TOPICWISE : ENGINEERING MATHEMATICS-1 (GATE - 2019) - REPORTS

Q. 1

Which of the following represents the solution to the system of equation?

$$\begin{bmatrix} 3 & 7.5 \\ -6 & 4.5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -90 \end{bmatrix}$$

Solution Video | Have any Doubt ?

A 12, –4 — Your answer is **Correct**

Solution :
(a)

$$\begin{bmatrix} 3 & 7.5 \\ -6 & 4.5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -90 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 7.5 & 6 \\ -6 & 4.5 & -90 \end{bmatrix}$$

$R_2 + 2R_1$

$$\begin{bmatrix} 3 & 7.5 & 6 \\ 0 & 19.5 & -78 \end{bmatrix}$$

$$19.5y = -78$$

or $$y = -4$$

$$3x + 7.5y = 6$$

$$3x + 7.5(-4) = 6$$

$$3x = 36$$

$\Rightarrow$ $$x = 12$$

$\therefore$ $$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ -4 \end{bmatrix}$$

B –12, –4

C –12, 4

D 12, 4

QUESTION ANALYTICS +

Q. 2

The normal distribution $N(\mu, \sigma^2)$ with mean $\mu \in R$ and variance $\sigma^2 > 0$ has probability distribution function:

$$N(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right) \text{ for } x \in R$$

The difference of median and mean is ________.

Solution Video | Have any Doubt ?

A μ

B σ

C $-\mu$

D 0 — Your answer is **Correct**

Solution :
(d)
Mean, median and mode are all same (μ) for normal distribution.

QUESTION ANALYTICS +

Q. 3

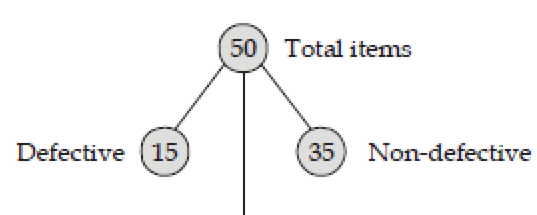
A bag contains 15 defective items and 35 non defective items. If three items are selected at random without replacement, what will be the probability that all three items are defective?

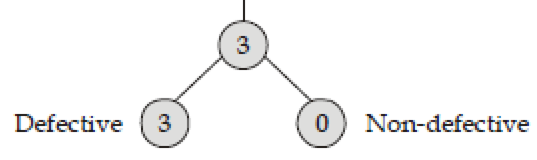
Solution Video | Have any Doubt ?

A $\frac{1}{40}$

B $\frac{13}{560}$ — Your answer is **Correct**

Solution :
(b)





$$\text{Required probability} = \frac{{}^{15}C_3 \times {}^{35}C_0}{{}^{50}C_3}$$

$$= \frac{15\times14\times13}{50\times49\times48} = \frac{13}{560}$$

C $\frac{15}{34}$

D $\frac{12}{499}$

QUESTION ANALYTICS +

Q. 4

Which one of the following represents the eigen vectors of matrix $\begin{bmatrix} 4 & 6 \\ 2 & 8 \end{bmatrix}$?

Solution Video | Have any Doubt ?

A $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

B $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

C $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Your answer is **Correct**

Solution :
(c)

The characteristic equation $|A - \lambda I| = 0$

$$\text{i.e.} \quad \begin{vmatrix} 4-\lambda & 6 \\ 2 & 8-\lambda \end{vmatrix} = 0$$

$$\begin{aligned}
\text{or} \quad (4 - \lambda)(8 - \lambda) - 12 &= 0 \\
\text{or} \quad 32 - 8\lambda - 4\lambda + \lambda^2 - 12 &= 0 \\
\Rightarrow \quad \lambda^2 - 12\lambda + 20 &= 0 \\
\Rightarrow \quad \lambda^2 - 10\lambda - 2\lambda + 20 &= 0 \\
\Rightarrow \quad (\lambda - 10)(\lambda - 2) &= 0 \\
\Rightarrow \quad \lambda &= 10, 2
\end{aligned}$$

Corresponding to $\lambda = 10$, we have

$$[A - \lambda I]x = \begin{bmatrix} -6 & 6 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{aligned}
\text{Which gives,} \quad -6x + 6y &= 0 \\
\Rightarrow \quad x &= y \\
2x - 2y &= 0 \\
\Rightarrow \quad x &= y
\end{aligned}$$

i.e. eigen vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Corresponding to $\lambda = 2$, we have

$$[A - \lambda I]x = \begin{bmatrix} 2 & 6 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Which gives, $2x + 6y = 0$ i.e. eigen vector $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$

D $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

QUESTION ANALYTICS +

Q. 5

Check whether the given system of equation has

$x + y + z = 8$
$2x + 3y + 5z = 8$
$4x + 5z = 2$

Solution Video | Have any Doubt ?

