SINGLE SUBJECT : ENGINEERING MATHEMATICS (GATE - 2019) - REPORTS

Q. 1

At the point $x = 2$, the function:

$$f(x) = \begin{cases} x^3 - 8 & 2 < x < \infty \\ x - 2 & -\infty < x \le 2 \end{cases} \text{ is}$$

Solution Video | Have any Doubt ?

A continuous and differentiable

B continuous and not differentiable — Correct Option

Solution :
(b)

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(x-2) = (2-2) = 0$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(x^3-8) = (2^3-8) = 0$$

Also $f(2) = 2 - 2 = 0$

Thus $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

$\therefore$ f is continuous at $x = 2$

$$f'(x) = \begin{cases} 3x^2 & 2 < x < \infty \\ 1 & -\infty < x \le 2 \end{cases}$$

and $Lf'(2) = 1$ and $Rf'(2) = 12$

$\therefore$ f is not differentiable at $x = 2$.

C discontinuous and differentiable

D discontinuous and not differentiable

QUESTION ANALYTICS +

Q. 2

Given that the determinant of the matrix $\begin{bmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{bmatrix}$ is –96, the determinant of the matrix $\begin{bmatrix} 4 & 12 & 0 \\ 8 & 24 & 16 \\ -4 & 0 & 8 \end{bmatrix}$ is

Solution Video | Have any Doubt ?

A 192

B 384

C –384

D –768 — Correct Option

Solution :
(d)
D = –96 for the given matrix

$$|A| = \begin{vmatrix} 4 & 12 & 0 \\ 8 & 24 & 16 \\ -4 & 0 & 8 \end{vmatrix} = 2^3 \begin{vmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{vmatrix}$$

(Taking 2 common from each row)

$\therefore$ $\text{Det}(A) = (2)^3 \times D$
$= 8 \times (-96)$
$= -768$

QUESTION ANALYTICS +

Q. 3

The value of $\lim_{x\to 4} \dfrac{(2x)^{1/3} - 2}{2x - 8}$ is

Solution Video | Have any Doubt ?

A $\frac{1}{3}$

B $\frac{1}{6}$

C $\frac{1}{24}$

D $\frac{1}{12}$ Correct Option

Solution :
(d)

$$\lim_{x\to 4}\frac{(2x)^{1/3}-2}{2x-8}$$

Above form is $\left(\frac{0}{0}\right)$ by putting the value $x = 4$

Applying L' Hospital rule

$$= \lim_{x\to 4}\frac{\frac{1}{3}(2x)^{\left(\frac{1}{3}-1\right)}\times 2}{2}$$

$$= \lim_{x\to 4}\frac{1}{3}(2x)^{\left(-\frac{2}{3}\right)}$$

$$= \frac{1}{3}(8)^{-2/3} = \frac{1}{12}$$

QUESTION ANALYTICS +

Q. 4

Which one of the following functions is strictly bounded?

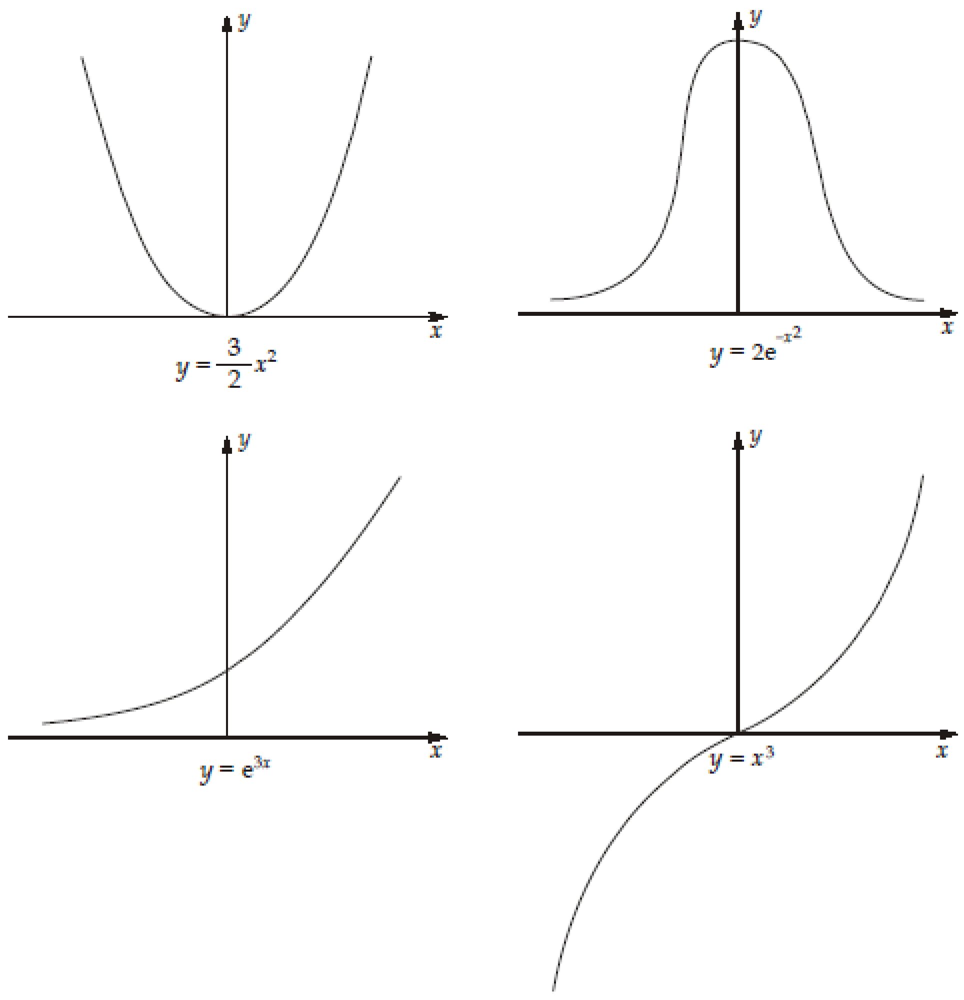
Solution Video | Have any Doubt ?

