FRTB Internal Model Approach (IMA): Technical Overview

I have calculated Regulatory trading book capital required based on IMA method using an excel based model. I used a multi asset portfolio containing rates, equity, commodity and credit products. I calculated 12 expected shortfalls. Below are the steps I followed in detail. Youtube and Github artefacts at below links

https://github.com/amitsharma90-source/FRTB-Internal-model-based-approach

https://www.youtube.com/watch?v=wCyxofJSFXw

Executive Summary

The FRTB Internal Model Approach represents the Basel III framework for calculating regulatory market risk capital using Expected Shortfall (ES) as the core risk metric. This approach applies to trading book positions and requires banks to demonstrate robust risk modelling capabilities that meet stringent regulatory standards for modellable risk factors.

1. Core Methodology: Expected Shortfall

Risk Metric: Expected Shortfall replaces Value-at-Risk as the primary measure, calculated at 97.5% one-tailed confidence level with a base horizon of 10 days.

Calculation Frequency: Daily computation is required for diversified ES (IMCC(C)), while risk class-specific undiversified ES can be calculated weekly.

2. Modellable Risk Factor Eligibility

Banks must demonstrate that risk factors used in their internal models meet the Risk Factor Eligibility Test (RFET) outlined in MAR 31.

Eligibility Criteria (Banks typically choose Criterion 2):

Criterion 1: At least 24 real price observations per year with:

- No more than one observation per day
- No 90-day period with fewer than 4 observations
- Monitored monthly

Criterion 2 (Commonly Used): At least 100 real price observations over the previous 12 months with no more than one observation per day

The second criterion is considered more lenient and is the preferred choice for most banks conducting the eligibility test.

3. Liquidity Horizon Framework

Concept

Liquidity Horizon (LH) represents the average time required for a bank to liquidate or hedge a risk factor position. Highly liquid instruments (rates, equities) receive lower LH values, while less liquid instruments receive higher values.

Liquidity Horizon Assignments (MAR 33)

Risk Factor Category	**LH (days)**
Interest rate: specified currencies (EUR, USD, GBP, AUD, JPY, SEK, CAD, domestic)	10
Interest rate: unspecified currencies	20
Interest rate: volatility	60
Interest rate: other types	60
Credit spread: sovereign (IG or IG equivalent)	20
Credit spread: sovereign (HY or HY equivalent)	40
Credit spread: corporate (IG)	40
Credit spread: corporate (HY)	60
Credit spread: volatility	120
Credit spread: other types	120
Equity price (small cap): volatility	60
Equity: other types	60
Foreign exchange (FX): specified currency pairs	10
FX rate: currency pairs	20
FX: volatility	40
FX: other types	40
Energy and carbon emissions trading price	20

Risk Factor Category	LH (days)
Precious metals and non-ferrous metals price	20
Other commodities price	60
Energy and carbon emissions trading price: volatility	60
Precious metals and non-ferrous metals price: volatility	60

4. ES Calculation Methodology

The ES calculation follows a sophisticated approach that accounts for different liquidity horizons:

Step 1: Risk factors are divided by their liquidity horizons

Step 2: ES is calculated by keeping lowest LH risk factors constant while applying scenarios only to higher LH risk factors

Step 3: Higher LH ES values are scaled to their actual liquidity horizon using the **square root rule**:

$$ES = \sqrt{(ES_1(P))^2 + \sum_{j=2}^{5} (ES_j(P,j)) \sqrt{\frac{LH_j - LH_{j-1}}{T}}}$$

Where:

- T = base horizon of 10 days
- LH_j = liquidity horizon for category j
- ES values are calculated for each liquidity horizon level (j=1 to 5)

5. Stressed ES Requirement

Rationale

Banks must calibrate ES to a stressed period representing maximum historical losses. This ensures the model captures tail risk during extreme market conditions.

Validation Test

Risk factors from the historical stressed period must explain at least 75% of the current ES:

$$\frac{ES(Stressed)}{ES(Current)} > 75\%$$

This ensures the stressed period remains relevant to the current portfolio composition.

