## **Fixed Income Tracking Error Attribution Engine**

## **Overview**

I've developed a comprehensive tracking error risk attribution engine for fixed income portfolios that addresses the growing need for precise, transparent risk analytics in portfolio management. Built on Python with the industry-standard QuantLib library, this tool delivers institutional-quality analytics with the flexibility and transparency that proprietary systems often lack.

## **Key Features:**

- Professional Accuracy: Uses QuantLib's proven pricing engines
- **Transparent Methodology**: Open-source foundation enables validation and customization
- Industry Compatibility: Output format matches Bloomberg/MSCI reporting standards

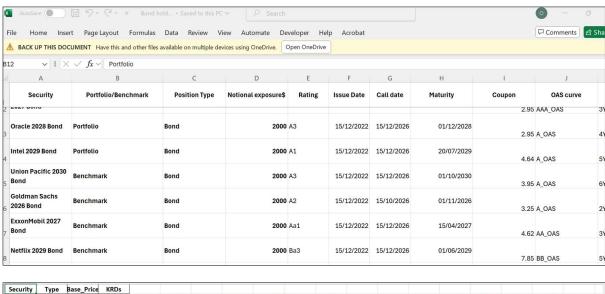
# **Portfolio Analytics Output**

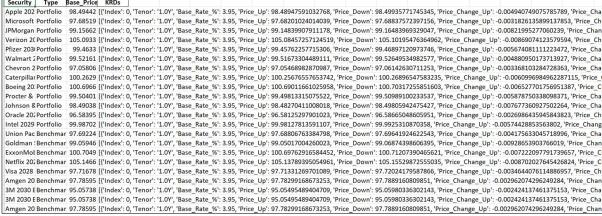
The engine produces comprehensive risk attribution tables showing how each yield curve tenor contributes to total tracking error:

Tenor	Portfolio_KRD	Benchmark_KRD	Net_KRD	Factor_V	TE_Isolat	TE_Correl	Marginal	Contribut	Pct_of_TE
				ol_%	ed_bps	ated_bps	_TE	ion_bps	
1Y	7.36	4.38	2.98	0.76	225.86	295.04	0.01	221.77	0.74
2Y	6.54	5.60	0.94	0.83	78.26	271.15	0.01	70.62	0.24
3Y	4.12	1.98	2.14	0.84	179.23	242.72	0.01	144.78	0.48
4Y	3.40	2.66	0.73	0.83	60.53	224.73	0.01	45.27	0.15
5Y	2.31	5.55	-3.24	0.83	267.46	203.03	0.01	-180.72	-0.60
7Y	0.16	0.19	-0.03	0.80	2.07	179.18	0.00	-1.24	0.00
10Y	0.00	0.00	0.00	0.77	0.00	159.14	0.00	0.00	0.00

What This Shows: Each row represents a different yield curve tenor (1Y, 2Y, 3Y, etc.), with columns showing portfolio exposure, benchmark exposure, net active positions, and multiple risk metrics. The percentage contributions sum to 100%, showing exactly where your tracking error originates.

# **Portfolio Composition Analysis**





**Key Insight:** This example demonstrates realistic portfolio management scenarios where:

- Portfolio and benchmark holdings differ significantly
- · Active positions create tracking error exposure
- Risk attribution helps identify concentration and hedging opportunities

## 1. Key Rate Duration (KRD)

**Definition:** Measures portfolio sensitivity to 1 basis point changes in specific yield curve tenors.

#### **Calculation Process:**

- 1. **Yield Curve Construction:** Create zero-coupon bond helpers using current market rates with proper day count conventions and holiday calendars
- 2. **Bond Modelling:** Generate coupon payment schedules and call schedules (for callable bonds)

### 3. Pricing Engine Setup:

- Callable bonds: Hull-White interest rate model with calibrated parameters
- o Non-callable bonds: Standard discounting engine
- 4. Base Price Calculation: Determine clean bond price using current yield curve
- 5. **Shock Application:** Shift target tenor rate by ±1 basis point while holding other rates constant
- 6. **KRD Computation:** KRD = (Price\_Down Price\_Up) / (2 × Base\_Price × 0.0001)

