Factor-Based Tracking Error Attribution Framework

A Complete Technical Guide to 8-Step Equity TE Decomposition

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30-Stock Active Equity Portfolio

7-Factor Fama-French + Momentum + Volatility Model

82 Monthly Observations | Reference Date: February 2026

Total Tracking Error: 460.4 basis points

Systematic: 277.4 bps (60.3%) | Idiosyncratic: 182.9 bps (39.7%)

1. Executive Summary

This document presents a complete factor-based tracking error attribution for a 30-stock active equity portfolio benchmarked against a diversified equity index. The model uses a 7-factor framework comprising Fama-French five factors (Market, Size, Value, Profitability, Investment), Momentum, and a custom Volatility factor.

Key Findings

Total TE: 460.4 bps, decomposed into Systematic (277.4 bps, 60.3%) and Idiosyncratic (182.9 bps, 39.7%)

Dominant factor exposures: Volatility factor (-0.258 active exposure, 35.0% of TE^2) and Value/HML (-0.127, 23.5%)

NVDA is the single largest security contributor: 303.9 bps (66.0% of total TE)

Top 5 securities account for 107.6% of total TE (offsets from diversifying positions)

Regression quality: Mean Adj R^2 = 0.50, with 26/30 stocks having market beta t-stat > 3.0

2. The 8-Step Pipeline

The attribution engine follows an 8-step pipeline that transforms raw price data and portfolio weights into a complete tracking error decomposition by both factor and security:

Step	Name	Description
Step 1	**Load Data**	Module 1 output: prices, weights, factor returns, fundamentals
Step 2	**Compute Stock Returns**	Convert 85 months of prices to 84 monthly returns
Step 3	**Active Weights**	h = portfolio weight − benchmark weight per security
Step 4	**Factor Regressions**	OLS: (R_i − RF) ~ 7 factors, producing 30×7 beta matrix
Step 5	**Active Factor Exposures**	$f = B^T \times h$ (aggregate factor tilts from active bets)
Step 6	**Factor Covariance**	7×7 covariance matrix from factor return time series
Step 7	**Idiosyncratic Risk**	$\Sigma(h^2 \times \sigma^2_\varepsilon)$ for stock-specific risk
Step 7b	**Security Attribution**	Marginal TE and Contribution to TE per security
Step 8	**Factor Decomposition**	Euler decomposition of TE^2 by factor

3. Steps 1–2: Data Loading and Return Computation

3.1 Data Sources

Module 1 provides a consolidated Excel workbook with 8 sheets: monthly prices for 32 tickers (30 securities + SPLV + SPY) spanning 85 months (February 2019 to February 2026), portfolio and benchmark weights, fundamentals from yfinance, and Fama-French factor returns from the Kenneth French Data Library.

3.2 Return Computation

Monthly returns are computed as simple percentage changes from consecutive month-end prices:

$$R\_t = (P\_t - P\_\{t-1\}) / P\_\{t-1\}$$

85 price observations yield 84 return observations. SPLV and SPY returns are used only for Vol factor construction (SPLV return minus SPY return) and are excluded from the security universe.

4. Step 3: Active Weights

4.1 Definition

The active weight measures the size of the active bet on each security:

