Fixed Income Tracking Error Attribution Engine

Overview

I've developed a comprehensive tracking error risk attribution engine for fixed income portfolios that addresses the growing need for precise, transparent risk analytics in portfolio management. Built on Python with the industry-standard QuantLib library, this tool delivers institutional-quality analytics with the flexibility and transparency that proprietary systems often lack.

Key Features:

- **Professional Accuracy**: Uses QuantLib's proven pricing engines
- **Transparent Methodology**: Open-source foundation enables validation and customization
- **Industry Compatibility**: Output format matches Bloomberg/MSCI reporting standards

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Portfolio Analytics Output

The engine produces comprehensive risk attribution tables showing how each yield curve tenor contributes to total tracking error:

Tenor	Portfolio_KRD	Benchmark_KRD	Net_KRD	Factor_Vol_%	TE_Isolated_bps	TE_Correlated_bps	Marginal_TE	Contribution_bps	Pct_of_TE
1Y	7.36	4.38	2.98	0.76	225.86	295.04	0.01	221.77	0.74
2Y	6.54	5.60	0.94	0.83	78.26	271.15	0.01	70.62	0.24
3Y	4.12	1.98	2.14	0.84	179.23	242.72	0.01	144.78	0.48
4Y	3.40	2.66	0.73	0.83	60.53	224.73	0.01	45.27	0.15
5Y	2.31	5.55	-3.24	0.83	267.46	203.03	0.01	-180.72	-0.60
7Y	0.16	0.19	-0.03	0.80	2.07	179.18	0.00	-1.24	0.00
10Y	0.00	0.00	0.00	0.77	0.00	159.14	0.00	0.00	0.00

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================================================================================
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TRACKING ERROR ANALYTICS SUMMARY (EMPIRICAL DATA ONLY)
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Tenor Portfolio_KRD Benchmark_KRD Net_KRD Factor_Vol_% TE_Isolated_bps TE_Correlated_bps Marginal_TE Contribution_bps Pct_of_TE
   1Y        7.3606        4.3802  2.9804          0.76          225.86            295.04      0.0074            221.77     73.8%
   2Y        6.5426        5.6009  0.9417          0.83           78.26            271.15      0.0075             70.62     23.5%
   3Y        4.1201        1.9817  2.1384          0.84          179.23            242.72      0.0068            144.78     48.2%
   4Y        3.3970        2.6639  0.7331          0.83           60.53            224.73      0.0062             45.27     15.1%
   5Y        2.3108        5.5495 -3.2387          0.83          267.46            203.03      0.0056           -180.72    -60.1%
   7Y        0.1615        0.1873 -0.0258          0.80            2.07            179.18      0.0048             -1.24     -0.4%
  10Y        0.0000        0.0000  0.0000          0.77            0.00            159.14      0.0041              0.00      0.0%

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TOTAL TRACKING ERROR: 300.48 basis points
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PORTFOLIO SUMMARY:
Portfolio Duration: 23.89 years
Benchmark Duration: 20.36 years
Active Duration: 3.53 years
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What This Shows: Each row represents a different yield curve tenor (1Y, 2Y, 3Y, etc.), with columns showing portfolio exposure, benchmark exposure, net active positions, and multiple risk metrics. The percentage contributions sum to 100%, showing exactly where your tracking error originates.

Portfolio Composition Analysis

	Security	Portfolio/Benchmark	Position Type	Notional exposure$	Rating	Issue Date	Call date	Maturity	Coupon	OAS curve	
2	2027 Bond								2.95	AAA_OAS	3Y
3	Oracle 2028 Bond	Portfolio	Bond	2000	A3	15/12/2022	15/12/2026	01/12/2028	2.95	A_OAS	4Y
4	Intel 2029 Bond	Portfolio	Bond	2000	A1	15/12/2022	15/12/2026	20/07/2029	4.64	A_OAS	5Y
5	Union Pacific 2030 Bond	Benchmark	Bond	2000	A3	15/12/2022	15/12/2026	01/10/2030	3.95	A_OAS	6Y
6	Goldman Sachs 2026 Bond	Benchmark	Bond	2000	A2	15/12/2022	15/10/2026	01/11/2026	3.25	A_OAS	2Y
7	ExxonMobil 2027 Bond	Benchmark	Bond	2000	Aa1	15/12/2022	15/12/2026	15/04/2027	4.62	AA_OAS	3Y
8	Netflix 2029 Bond	Benchmark	Bond	2000	Ba3	15/12/2022	15/12/2026	01/06/2029	7.85	BB_OAS	5Y

