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Computer Science Engineering(CS)

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TOPICWISE : ENGINEERING MATHEMATICS-2 (GATE - 2019) - REPORTS

OVERALL ANALYSIS | COMPARISON REPORT | SOLUTION REPORT

ALL(17) | CORRECT(2) | INCORRECT(6) | SKIPPED(9)

Q. 1

Consider the rank of matrix 'A' of size $(m \times n)$ is "$m - 1$". Then, which of the following is true?

Have any Doubt ?

A. AA^T will be invertible.

B. A have "$m - 1$" linearly independent rows and "$m - 1$" linearly independent column. — Correct Option

Solution :
(b)
Rank of matrix is "$m - 1$", so it must have "$m - 1$" linearly independent rows as well as "$m - 1$" independent columns.

C. A will have "m" linearly independent rows and "n" linearly independent columns.

D. A will have "$m - 1$" linearly independent rows and "$n - 1$" independent columns.

QUESTION ANALYTICS +

Q. 2

For function $f(x) = 4x^3 - 6x^2$, the maximum occurs in interval $[-1, 2]$ when x is equal to

Have any Doubt ?

A. 0 — Your answer is **Wrong**

B. - 1

C. 1

D. 2 — Correct Option

Solution :
(d)

$$f(x) = 4x^3 - 6x^2$$
$$\frac{d\,f(x)}{dx} = 12x^2 - 12x$$
$$12x^2 - 12x = 0$$
$$12x\,[x - 1] = 0$$
$$x = 0, 1$$
$$\frac{d\,f'(x)}{dx} = 24x - 12$$

At $x = 0$, $24 \times 0 - 12 = -12 < 0$ maxima
At $x = 1$, $24 \times 1 - 12 = 12 > 0$ minima
So, at

$$x = -1,\ f(-1) = 4(-1)^3 - 6(-1)^2 = -4 - 6 = -10$$
$$x = 0,\ f(0) = 4(0)^3 - 6(0)^2 = 0$$
$$x = 1,\ f(1) = 4(1)^3 - 6(1)^2 = 4 - 6 = -2$$
$$x = 2,\ f(2) = 4(2)^3 - 6(2)^2 = 32 - 24 = 8$$

So maximum value occurs at $x = 2$.

QUESTION ANALYTICS +

Q. 3

Find the limit?

$$\lim_{x \to \infty}\left[1 + \frac{3}{2x}\right]^{5x}$$

Have any Doubt ?

A. e^{15}

B. e^3

C. $e^{15/2}$ — Your answer is **Correct**

Solution :
(c)

$$\lim_{x \to \infty}\left[1 + \frac{3}{2x}\right]^{5x}$$

Put limit $x \to \infty$
1^∞ from create,
So, we know, for form 1^∞

$$\lim_{x\to\infty} f(x)^{g(x)} = e^{\left(\lim_{x\to\infty}(f(x)-1)\cdot g(x)\right)}$$

Apply in given function:

$$= e^{\lim_{x\to\infty}\left[1+\frac{3}{2x}-1\right]5x}$$

$$= e^{\lim_{x\to\infty}\left[\frac{3}{2x}\right]5x}$$

$$= e^{15/2}$$

D $e^{5/3}$

QUESTION ANALYTICS +

Q. 4

Consider the following function:

$$f(x)=\begin{cases}-1.5x^2, & x\le -2\\ 6x-5, & x>-2\end{cases}$$

Which of the following is true at $x = -2$?

Have any Doubt ?

A Continuous but not differentiable — Your answer is **Wrong**

B Differentiable and continuous both

C Differentiable but not continuous

D neither continuous nor differentiable — Correct Option

Solution :
(d)

Check for continuous:

$$\begin{aligned} f(-2) &= -1.5 \times (-2)^2 = -6 \\ f(-2^+) &= 6(-2) - 5 = -17 \\ f(-2^-) &= -1.5 \times (-2)^2 = -6 \\ f(-2^-) &\ne f(-2^+) \end{aligned}$$

Function is not continuous, hence cannot be differentiable i.e. differentiable $\rightarrow$ continuous.

