What characteristics impact car fuel consumption?

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Project for the "Regression Models" course at Coursera, January 2015 session. Source at: https://github.com/jmcastagnetto/regmod-jan2015-project/blob/master/project-regmod-jan2015-jmcastagnetto.Rmd

Note: This is an edited version of the Rmarkdown version, mainly due to the inability of IPython/Jupyter to grok latex label references, or embedding R code in between the markdown text. Compare the PDF generated from this notebook with the one generated from the Rmarkdown document.

1 Executive Summary

If we consider only the type of transmission, on average, manual trumps over automatic (24.39 mpg vs 17.15 mpg), with a difference that is statistically significant (p-value < 0.01, and confidence interval that does not contain zero). This is confirmed by a t-test and also the corresponding simple regression model "`mpg ~ am`" (Model 1). Nevertheless, this model only explains ~36% of the variance in fuel consumption. Further analysis produces a better model, where the fuel usage depends on: the type of transmission, the car weight, the engine's number of cylinders, and the engine power ("`mpg ~ cyl + hp + wt + am`", Model 2). This second model is capable of explaining ~86.6% of the variance in the data.

2 Exploratory data analysis

The `mtcars` dataset [1], contains information on 32 cars (1973-1974 models) and 11 characteristics recorded for each one. I decided to distinguish between numeric and factor variables, and recode the latter as such:

```
In [55]: library(ggplot2)
         library(dplyr)
         library(pander)
         library(xtable)
```

```
In [56]: data(mtcars)
         mtcars$cyl <- factor(mtcars$cyl)
         mtcars$vs <- factor(mtcars$vs, levels=c(0, 1), labels=c("V-engine", "Straight"))
         mtcars$am <- factor(mtcars$am, levels=c(0, 1), labels=c("Automatic", "Manual"))
         mtcars$gear <- factor(mtcars$gear)
         mtcars$carb <- factor(mtcars$carb)

         # correlation with mpg
         cmpg <- cor(mtcars[, c("mpg", "disp", "hp", "drat", "wt","qsec")])[1,]

         # comparison auto/manual
         mpg.auto <- subset(mtcars, am=="Automatic")$mpg
         mpg.manual <- subset(mtcars, am=="Manual")$mpg
```

[1] Henderson, H.V. and Velleman, P.F. (1981), "Building multiple regression models interactively". *Biometrics*, 37, 391-411.

```
mpg.summ <- mtcars %>% group_by(am) %>%
    summarise(Median=median(mpg),
              Avg=mean(mpg),
              SD=sd(mpg),
              Min=min(mpg),
              Max=max(mpg))
names(mpg.summ) <- c("Transm.", names(mpg.summ)[-1])

# t-test
t1 <- t.test(mpg.manual, mpg.auto)
```

Plots of `mpg` vs factor variables (Fig. 1), indicate that there are differences in fuel consumption between the different classes, e.g. cars with manual transmission seem to fare better than the ones with automatic. Similar trends are observed for those with 4 cylinder motors, and straight engines, which seem to have better mileage than their corresponding counterparts.

If we now look at plots of the numeric variables (Fig. 2), we observe strong negative correlations between `mpg` and `disp` (-0.85), `hp` (-0.78) and `wt` (-0.87), i.e. heavy cars with big and powerful engines consume more, which intutively makes sense. Positive correlations are lower in magnitude and occur between `mpg` and `drat` (0.68) or `qsec` (0.42).