A $\frac{3x^2}{2}$

B $2e^{-x^2}$ Correct Option

Solution :
(b)

From the graphs below, we can see that only $2e^{-x^2}$ is strictly bounded.



C e^{3x}

D x^3

QUESTION ANALYTICS +

Q. 5

A continuous random variable X has a probability density function $f(x) = e^{-2x}$, $0 < x < \infty$. Then P[X > 1] is

Solution Video | Have any Doubt ?

A 0.270

B 0.067 Correct Option

Solution :
(b)

$$P[X > 1] = \int_1^\infty f(x)\,dx = \int_1^\infty e^{-2x}\,dx = \left.\frac{-e^{-2x}}{2}\right|_1^\infty$$

$$= -\left(\frac{e^{-2\infty}}{2}-\frac{e^{-2}}{2}\right) \qquad [e^{-\infty} = 0]$$

$$= \frac{e^{-2}}{2} = 0.067$$

C 0.034

D 0.135

QUESTION ANALYTICS +

Q. 6

What are the eigen values of matrix given below:

$$A = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 2 \\ 0 & -6 & -8 \end{bmatrix}_{3\times 3}$$

Solution Video | Have any Doubt ?

A (0, –2, 6)

B (0, 2, 6)

C (0, –2, –6) Correct Option

Solution :
(c)

$$A = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 2 \\ 0 & -6 & -8 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -\lambda & 2 & 0 \\ 0 & -\lambda & 2 \\ 0 & -6 & -8-\lambda \end{vmatrix} = 0$$

$-\lambda\,(8\lambda + \lambda^2 + 12) + 2(0 - 0) = 0$

Either $\lambda = 0$ or $(\lambda^2 + 8\lambda + 12) = 0$

$\lambda = 0$ or $(\lambda + 2)(\lambda + 6) = 0$

Therefore $\lambda = 0, -2, -6$

So option (c) is correct answer.

D (0, 1, 3)

QUESTION ANALYTICS +

Q. 7

A diagnostic test has a probability 95% of giving a positive result when applied to a person suffering from a certain disease and probability 10% of giving a positive when applied to a nonsufferer. It is estimated that 5% of the population are sufferers. Suppose that the test is now administered to a person about whom we have no relevant information regarding to the disease.

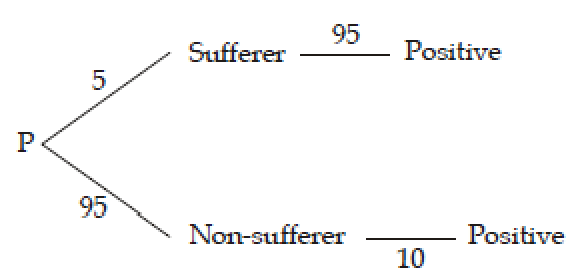
What is the probability that given a positive result, the person is a sufferer?

Solution Video | Have any Doubt ?

A 0.045

B 0.333 Correct Option

Solution :
(b)



$$P\left(\frac{\text{Sufferer}}{\text{Positive}}\right) = \frac{P(\text{Sufferer}) \times P(\text{Positive})}{P(\text{Sufferer}) \times P(\text{Positive}) + P(\text{Non-sufferer}) \times P(\text{Positive})}$$

$$= \frac{0.95 \times 0.05}{(0.95 \times 0.05) + (0.95 \times 0.1)}$$

$$= \frac{0.0475}{0.0475 + 0.095} = \frac{0.0475}{0.1425} = 0.333$$

C 0.099

D 0.435

QUESTION ANALYTICS +

Q. 8

What is the result of $\int_0^1 \log(1+x)dx$?

Solution Video | Have any Doubt ?

