

COMBINATORICS

1. INTRODUCTION TO PERMUTATIONS AND COMBINATIONS

1. INTRODUCTION

$\Rightarrow n!$ (or) n

$$\Rightarrow n! = 1 \cdot 2 \cdot 3 \cdots n \quad \left. \begin{array}{l} \\ \text{Factorial of Negative numbers does not exist.} \end{array} \right\}$$

$$\Rightarrow 0! = 1$$

$$\Rightarrow [n! = (n-1)! \times n]$$

$$\left. \begin{array}{l} \Rightarrow (n+1)! - n! \Rightarrow n!(n+1) - n! \\ \qquad \qquad \qquad \Rightarrow n! (n+1) \\ \boxed{(n+1)! - n! = n!(n+1)} \end{array} \right| \quad \left. \begin{array}{l} \Rightarrow (n-1)! + n! = (n-1)! + (n-1)! * n \\ \qquad \qquad \qquad = (n-1)! (1+n) \\ \boxed{(n-1)! + n! = (n+1)(n-1)!} \end{array} \right|$$

$$\left. \begin{array}{l} \Rightarrow n_{Pr} = \frac{n!}{(n-r)!} \Rightarrow 5_{P_3} = \underbrace{5 \times 4 \times 3}_{(3 \text{ factors})} = 60 \\ \Rightarrow 10_{P_4} = 10 \times 9 \times 8 \times 7 \\ \qquad \qquad \qquad = 5040. \end{array} \right| \quad \left. \begin{array}{l} \Rightarrow n_{Cr} = \frac{n!}{(n-r)! r!} \\ \boxed{n_{Pr} = n_{Cr} \times r!} \end{array} \right|$$

2. PROPERTIES OF n_{Cr}

$$\left. \begin{array}{l} \text{If } n_{Cr} = n_{Cy} \Rightarrow \text{let } n-r=x \Rightarrow n_{Cx} = n_{Cy} \end{array} \right| \quad \text{PROPERTIES OF } n_{Cr}$$

$$\left. \begin{array}{l} \frac{n!}{(n-r)! r!} = \frac{n!}{(n-x)! x!} \\ \qquad \qquad \qquad = \frac{n!}{(n-(n-x))! (n-x)!} \end{array} \right| \quad \left. \begin{array}{l} \text{If } n_{Cx} = n_{Cy} \text{ then } \left\{ \begin{array}{l} x=y \text{ or} \\ y=n-x \text{ or} \\ x=n-y. \end{array} \right. \end{array} \right\}$$

$$\left. \begin{array}{l} \frac{n!}{(n-r)! r!} = \frac{n!}{r! (n-r)!} \end{array} \right| \quad \therefore \boxed{n_{Cr} = n_{Cn-r}}$$

INTRODUCTION TO PERMUTATION

Now, consider n_r . Now the value of n_r is max if

a) n is even $\Rightarrow r = n/2$

b) n is odd $\Rightarrow r = \frac{n-1}{2}$ or $r = \frac{n+1}{2}$

c) $n_r = n_{r-y} \Rightarrow x+y=n$

3 PROPERTIES OF n_r -2

$$\Rightarrow n_r + n_{r-1} = (n+1)_{r-1}$$

$$\Rightarrow \frac{n!}{(n-r)!r!} + \frac{n!}{(n-r+1)!(r-1)!}$$

$$= \frac{n!}{(n-r)!(r-1)!x^r} + \frac{n!}{(n-r)!(n-r+1)(r-1)!}$$

$$= \frac{n!}{(n-r)!(r-1)!} \left[\frac{1}{r} + \frac{1}{n-r+1} \right]$$

$$= \frac{n!}{(n-r)!(r-1)!} \left[\frac{n-r+1+r}{nr-r^2+r} \right]$$

$$= \frac{n! (n+1)}{(n-r+1)! r!} = \frac{(n+1)!}{(n+1-r)! r!} = (n+1)_{r-1}$$

4. PROPERTIES OF n_r -3

$$\Rightarrow n_r = \frac{n}{r} (n-1)_{r-1}$$

$$\Rightarrow n_r = \frac{n(n-1)!}{(n-r)!(r-1)!r}$$

$$= n/r \left[\frac{(n-1)!}{(n-r)!(r-1)!} \right]$$

$$= n/r \left[\frac{(n-1)!}{((n-1)-(r-1))!(r-1)!} \right] = n/r \left[(n-1)_{r-1} \right]$$

$$10_3 = \frac{10!}{7! 3!} \Rightarrow \frac{7! \times 8 \times 9 \times 10}{7! \times 3 \times 2 \times 1}$$

$$10_3 = \frac{10}{3} \times 9_{c_3} = \frac{8 \times 9 \times 10}{3 \times 2 \times 1}$$

$$= \frac{10}{3} \times \frac{9}{2} \times 8_{c_1}$$

$$= \frac{10}{3} \times \frac{9}{2} \times \frac{8}{1} \times 7_{c_0} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1}$$

$$= n_r +$$

5. PROPERTIES

$$\Rightarrow \frac{n_r}{n_{r-1}} =$$

$$\frac{n_r}{n_{r-1}} =$$

$$\frac{n_r}{n_{r-1}} =$$

INTRODUCTORY

Ex:

$$10_{c_4} + 10_{c_3} = 11_{c_4}$$

$$= 10_{c_3} + 10_{c_4} = ? \text{ we think } r=3 \text{ but } x=r=4$$

$$(10_{c_4} + 10_{c_3}) = 11_{c_4}$$

$$10_{c_4} + 10_{c_3} = (11)_{c} \text{ more than } 10_{c}$$

\downarrow

largest of 4, 3

6. EXAMPLE

$$W.K.T = \frac{n}{r}$$

$$= \frac{21}{84}$$

$$= 7r =$$

$$= [10r]$$

SOL

7. EXAMPLE

$$90_9 + \sum_{r=0}^{10}$$

$$= 90_9 +$$

$$= n_r +$$

$$= 100_8$$

5. PROPERTIES OF NCR - 4

$$\Rightarrow \frac{n_{Cr}}{n_{Cr-1}} = \frac{(n-r+1)}{r}$$

$$\frac{n_{Cr}}{n_{Cr-1}} = \frac{n!}{(n-r)! r!} \times \frac{(n-r+1)! (r-1)!}{n!}$$

$$\frac{n_{Cr}}{n_{Cr-1}} = \frac{(n-r+1)}{r}$$

$$\begin{aligned} n_{C_0} + n_{C_1} + n_{C_2} + \dots + n_{C_n} &= 2^n \\ n_{C_0} + n_{C_1} + n_{C_2} + \dots &= 2^{n-1} \\ n_{C_1} + n_{C_2} + n_{C_3} + \dots &= 2^{13} \\ 13_{C_0} + 13_{C_1} + \dots + 13_{C_{13}} &= 2^{13} \\ 13_{C_0} + 13_{C_1} + 13_{C_2} + \dots &= 2^{12} \\ 13_{C_1} + 13_{C_2} + 13_{C_3} + \dots &= 2^{12}. \end{aligned}$$

6. EXAMPLE ONE NCR

$$W.K.T = \frac{n_{Cr}}{n_{Cr-1}} = \frac{(n-r+1)}{r} \quad | \quad \text{Given } n_{C_{r-1}} = 36, n_{Cr} = 84, n_{C_{r+1}} = 126$$

what is 'n' and 'r'?

$$\begin{aligned} &= \frac{217}{84} = \frac{n-r+1}{r} \\ &= 7r = 3n - 3r + 3 \\ &= [10r = 3n + 3] - ① \end{aligned}$$

$$\begin{aligned} \text{Now, } \frac{n_{Cr+1}}{n_{Cr}} &= \frac{n-(r+1)+1}{r+1} \\ &= \frac{n-r}{r+1} \therefore \left[\frac{n_{Cr+1}}{n_{Cr}} = \frac{n-r}{r+1} \right] \end{aligned}$$

$$\text{Now, } \frac{n-r}{r+1} = \frac{126}{84} = \frac{63}{42} = \frac{21}{14} = \frac{3}{2}$$

$$\Rightarrow 2n - 2r = 3r + 3$$

$$\Rightarrow [2n - 5r = 3] - ②$$

Solving ① and ② $n=9$ and $r=3$

7. EXAMPLE 2 ON NCR

$$90_{C_9} + \sum_{r=0}^{10} (100-r)_{C_8} = ?$$

$$= 90_{C_9} + \left(100_{C_8} + 99_{C_8} + 98_{C_8} + 97_{C_8} + 96_{C_8} + 95_{C_8} + 94_{C_8} + 93_{C_8} + 92_{C_8} + 91_{C_8} + 90_{C_8} \right)$$

$$= n_r + n_{C_{r-1}} = (n+1)_{C_r} \Rightarrow 90_{C_9} + 90_{C_8} = 100_{C_9} = 9^2_{C_9} = 93_{C_9} - 94_{C_9}$$

$$= 100_{C_8} + 100_{C_9} = 101_{C_9}$$

8. Example 3 on NCR

$$50_{c_{20}} + 3 \cdot 50_{c_{21}} + 3 \cdot 50_{c_{22}} + 50_{c_{23}}$$

$$= 50_{c_{20}} + 50_{c_{21}} + 50_{c_{21}} + 50_{c_{21}} + 50_{c_{22}} + 50_{c_{22}} + 50_{c_{22}} + 50_{c_{23}} + 50_{c_{21}} \quad \left. \begin{array}{l} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array} \right\} \text{This process doesn't work}$$

$$= 51_{c_{21}} + 51_{c_{22}} + 51_{c_{22}} + 51_{c_{22}} + 50_{c_{23}} \Rightarrow 51_{c_{21}} + 51_{c_{22}} + 51_{c_{22}} + 50_{c_{23}} + 51_{c_{22}}$$

$$= 50_{c_{20}} + 3(50_{c_{21}} + 50_{c_{22}}) + 50_{c_{23}}$$

$$= 50_{c_{20}} + 3(51_{c_{22}}) + 50_{c_{23}}$$

$$= 50_{c_{20}} + 51_{c_{22}} + 51_{c_{22}} + 51_{c_{22}} + 50_{c_{23}} \quad \left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} \text{This combination doesn't work}$$

$$\Rightarrow (50_{c_{20}} + 50_{c_{21}}) + 2(50_{c_{21}}) + 2(50_{c_{22}}) + (50_{c_{22}} + 50_{c_{23}})$$

$$\Rightarrow 51_{c_{21}} + 2(51_{c_{22}}) + 51_{c_{23}}$$

$$= (51_{c_{21}} + 51_{c_{22}}) + (51_{c_{22}} + 51_{c_{23}})$$

$$= 52_{c_{22}} + 52_{c_{23}} = 53_{c_{23}}$$

9. Example 4 on NCR

$1! + 4! + 7! + 10! + \dots + 30!$ what is the last digit of the sum

and 2 last digits of sum?

$$\text{Sol: } 1! + 4! + 7! + 10! + \dots + 30!$$

\Rightarrow Any thing after $10!$ will contain units digit as '0' and '00' as last 2 digits.