A Infinite solution

B No solution

C Unique solution

Your answer is **Correct**

Solution :
(c)

$$\begin{bmatrix} 1 & 1 & 1 & 8 \\ 2 & 3 & 5 & 8 \\ 4 & 0 & 5 & 2 \end{bmatrix} = M(A|B)$$

$$\text{Rank }(A) = 3$$
$$\text{Rank }(A|B) = 3$$
$$\text{Number of variables} = 3$$
So unique solution as
$$\rho(A|B) = \rho(A) = \text{Number of variables}$$

D Question incomplete

QUESTION ANALYTICS

Q. 6

The determinant of a 2 × 2 matrix is 30. If one eigen value of the matrix is 5, then other eigen value is ________.

Solution Video | Have any Doubt ?

6 — Your answer is **Correct**6

Solution :
6
The product of eigen values is always equal to the determinant value of the matrix.
$$\lambda_1 = 5, \lambda_2 = \text{Unknown}$$
$$|A| = 30$$
$$\lambda_1 \times \lambda_2 = 30$$
$$5 \times (\lambda_2) = 30$$
$$\Rightarrow \quad \lambda_2 = 6$$

QUESTION ANALYTICS

Q. 7

The value of x for which equation satisfied is ________. [Upto 1 decimal place]

$$e^x e^2 = \frac{e^4}{e^{x+1}}$$

Solution Video | Have any Doubt ?

0.5 — Correct Option

Solution :
0.5
Using the product and quotient properties of exponents we can rewrite the equation as
$$e^{x+2} = e^{4-(x+1)}$$
$$= e^{4-x-1}$$
$$= e^{3-x}$$
Since the exponential function e^x is one-to-one, we know the exponents are equal:
$$x + 2 = 3 - x$$
$$\Rightarrow \quad x = 0.5$$

Your Answer is 0.6

QUESTION ANALYTICS

Q. 8

Four vendors were asked to supply GPS instruments to the Indian Army. The respective probabilities of their meeting the strict technical specifications are 0.6, 0.7, 0.8 and 0.9. Each vendor supplies one instrument. The probability that out of the total four instruments supplied by the vendors, at least one will meet the design specification is________. (Upto 3 decimal places)

FAQ | Solution Video | Have any Doubt ?

0.997 (0.996 - 0.999) — Correct Option

Solution :
0.997 (0.996 - 0.999)
Probability of atleast one meeting the specification
$$= 1-\left(\bar{A}\cap\bar{B}\cap\bar{C}\cap\bar{D}\right)$$
$$= 1 - (0.4 \times 0.3 \times 0.2 \times 0.1)$$
$$= 1 - (0.0024)$$
$$= 0.9976$$

QUESTION ANALYTICS

Q. 9

A coin is tossed 5 times. The probability of getting exactly 3 heads is ________. (Upto 2 decimal place)

Solution Video | Have any Doubt ?

0.31 (0.30 - 0.33) — 0.31 Your answer is **Correct**

Solution :
0.31 (0.30 - 0.33)
Using binomial distribution formula $P = {}^nC_x\, P^x\, S^{(n-x)}$
$$P(x = 3) = {}^5C_3(0.5)^3\,(0.5)^{(5-3)}$$
$$= \frac{5!}{3! \times 2!}(0.5)^3(0.5)^2$$
$$= 0.3125 \approx 0.31$$

QUESTION ANALYTICS

Q. 10

Consider X be a random variable with E(X) = 10 and Var(X) = 25. What is the positive value of a and b such that Y = aX – b has expectation 0 and variance 1?

Solution Video | Have any Doubt ?

