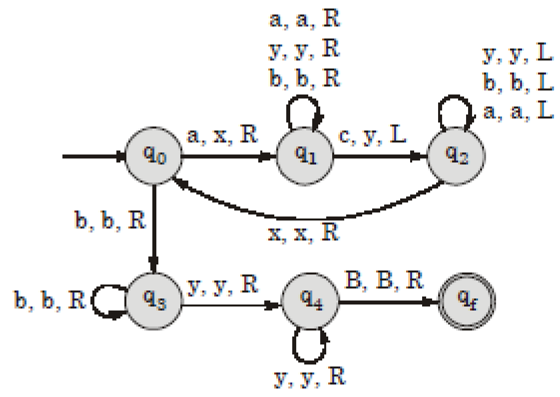
TOPICWISE : THEORY OF COMPUTATION-2 (GATE - 2019) - REPORTS

Q. 1

Consider the following TM:



Note: (a, b, c) represents: by reading input ‘a’, it replaces ‘a’ by ‘b’ and moves to ‘c’ direction.
Which of the following language accepted by above TM?

Solution Video | Have any Doubt ?

(A) $\{a^m b^n c^k \mid m, n, k \geq 0, m = k\}$

(B) $\{a^m b^n c^k \mid m, n, k \geq 0, m = n\}$

(C) $\{a^m b^n c^k \mid m, n, k > 0, m = k\}$ Correct Option

Solution :
(c)

$$L = \{a^m b^n c^k \mid m, n, k > 0 \text{ and } m = k\}$$

Here, a’s are replaced by x and c’s are replaced by y in every scan from $q_0 \to q_1 \to q_2 \to q_0$
To reach final state, atleast one b should appear and atleast one y (y represents c hence a also must appear) should appear.

$\therefore$ $L = a^i b^j c^i \mid i, j > 0\}$ is accepted by TM

So option (c) is correct.

(D) $\{a^m b^n c^k \mid m, n, k > 0, m = n\}$

QUESTION ANALYTICS +

Q. 2

Consider a Game played be between two players (Player-1, Player-2) repeatedly flip a coin:
On output as a head, **Player-1** get a point
On output as a tail, **Player-2** get a point
A player wins if his score reaches 2 points before the other player by reaching final state. Which of the following depicts NFA for above problem?

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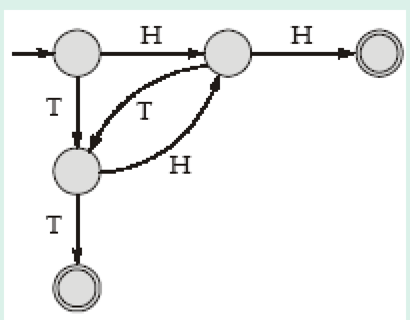
(A)

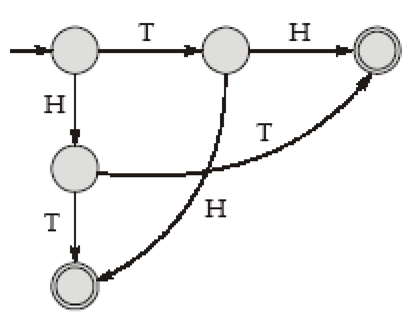


Your answer is **Correct**

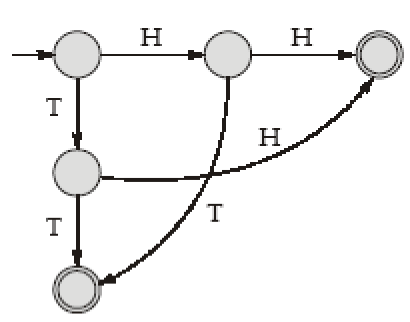
Solution :
(a)

In NFA given by option (a) possibilities at final state are:
H(T H)* H
T(H T)* T
Which shows atleast one player is winning i.e. getting two points 1st by reaching at final state.