A $\log\left(\frac{2}{e}\right)$

B $\log\left(\frac{4}{e}\right)$ Correct Option

Solution :
(b)

$\int_0^1 \log(1+x)dx$

Using integration by parts:

$$= \int_0^1 \log(1+x)\cdot 1\,dx$$

$$= \left|\log(1+x)\cdot x\right|_0^1 - \int_0^1 \frac{1}{1+x}\cdot x\,dx$$

$$= \log(2) - \int_0^1 \frac{x}{1+x}dx$$

$$= \log(2) - \int_0^1 \frac{1+x-1}{1+x}dx$$

$$= \log(2) - \int_0^1 1dx + \int_0^1 \frac{dx}{1+x}$$

$$= \log(2) - \left|x\right|_0^1 + \left|\log(1+x)\right|_0^1$$

$$= \log(2) - 1 + \log(2)$$

$$= \log(2^2) - 1$$

$$= \log(2^2) - \log_e(e)$$

$$= \log\left(\frac{4}{e}\right)$$

C $\log\left(\frac{e}{2}\right)$

D $\log\left(\frac{e}{4}\right)$

QUESTION ANALYTICS +

Q. 9

Consider Vamshi decides to toss a fair coin repeatedly until he gets a tail. He makes atmost 4 tosses. The value of variance (T) is ________ (variable T denotes the number of tosses).

Solution Video | Have any Doubt ?

A $\frac{15}{16}$

B $\frac{1}{16}$

C $\frac{252}{256}$ Correct Option

Solution :
(c)

x	1	2	3	4
$P(x)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$

$$V(x) = E(x^2) - (E(x))^2$$

$$E(x^2) = 1^2\times\frac{1}{2} + 2^2\times\frac{1}{4} + 3^2\times\frac{1}{8} + 4^2\times\frac{1}{16} = \frac{58}{16}$$

$$E(x) = 1\times\frac{1}{2} + 2\times\frac{1}{4} + 3\times\frac{1}{8} + 4\times\frac{1}{16} = \frac{26}{16}$$

$$V(x) = E(x^2) - (E(x))^2$$

$$= \frac{58}{16} - \left(\frac{26}{16}\right)^2 = \frac{252}{256}$$

D $\frac{16}{225}$

QUESTION ANALYTICS +

Q. 10

If A is 3 × 3 matrix and Trace $A = 9$, $|A| = 24$ and one of the eigen values is 3, then sum of other eigen values is ________.

Solution Video | Have any Doubt ?

A 5

B 8

C 6 Correct Option

Solution :
(c)

Given,

$$\text{Trace } A = 9$$

$$|A| = 24$$

$$\lambda_1 = 3$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 9$$

$$\Rightarrow \quad 3 + \lambda_2 + \lambda_3 = 9$$

$$\Rightarrow \quad \lambda_2 + \lambda_3 = 6$$

D 9

QUESTION ANALYTICS +

Q. 11

If the sum of diagonal elements of a 2×2 symmetric matrix is -8, then the maximum possible value of determinant of the matrix is ________.

Solution Video | Have any Doubt ?

16 Correct Option

Solution :
16

Consider a symmetric matrix $A = \begin{bmatrix} a & b \\ b & d \end{bmatrix}$

Given
$$a + d = -8$$
$$|A| = ad - b^2$$

Now since b^2 is always non-negative, maximum determinant will come when $b^2 = 0$.

So we need to maximize

$$|A| = ad$$
$$= ad = a \times (-(8 + a)) = -a^2 - 8a$$
$$\frac{d|A|}{da} = -2a - 8 = 0$$
$$\Rightarrow \quad a = -4 \text{ is the only stationary point}$$

Since, $\left[\frac{d^2|A|}{da^2}\right]_{a=-4} = -2 < 0$, we have a maximum at $a = -4$

Since, $a + d = -8$, $d = -4$. Now maximum value of determinant is $|A| = 16$

QUESTION ANALYTICS +

Q. 12

The probability density function of random variable X is

$$f_x(x) = \begin{cases} \left(\frac{1}{2}\right)e^{-x/2} & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

The expectation $E(x)$ is ________.

Solution Video | Have any Doubt ?

2 Correct Option

Solution :
(2)

$$E(x) = \int_0^\infty \left(\frac{1}{2}\right) x e^{-x/2} dx$$
$$= \frac{1}{2}\left[\int_0^\infty x e^{-x/2} dx\right]$$
$$= \frac{1}{2}\left\{\left[\frac{xe^{-x/2}}{-\frac{1}{2}}\right]_0^\infty - \left[\int_0^\infty \frac{e^{-x/2}}{-\frac{1}{2}} dx\right]\right\}$$
$$= \frac{1}{2}\left\{-\left[\frac{e^{-x/2}}{-\frac{1}{2} \times -\frac{1}{2}}\right]_0^\infty\right\}$$
$$= \frac{1}{2}\left[-0 + \frac{1}{\frac{1}{2} \times \frac{1}{2}}\right] = 2$$

QUESTION ANALYTICS +

Q. 13

Let A and B be 3×3 matrices such that $A' = -A$ and $B' = B$. Then matrix $3AB + \lambda BA$ is a skew symmetric matrix for λ equal to ________.

FAQ | Solution Video | Have any Doubt ?

