# Option Pricing using qGANs

June 30, 2022

[6]:
```python
import matplotlib.pyplot as plt
import numpy as np

from qiskit import Aer, QuantumRegister, QuantumCircuit
from qiskit.circuit import ParameterVector
from qiskit.circuit.library import TwoLocal
from qiskit.quantum_info import Statevector

from qiskit.utils import QuantumInstance
from qiskit.algorithms import IterativeAmplitudeEstimation, EstimationProblem
from qiskit_finance.applications.estimation import EuropeanCallPricing
from qiskit_finance.circuit.library import NormalDistribution
```

[7]:
```python
# Set upper and lower data values
bounds = np.array([0.0, 7.0])
# Set number of qubits used in the uncertainty model
num_qubits = 3

# Load the trained circuit parameters
g_params = [0.29399714, 0.38853322, 0.9557694, 0.07245791, 6.02626428, 0.
↪13537225]

# Set an initial state for the generator circuit
init_dist = NormalDistribution(num_qubits, mu=1.0, sigma=1.0, bounds=bounds)

# construct the variational form
var_form = TwoLocal(num_qubits, "ry", "cz", entanglement="circular", reps=1)

# keep a list of the parameters so we can associate them to the list of␣
↪numerical values
# (otherwise we need a dictionary)
theta = var_form.ordered_parameters

# compose the generator circuit, this is the circuit loading the uncertainty␣
↪model
g_circuit = init_dist.compose(var_form)
```

[8]:
```
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```
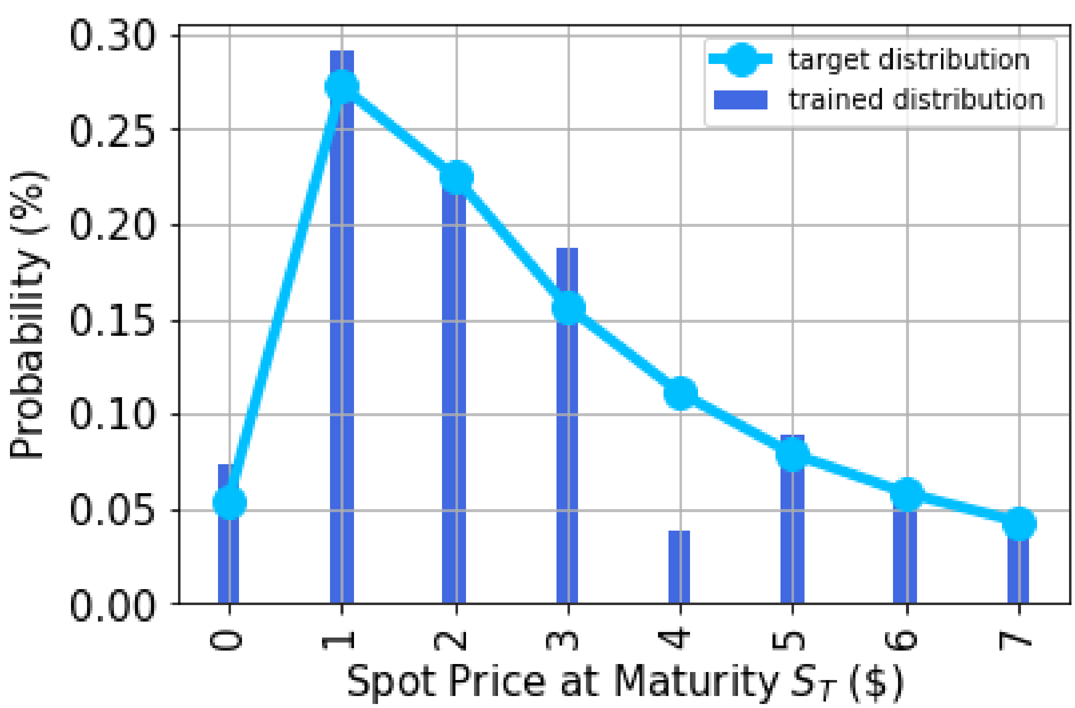

[9]:
```
# set the strike price (should be within the low and the high value of the␣
↪uncertainty)
strike_price = 2

