CS357

Optimization Algorithms and Techniques Lab

A Large Scale New Variant of Capacitated Clustering Problem (VCCP)

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Contents:-

1 Introduction

In this project, we will work to solve a problem encountered in a green energy investment project of the Champagne–Ardenne region of France. Here, the local government wants to build 4 plants in 6 potential sites to produce the ethanol from the wheat straw at the minimum total cost of opening plants and assigning suppliers to the plants. This problem (**VCCP**) is a new variant of the Capacitated Clustering Problem (CCP), which is a multi-objective problem with both equality and inequality constraints and it is one of the most widely studied location problems with various applications, for example in vehicle routing (Koskosidis & Powell, 1992) and political districting (Bozkaya, Erkut, & Laporte, 2003), etc.

2 Problem statement

In our problem (VCCP), we have to **minimize the cost of opening plants in some of potential sites and assigning suppliers to them**. Each plant is subject to a minimum capacity requirement, where each supplier is assigned to one plant at most and when a supplier is assigned to a facility, the former will supply its entire available volume to the latter. For an open plant, if its minimum capacity requirement has been attained, then assigning more suppliers to it will increase the cost. This case will never happen in the optimal solution of the problem. The total supply volume of all suppliers is greater than the total requirement of all plants.

3 Mathematical Formulation

Let S be a set of suppliers with positive supply volume SV_i and P = {1,...,n} denote a set of potential sites for plants (facilities) with an associated minimum capacity requirement CR_j and a fixed opening cost FP_j and we have to open exactly p Plants. C_{ij} denotes the cost of assigning supplier $i \in S$ to the plant located at site $j \in P$. We define a binary variable Y_j, which takes 1 if a plant is located at site $j \in P$ and 0 otherwise. Binary variable X_{ij} equals 1 if supplier $i \in S$ is assigned to plant located at site $j \in P$ and 0 otherwise.

The VCCP can then be formulated as the following binary integer program:

Model P: $$Z = \min \sum_{i \in S} \sum_{j \in P} C_{ij} . X_{ij} + \sum_{j \in P} FP_j . Y_j \qquad \text{...(1)}$$

St $$\sum_{j \in P} X_{ij} \leq 1 \quad \forall\, i \in S \qquad \text{...(2)}$$

$$\sum_{i \in S} SV_{ij} . X_{ij} \geq CR_j . Y_j \quad \forall\, j \in P \qquad \text{...(3)}$$

$$\sum_{j \in P} Y_j = p \quad \forall\, j \in P \qquad \text{...(4)}$$

$$X_{ij}, Y_j \in \{0,1\} \quad \forall\, i \in S, \forall\, j \in P \qquad \text{...(5)}$$

The objective function (1) is to minimize the total cost of assigning suppliers to plants and establishing such plants.

The assignment constraint (2) guarantee that each supplier is assigned to at most one plant.

The constraint (3) satisfies minimum supply requirement.

The constraint (4) ensures that p plants are exactly located.

4 Algorithm-1

As the above problem is NP Hard as if one plant is located (p = 1) at one potential site (n = 1), it reduces to a knapsack problem. Therefore, Heuristics are the only practical techniques for handling large scale instances of this problem. However, Lagrangian relaxation approaches are also have been successfully applied to CCP problems, such as the Capacitated Facility Location Problem. So, the authors of the paper have proposed **Lagrangian relaxation approach**, in which there are two phases of dual optimization, the subgradient deflection method in the first phase and the standard subgradient method in the second phase, to approximately solve the problem by relaxing the assignment constraints. In this, the best Lagrange multipliers and the best upper bound found in the first phase are used as inputs for the second phase. At each Lagrangian iteration, a feasible solution is constructed from the optimal solution of the Lagrangian relaxed problem by applying a greedy algorithm.

4.1 Langrangian Relaxation

We will relax the assignment constraint (2) and dualizing them into the objective function (1) by introducing a set of nonnegative langrange multipliers Λ. The langrange Relaxed Problem $P_{LR}(\Lambda)$ can be formulated as

$$z_{LR}(\lambda) = min \sum_{i\in s} \sum_{j\in P} C_{ij} \cdot x_{ij} + \sum_{j\in p} FP_j + \sum_{i\in s} \lambda_i \left(\sum_{j\in P} x_{ij} - 1 \right)$$

...(6)

$$\sum_{i \in S} SV_{ij} . X_{ij} \geq CR_j . Y_j \quad \forall j \in P \qquad ...(3)$$

$$\sum_{j \in P} Y_j = p \quad \forall j \in P \qquad ...(4)$$

$$X_{ij}, Y_j \in \{0,1\} \quad \forall i \in S, \forall j \in P \qquad ...(5)$$

$Z_{LR}(\Lambda)$ provides a lower bound for the VCCP for any $\Lambda_i \geq 0$.

