→ Game theory :-

* Intro [Combinatorial game theory]
  - perfect information
  - no randomization like coin toss
  - 3 cases → win, lose or tie
  - Impartial game: outcome does not depend on the turn first or not
  - Partisan game: moves are not same
  - Given ⇒ set of position (including initial position & whose turn is to move & terminal position honi compulsory hai)
  - Game should end at some time

Q- Ex:- ① 2 Player
② "N" coins in a pile
③ In each turn, a player can choose to remove one or two coins
④ whoever removes last coin, wins.

Ans Nayaak

Observation :→ Found out that first player is guaranteed to win unless "N" is a multiple of 3.

Strategy :-

* Finder - Keepers game:

A & B playing a game in which a certain no of coins are placed on table. Each player picks atleast 'a' & atmost 'b' coins in his turn unless there is less than 'a' coins left in which case the player has to pick all those left

→ (a) finders - Winners → [In this, who picks last coin wins]

Strategy is to reduce Opponent to a state containing $\boxed{(a+b) \times K}$ coins which is a loosing state for opponent

→ (b) Keepers - losers → [In this, who picks last coin, loses]

Strategy, to reduce opponent to a state containing $\boxed{(a+b) \times K + x} \rightarrow \boxed{1 \leq x \leq a}$

, which is a loosing state for opponent

Problems

Q-1 A and B play game of finder-winners with $a = 2$ & $b = 6$. If A starts the game & there are 74 coins on table initially, how many should A pick?

A will pick 2.
then, B will be left with 72 & ab
B kuch bhi karle, A can always pick $(8 - x)$ & wrap up

Q- In a game of Keeper - losers, B started the game when there were N coins on table. If B is confident of winning game & $a = 3$, $b = 5$, which of following cannot be 'N'?

(a) 94 (b) 92 (c) 76 (d) 66

In Keeper losers, aim is to give opponent

$(3+5)K + x$, $1 \le x \le 3$

$\therefore$ (66)

Coz B ko bhi atleast 3 uthane hai na, that's why

Date
DELTA Pg No

(3) A & B play finders winner with 56 coins, A plays first, a = 1, b = 6. What B should pick immediately after A?

S A will lose in any case. But ~~the no~~ the no of coins B picks, depends on A's pick

(4) In a game of Keeper - Losers, 126 coins. A plays first, a = 3, b = 6, who wins?

S If A want to win, he should leave $(3+6) \times K + x$
i.e $9K+1$, $9K+2$, $9K+3$ coins ~~on~~ on table.

He can do this by picking 6 coins & hence can win

Next is:-

(1) Game of Nim → Gfg / TopCoder

(2) Grundy Numbers / Mex function → Gfg / Topcoder

(3) Sprague - Grundy Theorem → Gfg

→ How to apply Sprague-grundy theorem - ?

Steps:

→ Break composite games into sub-games

→ Then for each sub-game, calculate grundy no at that position

→ Then calculate XOR of all calculated grundy numbers

⇒ If the XOR value is non-zero, then player who is going to make the turn (first player) will win else he is destined to lose, no matter what-

After this (Practice problems)

→ SPOJ (MLM game, RESNO4, GAME 2, GAME 31, NGM, NUMGAME, NIMGAME, MATGAME, REMGAME)