3 An initial model: Fuel consumption as a function of car's transmission type

In Fig. 3(a) we can see that there is a distinctive improvement in fuel usage for cars with manual transmission. The observed mean difference is 7.24 mpg, and a t-test (*vide infra*), gives results that are statistically significant: a p-value < 0.01, and a 95% confidence interval ([3.21, 11.28]) that does not include zero.

```
In [57]: pander(t1, caption="t-Test results: automatic vs manual", style="grid")
```

```
+------------------+-------+----------------+-------------------------+
|  Test statistic  |  df   |    P value     |  Alternative hypothesis |
+==================+=======+================+=========================+
|      3.767       | 18.33 | _0.001374_ * * |        two.sided        |
+------------------+-------+----------------+-------------------------+

Table: t-Test results: automatic vs manual
```

The simple regression model: "`mpg ~ am`", gives us coefficients that are statistically significant (see Table below): p-values < 0.01 as well as reasonable confidence intervals. In fact β_1 (=7.24) is, as expected, equal to the difference of the means calculated earlier, and indicates us that on average, there is an *improvement of 7.24 mpg* for cars with manual transmission. But this simple model only explains about 36% of the variance at best ($R^2 = 0.3598$, adjusted-$R^2 = 0.3385$)

```
In [58]: model1 <- lm(mpg ~ am, data=mtcars)
         model1.summ <- summary(model1)
         model1.table <- cbind(as.data.frame(model1.summ$coefficients),
                               as.data.frame(confint(model1)))
         pander(model1.table, caption="Linear model 1: `mpg ~ am`",
                split.tables=Inf, style="grid")
```

```
+-------------------+-----------+-------------+----------+-----------+--------+---------+
|              |  Estimate |  Std. Error |  t value |  Pr(>|t|) |  2.5 % |  97.5 % |
+===================+===========+=============+==========+===========+========+=========+
|  **(Intercept)**  |   17.15   |    1.125    |   15.25  | 1.134e-15 |  14.85 |  19.44  |
```

amManual	7.245	1.764	4.106	0.000285	3.642	10.85

Table: Linear model 1: ‘mpg ~ am‘

Diagnostic plots for this model (Fig. 3) indicate that the assumption of normality is warranted (Q-Q plot (a)), as well as the expected distribution of residuals vs predicted values for factor variables[2].

4 Finding a model that considers the effect of other variables

In order to simplify the generation of models, I used a stepwise model selection procedure, employing the algorithms implemented in R's `step()` function. The starting point was a saturated model (i.e. `mpg` vs the rest) not including interactions. In the end, the best model (selected by Akaike's Information Criterion, AIC) has the form: "`mpg ~ cyl + hp + wt + am`", containing 2 factors (number of cylinders and type of transmission) and 2 numeric (weight and power) variables. This expanded model explains at most 86.6% of the variance ($R^2 = 0.8659$, adjusted-$R^2 = 0.8401$). The model coefficients are listed in the table below.

```
In [59]: model.all <- lm(mpg ~ ., data=mtcars)
         model2 <- step(model.all, direction="both", trace=0)
         model2.summ <- summary(model2)
         model2.table <- cbind(as.data.frame(model2.summ$coefficients),
                               as.data.frame(confint(model2)))
         form2 <- as.character(formula(model2))
         form2.char <- paste(form2[2], form2[1], form2[3])
         pander(model2.table, caption=paste0("Linear model 2: ‘", form2.char,"‘"),
                split.tables=Inf, style="grid")
```

	Estimate	Std. Error	t value	Pr(>|t|)	2.5 %	97.5 %
(Intercept)	33.71	2.605	12.94	7.733e-13	28.35	39.06
cyl6	-3.031	1.407	-2.154	0.04068	-5.924	-0.1386
cyl8	-2.164	2.284	-0.9472	0.3523	-6.859	2.532
hp	-0.03211	0.01369	-2.345	0.02693	-0.06025	-0.003964
wt	-2.497	0.8856	-2.819	0.009081	-4.317	-0.6765
amManual	1.809	1.396	1.296	0.2065	-1.061	4.679

Table: Linear model 2: ‘mpg ~ cyl + hp + wt + am‘

In this model, the positive effect of the car's transmission is diminished ($\beta_1 = 1.81$) with respect to the simpler model, and instead negative effects appear due to the car's weight, the number of cylinders (related to engine size, perhaps), and (to a lesser degree) engine power. An ANOVA (see table below) comparing the two models indicate that the second model is indeed a significant improvement (p-value < 0.001) over the simple one discussed earlier.

```
In [60]: anova.m1.m2 <- anova(model1, model2)
         pander(cbind(Models=c("mpg ~ am", "mpg ~ cyl + hp + wt + am"), anova.m1.m2),
```

