COMPSCIX433.6-007- Homework

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Questions for homework 3

Instructions: For this homework use "Questions for homework 3.pdf" that is in the files section of this course. You have to solve the problems by hand and copy paste the pictures of your work in a Jupyter notebook> Make sure to convert the Jupyter notebook into a PDF file and then upload the PDF file. If you don't convert the Jupyter notebook to PDF then I will not be able to see your work and I will not be able to give you any grade.

1) Use the below table to find the following:
   a) Probability that a car is Yellow given that it is stolen
   b) Probability that a car is Red given that it is not stolen.

Example No.	Color	Type	Origin	Stolen?
1	Red	Sports	Domestic	Yes
2	Red	Sports	Domestic	No
3	Red	Sports	Domestic	Yes
4	Yellow	Sports	Domestic	No
5	Yellow	Sports	Imported	Yes
6	Yellow	SUV	Imported	No
7	Yellow	SUV	Imported	Yes
8	Yellow	SUV	Domestic	No
9	Red	SUV	Imported	No
10	Red	Sports	Imported	Yes

2) Find the eigen values and eigen vectors of A = [[3, 5], [3, 1]].

Mathematical Formula for Probability Caculation is as following

LaTeX equations

Question1.a: Probability that the car is Yellow given that it is stolen

We want to check the claim: Car is Yellow Given it is stolen.

$$P(Yellow|Stolen) = \frac{P(Stolen|Yellow)*P(Yellow)}{P(Stolen)}$$

From the table we know

$$p(Stolen) = 5/10 = 0.5$$

$$p(Yellow) = 5/10 = 0.5$$

Conditional probability of Stolen given Yellow is

$$P(Stolen|Yellow) = \frac{P(Stolen \cap Yellow)}{P(Stolen)}$$

$$P(Stolen|Yellow) = (2/10)/(5/10)$$
$$p(Stolen|Yellow) = 2/5 = 0.40$$

let's plug these values into the Bayes equation.

$$P(Yellow|Stolen) = \frac{0.40*0.50}{0.50} = 0.40 = 40.00$$

This probability is 40%.

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Question 1

Convert data to below table:-

Color	stolen	Not stolen	Total	Probab.
Red	3	2	5	5/10
yellow	2	3	5	5/10
Total	5	5	10	
Probab.	5/10	5/10		

a) Probability that the car is yellow given that it is stolen.

$$P(\text{yellow}|\text{stolen}) = \frac{P(\text{stolen}|\text{yellow}) \times P(\text{yellow})}{P(\text{stolen})}$$

$P(\text{yellow}/\text{stolen}) =$

$P(\text{yellow}) = 5/10$ ↳ ① $P(\text{stolen}) = 5/10$ ↳ ②

$$P(\text{stolen}/\text{yellow}) = \frac{P(\text{stolen} \cap \text{yellow})}{P(\text{yellow})} = \frac{2/\cancel{10}}{5/\cancel{10}} = \frac{2}{5} \quad -(3)$$

$$P(\text{yellow}|\text{stolen}) = \frac{2/5 \times \cancel{5/10}}{\cancel{5/10}} = \boxed{0.40} \text{ or } \boxed{40\%}$$

LaTeX equations

Question1.b: Probability that the car is red given that it is not stolen

We want to check the claim: car is red given that it is not stolen

$P(Red|NotStolen) = \frac{P(NotStolen|Red)*P(Red)}{P(NotStolen)}$

From the table we know

$p(NotStolen) = 5/10 = 0.5$

$p(Red) = 5/10 = 0.5$

Conditional probability of Not Stolen given Red is

$p(NotStolen|Red) = \frac{P(NotStolen \cap Red)}{P(NotStolen)}$

$p(NotStolen|Red) = (2/10)/(5/10)$
$p(NotStolen|Red) = 2/5 = 0.40$

let's plug these values into the Bayes equation.

$p(Red|NotStolen) = \frac{0.40*0.50}{0.50} = 0.40 = 40.00$

This probability is 40%.