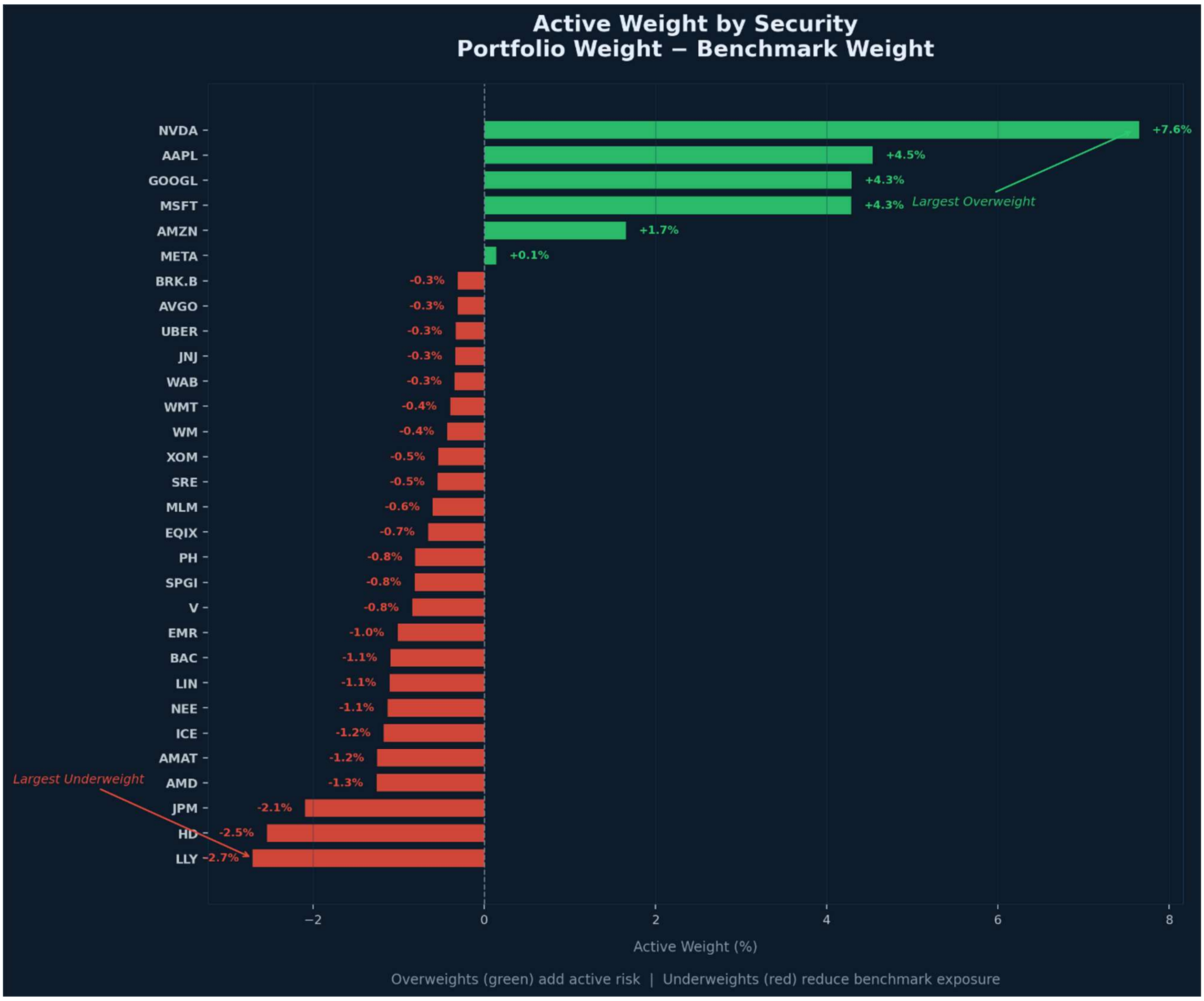
$$h\_i = w\_portfolio\_i - w\_benchmark\_i$$

A positive active weight means the portfolio overweights the security relative to benchmark. A negative active weight means the portfolio underweights it. Securities only in the portfolio have positive h; securities only in the benchmark have negative h. If both portfolio and benchmark weights sum to 1.0, the active weights sum to zero.

4.2 Results from Our Portfolio

Ticker	Active Weight	Note
NVDA	+7.648%	Largest overweight
AAPL	+4.538%	
GOOGL	+4.293%	
MSFT	+4.288%	
AMZN	+1.659%	
META	+0.145%	Negligible
LLY	−2.706%	Largest underweight
HD	−2.535%	
JPM	−2.089%	
AMD	−1.256%	

The portfolio is concentrated in mega-cap tech: NVDA, AAPL, GOOGL, and MSFT together account for +20.7% of active weight. This concentration will have significant implications for both systematic and idiosyncratic tracking error.



Active Weight by Security
Portfolio Weight − Benchmark Weight
NVDA +7.6%
AAPL +4.5%
GOOGL +4.3%
MSFT +4.3%
AMZN +1.7%
META +0.1%
BRK.B -0.3%
AVGO -0.3%
UBER -0.3%
JNJ -0.3%
WAB -0.3%
WMT -0.4%
WM -0.4%
XOM -0.5%
SRE -0.5%
MLM -0.6%
EQIX -0.7%
PH -0.8%
SPGI -0.8%
V -0.8%
EMR -1.0%
BAC -1.1%
LIN -1.1%
NEE -1.1%
ICE -1.2%
AMAT -1.2%
AMD -1.3%
JPM -2.1%
HD -2.5%
LLY -2.7%
Largest Overweight
Largest Underweight
−2
0
2
4
6
8
Active Weight (%)
Overweights (green) add active risk | Underweights (red) reduce benchmark exposure

5. Step 4: Time-Series Factor Regressions

5.1 The Regression Model

For each of the 30 securities, we run an ordinary least squares (OLS) regression of the stock's excess return against 7 factor returns:

$$(R\_i - RF) = \alpha\_i + \beta\_i1\times(Mkt-RF) + \beta\_i2\times SMB + \beta\_i3\times HML + \beta\_i4\times RMW + \beta\_i5\times CMA + \beta\_i6\times Mom + \beta\_i7\times Vol + \varepsilon\_i$$

5.2 Regression Outputs

Beta (β): The slope coefficient for each factor. Measures sensitivity — how much the stock's excess return moves per unit of factor return. A market beta of 1.5 means the stock moves 1.5% for every 1% market move.

Alpha (α): The y-intercept. Return not explained by any factor. Geometrically, it shifts the regression line up (positive alpha) or down (negative alpha) without changing the slope. Alpha is a diagnostic — it does not enter the TE calculation because it is a constant and contributes zero variance.