Security	Type	Base_Price	KRDs
Apple 202	Portfolio	98.49442	[{'Index': 0, 'Tenor': '1.0Y', 'Base_Rate_%': 3.95, 'Price_Up': 98.48947591032768, 'Price_Down': 98.49935771745345, 'Price_Change_Up': -0.004940749075785789, 'Price_Cha
Microsoft	Portfolio	97.68519	[{'Index': 0, 'Tenor': '1.0Y', 'Base_Rate_%': 3.95, 'Price_Up': 97.68201024014039, 'Price_Down': 97.68837572397156, 'Price_Change_Up': -0.0031826135899137853, 'Price_Ch
JPMorgan	Portfolio	99.15662	[{'Index': 0, 'Tenor': '1.0Y', 'Base_Rate_%': 3.95, 'Price_Up': 99.14839907911178, 'Price_Down': 99.16483969329047, 'Price_Change_Up': -0.00821995277060239, 'Price_Chan
Verizon 2(	Portfolio	105.0933	[{'Index': 0, 'Tenor': '1.0Y', 'Base_Rate_%': 3.95, 'Price_Up': 105.08457257124519, 'Price_Down': 105.10195476364962, 'Price_Change_Up': -0.00869074123579594, 'Price_Ch
Pfizer 203(	Portfolio	99.4633	[{'Index': 0, 'Tenor': '1.0Y', 'Base_Rate_%': 3.95, 'Price_Up': 99.45762257715306, 'Price_Down': 99.46897120973746, 'Price_Change_Up': -0.005674081111223472, 'Price_Cha
Walmart 2	Portfolio	99.52161	[{'Index': 0, 'Tenor': '1.0Y', 'Base_Rate_%': 3.95, 'Price_Up': 99.51673304489111, 'Price_Down': 99.52649534982577, 'Price_Change_Up': -0.004880950173713927, 'Price_Cha
Chevron 2	Portfolio	97.05806	[{'Index': 0, 'Tenor': '1.0Y', 'Base_Rate_%': 3.95, 'Price_Up': 97.05468982870987, 'Price_Down': 97.06142630711253, 'Price_Change_Up': -0.003368103284728363, 'Price_Cha
Caterpillar	Portfolio	100.2629	[{'Index': 0, 'Tenor': '1.0Y', 'Base_Rate_%': 3.95, 'Price_Up': 100.25676557653742, 'Price_Down': 100.26896547583235, 'Price_Change_Up': -0.0060996984962287115, 'Price_(
Boeing 20:	Portfolio	100.6966	[{'Index': 0, 'Tenor': '1.0Y', 'Base_Rate_%': 3.95, 'Price_Up': 100.69011661025958, 'Price_Down': 100.70317255851603, 'Price_Change_Up': -0.006527701756951387, 'Price_C
Procter &	Portfolio	99.50401	[{'Index': 0, 'Tenor': '1.0Y', 'Base_Rate_%': 3.95, 'Price_Up': 99.49813315075522, 'Price_Down': 99.50989100233537, 'Price_Change_Up': -0.005878750338098371, 'Price_Cha
Johnson &	Portfolio	98.49038	[{'Index': 0, 'Tenor': '1.0Y', 'Base_Rate_%': 3.95, 'Price_Up': 98.48270411008018, 'Price_Down': 98.49805942475427, 'Price_Change_Up': -0.007677360927502264, 'Price_Cha