\Rightarrow Now consider $5!$ to $7!$ it also contains unit digit as '0' because $5! = 120$.

\Rightarrow Now, the only thing that effects the sum is $1! + 4! = 25$,

$\therefore \boxed{\text{last digit in sum} = 5}$

$$1 + 24 + 5040 = 5065$$

$\therefore \boxed{\text{last 2 digits} = 65}$

10. FUNDI

Assume

3 Red bat

3 Black bat

\Rightarrow This pr

2) Assume

(arb)

2 shirts

2 pants

(1,2)

\Rightarrow This pr

\Rightarrow A

\Rightarrow 10 Bla

10 Bla

II. Row

\Rightarrow permu

\Rightarrow choo

\Rightarrow The

\Rightarrow The r

10 FUNDAMENTAL RULES OF COUNTING

Assume there are

3 Red balls of diff sizes } The no. of ways you can choose a ball out of
 3 Black balls of diff sizes } thus 6 balls is 6 ways. This problem can be
 subdivided into two parts as

$$\Rightarrow \text{The no. of ways of choosing Redball} = 3w$$

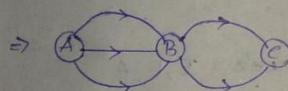
$$\Rightarrow \text{The no. of ways of choosing Blackball} = \frac{3w}{6w}$$

\Rightarrow This principle / Rule is called Addition principle of probability.

ii) Assume there are

2 shirts } The no. of ways you can dress up is 2 ways
 2 pants } (a₁₁)
 (a₁₂)
 (a₂₁)
 (a₂₂) } 4 ways = (2 × 2)
 (1/2)

\Rightarrow This principle is called multiplication theorem on probability.



\Rightarrow The no. of ways you can reach from 'A' to 'C' is
 $3 \times 2 = 6$ ways.

\Rightarrow 10 Blue suits } The no. of ways you can dress up = $10 + 10 = 20$,
 10 Black suits }

II. ROW ARRANGEMENTS WITHOUT REPETITIONS

\Rightarrow permutation = Arrangements

\Rightarrow choosing = combination.

\Rightarrow The no. of ways of arranging "abc" is abc, acb, bac, bca, cab, cba

\Rightarrow The no. of ways of arranging 'n' things is $\frac{n!}{\text{now}}$

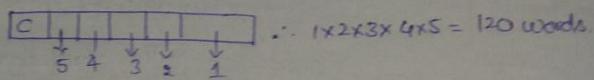
$$= 1 \cdot 2 \cdot 3 \cdots (n)$$

$$= n!$$

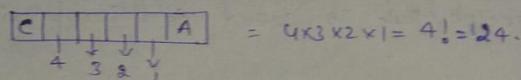
12 Examples on Row Arrangements without Repetitions

I) CINEMA \Rightarrow The no. of letters/arrangements of this word = $6! = 720$

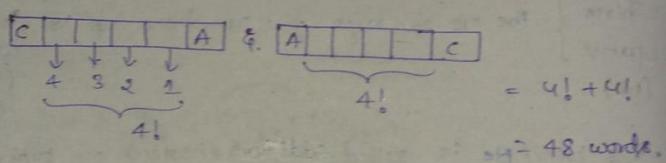
Restricted Permutations \Rightarrow The no. of words that are formed where the word always begin with 'C'.



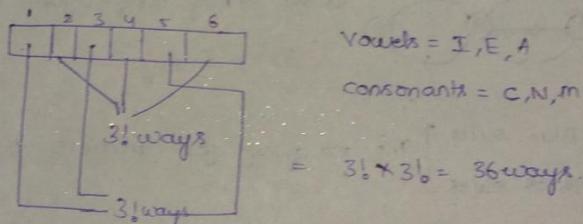
\Rightarrow Start with 'C' and end with 'A'.



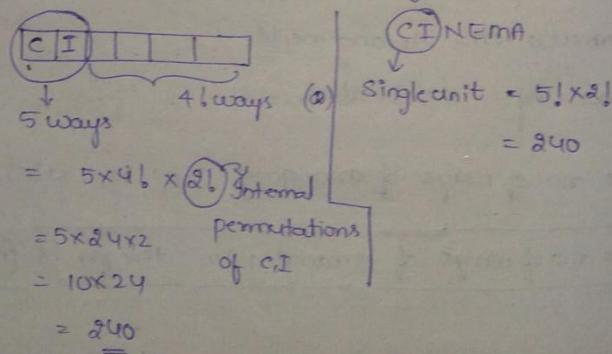
\Rightarrow 'C' and 'A' should be the Extremes.



\Rightarrow Vowels are at even positions



\Rightarrow 'C' and 'I' should always be together



\Rightarrow CINEMA

\Rightarrow C

4 groups
= 4!

\Rightarrow 'C' and 'A'

$192 = 4 \times 2! \times 4$

$144 = 3 \times 2^2$

96 \Leftarrow

$\frac{48}{480}$

\Rightarrow CINEMA

Arrange

The no. of

one idea

\Rightarrow we ha

kind

them

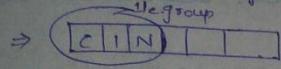
TIONS 6

$$! = 720$$

and always

words.

\Rightarrow CIN should always be together



7

$$\begin{aligned} & 4! \times 3! = 8 \times 6 = 124 \\ & 4 \text{ groups} \quad \rightarrow \text{Internal permutations} \\ & = 4! \text{ ways.} \end{aligned}$$

18

\Rightarrow 'C' and 'I' should never be together = Total Arrangements - CI come together

$$192 \leftarrow 4 \times 2! \times 4!$$

$$\Leftarrow \boxed{\text{C} \quad \text{I} \quad \quad \quad \quad}$$

$$144 \leftarrow 3 \times 2! \times 4!$$

$$\Leftarrow \boxed{\text{C} \quad \quad \text{I} \quad \text{I}}$$

$$96 \leftarrow 2 \times 2! \times 4!$$

$$\Leftarrow \boxed{\text{C} \quad \quad \quad \text{I}}$$

$$48 \leftarrow 1 \times 2! \times 4!$$

$$\Leftarrow \boxed{\text{C} \quad \quad \quad \text{I}}$$

$$\underline{480}$$

$$\Rightarrow 6! - (5! \times 2!) = 480$$

↓

This procedure is applicable
only for 2 letters not for 3
Alphabets,

\Rightarrow CIN should always be together and 'S' should always be 'C' and 'N'.

words.

$$\boxed{\text{C} \quad \text{I} \quad \text{N} \quad \quad}$$

it gives 12 words arrangements (as shown above)

$2 \times 4 \times 1 \times 3! = 4 \times 6 = 24$ words $\times 2$ (CIN can be changed).

$$= 48 \text{ words}$$

13

ROW ARRANGEMENTS WITH REPETITIONS

Arrangements with Repetition. (some objects are identical)

The no. of ways of arranging 'n' elements in a row in which 'r' elements are identical is called $= \frac{n!}{r!}$

\Rightarrow we have 'n' objects out of which 'm' are of one kind, 'r' are of another kind and 'p' objects are of other kind. Then the no. of ways to arrange them is " $\frac{n!}{m! \times r! \times p!}$ "

14. EXAMPLES ON ROW ARRANGEMENTS WITH REPETITIONS

RAVINDRA BABU

R - 2 times

A - 3 times

B - 2 times.

⇒ No. of 12 lettered words that can be formed is

$$\frac{12!}{2! 3! 2!}$$

Restricted Permutations ⇒ Start with R

$$\frac{11!}{3! 2!}$$

⇒ start with R' and end with U

$$\frac{10!}{3! 2!}$$

⇒ vowels and consonants should come together.

Vowels = A, I, A, A, U

} 2 Groups 2! ways

Consonants = R, V, N, D, R, B, B.

$$\frac{5!}{3!} \times \frac{5!}{2!} \times 2! \times \frac{10!}{2! 3! 2!}$$

15. EXAMPLES ON ROW ARRANGEMENTS WITH REPETITIONS - 2

RAVINDRA BABU

R - 2 times

B - 2 times

A - 3 times.

⇒ All A's should come together

$$[AAA] - - - - -$$

$$= \frac{10!}{2! 2! 2!} \times \frac{3!}{3!} = \frac{10!}{2! 2!} = \binom{AAA BB}{AAA, B-B}$$

⇒ All A's should be together and B's should not be together

$$= [AAA]$$

= (All A's coming together) - (All B's not coming together).

$$= \left(\frac{10!}{2! 2!} - \frac{9!}{2!} \times \frac{3!}{3!} \times \frac{2!}{2!} \right)$$

$\boxed{AAA} \boxed{BB}$ R, V, N, D, R, U

16. EXAMPLES ON

R's → not together

B's → should not be

Represent together & on 'B'

$$\frac{12!}{2! 3! 2!}$$

The N

S - 2 times

17. EXAMPLE

2, 3, 4, 4, 5

f 8

18. EXAMPLE

2, 3, 4, 4, 5.