A $a = 1, b = 2$

B $a = 0.2, b = 2$ Correct Option

Solution :
(b)

We know that, $E(X) = 10$

and $Var(X) = 25$

Now, $E(Y) = E(aX - b) = 0$

$aE(X) - b = 0$

$\Rightarrow$ $a(10) - b = 0$

$10a - b = 0$ ...(i)

Given, $Var(Y) = 1$

$Var(aX - b) = a^2\, Var(X) = 1$

$\Rightarrow$ $25a^2 = 1$

i.e $a = \pm\frac{1}{5}$

$a = \frac{1}{5}$ (taking positive values only)

By putting value of 'a' in equation (i)

We get $b = 2$

C $a = 0.2, b = 1$

D $a = 0.2, b = 0.5$

QUESTION ANALYTICS +

Q. 11

For a given matrix $M = \begin{bmatrix} 12+9i & -i \\ i & 12-9i \end{bmatrix}$ where $i = \sqrt{-1}$, the inverse of matrix M is

Solution Video | Have any Doubt ?

A $\frac{1}{225}\begin{bmatrix} 12+9i & -i \\ i & 12-9i \end{bmatrix}$

B $\frac{1}{225}\begin{bmatrix} i & 12-9i \\ 12+9i & -i \end{bmatrix}$

C $\frac{1}{224}\begin{bmatrix} 12-9i & i \\ -i & 12+9i \end{bmatrix}$ Correct Option

Solution :
(c)

Given matrix is $M = \begin{bmatrix} 12+9i & -i \\ i & 12-9i \end{bmatrix}$

Determinant of $M = \begin{vmatrix} 12+9i & -i \\ i & 12-9i \end{vmatrix} = (12 + 9i)(12 - 9i) + i^2$

$= (12^2 - 9^2i^2) + i^2$

$= 225 - 1 = 224$

$\therefore$ Inverse of $M = M^{-1} = \frac{1}{|M|}(adjM)$

$= \frac{1}{224}\begin{bmatrix} 12-9i & i \\ -i & 12+9i \end{bmatrix}$

D $\frac{1}{224}\begin{bmatrix} 12+9i & -i \\ i & 12-9i \end{bmatrix}$

QUESTION ANALYTICS +

Q. 12

What is the standard deviation of a uniformly distributed variable between 0 and $\frac{1}{2}$?

Have any Doubt ?

A $\frac{1}{2\sqrt{12}}$ Your answer is **Correct**

Solution :
(a)

For rectangular distribution

Variance $= \frac{(b-a)^2}{12}$

Here, $a = 0, b = \frac{1}{2}$

$\therefore$ Variance $= \frac{\left(0-\frac{1}{2}\right)^2}{12} = \frac{\frac{1}{4}}{12} = \frac{1}{4\times 12}$

Then standard deviation $= \sqrt{\text{Variance}}$

$$= \sqrt{\frac{1}{4 \times 12}} = \frac{1}{2\sqrt{12}}$$

B $\frac{1}{\sqrt{12}}$

C $\frac{2}{\sqrt{12}}$

D $\frac{1}{\sqrt{6}}$

QUESTION ANALYTICS +

Q. 13

Multiplication of matrices A and B is C. Matrices A and C are

$$A = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

What is the matrix B?

Solution Video | Have any Doubt ?

A $\begin{bmatrix} \cos\theta & \cos\theta & 0 \\ -\cos\theta & \sin\theta & 0 \\ 0 & 0 & -1 \end{bmatrix}$

B $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Correct Option

Solution :

(b)

According to question $A \times B = C$

Matrix C is a unit matrix. So matrix B will be inverse of A

$$A^{-1} = \frac{\text{Adj (A)}}{|A|}$$

$$|A| = 1 \times 1 = 1$$

$$\text{Adj (A)} = (\text{Cofactor (A)})^T$$

Solve to get, $\text{Adj (A)} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now substitute the values of $|A|$ and Adj (A) to get,

$$B = A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

C $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Your answer is **Wrong**

D $\begin{bmatrix} \sin\theta & -\cos\theta & 0 \\ \cos\theta & \sin\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

QUESTION ANALYTICS +

Q. 14

Consider ‘A’ is a set containing n elements. A subset ‘P’ of ‘A’ is chosen at random. The set ‘A’ is reconstructed by replacing the elements of ‘A’. A subset ‘Q’ of ‘A’ is again chosen at random. What is the probability that ‘P’ and ‘Q’ have no common element?

FAQ | Solution Video | Have any Doubt ?