3 Correct Option

Solution :
3

Since it is skew symmetric, $(3AB + \lambda BA)' = -(3AB + \lambda BA)$

$$\text{LHS} = 3(AB)' + \lambda(BA)'$$
$$= 3B'A' + \lambda A'B'$$

Now
$$B' = B \text{ and } A' = -A$$
$$= -3BA - \lambda AB = -(\lambda AB + 3BA)$$
$$\text{RHS} = -(3AB + \lambda BA)$$
$$= 3BA + \lambda AB = 3AB + \lambda BA$$
$$(\lambda - 3)AB = (\lambda - 3)BA$$
$$\lambda = 3$$

QUESTION ANALYTICS +

Q. 14

Let $A_n = 1 + 2 + 3 + \ldots\ldots + n$ and $T_n = \frac{A_2}{A_2 - 1} \times \frac{A_3}{A_3 - 1} \times \frac{A_4}{A_4 - 1} \ldots\ldots \frac{A_n}{A_n - 1}$, where $n \in N\,(n \geq 2)$.

Then the value of $\lim_{n \to \infty} T_n$ is ________.

Solution Video | Have any Doubt ?

3 — Correct Option

Solution :
3

$$A_n = \frac{n(n+1)}{2}$$

$$A_n - 1 = \frac{n(n+1)}{2} - 1 = \frac{n^2+n-2}{2}$$

$$= \frac{(n+2)(n-1)}{2}$$

$$\frac{A_n}{A_n - 1} = \frac{n(n+1)}{(n+2)(n-1)} = \left(\frac{n}{n-1}\right)\left(\frac{n+1}{n+2}\right)$$

$$T_n = \left(\frac{2}{1}\times\frac{3}{2}\times\frac{4}{3}\cdots\cdots\frac{n}{n-1}\right)\left(\frac{3}{4}\times\frac{4}{5}\cdots\cdots\frac{n+1}{n+2}\right)$$

$$\frac{n}{1}\times\frac{3}{n+2} = \frac{3n}{n+2}$$

$$\lim_{n\to\infty} T_n = \lim_{n\to\infty}\frac{3n}{n+2} = 3$$

QUESTION ANALYTICS +

Q. 15

A fair coin is tossed till a head appears for the first time. The probability that the number of required tosses is odd will be ________. (Upto 2 decimal places)

Solution Video | Have any Doubt ?

0.67 [0.65 - 0.68] — Correct Option

Solution :
0.67 [0.65 - 0.68]
P(E) = Probability of head appearing in odd number of tosses
= P(H) + P(TTH) + ...

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) + \ldots\ldots$$

$$= \frac{1/2}{1-\frac{1}{4}} = \frac{2}{3} = 0.67$$

QUESTION ANALYTICS +

Q. 16

Let M be a matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Then the sum of all elements of M^{18} is ________.

Solution Video | Have any Doubt ?

60 — Correct Option

Solution :
60

$$M = \begin{bmatrix} I & I \\ 0 & I \end{bmatrix}$$

$$M^2 = \begin{bmatrix} I & I \\ 0 & I \end{bmatrix}\begin{bmatrix} I & I \\ 0 & I \end{bmatrix}$$

$$= \begin{bmatrix} I^2 + 0\times I & I^2 + I^2 \\ 0\times I + I\times 0 & 0\times I + I\times I \end{bmatrix}$$

$$= \begin{bmatrix} I^2 & 2I^2 \\ 0 & I^2 \end{bmatrix} \text{ since } [I^2 = I]$$

$$= \begin{bmatrix} I & 2I \\ 0 & I \end{bmatrix}$$

$$M^3 = \begin{bmatrix} I & 2I \\ 0 & I \end{bmatrix}\begin{bmatrix} I & I \\ 0 & I \end{bmatrix} = \begin{bmatrix} I^2 & 2I^2 + I^2 \\ 0 & I\times I + I\times 0 \end{bmatrix}$$

$$= \begin{bmatrix} I^2 & 3I^2 \\ 0 & I^2 \end{bmatrix}$$

$$M^3 = \begin{bmatrix} I & 3I \\ 0 & I \end{bmatrix}$$

$$M^K = \begin{bmatrix} I & KI \\ 0 & I \end{bmatrix}$$

$$M^{18} = \begin{bmatrix} I & 18I \\ 0 & I \end{bmatrix}$$

$$= 3\times 1 + 3\times 1 + 18\times 3\times 1$$

$$= 6 + 54 = 60$$

QUESTION ANALYTICS +

Q. 17

What is the value of $\lim\limits_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{1/x}$?

Solution Video | Have any Doubt ?

A abc

B $\sqrt[2]{abc}$

C $\sqrt[3]{abc}$ — Correct Option

Solution :
(c)

$$\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{1/x}$$

$$= \lim_{x\to 0}\left(\frac{3+a^x+b^x+c^x-3}{3}\right)^{1/x}$$

$$= \lim_{x\to 0}\left(1+\frac{a^x+b^x+c^x-3}{3}\right)^{1/x}$$

$$= \lim_{x\to 0}\left(1+\frac{(a^x-1)+(b^x-1)+(c^x-1)}{3}\right)^{1/x}$$

We know that:

$$\lim_{x\to 0}(1+\lambda x)^{1/x} = e^{\lambda}$$

$$= e^{\lim\limits_{x\to 0}\frac{(a^x-1)}{3x}+\frac{(b^x-1)}{3x}+\frac{(c^x-1)}{3x}}$$

$$= e^{\lim\limits_{x\to 0}\frac{1}{3}\left(\frac{a^x-1}{x}+\frac{b^x-1}{x}+\frac{c^x-1}{x}\right)}$$

