Teleportation

Goal: Alice wants to send her (unknown) state $|\phi\rangle_S = \alpha|0\rangle_S + \beta|1\rangle_S$ to Bob. She can only send him two classical bits though.

They both share the maximally entangled state: $|\psi^{00}\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB})$

Initial state of the total system

$$|\phi\rangle_S \otimes |\psi^{00}\rangle_{AB} = \frac{1}{\sqrt{2}}\left(\alpha|000\rangle_{SAB} + \alpha|011\rangle_{SAB} + \beta|100\rangle_{SAB} + \beta|111\rangle_{SAB}\right)$$

$$= \frac{1}{2\sqrt{2}}\Big[\left(|00\rangle_{SA} + |11\rangle_{SA}\right) \otimes \left(\alpha|0\rangle + \beta|1\rangle\right)$$

$$+ \left(|01\rangle_{SA} + |10\rangle_{SA}\right) \otimes \left(\alpha|1\rangle_B + \beta|0\rangle_B\right)$$

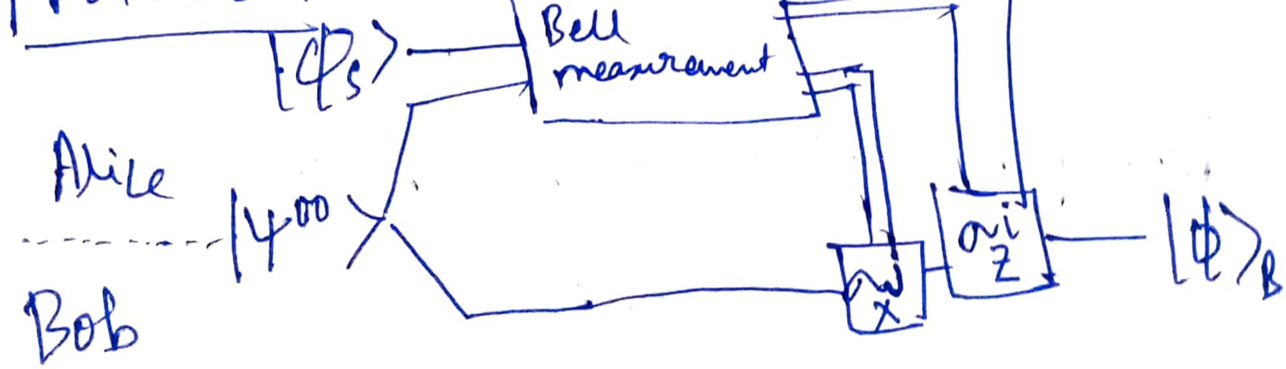
$$+ \left(|00\rangle_{SA} - |11\rangle_{SA}\right) \otimes \left(\alpha|0\rangle_B - \beta|1\rangle_B\right)$$

$$+ \left(|01\rangle_{SA} - |10\rangle_{SA}\right) \otimes \left(\alpha|1\rangle_B - \beta|0\rangle_B\right)\Big]$$

$$= \frac{1}{2}\Big[|\psi^{00}\rangle_{SA} \otimes |\phi\rangle_B + |\psi^{01}\rangle_{SA} \otimes \sigma_x |\phi\rangle_B$$

$$+ |\psi^{10}\rangle_{SA} \otimes \sigma_z |\phi\rangle_B + |\psi^{11}\rangle_{SA} \otimes \sigma_x \sigma_z |\phi\rangle_B\Big]$$

PROTOCOL:



Steps:

1) Alice measures on ~~S~~ A in Bell Basis

Alice's measurement → Bob's state

$|\psi^{00}\rangle \rightarrow |\phi\rangle_B$

$|\psi^{01}\rangle \rightarrow \sigma_x|\phi\rangle_B$

$|\psi^{10}\rangle \rightarrow \sigma_z|\phi\rangle_B$

$|\psi^{11}\rangle \rightarrow \sigma_x\sigma_z|\phi\rangle_B$

2) She sends her classical outputs i, j to Bob

Alice sends

i,	j
0	0
0	1
1	0
1	1

3) Bob applies $\sigma_z^i \sigma_x^j$ to his qubit and gets $|\phi\rangle$

Alice	Bob	
0 0	I	$\lvert\phi\rangle_B$
0 1	σ_x	$\lvert\phi\rangle_B$
1 0	σ_z	$\lvert\phi\rangle_B$
1 1	$\sigma_z \sigma_x$	$\lvert\phi\rangle_B$

Alice's state collapses during the measurement, so she does not have the initial state $\lvert\phi\rangle$ ~~any~~ now.