Oracle 20:	Portfolio	96.58395	[{'Index': 0, 'Tenor': '1.0Y', 'Base_Rate_%': 3.95, 'Price_Up': 96.58125297901023, 'Price_Down': 96.58665048605951, 'Price_Change_Up': -0.0026986435945843823, 'Price_Ch
Intel 2029	Portfolio	99.98702	[{'Index': 0, 'Tenor': '1.0Y', 'Base_Rate_%': 3.95, 'Price_Up': 99.98127813591107, 'Price_Down': 99.9925310870358, 'Price_Change_Up': -0.00574428853563802, 'Price_Chang
Union Pac	Benchmar	97.69224	[{'Index': 0, 'Tenor': '1.0Y', 'Base_Rate_%': 3.95, 'Price_Up': 97.68806763384798, 'Price_Down': 97.69641924622543, 'Price_Change_Up': -0.004175633045718996, 'Price_Cha
Goldman !	Benchmar	99.05946	[{'Index': 0, 'Tenor': '1.0Y', 'Base_Rate_%': 3.95, 'Price_Up': 99.05017004260023, 'Price_Down': 99.06874398606395, 'Price_Change_Up': -0.00928653903766019, 'Price_Chan
ExxonMot	Benchmar	100.7049	[{'Index': 0, 'Tenor': '1.0Y', 'Base_Rate_%': 3.95, 'Price_Up': 100.69762916584452, 'Price_Down': 100.71207390465621, 'Price_Change_Up': -0.007222097791739657, 'Price_C
Netflix 20:	Benchmar	105.1466	[{'Index': 0, 'Tenor': '1.0Y', 'Base_Rate_%': 3.95, 'Price_Up': 105.13789395054961, 'Price_Down': 105.15529872555035, 'Price_Change_Up': -0.008702027645426824, 'Price_C
Visa 2028	Benchmar	97.71678	[{'Index': 0, 'Tenor': '1.0Y', 'Base_Rate_%': 3.95, 'Price_Up': 97.71331269701089, 'Price_Down': 97.72024179587866, 'Price_Change_Up': -0.0034644076114886957, 'Price_Ch
Amgen 20	Benchmar	97.78595	[{'Index': 0, 'Tenor': '1.0Y', 'Base_Rate_%': 3.95, 'Price_Up': 97.78299168673253, 'Price_Down': 97.7889160809851, 'Price_Change_Up': -0.002962074296249284, 'Price_Chan
3M 2030 E	Benchmar	95.05738	[{'Index': 0, 'Tenor': '1.0Y', 'Base_Rate_%': 3.95, 'Price_Up': 95.05495489404709, 'Price_Down': 95.05980336302143, 'Price_Change_Up': -0.002424137461375153, 'Price_Cha
3M 2030 E	Benchmar	95.05738	[{'Index': 0, 'Tenor': '1.0Y', 'Base_Rate_%': 3.95, 'Price_Up': 95.05495489404709, 'Price_Down': 95.05980336302143, 'Price_Change_Up': -0.002424137461375153, 'Price_Cha
Amgen 20	Benchmar	97.78595	[{'Index': 0, 'Tenor': '1.0Y', 'Base_Rate_%': 3.95, 'Price_Up': 97.78299168673253, 'Price_Down': 97.7889160809851, 'Price_Change_Up': -0.002962074296249284, 'Price_Chan