$$= e^{1/3\,(\log a+\log b+\log c)} \quad \left[\because \lim_{x\to 0}\frac{a^x-1}{x}=\log a\right]$$

$$= e^{1/3\log(abc)} = e^{\log(abc)^{1/3}} = (abc)^{1/3}$$

$$= \sqrt[3]{abc}$$

D $(abc)^3$

QUESTION ANALYTICS +

Q. 18

An artillery target may be either at point 1 with probability $\frac{8}{9}$ or at point 2 with probability $\frac{1}{9}$. We have 21 shells, each of which can be fired at point 1 or point 2. Each shell may hit the target, independently of other shells, with probability $\frac{1}{2}$. If 12 shells are fired at point 1 and 9 shells are fired at point 2, what is the probability that the target is hit?

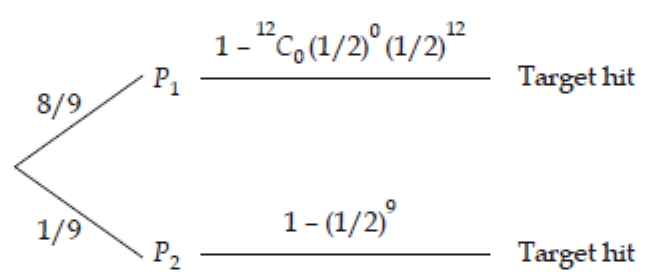
Solution Video | Have any Doubt ?

A $\frac{8}{9}2^{12}+\frac{1}{9}2^{9}$

B $\frac{8}{9}\left(\frac{1}{2^{12}}\right)+\frac{1}{9}\left(\frac{1}{2^{9}}\right)$

C $\frac{8}{9}\left(1-\frac{1}{2^{12}}\right)+\frac{1}{9}\left(1-\frac{1}{2^{9}}\right)$ — Correct Option

Solution :
(c)



$$P(\text{Target hit}) = \frac{8}{9}\left(1-\frac{1}{2^{12}}\right)+\frac{1}{9}\left(1-\frac{1}{2^{9}}\right)$$

So option (c) is correct answer.

D None of these

QUESTION ANALYTICS +

Q. 19

Which of the following is result of $\int_0^{\pi/3}\sin^3\theta\, d\theta$?

Solution Video | Have any Doubt ?

A 0

B 1

C $\frac{1}{3}$

D $\frac{5}{24}$ Correct Option

Solution :
(d)

$$I = \int_0^{\pi/3} (1-\cos^2\theta)\sin\theta\, d\theta$$

Let, $\cos\theta = t$

$$-\sin\theta \frac{d\theta}{dt} = 1$$

$$-\sin\theta\, d\theta = dt$$

at $\theta = 0, \cos 0 = 1$

at $\theta = \frac{\pi}{3}, \cos\frac{\pi}{3} = \frac{1}{2}$

$$I = \int_1^{1/2} -(1-t^2)\,dt$$

$$= -\left[t - \frac{t^3}{3}\right]_1^{1/2}$$

$$= -\left(\frac{11}{24}\right) - \left(-\frac{2}{3}\right)$$

$$= \frac{5}{24}$$

QUESTION ANALYTICS +

Q. 20

Cayley Hamilton theorem states that a square matrix satisfies its own characteristic equation. Consider a matrix, $A = \begin{bmatrix} 3 & 1 \\ -2 & 0 \end{bmatrix}$.

The relation satisfied by the matrix A is

Solution Video | Have any Doubt ?

A $A + 3I + 2A^{-1} = 0$

B $(A - I)(A + 2I) = 0$

C $A - 3I - 2A^{-1} = 0$

D $A - 3I + 2A^{-1} = 0$ Correct Option

Solution :
(d)

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 3 & 1 \\ -2 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\begin{vmatrix} 3-\lambda & 1 \\ -2 & -\lambda \end{vmatrix} = 0$$

$$-3\lambda + \lambda^2 + 2 = 0$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$A^2 - 3A + 2 = 0$$

$$A - 3I + 2A^{-1} = 0 \quad (\times \text{ by } A^{-1} \text{ on both sides})$$

QUESTION ANALYTICS +

Q. 21

A matrix has eigen values –6 and –3, the corresponding eigen vectors are $\begin{bmatrix} 3 \\ -6 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ -3 \end{bmatrix}$ respectively. The matrix is

Solution Video | Have any Doubt ?

A $\begin{bmatrix} -3 & 0 \\ 0 & -6 \end{bmatrix}$

B $\begin{bmatrix} 3 & 3 \\ -3 & -6 \end{bmatrix}$

C $\begin{bmatrix} 0 & 3 \\ -6 & -9 \end{bmatrix}$ Correct Option

Solution :
(c)

$$AX = \lambda X$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 \\ -6 \end{bmatrix} = (-6)\begin{bmatrix} 3 \\ -6 \end{bmatrix}$$

$$3a - 6b = -18 \quad \text{... (i)}$$

$$3c - 6d = 36 \quad \text{... (ii)}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 \\ -3 \end{bmatrix} = (-3)\begin{bmatrix} 3 \\ -3 \end{bmatrix}$$

$$3a - 3b = -9 \quad \text{... (iii)}$$
$$3c - 3d = 9 \quad \text{... (iv)}$$

From equation (i) and (iii), $a = 0$ and $b = 3$.
From equation (ii) and (iv), $c = -6$ and $d = -9$.

$$\therefore \quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -6 & -9 \end{bmatrix}$$

D $\begin{bmatrix} 3 & 6 \\ -6 & -12 \end{bmatrix}$

QUESTION ANALYTICS +

Q. 22

A function $y = 7x^2 + 12x$ is defined over an open interval $x = (1, 3)$. At least at one point is this interval, $\frac{dy}{dx}$ is exactly

Solution Video | Have any Doubt ?