(B)



(C)



(D) None of these

QUESTION ANALYTICS +

Q. 3

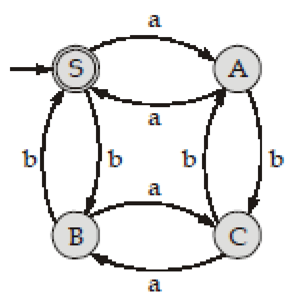
Which of the following represents the grammar for language L = {w | $n_a(w)$ and $n_b(w)$ are both even}?

Solution Video | Have any Doubt ?

A
$S \to aA \mid bB$
$A \to bC \mid aS$
$B \to aC \mid bS$
$C \to aB \mid bA$

B — Correct Option
$S \to aA \mid bB \mid \epsilon$
$A \to bC \mid aS$
$B \to aC \mid bS$
$C \to aB \mid bA$

Solution :
(b)



Option (b) can be obtained from the DFA given above.
Therefore (b) is correct.

C
$S \to aA \mid bB \mid \epsilon$
$A \to bS \mid aS$
$B \to aS \mid bS$

D — Your answer is **Wrong**
$S \to aA \mid bB \mid \epsilon$
$A \to bS \mid aC$
$B \to bC \mid aS$
$C \to aB \mid bA$

QUESTION ANALYTICS +

Q. 4

Consider <M> be the encoding of a Turing Machine as a string over alphabet $\Sigma = \{0, 1\}$. Consider L = {<M> | M is TM that halt on all input and L(M) = L′ for some undecidable language L′}. Then L is

FAQ | Solution Video | Have any Doubt ?

A — Decidable and recursive — Your answer is **Correct**

Solution :
(a)
Since M is a TM that halts on all input, so L(M) is decidable. So, L(M) ≠ L′. Since decidable language cannot be equal to some undecidable language.
So, $L = \phi$
Hence decidable and recursive.

B — Decidable and non-recursive

C — Undecidable and recursively enumerable

D — Undecidable and non-recursively enumerable

QUESTION ANALYTICS +

Q. 5

Which of the decision problems are decidable?

FAQ | Solution Video | Have any Doubt ?

A — Given a RE grammar G, is $L(G) = \Sigma^*$?

B — Given two deterministic CFG G_1 and G_2, is $L(G_1) \cap L(G_2) = \phi$?

C — Given two deterministic CFG G_1 and G_2, is $L(G_1) = L(G_2)$? — Your answer is **Correct**

Solution :
(c)
- For RE grammar, $L(G) = \Sigma^*$ i.e. RE grammar accept everything is undecidable.
- For two DCFG, $L(G_1) \cap L(G_2) = \phi$ is undecidable since $L(G_1) \cap L(G_2) = \phi \equiv \overline{L(G_1)} \cup \overline{L(G_2)} = \phi$ i.e. DCFL ∪ DCFL = ϕ, we know DCFL is not closed under union, so, undecidable.
- Equivalence problem for DCFG are decidable but for CFG is undecidable.

D — Given two CFG G_1 and G_2, is $L(G_1) = L(G_2)$?

QUESTION ANALYTICS +

Q. 6

Consider the following language:

$L_1 = \{0^l 1^m 0^{l+m} \mid l, m \geq 0\}$ $\quad$ $L_2 = \{0^l 1^{2l} 0^n \mid l \geq 0, n \geq 0\}$

$L_3 = \{0^l 1^{2l} 0^{l+n} \mid l \geq 0, n \geq 0\}$ $\quad$ $L_4 = \{0^m 1^n 2^m 3^n \mid m, n > 0\}$

The number of languages are DCFL ________.

Solution Video | Have any Doubt ?

2 Correct Option

Solution :
2

L_1 : is DCFL, push all 0′s and 1′s in the stack the for every 0 of the string, start popping from the stack.

L_2 : is DCFL, for every 0 in the string push two 0′s in the stack, for every '1', pop a '0' from the stack, then skip operation will be applied on all 0′s.

L_3 : $0^l 1^{2l} 0^l 0^n$, this is not even a CFG. Due to three level dependency, it can't be solved using single stack.