QUESTION ANALYTICS +

Q. 5

Consider a man is known to speak truth 3 out of 5 times, he throw a die and reports the number obtained is 2. What is the probability that the number obtained is actually 2?

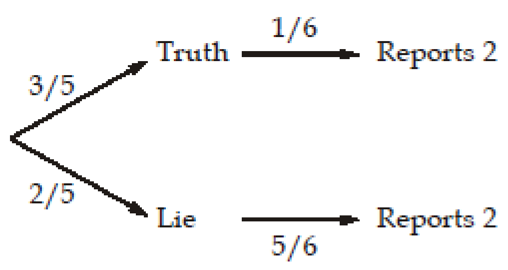
Have any Doubt ?

A $\frac{13}{30}$ — Your answer is **Wrong**

B $\frac{3}{13}$ — Correct Option

Solution :
(b)

Applying Bayes Theorem:



So,

$$\text{P(spoke truth/reports 2)} = \frac{\text{P(spoke truth} \cap \text{reports 2)}}{\text{P(reports 2)}}$$

$$= \frac{\frac{3}{5}\times\frac{1}{6}}{\frac{3}{5}\times\frac{1}{6}+\frac{2}{5}\times\frac{5}{6}} = \frac{3}{13}$$

C $\frac{1}{10}$

D None of the above

QUESTION ANALYTICS +

Q. 6

The value of

$$\int_0^2 \frac{1}{(3+2x)^2}\,dx = \_\_\_\_\_\_\_.\ \text{(Upto 3 decimal places)}$$

Have any Doubt ?

0.095 (0.095 - 0.096) — Correct Option

Solution :

0.095 (0.095 - 0.096)

Consider, $u = 3 + 2x$

$$\frac{du}{dx} = 2$$

$$dx = \frac{du}{2}$$

Calculate new limits:

$$x = 0, u = 3 + 2 \cdot x = 3 + 0 = 3$$
$$x = 2, u = 3 + 2 \cdot x = 3 + 2 \times 2 = 7$$

By substitution:

$$= \int_3^7 \frac{1}{u^2} \cdot \frac{1}{2} du$$
$$= \frac{1}{2}\left[-u^{-1}\right]_3^7$$
$$= \frac{1}{2}\left[\frac{1}{3} - \frac{1}{7}\right]$$
$$= \frac{1}{2}\left[\frac{4}{21}\right] = \frac{2}{21}$$
$$= 0.095$$

QUESTION ANALYTICS +

Q. 7

The maximum value of the function:

$f(x) = x^3 - 9x^2 + 24x + 5$

in interval of [–3 to 3] is ________.

FAQ | Have any Doubt ?

25 Correct Option

Solution :
25

$$f(x) = x^3 - 9x^2 + 24x + 5$$
$$\frac{df}{dx} = 3x^2 - 18x + 24$$

Function attains local minimum or maximum at critical points.
Critical points are those where $f'(x) = 0$

$$\begin{aligned} 3x^2 - 18x + 24 &= 0 \\ x^2 - 6x + 8 &= 0 \\ x^2 - 4x - 2x + 8 &= 0 \\ x(x-4) - 2(x-4) &= 0 \\ (x-2)(x-4) &= 0 \\ x &= 2, 4 \end{aligned}$$

$$\frac{d\,f'(x)}{dx} = 6x - 18$$
$$f''(2) = 12 - 18 = -6 < 0 \text{ (local maximum)}$$
$$f''(4) = 24 - 18 = 6 > 0 \text{ (local minimum)}$$

In given interval:

x	$f(x)$
–3	Some value in negative
2	25
3	23

Hence maximum value is 25 at $x = 2$.

QUESTION ANALYTICS +

Q. 8

Consider

$$f(x) = \begin{cases} -x, & x \le 1 \\ 1+x, & x \ge 1 \end{cases} \text{ and } g(x) = \begin{cases} 1-x, & x \le 0 \\ x^2, & x > 0 \end{cases}$$

The composition of f and g i.e. $gof(x) = g(f(x))$. Then out of $f(x)$, $g(x)$ and $gof(x)$ in the interval $(-\infty, 0)$, how many are discontinuous ________.

Have any Doubt ?