A 26

B 40 — Correct Option

Solution :
(b)

$$y = 7x^2 + 12x$$

Using Lagrange's mean value theorem:

At $\quad x = 1, y = 7 + 12 = 19$

$x = 3, y = 63 + 36 = 99$

$$f'(x) = \frac{f(b) - f(a)}{b - a}$$
$$= \frac{99 - 19}{3 - 1} = 40$$

So option (b) is correct answer.

C 62

D 54

QUESTION ANALYTICS +

Q. 23

A random variable x has the following probability distribution.

x	0	1	2	3	4
$P(x)$	c	$2c$	$2c$	c^2	$5c^2$

The mean and variance of x is

Solution Video | Have any Doubt ?

A 1.638, 1.45 — Correct Option

Solution :
(a)

Since, $\quad \sum_{x=0}^{4} P(x) = 1$

$$c + 2c + 2c + c^2 + 5c^2 = 1$$
$$6c^2 + 5c - 1 = 0$$
$$c = \frac{1}{6}, -1$$

Since $P(x) \geq 0$, the possible value of

$$c = \frac{1}{6}$$

x	0	1	2	3	4
$P(x)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{1}{36}$	$\frac{5}{36}$
$xP(x)$	0	$\frac{2}{6}$	$\frac{4}{6}$	$\frac{3}{36}$	$\frac{20}{36}$

$$\text{Mean} = \sum_{x=0}^{4} xP(x) = 0 + \frac{2}{6} + \frac{4}{6} + \frac{3}{36} + \frac{20}{36}$$
$$= \frac{59}{36} = 1.638$$
$$\text{Variance} = \sigma^2 = E(x^2) - [E(x)]^2$$
$$= \left[0\left(\frac{1}{6}\right) + 1\left(\frac{2}{6}\right) + 4\left(\frac{2}{6}\right) + 9\left(\frac{1}{36}\right) + 16\left(\frac{5}{36}\right) - \left(\frac{59}{36}\right)^2\right] = 1.45$$

B 1.638, 1.204

C 1.204, 1.45

D 1.45, 1.638

QUESTION ANALYTICS +

Q. 24

Consider function $f(x) = (x^2 - 4)^2$ where x is a real number. Which of the following is true about given function?

Solution Video | Have any Doubt ?

A Has only one minima

B Has only two minima Correct Option

Solution :
(b)

$$\begin{aligned} f(x) &= (x^2-4)^2 \\ f'(x) &= 2(x^2-4)\times 2x \\ &= 4x(x^2-4)=0 \\ x &= 0, x=2 \text{ and } x=-2 \text{ are the stationary points} \\ f''(x) &= 4[x(2x)+(x^2-4)\times 1] \\ &= 4[(2x^2+(x^2-4]=4[3x^2-4] \\ &= 12x^2-16 \\ f''(0) &= -16<0 \quad \text{(So maxima at } x=0) \\ f''(2) &= (12)2^2-16=32>0 \quad \text{(So minima at } x=2) \\ f''(-2) &= 12(-2)^2-16=32>0 \quad \text{(So minima at } x=-2) \end{aligned}$$

$\therefore$ There is only one maxima and only two minima for this function.

C Has three minima

D Has three maxima

QUESTION ANALYTICS +

Q. 25

Consider the following linear equations of system :

$$\begin{aligned} x+y+z &= 6 \\ x+2y+5z &= 10 \\ 2x+3y+\lambda z &= \mu \end{aligned}$$

Which of the following is correct about system?

Solution Video | Have any Doubt ?

A System has unique solution for $\lambda \neq 6$

B System has no solution for $\lambda = 6$ and $\mu \neq 16$

C System has infinite solution for $\lambda = 6$ and $\mu = 16$

D All of the above Correct Option

Solution :
(d)

$$AX = B$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 2 & 3 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

$$C = (A, B) = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & 2 & 5 & : & 10 \\ 2 & 3 & \lambda & : & \mu \end{bmatrix}$$

After performing $R_2 \leftarrow R_2 - R_1$ and $R_3 \leftarrow R_3 - 2R_1$

$$C = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 4 & : & 4 \\ 0 & 1 & \lambda-2 & : & \mu-12 \end{bmatrix}$$

After performing $R_3 \leftarrow R_3 - R_2$

$$C = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 4 & : & 4 \\ 0 & 0 & \lambda-6 & : & \mu-16 \end{bmatrix}$$

Since $R(A) = R(C)$ for unique solution

So $\lambda - 6 \neq 0, \lambda \neq 6$.

