SINGLE SUBJECT : DISCRETE MATHEMATICS (GATE - 2019) - REPORTS

Q. 1

Consider the following statements:

$S_1: \forall x \forall y((x < 0) \wedge (y < 0) \Rightarrow (xy > 0))$

$S_2: \forall x \exists y(x + y = 0)$

$S_3: \forall x \forall y((x < 0) \wedge (y \geq 0) \Rightarrow (x\,y < 0)$

Assuming the domain to be the set of all integers. Which of the following statements is/are true?

FAQ | Solution Video | Have any Doubt ?

A S_1 and S_2 only — Correct Option

Solution :
(a)
S_1 is true as product of negative numbers is always positive.
S_2 is also true as for every number, these exists an additive inverse.
S_3 is false because, if $y = 0$, no matter what the value of x is; the product will be zero, however S_3 says it should always be negative, which is false.

B S_2 and S_3 only

C S_1 and S_3 only

D S_1, S_2 and S_3 only — Your answer is **Wrong**

QUESTION ANALYTICS

Q. 2

Consider a tree T with n vertices and $(n - 1)$ edges. We define a term called cyclic cardinality of a tree (T) as the number of cycles created when any two vertices of T are joined by an edge. Given a tree with 10 vertices, what is the cyclic cardinality of this tree?

Solution Video | Have any Doubt ?

A 10

B 100

C 45 — Your answer is **Correct**

Solution :
(c)
For tree with n vertices, cyclic cardinality is equal to ${}^{N}C_2$.

Hence $\quad {}^{10}C_2 = \dfrac{10(10-1)}{2} = 45$

D 90

QUESTION ANALYTICS

Q. 3

A degree sequence $(d_1, d_2 \,\ldots\ldots\, d_n)$ is graphical $(d_1 \geq d_2 \geq \ldots\ldots \geq d_n)$ if there exists a simple undirected graph with n vertices having degrees, $d_1, d_2 \,\ldots\ldots\, d_n$. Consider the following sequences:

$S_1: (2^8, 2^7, 2^6, 2^5, 2^4, 2^3, 2^2, 2^1)$

$S_2: (8^0, 7^0, 6^0, 5^0, 4^0, 3^0, 2^0, 1^0)$

Which of the sequence(s) given above is graphical?

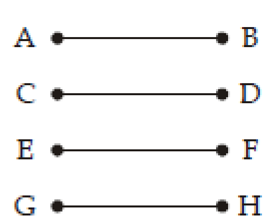
Solution Video | Have any Doubt ?

A S_1 and S_2 only

B Only S_1

C Only S_2 — Correct Option

Solution :
(c)
In S_1, all degrees are distinct. Hence sequence can't be graphical.
In S_2, degree of all vertices = 1 and since number of vertices with odd degree is even, S_2 is graphical.
Also a possible graph for S_2 looks like this (for elaboration):



Graph for S_2

D None of these

D None of these

QUESTION ANALYTICS

Q. 4

Let $M(n)$ denotes the number of n bit binary strings in which no two 1's are consecutive. Which of the following correctly represents the recurrence relation for $M(n)$?

Solution Video | Have any Doubt ?

A $M(n) = 2M(n-1) + M(n-2);\ M(1) = 2,\ M(2) = 3$

B $M(n) = M(n-1) + M(n-2);\ M(1) = 2,\ M(2) = 3$ — Your answer is **Correct**

Solution :
(b)

For $n = 2$, there are 3 strings 00, 01, 10 which don't have consecutive is.
Similarly for $n = 3$, we have 5 bit strings (000, 001, 010, 100, 101)
And for $n = 4$, we have 8 strings.
Thus, if we observe the pattern, $M(n)$ actually is equivalent to the n^{th} term of the Fibonacci sequence as the sum of the previous Z values equal the current value. Hence the answer is (b).

C $M(n) = \dfrac{M(n-1)+M(n-2)}{2};\ M(1) = 2,\ M(2) = 3$

D $M(n) = M(n-1) - M(n-2);\ M(1) = 2,\ M(2) = 3$

QUESTION ANALYTICS

Q. 5

Consider 3 sets A, B and C. Now consider the following statements:

S_1 : If $A \cup C = B \cup C$, then $A = B$.
S_2 : If $A \cap C = B \cap C$, then $A = B$.

Which of the above statements is/are true?

Solution Video | Have any Doubt ?

