Coordinate Hit-and-run

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Given a well-guaranteed membership oracle for K, the problem is to approximately uniformly sample points for K.

The approach in broad strokes

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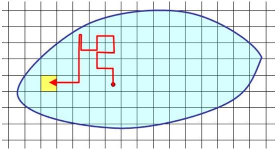

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Martin Dyer, Alan Frieze, and Ravi Kannan.

A random polynomial-time algorithm for approximating the volume of convex bodies.

J. ACM, 38(1):1–17, January 1991.