0 Correct Option

Solution :
0

For interval $(-\infty, 0)$

$$f(x) = -x; x < 0$$
$$g(x) = 1 - x; x \le 0$$

Both are continuous for $x < 0$ and we know composition of two continuous function is also continuous. So, $gof(x)$ is also continuous.
Hence no function is discontinuous.

Your Answer is 2

QUESTION ANALYTICS +

Q. 9

Consider a 3 × 3 matrix 'A' having det(A) = – 5. The value of det(4A) is ________.

Have any Doubt ?

-320 Correct Option

Solution :
-320

We know that,

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$4A = \begin{bmatrix} 4a & 4b & 4c \\ 4d & 4e & 4f \\ 4g & 4h & 4i \end{bmatrix}$$

$$\det(4A) = 4^3 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$= 4^3 \times (-5)$$

$$= -320$$

Your Answer is -20

QUESTION ANALYTICS +

Q. 10

Consider the following function:

$$f(x) = \begin{cases} \frac{x-c}{1+c}, & \text{if } x \le 0 \\ x^2 + c, & \text{if } x > 0 \end{cases}$$

Which of the following value of c, for which function is continuous for every 'x'?

Have any Doubt ?

A 2

B - 2

C 0

D Both (b) and (c) Correct Option

Solution :
(d)

function $f(x)$ is continuous for every $x \ne 0$ (since $\frac{x-c}{1+c}$ and $x^2 + c$ are polynomials, and polynomials are continuous).

$$f(0) = \frac{0-c}{1+c} = \frac{-c}{1+c}$$

$$\lim_{x \to 0^-} \frac{0-c}{1+c} = \frac{-c}{1+c}$$

$$\lim_{x \to 0^+} 0^2 + c = c$$

Since $f(x)$ is continuous for every x, hence continuous for $x = 0$.

$$\Rightarrow \quad f(0) = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$$

$$\Rightarrow \quad \frac{-c}{1+c} = c$$

$$\Rightarrow \quad -c = c(1 + c)$$

$$c^2 + 2c = 0$$

$$c = -2 \text{ or } c = 0$$

So option (d) is correct answer

QUESTION ANALYTICS +

Q. 11

Which of the following matrix is LU decomposible?

Have any Doubt ?

A $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 1 & 3 & 4 \end{bmatrix}$

B $\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$ Correct Option

Solution :
(b)

To check matrix is LU decomposible by checking if principal minors have non-zero determinants.

Check (a):

$$|A_1| = |1| = 1 \ne 0$$

Now $$|A_2| = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} = 0$$

So option (a) is not LU decomposible.

Check (b):

$$\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix} \text{ here } |A_1| = 3, |A_2| = \begin{vmatrix} 3 & 2 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$$

So LU decomposible.

Check (c):

$\begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}$ here $|A_1| = 0$

So not LU decomposible.

Check (d):

$\begin{bmatrix} 1 & -3 & 7 \\ -2 & 6 & 1 \\ 0 & 3 & -2 \end{bmatrix}$ here $|A_1| = 1 \neq 0$ but

$$|A_2| = \begin{vmatrix} 1 & -3 \\ -2 & 6 \end{vmatrix} = |6-6| = 0$$

So not LU decomposible.

C $\begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}$

D $\begin{bmatrix} 1 & -3 & 7 \\ -2 & 6 & 1 \\ 0 & 3 & -2 \end{bmatrix}$

QUESTION ANALYTICS +

Q. 12

Consider the following table with data recorded over a month with 30 days:

		Weather	
		Sunny	Not sunny
Mood	Good	12	9
	Not Good	4	5

If Rahul recorded on each day, whether it was sunny or not sunny and whether Rahul's mood was good or not good. If given day is sunny, then what is the probability that on given day Rahul's mood is good?

Have any Doubt ?

A $\frac{1}{4}$

B $\frac{3}{4}$

Correct Option

Solution :

(b)

Let, P(G) represent given day mood is good.

P(S) represent given day is sunny.

