MEASUREMENTS

→ We choose orthogonal bases

→ Doing a measurement onto the basis $\{|0\rangle, |1\rangle\}$, the state will collapse into either of the states.

either $|0\rangle$ or $|1\rangle$

Why?

Ans: Because those are the eigen states of the σ_z

$\sigma_z \rightarrow$ Z-measurement.

$\rightarrow$ There are infinitely many bases, but common bases are (other than orthogonal):

$$\left\{ |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle),\ |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \right\} \text{ and}$$

$$\boxed{\langle + | - \rangle = 0 \text{ (Orthogonal)}}$$

$$\underbrace{\left\{ |+i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle),\ |-i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle) \right\}}$$

$\Downarrow$

Eigenstates of σ_x, σ_y, respectively.

BORN RULE:

The probability that a state $|\psi\rangle$ collapses during a projective measurement onto the basis

$\{(\underbrace{|x\rangle, |x^{+}\rangle}_{\text{Orthogonal}})\}$ to

the state $|x\rangle$ is given by:

$$P(x) = |\langle x|\psi\rangle|^2, \qquad \sum_i P(x_i) = 1$$

<u>Eg.</u> 1)

$$|\psi\rangle = \frac{1}{\sqrt{3}}\left(|0\rangle + \sqrt{2}\,|1\rangle\right) \rightarrow \text{measured in the basis } \{|0\rangle, |1\rangle\}$$

$$P(0) = \left| |0\rangle \frac{1}{\sqrt{3}}\left(|0\rangle + \sqrt{2}\,|1\rangle\right)\right|^2$$

$$= \left| \frac{1}{\sqrt{3}} \underbrace{\langle 0|0\rangle}_{1} + \frac{\sqrt{2}}{\sqrt{3}} \underbrace{\langle 0|1\rangle}_{0} \right|^2$$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}$$

$$P(1) = \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2 = \frac{2}{3}$$

2) $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \rightarrow$ measured in basis $\{|+\rangle, |-\rangle\}$

$$P(+) = |\langle +|\psi\rangle|^2$$

$$= \left| \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \right|^2$$

$$= \frac{1}{4} \left| \underbrace{\langle 0|0\rangle}_{1} - \cancel{\langle 0|1\rangle} + \cancel{\langle 1|0\rangle} - \underbrace{\langle 1|1\rangle}_{1} \right|^2$$

$$= 0$$

Since $P(+) = 0$, $P(-) = 1$