4.2 Resolution of Langrange Dual Problem

To find best lower bound, we need to solve langrangian dual problem to determine optimal langrangian multipliers Λ* such that

$$z_D = \max_{\lambda \geq 0 \cdot} z_{LR}(\lambda)$$

The subgradient method is used to solve the dual problem which repeatedly solves the langrangian relaxed problem $P_{LR}(\Lambda)$ and update the langrangian multiplier based on subgradient of $Z_{LR}(\Lambda)$ at the current value of Λ.

4.3 Standard Subgradient Method

Initially take $\Lambda^0=0$, Θ=1, k=0 (iterator), $Z_{UB}=\infty$

In each Iteration, solve the **relaxed problem $P_{LR}(\Lambda_k)$, construct feasible solution** (discussed in 4.5) and update Z_{UB}.

Then Calculate:-

$$g_i^k = \sum_{j \in P} x_{ij} - 1 \quad \text{the } i^{th} \text{ component of subgradient of } Z_{LR}(\Lambda) \text{ at } \Lambda^k$$

$$t_k = \theta_k [Z_{UB} - Z_{LR}(\lambda^k)] / \|g^k\|^2 \quad \text{the step size}$$

$$\Lambda_i^{k+1} = \max \{ 0 , \Lambda_i^k + t_k * g_i^k \} \quad \forall i \in S$$

If in a given number of iterations, no improvement of lower bound is observed, then assign $\Theta^{(k+1)} = \Theta^k / 2$ else it remains same.

Check stopping criteria:-

(i) The lower bound is not improving for a given number of iterations.
(ii) Maximum iterations reached.

Else continue iterations k=k+1.

4.4 Subgradient deflection Method

It is same as previous phase but, it defines moving direction (h^k) as a combination of previous direction and current subgradient (g^k).

$$h^k = (g^k + 0.3*h^{k-1} + 0.1*h^{k-2})/1.4$$

$$t_k = \theta_k[Z_{UB} - Z_{LR}(\lambda^k)]/\|h^k\|^2 \text{ the step size}$$

$$\Lambda_i^{k+1} = \max \{ 0 , \Lambda_i^k + t_k * h_i^k \} \quad \forall\, i \in S$$

We combine both phases as :-

(i) In first phase, the value of theta becomes very small and tends to zero, so langrange multipliers become stable, so to come out of this dilemma, second phase with reset value of theta is taken.

(ii) Subgradient deflection improves the convergence of subgradient algorithm.

(iii) Good initial Lagrange multipliers and different ways of updating the multipliers can augment the probability of finding a better lower bound

4.5 To find optimal solution of $P_{LR}(\Lambda)$

We have Λ^k from iteration, we can set value of Y_j=1 for all j={1,2,..,n}. Then $P_{LR}(\Lambda)$ can be decomposed into n independent 0-1 knapsack problems, these problem can be solved effectively by pisinger's MINKNAP.

The j^{th} knapsack problem is

$$Z_{KP}^{j}(\lambda) = min \sum_{i \in S} C_{ij} \cdot X_{ij} + FP_j + \sum_{i \in S} \lambda_i \cdot X_{ij}$$

...(7)

$$\sum_{i \in S} SV_{ij} . X_{ij} \geq CR_j . Y_j \quad \forall\, j \in P \qquad ...(3)$$

$$X_{ij}, Y_j \in \{0,1\} \quad \forall\, i \in S , \forall\, j \in P \qquad ...(5)$$

Let π be the permutation of number 1,2,..n such that

$$Z_{KP}^{\pi(1)}(\Lambda) \leq Z_{KP}^{\pi(1)}(\Lambda) \leq \ ... \leq Z_{KP}^{\pi(n)}(\Lambda)$$

As p plants have to be located, first p knapsack problem solution with smallest value make optimal solution $P_{LR}(\Lambda)$. The optimal solution of langrangian relaxed problem is defined as:-

