Assignment 0

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Date

CS 512 - Homework 0

A. given : $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$, $C = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$

Solutions

1. $2A - B =$

$$\begin{bmatrix} 2\times 1 - 4 \\ 2\times 2 - 5 \\ 2\times 3 - 6 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

2. $\|A\|$ and the angle of A relative to the positive x axis =

$\|A\| = (x, y, z) = (1, 2, 3)$

Length of point A from origin $= \sqrt{x^2+y^2+z^2}$

$= \sqrt{1^2+2^2+3^2}$

$= \sqrt{14}$

Angle from x axis $= \cos\alpha = \dfrac{x}{\sqrt{x^2+y^2+z^2}}$

$= \dfrac{1}{\sqrt{14}} = 75°$

Date

A. 3. A, a unit vector in direction of A.

$$\|A\| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$$

$$\therefore U_v = \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$$

$$= \left(\frac{\sqrt{14}}{14}, \frac{2\sqrt{14}}{14}, \frac{3\sqrt{14}}{14}\right)$$

$$= \left(\frac{\sqrt{14}}{14}, \frac{\sqrt{14}}{7}, \frac{3\sqrt{14}}{14}\right)$$

4. The direction cosine of A.

A w.r.t x = $\boxed{75^\circ}$ A w.r.t Y = $\cos^{-1}\left(2/\sqrt{14}\right) = \boxed{58^\circ}$

$\cancel{= \frac{1}{\sqrt{1^2+2^2+3^2}}}$ $\cancel{= \frac{1}{\sqrt{14}}}$

A w.r.t Z = $\cos^{-1}\left(3/\sqrt{14}\right) = \boxed{37^\circ}$

5. A.B and B.A

$$A\cdot B = 1\times 4 + 2\times 5 + 3\times 6 = 4+10+18$$
$$= 32$$

$$B\cdot A = A\cdot B = 32$$

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A. 6, The angle b/w A and B.

$A \cdot B = 32$

$\|A\| = \cancel{\sqrt{14}}\ \sqrt{14}$ $\quad \|B\| = \sqrt{4^2+5^2+6^2} = \sqrt{77}$

$\therefore \cos\theta = \dfrac{32}{\sqrt{14}\cdot\sqrt{77}} = \dfrac{32}{7\sqrt{22}} = 13^\circ$

7. A vector which is $\perp$ to A.

Let $\perp$ vector be $= V = (v_1, v_2, v_3)$

$\Rightarrow A \cdot V = v_1 + 2v_2 + 3v_3$

since its $\perp$, $A \cdot V = 0$

$\therefore v_1 + 2v_2 + 3v_3 = 0$

If v_1 & $v_2 = 1$

$v_3 = \dfrac{-3}{3} = -1$

$\therefore \perp$ vector is $(1, 1, -1)$ to A.

A. 8. $A \times B$ & $B \times A$:

$$A = (1, 2, 3)^T$$
$$B = (4, 5, 6)^T$$

$$A \times B = (2\times6 - 3\times5,\ 3\times4 - 1\times6,\ 1\times5 - 4\times2)^T$$
$$= (12 - 15,\ 12 - 6,\ 5 - 8)^T$$
$$= (-3,\ 6,\ -3)^T$$

$$A \times B = \begin{pmatrix} -3 \\ 6 \\ -3 \end{pmatrix}$$

$$A \times B = -(B \times A)$$
$$\therefore B \times A = \begin{pmatrix} 3 \\ -6 \\ 3 \end{pmatrix}$$

9. A vector which is $\perp$ to both A & B.

Cross product of vector A & B is $\perp$ to each vector A & B.

$\therefore$ $\perp$ vectors is $\begin{pmatrix} -3 \\ 6 \\ -3 \end{pmatrix}$

A. 10. The linear dependancy between A, B, C.

$$\left[\begin{array}{ccc|c} 1 & 4 & -1 & 0 \\ 2 & 5 & 1 & 0 \\ 3 & 6 & 3 & 0 \end{array}\right]$$

$\Rightarrow$ 2nd row − 2x 1st row & 3rd row − 3x 1st row

$$\left[\begin{array}{ccc|c} 1 & 4 & -1 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & -6 & 6 & 0 \end{array}\right]$$

$\Rightarrow$ 3rd row − 2x 2nd row

$$\left[\begin{array}{ccc|c} 1 & 4 & -1 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore$ A, B, C are linearly dependent.