⇒ Even digits

16. EXAMPLES ON ROW ARRANGEMENTS WITH REPETITIONS - 3

(8)

18

R's → not together

B's → should not be together

Represent R's come
together & no restriction
on B's

$$\therefore \frac{12!}{2!3!2!} - \left[\frac{11! \times 2!}{2!3!2!} + \left(\frac{11! \times 10!}{3!2!} \right) \right]$$

[Total method]

- (RR, BB)
- (R-R, BB)
- (RR, B-B)
- (R-R, B-B)

[No. of Arrangements in
which R/B come together)

+ (R's not together &
B's come together) + (R together
& B. not together)

(B's come together) + (R together
& B. not together)

$$\text{Total} = \left(R \text{ come together} \right) \\ \left(\frac{11!}{3!2!} - \frac{10!}{3!} \right)$$

$$\therefore \text{The No. of ways} = \frac{12!}{2!3!2!} - \left[\frac{11! \times 2!}{2!3!2!} + \left(\frac{11!}{3!2!} - \frac{10!}{3!} \right) \right]$$

17. EXAMPLES ON ROW ARRANGEMENTS WITH REPETITIONS - 4

$$2, 3, 4, 4, 5 \Rightarrow \text{Total no. of nos} = \frac{5!}{2!} = 60$$

⇒ Even nos:

$$\frac{4!}{2!} = 12 \quad \left(\begin{array}{c} 2, 4, 4 \\ 2, 4 \end{array} \right) \quad \frac{4!}{1!} = 24 \quad \left(\begin{array}{c} 2, 4 \\ 2, 4 \end{array} \right)$$

= odd nos:

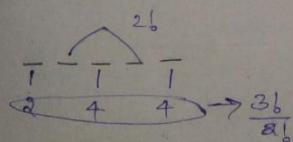
$$\frac{4!}{2!} \cdot \left(\begin{array}{c} 3 \\ 3 \end{array} \right) \quad \frac{4!}{2!} \quad \Rightarrow 36, \text{ even nos.}$$

$$= \frac{2 \times 4!}{2!} = 24 \text{ odd nos.}$$

18. EXAMPLES ON ROW ARRANGEMENTS WITH REPETITIONS - 5

2, 3, 4, 4, 5.

⇒ Even digits should be at odd places. (2, 4, 4)



$$\therefore \frac{3!}{2!} \times 2! = 6 \text{ ways.}$$

⇒ Find the no. of nos. that are < 40000.

$$\begin{array}{|c|c|c|c|} \hline 1 & & & \\ \hline \end{array} \quad \frac{3!}{2} \quad \frac{4!}{2!} = \frac{2 \times 4!}{2!} = 24 \text{ words.}$$

⇒ The nos. should be < 50000

$$\begin{array}{|c|c|c|c|} \hline & & & \\ \hline \end{array} \quad \begin{array}{l} \text{Now, } \frac{4!}{2!} \text{ nos.} \\ 2 - \text{fix it then} \\ 3 - \text{fix it then} \rightarrow \text{Now, } \frac{4!}{2!} \text{ nos.} \\ 4 - \text{fix it then} \rightarrow \text{Now, } 4! \text{ nos.} \end{array} = 2\left(\frac{4!}{2!}\right) + 4!$$

19. Examples on Row Arrangements with Repetitions - 6

2,3,4,4,5

→ Find the 5 digit nos. which are divisible by 2. = 36 even nos.

⇒ file by 3 → {A no. is divisible by 3 if the total sum is divisible by 3}

$$\boxed{\quad \quad \quad \quad} \quad 2+3+4+4+5=18$$

$$\therefore \text{The nos. are } = \frac{5!}{2!}$$

⇒ Divisible by 4 (last 2 digits should be divisible by 4)

$$\begin{aligned} &= \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \\ &- - - (2,4) = 3! \\ &- - - (4,4) = 3! \quad \therefore \text{The total nos.} = 3! + 3! + 2\left(\frac{3!}{2!}\right) \\ &- - - (3,2) = 3!/2! \\ &- - - (5,2) = 3!/2! = 3(3!) \\ &\qquad\qquad\qquad = 18! \end{aligned}$$

⇒ divisible by 5

$$\begin{array}{c} \underline{\quad \quad \quad \quad} \\ \underbrace{\quad \quad \quad \quad}_{5} \\ \frac{4!}{2!} \times 1 \quad \therefore \text{The no. of words} = 12! \end{array}$$

20. Example

0,2,3,4,5 ⇒

⇒

↓

$$\boxed{\quad \quad} \quad 4$$

$$\boxed{\quad \quad} \quad 3$$

(S13M)

(10)

20. EXAMPLES ON ROW ARRANGEMENTS WITH REPETITIONS

0, 1, 2, 3, 4, 5 \Rightarrow Find no. of 5 digit numbers

$$\Rightarrow \boxed{\quad \quad \quad \quad \quad} = (\text{Total No's}) - (\text{No's having '0' in first place}) \\ = 5! - 4! \\ = \underline{\underline{96}}$$

(Q1)

$$\begin{array}{|c|c|c|c|c|} \hline & & & & \\ \hline \downarrow & & & & \\ \hline (2, 3, 4, 5) & & & & 4! \\ \hline 4 & & & & \\ \hline \end{array} = 4 \times 4! \text{ nos} \\ = 96 \text{ nos}$$

=

\Rightarrow Find the total no. of 5 digit nos that are even?

$$= \boxed{\quad \quad \quad \quad \quad} \\ (0, 2, 4) = 3 \text{ choices}$$

{3}

Now, $\begin{array}{|c|c|c|c|c|} \hline & & & & 0 \\ \hline \downarrow & & & & \\ \hline (3, 4, 5) & & & & 3! \\ \hline 3 & & & & \\ \hline \end{array}$

No's = $4! \text{ nos.}$ $\begin{array}{|c|c|c|c|c|} \hline & & & & 2 \\ \hline \downarrow & & & & \\ \hline (3, 4, 5) & & & & 3! \\ \hline 3 & & & & \\ \hline \end{array}$

$\therefore \text{nos} = 3 \times 3!$

$$\begin{array}{|c|c|c|c|c|} \hline & & & & 4 \\ \hline \downarrow & & & & \\ \hline (2, 3, 5) & & & & 3! \\ \hline 3 & & & & \\ \hline \end{array} = 3 \times 3! \text{ nos}$$

$$\therefore \text{Total No's} = 4! + (3 \times 3!) + (3 \times 3!)$$

$$= 24 + 3(6) + 3(6)$$

$$= 24 + 36 \\ = \underline{\underline{60}}$$

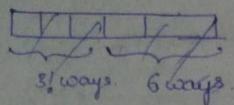
2(3! / 2!)

\Rightarrow How many 5 digit nos are present that are divisible by 5?

$$\begin{array}{|c|c|c|c|c|} \hline & & & & 0 \\ \hline \downarrow & & & & \\ \hline (2, 3, 4) & & & & 3! \\ \hline 3 & & & & \\ \hline \end{array} = 4! \text{ nos} \Rightarrow \boxed{\text{Total No's} = 4! + 3 \times 3!}$$

$$\begin{array}{|c|c|c|c|c|} \hline & & & & 5 \\ \hline \downarrow & & & & \\ \hline (2, 3, 4) & & & & 3! \text{ ways} \\ \hline 3 & & & & \\ \hline \end{array} = 3 \times 3! \text{ nos}$$

⇒ Find no. of 5 digit nos. that are divisible by 4.



Given no. = 92345

3! ways. 6 ways.

- (20) (40)
- (24) (52)
- (32)
- (04)

$$\begin{array}{|c|c|c|c|c|} \hline & & 2 & 0 \\ \hline \end{array} = 3!$$

3! 1

$$\begin{array}{|c|c|c|c|c|} \hline & & 0 & 4 \\ \hline \end{array} = 3! \text{ nos}$$

3! 1

$$\begin{array}{|c|c|c|c|c|} \hline & & 2 & 4 \\ \hline \end{array} = 2 \times 3!$$

2! 2!

$$\begin{array}{|c|c|c|c|c|} \hline & & 4 & 0 \\ \hline \end{array} = 3!$$

3! 1

$$\begin{array}{|c|c|c|c|c|} \hline & & 3 & 2 \\ \hline \end{array} = 2 \times 2!$$

(2) 2!

$$\begin{array}{|c|c|c|c|c|} \hline & & 5 & 2 \\ \hline \end{array} = 2 \times 2!$$

2! 2!

$$\begin{aligned} \therefore \text{Total nos. divisible by 4} &= 3(3!) + 3(2 \times 2!) \\ &= 3(6) + 3(4) \\ &= 18 + 12 = \underline{\underline{30}} \end{aligned}$$

21. RANK IN A DICTIONARY

⇒ RAVI - 4! words can be formed. (Find the rank of RAVI)

⇒ AIRV - Alphabetical order

⇒ A - - - = 3! words

I - - - = 3! words

R I V = 1! word

R A V I = 1 word

$$\begin{array}{r} 14 \\ \hline \end{array}$$

⇒ RANK (Find the rank of the word Rank)

AKNR

A - - - = 3!

K - - - = 3!

N - - - = 3!

~~R A K N~~ = 3!