$Y_{\pi(j)} = 1 \quad \forall\, j \leq p$

$Y_{\pi(j)} = 0 \quad \forall\, j > p$

$X_{i,\pi(j)}$ $\forall j \leq p$, $\forall i \in S$ takes the solution value of corresponding knapsack problem and

$X_{i,\pi(j)} = 0$ $\forall j > p$, $\forall i \in S$

The optimal solution of $P_{LR}(\Lambda)$ is given by

$$Z_{LR}(\lambda) = \sum_{i=1}^{p} Z_{KP}^{\pi(i)}(\lambda) - \sum_{i \in S} \lambda_i$$

4.6 Construction of Feasible Solution

As an upper bound is required in calculating the step size t_k, the optimal solution of relaxed langrangian problem $P_{LR}(\Lambda^k)$ may be infeasible, but it provides some useful information for construction of feasible solution like

(i) The location of open p plants.
(ii) If supplier i is assigned to exactly one open plant j , then we can set xij=1 in feasible solution.

Now the remaining problem is to select the enough suppliers from the unassigned ones to satisfy the residual capacity requirements of all open plants.

The assignment of supplier i to an open plant j is calculated by weight function $f(i,j)=c_{ij}/sv_i$.

For each open plant j, we can calculate its desirability(rho) which is difference of second smallest and smallest value of weight.

$$\rho_j = \min_{s \neq i_j} f(s,j) - f(i_j,j)$$

Now the supplier can be served by decreasing order of their desirability.

The solution obtained by greedy algorithm is feasible and can be improved by tabu search algorithm but it is time consuming, so we avoid using it.

We have written the c++ code of this algorithm, dataset is generated by specifications given in the research paper itself.

5 Algorithm-2

I found one research paper on Scatter search for the single source capacitated facility location problem which is very similar to our problem. Scatter Search is a population-based algorithm that stores solutions in a so called reference set and constructs new solutions by combining existing ones. First, an initial solution is created and it is improved. Then taking this as seed solution, the set of diverse and new solution is created, which considered as a candidate solution for the reference set. Then the search is carried out in a loop until stopping criteria is met. In each iteration, it constructs set of the reference solutions combined by some combining methods like path relinking, etc.

The proposed approach uses GRASP to initialize the Reference Set. Solutions of the Reference Set are combined using a procedure that consists of two phases: (1) the initialization phase and (2) the improvement phase. During the initialization phase each client is assigned to an open facility to obtain a solution that is then improved with the improvement phase.

In that paper, we have to open some facilities from set of possible locations and we have to satisfy the demand of each client with restriction to be supplied from single facility. We have to minimize the cost of opening facility (FP_j) and assigning facilities to clients(C_{ij}). We can assign only one facility to an client and we have to satisfy the demand of client.

Scatter Search consists of five methods:-

Diversification method:- provide trial solutions done using GRASP method which consists of construction phase and local search phase. It consists of:-

- Construction phase:- one potential site is selected and several clients are assigned to it (feasible wrt to capacity). The choice of the next site to open a facility is determined by randomly selecting one element. The construction phase terminates when all clients are assigned to an open facility.
- Local Search Phase:- in this phase, we try to shift assignment of one client to other facility or swap the assignment of two clients and repeat these till no further improvement is there.

Improvement Phase:- in this phase also, local search phase algorithm is applied additionally with tabu search, the best admissible move (with respect to the objective function value) in shift or swap process is performed.

Reference Set Update method:- When the Solution Combination Method is applied, new solutions are generated and are added to a Pool of solutions called Refence Set using reference set update method.

Subset Generation method:- It is used to generate subsets of reference solutions to be combined by the Solution Combination Method.

Solution Combination method:- At each iteration a different reference set of solutions is considered. It consist of:-

- Intialization phase:- . We use three different criteria to decide the initial assignment of each client.
- Improvement phase:- Also we use reassignment procedure if feasibility condition violates. Then elimination procedure is used to close one facility and client is reassign to other open facilities.

I have not applied the scatter search in the original problem as I am not able to make methods of assigning suppliers to plants or other methods as they are applied for the single source capacitated location problem which is different from our problem.

6 Conclusion

In this project, we first introduced the VCCP problem and then we formulate it as an optimization problem. Then we go through the Langrangian relaxation approach, which consists of two phases, Standard subgradient and subgradient deflection, in each iteration we solve the problem by relaxing assignment constraints and update the langrangian multipliers. Then, we construct feasible solution using greedy approach. We have also explored the possibility of applying scatter search in the problem.

7 Acknowledgement

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8 References

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