
MA 412: COMPLEX ANALYSIS TUTORIALS

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§1. Assignment 4

Exercise 1.1. Let $\gamma : [a, b] \rightarrow \mathbb{R}$ be non-decreasing. Show that γ is of bounded variation and $V(\gamma) = \gamma(b) - \gamma(a)$.

Solution

For any partition $P = \{a = t_0 < t_1 < \cdots < t_m = b\}$ of $[a, b]$,

$$\begin{aligned} v(\gamma; P) &= \sum_{k=1}^m |\gamma(t_k) - \gamma(t_{k-1})| \\ &= \sum_{k=1}^m \gamma(t_k) - \gamma(t_{k-1}) \\ &= \gamma(t_m) - \gamma(t_0) = \gamma(b) - \gamma(a). \end{aligned}$$