R A K - - - 1! word

R A N K - 1 word

R A N K - 1 word

∴ Rank of the word

RANK = 20

⇒ Find the Ran

⇒ Alphabetical ord

A -
O -
R -
S A
S O
S C
S D
S E

⇒ Find the
Alphabetical ord

E - - -

G - - -

I E - -

I G E -

I G I -

I G N -

I G N -

I G N -

22. RANK

⇒ Given +

Now find

⇒ Find the Rank of Sourav. = $6!$ words are formed

⇒ Alphabetical order = A O R S O V

$$A = 5!$$

$$O = 5!$$

$$R = 5!$$

$$\boxed{S} \ A = 4!$$

$$\boxed{S} \ \boxed{O} \ A = 3!$$

$$\boxed{S} \ \boxed{O} \ \boxed{R} = 3!$$

$$\boxed{S} \ \boxed{O} \ \boxed{U} \ A = 2!$$

$$\boxed{S} \ \boxed{O} \ \boxed{U} \ \boxed{R} \ \boxed{A} \ \boxed{V} = 1$$

$$\boxed{399}$$

⇒ find the Rank of IGNITE

Alphabetical order = E G I I N T.

$$E - - - - = 5! / 2! = 60$$

$$G - - - - = 5! / 2! = 60$$

$$\boxed{I} \ E - - - = 4! / 2! = 24$$

$$\boxed{I} \ \boxed{G} \ E - - = 3! = 6$$

$$\boxed{I} \ \boxed{G} \ \boxed{I} - - = 3! = 6$$

$$\boxed{I} \ \boxed{G} \ \boxed{N} \ E - - = 2! = 2$$

$$\boxed{I} \ \boxed{G} \ \boxed{N} \ \boxed{I} \ \boxed{E} - - = 2! = 2$$

$$\boxed{I} \ \boxed{G} \ \boxed{N} \ \boxed{I} \ \boxed{I} \ \boxed{E} - -$$

$$\frac{160}{160}$$

22. RANK IN A DICTIONARY - 2

⇒ Given the Alphabets A, I, R, V. Find the word whose rank is 16

Now find the word whose rank is 14?

x word

$$\boxed{A} A \dots = 3! = 6 (1-6)$$

$$I \dots = 3! = 6 (7-12)$$

$$R A \dots = 2! (13-14)$$

RAIV - 13

$$\boxed{RAVI} = 14^{\text{th}}$$

⇒ Find the word whose rank is 20.

$$A \dots = 3! = 6 (1-6)$$

$$I \dots = 3! = 6 (7-12)$$

$$R \dots = 3! = 6 (13-18)$$

$$\boxed{V} \boxed{A} \dots = 2! (19, 20)$$

V A I R - 19

$$\boxed{VARI} = 20$$

Rank = 18

$$A \dots = 3! = 6$$

$$I \dots = 3! = 6$$

$$\boxed{R} \boxed{V} \dots = 2!$$

$$\boxed{R} \boxed{I} \dots = 2$$

$$\boxed{R} \boxed{V} \dots = 2$$

$$R V A I = 17$$

$$R V I A = 18$$

14

23) CIRCULAR

→ The no. of words

a, b, c

a b c

a c b

b a c

b c a

c a b

c b a

24. CIRCULAR

How many words can be formed in a circle?

RAVIN DRA

CIRCULAR

25. Examples

A Family

how many words can be formed in a circle?

and father

⇒ Find the word whose rank is 160. (E, G, I, I, N, R)

$$E \dots = 5! / 2! = 60 (1-60)$$

$$G \dots = 5! / 2! = 60 (61-120)$$

$$\boxed{I} E \dots = 4! = 24 (121-144)$$

$$\times \boxed{I} \boxed{G} E \dots = 4! = 24 (145-168) \quad 160 \text{ is in the range } 100-120 \\ \text{our reqd word starts with IG.}$$

$$\boxed{I} \boxed{G} E \dots = 3! = 6 (145-150)$$

$$\boxed{I} \boxed{G} I \dots = 3! = 6 (151-156)$$

$$\boxed{I} \boxed{G} N \dots = 3! = 6 (\text{excess } 160) \therefore \text{The word starts with IGN}$$

$$\boxed{I} \boxed{G} \boxed{N} E \dots = 2! = 2! (157-158)$$

$$\boxed{I} \boxed{G} \boxed{N} \boxed{I} \boxed{R} = 160$$

LET US TRY A DICTIONARY

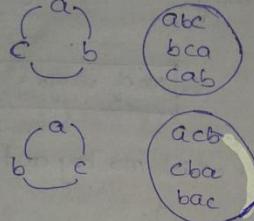
(14)

23. CIRCULAR PERMUTATIONS WITHOUT REPETITIONS.

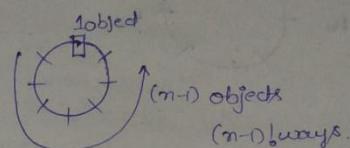
→ The no. of arrangements of 'n' distinct objects around a circle is $(n-1)!$.

abc

abc
acb
bac
bca
cab
cba.



(15)



∴ No. of permutations of 'n' distinct objects = $(n-1)!$.

24. CIRCULAR PERMUTATIONS WITH REPETITIONS

IA = 18

How many ways you can arrange letters of the word RAVINDRA in a circle?

RAVIN DRA = 8 letters \Rightarrow No. of Arrangements = $7!$ / AIR are repeated two times

$$\frac{7!}{2! 2!}$$

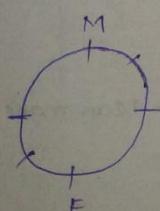
CIRCULAR : $\frac{8!}{2! 2!}$

PERMUTATIONS : $\frac{12!}{2!}$

25. EXAMPLES ON CIRCULAR PERMUTATIONS - I

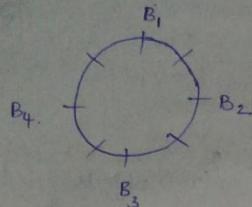
A family has 6 mem, Mother, Father, sons (S_1, S_2), Daughters (D_1, D_2). Now

how many ways you can make them sit in a table such that Mother and Father faces each other.



Then 4 mem are remaining and 4 objects are present $\therefore 4!$ ways.

26. CIRCULAR PERMUTATIONS MULTIONES
 We have 4 Boys - B_1, B_2, B_3, B_4 } what are the no. of ways in which no two boys are together and not two girls are together.
 4 Girls - G_1, G_2, G_3, G_4 }



\Rightarrow 4 boys can be arranged in circle in $3!$ ways.

\Rightarrow In the remaining 4 blanks 4 girls can be seated in $4!$ ways

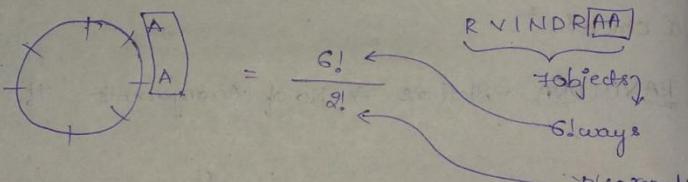
$$\therefore \text{No. of ways} = 4! \times 3!$$

\Rightarrow No. of ways in which no boy and girl are together is $7! - (4! \times 3!)$

26. EXAMPLE ON CIRCULAR PERMUTATIONS -- 2

RAVINDRA \Rightarrow Arrange in circle and ① Two a's always come together

$$\frac{8!}{2!2!} \text{ ways.}$$



\Rightarrow Two a's should never come together.

$$\text{No. of a's come together} = \left(\frac{8!}{2!2!} \right) - \left(\frac{6!}{2!} \right)$$

28. INTROD

The no. of ways in which n objects are arranged in a straight line is $n^r =$

$$3^2 = 3 =$$

29. EXAMP

10 Boys, 8 Girls group shown

$$= 10C_3 \times 8C_5$$

= Majority of

\Rightarrow Majority of

\Rightarrow We have

2 wicket

27. EXAMPLES ON GARLAND MODEL QUESTIONS

The no. of ways in which we can make garland from n flowers is $\frac{(n-1)!}{2}$.

There are 5 flowers. Now the no. of ways that I can make garland

$$\text{is } \frac{(5-1)!}{2} = \frac{(5-1)!}{2} = \frac{4!}{2} = 12$$

which mature

28. INTRODUCTION TO COMBINATIONS

The no. of ways of choosing 'n' objects from a set of 'n' distinct objects
in $n!$ ways is $n_r = \frac{n!}{(n-r)!r!}$

be stated

4 ways

$\frac{4!}{3!}$

core together

$$3^3 = 3 = \begin{array}{l} abc \\ ab \\ bc \\ ac \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} 3 \text{ ways.}$$

$$5^2 = 10 = \begin{array}{l} abcde \\ ab, ac, ad, ae \\ bc, bd, be \\ cd, ce, de \end{array}$$

29. EXAMPLE ON COMBINATION--!

10 Boys, 8 Girls. In how many ways can you choose 5 of them such that the group should contain 3 Boys and 2 Girls?

$$= 10C_3 \times 8C_2$$

$\frac{AA}{\downarrow}$
objects
ways

R is repeated
two times.

$$\Rightarrow \text{Majority are Boys} \Rightarrow (3 \text{ Boys} + 2 \text{ G}) + (4 \text{ B} + 1 \text{ G}) + (5 \text{ B}, 0 \text{ G})$$
$$= (10C_3 + 8C_2) + (10C_4 + 8C_1) + (10C_5)$$

$$\Rightarrow \text{Majority are Girls} = (3G + 2B) + (4G + 1B) + (5G + 0B)$$
$$= (8C_3 + 10C_2) + (8C_4 + 10C_1) + (8C_5)$$

\Rightarrow We have 22 players, form a team of 11, we have 12 Batsmen, 8 Bowlers, 2 wicket keepers, the team should contain (5 Batsmen, 5 bowlers, 1 wk) atleast.

12	8	2wk	ways
$12C_5$	$8C_5$	$2C_1$	$12C_5 \times 8C_5 \times 2C_1$
$12C_6$	$8C_4$	$2C_1$	$12C_6 \times 8C_4 \times 2C_1$
$12C_6$	$8C_5$	1	$12C_6 \times 8C_5 \times 1$

30. EXAMPLE ON COMBINATIONS

⇒ A mathematics question paper contains 12 Questions and we are supposed to ans 8 Then how many ways you can answer the Question paper? ${}^{12}C_8$

⇒ The QP has 2 parts, the first 2 parts contains questions from (1-6) and 2nd part contains questions from (7-12). The Restriction is you have to answer atleast 3 from part 1 and In total you have to ans 8 questions?