L_4 : Here we need to compare each 2 with 0 and each 3 with 1. However, in both the cases top of stack contains 1′s and 2′s respectively. So, can't be solved using single stack.

Your Answer is 1

QUESTION ANALYTICS +

Q. 7

Let L = $\{(a^P)^* | P$ is a prime number$\}$ and $\Sigma = \{a\}$. The minimum number of states in NFA that accepts the language L are ________.

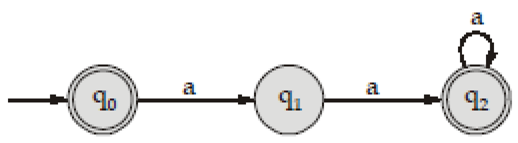
FAQ | Solution Video | Have any Doubt ?

3 Correct Option

Solution :
3

$$\begin{aligned} L &= \{(a^P)^* | P \text{ is a prime}\} \\ &= (a^2)^* \cup (a^3)^* \cup (a^5)^* \cup .... = \{\varepsilon, a^2, a^3, a^4, a^5, a^6, ...\} \\ &= \text{All strings of a's except the string a} \\ &= \{a^n | n = 0 \text{ or } n \geq 2\} \end{aligned}$$

NFA:



Number of states = 3.

QUESTION ANALYTICS +

Q. 8

Consider the context free grammars over the alphabet {a, b} given below. S is non-terminal:

$G_1 : S \to aSa \mid bSb \mid \in$

$G_2 : S \to aaS \mid bbS \mid \in$

The shortest length strings which does not belongs to L(G_1) but belongs to L(G_2) is ________.

Solution Video | Have any Doubt ?

4 Correct Option

Solution :
4

$$\begin{aligned} L(G_1) &= \text{Set of even palindromes} \\ L(G_2) &= (aa + bb)^* \end{aligned}$$

So, string "aabb" or "bbaa" belongs to L(G_2) but not to L(G_1).

Hence 4 is answer.

QUESTION ANALYTICS +

Q. 9

Consider the following two statements with respect to countability?

Statement 1: If A ∪ B is uncountable, then both set A and set B must be uncountable.

Statement 2: The Cartesian product of two countable sets A and B is countable.

The number of the above two statements correct are _______.

Solution Video | Have any Doubt ?

1 Your answer is **Correct**1

Solution :
1

Statement-1: This is incorrect i.e. atleast one set can be uncountable but need not be both.

Statement-2: Cartesian product of two countable sets A and B is countable.

QUESTION ANALYTICS +

Q. 10

Consider the following statements:

S_1 : Pumping lemma can be used to prove given language is regular.

S_2 : Given a grammar, checking if the grammar is not regular is decidable problem.

S_3 : If L is a regular and M is not a regular language then L.M. is necessarily non-regular.

S_4 : The number of derivations step for any strings W of length n is grammar is CNF and GNF form is $(2n - 1)$ and (n) respectively.

Which of the following statement is correct ?

Solution Video | Have any Doubt ?

A. Only S_1, S_3 is correct

B. Only S_2, S_4 is correct — Correct Option

Solution :
(b)

S_1 : Pumping lemma can prove that language is not regular but can't prove that the language is regular. Hence this is false.

S_2 : We can check regular grammar by following productions $V \to T^* V + T^*$ or $V \to V T^* + T^*$

S_3 : Consider 'L' to be ϕ and 'M' to $\{a^n b^n \mid n \le 0\}$
L.M. = ϕ, which is regular
S_4 : In case of CNF, $(n - 1)$ derivations are required to generate a string with (n) Non-Terminals, since only one Non-Terminals is added during each derivation.
Further, (n) derivations are required to convert those Non-Terminals to terminals.
So, in total, to generate a string of n terminals:

$$\underbrace{(n-1)}_{\substack{\text{To generate}\\ \text{string with } n\\ \text{Non-Terminals}}} + \underbrace{(n)}_{\substack{\text{To convert}\\ \text{NT} \to \text{T}}} = \underbrace{(2n-1)}_{\text{Total}}$$

However, in case of GNF: In a single derivation, we get a terminal in addition to our Non-Terminals. $S \to T(NT)^*$
Therefore, no need for $(n - 1)$ derivations to increase length.
Hence, only (n) derivations are required.