[2]http://www.itl.nist.gov/div898/handbook/pri/section2/pri24.htm

```
                caption="Comparison of the simple and extended linear models",
                split.tables=Inf, style="grid")
```

```
+--------------------------+----------+-------+------+-------------+-------+-----------+
|          Models          |  Res.Df  |  RSS  |  Df  |  Sum of Sq  |   F   |  Pr(>F)   |
+==========================+==========+=======+======+=============+=======+===========+
|         mpg ~ am         |    30    | 720.9 |      |             |       |           |
+--------------------------+----------+-------+------+-------------+-------+-----------+
| mpg ~ cyl + hp + wt + am |    26    |  151  |  4   |    569.9    | 24.53 | 1.688e-08 |
+--------------------------+----------+-------+------+-------------+-------+-----------+

Table: Comparison of the simple and extended linear models
```

This model tells us that, keeping all other variables constant, we now expect to have in *improvement of about 1.81 mpg* for manual over automatic. Considering the car's weight, there is an expected loss (assuming all other variables constant) of -2.5 mpg for each 1000 lb increase. A smaller loss of -0.03 mpg is expected for each HP increase in power. Finally, we expect also a decrease in mileage when comparing 4-cylinder cars with those with 6-cylinder (-3.03 mpg) and 8-cylinders (-2.16 mpg)

Diagnostic plots for this model (Fig. 4) indicate that in general the assumption of normality can be accepted, even though there is a slight deviation from ideal in the Q-Q plot (a). Also, as expected there is a random distribution of residuals when plotted agains the predicted values (b).

5 Appendix:

Figure 1: Variation of `mpg` vs factor variables

```
In [61]: par(mfrow=c(2,3))
         boxplot(mpg ~ am, mtcars, col=c("lightgreen", "cyan"),
                 ylab="Miles per gallon", xlab="Transmission type", sub="(a)")
         boxplot(mpg ~ vs, mtcars, col=c("lightgreen", "cyan"),
                 ylab="Miles per gallon", xlab="Engine type", sub="(b)")
         boxplot(mpg ~ cyl, mtcars, col=c("lightgreen", "cyan", "yellow"),
                 ylab="Miles per gallon", xlab="Number of Cylinders", sub="(c)")
         boxplot(mpg ~ gear, mtcars, col=c("lightgreen", "cyan", "yellow"),
                 ylab="Miles per gallon", xlab="Number of Gears", sub="(d)")
         boxplot(mpg ~ carb, mtcars, col=c("lightgreen", "cyan", "yellow", "red", "maroon", "grey"),
                 ylab="Miles per gallon", xlab="Number of Carburators", sub="(e)")
         par(mfcol=c(1,1))
```

