Introduction

We assume that the definitions of the following are known.

1. T_n, the nth Chebyshev polynomial of the first kind.
2. U_n, the nth Chebyshev polynomial of the second kind.
3. f_n, the nth Fibonacci number.

Problem Sheet 2

1. Show that
$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x).$$
when $n \geq 2$ and $T_0(x) = 1, T_1(x) = x$.
2. Show that
   (a) $T_n(1) = 1$ and
   (b) $T_n(-1) = (-1)^n$.
3. Show that
   (a) $U_n(1) = n + 1$.
   (b) $U_n(-1) = (-1)^n(n + 1)$
4. Show that
$$\frac{1}{\iota^n} U_n(\iota/2) = f_{n+1}.$$
5. Show that if $m, n \geq 1$,
$$T_{m+n}(x) = T_m(x)U_n(x) - T_{m-1}(x)U_{n-1}(x).$$
6. Similar to the tiling combinatorial model for $U_n(x)$, get a combinatorial model for the polynomial $T_n(x)$.