TOPICWISE : DISCRETE MATHEMATICS-1 (GATE - 2019) - REPORTS

Q. 1

Consider two well formed formulas in propositional logic:

$F_1 : (p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$
$F_2 : (p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$

Which of the following is correct?

Solution Video | Have any Doubt ?

A. F_1 is satisfiable, F_2 is valid

B. F_1 is unsatisfiable, F_2 is satisfiable — Correct Option

Solution :
(b)

$F_1 :\ (p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$
we know that $\neg (p \leftrightarrow q) \equiv (\neg p \leftrightarrow q)$
So, if $(p \leftrightarrow q)$ is assumed of A.
Then $A \wedge A' = 0$, means unsatisfiable.

$F_2 :\ (p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$
$= (p + q')\,(p' + q)\,(p' + q')$
$= p + q'\,(p' + qq')$
$= (p + q')\,p'$
$\equiv p'q'$ which is not valid but satisfiable.

So, F_1 is unsatisfiable but F_2 is satisfiable.

C. F_1 is unsatisfiable, F_2 is valid

D. F_1 and F_2 both are unsatisfiable — Your answer is **Wrong**

QUESTION ANALYTICS +

Q. 2

If $f(x) = \frac{x}{x-1},\ x \neq 1$, then which of the following represent $\underbrace{(f \circ f \circ f \circ \ldots\ldots f)}_{21 \text{ times}}(x)$?

Solution Video | Have any Doubt ?

A. $\left(\frac{x}{x-1}\right)^{21}$

B. $\frac{21x}{x-1}$

C. x — Your answer is **Wrong**

D. $\frac{x}{x-1}$ — Correct Option

Solution :
(d)

$$f(x) = \frac{x}{x-1}$$

$$f \circ f(x) = \frac{\left(\frac{x}{x-1}\right)}{\left(\frac{x}{x-1}\right) - 1} = \frac{\frac{x}{x-1}}{\frac{x-x+1}{x-1}} = \frac{\frac{x}{x-1}}{\frac{1}{x-1}}$$

i.e. $\underbrace{f \circ f(x)}_{2 \text{ times}} = x$

So, $f \circ \underbrace{(f \circ f \circ f \circ \ldots\ldots f)}_{20 \text{ times}}(x) = f(x)$

$$= \frac{x}{x-1}$$

QUESTION ANALYTICS +

Q. 3

Consider R is real number and S and R are subsets of R × R define as:

S = {(x, y) : y = x + 1 and 0 < x < 2}
T = {(x, y) : x – y is an integer}

Which one of the following is true?

Solution Video | Have any Doubt ?

A T is an equivalence relation on R but S is not — Your answer is **Correct**

Solution :
(a)

1. S = {$(x, y) : y = x + 1$ and $0 < x < 2$}

- **Check for Reflexive Relation:**
  $(x, x) : x = x + 1$ but $x \neq x + 1$
  Hence cannot be reflexive S is not equivalence relation on R.

2. T = {$(x, y) : x - y$ is an integer}

- **Check for Reflexive Relation:**
  $(x, x) : x - x$ is integer $x - x = 0$ and $0 \in$ integer
  So, T is reflexive.
- **Check for Symmetric Relation:**
  $(x, y) : x - y$ is integer and $(y, x) : y - x$ also an integer.
  So, T is symmetric relation.
- **Check for Transitive Relation:**
  $(x, y) : x - y$ is integer and $(y, z) : y - z$ is integer then $(x, z) : x - z$ is also integer.
  So, T is transitive.
  Hence T is equivalence relation but S is not.

B S is an equivalence relation on R but T is not

C Both S and T are an equivalence relation on R

D Neither S nor T is an equivalence relation on R

QUESTION ANALYTICS +

Q. 4

Consider a mapping f : $n \rightarrow$ N, where N is the set of natural numbers is defined as

$$f(n) = \begin{cases} n^2, & \text{for } n \text{ odd} \\ 2n+1, & \text{for } n \text{ even} \end{cases}$$

for $n \in$ N. Which of the following is true about 'f'?

