BLOCH SPHERE

We can write any normalized (pure) state as

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

(General)

$\phi \in [0, 2\pi) \leftarrow$ describes the relative phase

$\theta \in [0, \pi] \leftarrow$ ~~determines~~ determines the probability to measure $|0\rangle / |1\rangle$

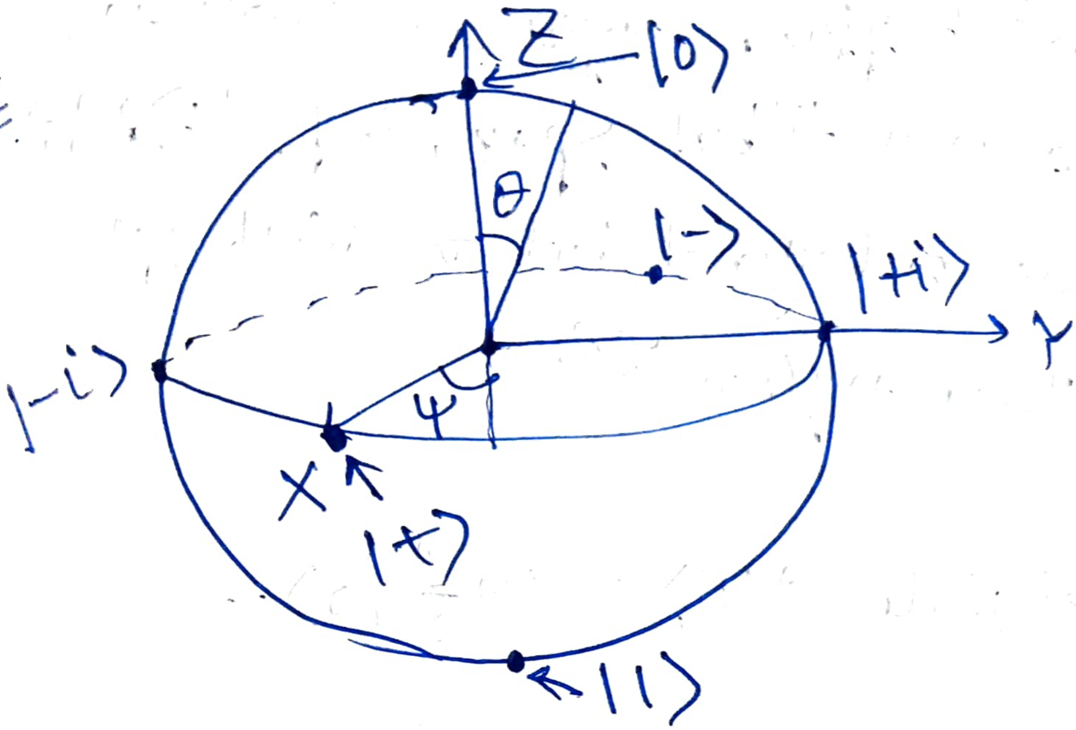
$$P(0) = \cos^2\frac{\theta}{2} \qquad P(1) = \sin^2\frac{\theta}{2}$$

→ All normalized pure states can be illustrated on the surface of a sphere with radius $|\vec{r}| = 1$, which is called Bloch sphere.

→ The coordinates of such a state are given by the Bloch vector.

$$\vec{r} = \begin{pmatrix} \sin\theta\cos\theta \\ \sin\theta\sin\psi \\ \cos\theta \end{pmatrix}$$

E.g.



$|0\rangle$: From general state: $\theta = 0$

φ arbitrary $\Rightarrow \vec{r} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$|1\rangle$: $\theta = \pi$, φ arbit. $\Rightarrow \vec{r} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$

$|+\rangle$: $\theta = \pi/2$, $\varphi = 0$, $\vec{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$|-\rangle$: $\theta = \pi/2$, $\varphi = \pi$, $\vec{r} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$

$|+i\rangle$: $\theta = \pi/2$, $\varphi = \pi/2$, $\vec{r} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$|-i\rangle$: $\theta = \pi/2$, $\varphi = 3\pi/2$, $\vec{r} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$

On the Bloch sphere, angles are twice as ~~large~~ big as in Hilbert Space, e.g. $|0\rangle$ & $|1\rangle$ are orthogonal but on Bloch sphere, their angle is 180°.

For general state $|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + \ldots$

$\theta \rightarrow$ angle on Bloch sphere

$\frac{\theta}{2} \rightarrow$ angle on Hilbert space.

$\rightarrow$

Z-measurement corresponds to the projection onto the z-axis and analogously for X & Y axis.