Entanglement

If a pure state $|\psi\rangle_{AB}$ on systems A & B cannot be written as $|\psi\rangle_A \otimes |\Phi\rangle_B$, it is entangled

Bell States (Maximally Entangled and build an orthonormal basis)

$$|\psi^{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

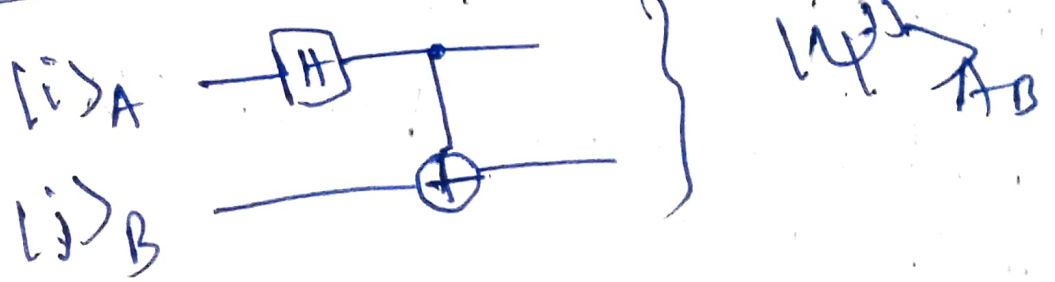
$$|\psi^{01}\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|\psi^{10}\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|\psi^{11}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

In general: $|\psi^{ij}\rangle = (I \otimes \sigma_x^j \sigma_z^i)|\psi^{00}\rangle$

Creation of Bell States



Initial state

$|ij\rangle_{AB}$

$(H_A \otimes I_B)$

$|00\rangle$ → $(|00\rangle + |10\rangle)/\sqrt{2}$

$|01\rangle$ → $(|01\rangle + |11\rangle)/\sqrt{2}$

$|10\rangle$ → $(|00\rangle - |10\rangle)/\sqrt{2}$

$|11\rangle$ → $(|01\rangle - |11\rangle)/\sqrt{2}$

H_A

↓ $CNOT_{A,B}$ ← Acting on

↑ Controlled on

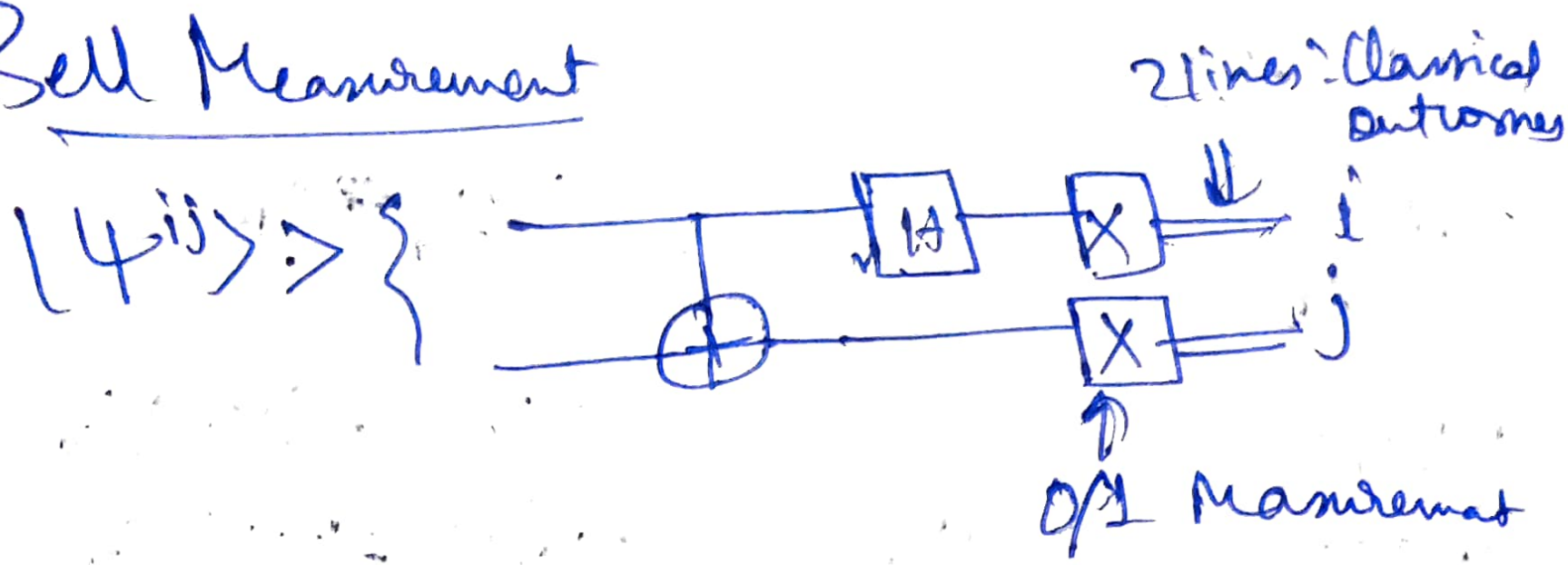
$|\psi^{00}\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$

$|\psi^{01}\rangle = (|01\rangle + |10\rangle)/\sqrt{2}$

$|\psi^{10}\rangle = (|00\rangle - |11\rangle)/\sqrt{2}$

$|\psi^{11}\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$

Bell Measurement



Classical outcomes i, j, ~~[illegible]~~ corresponding to a measurement of state $|\psi^{ij}\rangle$