A S_1 and S_2 only

B Only S_1

C Only S_2 — Your answer is **Wrong**

D None of these — Correct Option

Solution :
(d)

S_1 is false, take this counter example.

$$A = \{1\},\ C = \{2, 3\} \text{ and } B = \{1, 2\}$$

$$\underbrace{\overset{A}{\{1\}} \cup \overset{C}{\{2, 3\}}}_{\{1, 2, 3\}} = \underbrace{\overset{B}{\{1, 2\}} \cup \overset{C}{\{2, 3\}}}_{\{1, 2, 3\}}$$

Clearly $A \neq B$, hence S_1 is false.
For S_2 to be false, let $C = \phi$.
Let $A = \{1\},\ B = \{2\}$ (doesn't matter, A and B can be anything)

$$\underbrace{\overset{A}{\{1\}} \cap \overset{C}{\phi}}_{\phi} = \underbrace{\overset{B}{\{2\}} \cap \overset{C}{\phi}}_{\phi}$$

Hence $A \neq B$. Thus S_2 is also false.

QUESTION ANALYTICS

Q. 6

How many 3 digit numbers can be formed by using the digits 1, 2, 3, 4, 5 which are divisible by 6 without repetition?

Solution Video | Have any Doubt ?

A 6

B 8 — Correct Option

Solution :
(b)

For a number to be divisible by 6, it should be divisible by both 2 and 3.
To be divisible by 2, unit digit should be 2 or 4.
To divisible by 3 sum of all 3 digit should be divisible by 3. So sum should be either [3, 6, 9, 12 or 15].
Now, only 2 possibilities for unit digit are,
(i) $2 = \{3, 1, 2\}, \{1, 3, 2\}, \{4, 3, 2\}, \{3, 4, 2\}$
(ii) $4 = \{2, 3, 4\}, \{3, 2, 4\}, \{3, 5, 4\}, \{5, 3, 4\}$
So, total combination possible is 8.

C 10

D 12 — Your answer is **Wrong**

QUESTION ANALYTICS

Q. 7

Consider the following statements:
P_1: Sachin Tendulkar gets out before the tea break only if Ishant Sharma comes out to bat.
P_2 : Ishant Sharma won't come out to bat, if Lasith Malinga is not called to bowl.
P_3 : Sachin Tendulkar got out before the tea break.
Which of the following does not follow from P_1, P_2, P_3?

Solution Video | Have any Doubt ?

A Lasith Malinga is called to bowl.

B Ishant Sharma come out to bat.

C Sachin Tendulkar got out after the tea break. Correct Option

Solution :
(c)
It's quite easy to see how (c) does not follow from P_1, P_2, P_3. As P_3 is true, (c) is negation of P_3 hence (c) is the appropriate option.

D None of these

QUESTION ANALYTICS

Q. 8

The function are given below:

$f(x) = x - 1,\ g(x) = \dfrac{1}{\left(\dfrac{x}{x+1}\right)}$ then what is the value of $\dfrac{f(g(x))}{g(f(x))}$

Solution Video | Have any Doubt ?

A $\dfrac{-(1-x)}{x^2}$ Your answer is **Correct**

Solution :
(a)

$$\frac{f(g(x))}{g(f(x))} = \frac{f\left(\dfrac{x+1}{x}\right)}{g(x-1)}$$

$$= \frac{\dfrac{x+1}{x}-1}{\dfrac{x-1+1}{x-1}} = \frac{\dfrac{x+1-x}{x}}{\dfrac{x-1+1}{x-1}}$$

$$= \frac{\dfrac{1}{x}}{\dfrac{x}{x-1}} = \frac{(x-1)}{x^2}$$

$$= \frac{-(1-x)}{x^2}$$

So option (a) is correct.

B $\dfrac{1}{x^2}$

C $\dfrac{1-x}{x}$

D $\dfrac{-(x-1)}{x^2}$

QUESTION ANALYTICS

Q. 9

Let $f(x, y) = (x + y, x - y)$. What is $f^{-1}(x, y)$?

Solution Video | Have any Doubt ?

A $(x - y, x + y)$

B $(x - 2y, x + 2y)$

C $\left(\dfrac{x-y}{2}, \dfrac{x+y}{2}\right)$

D $\left(\dfrac{x+y}{2}, \dfrac{x-y}{2}\right)$ Correct Option

Solution :
(d)
Put, $x = 1$ and $y = 2$
$f(1, 2) = (3, -1)$
$\Rightarrow f^{-1}(3, -1) = (1, 2)$
Now substituting in options, (a), (b) and (c) will be ruled out, however (d) is correct.

$$f^{-1}(3, -1) = \left(\frac{3+(-1)}{2}, \frac{3-(-1)}{2}\right) = (1, 2)$$

Hence (d) is most appropriate.

