ISyE6669-OAN Homework Week 10

Fall 2021

Column Generation

In this problem, we want to walk you through the column generation algorithm to solve the cutting stock problem. Consider the following formulation

$$\begin{aligned} \min \quad & \sum_{j=1}^{n} x_i \\ \text{s.t.} \quad & \sum_{j=1}^{N} \boldsymbol{A}_j x_j = \boldsymbol{b} \\ & x_j \geq 0, \qquad \forall j = 1, \ldots, N. \end{aligned}$$

The problem has the following data. Customers need three types of smaller widths: $w_1 = 7, w_2 = 11, w_3 = 16$ with quantities $b_1 = 15, b_2 = 30, b_3 = 20$. The width of a big roll is $W = 80$.

1. Assume the column generation algorithm starts from the following initial patterns:

$$\boldsymbol{A}_1 = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}, \boldsymbol{A}_2 = \begin{bmatrix} 0 \\ 7 \\ 0 \end{bmatrix}, \boldsymbol{A}_3 = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}.$$

   Write down the restricted master problem (RMP) using these patterns. Solve this RMP by hand. Find the optimal basis $\boldsymbol{B}$ and its inverse $\boldsymbol{B}^{-1}$. Find the optimal dual solution $\hat{\boldsymbol{y}}^\top = \boldsymbol{c}_B^\top \boldsymbol{B}^{-1}$.

2. Solve RMP in Python CVX. Write down the optimal solution, the optimal basis $\boldsymbol{B}$, and its inverse $\boldsymbol{B}^{-1}$. Find the optimal dual solution $\hat{\boldsymbol{y}}^\top = \boldsymbol{c}_B^\top \boldsymbol{B}^{-1}$. To take the inverse of $\boldsymbol{B}$, you can use a calculator or computer program. In this iteration, you should be able to solve this LP by hand. But we ask you to set up the code in CVX and solve it using CVX. This code will be used in later iterations.

3. Write down the pricing problem, i.e. the knapsack problem using the above data and the optimal dual solution you found.

4. Solve the pricing problem in CVX. Should we terminate the column generation algorithm at this point? Explain. If the column generation should continue, what is the new pattern generated by the pricing problem?

5. If the column generation should continue, then augment (RMP) with the new column and solve it in CVX again. You can easily modify your CVX code by incorporating the new column. Write down the optimal solution, the optimal basis $\boldsymbol{B}$, and the inverse $\boldsymbol{B}^{-1}$. Compute the dual variable. Then solve the pricing problem again by modifying the date in your code. Should you terminate the column generation at this iteration? Explain. If the column generation should continue, do the same for all the following iterations until the column generation terminates.

6. Write down the final optimal solution, the optimal basis, and the optimal objective value.

7. For this problem, you need to submit all your codes for all the steps separately. Name them as question1 RMP step1, question1 Pricing step1, question1 RMP step2, and so on.