6. Multiple ES Calculations Required

Banks must calculate **12 different ES versions** (or more depending on portfolio composition):

#	ES Type	Diversified?	Risk Classes	Risk Factors Period	Full/Reduced
1	ES_F,C	Yes	All	Current	Full
2	ES_R,C	Yes	All	Current	Reduced
3	ES_R,S	Yes	All	Stressed	Reduced
4	ES_F,C	No	Equity	Current	Full
5	ES_R,C	No	Equity	Current	Reduced
6	ES_R,S	No	Equity	Stressed	Reduced
7	ES_F,C	No	Credit Spread	Current	Full
8	ES_R,C	No	Credit Spread	Current	Reduced
9	ES_R,S	No	Credit Spread	Stressed	Reduced
10	ES_F,C	No	Interest Rates	Current	Full
11	ES_R,C	No	Interest Rates	Current	Reduced
12	ES_R,S	No	Interest Rates	Stressed	Reduced

Note: Portfolios with additional broad risk classes (e.g., Commodities, FX) require additional ES calculations.

7. Diversified ES Formula

The diversified ES incorporates a scaling factor that is **floored at 1.0**, preventing any reduction in capital:

$$ES = ES_{R,S} \times \frac{ES_{F,C}}{ES_{R,C}}$$

Where the ratio $\frac{ES_{F,C}}{ES_{R,C}} \geq 1.0$

This ensures that moving from reduced to full risk factors, or from current to stressed scenarios, never reduces the capital requirement.

8. Final Capital Calculation (IMCC Formula)

The Internal Model Capital Charge combines diversified and undiversified ES using a 50/50 weighting (ρ = 0.5):

$$IMCC = \rho(IMCC(C)) + (1-\rho)(\sum_{i=1}^{B} IMCC(C_i))$$

Where:

- **IMCC(C)** = Diversified ES incorporating all correlations across risk classes

$$IMCC(C) = ES_{R,S} \times \frac{ES_{F,C}}{ES_{R,C}}$$

- **IMCC(C_i)** = Undiversified ES for individual broad risk class i

$$IMCC(C_i) = ES_{R,S,i} \times \frac{ES_{F,C,i}}{ES_{R,C,i}}$$

- **ρ = 0.5** = Diversification weight parameter
- **B** = Number of broad risk classes

Key Insights:

1. **50% diversified / 50% undiversified**: Half the capital comes from portfolio-level diversified ES (capturing inter-risk class correlations), while half comes from the sum of undiversified risk class-specific ES
2. **Calculation frequency**:
    - IMCC(C): Daily calculation required
    - IMCC(C_i): Weekly calculation sufficient
    - Ratio $\frac{IMCC(C)}{IMCC(All\ asset\ classes)}$: Weekly calculation acceptable
3. **Purpose**: This blended approach balances recognition of diversification benefits while maintaining conservative capital levels by preventing over-reliance on correlation assumptions

9. Regulatory References

All methodologies reference the Basel Framework MAR standards:

- **MAR 31**: Risk factor eligibility test
- **MAR 33**: Expected shortfall risk measure and liquidity horizons

Implementation Considerations

For banks implementing the IMA:

1. **Data Infrastructure**: Robust systems for capturing 100+ daily price observations per risk factor
2. **Stress Period Identification**: Historical analysis to identify maximum loss periods meeting the 75% threshold
3. **Computational Capacity**: Daily ES calculations across multiple scenarios and liquidity horizons
4. **Model Validation**: Ongoing backtesting and P&L attribution testing
5. **Reduced vs Full Risk Factors**: Clear methodology for determining which risk factors to include in reduced models
6. **Documentation**: Comprehensive audit trails for regulatory review

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This framework represents a significant advancement from the previous VaR-based approach, with Expected Shortfall providing better tail risk capture and the liquidity horizon framework ensuring appropriate capital for less liquid positions.