Residual Variance (σ^2_ε): Variance of the regression residuals. This is the stock-specific risk that factors cannot explain — earnings surprises, management changes, product launches. Enters the TE calculation through Step 7.

R-squared (R^2): Proportion of return variance explained by the 7 factors. Higher R^2 means the factor model captures more of the stock's behaviour; the remainder is idiosyncratic.

t-statistic: Tests whether a beta is statistically significant. $|t| > 2.0$ means the factor exposure is real with ~95% confidence. $|t| < 1.0$ means the exposure could be noise.

5.3 Interpreting the Results: Selected Examples

NVDA — High explanatory power, strong factor profile

Adj R^2 = 0.617 — the factor model explains 61.7% of NVDA's monthly return variation. Market beta is 0.996 (t = 3.82), confirming NVDA moves nearly 1:1 with the market but with modest statistical confidence. The most significant exposure is the Vol factor at −2.308 (t = −4.89), meaning NVDA strongly moves opposite to the low-vol-minus-market spread. HML beta is −0.773 (t = −2.29), confirming NVDA is an anti-value (growth) stock. RMW beta is +1.074 (t = 2.26), reflecting NVDA's strong profitability.

Annual alpha is +2,654 bps — NVDA outperformed its factor prediction by ~26.5% per year over this period. However, this is backward-looking and reflects the AI boom rather than a stable, forward-looking return source.

BAC — Highest R^2, classic bank factor profile

Adj R^2 = 0.853 — the best-explained stock in the portfolio. The factor model captures 85.3% of BAC's return variation. Market beta is 1.198 (t = 11.78, the highest t-stat in the universe), making this the most reliable market beta estimate. HML beta is +1.189 (t = 9.03), the strongest value exposure — BAC behaves like a deep value stock, consistent with bank valuations. RMW beta is −0.444 (t = −2.39), reflecting banks' lower accounting profitability metrics.

LLY — Lowest R^2, weak factor explanation

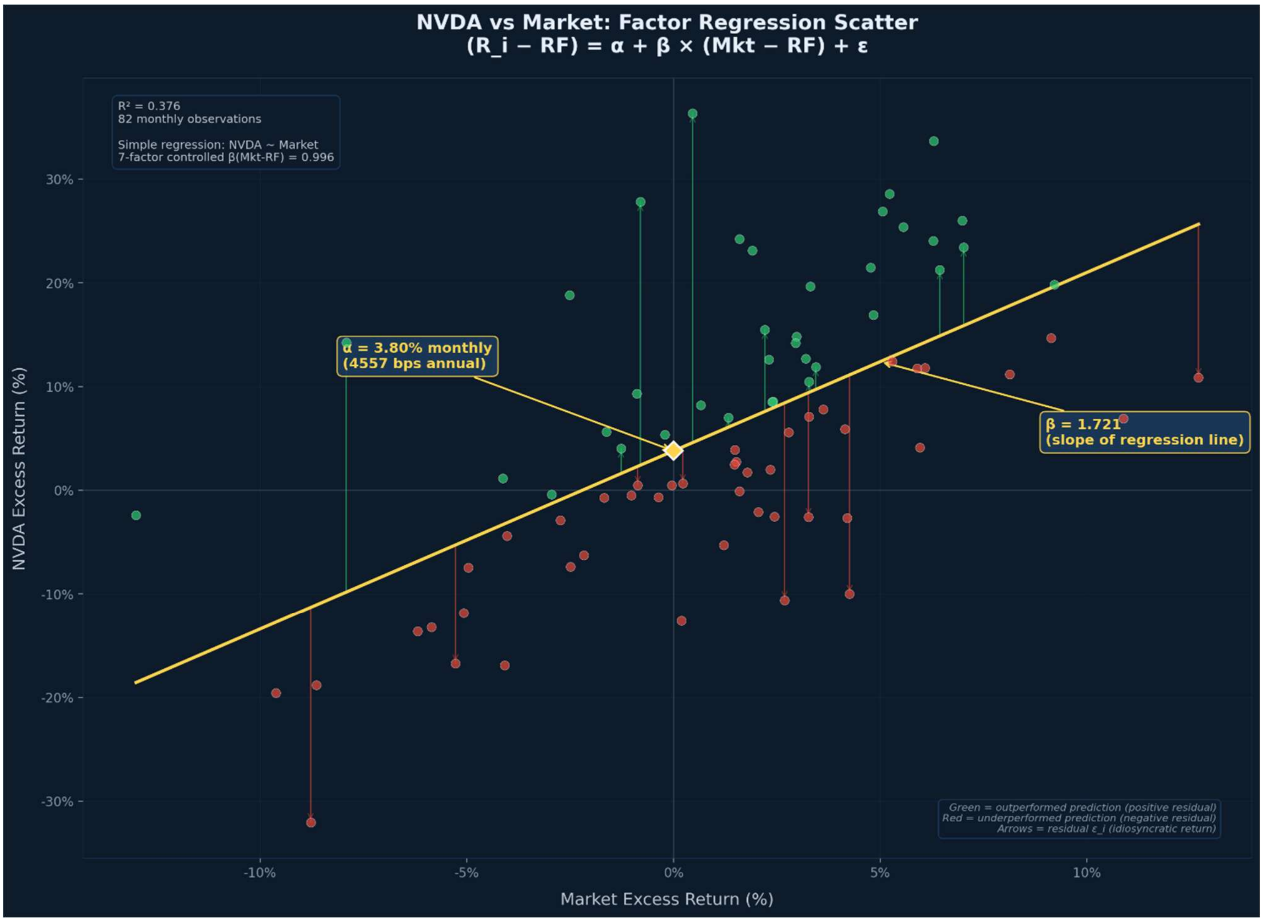
Adj R^2 = 0.064 — the factor model explains only 6.4% of LLY's return. No factor beta has $|t| > 2.0$. This means LLY's return is almost entirely idiosyncratic — driven by drug pipeline news (GLP-1 agonists), FDA approvals, and clinical trial results rather than broad market factors. The residual volatility is 30.1% annualized, among the highest. Despite the weak factor fit, LLY's alpha is +2,862 bps, the highest in the universe — but with R^2 this low, the alpha estimate itself is unreliable.