# set the approximation scaling for the payoff function
c_approx = 0.25
```

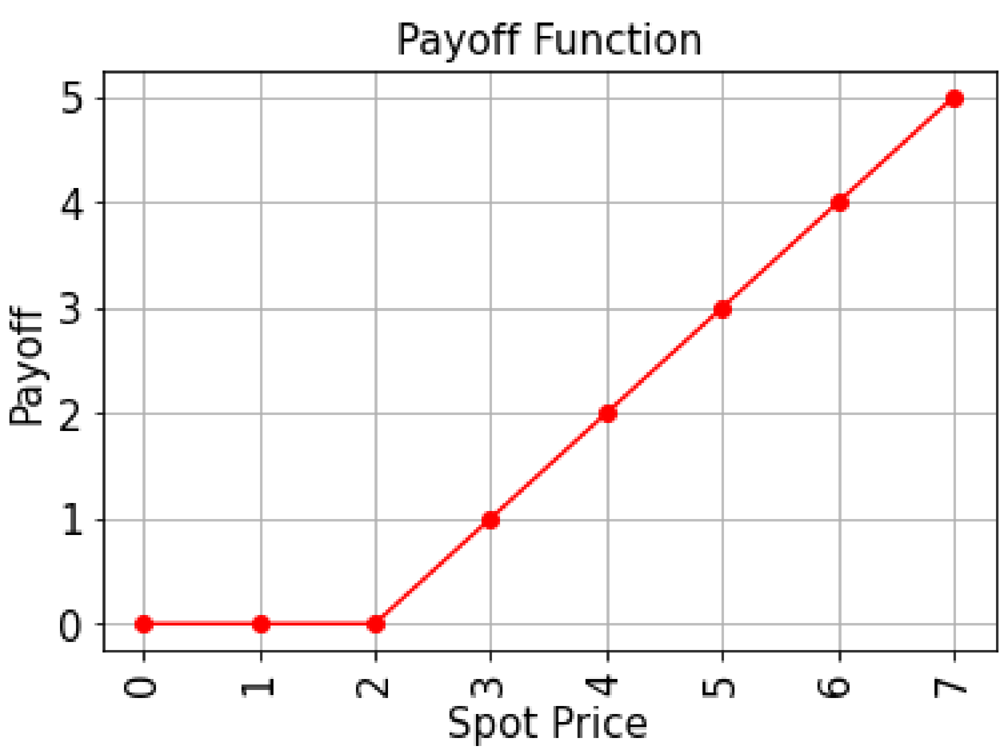
[10]:
```
# Evaluate trained probability distribution
values = [
    bounds[0] + (bounds[1] - bounds[0]) * x / (2**num_qubits - 1) for x in␣
↪range(2**num_qubits)
]
uncertainty_model = g_circuit.assign_parameters(dict(zip(theta, g_params)))
amplitudes = Statevector.from_instruction(uncertainty_model).data

x = np.array(values)
y = np.abs(amplitudes) ** 2

# Sample from target probability distribution
N = 100000
log_normal = np.random.lognormal(mean=1, sigma=1, size=N)
log_normal = np.round(log_normal)
log_normal = log_normal[log_normal <= 7]
```

```
log_normal_samples = []
for i in range(8):
    log_normal_samples += [np.sum(log_normal == i)]
log_normal_samples = np.array(log_normal_samples / sum(log_normal_samples))

# Plot distributions
plt.bar(x, y, width=0.2, label="trained distribution", color="royalblue")
plt.xticks(x, size=15, rotation=90)
plt.yticks(size=15)
plt.grid()
plt.xlabel("Spot Price at Maturity $S_T$ (\$)", size=15)
plt.ylabel("Probability ($\%$)", size=15)
plt.plot(
    log_normal_samples,
    "-o",
    color="deepskyblue",
    label="target distribution",
    linewidth=4,
    markersize=12,
)
plt.legend(loc="best")
plt.show()
```



```
[11]: # Evaluate payoff for different distributions
payoff = np.array([0, 0, 0, 1, 2, 3, 4, 5])
ep = np.dot(log_normal_samples, payoff)
print("Analytically calculated expected payoff w.r.t. the target distribution: ␣
 ↪%.4f" % ep)
ep_trained = np.dot(y, payoff)
print("Analytically calculated expected payoff w.r.t. the trained distribution:␣
 ↪%.4f" % ep_trained)

# Plot exact payoff function (evaluated on the grid of the trained uncertainty␣
 ↪model)
x = np.array(values)
y_strike = np.maximum(0, x - strike_price)
plt.plot(x, y_strike, "ro-")
plt.grid()
plt.title("Payoff Function", size=15)
plt.xlabel("Spot Price", size=15)
plt.ylabel("Payoff", size=15)
plt.xticks(x, size=15, rotation=90)
plt.yticks(size=15)
plt.show()
```

```
Analytically calculated expected payoff w.r.t. the target distribution:  1.0636
Analytically calculated expected payoff w.r.t. the trained distribution: 0.9805
```



[12]:
```
# construct circuit for payoff function
european_call_pricing = EuropeanCallPricing(
    num_qubits,
    strike_price=strike_price,
    rescaling_factor=c_approx,
    bounds=bounds,
    uncertainty_model=uncertainty_model,
)
```

[13]:
```
# set target precision and confidence level
epsilon = 0.01
alpha = 0.05

qi = QuantumInstance(Aer.get_backend("aer_simulator"), shots=100)
problem = european_call_pricing.to_estimation_problem()
# construct amplitude estimation
ae = IterativeAmplitudeEstimation(epsilon, alpha=alpha, quantum_instance=qi)
```

[14]:
```
result = ae.estimate(problem)
```

[15]:
```
conf_int = np.array(result.confidence_interval_processed)
print("Exact value:        \t%.4f" % ep_trained)
print("Estimated value:    \t%.4f" % (result.estimation_processed))
print("Confidence interval:\t[%.4f, %.4f]" % tuple(conf_int))
```

```
Exact value:            0.9805
Estimated value:        0.9638
Confidence interval:    [0.8495, 1.0781]
```

[ ]:
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