For no solution $R(A) \neq R(C)$ then $R(A) = 2$ and $R(C) = 3$

$$\lambda - 6 = 0$$

$\Rightarrow \quad \lambda = 6$ and $\mu - 16 \neq 0 \Rightarrow \mu \neq 16$

For infinite solution $R(A) = R(C) = 2$

then $\lambda - 6 = 0$ and $\mu - 16 = 0$

$\lambda = 6$ and $\mu = 16$

So all of options are true.

QUESTION ANALYTICS +

Q. 26

Probability density function of a random variable X is distributed uniformly between 0 and 10. The probability that X lies between 2.5 to 7.5 and the mean square value of X are respectively

Solution Video | Have any Doubt ?

A $\frac{1}{2}$ and $\frac{100}{3}$ Correct Option

Solution :
(a)

In uniform distribution $[a, b]$

$$k = \frac{1}{b-a}$$

$$= \frac{1}{10-0} = \frac{1}{10}$$

$$P(2.5 \le X \le 7.5) = \int_{2.5}^{7.5} \frac{1}{10} dx = \frac{1}{10} x \Big|_{2.5}^{7.5} = \frac{1}{10}(7.5-2.5) = \frac{1}{2}$$

$$E(x^2) = \text{Mean square value} = \int_0^{10} x^2 f(x) dx$$

$$\int_0^{10} \frac{1}{10} x^2 dx = \frac{1}{10} \frac{x^3}{3}\Big|_0^{10} = \frac{10^3 - 0^3}{30} = \frac{1000}{30} = \frac{100}{3}$$

B 5 and 100

C 5 and $\frac{100}{3}$

D $\frac{1}{2}$ and 100

QUESTION ANALYTICS +

Q. 27

Assume A and B are matrix of size $n \times n$, which of the following is true?

Solution Video | Have any Doubt ?

A If A is invertible, the $ABA^{-1} = B$.

B If A is an indempotent non-singular matrix, then A must be the identity matrix. Correct Option

Solution :
(b)

- $ABA^{-1} = B$ given,

$\Rightarrow$ $AB = BA$ since matrix multiplication is not commutative. So false even if A is invertible.

- A is idempotent, so $A^2 = A$, since A is non-singular, so it is invertible i.e. A^{-1} exist.

$$I = A^{-1} . A = A^{-1} . A^2 = IA = A$$

So A must be identity matrix. So true.

- If coefficient matrix A is invertible for $Ax = b$ then $x = A^{-1}$ unique solution exist. So false
- If B is zero matrix, then also $AB = B$ = zero matrix. So false

C If the coefficient matrix A of the system $Ax = b$ is invertible, then the system has infinitely many solution

D If $AB = B$ then B is identity matrix.

QUESTION ANALYTICS +

Q. 28

Consider a matrix P :

$$P = \begin{bmatrix} 4 & -6 \\ -2 & 8 \end{bmatrix}$$

If $P \times Q = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$, then which of the following represent the matrix Q?

Solution Video | Have any Doubt ?

A $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$

B $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$

C $\begin{bmatrix} 2 \\ 0.5 \end{bmatrix}$ Correct Option

Solution :
(c)

$$P \times Q \Rightarrow \begin{bmatrix} 4 & -6 \\ -2 & 8 \end{bmatrix} \times Q = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad Q = \begin{bmatrix} 4 & -6 \\ -2 & 8 \end{bmatrix}^{-1} \times \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad Q = \frac{1}{20}\begin{bmatrix} 8 & 6 \\ 2 & 4 \end{bmatrix} \times \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{8}{20} & \frac{6}{20} \\ \frac{2}{20} & \frac{4}{20} \end{bmatrix} \times \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{8}{20} \times 5 \\ \frac{2}{20} \times 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 0.5 \end{bmatrix}$$

D $\begin{bmatrix} 0.5 \\ 2 \end{bmatrix}$

QUESTION ANALYTICS +

Q. 29

Consider $f(x)$ be a function defined by

$$f(x)=\begin{cases}4x-5, & \text{if } x\le 2\\ x-\lambda, & \text{if } x>2\end{cases}$$

In $\lim_{x\to 2} f(x)$ exist, then the value of λ is ________.

Solution Video | Have any Doubt ?

-1 Correct Option

Solution :
(-1)

$$f(x) = \begin{cases}4x-5, & \text{if } x\le 2\\ x-\lambda, & \text{if } x>2\end{cases}$$

$$\begin{aligned}\therefore \quad \lim_{x\to 2^-} f(x) &= \lim_{h\to 0} f(2-h)\\ &= \lim_{h\to 0}[4(2-h)-5]\\ &= \lim_{h\to 0}[8-4h-5]\\ &= 3\\ \lim_{x\to 2^+} f(x) &= \lim_{h\to 0} f(2+h)\\ &= \lim_{h\to 0}[2+h-\lambda]\\ &= 2-\lambda\end{aligned}$$

Since limit exist, so,

$$\begin{aligned}\lim_{x\to 2^-} f(x) &= \lim_{x\to 2^+} f(x)\\ 2-\lambda &= 3\\ \lambda &= -1\end{aligned}$$

QUESTION ANALYTICS +

Q. 30

A manufacturer makes condensers which on an average are 1% defective. He packs them in boxes of 100. The probability that a box picked at random will contain 3 or more faulty condensers is ________. (Upto 2 decimal places)

Solution Video | Have any Doubt ?