QUESTION ANALYTICS +

Q. 10

Given below is the matrix representation (M_R) of a relation R, with 4 elements. {1, 2, 3, 4} respectively.

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Which of the following correctly represents R^3 in set builder notation?

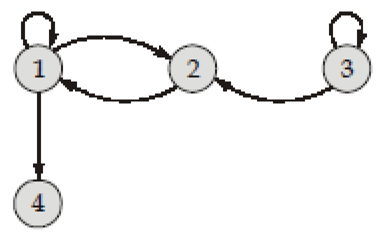
Solution Video | Have any Doubt ?

A {(1, 1) (1, 2) (1, 4) (2, 1) (2, 2) (2, 4) (3, 1) (3, 2) (3, 3)}

B {(1, 1) (1, 2) (1, 4) (2, 1) (2, 2) (2, 4) (3, 1) (3, 2) (3, 3) (3, 4)} Correct Option

Solution :
(b)

Converting matrix representation to digraph:



Now we can easily find R^3.

$$R^3 = \{(1, 1)\ (1, 2)\ (1, 4)\ (2, 1)\ (2, 2)\ (2, 4)\ (3, 1)\ (3, 2)\ (3, 3)\ (3, 4)\}$$

Hence (b) is correct.

C {(1, 1) (1, 2) (1, 4) (2, 2) (2, 4) (3, 1) (3, 2) (3, 3)}

D {(1, 1) (1, 2) (1, 4) (2, 1) (2, 2) (2, 4) (3, 1) (3, 2) (3, 4)}

QUESTION ANALYTICS +

Q. 11

Let M be a set of integers whose cardinality is 5. Let x, y and z be one of the integers belonging to M. Further, then how many subsets of M contain at least one of x, y and z ________.

Solution Video | Have any Doubt ?

28 Correct Option

Solution :
28

We will use inclusion exclusion principle.

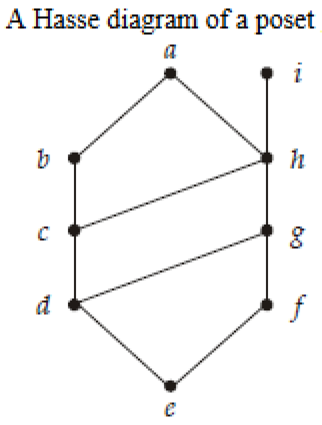
$$\begin{aligned} n(x \text{ or } y \text{ or } z) &= n(x) + n(y) + n(z) - n(x \cap y) - n(y \cap z) - n(x \cap z) + n(x \cap y \cap z) \\ &= {}^3C_1 \cdot 2^4 - {}^3C_2 \cdot 2^3 + {}^3C_3 \cdot 2^2 \\ &= 3(16) - 3(8) + 4 \\ &= 28 \end{aligned}$$

Your Answer is 31

QUESTION ANALYTICS +

Q. 12

A Hasse diagram of a poset given below:



Then find the number of upper bounds of the subset $\{e, f, c, h\}$ is ________.

FAQ | Solution Video | Have any Doubt ?

3 Correct Option

Solution :
3

Upper bound of $\{e, f, c, h\}$ is a, h, i.

So total upper bound is 3.

So correct answer is (3).

QUESTION ANALYTICS +

Q. 13

Let x denote the number of relations on a set with 100! elements which are both symmetric and asymmetric. Then the value of 2^X is ________.

Solution Video | Have any Doubt ?

2 Your answer is Correct2

Solution :
2
ϕ is the only relation which is both symmetric and asymmetric. Therefore X = 1.
Thus, $2^X = 2^1 = 2$

QUESTION ANALYTICS

Q. 14

The number of edge disjoint Hamiltonian cycles are present in K_{101} (complete graph with 101 vertices) are ________.

Solution Video | Have any Doubt ?

50 Correct Option

Solution :
50
The number of edge disjoint Hamiltonian cycle in K_n

$$= \left\lfloor \frac{n-1}{2} \right\rfloor$$

For $K_{101} \Rightarrow \left\lfloor \frac{101-1}{2} \right\rfloor = 50$

QUESTION ANALYTICS

Q. 15

Let H be a cyclic group of order 20, having a as its generator. Then the order of a^8 will be ________.

Solution Video | Have any Doubt ?