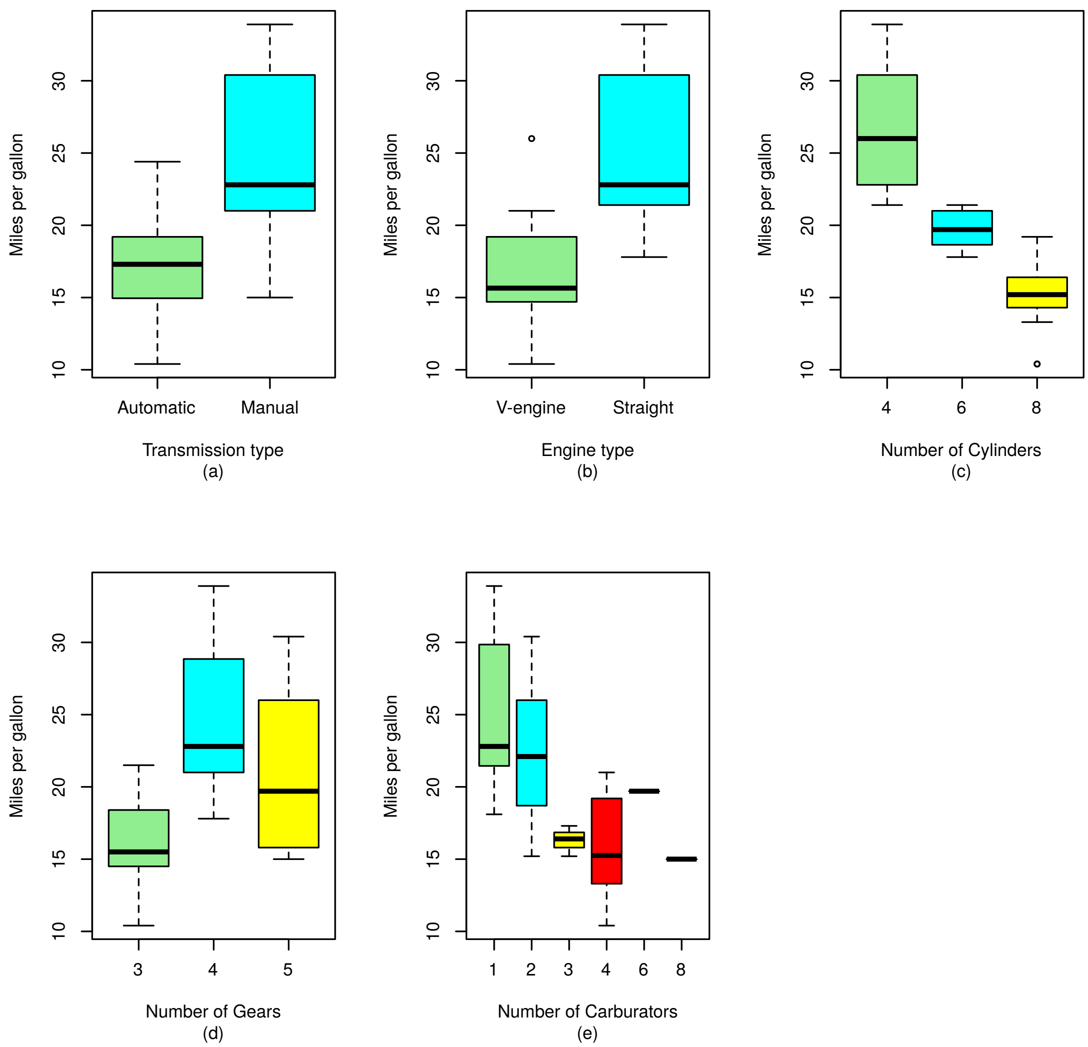


Figure 2: Scatterplots of `mpg` vs numeric variables (including correlation)\label{fig:mpg-num}

In [62]:
```
library(car)
panel.cor <- function(x, y, digits = 2, prefix = "", cex.cor, ...){
     usr <- par("usr"); on.exit(par(usr))
     par(usr = c(0, 1, 0, 1))
     r <- cor(x, y)
     txt <- paste0(prefix, round(r, digits))
     text(0.5, 0.5, txt, cex = 3 * abs(r), col=ifelse(r < 0, "red", "blue"))
}
spm(mtcars[,c("mpg", "disp", "hp", "drat", "wt","qsec")], smoother=FALSE,
    cex.labels=1.5, upper.panel=panel.cor)
```