11. A^TB & AB^T

$$A\cdot B = A^TB = \cancel{AB^T} = (1\times 4 + 2\times 5 + 3\times 6)$$

$$= 32$$

$$A\cdot B^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{bmatrix}$$

B. Given: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{bmatrix}$

Solutions

1. $2A - B$:

$$= \begin{bmatrix} 2\times1 - 1 & 2\times2 - 2 & 2\times3 - 1 \\ 2\times4 - 2 & 2\times-2 - 1 & 2\times3 + 4 \\ 2\times0 - 3 & 2\times5 + 2 & 2\times-1 - 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 5 \\ 6 & -5 & 10 \\ -3 & 12 & -3 \end{bmatrix}$$

2. AB and BA:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1+4+9 & 2+2-6 & 1-8+3 \\ 4-4+9 & 8-2-6 & 4+8+3 \\ 0+10-3 & 0+5+2 & 0-20-1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}$$

B. $= \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} = \begin{bmatrix} 1+8+0 & 2\cancel{+4}^{-4}+5 & 3+6- \\ 2+4+0 & 4-2-20 & 6+3+ \\ 3-8+0 & 6+4+5 & 9-6- \end{bmatrix}$

$$BA = \begin{bmatrix} 9 & 3 & 8 \\ 6 & -18 & 13 \\ -5 & 15 & 2 \end{bmatrix}$$

3. $(AB)^T$ and $B^T A^T$

$$(AB)^T = B^T A^T = \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}^T$$

$$= \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

4. $|A|$ and $|C|$

$|A| = 1(2-15) - 2(-4-0) + 3(20+0)$

$= -13 + 8 + 60$

$= 55$

$|C| = 1(15-6) - 2(12+6) + 3(4+5)$

$= 9 - 36 + 27$

$= 0$

Date

B. 5. the matrix (A, B or C) in which the row vectors form an orthogonal set

$$B \cdot B^{-1} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & \cancel{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 1 & -4 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 14 \end{bmatrix}$$

6. A^{-1} & B^{-1}

$$A^{-1} = \frac{1}{|A|} \times \text{Adjugated}(A)$$

$$\text{Adj } A = (-)\begin{bmatrix} 2-15 & -4 & 20 \\ -2-15 & -1 & +5 \\ 6+6 & 3-12 & -2-8 \end{bmatrix}^T = \begin{bmatrix} -13 & -4 & 20 \\ -17 & -1 & 5 \\ 12 & -9 & -10 \end{bmatrix}^T$$

$$= \begin{bmatrix} -13 & 4 & 20 \\ 17 & -1 & -5 \\ 12 & 9 & -10 \end{bmatrix}^T = \begin{bmatrix} -13 & 17 & 12 \\ 4 & -1 & 9 \\ 20 & -5 & -10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{55} \begin{bmatrix} -13 & 17 & 12 \\ 4 & -1 & 9 \\ 20 & -5 & -10 \end{bmatrix}$$

Date

6. $$B^{-1} = \frac{1}{|B|} \cdot Adj(B)$$

$$Adj(B) = (-)\begin{bmatrix} 1-8 & 2+12 & -2-3 \\ 2+2 & 1-3 & -2-6 \\ -4-1 & -4-2 & 1-4 \end{bmatrix}^T$$

$$= (-)\begin{bmatrix} -7 & 14 & -5 \\ 4 & -2 & -8 \\ -5 & -6 & -3 \end{bmatrix}^T = \begin{bmatrix} -7 & -14 & -5 \\ -4 & -2 & 8 \\ -5 & 6 & -3 \end{bmatrix}$$

$$Adj(B) = \begin{bmatrix} -7 & -4 & -5 \\ -14 & -2 & 6 \\ -5 & 8 & -3 \end{bmatrix}$$

$$|B| = 1(-7) - 2(14) + 1(-5)$$
$$= -7 - 28 - 5$$
$$= -40$$

$$B^{-1} = -\frac{1}{40}\begin{bmatrix} -7 & -4 & -5 \\ -14 & -2 & 6 \\ -5 & 8 & -3 \end{bmatrix}$$

$$= \frac{1}{40}\begin{bmatrix} 7 & 4 & 5 \\ 14 & 2 & -6 \\ 5 & -8 & 3 \end{bmatrix}$$

Date

C. Given: $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$

Solution.