5 Correct Option

Solution :
5
We have to find $O(a^8)$.
$\Rightarrow$ To find smallest x such that $(a^8)^x = e$.
As per question, we know

$$a^{20} = e[O(G) = 20]$$

$\Rightarrow$ $a^{40} = e$

Hence, $a^{8x} = a^{40} \Rightarrow x = 5$

Hence, $O(a^8) = 5$

Alternate Method:

If a is a generator, $O(a^x) = \frac{n}{\gcd(x,n)} = \frac{20}{\gcd(8,20)} = \frac{20}{4} = 5$

Your Answer is 10

QUESTION ANALYTICS

Q. 16

Consider a complete graph on $2^{\log_2 2^{10}} + 1$ vertices. Then the minimum number of edge removal operations needed to make the graph disconnected is ________.

Solution Video | Have any Doubt ?

1024 1024
Your answer is Correct

Solution :
1024
In complete graph with n vertices, the degree of each vertex is $(n - 1)$.
So number of edge removals = $n - 1$ (in case of complete graph)
Given in the question,

$$n = (2^{\log_2 2^{10}} + 1)$$

$$n = 2^{10} + 1$$

Degree of each vertex $= (2^{10} + 1) - 1 = 2^{10}$

Hence minimum number of edges to be removed

$$= 2^{10} = 1024$$

QUESTION ANALYTICS

Q. 17

We define a new operator, called the descendant of a given set A. The definition is as follows.
Descendant of a set A is defined as the set A ∪ {A}. Which of the following is correct?

Solution Video | Have any Doubt ?

A The set and its descendant can never have the same cardinality. Correct Option

Solution :
(a)
A is the correct option.
If x is the cardinality of set A, and y is the cardinality of descendant(A), then $y - x = 1$.
But option C says otherwise.
Hence C is also false. It also means that A is true, and B is false.
Here's some more explanation.
Let A = {1, 2, 3}
Then descendant(A) = {1, 2, 3, {1, 2, 3}}
Here |A| = 3, |descendant(A)| = 3 + 1 = 4

Let A = { }

Then descendant(A) = {{ }}

Here also it is 1 more than A's cardinality.

So it's quite to easy to see why option A is most appropriate choice. Irrespective of the set A, descendant(A) will always be greater in cardinality than the set A by 1 and |A| ≠ |descendant(A)|.

B The set and its descendant may have the same cardinality. Your answer is **Wrong**

C If x is the cardinality of the set A and y is the cardinality of descendant(A), then $x - y = 1$.

D None of these

QUESTION ANALYTICS +

Q. 18

Consider the following functions:

$f(x) = \ln x + x$

$g(x) = x^2 \sin x$

$h(x) = x^3 - x$

Which of the functions given above are many-one?

Solution Video | Have any Doubt ?

A $f(x), g(x)$

B $g(x), h(x)$ Your answer is **Correct**

Solution :

(b)

There are some one way theorems for checking if a function is many one.

One of them is used here.

Theorem A function has multiple roots ⇒ The function is many one.

[As for every root, function reaches at 0 value]

$$\begin{aligned} h(x) &= x^3 - x \\ &= x(x^2 - 1) \\ &= x(x-1)(x+1) \end{aligned}$$

So, $h(x)$ has multiple roots ⇒ $h(x)$ is many one.

$g(x)$ is also many one using the same property although not very obvious.

$x^2 \sin x$ will be zero (0),

If either $x^2 = 0$ or $\sin x = 0$

$\Downarrow \qquad \Downarrow$

$x = 0 \qquad x = \text{(odd multiples of } \pi)$

Hence at $x = 0, \pi, 2\pi, \ldots$

$g(x)$ will be zero.

⇒ $g(x)$ has multiple roots ⇒ $g(x)$ is many one

$f(x)$ is one-one. The reason is that, if a function is either strictly increasing (↑) or strictly decreasing (↓) then $f(x)$ is surely one-one and summation of 2 or more ↑ing functions is also ↑ing.

$$f(x) = \underbrace{\underset{\substack{\Downarrow \\ \text{Strictly} \\ \uparrow\text{ing}}}{x} + \underset{\substack{\Downarrow \\ \text{Strictly} \\ \uparrow\text{ing}}}{\ln x}}_{\substack{\Downarrow \\ \text{Strictly } \uparrow\text{ing}}}$$

Hence answer is (b).

C All of the above

D None of these

QUESTION ANALYTICS +

Q. 19

A graph G is said to be separable if G is either disconnected or can be disconnected by removing one vertex in G. Consider the following statements:

S_1 : Every k regular connected graph is non separable for all $k \geq 3$.

S_2 : Every k regular graph is connected.

Which of the above statement(s) is/are true?

Solution Video | Have any Doubt ?