Solution Video | Have any Doubt ?

A Surjective but not injective

B Injective but not surjective

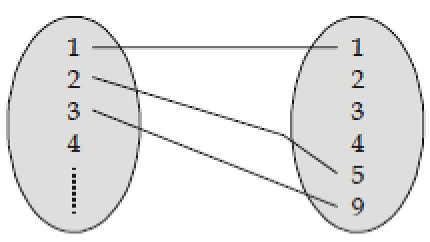
C Bijective

D Neither surjective nor injective — Correct Option

Solution :
(d)
'N' is given as {1, 2, 3 .....}

$$f(n) = \begin{cases} n^2, & \text{for } n \text{ odd} \\ 2n+1, & \text{for } n \text{ even} \end{cases}$$

- **Check for Injective:**
  for $f(3) = n^2 = 9$
  for $f(4) = 2n + 1$
  $= 2 \times 4 + 1$
  $= 8 + 1 = 9$
  Since both $f(3)$, $f(4)$ maps to same element 9.
  Hence cannot be injective.
- **Check for Surjective:**



Hence for domain elements 2, 4 ..... are not mapped to any elements. Hence cannot be surjective.

QUESTION ANALYTICS +

Q. 5

Which of the formula is correct for given sentence:
"No students are allowed to carry smartphone"

Solution Video | Have any Doubt ?

A $\exists x(\neg$ student $(x) \rightarrow$ carry_smartphone $(x))$

B $\forall x($student $(x) \rightarrow \neg$ carry_smartphone $(x))$ — Your answer is **Correct**

Solution :
(b)
"No students are allowed to carry smartphone"
Can be written as: Not a student are allowed to carry smartphone
$\equiv \neg[\exists x($student $(x) \wedge$ carry_smartphone $(x))]$
$\equiv \forall x(\neg$ student $(x) \vee \neg$ carry_smartphone $(x))$
$\equiv \forall x($student $(x) \rightarrow \neg$ carry_smartphone $(x))$
So, option (a) is correct representation only.

C $\forall x(\neg$ student $(x) \rightarrow$ carry_smartphone $(x))$

D $\forall x(\neg$ student $(x) \rightarrow \neg$ carry_smartphone $(x))$

QUESTION ANALYTICS +

Q. 6

The minimum number of ordered pair of integers (a, b) are needed to guarantee that there are two ordered pairs (a_1, b_1) and (a_2, b_2) such that a_1 mod 4 = a_2 mod 4 and b_1 mod 6 = b_2 mod 6

________.

FAQ Solution Video Have any Doubt ?

25 Correct Option

Solution :
25

For a in (a, b), there are 4 different congruence classes possible for mod 4 i.e. 0, 1, 2 and 3 and 6 different congruence classes possible for mod 6 i.e. 0, 1, 2, 3, 4 and 5.
So number of different ordered pair where (a_1, b_1) and (a_2, b_2) such that a_1 mod 4 = a_2 mod 4 and b_1 mod 6 = b_2 mod 6 not possible are $4 \times 6 = 24$.
So to get two pair with given condition we need 24 + 1 = 25 ordered pairs.

QUESTION ANALYTICS +

Q. 7

Consider the following well formed formula:

$(p \vee \neg q \vee \neg r \vee s) \rightarrow t \vee \neg u$

The maximum number of rows in truth table of above formula which evaluate to true are ________.

FAQ Solution Video Have any Doubt ?

