Sum-of-squares Exercises

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§1. Homework 1

Exercise 1.1. Let A be symmetric and $f(x) = x^\top Ax$. Prove that $\|A\|_2\|x\|_2^2 - f(x)$ is a degree 2 sum-of-squares polynomial.

Solution

We have

$$\|A\|_2\|x\|_2^2 - f(x) = x^\top(\|A\|_2\mathbb{I} - A)x,$$

and the matrix $\|A\|_2\mathbb{I} - A$ is PSD by the definition of $\|A\|_2$, so we are done.

Exercise 1.2.