A Both S_1 and S_2 only

B Only S_1 Your answer is **Wrong**

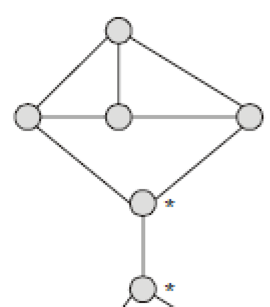
C Only S_2

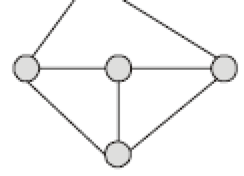
D None of these Correct Option

Solution :

(d)

S_1 is false; here is the counter example.





Vertices marked * are cut vertices. Hence S_1 is false, as the graph above is separable.
For S_2 consider the following graph:

Given graph is 2 regular and is not a connected graph thus S_2 is also false.

QUESTION ANALYTICS +

Q. 20

Let spider(x) denote, x is a spider. Then which of the following first order logic formulae are equivalent?

$S_1 : \forall x \forall y [\text{spider}(x) \wedge \text{spider}(y) \Rightarrow x = y]$

$S_2 : \forall x [\sim \text{spider}(x)] \vee \exists x (\text{spider}(x) \wedge \forall y (\text{spider}(y) \Rightarrow y = x))$

$S_3 : \sim \exists x (\text{spider}(x)) \vee \exists x (\text{spider}(x) \wedge \forall y (\sim \text{spider}(y) \vee y = x))$

FAQ Solution Video Have any Doubt ?

A S_1 and S_2 only

B S_3 and S_3 only

C All S_1, S_2, S_3 are equivalent — Correct Option

Solution :
(c)
All are equivalent.
$S_1 \rightarrow$ Atmost 1 spiders
$S_2 \rightarrow$ (Exactly 0) or (Exactly 1) spider ≡ Atmost 1 spiders
$S_3 \rightarrow$ Same as S_2. Obtain S_3 from S_2 using
(1) $p \Rightarrow q \Leftrightarrow \sim p \vee q$, and
(2) $\forall x (\sim \text{spider}(x)) \Leftrightarrow \sim \exists x (\text{spider}(x))$

D None of these

QUESTION ANALYTICS +

Q. 21

Let P, Q, R, S be 4 sets respectively. Which of the following laws always holds good?

Solution Video Have any Doubt ?

A P × (Q × R) = P × Q × R

B (P × Q) × (R × S) = P × (Q × R) × S — Your answer is **Wrong**

C P × Q = Q × R

D None of these — Correct Option

Solution :
(d)
Let's consider choice (a)
P × Q × R will have elements of the from (x, y, z) where $x \in$ P, $y \in$ Q and $z \in$ R.
However P × (Q × R) has elements of the from $(x, (y, z))$.
Moreover, P × Q × R is a triplet cartesian product, whereas P × (Q × R) is a binary cartesian product.
So either way it's easy to see why both aren't equal.
For option (b), take the following counter example.
Let P, Q, R, S are all ({ϕ})
(P × Q) × (R × S) will be, {((ϕ, ϕ), (ϕ, ϕ))}
And P × (Q × R) × S) will be, {(ϕ, (ϕ, ϕ), ϕ)}
Clearly both are not equal hence (b) is wrong.
(c) can only be true if either A = B or one of A and B is ϕ.
Hence (d) is the right choice.

QUESTION ANALYTICS +

Q. 22

Define a group (A, *) as follows:
Let A = {0, 1, 2, 3, ..., 23}
Given, (a * b) = (a + b) mod 24
The number of proper subgroups of A will be equal

FAQ Solution Video Have any Doubt ?

A 5

B 6 — Correct Option

Solution :
(b)

$24 = 3 \times 2$

$$\text{(Proper subgroups)} = [(1+1)(3+1) - \underbrace{2}_{\text{Trivial subgroups}}] = 6$$

C 8

D 7

QUESTION ANALYTICS

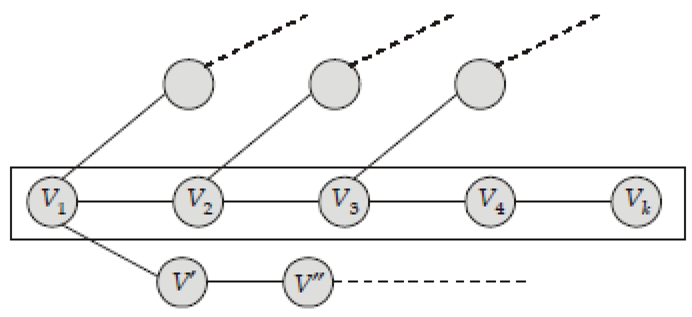
Q. 23

Consider the following statement:
S_1 : In a non-trival tree, there exits at least one vertex of degree 1.
S_2 : Every non trival tree is bichromatic.
Which of the above statements is/are true?