No Cloning Theorem (cannot copy state, collapse and send)
destroy.

Q-Sphere

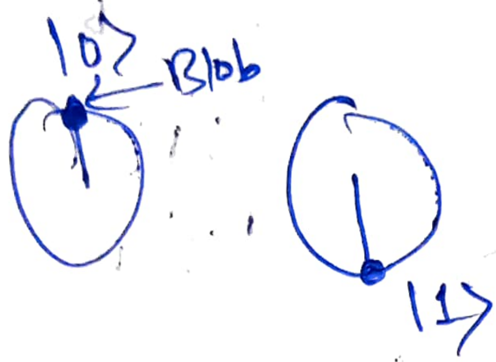
Bloch sphere can only illustrate the state of 1 qubit.

For multiple qubits → Q-sphere.

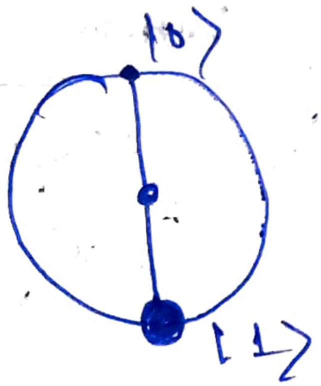
For one qubit : North pole : $\lvert 0\rangle$
South pole : $\lvert 1\rangle$

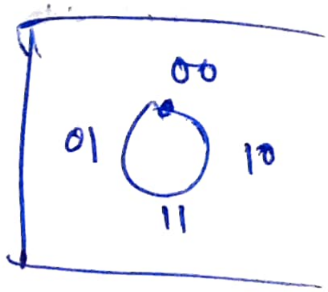
The size of blob is proportional to the probability of measuring the respective state.

- There are colors : The color indicates the relative phase compared to state $|0\rangle$.

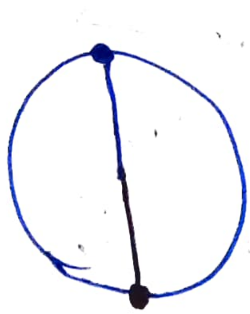


$$\frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}\,|1\rangle$$





$$(|0\rangle - |1\rangle)/\sqrt{2} \qquad e^{i\phi} = -1 \ (\text{phase} = \pi)$$



For n qubits 2^n Basis States

$n=3$

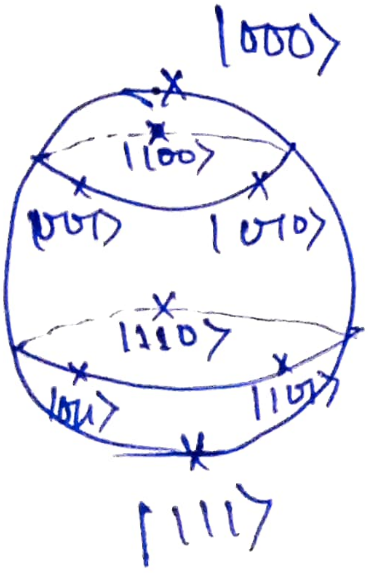
Basis: 000
001, 010, 100
011, 101, 110
111

} 8 (Equally distributed on Q-sphere)

Northpole: $0^{\otimes n}$ South pole: $1^{\otimes n}$.

All other states are aligned on parallels such that the number of '1's' on each latitude is constant and increasing from North to South.

Eg1
$n=3$



Eg 2

$$\frac{1}{2}\left(|000\rangle - |011\rangle + \sqrt{2}\, i \cdot |101\rangle\right)$$