A $(0.75)^n$ Correct Option

Solution :

(a)

Let, $‘A’ = \{a_1, a_2, a_3 \,........\, a_n\}$

There is an element a_1 of ‘A’ and two subsets ‘P’ and ‘Q’, then four possibilities

$$\left.\begin{array}{l} \text{(a) } a_1 \in P \text{ and } a_1 \in Q \\ \text{(b) } a_1 \in P \text{ and } a_1 \notin Q \\ \text{(c) } a_1 \notin P \text{ and } a_1 \in Q \\ \text{(d) } a_1 \notin P \text{ and } a_1 \notin Q \end{array}\right\} \text{4 choices}$$

Total number of ways selecting ‘P’ and ‘Q’ = 2^n

$\Rightarrow \quad 2^n \times 2^n = 4^n$ ways

$\Rightarrow \quad n(S) = 4^n$

Number of favorable elements = 3^n

$$P(E) = \frac{n(E)}{n(S)} = \frac{3^n}{4^n}$$

$$= (0.75)^n$$

B (0.85)^n

C (0.95)^n

D None of these

QUESTION ANALYTICS +

Q. 15

The eigen vectors of the matrix $\begin{bmatrix} 4 & 1 \\ 0 & 7 \end{bmatrix}$ are written in the form $\begin{bmatrix} 1 \\ p \end{bmatrix}$ and $\begin{bmatrix} 1 \\ q \end{bmatrix}$. What is p + q?

Solution Video | Have any Doubt ?

3 Correct Option

Solution :
3

$$\begin{bmatrix} (4-\lambda) & 1 \\ 0 & (7-\lambda) \end{bmatrix} = 0$$

$$\Rightarrow \quad (4-\lambda)(7-\lambda) = 0$$

$$\therefore \quad \lambda = 4, 7$$

Putting the value of $\lambda = 4$

$$\Rightarrow \quad \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ p \end{bmatrix} = 0$$

$$\Rightarrow \quad p = 0$$

Putting the value of $\lambda = 7$

$$\Rightarrow \quad \begin{bmatrix} -3 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ q \end{bmatrix} = 0$$

$$\Rightarrow \quad q = 3$$

$$\therefore \quad p + q = 3$$

QUESTION ANALYTICS +

Q. 16

Perform the following operations on the matrix $\begin{bmatrix} 1 & \frac{4}{3} & 15 \\ \frac{7}{3} & 3 & 35 \\ \frac{13}{3} & \frac{2}{3} & 65 \end{bmatrix}$

1. Add the third row to the second row.
2. Subtrace the third column from the first column.
The determined of the resultant matrix is ________.

FAQ | Have any Doubt ?

0 Correct Option

Solution :
0

Since operations 1 and 2 are elementary operations of the type of $R_i \pm kR_j$ and $C_i \pm kC_j$ respectively the determinant will be unchanged from the original determinant.
So the required determinant

$$= \begin{vmatrix} 1 & \frac{4}{3} & 15 \\ \frac{7}{3} & 3 & 35 \\ \frac{13}{3} & \frac{2}{3} & 65 \end{vmatrix} \xrightarrow{C_3 - 15C_1} \begin{vmatrix} 1 & \frac{4}{3} & 0 \\ \frac{7}{3} & 3 & 0 \\ \frac{13}{3} & \frac{2}{3} & 0 \end{vmatrix} = 0$$

Your Answer is 580.61

QUESTION ANALYTICS +

Q. 17

The number of satellites launched worldwide in a month follows Poisson distribution with mean as 6.8. The probability of launch of less than 3 satellites during a randomly selected month is________. (Upto 2 decimal places)

FAQ | Solution Video | Have any Doubt ?

0.03 Correct Option

Solution :
0.03

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$P(x < 3) = P(0) + P(1) + P(2)$$

$$= \frac{\lambda^0 e^{-\lambda}}{0!} + \frac{\lambda^1 e^{-\lambda}}{1!} + \frac{\lambda^2 e^{-\lambda}}{2!}$$

$$= \frac{1}{e^\lambda} + \frac{\lambda}{e^\lambda} + \frac{\lambda^2}{2e^\lambda}$$

As $\quad \lambda(\text{mean}) = 6.8$

$$\therefore \quad P(x < 3) = \frac{1 + 6.8 + \left(\frac{6.8^2}{2}\right)}{e^{6.8}} = \frac{30.92}{897.85} = 0.0344$$

Your Answer is 0.04

QUESTION ANALYTICS