#### **Business Uses:**

- Portfolio Construction: Ensure desired duration profile matches investment objectives
- Hedging Decisions: Identify which tenors need hedging to reduce unwanted interest rate risk
- Risk Budgeting: Allocate tracking error limits across different yield curve segments
- Performance Attribution: Understand how yield curve movements impact portfolio vs benchmark returns

### 2. Net KRD

**Definition:** Net KRD = Portfolio KRD - Benchmark KRD

### **Business Uses:**

 Active Risk Assessment: Quantify your actual interest rate bets relative to benchmark

- Position Sizing: Determine optimal position sizes to achieve target active duration
- Risk Monitoring: Track adherence to duration mandates and risk limits
- **Scenario Analysis:** Model portfolio performance under various interest rate environments

## 3. Factor Volatility

**Definition:** Annualized standard deviation of daily rate changes for each tenor, calculated from historical Treasury data.

Data Sources: Federal Reserve Economic Data (FRED) constant maturity series:

1Y Treasury

https://fred.stlouisfed.org/series/GS1

2Y Treasury

https://fred.stlouisfed.org/series/GS2

- 3Y Treasury
- 5Y Treasury
- <u>7Y Treasury</u>
- 10Y Treasury

**Calculation:** Uses last 5 years of monthly data for current market relevance: Annual Vol = Monthly Std Dev  $\times \sqrt{12}$ 

### **Business Uses:**

- Risk Scaling: Understand which parts of the yield curve are most volatile
- VaR Calculations: Input for Value-at-Risk models and stress testing
- Option Pricing: Volatility inputs for embedded option valuations
- Risk-Adjusted Returns: Compare returns relative to underlying factor volatilities

# 4. TE Impact of Isolated 1σ Change

Formula: TE Impact = |Net KRD| × Factor Volatility

**Interpretation:** Shows tracking error contribution if ONLY this tenor moves by its typical daily amount (1 standard deviation), with all other rates held constant.

#### **Business Uses:**

- Single-Factor Risk Assessment: Identify which curve segments pose the greatest individual risk
- Limit Setting: Establish position limits based on maximum acceptable singlefactor impact
- Stress Testing: Model "what-if" scenarios for specific yield curve movements
- Risk Reporting: Provide clear, understandable risk metrics to investment committees

## **5. TE Impact of Correlated 1σ Change**

**Definition:** Realistic tracking error impact when rates move according to their historical correlations.

Why It's Higher: When 5Y rates move, 3Y and 7Y rates typically move in the same direction with high correlation (0.90+), amplifying the total portfolio impact.

#### **Business Uses:**

- Realistic Risk Assessment: More accurate risk estimates than isolated analysis
- Portfolio Optimization: Account for correlation benefits when constructing hedges
- Diversification Analysis: Understand true concentration risk across correlated factors
- **Client Reporting:** Provide realistic risk expectations based on market relationships

### 6. Marginal Contribution to TEV

Formula: Marginal TE = (Covariance Matrix × Net KRD Vector) / Total TE

**Interpretation:** Shows how much total tracking error would increase if you added one unit of exposure to this factor(tenor).

## **Business Uses:**

- Incremental Risk Analysis: Evaluate risk impact of proposed trades before execution
- Risk Budgeting: Optimally allocate risk across different factors
- Trade Implementation: Sequence trades to minimize interim risk
- Portfolio Rebalancing: Identify most efficient ways to reduce tracking error

### 7. Contribution to TEV

Formula: Contribution = Net KRD × Marginal TE

**Key Property:** All contributions sum to total tracking error (Euler's theorem).

#### **Business Uses:**

- Risk Attribution: Pinpoint exactly where tracking error originates
- Performance Explanation: Decompose period returns by risk factor
- Risk Reduction: Identify which positions to adjust for maximum TE reduction
- Mandate Compliance: Demonstrate adherence to risk budgets and guidelines