The no. of ways are,

$$(123456) (789101112) \Rightarrow {}^6C_3 \times {}^6C_5 = \left\{ \begin{array}{l} \text{PART 1} \quad \text{PART 2.} \\ \Rightarrow {}^6C_4 \times {}^6C_4 = \\ \Rightarrow {}^6C_5 \times {}^6C_3 = \\ \Rightarrow {}^6C_6 \times {}^6C_2 = \end{array} \right. \begin{array}{l} \text{The total ways are} \\ ({}^6C_3 \times {}^6C_5) + ({}^6C_4 \times {}^6C_4) \\ + ({}^6C_5 \times {}^6C_3) + ({}^6C_6 \times {}^6C_2) \end{array}$$

31. EXAMPLES ON COMBINATIONS

There are 10 couples (10m, 10w) Now, i want to form a team of 4 mem such that i have (2m and 2w) then how many ways we can form it.

No. of ways $\boxed{{}^{10}C_2 \times {}^{10}C_2}$

⇒ Now In the team of 4 people I should see that no couple is included.

$$\Rightarrow \boxed{{}^{10}C_2 \times {}^8C_2}$$

⇒ Now In the 4mem team we need to include exactly one couple then

⇒ Now choose a couple from the 10 couples = ${}^{10}C_1$ (I have included

2mem since

i have chosen
a couple)

⇒ Now, among the 9m and 9w

choose 1 men = 9C_1

⇒ Since I need to include exactly one couple the I should not

include the wife of the person chosen above, so

im left with 8mem, so choose 1 from them = 8C_1 ways.

∴ The tot

⇒ Now In the 4 selected.

case 1: H_1 left

case 2: H_1

⇒ If H_1 is not se
may or may not

ways =

∴ No. of

w

⇒ w_2, w_3 show

<u>w_2</u>	<u>w_3</u>
✓	✓
✓	✗
✗	✓
✗	✗

32. EXAMPLES

⇒ There are 10
have shak

⇒ There are 2
persons =

(18)

$$\therefore \text{The total no. of ways} = 10C_1 \times 9C_1 \times 8C_1$$

(19)

Question

\Rightarrow Now in the 4 member team, If w_1 is selected then w_5 should be selected.

form (6) and
have to
tions?

case 1: w_1 is selected $\Rightarrow w_5$ should be included, Now 9 men, 9 women left \Rightarrow No. of ways of selecting = $9C_1 \times 9C_1$

case 2: w_1 is not included then, among 9 men choose 2 = $9C_2$ ways.

\Rightarrow If w_1 is not selected it doesn't mean that you should not select w_5 , it may or may not be selected. $\Rightarrow 10C_2$

$$\boxed{\text{ways} = 9C_2 \times 10C_2}$$

sample

$$(6C_4 \times 6C_4)$$

$$+ (6C_6 \times 6C_2)$$

$$\therefore \text{No. of ways} = \text{case 1} + \text{case 2}$$

$$\boxed{\text{ways} = 9C_1 \times 9C_1 + 9C_2 \times 10C_2}$$

$\Rightarrow w_2, w_3$ should not be selected simultaneously.

of 4 mem
n it.

cluded.

then

included
mem since
are chosen
a couple)

not
ve, so
ways.

$$\begin{array}{c} w_2 \\ \equiv \\ \checkmark \end{array} \quad \begin{array}{c} w_3 \\ \equiv \\ \checkmark \end{array} = 10C_2$$

$$\begin{array}{c} \checkmark \\ \times \end{array} = 10C_2 \times 8C_1 \quad \left. \begin{array}{c} \checkmark \\ \times \end{array} = 10C_2 \times 8C_1 \right\}$$

$\times \quad \checkmark = 10C_2 \times 8C_1$ These are the cases that we need

$$\times \quad \times = 10C_2 \times 8C_2 \quad \left. \begin{array}{c} \times \\ \times \end{array} \right\} = 10C_2 (8C_1 + 8C_1 + 8C_2),$$

QUREER

32. EXAMPLES ON COMBINATIONS - 4

\Rightarrow There are 10 couples (10m, 10w). How many different ways we can have shakehands between them?

\Rightarrow There are 20 people and we can have shakehands between any two

persons = \therefore The total no. of shake hands = $20C_2$

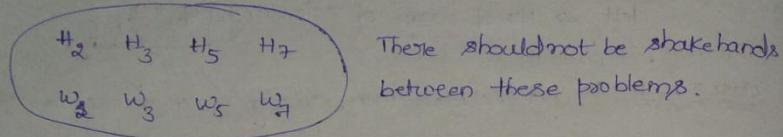
$$= \frac{20 \times 19}{2} = 190 \text{ Shake Hands.}$$

⇒ find the no. of shake hands that does not include a couple

$$\text{No. of shakehands} = \binom{20}{2} - 10 \quad \text{No. of shakehands of couples}$$

10-couples = 10-shakehands

⇒ Let us say $H_1, H_2, H_3, \dots, H_{10}$ } The condition is that there should
 $w_1, w_2, w_3, \dots, w_{10}$ } not be hand shake between partners.



$$\therefore \text{The No. of shakehands} = \binom{20}{2} - 8\binom{8}{2}$$

33. REPETITIONS IN COMBINATIONS

$$aab = \frac{3!}{2!} = 3 \text{ nds.}$$

$$--- (\binom{3}{2} \times 1)$$

$$= (3 \text{ ways})$$

$$aabcd = \frac{5!}{2!}$$

$$= \binom{5}{2} \times 3! \quad (\text{take two objects and fix them})$$

aa
and remaining objects can be placed in 3! ways.

There are 'n' numbers/objects in which 'r' are of similar kind. Then the no. of words that can be formed = $\frac{n!}{r!} = \binom{n}{r} (n-r)!$ → Arrange the remaining ones
 choose like Repeating objects

34. ORDER

⇒ we have abc & have to form all 3 lettered words such that a should occur before b. $\textcircled{a} \textcircled{b}, a-b$ → 1 letter between a and b.

a	b	c
---	---	---

3 positions

c a b →

a c b → } 3 ways.

a b c →

⇒ If choose 2 positions from available 3 positions = $\binom{3}{2} \left(\begin{matrix} 12 \\ 13 \end{matrix} \right) \begin{matrix} abc \\ acb \\ cab \end{matrix} \right\} 3$

⇒ PERMUTATION

in Alphabetical

vowels =

⇒ Now choose and place

⇒ Remaining

⇒ We have "

'n' lettered words

of words = n!

35. Examples

RAVIND →

R should

→ Fix (RiA).

R should

R → A → V

⇒ R should

RA

RA

- (20)
- ⇒ PERMUTATIONS (The condition is that vowels should always be in Alphabetical order)
- vowels = (A E I O U) = This is the order in which vowels should occur
- ⇒ Now choose 5 positions among 11 positions = ${}^{11}C_5$ ways (vowels cannot be permuted because order is specified here)
- ⇒ Remaining '6' letters can be placed in $\frac{6!}{2!}$ ways ('T' Repeated 2 times)

$$\therefore \text{No of ways} = \frac{11!}{6!5!} \times \frac{6!}{2!}$$

$$\boxed{\text{No of ways} = \frac{11!}{5!2!}}$$

we want
to find all
n lettered words in which n letters should be in some order then no. of words: $n! / r!$

35. EXAMPLE ON ORDER

RAYIND → The no. of possible words that can be formed in which R should always occur before A?

⇒ Fix (R, A) ⇒ out of 6 positions choose any two = ${}^6C_2 \times 4!$ Remaining words.

$$= \frac{6!}{2!4!} \times 4! = \frac{6!}{2!}$$

⇒ R should come before A' and A' should come before V.

R → A → V → ${}^6C_3 \times 3!$ Remaining words arrangements.

⇒ R should come before A and V

$$\frac{6!}{2!} = 180$$

ways

R	A	V	I	I	E	I
---	---	---	---	---	---	---

→ 4! ${}^6C_2 = 15$

ways

$$\therefore \frac{6!}{2!} + \frac{6!}{3!} = \text{ways}$$

36 Examples on ORDER - 2

(2)

RAVINDR → Find all words such that 'R' occurs before A or 'R' occurs before V?

$$m_{P_0} = m_0 \times r!$$

→ Find the total words and subtract the ways in which R and V occur before R. ($A \rightarrow V \rightarrow R, V \rightarrow A \rightarrow R$)

$$= 6! - \left(\binom{6}{3} \times 3! + \left(\binom{6}{3} \times 3! \right) \right)$$

37. SELECTIONS WITH REPETITIONS

$$abcd = 4C_2 = 6$$

$$aabc = \frac{4!}{2!} = 12$$

$$\begin{aligned} &= ab \\ &= ac \\ &= ad \\ &= bc \\ &= bd \\ &= cd \end{aligned}$$

$$\begin{array}{l} aa \\ ab \\ ac \\ bc \\ \hline \end{array} \left. \begin{array}{l} ab \\ ac \\ bc \end{array} \right\} 4 = 4C_2$$

1. INTRODUCTION

$$T(n) = T(n-1) +$$

$$T(n) = aT(n/b)$$

2. RECURRANCE

Sequence: Any

$$T(n)$$

$$n$$

$$T(0)$$

Recurrence Relation

present the relation of previous +

1).

38. EXAMPLE ON SELECTIONS WITH REPETITIONS

→ aabc → Find/choose two objects

Case 1: Two letters are same → one way = $2C_2$. Total =

Case 2: Two letters are different = $3C_2 = 3$ ways = (ab, ba)

→ RAVINDR → Choose two letters from this name

R-2

A-1

V-1

I-1

N-1

D-1

Case 1: Both the letters are same = 1 way. Total =

Case 2: Both the letters are different = $6C_2 = 6$ ways + 1 way

→ choose 3 letters from the given letters.

Case 1: 3 different letters = $6C_3$.

Case 2: 2 same + 1 different = $1 * 5C_2$ } = $(6C_3 + 5C_2)$ ways

→ RAVINDRA → choose two

1) 2 same - 2

2) 2 different - $6C_2$ } Total = $(2 + 6C_2)$

Q. Seq: 1, 3, 5

$$n = 0, 1, 2$$

$$T(0) = 1, T(1) =$$

$$T(n) =$$

(22)

$$m_B = m_C \times r_b$$

before B?

and V.