		Weather		
		Sunny	Not sunny	
Mood	Good	12	9	21
	Not Good	4	5	9
		16	14	30

So, $$P(G|S) = \frac{P(G \cap S)}{P(S)}$$

$$P(G \cap S) = \frac{12}{30}$$

$$P(S) = \frac{16}{30}$$

So, $$P(G|S) = \frac{\frac{12}{30}}{\frac{16}{30}} = \frac{12}{16}$$

$$= \frac{3}{4}$$

C $\frac{5}{16}$

D $\frac{16}{30}$

QUESTION ANALYTICS +

Q. 13

The value of the integral given below is:

$$\int_{\pi/6}^{\pi/3} \frac{\operatorname{cosec}^2 x}{\cot^2 x} dx$$

Have any Doubt ?

A $\frac{2}{3}$

B $\frac{2}{\sqrt{3}}$

Your answer is **Correct**

Solution :

(b)

Consider, $$u = \cot x$$

$$\frac{du}{dx} = -\operatorname{cosec}^2 x$$

$$du = -\operatorname{cosec}^2 x\, dx$$

$$-du = \operatorname{cosec}^2 x\, dx$$

Now new limits:

$$x = \frac{\pi}{6} \rightarrow u = \cot\frac{\pi}{6} = \sqrt{3}$$

$$x = \frac{\pi}{3} \rightarrow u = \cot\frac{\pi}{3} = \frac{1}{\sqrt{3}}$$

Substitute new limits and $\operatorname{cosec}^2 x\, dx$

$$\int_{\sqrt{3}}^{1/\sqrt{3}} \frac{-du}{u^2} = \left[\frac{u^{-2+1}}{-2+1}\right]_{\sqrt{3}}^{1/\sqrt{3}}$$

$$= \left[u^{-1}\right]_{\sqrt{3}}^{1/\sqrt{3}}$$

$$= \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{3-1}{\sqrt{3}}$$

$$= \frac{2}{\sqrt{3}}$$

C $\frac{3}{2}$

D $2\sqrt{3}$

QUESTION ANALYTICS +

Q. 14

If the determinant of matrix:

$$A = \begin{bmatrix} 0 & 4 & 2 & 1 \\ 3 & -1 & 0 & 2 \\ 5 & 2 & x & 4 \\ 6 & 1 & -1 & 0 \end{bmatrix}$$

is 245, then which of the following represents the value of 'x'?

Have any Doubt ?

A -6

B 4

C -4

D 6 Correct Option

Solution :
(d)

$$A = \begin{vmatrix} 0 & 4 & 2 & 1 \\ 3 & -1 & 0 & 2 \\ 5 & 2 & x & 4 \\ 6 & 1 & -1 & 0 \end{vmatrix} = 245$$

$$\Rightarrow 5\begin{vmatrix} 4 & 2 & 1 \\ -1 & 0 & 2 \\ 1 & -1 & 0 \end{vmatrix} - 2\begin{vmatrix} 0 & 2 & 1 \\ 3 & 0 & 2 \\ 6 & -1 & 0 \end{vmatrix} + x\begin{vmatrix} 0 & 4 & 1 \\ 3 & -1 & 2 \\ 6 & 1 & 0 \end{vmatrix} - 4\begin{vmatrix} 0 & 4 & 2 \\ 3 & -1 & 0 \\ 6 & 1 & -1 \end{vmatrix} = 245$$

$\Rightarrow$ 5[1[4] + 1[8 + 1]] – 2[6[4] + 1[–3]] + x[6[8 + 1] – 1[–3]] –4[–4[–3] + 2[3 + 6]] = 245

$\Rightarrow$ 5[4 + 9] – 2[24 – 3] + x[54 + 3] –4[12 + 18] = 245

$\Rightarrow$ 65 – 42 + 57x – 120 = 245

57x = 245 + 120 + 42 – 65

57x = 342

x = 6

Alternate method:

For a shorter method, kindly refer to the video solution corresponding to this question.

QUESTION ANALYTICS +

Q. 15

The value of

$$\lim_{x\to 0}\left[2+\left(\frac{\log\cos x}{\log\cos(x/2)}\right)^2\right]^3 = \_\_\_\_\_\_.$$

Have any Doubt ?