Have any Doubt ?

A S_1 and S_2 only — Correct Option

Solution :
(a)
S_1 : Every non-trival tree ($n \geq 2$) must have atleast are vertex of degree 1. We'll prove this constructively. Let's say we're building the tree with vertices V_1, V_2, V_3 .... and so on.



Now we reach a vertex called V_k. Now we have 2 choices. Either we continue by adding another vertex to the tree or we make an edge from V_k to V_i such that $i < k$. However 2nd case is not possible as that will create cycle and tree can't have cycle in it. Now case 1 can't continue forever as vertices can't be ∞. Hence the moment we stop, there will always be at least one vertex with degree 1. Hence proved.

S_2 : Every non trival tree is bichromatic as tree is always bipartite. Hence S_2 is also true.

B Only S_1 — Your answer is **Wrong**

C Only S_2

D None of these

QUESTION ANALYTICS

Q. 24

Consider the following statement:
S_1 : The relation $R = \Phi$ on empty set is symmetric and transitive, but not reflexive.
S_2 : The relation R defined as, $x\,R\,y$ iff $xy \geq 1$ on the set of real numbers is symmetric and transitive.
Which of the above statements is/are true?

Solution Video | Have any Doubt ?

A Both S_1 and S_2

B Only S_2 — Your answer is **Wrong**

C Only S_1

D None of these — Correct Option

Solution :
(d)
On empty set, ϕ is an equivalence relation. Therefore S_1 is false.
In S_2 the relation is not transitive. Take this counter example.

$$(3, 7) \in R \text{ as } 21 \geq 1$$

$$\text{and} \quad \left(7, \frac{1}{6}\right) \in R \text{ as } \frac{7}{6} \geq 1$$

$$\text{but} \quad \left(3, \frac{1}{6}\right) \notin R \text{ as } \frac{1}{2} < 1 \quad \Rightarrow \text{Not transitive}$$

QUESTION ANALYTICS

Q. 25

In how many ways can we chose a cricket team of 11 players out of 10 batsman, 5 bowlers and 2 keepers such that the team has at least 4 bowlers?

FAQ | Solution Video | Have any Doubt ?

A 1284 — Correct Option

Solution :
(a)

$$(\geq 4 \text{ bowlers}) = (\text{Exactly 4 bowlers}) + (\text{Exactly 5 bowlers})$$

$$= {}^5C \times {}^{12}C + {}^5C \times {}^{12}C$$

Selecting 4 bowlers from 5 bowlers | Selecting the rest (Note that there is no restriction on number of keepers playing in the team) | Select 5 bowlers | Selecting the rest of the team

Solve to get = 1284

B 6! * 7!

C 504

D None of these — Your answer is **Wrong**

QUESTION ANALYTICS +

Q. 26

Let $\lfloor x \rfloor$ denote the smallest integer greater than or equal to x and $\lceil x \rceil$ denote the greatest integer smaller than or equal to x. Consider the following statements:

$S_1 : \left\lfloor \frac{x}{2} \right\rfloor = \left\lceil \frac{x+1}{2} \right\rceil$

$S_2 : \lceil 2x \rceil = 2\lceil x \rceil$

$S_3 : \lfloor \lceil x \rceil \rfloor = \lceil x \rceil$

$S_4 : \lfloor xy \rfloor = \lfloor x \rfloor \lfloor y \rfloor$

How many statements above is/are correct?

FAQ | Solution Video | Have any Doubt ?

A 0

B 1 — Correct Option

Solution :
(b)

Notice that we're reversed the convention for ceil and floor. Now use the usual notation for ceil and floor and flip the ones used in options, get the correct statement and mark it.