5.4 Statistical Significance Across the Universe

The t-statistic determines which factor exposures can be trusted for portfolio construction versus which are noise:

Factor	$\|t\| > 2.0$	Interpretation
Mkt-RF	30/30	All stocks have significant market exposure
Vol	16/30	Strong for low-vol names (WM, ICE, NEE, BRK.B); weak for others
HML	11/30	Clear signal for banks (BAC, JPM) and tech (MSFT, AMZN, NVDA)
SMB	5/30	Mostly noise; PH and BRK.B are notable exceptions
Mom	4/30	Only HD, META, UBER, JPM show significant momentum exposure
RMW	5/30	HD, NVDA, LIN, UBER show significant profitability exposure
CMA	4/30	Weakest factor; MLM and WM are the only clear signals

The market factor is universally significant, which validates the regression framework. The Vol and HML factors show good discriminating power across the universe. CMA and RMW are the weakest — their exposures should be interpreted cautiously.


NVDA vs Market: Factor Regression Scatter
(R_i − RF) = α + β × (Mkt − RF) + ε
R² = 0.376
82 monthly observations
Simple regression: NVDA ~ Market
7-factor controlled β(Mkt-RF) = 0.996
α = 3.80% monthly
(4557 bps annual)
β = 1.721
(slope of regression line)
Green = outperformed prediction (positive residual)
Red = underperformed prediction (negative residual)
Arrows = residual ε_i (idiosyncratic return)
NVDA Excess Return (%)
30%
20%
10%
0%
-10%
-20%
-30%
Market Excess Return (%)
-10%
-5%
0%
5%
10%

6. Step 5: Active Factor Exposures

6.1 Definition and Formula

Active factor exposures aggregate the 30 security-level bets into 7 factor-level tilts:

```
f_j = Σ(h_i × β_ij)   for each factor j
```

This is matrix multiplication: $f = B^T \times h$. The transposed beta matrix (7×30) multiplied by the active weight vector (30×1) produces a 7×1 vector. Each element is the weighted sum of betas across all securities, weighted by active weights. It answers: given all my stock bets, what is my aggregate tilt toward each factor?

6.2 Results

Factor	Active Exp.	Direction	% of TE²	Interpretation
Vol	−0.2584	Strongly SHORT low-vol	35.0%	Portfolio heavily tilts toward high-beta, high-vol names (NVDA)
HML	−0.1273	SHORT value	23.5%	Tech overweights drive anti-value tilt
RMW	+0.1011	LONG profitability	0.1%	Overweighting profitable companies (NVDA, MSFT)
SMB	−0.0640	SHORT small-cap	3.4%	Mega-cap overweights drive large-cap tilt
Mom	−0.0270	SHORT momentum	0.7%	Mild contrarian positioning
Mkt-RF	−0.0206	SHORT market	−2.0%	Slightly less market exposure than benchmark
CMA	+0.0050	LONG conservative inv.	−0.5%	Negligible; effectively neutral

The portfolio's two dominant factor tilts are short Vol (−0.258) and short Value/HML (−0.127). Together they account for 58.5% of systematic TE². The short-Vol tilt is driven by the large NVDA overweight (Vol beta = −2.308); the short-Value tilt reflects the tech-heavy positioning where NVDA, MSFT, AMZN, and AAPL all have negative HML betas.

7. Step 6: Factor Covariance Matrix

7.1 Why Not Estimate Security Covariance Directly?

A 30×30 security covariance matrix has 465 unique entries. Estimating 465 parameters from 82 monthly observations produces a noisy, unstable matrix. Instead, we construct the security covariance matrix through the factor model:

$$V = B \times \Sigma\_f \times B^T + D$$

B = 30×7 beta matrix from Step 4. **Σ_f** = 7×7 factor covariance matrix (28 unique parameters, well-estimated from 82 observations). **D** = 30×30 diagonal matrix of residual variances (30 parameters). Total: ~268 parameters versus 465, each estimated with greater statistical confidence.