49 Correct Option

Solution :
49

Case 1: $(p \vee \neg q \vee \neg r \vee s) \rightarrow t$
When $t = 1$ and $u = 1$ then p, q, r, s take any value i.e. either 0 or 1 because $A \rightarrow$ True is always tautology.
So, number of values: $2^4 = 16$.
Case 2: $(p \vee \neg q \vee \neg r \vee s) \rightarrow \neg u$
When $t = 0$ and $u = 0$ then p, q, r, s take any value i.e. either 0 or 1 because $A \rightarrow$ True is always tautology.
So, number of values: $2^4 = 16$.
Case 3: $(p \vee \neg q \vee \neg r \vee s) \rightarrow t \vee \neg u$
When $t = 1$ and $u = 0$ then p, q, r, s take any value i.e. either 0 or 1 because $A \rightarrow$ True is always tautology.
So, number of values: $2^4 = 16$.
Case 4: $(p \vee \neg q \vee \neg r \vee s) \rightarrow$ False
When $t = 0$ and $u = 1$, the $p = 0, p = 1, r = 1$ and $s = 0$ make form False $\rightarrow$ False i.e. tautology.
So, number of values : 1.

$$\begin{aligned}\text{Total number of values} &= 16 \times 3 + 1 \\ &= 48 + 1 = 49\end{aligned}$$

QUESTION ANALYTICS +

Q. 8

The n^{th} term independent of x in expansion of $\left(x+\frac{1}{x^2}\right)^{15}$. The coefficient of n^{th} term is ________.

Solution Video Have any Doubt ?

3003 Correct Option

Solution :
3003

By using binomial expansion of $\left(x+\frac{1}{x^2}\right)^{15}$ i.e.,

$$\begin{aligned}E(x) &= \left({}^{15}C_r\right) \times (x^2)^{15-r} \times \left(\frac{1}{x^2}\right)^r \\ &= {}^{15}C_r \times x^{15-r} \times x^{-2r} \\ &= {}^{15}C_r \times x^{15-3r}\end{aligned}$$

Since $E(x)$ must be free from x, so $15 - 3r = 0$.

$$r = 5$$

Hence, by putting $r = 5$ in equation (1)

$$\begin{aligned}E(4) &= {}^{15}C_5 \times x^{15-15} \\ &= {}^{15}C_5 \\ &= 3003\end{aligned}$$

QUESTION ANALYTICS +

Q. 9

The number of seven digit integers possible with sum of the digits equal to 11 and formed by using the digits 1, 2 and 3 only are ________.

Solution Video Have any Doubt ?

161 Correct Option

Solution :
161

Total possibility with sum = 11 and 7 digits

3,3,1,1,1,1,1

3,2,2,1,1,1,1

2,2,2,2,1,1,1

$$3,3,1,1,1,1,1 \Rightarrow \frac{7!}{2! \times 5!} = 21 \text{ numbers}$$

$$3,2,2,1,1,1,1 \Rightarrow \frac{7!}{2! \times 4!} = 105 \text{ numbers}$$

$$2,2,2,2,1,1,1 \Rightarrow \frac{7!}{4! \times 3!} = 35 \text{ numbers}$$

$\therefore$ The number of 7 digit integers

$$= 21 + 105 + 35 = 161$$

Your Answer is 78125

QUESTION ANALYTICS +

Q. 10

Consider there are two tribes living on the Island: Knights and knaves. Knights always tell truth while Knaves always tells lie. Let we counter two random people A and B, upon asking a question to 'A', A says "If B is Knight then I am a Knave". What we can conclude about person A and B?

FAQ Solution Video Have any Doubt ?

A A is Knight and B is Knave — Your answer is **Correct**

Solution :

(a)

Option (a) is correct. Let's see why:

A says "If B is Knight then I am a Knave", which is equivalent to the propositional logic statement

B is Knight $\Rightarrow$ A is Knave

Taking contrapositive, we get

A is Knight $\Rightarrow$ B is Knave

So option (a) is consistent with the above statement (as by assuming A as Knight and B as Knave, we get a true $\Rightarrow$ true assignment).

And similarly we can verify that the other options won't be consistent as they will lead to a contradiction.

In case you want a more detailed explanation, you can refer the video solution of this question.

B A is Knave and B is Knave

C Both A and B are Knight

D Both A and B are Knave

QUESTION ANALYTICS +

Q. 11

Which of the following is an uncountable set?