S_1: is actually $\left\lceil \frac{x}{2} \right\rceil = \left\lfloor \frac{x+1}{2} \right\rfloor$

Put $x = 2.1 \Rightarrow \text{LHS} \neq \text{RHS} \Rightarrow \text{false}$

S_2: $\lfloor 2x \rfloor = 2\lfloor x \rfloor$

Put $x = 0.9 \Rightarrow \text{LHS} \neq \text{RHS} \Rightarrow \text{false}$

S_4: $\lceil xy \rceil = \lceil x \rceil \lceil y \rceil$

Put $x = y = 1.1$

$$\text{LHS} = \lceil 1.21 \rceil \qquad \text{RHS} = \lceil 1.1 \rceil \lceil 1.1 \rceil$$
$$\text{LHS} = 2 \qquad = 2^2$$
$$\text{LHS} = 2 \qquad \text{RHS} = 4$$

$\text{LHS} \neq \text{RHS} \Rightarrow S_4$ is false

Now take S_3:

$$\underbrace{\lceil \lfloor x \rfloor \rceil}_{\text{INTEGER}} = \underbrace{\lfloor x \rfloor}_{\text{INTEGER}}$$

This will be true, because irrespective of whether x contains fractional part or not, $\lfloor x \rfloor$ will be integer and $\lceil \text{integer} \rceil$ = integer always holds true.

Hence S_3 is true.

C 2

D 3 — Your answer is **Wrong**

QUESTION ANALYTICS +

Q. 27

The value of $({}^nC_1) + 2({}^nC_2) + 3({}^nC_3) + 4({}^nC_4) + \ldots\ldots + n\,({}^nC_n)$ is equal to

Solution Video | Have any Doubt ?

A 2^n

B 2^{n-1}

C 3^n

D $n2^{n-1}$ — Your answer is **Correct**

Solution :
(d)

We know, $(1+x)^n = {}^nC_0 + {}^nC_1\,x + {}^nC_2\,x^2 + \ldots\ldots + {}^nC_n\,x^n$

Differentiate both sides,

$$n\,.\,(1+x)^{n-1} = [0 + {}^nC_1 + 2\,{}^nC_2\,x + 3\,{}^nC_3\,x^2 + \ldots\ldots\ n\,.\,{}^nC_n\,x^{n-1}]$$

Put $x = 1$ to get

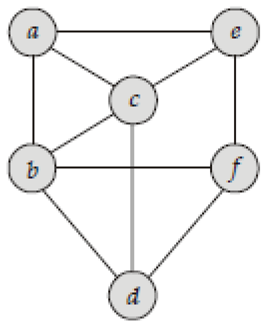
$$n \cdot 2^{n-1} = ({}^nC_1 + 2\,{}^nC_2 + 3\,{}^nC_3 + \ldots\ldots + n \cdot {}^nC_n)$$

Hence correct answer is (d).

QUESTION ANALYTICS

Q. 28

The chromatic number of the given graph is

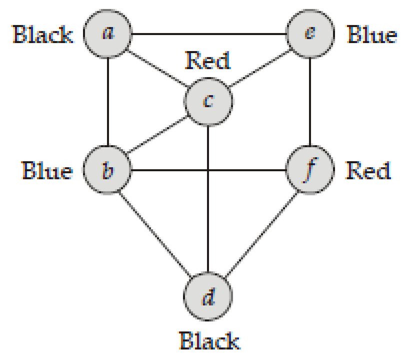


Solution Video | Have any Doubt ?

A 2

B 3 — Your answer is **Correct**

Solution :
(b)



Using 3 colours we can colour the above graph as shown.
Hence, answer is (b).

C 4

D 5

QUESTION ANALYTICS

Q. 29

Let $f(x)$ satisfies the equation:

$f(x) + 2f(1 - x) = 3x \;\forall x \in R.$

Then $f(-3) + f(-2)$ will be equal to ________.

Solution Video | Have any Doubt ?

19 — Correct Option

Solution :
19

Given $\quad f(x) + 2f(1 - x) = 3x \qquad$ ...(i)

Put $\quad x \to 1 - x$

$f(1 - x) + 2f(x) = 3 - 3x \qquad$ ...(ii)

Solving equation (i) and (ii), we get

$$f(x) = (2 - 3x)$$

Now we can easily find $f(-3) + f(-2)$.

$$\left.\begin{matrix} f(-3) = 11 \\ f(-2) = 8 \end{matrix}\right\} \Rightarrow f(-3) + f(-2) = 19$$

QUESTION ANALYTICS

Q. 30

Let S be a set of 5 elements:

$S = \{\alpha, \beta, \Gamma, \delta, ₹\}$

Let X be number of pairs (S_1, S_2) that satisfy following conditions.

(a) S_1 and S_2 are disjoint.

(b) $S_1, S_2 \subseteq S$

Then the value of $\log_3 X$ will be ________.

FAQ | Solution Video | Have any Doubt ?