5832 Correct Option

Solution :
5832

$$\Rightarrow \lim_{x\to 0}\left[2+\left(\frac{\log\cos x}{\log\cos(x/2)}\right)^2\right]^3$$

$$\Rightarrow \left[2+\left(\lim_{x\to 0}\frac{\log\cos x}{\log\cos(x/2)}\right)^2\right]^3$$

$\Rightarrow$ Apply L'Hospital rule since cos 0 = 1 and log(1) = 0 which form indeterminant form i.e. 0/0

$$\Rightarrow \left[2+\left(\lim_{x\to 0}\frac{\frac{\sin x}{\cos x}}{\frac{1}{2}\frac{\sin(x/2)}{\cos(x/2)}}\right)^2\right]^3$$

$$\Rightarrow \left[2+\left(2\times\lim_{x\to 0}\frac{\tan x}{\tan(x/2)}\right)^2\right]^3$$

$\Rightarrow$ Apply L'Hospital rule again make 0/0 form.

$$\Rightarrow \left[2+\left(4\times\lim_{x\to 0}\frac{\sec^2 x}{\sec^2(x/2)}\right)^2\right]^3$$

$$\Rightarrow \left[2+\left(4\times\frac{\sec^2 0}{\sec^2 0}\right)^2\right]^3$$

$$\Rightarrow \left[2+\left(4\times\frac{1}{1}\right)^2\right]^3$$

$$\Rightarrow \left[2+4^2\right]^3$$

$\Rightarrow$ $[2 + 16]^3 = [18]^3$

$\Rightarrow$ 5832

Your Answer is 8

QUESTION ANALYTICS

Q. 16

Consider Kuldeep purchase a product of company X. The manual on it states that the lifetime T of product is defined as the amount of time (in years) the product works properly until it breaks down, satisfy following equation:

$P(T \geq t) = e^{-t/4}$, for all $t \geq 0$

The probability that it breaks down in 3rd year is ________. (Upto 2 decimal places)

FAQ Have any Doubt ?

0.13 (0.11 - 0.16) Correct Option

Solution :

0.13 (0.11 - 0.16)

Consider 'A' be an event that product break down in 3rd year and

So,

$$\begin{aligned} P(B) &= P(T \geq 2) \\ &= e^{-2/4} \\ P(A) &= P(2 \leq T \leq 3) \\ &= P(T \geq 2) - P(T \geq 3) \\ &= e^{-2/4} - e^{-3/4} \\ &= e^{-1/2} - e^{-3/4} \\ &= 0.134 \text{ (approx.)} \end{aligned}$$

QUESTION ANALYTICS

Q. 17

Consider there are 3 true coins and 1 false coin with tail on both sides. A coin is chosen at random and tosses 4 times. If tail occurs all the 4 times, then the probability that false coin is chosen is ________. (Upto 2 decimal places)

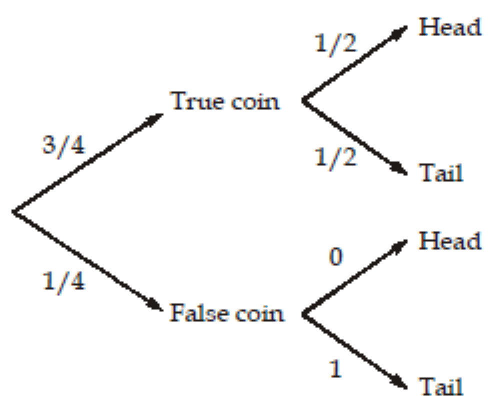
Have any Doubt ?

0.84 (0.84 - 0.85) Correct Option

Solution :

0.84 (0.84 - 0.85)

According to Bayes theorem:



$$\text{So, probability of obtaining tail} = \frac{1}{4}\times 1+\frac{3}{4}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$

$$= \frac{1}{4}+\frac{3}{4}\times\frac{1}{16}$$

$$= \frac{1}{4}+\frac{3}{64}$$

$$= \frac{16+3}{64} = \frac{19}{64}$$

$$\text{So, P(False coin/Tail on 4 tosses)} = \frac{\frac{1}{4}\times 1}{\frac{19}{64}} = \frac{\frac{1}{4}}{\frac{19}{64}}$$

$$= \frac{64}{19\times 4} = \frac{16}{19}$$

= 0.842

QUESTION ANALYTICS