

# Gamma Function

## Problem 1

### 1 Introduction

**Gamma Function:** It is commonly referred as factorial function for complex numbers. It is derived by Daniel Bernoulli. The gamma function  $\gamma(z)$  is defined for all complex values of  $z$  larger than zero. Complex number can be consist of real and imaginary number, like  $z = a + ib$  in which  $a$  and  $b$  can real numbers. A complex number is typically written in the form where  $a$  is the real part and  $b$  is the imaginary part.

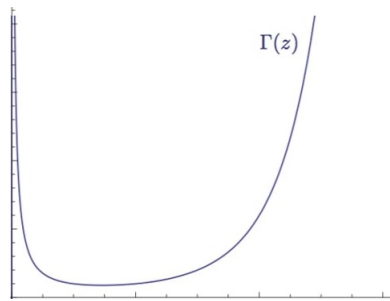


Figure 1: Gamma Function

### 2 Overall Description

This is a project based on gamma function in which we are making calculator for gamma value. User can insert any real value and expect real value except on boundary conditions.

### 3 Stakeholders

**Users 1:** This function is mostly used in physics calculations. So, most important stakeholders are scientists for their calculations.

**Users 2:** This function is also used in basic maths calculations or any analytically field.

### 4 Related to Function

#### 4.1 Formulas

- **Formula1:**  $\Gamma(x) = \int_0^{\infty} s^{x-1} e^{-s} ds \quad \forall \operatorname{Re}(x) > 0$
- **Formula2:**  $\Gamma(1/2) = \sqrt{\pi}$
- **Formula3:**  $n! = n * (n - 1)!$

- **Formula4:**  $\Gamma(x) = x\Gamma(x-1)$
- **Formula5:**  $\Gamma(0) = \text{undefined}$

## 4.2 Popular Constant Values of Function

- **Constant 1:**  $\Gamma(0) = \text{undefined}$
- **Constant 2:**  $\Gamma(1) = 1$
- **Constant 3:**  $\Gamma(2) = 1$
- **Constant 4:**  $\Gamma(3) = 6$
- **Constant 5:**  $\Gamma(3/2) = 0.886$
- **Constant 6:**  $\Gamma(-3/2) = 2.36$
- **Constant 7:**  $\Gamma(-1/2) = -3.54$

## 4.3 Domain of Function

$\forall$  Real numbers excluding negative values  
 $[0, \infty)$

## 4.4 Co domain of Function

- It ranges from  $(0, \infty)$
- For positive integers, we return integer value as normal factorial
- For other real numbers, we use integral function.

# Problem 2

## 5 Requirements/Constraints of Function

### 5.1 Requirements

- **Req1:** For **Large input** in positive value, it will return infinity as **Constraint 3**.
- **Req2:** For **negative input**  $\forall x < 0$ , **Function** will return **input error**, keeping in mind **Constraint 1** and **Constraint 2**
- **Req3:** For  $x = 0$ , **Function** will return **1**, keeping in mind **Constraint 1**
- **Req4:** For  $Re(x) > 0$ , **Function** will return positive real value, keeping in mind **Constraint 1**

### 5.2 Constraints

- **Constraint 1:** For Input, types must be Integer, Double, Float data types
- **Constraint 2:** We cannot input value of **non negative values**
- **Constraint 3:** We cannot input the value large positive number i.e greater than 170 as it will return infinity as a programming language constraint

## Problem 3

### 6 Algorithms

#### 6.1 Pseudo Code 1

##### 6.1.1 Algorithm

This algorithm is based on calculating based on using core integral using graph like dividing whole graph in small parts and calculating each part using formula of trapezium ( $1/2 * (base1 + base2) * height$ ) and combining it at the end.

```
function yAxisValue(Argument x, Argument s) {
    Calculate the value using  $value = s^{x-1}e^{-s}$ 
    return value
end
}
function gammaFunction(Argument x) {
    if x < 0
        then raise Input Error
    if x > 170
        then return "Infinity"

    Initialize finalData with 0
    Set Interval for gap =  $10^{-3}$ 

    while loop i for range(0,Infinity)
        Add the finalData by using formula of trapezium using
         $1/2 * gap * (yAxisValue(i) + yAxisValue(i - gap))$ 
        increment i with gap value
    return finalData
end
}
{
In main function
    Take a input of x
    Call gammaFunction with input x-1 as a argument
end
}
```

##### 6.1.2 Advantages

- Get More Precise Values for input as tested with existing results
- Using basic core approach of integration

##### 6.1.3 Disadvantages

- We are iterating the loop at a large value so it takes time

## 6.2 Pseudo Code 2

### 6.2.1 Algorithm

This algorithm is based on calculating using pre-calculated values which act as a coefficients, iterating and calculating each values using input and coefficients

```
function gammaFunction(Argument x) {
    if x<0
        then raise Input Error
    if x>170
        then return "Infinity"

    declare array of finalArray precalculated values
    temp = x + constant
    Calculate temp value using log of temp value
    declare finalValue with value 1

    while loop i for range(0,length of finalArray)
        finalValue+ = finalArray[index]/ + +temp
    return finalValue
end
}
{
In main function
    Take a input of x
    Call gammaFunction with input x-1 as a argument
end
}
```

### 6.2.2 Advantages

- We are using constant pre-calculated value which increase the speed of algorithm
- This algorithm has less calculation which reduce complexity of code

### 6.2.3 Disadvantages

- We have generated constant value which not always give accurate value

## 6.3 Final Chosen Algorithm

**Pseudo code 1** is chosen finally for implementation over **pseudo code 2** because I am using core implementation of integration using graph which improve the accuracy of over results over using constant coefficient. Pseudo code 2 is undoubtedly fast then than pseudo code 1 but there is no much execution time difference. So overall **Pseudo code 1** is best option

## 7 References

- <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4247832/>
- <https://medium.com/cantors-paradise/the-riemann-hypothesis-explained-fa01c1f75d3f>