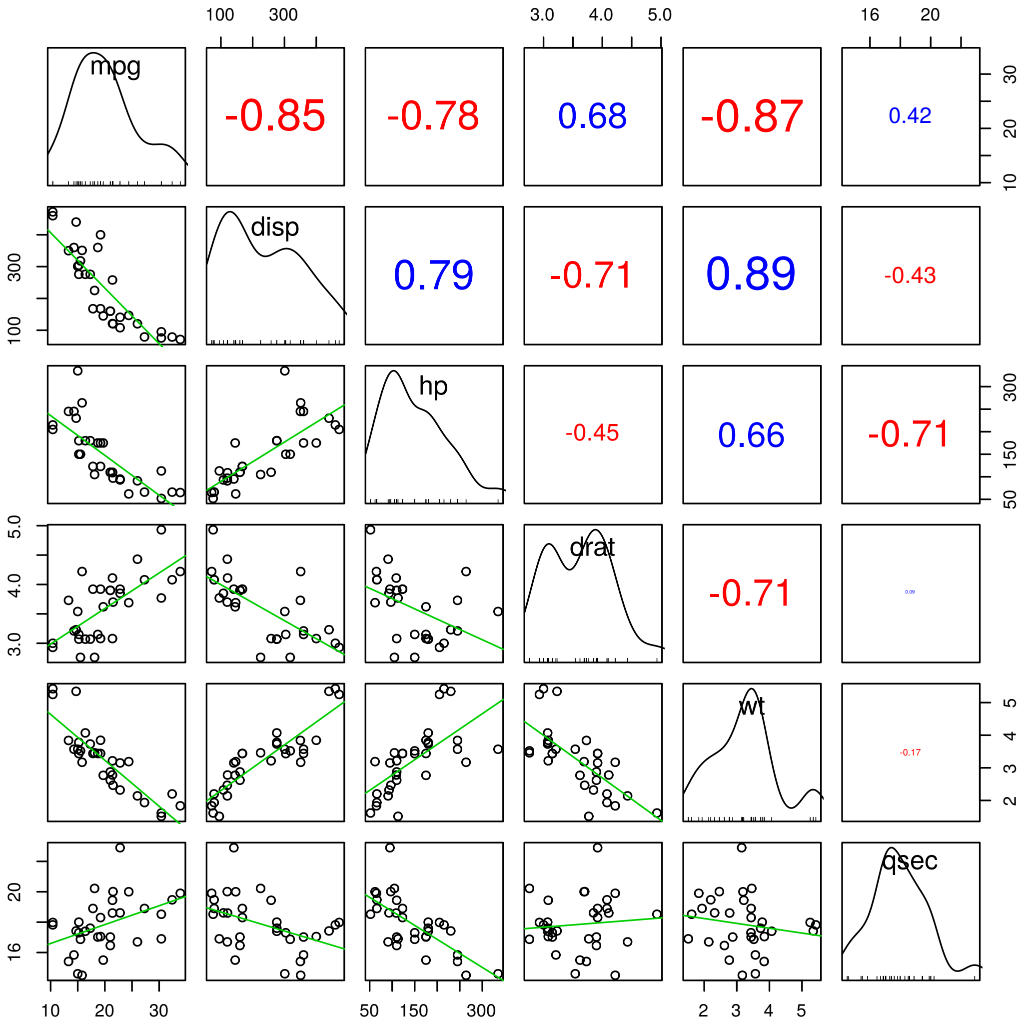


Figure 3: Diagnostic plots for model 1: `mpg ~ am` (a) Q-Q plot (black: automatic, red: manual) (b) Residual vs Predicted plot (black: automatic, red: manual, blue: linear regression of residuals vs predicted)

```
In [63]: par(mfrow=c(1,2))
         set.seed(123)
         qqp1 <- qqPlot(model1, main="(a)", id.n=4,
                        ylab="Studentized Residuals",
                        pch=19, cex=0.8, id.cex=0.6, id.col="grey", col.lines="cyan",
                        col=mtcars$am)
         plot(resid(model1) ~ predict(model1), main="(b)",
                        ylab="Residuals", xlab="Predicted value (mpg)",
                        pch=1, cex=0.8, col=mtcars$am)
         abline(lm(resid(model1) ~ predict(model1)), lwd=2, col="blue")
         par(mfrow=c(1,1))
```