7.2 Derivation

Starting from the factor model: $R = \alpha + B \times F + \varepsilon$. Taking the covariance of both sides (alpha drops out as a constant):

$$\mathrm{Cov}(R) = B \times \mathrm{Cov}(F) \times B^T + \mathrm{Cov}(\varepsilon)$$

By construction, residuals are uncorrelated across securities (each stock's idiosyncratic noise is independent), so $\mathrm{Cov}(\varepsilon)$ is diagonal. This gives $V = B \times \Sigma\_f \times B^T + D$.

7.3 Factor Correlation Matrix

	Mkt-RF	SMB	HML	RMW	CMA	Mom	Vol
Mkt-RF	1.000	0.323	0.020	0.025	−0.189	−0.410	−0.602
SMB		1.000	0.377	−0.310	0.041	−0.501	−0.211
HML			1.000	0.189	0.629	−0.278	0.269
RMW				1.000	0.216	−0.079	0.265
CMA					1.000	0.109	0.456
Mom						1.000	0.356
Vol							1.000

Notable correlations: Mkt-RF and Vol are strongly negatively correlated (−0.602) — when the market rises, the low-vol spread compresses. HML and CMA show the highest positive correlation (0.629) — value and conservative investment tend to co-move. The condition number is 16.3, indicating a well-conditioned, numerically stable matrix.

8. Step 7: Idiosyncratic Risk

8.1 Formula

Idiosyncratic tracking error captures stock-specific risk not explained by the 7 factors:

$$TE^2\_idio = \Sigma(h\_i^2 \times \sigma^2\_\varepsilon i)$$

Each security's contribution is its squared active weight times its residual variance. No cross-terms exist because idiosyncratic risks are uncorrelated by construction — AAPL's earnings surprise is independent of NVDA's earnings surprise.

Squaring the active weight means doubling your bet on a stock quadruples its idiosyncratic risk contribution. This is why concentrated portfolios are punished disproportionately.

8.2 Top Idiosyncratic Risk Contributors

Ticker	Active Wt	Resid Vol	Idio TE (bps)	% of Idio	Driver
NVDA	+7.648%	29.4%	225.2	48.7%	Largest position + high residual vol
GOOGL	+4.293%	19.9%	85.4	7.0%	Large position, moderate residual vol
AAPL	+4.538%	18.5%	84.1	6.8%	Large position, moderate residual vol
LLY	−2.706%	30.1%	81.4	6.4%	Highest residual vol + underweight
AMD	−1.256%	42.5%	53.4	2.7%	Highest residual vol in universe
MSFT	+4.288%	11.7%	50.3	2.4%	Large position, low residual vol

NVDA alone accounts for 48.7% of idiosyncratic TE^2. This is driven by the combination of the largest active weight (+7.6%) and high residual volatility (29.4% annualized). The top 6 securities account for 74.1% of idiosyncratic risk and the top 5 account for 107.6% of total TE, highlighting that the portfolio is aggressively concentrated in a handful of names. This concentration amplifies all risk — factor-driven and stock-specific alike — because tracking error scales with the square of the active weight.

9. Step 7b: Security-Level TE Attribution

9.0 Total Tracking Error

Before decomposing TE by security, we first compute the total TE that all subsequent calculations reference:

Systematic $TE^2 = f' \times \Sigma\_f \times f$

Idiosyncratic $TE^2 = \Sigma(h^2_i \times \sigma^2\_\varepsilon_i)$

Total TE^2 = Systematic TE^2 + Idiosyncratic TE^2

Total TE = $\sqrt{(\text{Total } TE^2)}$

This combines the active factor exposures (Step 5), factor covariance (Step 6), and residual variances (Step 7) into a single portfolio-level risk number. Variances add because systematic and idiosyncratic risks are uncorrelated by OLS construction. For our portfolio: Total TE = $\sqrt{(277.4^2 + 182.9^2)}$ = 460.4 bps.

9.1 Marginal TE — Definition and Derivation

Marginal TE measures the sensitivity of total TE to a change in active weight for a specific security:

```
Marginal TE_i = (V × h)_i / Total TE
```

This is the partial derivative of total TE with respect to h_i. It comes from differentiating TE = $\sqrt{(h^TVh)}$:

```
d(TE)/d(h_i) = d(√Q)/d(h_i) = (1/2√Q) × d(Q)/d(h_i)
```

Since Q = h^TVh is a quadratic form, d(Q)/d(h_i) = 2×(V×h)_i. The factor of 2 cancels, giving:

```
Marginal TE_i = (V × h)_i / √(hᵀVh) = (V × h)_i / TE
```

The numerator (V × h)_i captures how security i interacts with the entire portfolio's risk — both its own variance and its covariance with every other active bet. Dividing by TE converts from variance space to TE space.

