1. **Objective**: To perform ex ante tracking error(TE) attribution for a fixed income multi-currency portfolio. We will quantify:

   1.1. Marginal TE of different TE components i.e.
   a) 1-10 year curve rates
   b) Credit spread to government bonds
   c) FX exposure for non-base currency bonds

   1.2. Contribution of different TE components i.e.
   d) 1-10 year curve rates
   e) Credit spread to government bonds
   f) FX exposure for non-base currency bonds

2. **Observations**:

   2.1. FX is most volatile factor and is more volatile then rates and OAS. This is evident in below screenshot.

Bond Type	Curve TE	Spread TE	Ratio	Why
EUR	~7 bps	~2 bps	1:3	OAS var ≈ 1/3 rate var, EUR bonds have no FX dilution effect Therefore: Spread TE ≈ 1/3 of Curve TE
USD	~0.1 bps	~0.1 bps	1:1	• FX variance is 100× larger • FX exposure dominates the TEV - Curve and spread both become small relative to FX • Their ratio becomes less meaningful

2.2. OAS factor has least volatility while FX factor has highest volatility and hence highest contribution to tracking error

	1Y	2Y	3Y	4Y	5Y	7Y	10Y	OAS	FX
1Y	0.00005743	0.00005741	0.00005290	0.00004961	0.00004633	0.00004163	0.00003697	0.00000561	0.00016
2Y	0.00005741	0.00006905	0.00006815	0.00006542	0.00006269	0.00005714	0.00005088	0.00000390	0.00024
3Y	0.00005290	0.00006815	0.00007025	0.00006868	0.00006712	0.00006233	0.00005628	0.00000309	0.00026
4Y	0.00004961	0.00006542	0.00006868	0.00006817	0.00006766	0.00006395	0.00005859	0.00000252	0.00026
5Y	0.00004633	0.00006269	0.00006712	0.00006766	0.00006820	0.00006556	0.00006090	0.00000195	0.00025
7Y	0.00004163	0.00005714	0.00006233	0.00006395	0.00006556	0.00006465	0.00006122	0.00000117	0.00023
10Y	0.00003697	0.00005088	0.00005628	0.00005859	0.00006090	0.00006122	0.00005934	0.00000102	0.00021
OAS	0.00000561	0.00000390	0.00000309	0.00000252	0.00000195	0.00000117	0.00000102	0.00002006	0.00014
FX	0.00016237	0.000236903	0.000260641	0.00025511	0.00024959	0.000231452	0.0002058	0.00013627	0.00518

TEV Attribution | Validation | Covariance Matrix | Program Log

3. Data inputs:

3.1. Last 5 year monthly rates from FRED site. Example link for 10 year rate below. Only USD curve has been used.
https://fred.stlouisfed.org/series/GS10

3.2. OAS spread for 5 years. We have assumed that all bonds are A rated, but we can extend this model to multiple credit rating bonds.
https://fred.stlouisfed.org/series/BAMLC0A3CA

3.3. USD/EUR rates for 5 years.
USD/EUR rates: https://fred.stlouisfed.org/series/DEXUSEU

3.4. Portfolio holdings and benchmark constituents

4. Functionality:

4.1. Determine absolute changes on monthly bases for all tenors and OAS spread.
4.2. Determine % changes for monthly FX rates.
4.3. Calculate Covariance matrix for last 5 years of risk factor changes.
4.4. Determine key rate durations for security
4.5. Assume that credit spread duration is same as sum of all KRDs with scaling by 0.95 for callable bonds because callable bonds have lesser rate sensitivity because of shorter effective maturity.
4.6. Determine FX sensitivity
4.7. Determine net weight of position.
4.8. Determine net exposure by multiplying net weight with risk factor sensitivity
4.9. Calculate Total TEV by calculating quadratic dot product of net exposure and covariance matrix.

$$\boldsymbol{TE = \sqrt{Combined\ factor\ vector^{T} \times \Sigma \times Combined\ factor\ vector}}$$

4.10. Determine marginal tev by dividing matrix dot product of net exposure and factor covariance matrix by total tev.

Marginal TE = (Σ × Net exposure vector) / Total_TEV

a) Σ = Covariance matrix of 9 factors
b) Net exposure vector = Net weight of security × Sensitivity vector of 9 factors i.e. 7 KRDs, 1 OAS sensitivity, and 1 FX sensitivity
c) Total TEV comes from step 9

4.11. Calculate contribution to tev for each factor.

Contribution to TEV = net_exposure_vector * marginal_te_vector

5. Future improvements in model:
   5.1. Separate curves for different currencies
   5.2. Bonds with different ratings and hence different OAS time series
   5.3. Addition of Equity holdings to show TEV attribution of multi asset portfolio.