
Theory
practical

F
Froic
Month 1
$S_1$
$S_2$
$S_3$
11K 10K
3K 20K
6.5K 5K
I
I

| ANOVA | | | | | | |
|---|---|---|---|---|---|---|
| Source of Variation | SS | df | MS | F | P-value | F crit |
| ~~Sample~~ R | 1.0417 | 1 | 1.0417 | 2.4194 | 0.1373 | 4.4139 |
| Columns C | 14.083 | 2 | 7.0417 | 16.355 | 9E-05 | 3.5546 |
| Interaction I | 0.0833 | 2 | 0.0417 | 0.0968 | 0.9082 | 3.5546 |
| Within | 7.75 | 18 | 0.4306 | | | |
| | | | | | | |
| Total | 22.958 | 23 | | | | |




R Accept $H_0$ → All $\mu$ are equal

C Reject $H_0$ → 16.35 → Atleast 1 $\mu$ is not equal

I Accept $H_0$ → There is No Interaction