## 8. Percentage of Total Tracking Error

Formula: % of TE = (Contribution to TE / Total TE) × 100

**Validation:** All percentages sum to 100% (including negative contributions from hedging positions).

### **Business Uses:**

- Risk Concentration Analysis: Identify if portfolio risk is overly concentrated
- **Diversification Assessment:** Ensure risk is appropriately spread across factors
- Effective Number of Bets: Calculate risk diversification efficiency
- Board Reporting: Present risk in easily understood percentage terms

### **Advanced Applications**

### Risk Management

- **Daily Monitoring:** Track risk metrics against predetermined limits
- Stress Testing: Model portfolio behaviour under historical crisis scenarios

#### **Portfolio Construction**

- **Optimization:** Use marginal contributions to build mean-variance efficient portfolios
- Transition Management: Minimize tracking error during portfolio rebalancing
- Benchmark Replication: Achieve precise benchmark matching with minimal holdings

## **Performance Attribution**

• Risk-Adjusted Performance: Evaluate returns relative to tracking error taken

### **Technical Implementation Notes**

**Correlation Matrix:** Uses empirical correlations from 5-year historical data rather than assumptions, providing more accurate risk estimates for current market conditions.

```
2. Calculating covariance matrix from last 5 years...
Using data from 2020-04-01 to 2025-04-01
Number of observations: 61
Monthly changes calculated: 60 observations
 Annual volatilities:
 1Y: 0.76%
 2Y: 0.83%
 3Y: 0.84%
 4Y: 0.83%
 5Y: 0.83%
 7Y: 0.80%
 10Y: 0.77%
 Empirical correlations (sample):
 1Y-2Y: 0.912
 2Y-5Y: 0.914
 5Y-10Y: 0.957
```

## **Calculated Correlation matrix**

	1Y	2Y	3Y	4Y	5Y	7Y	10Y
1Y	1	0.91159	0.83279	0.79292	0.74028	0.6833	0.63328
2Y	0.91159	1	0.97855	0.95354	0.91351	0.85529	0.79489
<b>3Y</b>	0.83279	0.97855	1	0.99249	0.96964	0.92495	0.87161
4Y	0.79292	0.95354	0.99249	1	0.99227	0.96327	0.92113
5Y	0.74028	0.91351	0.96964	0.99227	1	0.98739	0.95724
7Y	0.6833	0.85529	0.92495	0.96327	0.98739	1	0.98851
10Y	0.63328	0.79489	0.87161	0.92113	0.95724	0.98851	1

### **Calculated Covariance matrix**

	1Y	2Y	3Y	4Y	5Y	7Y	10Y
1Y	5.7E-05	5.7E-05	5.3E-05	5E-05	4.6E-05	4.2E-05	3.7E-05
2Y	5.7E-05	6.9E-05	6.8E-05	6.5E-05	6.3E-05	5.7E-05	5.1E-05
3Y	5.3E-05	6.8E-05	7E-05	6.9E-05	6.7E-05	6.2E-05	5.6E-05
4Y	5E-05	6.5E-05	6.9E-05	6.8E-05	6.8E-05	6.4E-05	5.9E-05
5Y	4.6E-05	6.3E-05	6.7E-05	6.8E-05	6.8E-05	6.6E-05	6.1E-05
7Y	4.2E-05	5.7E-05	6.2E-05	6.4E-05	6.6E-05	6.5E-05	6.1E-05
10Y	3.7E-05	5.1E-05	5.6E-05	5.9E-05	6.1E-05	6.1E-05	5.9E-05

**Callable Bond Handling:** Hull-White model parameters (mean reversion: 0.03, volatility: 0.015) are industry-standard calibrations that can be customized for specific market environments.

**Validation Features:** Built-in checks ensure mathematical consistency (e.g., sum of KRDs  $\approx$  total duration, sum of TE contributions = total TE).

#### Conclusion

This tracking error attribution engine provides institutional-quality analytics with the transparency and flexibility that modern portfolio management demands. By combining rigorous quantitative methods with practical business applications, it enables more informed investment decisions and superior risk management outcomes.

The tool's foundation on open-source libraries ensures both cost-effectiveness and the ability to customize analytics for specific investment strategies or regulatory requirements.