Maps Between Topological Spaces

Def Continuity
Let X and Y be topological spaces. A function $f: X \to Y$ is said to be continuous at $b \in X$ if for any open $V \subseteq Y$ with $f(b) \in V$, there exists open $U \ni b$ (in X) such that $f(U) \subseteq V$.
f is continuous if for any open V in Y, $f^{-1}(V)$ is open in X.

Note that f is continuous iff it is continuous at all $b \in X$.
(How? Use the fact that an arbitrary union of open sets is open)

Recall that this is equivalent to the usual definition of continuity for metric spaces (taking the metric topology here).

Since the topologies matter as well, note that even the identity map from $\mathbb{R}_c$ to $\mathbb{R}$ is not continuous.

If the topology of Y is given by a basis $\mathcal{B}$ and we want to determine continuity, it suffices to check the pre-images of basis elements of Y.
Indeed, use the fact that an arbitrary union of open sets is open.

Further, it suffices to just check subbasis elements!
Indeed, the set of finite intersections of subbasis elements form a basis.
(and a finite intersection of open sets is open)