0.08 (0.08 - 0.09) Correct Option

Solution :
0.08 (0.08 - 0.09)

$$\begin{aligned}P &= 1\% = 0.01\\ n &= 100\\ \lambda &= nP = 100\times 0.01 = 1\\ P(r) &= \frac{e^{-\lambda}\lambda^r}{r!}\end{aligned}$$

P(3 or more faulty condensers)

$$\begin{aligned}&= P(3)+P(4)+\ldots\ldots+P(100)\\ &= 1-[P(0)+P(1)+P(2)]\\ &= 1-\left[\frac{e^{-1}1^0}{0!}+\frac{e^{-1}1^1}{1!}+\frac{e^{-1}1^2}{2!}\right]=1-\left[e^{-1}+e^{-1}+\frac{e^{-1}}{2}\right]\\ &= 1-e^{-1}\left[\frac{5}{2}\right]=0.0803\end{aligned}$$

QUESTION ANALYTICS +

Q. 31

Consider two matrices given below:

$$P=\begin{bmatrix}a & b\\ c & d\end{bmatrix};\ Q=\begin{bmatrix}a^2+b^2 & ac+bd\\ ac+bd & c^2+d^2\end{bmatrix}$$

If the rank of matrix P is 2, then the rank of matrix Q will be ________.

Solution Video | Have any Doubt ?

2 Correct Option

Solution :
(2)

$$\begin{aligned}P &= \begin{bmatrix}a & b\\ c & d\end{bmatrix}\\ Q &= \begin{bmatrix}a^2+b^2 & ac+bd\\ ac+bd & c^2+d^2\end{bmatrix}\\ &= P\times P^T=\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}a & c\\ b & d\end{bmatrix}\\ &= \begin{bmatrix}a^2+b^2 & ac+bd\\ ac+bd & c^2+d^2\end{bmatrix}=Q\end{aligned}$$

i.e. $\quad P\,.\,P^T = Q$

So, rank of matrix Q is same as rank of P.

QUESTION ANALYTICS +

Q. 32

A system matrix is given as follows:

$$A = \begin{bmatrix} 0 & 2 & -2 \\ -12 & -22 & 12 \\ -12 & -22 & 10 \end{bmatrix}$$

The value of the ratio of the absolute maximum eigen value to the absolute minimum eigen value is ________. (It is known that one of the eigen value is –6)

Solution Video | Have any Doubt ?

3 — Correct Option

Solution :
3

Characteristic equation is $|A - \lambda I| = 0$

$$\begin{vmatrix} -\lambda & 2 & -2 \\ -12 & -22-\lambda & 12 \\ -12 & -22 & 10-\lambda \end{vmatrix} = 0$$

$\Rightarrow -\lambda[-220 + 22\lambda - 10\lambda + \lambda^2 + 264] - 2[-120 + 12\lambda + 144] - 2\,[264 - 264 - 12\lambda] = 0$

$\Rightarrow -\lambda(\lambda^2 + 12\lambda + 44) - 2(12\lambda + 24) + 24\lambda = 0$

$\Rightarrow -\lambda^3 - 12\lambda^2 - 44\lambda - 24\lambda - 48 + 24\lambda = 0$

$\Rightarrow -\lambda^3 - 12\lambda^2 - 44\lambda - 48 = 0$

$\Rightarrow \quad \lambda^3 + 12\lambda^2 + 44\lambda + 48 = 0$

$\Rightarrow \quad (\lambda + 2)(\lambda + 4)(\lambda + 6) = 0$

(one eigen value is –6, so $\lambda + 6$ is factor of this equation so we can obtain the other roots by polynomial division)

$\Rightarrow \quad \lambda = -2, -4, -6$

$$\text{Ratio} = \frac{6}{2} = 3$$

QUESTION ANALYTICS

Q. 33

Consider $f(x) = 3x^3 - 7x^2 + 5x + 6$. The minimum value of $f(x)$ over the interval [0, 2] is ________.
(Upto 1 decimal place)

Solution Video | Have any Doubt ?

6 — Correct Option

Solution :
(6)

$$\begin{aligned}
f(x) &= 3x^3 - 7x^2 + 5x + 6 \\
f'(x) &= 9x^2 - 14x + 5 \\
f'(x) &= 0 \\
&= 9x^2 - 14x + 5 \\
&= 9x^2 - 9x - 5x + 5 \\
&= 9x[x-1] - 5[x-1] \\
&= (9x-5)\,(x-1) \\
x &= 1 \text{ and } x = \frac{5}{9} = 0.55 \\
f''(x) &= 18x - 14 \\
f''(1) &= 18 - 14 \\
&= 4 > 0 \text{ (minima)} \\
\text{Minimum } \{f(0), f(1), f(2)\} &= \text{minimum } \{6, 7, 12\} \\
&= 6
\end{aligned}$$

QUESTION ANALYTICS