Interpretation: NVDA's marginal TE of 3,973 bps means increasing NVDA's active weight by 1% (from 7.6% to 8.6%) would increase total TE by approximately 39.7 bps. This is the portfolio construction signal — it identifies which securities are most expensive to add risk to.

9.2 Contribution to TE — Euler Decomposition

Contribution to TE multiplies marginal TE by the actual active weight:

```
CTR_i = h_i × Marginal TE_i
```

By Euler's theorem for homogeneous functions, these contributions sum exactly to total TE — not approximately, but with mathematical certainty. This works because TE is homogeneous of degree 1: scaling all active weights by a constant λ scales TE by exactly λ. Euler's theorem guarantees that for any such function, f(h) = Σ h_i × (df/dh_i).

9.3 Security Attribution Results

Ticker	Active Wt	Marg TE	CTR (bps)	% of TE	Sys bps	Idio bps

NVDA	+7.648%	3,973	**+303.9**	66.0%	+193.7	+110.1
GOOGL	+4.293%	1,367	**+58.7**	12.7%	+42.8	+15.8
AAPL	+4.538%	1,216	**+55.2**	12.0%	+39.8	+15.4
MSFT	+4.288%	1,172	**+50.2**	10.9%	+44.7	+5.5
AMZN	+1.659%	1,652	**+27.4**	6.0%	+25.6	+1.8
AMD	−1.256%	1,768	**−22.2**	−4.8%	−28.4	+6.2
AMAT	−1.246%	905	**−11.3**	−2.5%	−13.5	+2.2
AVGO	−0.310%	1,303	**−4.0**	−0.9%	−4.2	+0.2
SPGI	−0.810%	475	**−3.8**	−0.8%	−4.2	+0.3
EQIX	−0.652%	495	**−3.2**	−0.7%	−3.5	+0.3

NVDA dominates with 66.0% of total TE. Its 303.9 bps contribution splits into 193.7 bps systematic (factor-driven) and 110.1 bps idiosyncratic (stock-specific). AMD and AMAT show negative contributions — their underweights are diversifying risk relative to the portfolio's overall positioning.

The top 5 securities (NVDA, GOOGL, AAPL, MSFT, AMZN) contribute 495.4 bps, which is 107.6% of the 460.4 bps total TE. The excess over 100% is offset by diversifying positions that reduce TE.

10. Step 8: Factor-Level TE Decomposition

10.1 The Universal Quadratic Form

Systematic tracking error is computed using the same formula used in fixed income TE (with KRDs), VaR, and every risk model built on factor decomposition:

$$TE^2\_systematic = f^T \times \Sigma\_f \times f$$

Where f is the 7×1 active factor exposure vector and Σ_f is the 7×7 factor covariance matrix. This is the quadratic form — active exposures sandwiching the covariance matrix. Total TE² adds the idiosyncratic component:

$$TE^2\_total = f^T \times \Sigma\_f \times f + \Sigma(h\_i^2 \times \sigma^2\_\varepsilon i)$$

10.2 Euler Decomposition by Factor

Each factor's contribution to systematic TE² is computed in variance space:

$$Marginal\_j = (\Sigma\_f \times f)\_j \qquad \text{[per-unit sensitivity in variance space]}$$

$$Contribution\_Var\_j = f\_j \times Marginal\_j \qquad \text{[actual share of TE}^2\text{]}$$