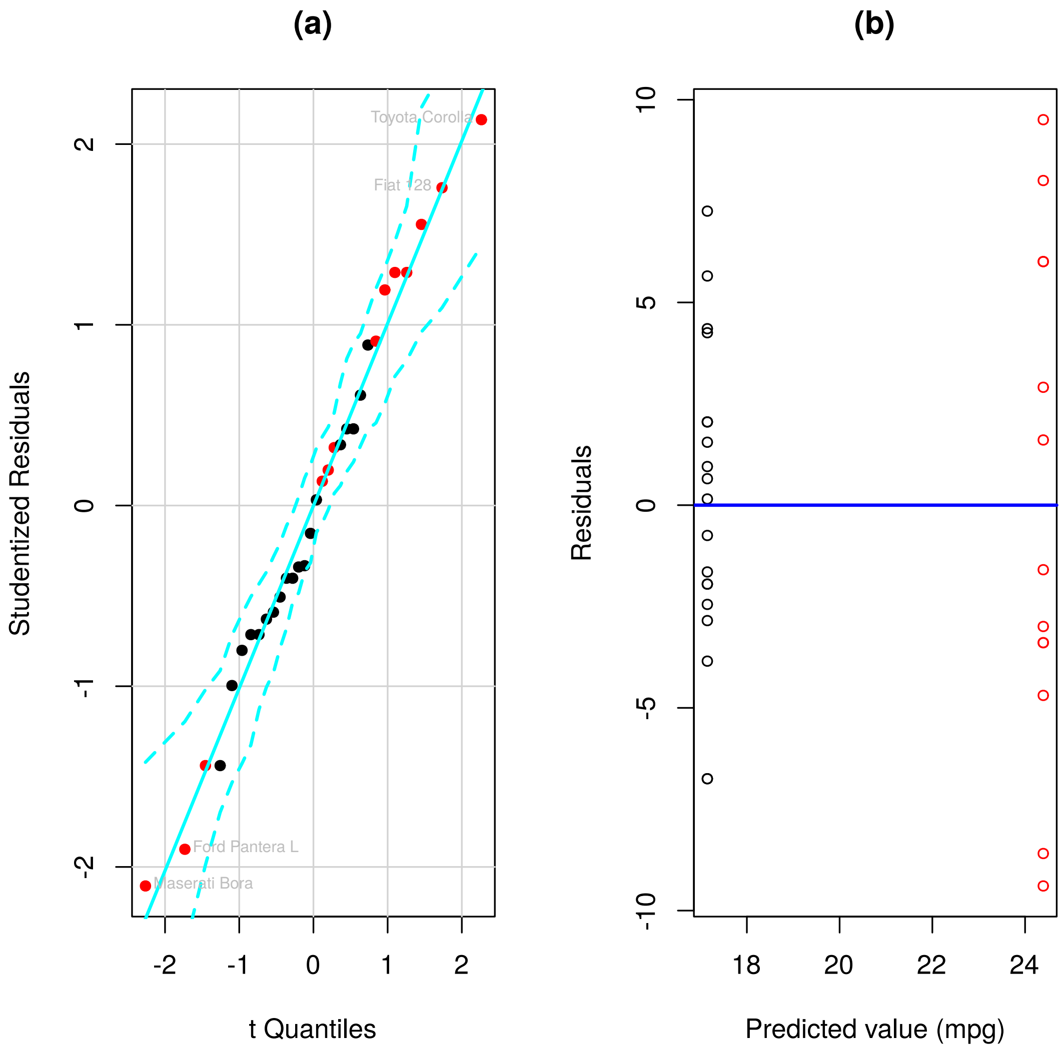
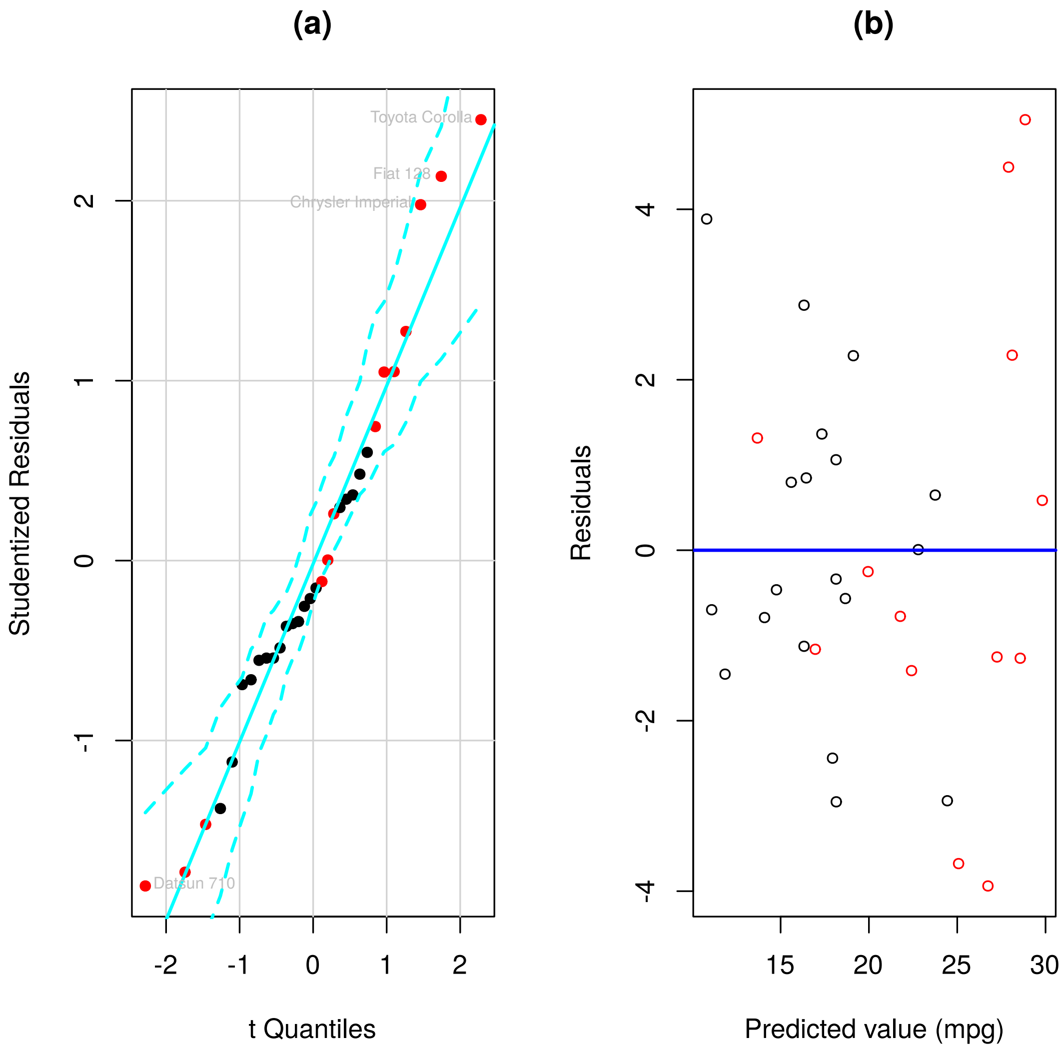


Figure 4: Diagnostic plots for model 2: `mpg ~ cyl + hp + wt + am` (a) Q-Q plot (black: automatic, red: manual) (b) Residual vs Predicted plot (black: automatic, red: manual, blue: linear regression of residuals vs predicted)

```
In [64]: par(mfrow=c(1,2))
         set.seed(123)
         qqp2 <- qqPlot(model2, main="(a)", id.n=4,
                        ylab="Studentized Residuals",
                        pch=19, cex=0.8, id.cex=0.6, id.col="grey", col.lines="cyan",
                        col=mtcars$am)
         plot(resid(model2) ~ predict(model2), main="(b)",
                        ylab="Residuals", xlab="Predicted value (mpg)",
                        pch=1, cex=0.8, col=mtcars$am)
         abline(lm(resid(model2) ~ predict(model2)), lwd=2, col="blue")
         par(mfrow=c(1,1))
```



(a)
(b)
Toyota Corolla
Fiat 128
Chrysler Imperial
Datsun 710
Studentized Residuals
t Quantiles
Residuals
Predicted value (mpg)