Key Insight: This example demonstrates realistic portfolio management scenarios where:

- Portfolio and benchmark holdings differ significantly
- Active positions create tracking error exposure
- Risk attribution helps identify concentration and hedging opportunities

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Risk Metrics Explained

1. Key Rate Duration (KRD)

Definition: Measures portfolio sensitivity to 1 basis point changes in specific yield curve tenors.

Calculation Process:

1. **Yield Curve Construction:** Create zero-coupon bond helpers using current market rates with proper day count conventions and holiday calendars
2. **Bond Modelling:** Generate coupon payment schedules and call schedules (for callable bonds)
3. **Pricing Engine Setup:**
    - Callable bonds: Hull-White interest rate model with calibrated parameters
    - Non-callable bonds: Standard discounting engine
4. **Base Price Calculation:** Determine clean bond price using current yield curve
5. **Shock Application:** Shift target tenor rate by ±1 basis point while holding other rates constant
6. **KRD Computation:** KRD = (Price_Down - Price_Up) / (2 × Base_Price × 0.0001)

Business Uses:

- **Portfolio Construction:** Ensure desired duration profile matches investment objectives
- **Hedging Decisions:** Identify which tenors need hedging to reduce unwanted interest rate risk
- **Risk Budgeting:** Allocate tracking error limits across different yield curve segments
- **Performance Attribution:** Understand how yield curve movements impact portfolio vs benchmark returns

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2. Net KRD

Definition: Net KRD = Portfolio KRD - Benchmark KRD

Business Uses:

- **Active Risk Assessment:** Quantify your actual interest rate bets relative to benchmark

- **Position Sizing:** Determine optimal position sizes to achieve target active duration
- **Risk Monitoring:** Track adherence to duration mandates and risk limits
- **Scenario Analysis:** Model portfolio performance under various interest rate environments

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3. Factor Volatility

Definition: Annualized standard deviation of daily rate changes for each tenor, calculated from historical Treasury data.

Data Sources: Federal Reserve Economic Data (FRED) constant maturity series:

- 1Y Treasury

  https://fred.stlouisfed.org/series/GS1
- 2Y Treasury

  https://fred.stlouisfed.org/series/GS2
- 3Y Treasury
- 5Y Treasury
- 7Y Treasury
- 10Y Treasury

Calculation: Uses last 5 years of monthly data for current market relevance: Annual Vol = Monthly Std Dev × $\sqrt{12}$

Business Uses:

- **Risk Scaling:** Understand which parts of the yield curve are most volatile
- **VaR Calculations:** Input for Value-at-Risk models and stress testing
- **Option Pricing:** Volatility inputs for embedded option valuations
- **Risk-Adjusted Returns:** Compare returns relative to underlying factor volatilities

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4. TE Impact of Isolated 1σ Change

Formula: TE Impact = |Net KRD| × Factor Volatility

Interpretation: Shows tracking error contribution if ONLY this tenor moves by its typical daily amount (1 standard deviation), with all other rates held constant.

Business Uses:

- **Single-Factor Risk Assessment:** Identify which curve segments pose the greatest individual risk
- **Limit Setting:** Establish position limits based on maximum acceptable single-factor impact
- **Stress Testing:** Model "what-if" scenarios for specific yield curve movements
- **Risk Reporting:** Provide clear, understandable risk metrics to investment committees

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5. TE Impact of Correlated 1σ Change

Definition: Realistic tracking error impact when rates move according to their historical correlations.

Why It's Higher: When 5Y rates move, 3Y and 7Y rates typically move in the same direction with high correlation (0.90+), amplifying the total portfolio impact.

Business Uses:

- **Realistic Risk Assessment:** More accurate risk estimates than isolated analysis
- **Portfolio Optimization:** Account for correlation benefits when constructing hedges
- **Diversification Analysis:** Understand true concentration risk across correlated factors
- **Client Reporting:** Provide realistic risk expectations based on market relationships

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6. Marginal Contribution to TEV

Formula: Marginal TE = (Covariance Matrix × Net KRD Vector) / Total TE

Interpretation: Shows how much total tracking error would increase if you added one unit of exposure to this factor(tenor).

Business Uses:

- **Incremental Risk Analysis:** Evaluate risk impact of proposed trades before execution
- **Risk Budgeting:** Optimally allocate risk across different factors
- **Trade Implementation:** Sequence trades to minimize interim risk
- **Portfolio Rebalancing:** Identify most efficient ways to reduce tracking error

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7. Contribution to TEV

Formula: Contribution = Net KRD × Marginal TE

Key Property: All contributions sum to total tracking error (Euler's theorem).

Business Uses:

- **Risk Attribution:** Pinpoint exactly where tracking error originates
- **Performance Explanation:** Decompose period returns by risk factor
- **Risk Reduction:** Identify which positions to adjust for maximum TE reduction
- **Mandate Compliance:** Demonstrate adherence to risk budgets and guidelines

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8. Percentage of Total Tracking Error

Formula: % of TE = (Contribution to TE / Total TE) × 100

Validation: All percentages sum to 100% (including negative contributions from hedging positions).