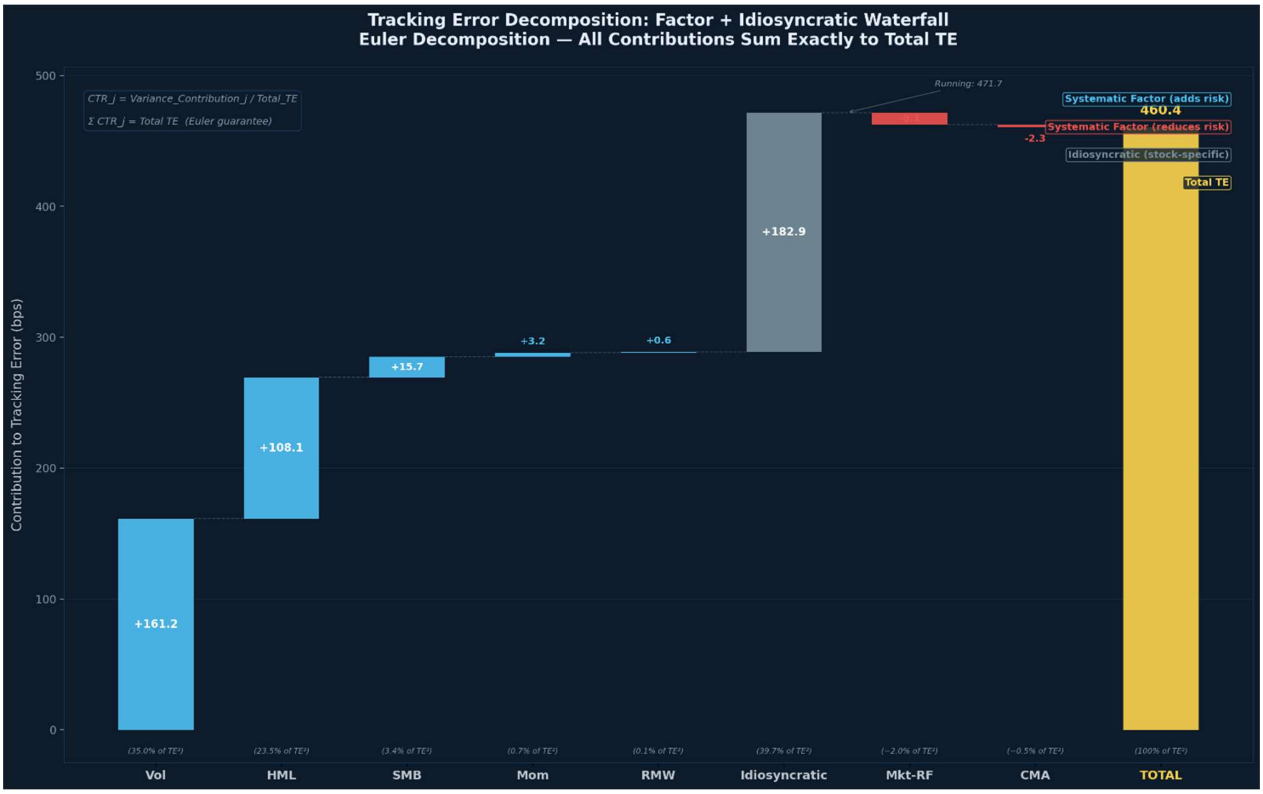
These variance contributions sum exactly to systematic TE². To convert to TE space (basis points), we divide each variance contribution by total TE:

$$Contribution\_TE\_j = Contribution\_Var\_j \;/\; Total\_TE$$

This is the same Euler method used in the security-level attribution (Step 7b). Dividing each variance piece by total TE guarantees that contributions sum exactly to total TE. The alternative — taking square roots of each factor's variance contribution independently — does not sum correctly because $\sqrt{a} + \sqrt{b} \neq \sqrt{(a+b)}$. The Euler method avoids this problem entirely.

10.3 Factor Attribution Results

Factor	Active Exp	CTR (bps)	% of TE²	Direction	Interpretation
Vol	−0.258	161.2	35.0%	↑ Adds risk	Dominant factor — driven by NVDA
HML	−0.127	108.1	23.5%	↑ Adds risk	Anti-value tilt from tech overweights
SMB	−0.064	15.7	3.4%	↑ Adds risk	Large-cap tilt
Mom	−0.027	3.2	0.7%	↑ Adds risk	Mild contrarian positioning
RMW	+0.101	0.6	0.1%	↑ Adds risk	Negligible despite sizable exposure
Mkt-RF	−0.021	−9.1	−2.0%	↓ Reduces risk	Slightly defensive positioning
CMA	+0.005	−2.3	−0.5%	↓ Reduces risk	Diversifying effect
Idiosyncratic	—	**182.9**	39.7%		Stock-specific risk
TOTAL	—	**460.4**	100.0%		All contributions sum exactly


Tracking Error Decomposition: Factor + Idiosyncratic Waterfall
Euler Decomposition — All Contributions Sum Exactly to Total TE
CTR_j = Variance_Contribution_j / Total_TE
Σ CTR_j = Total TE (Euler guarantee)
Contribution to Tracking Error (bps)
500
400
300
200
100
0
+161.2
+108.1
+15.7
+3.2
+0.6
+182.9
Running: 471.7
-2.3
460.4
Systematic Factor (adds risk)
Systematic Factor (reduces risk)
Idiosyncratic (stock-specific)
Total TE
(35.0% of TE²)
(23.5% of TE²)
(3.4% of TE²)
(0.7% of TE²)
(0.1% of TE²)
(39.7% of TE²)
(−2.0% of TE²)
(−0.5% of TE²)
(100% of TE²)
Vol
HML
SMB
Mom
RMW
Idiosyncratic
Mkt-RF
CMA
TOTAL

11. Portfolio Manager Action Plan

11.1 Diagnosis: Where Is the Risk?

The portfolio has three primary risk concentrations:

1. NVDA concentration (66% of total TE). NVDA contributes 303.9 bps out of 460.4 bps total. This is an extreme single-name concentration. The contribution splits roughly 64% systematic / 36% idiosyncratic, meaning even the factor exposure component is dominated by NVDA's extreme Vol factor beta (−2.308).

2. Volatility factor tilt (35% of TE^2). The active Vol exposure is −0.258, the largest of any factor. This is almost entirely from NVDA's Vol beta. The portfolio is implicitly short low-volatility and long high-volatility stocks relative to benchmark.