C Only S_3 is correct — Your answer is **Wrong**

D Only S_2, S_3 is correct

QUESTION ANALYTICS +

Q. 11

Consider L_1, L_2 be any two context sensitive languages and R be any regular language. Then which of the following is/are correct?

I. $L_1 \cup R$ is regular.
II. $\overline{L_2}$ is context sensitive language.
III. $L_1 \cap L_2$ is context sensitive.
IV. $L_1 - L_2$ is non-CSL.

Solution Video | Have any Doubt ?

A I, II and IV only

B II and III only — Correct Option

Solution :
(b)

- $L_1 \cup R$ = CSL $\cup$ Reg = CSL but need not regular.
- $\overline{L_2} = \overline{\text{CSL}}$ = CSL, since CSL closed under complement.
- $L_1 \cap L_2$ = CSL $\cap$ CSL = CSL, since CSL closed under intersection.
- $L_1 - L_2$ = CSL − CSL = CSL $\cap$ $\overline{\text{CSL}}$ = CSL, since CSL are closed under intersection and complement.

So, only II and III are true.

C I and IV only

D II, III and IV only — Your answer is **Wrong**

QUESTION ANALYTICS +

Q. 12

Which of the following are context free?
$L_1 : \{a^n b^m a^k \mid k = mn \text{ and } k, m, n \ge 1\}$
$L_2 : \{a^{m+n} b^{n+m} c^m \mid n, m \ge 1\}$
$L_3 : \{a^n b^n c^m \mid m < n \text{ and } m, n \ge 1\}$

FAQ | Solution Video | Have any Doubt ?

A L_1 and L_2 only

B L_2 and L_3 only

C L_1, L_2 and L_3 only

D None of the language — Your answer is **Correct**

Solution :
(d)

$L_1 : \{a^n b^m a^k \mid k = mn\}$ is not CFL, since we can not implement it with single stack.
$L_2 : \{a^{m+n} b^{n+m} c^m \mid n, m \ge 1\}$ is non-CFL since here more than 1 comparison present i.e., $\{a^m a^n b^n b^m c^m\}$. Hence cannot be implement by single stack.
$L_3 : \{a^n b^n c^m \mid m < n \text{ and } m, n \ge 1\}$ is non-CFL since more than 1 comparison are present simultaneously. i.e. after comparison of n = n, we left with only c^m and we cannot compare m < n or not.
So, none of the language is CFL.

QUESTION ANALYTICS +

Q. 13

Identify the language generated by the following grammar where S is start variable?

$S \to S_1 \mid S_2$
$S_1 \to S_1 c \mid A$
$A \to aAb \mid \epsilon$
$S_2 \to aS_2 \mid B$
$B \to bBc \mid \epsilon$

Solution Video | Have any Doubt ?

A $\{a^n b^n c^m ân, m \geq 0\}$

B $\{a^n b^m c^k ân, m, k \geq 0\}$

C $\{a^n b^m c^m ân, m \geq 0\}$

D $\{a^n b^n c^m ân, m \geq 0\} \cup \{a^n b^m c^m ân, m \geq 0\}$ Your answer is **Correct**

Solution :
(d)

$$L_1 : S_1 \to S_1 c \mid A \Leftarrow \{a^n b^n c^m \mid n, m \geq 0\}$$
$$A \to aAb \mid \epsilon = \{a^n b^n \mid n \geq 0\}$$
$$L_2 : S_2 \to aS_2 \mid B \Leftarrow \{a^n b^m c^m \mid n, m \geq 0\}$$
$$B \to bBc \mid \epsilon \Rightarrow \{b^m c^m \mid m \geq 0\}$$

So, $L = L_1 \cup L_2 = \{a^n b^n c^m \mid n, m \geq 0\} \cup \{a^n b^m c^m \mid n, m \geq 0\}$.

