Dirac Notation (Used for describing Quantum states)

$a, b \in \mathbb{C}^2$

1) KET: $|a\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$

2) BRA: $\langle b| = |b\rangle^\dagger = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}^\dagger = \begin{pmatrix} b_1^* & b_2^* \end{pmatrix}$

3) BRA-KET: $\langle b|a\rangle = a_1 b_1^* + a_2 b_2^*$

$= \langle a|b\rangle^* \in \mathbb{C}$

4) KET-BRA: $|a\rangle\langle b| = \begin{pmatrix} a_1 b_1^* & a_1 b_2^* \\ a_2 b_1^* & a_2 b_2^* \end{pmatrix}$

States: $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$,

which are orthogonal:

$\langle 0|1\rangle = \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$= 1.0 + 0.1 = 0$

→ All quantum states are normalized!

$$\langle \psi | \psi \rangle = 1$$

Eg. $|\psi\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 1/\sqrt{2} \end{pmatrix}$

↑ for the normalization constraint ($\langle \psi | \psi \rangle = 1$)