(23)

= 18

2. RECURRENCE RELATIONS

1. INTRODUCTION

$$T(n) = T(n-1) + T(n-2) + T(n-3) + K.$$

$$T(n) = aT(n/b) + K \rightarrow \text{master theorem}$$

2. RECURRENCE RELATION AND SOLUTION

Sequence: Any order in which you write a number.

$$T(n) = 0, 2, 4, 6, 8, 10, \dots$$

$$n = 0, 1, 2, 3, 4, 5$$

$$T(0) = 0, T(1) = 2, T(2) = 4, T(3) = 6.$$

Recurrence Relation: It is a relation, in which we are possible to write present the n th term / any term in the sequence as an mathematical expression of previous terms - then it is called RR.

Total =
= 4 ways

1.

$$\boxed{T(n) = T(n-1) + 2} \quad [\text{For above example}]$$

$$\left. \begin{array}{l} T(0) = 0 \\ T(1) = 2 \end{array} \right\} \quad \text{Initial conditions}$$

The main aim of topic
RR is to find such sol to the RR's.

Sol to above RR $T(n) = 2n$.

$$\text{LHS of RR} = T(n) = 2n$$

$$\begin{aligned} \text{RHS of RR} &= T(n-1) + 2 = 2(n-1) + 2 \\ &= 2n. \end{aligned}$$

Seq: 1, 3, 5, 7, 9, 11, ...

$$n = 0, 1, 2, 3, 4, 5$$

$$T(0) = 1, T(1) = 3, T(2) = 5 \quad \boxed{T(n-1) + 2 = T(n)}$$

$$T(n) = 2n + 1$$

$$\text{LHS of RR} = T(n) = 2n + 1$$

$$\text{RHS of RR} = T(n-1) + 2$$

$$= [T(n-1) + 1] + 2 = 2(n-1) + 1 + 2$$

$$= 2n + 1.$$

$$\boxed{T(n) = 2n + 1} \quad n \geq 1$$

$$T(0) = 1 \quad (\text{Initial condition})$$

Sequence: 1, 2, 4, 8, 16----

$n: 0, 1, 2, 3, 4$

$$T(0)=1, T(1)=2, T(2)=4, T(3)=8$$

$$\boxed{T(n)=2T(n-1), n \geq 0}$$

$T(0)=0, n=0 \rightarrow [Initial condition]$

$$Sol: T(n)=2^n \quad (2)$$

$$LHS = T(n)=2^n$$

$$RHS = 2T(n-1)$$

$$= 2^1 2^{n-1}$$

$$= 2^n \quad LHS=RHS$$

4. GATE 2004 QUES
The Recurrence relation

$$a) \sqrt{2^{n+1}-2} \rightarrow 2^n$$

$$T(0)=1$$

$$OPA: - 2^{1+1}-1-2 = 1$$

$$OPB: 2^{1-1} = 1.$$

$$T(0)=2T(1)+2 \Rightarrow$$

$$OPA: 2^3-2-2 = 4$$

$$OPB: 2^{3-2} = 2$$

5. GATE 2002 QUES

The solution to the

$$a) 2^K \checkmark (3^{K+1}-1)$$

$$T(0)=1$$

a) Now if the
in Bigoh notation

3. EXAMPLES

For the recurrence relation $T(n)=2T(n-1)-T(n-2), n \geq 2$, find whether the following are solutions?

a) $T(n)=3n$ b) $T(n)=2^n$ c) $T(n)=5$.

OPVM,

a) $T(n)=3n$.

$$T(n)=2T(n-1)-T(n-2)$$

$$LHS = T(n)=3n$$

$$RHS = 2T(n-1)-T(n-2)$$

$$= 2[3(n-1)] - [3(n-2)]$$

$$= 6(n-1) - [3n-6]$$

$$(Or) = 6n-6-3n+6 = 3n. \checkmark \text{ This is } \underline{\text{solution}}$$

$$T(n)=3n$$

$$T(2)=2T(1)-T(0)$$

$$T(0)=0$$

$$= 2 \times 3 - 0$$

$$T(1)=3$$

$$= 6 \checkmark$$

$$T(2)=6$$

$$T(3)=2[T(2)-T(1)]$$

$$T(3)=9$$

$$= 2(6)-3$$

$$= 12-3=9.$$

b) $T(n)=2^n$

$$T(0)=1$$

$$T(2)=2T(1)-T(0)$$

$$= 2(2)-1$$

$$T(1)=\cancel{2}$$

$$T(2)=\cancel{4}$$

Not equal

$\rightarrow (3)$

$$T(3)=8$$

$\therefore 2^n$ is not solution.

c) $T(n)=5$

$$T(0)=5$$

$$T(1)=5$$

$$T(2)=5$$

$$T(3)=5$$

$$T(2)=2T(1)-T(0)=(2 \times 5)-5=5$$

$$T(3)=T(2)=2T(2)-T(1)$$

$$= 2 \times 5 - 5 = 5 \checkmark$$

$$OPVM: - T(0)$$

$$T(1)=1$$

4. GATE 2004 QUESTION

(24)

The Recurrence relation

$$T(n) = 2T(n-1) + n, n \geq 2$$

$$T(1) = 1$$

Evaluates to

(25)

e 1/8

LHS = RHS

102 TMI

$$T(n) = 1$$

$$\text{optA: } 2^{n+1} - 1 - 2 = 1 \quad \Leftrightarrow 2^n - 2 = 0 \times$$

$$\text{optB: } 2^n - 1 = 1 \quad \Leftrightarrow 2^n + n = 2^{n+1} = 3 \times$$

either

$$T(2) = 2T(1) + 2 \Rightarrow 2(1) + 2 = 4$$

$$\text{optA: } 2^2 - 2 - 2 = 4 \quad \checkmark$$

$$\text{optB: } 2^2 - 2 = 2$$

$$\boxed{\text{optA: } 2^{n+1} - n - 2}$$

5. GATE 2002 QUESTION

The Solution to the Recurrence Relation $T(2^K) = 3T(2^{K-1}) + 1$

$$T(1) = 1$$

$$\text{a) } 2^K \quad \cancel{\text{b) } (3^{K+1} - 1)/2 \quad \rightarrow 3^{\log_2 K}}$$

$$\text{d) } 2^{\log_3 K}$$

102 TMI 2002 EDITION

$$T(1) = 1$$

a) Now if the question is $T(2^K) = 3T(2^{K-1}) + 1$ and options are given

in Bigoh notations then how to solve them is.

This is 80h

$$T(2^K) = 3T(2^{K-1}) + 1 \Rightarrow \text{let } 2^K = n$$

$$2^K/2 = n/2 \Rightarrow \boxed{2^{K-1} = n/2}$$

$$T(n) = 3T(n/2) + 1$$

$$T(n) = O(n^{\log_2 3}) + O(n^{\log_2 \log_2 3})$$

$$a=3, b=2, K=0, P=0.$$

$$a > b^K \Rightarrow T(n) = O(n^{\log_2 3})$$

$$\Rightarrow T(n) = O(n^{\log_2 3})$$

$$= O((2^K)^{\log_2 3}) = O(2^{K \cdot \log_2 3}) = O(2^{\log_2 3^K})$$

$$\boxed{T(n) = O(3^K).}$$

$$\text{Given: } T(2^K) = (3^{K+1} - 1)/2.$$

∴

$$T(1) = 1 \Rightarrow \text{put } K=0 \text{ in } T=1$$

$$= (3^1 - 1)/2 = 1. \quad \checkmark$$

$$\text{LHS: } T(2^K) = (3^{K+1} - 1)/2$$

$$\text{RHS: } 3T(2^{K-1}) + 1$$

$$= 3 \left[\frac{(3^{K+1} - 1)}{2} \right] = \frac{3(3^{K+1} - 1)}{2} = \frac{3}{4}(3^K - 1)$$

Now, if the questions are given of this form $T(n) = 3T(\sqrt{n}) + 1$ then convert it as $T(n) = 3T(2^k) + 1$

$$\text{let } n = 2^k \Rightarrow T(2^k) = 3T(2^{k/2}) + 1$$

$$\Rightarrow \text{let } 2^k = p \text{ then, } T(p) = 3T(p/2) + 1$$

Apply master theorem.

6. GATE 2008 QUESTION

When $n = 2^k$ for some $k \geq 0$ the recurrence relation $T(n) = \sqrt{2}T(n/2) + \sqrt{n}$ evaluates to

- a) $\sqrt{n}(\log n + 1)$ b) $\sqrt{n} \log n$ c) $\sqrt{n} \log \sqrt{n}$ d) $n \log \sqrt{n}$

$$T(1) = 1 \Rightarrow T(2) = \sqrt{2} + 1 + \sqrt{2} \\ = 2\sqrt{2}$$

OPA: $\sqrt{n}(\log n + 1) = T(n)$

T(1) = $\sqrt{1}(\log 1 + 1) = 1$

$T(2) = \sqrt{2}(\log 2 + 1)$

$$= \sqrt{2}(2) = 2\sqrt{2}$$

NOTES FOR 2008 STEP 3

7. GATE 2008 QUESTION

Let x_n denote the no. of binary strings of length n that contain no consecutive ones. Which of the following recurrence relation does x_n satisfy?

- a) $x_n = 2x_{n-1}$ b) $x_n = x_{n/2} + 1$ c) $x_n = x_{n/2} + n$ d) $x_n = x_{n-1} + x_{n-2}$

OPUM

x_1 = The no. of binary strings that contain no consecutive ones = $\{1, 0\}$

$$\begin{aligned} \text{OPA: } x_1 &= 2x_0 = 0 // X & \Rightarrow x_1 = 2 \\ \text{OPB: } x_2 &= x_{2/2} + 1 = 2+1=3. X & x_2 = \{01, 10, 11\} \\ \text{OPC: } x_2 &= x_{2/2} + 1 = x_2 = 2+2=4. X & = 3 \\ \text{OPD: } x_2 &= x_{\text{one}} + x_0 = 2+0=2. (X) & x_3 = \{5\} \end{aligned}$$

OPD: $x_3 = x_2 + x_1$

$$= 3+2 \\ = 5. V.$$

SETS

1. SETS AND

Set: well

Ex: A =

S =

Nullset: S

subset: P

of B.