S_1 : $A = \{x \in Q \mid -100 \le x \le 100\}$ where Q represent set of rational numbers

S_2 : B = set of all real number between (0, 0.1]

S_3 : $C = \{(x, y) \mid x \in N, y \in Z\}$ where N represent set of natural numbers and Z represent set of integers

$S_4 : D = \left\{\frac{1}{n} \middle| n \in N\right\}$

FAQ Solution Video Have any Doubt ?

A S_1 and S_2 only

B S_2 only — Correct Option

Solution :

(b)

• Set A is countable. Since Q (set of rational numbers) is countable and every subset of countable set is also countable.

• Set B is uncountable. Since every subset of real number is uncountable.

• Set C is countable because it is Cartesian product of two countable sets i.e. N × Z.

• Set D is countable. Since one to one correspondence with set of natural number Cantor's theorem

C S_2 and S_4 only — Your answer is **Wrong**

D S_1 and S_3 only

QUESTION ANALYTICS +

Q. 12

Assume among 75 children who went to an water park, where they could ride on merry-goround, roller coaster and ferris wheel. It is known that, 20 of them had taken all 3 rides and 55 had taken atleast 2 of the 3 rides. Each ride costs ₹ 0.50 and total receipt park is ₹ 70. How many number of children who did not try any of the rides?

Solution Video Have any Doubt ?

A 10 — Correct Option

Solution :

(a)

Total children = 75

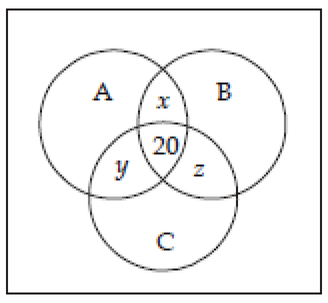
∴ Total receipt = ₹ 70 (₹ 0.50/ride)

∴ Total rides = 70 × 2 = 140

20 children had taken all the 3 rides

∴ 55 had taken at least 2 rides (2 or 3 rides).

So, 55 – 20 = 35 had taken exactly 2 rides.



Let, $x + y + z = 35$

Children who had taken exactly one ride

Total single ride = 140 – (35 × 2 + 20 × 3)

= 140 – (70 + 60) = 10

So, total number of students who took exactly singe ride = 10

Children who took no ride = 75 – (35 + 20 + 10)

= 75 – (65) = 10

B 12

C 15

D 25

QUESTION ANALYTICS +

Q. 13

Consider the following two statements:

S_1 : All clear explanations are satisfactory.
S_2 : Some excuses are unsatisfactory.

Which one of the following statement follows from S_1 and S_2 as per inference rules of logic?

Solution Video | Have any Doubt ?

A Every excuses are not clear explanations.

B Some excuses are clear explanations. Your answer is **Wrong**

C Some excuses are not clear explanations. Correct Option

Solution :
(c)

$S_1 : \forall x$ (clear $(x) \rightarrow$ (satisfactory (x))

$S_2 : \exists x$ (Excuse $(x) \wedge \neg$ satisfactory (x))

S_1 by using contrapositive rule

$S_3 : \forall x(\neg$ satisfactory $(x) \rightarrow \neg$ clear (x))

By using S_3 and S_3

$\exists x$ (Excuse $(x) \rightarrow$ not clear (x)) i.e. some excuses are not clear explanation.

D Some explanations are clear excuses.

QUESTION ANALYTICS +

Q. 14

Consider the relation 'R' on the power set P(A) of a set A as,
$\forall a, b \in P(A) \{(a, b) \in R \leftrightarrow a \cap b \neq \phi\}$

Which of the following is true?

FAQ | Solution Video | Have any Doubt ?

A R is reflexive, transitive but not symmetric.