5 — Correct Option

Solution :
5

Possibilities:

S_1	S_2	
ϕ	Power set of $\{\alpha, \beta, \Gamma, \delta, ₹\}$	2^5
One element subsets of S	Power set of $(S - S_1)$	${}^5C_1 * 2^4$
2 element subsets of S	Power set of $(S - S_1)$	${}^5C_2 * 2^3$
3 element subsets of S	Power set of $(S - S_1)$	${}^5C_3 * 2^2$
4 element subsets of S	Power set of $(S - S_1)$	${}^5C_4 * 2^1$
5 element subsets of S	ϕ	${}^5C_5 * 1$

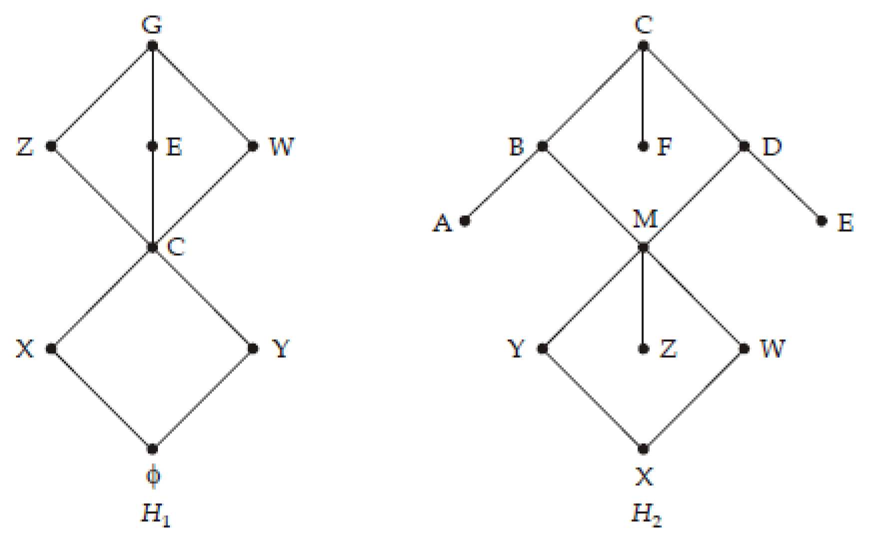
Add these to get, $\quad X = 243$

Now $\quad \log_3 X = 5$

Your Answer is 2.7

QUESTION ANALYTICS

Q. 31



Let X denote the number of topological orders possible in H_1. Let Y denote the number of minimal elements present in H_2. Then the value of X + 10 Y = ________.

Solution Video | Have any Doubt ?

62 Correct Option

Solution :
62

$$\begin{aligned} X &= 2! \times 3! = 12 \\ Y &= 5 \text{ (X, Z, A, F, E)} \\ \Rightarrow \quad X + 10\,Y &= 12 + 50 = 62 \end{aligned}$$

QUESTION ANALYTICS

Q. 32

Let X be the number of subsets of a set of size N containing even number of elements. Let Y be the number of functions possible from a set with N elements to

{0, 1}. Then the quantity $\frac{X}{Y}$ is equal to ________.

Solution Video | Have any Doubt ?

0.5 Correct Option

Solution :
0.5

We know,

$$X = ({}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots.. + {}^nC_n)$$

We know that this is a standard identity and is equal to 2^{n-1}.

Hence $\quad X = 2^{n-1}$

We can easily see that $\quad Y = \underbrace{2 \cdot 2 \cdot 2 \ldots\ldots.. 2}_{n \text{ times}} = 2^n$

Hence $\quad \left(\frac{X}{Y}\right) = \frac{2^{n-1}}{2^n} = \frac{1}{2} = 0.5$

Your Answer is 1

QUESTION ANALYTICS

Q. 33

Let G be a graph with 5! vertices, with each vertex labelled by a distinct permutation of the numbers 1, 2, 3, 4, 5. There is an edge between vertices u and v if and only if the label of u can be obtained by swapping two adjacent numbers in the label of v. Let y denote the degree of a vertex in G, z denote the number of connected components in G, and w denote the number of edges in G. Then y + z + w = ________.

Solution Video | Have any Doubt ?

245 Correct Option

Solution :
245

The degree of each vertex will be (5 – 1) = 4, as the number of vertices adjacent to it according to the question will be equal to the number of adjacent swappable pairs, which will be 4. There will be just 1 component, as every vertex will be reachable.

To find w, use the handshaking theorem.

$$\begin{aligned} 5! * 4 &= 2 * e \\ e &= 240 \Rightarrow w = 240 \\ \text{Hence required answer} &= 240 + 4 + 1 = 245 \end{aligned}$$

QUESTION ANALYTICS