3. Value factor tilt (23.5% of TE^2). The active HML exposure is −0.127, driven by the tech-heavy overweight (NVDA, MSFT, AMZN, AAPL all have negative HML betas). The portfolio is positioned against value stocks.

11.2 Why Is Idiosyncratic TE High?

Idiosyncratic risk contributes 182.9 bps to total TE (39.7% of total TE^2). This is elevated because:

Concentrated active weights: Idiosyncratic risk scales with h^2 (squared active weight). NVDA at +7.6% contributes $h^2 = 0.0058$, which is larger than the combined h^2 of the 20 smallest active positions. Concentration amplifies stock-specific noise.

High residual volatility stocks: NVDA (29.4%), AMD (42.5%), LLY (30.1%), and META (30.5%) all have residual volatility above 25% annualized. These stocks are inherently noisy even after removing factor effects. When combined with non-trivial active weights, they drive idiosyncratic TE.

Low R^2 stocks with active bets: LLY has R^2 of 0.064 and active weight of −2.7%. Its return is almost entirely unexplained by factors. Any active bet on LLY is mostly an idiosyncratic bet.

11.3 Actions to Reduce Total TE

Action	Priority	Est. Impact	Rationale
1. Trim NVDA overweight	High	~150–200 bps	Reducing from +7.6% to +4.0% would cut NVDA's TE contribution roughly in half. This addresses both systematic (Vol, HML) and idiosyncratic risk simultaneously.
2. Diversify tech exposure	High	~50–100 bps	Spread tech overweight across MSFT, GOOGL, AMZN instead of concentrating in NVDA. All have similar factor profiles but lower residual volatility.
3. Add value exposure	Medium	~30–60 bps	Increase positions in XOM, BAC, or JPM (high HML beta). This directly offsets the −0.127 active HML exposure and reduces systematic TE from the value factor.
4. Reduce LLY underweight	Low	~10–20 bps	LLY's R^2 is 0.064 — the factor model cannot explain its risk. Any active bet on LLY is almost purely idiosyncratic. Moving toward benchmark weight reduces unexplained risk.
5. Use marginal TE as guide	—	Ongoing	Before adding to any position, check marginal TE. NVDA at 3,973 bps/unit is 33× more expensive than WM at 67 bps/unit. Route new capital toward securities with low marginal TE.

11.4 Interpreting Marginal TE for Portfolio Construction

Marginal TE is the portfolio construction signal. It tells the PM the cost of adding one more unit of active weight to each security. The spread between the most expensive and cheapest securities is enormous:

Ticker	Marginal TE (bps)	Signal
NVDA	**3,973**	Most expensive — adding here amplifies TE significantly
AMD	**1,768**	High — volatile semiconductor with factor overlap to NVDA
AMZN	**1,652**	High — but lower than NVDA due to lower residual vol
JNJ	**−242**	Negative — adding JNJ would reduce total TE (diversifier)
XOM	**−546**	Strong diversifier — value exposure offsets portfolio's growth tilt

Negative marginal TE securities (JNJ, XOM, LLY, HD, NEE) are natural hedge candidates. Adding to these positions would reduce total TE. The PM can use marginal TE as a real-time screening tool when deploying capital.

12. Mathematical Appendix

12.1 The Quadratic Form: $f^T \Sigma f$

This formula appears in every risk model because the variance of any linear combination of random variables takes this form. If portfolio active return = $f^T \times F$ (exposure vector times factor returns), then:

$$\mathrm{Var}(f^T F) = f^T \times \mathrm{Cov}(F) \times f = f^T \times \Sigma \times f$$

The formula is universal across asset classes. In equity TE: f = active factor exposures, Σ = factor covariance. In fixed income TE: f = active KRD exposures, Σ = yield curve covariance. In VaR: f = portfolio sensitivities, Σ = risk factor covariance. The structure is identical; only the inputs change.

12.2 Euler's Theorem and Exact Decomposition

$TE(h) = \sqrt{(h^T V h)}$ is homogeneous of degree 1: $TE(\lambda h) = \lambda TE(h)$. Euler's theorem states that for any degree-1 homogeneous function:

$$TE = \Sigma\, h\_i \times (dTE/dh\_i) = \Sigma\, CTR\_i$$

This is why Contribution to TE sums exactly to Total TE with mathematical certainty — zero approximation error. The same property holds for factor-level decomposition. This guarantee comes from the mathematical structure, not from backtesting or empirical validation.

12.3 Outer Product in Covariance-to-Correlation Conversion

The correlation matrix is derived from the covariance matrix by normalizing each element by the product of standard deviations:

$$\rho\_ij = \mathrm{Cov}(i,j) / (\sigma\_i \times \sigma\_j)$$

In matrix form: Corr = Cov / outer(std, std). The outer product creates a matrix where cell (i,j) = σ_i × σ_j. Element-wise division applies the correlation formula to all pairs simultaneously.

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