B R is not reflexive and transitive but symmetric. Correct Option

Solution :
(b)

- R is not reflexive since ϕ is an element of power, set of any subset of A and $\phi \cap \phi = \phi$ and be to R.
- R is symmetric because intersection $(\cap)$ is commutative, thus $a \cap b \neq \phi$ the $b \cap a \neq \phi$.
- R is not transitive because $a \cap b \neq \phi$ and $b \cap c \neq \phi$ does not assure $a \cap c \neq \phi$. **e.g.**, a = {
  $b = \{2, 3\}$ and $c = \{3, 4\}$

So, $\{1, 2\} \cap \{2, 3\} \neq \phi$

$\{2, 3\} \cap \{3, 4\} \neq \phi$

but $\{1, 2\} \cap \{3, 4\} = \phi$ so **fail**.

C R is reflexive, symmetric and transitive. Your answer is **Wrong**

D R is reflexive but not symmetric and transitive.

QUESTION ANALYTICS +

Q. 15

Consider $A_1, A_2, A_3, \ldots A_{45}$ are forty-five sets each having 7 elements and $B_1, B_2, B_3, \ldots B_n$ are n sets each having 4 elements. Let $\bigcup_{i=1}^{45} A_i = \bigcup_{i=1}^{n} B_i = S$ and each elements of S belongs to exactly 15 of A_i's and exactly 12 of B_i's. Then the value of n is ________ [Assume elements are not repeated]

Solution Video | Have any Doubt ?

63 — Correct Option

Solution :
63
Total number of elements in Ai = 45 × 7 = 315
Each element is used 15 times, so

$$S = \frac{315}{15} = 21$$

Similarly element in $B_i = n \times 4$
Each element is used 12 times, so

$$S = \frac{4n}{12}$$

So, $$\frac{4n}{12} = 21$$
$$4n = 21 \times 12$$
$$n = 21 \times 3$$
$$n = 63$$

QUESTION ANALYTICS +

Q. 16

The number of non-negative integer solutions for following pairs of equation are ________.

$$x_1 + x_2 + x_3 = 8$$
and $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 20$

Solution Video | Have any Doubt ?

4095 — Correct Option

Solution :
4095
Number of solution for equation (1)

$$x_1 + x_2 + x_3 = 8$$
$$\Rightarrow \binom{8+3-1}{8}$$
$$\Rightarrow {}^{10}C_8 \Rightarrow \frac{10\times 9\times 8!}{8!\times 2!}$$
$$\Rightarrow 45$$

Number of solution for equation (2)

$$\underbrace{x_1 + x_2 + x_3}_{y_1} + x_4 + x_5 + x_6 = 20$$
$$\Rightarrow y_1 + x_4 + x_5 + x_6 = 20$$
$$\Rightarrow 8 + x_4 + x_5 + x_6 = 20$$
$$\Rightarrow x_4 + x_5 + x_6 = 12$$
$$\Rightarrow \binom{12+3-1}{12}$$
$$\Rightarrow {}^{14}C_{12} \Rightarrow \frac{14\times 13\times 12!}{12!\times 2!}$$
$$\Rightarrow 91$$

So, total number of solutions = 45 × 91 = 4095

QUESTION ANALYTICS +

Q. 17

Consider a set S = {1000, 1001, 1002, ... 9999}. The numbers in set 'S' have atleast one digit as 2 and atleast one digit as 5 are ________.

FAQ | Solution Video | Have any Doubt ?

920 — Correct Option

Solution :
920

$$\text{Size of (S)} = |S|$$
$$= 9999 - 1000 + 1 = 9000$$

Let X is set which do not have any '2':

$$|X| = 8 \times 9 \times 9 \times 9$$
$$= 5832$$

Let Y is set which do not have any '5':

$$|Y| = 8 \times 9 \times 9 \times 9$$
$$= 5832$$

Then X ∩ Y is set which does not contain any '2' and any '5':

$$|X \cap Y| = 7 \times 8 \times 8 \times 8$$
$$= 3584$$

So, |having atleast one '2' and atleast one '5'|

$$= |S| - |X \cup Y|$$
$$= |S| - (|X| + |Y| - |X \cap Y|)$$
$$= 9000 - (2 \times 5832 - 3584)$$
$$= 920$$

QUESTION ANALYTICS +