Business Uses:

- **Risk Concentration Analysis:** Identify if portfolio risk is overly concentrated
- **Diversification Assessment:** Ensure risk is appropriately spread across factors
- **Effective Number of Bets:** Calculate risk diversification efficiency
- **Board Reporting:** Present risk in easily understood percentage terms

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Advanced Applications

Risk Management

- **Daily Monitoring:** Track risk metrics against predetermined limits
- **Stress Testing:** Model portfolio behaviour under historical crisis scenarios

Portfolio Construction

- **Optimization:** Use marginal contributions to build mean-variance efficient portfolios
- **Transition Management:** Minimize tracking error during portfolio rebalancing
- **Benchmark Replication:** Achieve precise benchmark matching with minimal holdings

Performance Attribution

- **Risk-Adjusted Performance:** Evaluate returns relative to tracking error taken

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Technical Implementation Notes

Correlation Matrix: Uses empirical correlations from 5-year historical data rather than assumptions, providing more accurate risk estimates for current market conditions.

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2. Calculating covariance matrix from last 5 years...
   Using data from 2020-04-01 to 2025-04-01
   Number of observations: 61
   Monthly changes calculated: 60 observations
   Annual volatilities:
   1Y: 0.76%
   2Y: 0.83%
   3Y: 0.84%
   4Y: 0.83%
   5Y: 0.83%
   7Y: 0.80%
   10Y: 0.77%

   Empirical correlations (sample):
   1Y-2Y: 0.912
   2Y-5Y: 0.914
   5Y-10Y: 0.957
```

<u>**Calculated Correlation matrix**</u>

	1Y	2Y	3Y	4Y	5Y	7Y	10Y
1Y	1	0.91159	0.83279	0.79292	0.74028	0.6833	0.63328
2Y	0.91159	1	0.97855	0.95354	0.91351	0.85529	0.79489
3Y	0.83279	0.97855	1	0.99249	0.96964	0.92495	0.87161
4Y	0.79292	0.95354	0.99249	1	0.99227	0.96327	0.92113
5Y	0.74028	0.91351	0.96964	0.99227	1	0.98739	0.95724
7Y	0.6833	0.85529	0.92495	0.96327	0.98739	1	0.98851
10Y	0.63328	0.79489	0.87161	0.92113	0.95724	0.98851	1

Calculated Covariance matrix

	1Y	2Y	3Y	4Y	5Y	7Y	10Y
1Y	5.7E-05	5.7E-05	5.3E-05	5E-05	4.6E-05	4.2E-05	3.7E-05
2Y	5.7E-05	6.9E-05	6.8E-05	6.5E-05	6.3E-05	5.7E-05	5.1E-05
3Y	5.3E-05	6.8E-05	7E-05	6.9E-05	6.7E-05	6.2E-05	5.6E-05
4Y	5E-05	6.5E-05	6.9E-05	6.8E-05	6.8E-05	6.4E-05	5.9E-05
5Y	4.6E-05	6.3E-05	6.7E-05	6.8E-05	6.8E-05	6.6E-05	6.1E-05
7Y	4.2E-05	5.7E-05	6.2E-05	6.4E-05	6.6E-05	6.5E-05	6.1E-05
10Y	3.7E-05	5.1E-05	5.6E-05	5.9E-05	6.1E-05	6.1E-05	5.9E-05

Callable Bond Handling: Hull-White model parameters (mean reversion: 0.03, volatility: 0.015) are industry-standard calibrations that can be customized for specific market environments.

Validation Features: Built-in checks ensure mathematical consistency (e.g., sum of KRDs ≈ total duration, sum of TE contributions = total TE).

Conclusion

This tracking error attribution engine provides institutional-quality analytics with the transparency and flexibility that modern portfolio management demands. By combining rigorous quantitative methods with practical business applications, it enables more informed investment decisions and superior risk management outcomes.

The tool's foundation on open-source libraries ensures both cost-effectiveness and the ability to customize analytics for specific investment strategies or regulatory requirements.