A

Note: F

proper subset

proper sup

Not

2. POWER

Denoted by

\Rightarrow If A is

power set

Ex: A =

Subs

\Rightarrow If a

9. GATE 2015 QUESTION

Let a_n represent the no. of bit strings of length n containing 2 consecutive 1's what is the recurrence relation for a_n ?

(a) $a_{n-2} + a_{n-1} + 2^{n-2}$ (b) $a_{n-2} + 2a_{n-1} + 2^{n-2}$

(c) $2a_{n-2} + a_{n-1} + 2^{n-2}$ (d) $2a_{n-2} + 2a_{n-1} + 2^{n-2}$

$a_1 = 0$

$a_2 = 1 \{11\}$

OPA: $a_3 = 8$ (from side analysis)

$a_3 = 3 = \{011, 110, 111\}$

$a_n = a_{n-2} + a_{n-1} + 2^{n-2}$

$a_4 = 8 = \{___\}$

$a_3 = a_1 + a_2 + 2 = 0 + 1 + 2 = 3. \checkmark$

OPB: $a_1 + 2a_2 + 2 = 4 \times$

OPC: $2a_1 + a_2 + 2^1 = 3 \checkmark$

OPD: $a_3 = 4 \times$

Now, $a_4 = 8 \rightarrow$ OPA: $a_2 + a_3 + 2^2 = 8 \checkmark$

→ OPC: $2 \times 1 + 3 + 4 = 9$

∴ OPA

3. GENERATING FUNCTIONS

1. INTRODUCTION

Let us say we have a series a_1, a_2, a_3, a_4, a_5 now what is the generating function for the sequence. GF is nothing but we take each ele in the sequence and assign some weights. Now, the GF for the above sequence will be $a_1x^0 + a_2x^1 + a_3x^2 + a_4x^3 + a_5x^4$

Now, The Seq: 1, 1, 1, 1, 1

Seq: - 1, 1, 1, 1, ...

GF: $1x^0 + 1x^1 + 1x^2 + 1x^3 + 1x^4 + 1x^5$

GF = $1 \cdot x^0 + 1 \cdot x^1 + 1 \cdot x^2 + 1 \cdot x^3 + \dots$

= $1 + x + x^2 + x^3 + x^4 + x^5$

GF = $1 + x + x^2 + x^3 + \dots$

2. EXAMPLE - 1

$$1 > 1 \cdot x^0 + 1 \cdot x^1 = 1 + x = \frac{(x^2 - 1)}{(x - 1)} = \left[\frac{1 - x^2}{1 - x} \right] \text{ Given Seq = 1, 1}$$

$$2 > \text{Seq: } 1, 1, 1 = 1 + x + x^2 = 1 + x + x^2 * \frac{(x - 1)}{(x - 1)} = \frac{(x^3 - 1)}{(x - 1)} = \left[\frac{1 - x^3}{1 - x} \right]$$

3 > 1, 1, 1, ... n 18

GF: $1+x+x^2+\dots+x^n = \frac{x^n-1}{x-1} = \left[\frac{1-x^n}{1-x} \right]$ derivative of Eqs (28)

(Suppose) $n = m : np$

$x^n - 1 = 0$

$x^n - 1 = x^m \cdot x^{m-1} \cdots x^1 - 1 = 0$

$x^m - 1 = 0 \Rightarrow x = 1$

$x^{m-1} = 1$

3. Generating Functions

INTRODUCTION

4. INTRODUCTION TO PROPOSITIONAL CALCULUS.

1. INTRODUCTION

(2)

Proposition: Any declarative statement with a truth value assigned to it is called proposition. Ex: 1) My name is Ravindra Babu Ravula.

2) I am hungry [Yes/no is truth value]

3) $x = y + 1$ [No truth value]

(2)

(2)

2. CONNECTIVES \wedge AND \vee AND \sim

Propositional variable: Variable used to denote a proposition is called pv.

Connectives: These are the things which connect more than one propositional

P: My name is RBR

variable
(propositions)

Negation: $\neg P$: My name is not RBR

i.e. It is not the case that my name is not RBR.

\wedge : p; I got 80% marks

Q: I got A Grade } $P \wedge Q$ = I got 80% marks AND I got 'A' grade.

\vee :

P: } $P \vee Q$ = I got 80% marks OR A grade.

Q:

		P	Q	$P \wedge Q$	P	Q	$P \vee Q$
		T	T	T	T	T	T
		T	F	F	T	F	T
		F	T	F	F	T	T
		F	F	F	F	F	F

3. IMPLICATION

p: you get 60 marks in Gate

q: you will go to its.

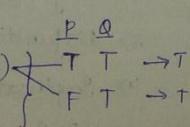
$p \rightarrow q$: If you get 60 marks in Gate then you will go to its.

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

p: Today is Monday (T/F)

Q:

$5 \times 2 = 10$ (T)



$\therefore p \rightarrow q$ is always a

The various ways of Representing $p \rightarrow q$ are:

- 1) If p then q . \rightarrow A necessary condition for ' p ' is ' q '.
- 2) ' p ' is sufficient for q . \rightarrow q unless $\sim p$.
- 3) q if p . \rightarrow ' p implies q '.
- 4) q when p . \rightarrow ' p ' only if ' q '.
- 5) $\sim q$ whenever $\sim p$. \rightarrow A sufficient condition for ' q ' is ' p '.
- 6) q is necessary for p . \rightarrow ' q ' follows from ' p '.

4. QUESTION ON IMPLICATION

$p \rightarrow q$: If today is Friday then $2 \times 3 = 6$.

Converse: $q \rightarrow p$: If $2 \times 3 = 6$ then today is Friday

Contrapositive: $\sim q \rightarrow \sim p$: If $2 \times 3 \neq 6$ then today is not Friday

Inverse: $\sim p \rightarrow \sim q$: If today is not Friday then $2 \times 3 \neq 6$.

i) $p \rightarrow q \not\equiv \neg p$.

$p \rightarrow q \equiv \neg q \rightarrow \neg p$ (conditional & contrapositive)

$p \rightarrow q \equiv [\neg p \vee q]$

$q \rightarrow p \equiv \neg p \rightarrow \neg q$ (Inverse and converse)

$\equiv (\neg q \rightarrow \neg p)$

5. BI-CONDITIONAL

P	Q	$P \leftrightarrow Q$	$(p \rightarrow q) \wedge (q \rightarrow p)$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	T	T

DM: 0.1

$P \leftrightarrow Q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Various Representations of Bi-conditional are: if and only if

if it is necessary and sufficient.

precedence of operations : 1) Negation

2) Conjunction $\Rightarrow \neg p \wedge q$ vi

3) Disjunction

4) Implication $\Rightarrow ((\neg p) \vee q)$ vi

5) Bi Implication

6. Example 1

You can access the internet only if you are a computer science major or you are not a freshman.

(26) There are 3 statements here

- P: you can access the internet $\rightarrow p$.
- Q: you are a computer science major } $(Q \vee R)$
- R: you are not a freshman.

$$P \rightarrow (Q \vee R)$$

\Rightarrow what ever before "only if" is called hypothesis and what is given after that is called conclusion.

I. Example 2

You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years.

P: You can ride the roller coaster

q: You are under 4 feet tall

r: You are older than 16 years.

Now, we can replace 'unless' with 'And' and Negate the statement.

$$(Q \wedge \neg r)$$

$$(\neg p)$$

$$(Q \wedge \neg r)$$

$$P \rightarrow Q \quad (\text{if } p)$$

Conclusion Hypothesis

(After it)

present after if \rightarrow Hypothesis.

Before
= conclusion

\therefore final result = Hypothesis \rightarrow Conclusion

$$\Rightarrow [(Q \wedge \neg r) \rightarrow p]$$

8. EXAMPLE - 3

The automated reply cannot be sent (when) the file system is full.

$\sim P$ when Q .

$$\therefore [Q \rightarrow \sim P]$$

p: The automated reply can be sent

q: The file system is full

if $P \rightarrow Q = Q$ when (P)

Conclusion Hypothesis

9. CONSISTENT SYSTEM

Determine whether these specifications are consistent.

→ The diagnostic message is stored in the buffer or it is retransmitted

→ The diagnostic message is not stored in the buffer

or → "If the diagnostic message is stored in the buffer, then it is retransmitted."

1) The diagnostic message is stored in the buffer or it is retransmitted

$$P \vee Q \quad P \vee Q$$

2) The diagnostic message is not stored in buffer ($\neg P$)

3) If the diagnostic message is stored in the buffer, then it is retransmitted. ($P \rightarrow Q$)

$$\therefore (P \vee Q)$$

$$\neg (P)$$

$$\therefore (P \rightarrow Q)$$

Now we should find some assignment of truth values to 'P' and 'Q' such that all the 3 will be True simultaneously

$$P \vee Q$$

$$\neg P$$

$$P \rightarrow Q$$

This should be true $\Rightarrow \neg P = T$

$$\Rightarrow P = F$$

Now, $P \vee Q$ should be $\Rightarrow P = F$ already known

$$\text{True } Q = T$$

SETS

1. SETS AND SUBSETS

Set: well-defined collection of objects

Ex: $A = \{1, 2, 3\}$

$S = \{\}$

Null set: set containing no elements

subset: If

of B .