1. The eigenvalues & corresponding eigenvectors of A.

i. Let λ be eigenvalue of A

$$\det\left(\lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} \lambda - 1 & -2 \\ -3 & \lambda - 2 \end{bmatrix}\right) = 0$$

$$(\lambda - 1)(\lambda - 2) - 6 = 0$$

$$\lambda^2 - 2\lambda - \lambda + 2 - 6 = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

$$(\lambda - 4)(\lambda + 1) = 0$$

$$\lambda = 4 \text{ or } -1$$

$A\vec{v} = \lambda\vec{v}$ where $\lambda = 4 \;\&\; -1$

assuming non-zero eigenvector

$\vec{0} = \lambda\vec{v} - A\vec{v}$

$\vec{0} = \lambda I_n\vec{v} - A\vec{v}$

$\vec{0} = (\lambda I_n - A)\vec{v}$

$$\vec{0} = \left(\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}\right)\vec{v}$$

$$\vec{0} = \left(\begin{bmatrix} 3 & -2 \\ -3 & 2 \end{bmatrix}\right)\vec{v}$$

$$\vec{0} = \left(\begin{bmatrix} 3 & -2 \\ 0 & 0 \end{bmatrix}\right)\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$3v_1 - 2v_2 = 0$$

$$3v_1 = 2v_2$$

$$v_1 = \frac{2}{3}v_2$$

$$E_4 = \left\{\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = t\begin{bmatrix} 2/3 \\ 1 \end{bmatrix},\ t \in \mathbb{R}\right\}$$

Date

$$E_{-1} = N\left(\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}\right)$$

$$E_{-1} = N\left(\begin{bmatrix} -2 & -2 \\ -3 & -3 \end{bmatrix}\right)$$

$$-v_1 = v_2$$

$$E_{-1} = \left\{ \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = t\begin{bmatrix} -1 \\ 1 \end{bmatrix},\ t \in \mathbb{R} \right\}$$

C. 2. $\begin{bmatrix} -2 & -2 \\ -3 & -3 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} -2 & -2 \\ -3 & -3 \end{bmatrix} \cdot$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$$

C. 3. $\begin{bmatrix} -2 & -2 \\ -3 & -3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} -6 & 4 \\ -9 & 6 \end{bmatrix}$

C. 4. $\det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}\right) = 0$

$$\det\left(\begin{bmatrix} \lambda - 2 & 2 \\ 2 & \lambda - 5 \end{bmatrix}\right) = 0$$

$$(\lambda - 2)(\lambda - 5) - 4 = 0$$

Date

$$\lambda^2 - \lambda 5 - 2\lambda + 10 - 4 = 0$$

$$\lambda^2 - 7\lambda + 6 = 0$$

$$(\lambda - 6)(\lambda - 1) = 0$$

$$\lambda = 6 \text{ or } 1$$

$$E_6 = N\left(\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} - \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}\right)$$

$$E_6 = N\left(\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}\right)$$

$$E_6 = N\left(\begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}\right)$$

$$2V_1 + V_2 = 0$$
$$V_1 = -\frac{1}{2} V_2$$

$$E_1 = N\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}\right)$$

$$E_1 = N\left(\begin{bmatrix} -1 & 2 \\ 2 & -4 \end{bmatrix}\right)$$

$$-V_1 + 2V_2 = 0$$
$$\frac{1}{2} V_1 = V_2$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix} = -2 + 2 = 0$$

$$\cancel{= \begin{bmatrix} -2 & 4 \\ 0 & 0 \end{bmatrix} =}$$

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C. 5. The dot product of ~~eigh~~ eigenvectors of B is 0, therefore they are orthogonal vector.

D. given : $f(x) = x^2+3$; $g(x,y) = x^2+y^2$,

solution

1. $$f(x) = x^2+3$$
$$f'(x) = 2x$$
$$f''(x) = 2$$

2. $$g(x,y) = x^2+y^2$$

$$\frac{\partial g}{\partial x} = 2x \qquad \frac{\partial g}{\partial y} = 2y$$
$$= 2 \qquad\qquad = 2$$

3. $$\nabla g(x,y) = x^2+y^2$$
$$g'(x,y) = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$$
$$g''(x,y) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

Date

4. $p\left(x \mid \mu, \sigma^2\right) = \mathcal{N}(x; \mu, \sigma^2) = \frac{1}{Z} \exp\left(-\frac{(x-}{2\sigma^2}\right.$

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