QUESTION ANALYTICS +

Q. 14

If $L_1 = \{a^n b^n \mid n \geq 0\}$ and $L_2 = \{b^n c^n \mid n \geq 0\}$, consider

I. $L_1 \cdot L_2$ is non CFL

II. $L_1 \cdot L_2 = \{a^n b^{2n} c^n \mid n \geq 0\}$

Which one of the following is correct?

Solution Video | Have any Doubt ?

A Only I Your answer is **Wrong**

B Only II

C Both I and II

D Neither I nor II Correct Option

Solution :
(d)

$$L_1 = \{a^n b^n \mid n \geq 0\} \text{ is DCFL and CFL also.}$$
$$L_2 = \{b^n c^n \mid n \geq 0\} \text{ is DCFL and CFL also.}$$

We know that CFL · CFL = CFL

So, $L_1 \cdot L_2 = \{a^n b^n b^m c^m \mid n, m \geq 0\}$ which is CFL and we can see that $L_1 \cdot L_2$ is clearly not equal to $\{a^n b^{2n} c^n \mid n \geq 0\}$.

So II is not true.

So answer is option (d).

QUESTION ANALYTICS +

Q. 15

The number of strings present of length 10 in language $L = \{a^{2n+1} b^{2m+1} \mid n \geq 0, m \geq 0\}$ are ______.

FAQ Solution Video | Have any Doubt ?

5 Your answer is **Correct**5

Solution :
5

$$\text{Language } L = \{a^{2n+1} b^{2m+1} \mid n \geq 0, m \geq 0\}$$
$$\text{Regular expression} = (aa)^* a (bb)^* b$$

Since we need to find number of strings of length 10,

$$|a^{2n+1} b^{2m+1}| = 2n + 1 + 2m + 1$$
$$= 2(m + n) + 2$$

Now $$2(m + n) + 2 = 10$$
$$m + n = 4$$

$\therefore$ Number of solutions of this equation = 5

QUESTION ANALYTICS +

Q. 16

Consider the following Problems:

P_1: {<M, x, k> | M is a TM and M does not halt on x within k steps}

P_2: {<M> | M is a TM and M accepts atleast two strings of different length}

P_3: {<M> | M is a TM and there exist an input whose length is less than 100, on which M halts}

The number of problems which is RE but not REC is ______.

FAQ Solution Video | Have any Doubt ?

2 Correct Option

Solution :
2

P_1: T_{Yes} : When machine does not halt on x until k steps.

T_{No}: When machine halt on x within k steps.

So, recursive.

P_2: T_{Yes} : When machine accepts atleast two strings of different length.

T_{No}: Not exist, since machine may go into infinite loop

So, Re but not REC.

P_3: T_{Yes} : Run all strings till 100 steps, if machine halt.

T_{No}: Does not exist, since machine may go into infinite loop.

So, RE but not REC.

Your Answer is 1

QUESTION ANALYTICS +

Q. 17

Consider the following CFG:

$S \rightarrow aSa \mid bSb \mid a \mid b \mid \varepsilon$

For the above CFG, the total number of strings generated whose length is less than or equal to 6 is ________.

FAQ Solution Video Have any Doubt ?

29 Correct Option

Solution :
29

The grammar generates the set of all palindromes possible over $\{a, b\}$.

Lets first find the number of even palindromes of length atmost 6 (0, 2, 4, 6 length respectively).

$$\begin{aligned} 0 \text{ length palindromes} &= 2^{0/2} = 1 \\ 2 \text{ length palindromes} &= 2^{2/2} = 2 \\ 4 \text{ length palindromes} &= 2^{4/2} = 4 \\ 6 \text{ length palindromes} &= 2^{6/2} = 8 \end{aligned}$$

So total number of even palindromes of length atmost 6 = 1 + 2 + 4 + 8 = 15

Similarly number of odd palindromes of length atmost 6 = 2 + 4 + 8 = 14

So total palindromes = 29

Your Answer is 21

QUESTION ANALYTICS +