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b) Probability that a car is Red given that it is not stolen.

$$P(\text{Red} \mid \text{Not stolen}) = \frac{P(\text{Not stolen} \mid \text{Red}) \times P(\text{Red})}{P(\text{Not stolen})}$$

$P(\text{Not stolen}) = 5/10$

$P(\text{Red}) = 5/10$

$$P(\text{Not stolen} \mid \text{Red}) = \frac{P(\text{Not stolen} \cap \text{Red})}{P(\text{Red})} = \frac{2/\cancel{10}}{5/\cancel{10}} = 2/5$$

$$P(\text{Red} \mid \text{Not stolen}) = \frac{2/5 \times 5/10}{5/10} = 2/5 \; \boxed{0.40} \text{ or } \boxed{40\%}$$

LaTeX equations

Eigen vectors - v is an eigen vector of A is $Av = \lambda v$ where λ is a scalar. Calculate the eigen values and eigen vectors

$det(A - \lambda I) = 0$

$(A - \lambda I)v = 0$

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Q2|| Find the eigen values and eigen vectors of A $= [[3,5],[3,1]]$

$A = \begin{bmatrix} 3 & 5 \\ 3 & 1 \end{bmatrix}$

$A - \lambda I = 0 = \begin{bmatrix} 3 & 5 \\ 3 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$0 = \begin{bmatrix} 3 & 5 \\ 3 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$0 = \begin{bmatrix} (3-\lambda) & 5 \\ 3 & (1-\lambda) \end{bmatrix}$

$= ((3-\lambda)(1-\lambda)) - (5 \times 3)$

$= (3 - 3\lambda - \lambda + \lambda^2) - 15$

$= 3 - 4\lambda + \lambda^2 - 15$

$= \lambda^2 - 4\lambda - 12$

$= (\lambda - 6)(\lambda + 2)$

so either

$(\lambda - 6) = 0$ or $(\lambda + 2) = 0$ $\Rightarrow$ $\lambda = 6$, $\lambda = -2$

Eigen value of A $\begin{bmatrix} 3 & 5 \\ 3 & 1 \end{bmatrix}$

are

$\boxed{\lambda = 6 \text{ or } -2}$

Eigen value for $\lambda = 6$

$Av = \lambda v$

$Av - \lambda v = 0$

$(A - \lambda) v = 0$

$\begin{bmatrix} 3 & 5 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$\boxed{\lambda = 6}$

$\begin{bmatrix} 3-6 & 5 \\ 3 & 1-6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$\begin{bmatrix} -3 & 5 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$-3x + 5y = 0$

$3x = 5y$

$y = 3/5\, x$

$\boxed{\text{for } x = 1 \quad y = 0.6}$

verify

$\begin{bmatrix} 3 & 5 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \lambda \begin{bmatrix} x \\ y \end{bmatrix} = 6 \begin{bmatrix} x \\ y \end{bmatrix}$

$\begin{bmatrix} 3 & 5 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0.6 \end{bmatrix} = \begin{bmatrix} 6 \\ 3.6 \end{bmatrix}$

which is $= 6 \begin{bmatrix} 1 \\ 0.6 \end{bmatrix} = 6 \begin{bmatrix} 1 \\ 0.6 \end{bmatrix}$

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$\lambda = 2$

$$A = \begin{bmatrix} 3 & 5 \\ 3 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$0 = \begin{bmatrix} 3-\lambda & 5 \\ 3 & 1-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$\boxed{\lambda = -2}$

$$\begin{bmatrix} 5 & 5 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$(5x + 5y)\ (3x + 3y) = 0$$

$$x = -y \quad — (1)$$

$$y = -x \quad — (2)$$

$\boxed{\text{for } x = 1,\ y = -1}$ eigen vector for $\lambda = (-2)$

Verification

$$\begin{bmatrix} 3 & 5 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \lambda x$$

$$\begin{bmatrix} -2 \\ +2 \end{bmatrix} = \lambda x = -2 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$