A

Note: for

Proper subset

proper sub

Note

2. POWER SET

Denoted by

\Rightarrow If A is

power set

Ex: $A =$

Subs

\Rightarrow If a set

10. EQUIVALENCES

$$1) p \rightarrow q = \sim p \vee q$$

$$2) p \rightarrow q = \sim q \rightarrow \sim p$$

$$3) p \vee q = \sim p \rightarrow q$$

$$4) p \wedge q = \sim (\sim p \rightarrow \sim q)$$

$$5) \sim (p \rightarrow q) = p \wedge \sim q$$

$$6) (\sim p \rightarrow q) \wedge (p \rightarrow q) = p \rightarrow (q \wedge r)$$

$$7) (p \rightarrow q) \wedge (q \rightarrow r) = (p \vee q) \rightarrow r$$

$$8) (p \rightarrow q) \vee (p \rightarrow r) = p \rightarrow (q \vee r)$$

$$9) (p \rightarrow q) \vee (q \rightarrow r) = (p \wedge q) \rightarrow r$$

$$10) (p \leftrightarrow q) = (p \rightarrow q) \wedge (q \rightarrow p)$$

$$11) \sim (p \leftrightarrow q) \equiv (p \leftrightarrow \sim q)$$

SETS

1. SETS

Set: w

Ex: A

S

Nullset

subset:

of B.

Note:

Proper

proper

& Pow

Denote

→ If

pow

Ex:

11. DE MORGAN'S LAW

$$1) \sim (p \vee q) \equiv \sim p \wedge \sim q$$

$$2) \sim (p \wedge q) \equiv \sim p \vee \sim q$$

Ex: Ravi has a computer and a phone



If I apply Negation to this what will
be the output??

Let p: Ravi has a computer

q: Ravi has a phone

Now, $\sim(p \wedge q) = \sim p \vee \sim q$

→ Ravi does not have a computer

or Ravi does not have a phone

12. PROPOSITIONAL FUNCTION

→ proposition is a statement to which we can assign a truth value.

Ex: 2 is greater than 10 (False) ← proposition.

{
x is greater than 10 } ← cannot take T/F unless
variable predicate the value of x is known.

This is called propositional
function, p(x)

∴ p(x): x is greater than 10

p(2): 2 is > 10 (F)

p(12): 12 is > 10 (T)

→ The propositional
function transforms itself
into proposition.

p(x), computer x is connected to network y.

⇒ If

No 13 QUANTIFIERS

if $p(x) : x > 2$

Domain: $\{1, 2, 3, 4\}$ $p(1)=F, p(2)=F, p(3)=T, p(4)=T$

(34)

Now, $\forall x p(x)$ is true, if $p(x)$ is true for all values of x .

6. Universal Quantifier / universal Quantification.

ex Now, let $p(x) = x > 2$

a) Domain: $\{1, 2, 3, 4\}$

If $p(1)=F, p(2)=F, p(3)=T, p(4)=T$ $\therefore \forall x p(x)$ is true if there exists atleast one value of x in the domain, for which $p(x)=\text{True}$

In this case $\exists x p(x)=\text{True}$

↓
Existential Quantifier.

14. RELATION BETWEEN THE TWO QUANTIFIERS

Let If $\forall p(x)$ is true then $\exists x p(x)$ is True

$p(x) = x > 3$

cor If $\forall x p(x)$ is false then $\exists x p(x)$ is (false/True)

Domain: $\{2, 3, 4\}$

a) If $\exists x p(x)$ is true then $\forall x p(x)$ is true

$\exists x p(x) = T \leftarrow$

Opv If $\exists x p(x)$ is false then $\forall x p(x)$ is false

$\forall x p(x) = F$

$\forall x$: → The precedence of \forall, \exists will be higher than any other logical connectives.

$$\forall x p(x) \vee Q(x) \Rightarrow [\forall x p(x)] \vee Q(x).$$

15. DISTRIBUTING QUANTIFIERS

$$1) \forall x (x+y+z) = (\forall x (x)) + y + z = 10$$

$$2) \exists x p(x) \vee \forall x Q(x) = [\exists x p(x)] \vee [\forall x Q(x)]$$

$$3) \forall x (p(x) \wedge Q(x))$$

$$\text{Domain: } \{x_1, x_2\} = [p(x_1) \wedge Q(x_1) \wedge Q(x_1) \wedge Q(x_2)] \quad \therefore \forall \text{ can be}$$

SETS

SETS AND S

Set: well de-

Ex: $A = \{1, 2$

$S = \{3\}$

Nullset: set o

subset: If e

of B . $A = \emptyset$

$A = \emptyset$

Note: for e

Proper Subset

proper subset

Note:

d. POWER SET

Denoted by

⇒ If A is f

power set

Ex: $A = \{a$

subsets

P

⇒ If a set

④ $\forall x$ cannot be "distributed over disjunction." that is, $\forall x(p(x) \vee q(x)) \neq [\forall x p(x)] \vee [\forall x q(x)]$ that is, $\forall x(p(x) \vee q(x)) \neq [\forall x p(x)] \vee [\forall x q(x)]$ (5)

$$\forall x(p(x) \vee q(x)) = [p(x_1) \vee q(x_1)] \wedge [p(x_2) \vee q(x_2)] \quad (35)$$

D: $\{x_1, x_2\}$

$$[\forall x p(x)] \vee [\forall x q(x)] = [p(x_1) \wedge p(x_2)] \vee [q(x_1) \wedge q(x_2)]$$

Not Equal

⑤ $\exists x(p(x) \vee q(x)) = [\exists x p(x)] \vee [\exists x q(x)] \rightarrow$ There exists can be distributed over disjunction.

16. QUANTIFIERS WITH NEGATION

1) Every student in the class has taken a course in CS

$p(x)$: x has taken a course in CS

of $\forall x p(x)$:

x : Students of the class.

Now, $\sim(\text{Statement 1})$: There exist at least one student in the class

2) who has not taken the course in CS.

Let

con-

a)

opm

$$\begin{aligned} \sim(\forall x p(x)) &\equiv \exists x (\sim p(x)) \\ \sim \exists x p(x) &\equiv \forall x \sim p(x) \end{aligned}$$

$x_1 =$

17. EXAMPLES ON NEGATING THE QUANTIFIERS

→ There is ^{an} honest politician

$H(x)$: x is honest

$\exists x H(x)$: There is an honest politician

x : {All politicians}

Now, $\sim(\exists x H(x)) = \forall x \sim(H(x))$

opD

= All the politicians are dishonest.

→ All Americans eat cheese burgers.

$C(x)$: x eats cheese burgers $\Rightarrow \forall x C(x)$ and x : {All Americans}, Now

$\sim(\forall x C(x)) = \exists x \sim(C(x)) \Rightarrow$ "There exists an American who don't eat cheese burgers!"

SETS

1. SETS AND SUBSETS

Set: well def.

Ex: $A = \{1, 2, 3\}$

$S = \{\text{set}\}$

Nullset: set w/

subset: If ever

of B .

$A = \{1, 2, 3\}$

Note: For every

proper subset: Any proper subset of

Note: If A

POWER SET

denoted by $P(A)$

If A is finite set

power set of A : If

$x: A = \{a, b\}$

Subsets of A :

$P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

If a set contains n

18. IMPORTANT EXAMPLES

$$\Rightarrow \neg \forall x (P(x) \wedge Q(x)) \equiv \exists x (\neg P(x) \vee \neg Q(x)) \quad (6)$$

$$\Rightarrow \neg \forall x [R(x)] \text{ where } R(x) = P(x) \wedge Q(x)$$

$$\Rightarrow \exists x \neg (R(x)) \equiv \exists x (\neg P(x) \vee \neg Q(x)) \rightarrow \neg P(x) \vee \neg Q(x)$$

6. $\Rightarrow \exists x \neg (P(x) \wedge Q(x))$

With evn
ex. $\Rightarrow \exists x [\neg P(x) \vee \neg Q(x)]$

a) $\Rightarrow \neg \forall x (P(x) \rightarrow Q(x))$

T(1) $\Rightarrow \neg \forall x (\neg P(x) \vee Q(x))$

$\Rightarrow \exists x [P(x) \wedge \neg Q(x)]$

OPA: 3) $\neg \exists x (P(x) \wedge Q(x)) = \forall x (\neg P(x) \vee \neg Q(x)) \equiv \forall x (P(x) \rightarrow \neg Q(x))$

4) $\forall x (P(x) \rightarrow \neg Q(x)) = \forall x (\neg P(x) \vee \neg Q(x))$

Q. E.

19. TRANSLATING ENGLISH STATEMENTS IN PROPOSITIONAL FUNCTION

Let cons.

Every student in the class has taken the course operating system.

Ques:

$O(x)$: x has taken the course OS.

x : {Students in the class}

=

$\forall x O(x)$

Now let $S(x)$: x is a student in the class.

\therefore If x is a student in the class then he has taken OS

$$\forall x (S(x) \rightarrow O(x)). \quad (\text{domain is expanded here})$$

For all people in the world

If he has a student in the class then he has taken the course OS.

x : {All the people in the world}

Hint: Now, if you see the word Every in the question the option ans must contain \forall and \rightarrow .

Now, $\neg \forall x (S(x) \rightarrow O(x)) = \exists x (S(x) \wedge \neg O(x)) \rightarrow \text{check}$

SETS

SETS AND SUBSETS

Set: well-defined

Ex: $A = \{1, 2, 3\}$

$S = \{\text{set of}$

null set: set with

subset: If every element of A is also an element of B.

$A = \{1, 2, 3\}$

Note: For every

proper subset: Any

proper subset of

Note: If A

POWER SET

denoted by $P(A)$

If A is finite set

power set of A: S

$\therefore A = \{a, b\}$

Subsets of A:

$P(A) = \{\emptyset,$

If a set contains

Now, if the questions are given of this form $T(n) = 2T(\sqrt{n})$
it as \dots

20. TRANSLATION CONTINUED

(6)

Some student in the class has taken OS course

(37)

$O(x)$: x has taken OS.

$\Rightarrow \exists x O(x)$ —————
 x : Students in the class



Now, $S(x)$: x is student in the class

$\exists x (S(x) \wedge O(x)) \Rightarrow$ Now, if you see "some" then Ans
should contain (\exists and AND).

Now, $\sim [\exists x (S(x) \wedge O(x))]$

$\sim (\text{some student in the class has taken OS course}) = (\text{No student in the class has taken OS course})$

$\therefore \sim [\exists x (S(x) \wedge O(x))]$

$= \forall x [\sim S(x) \vee \sim O(x)]$

$= \forall x [S(x) \rightarrow \sim O(x)]$