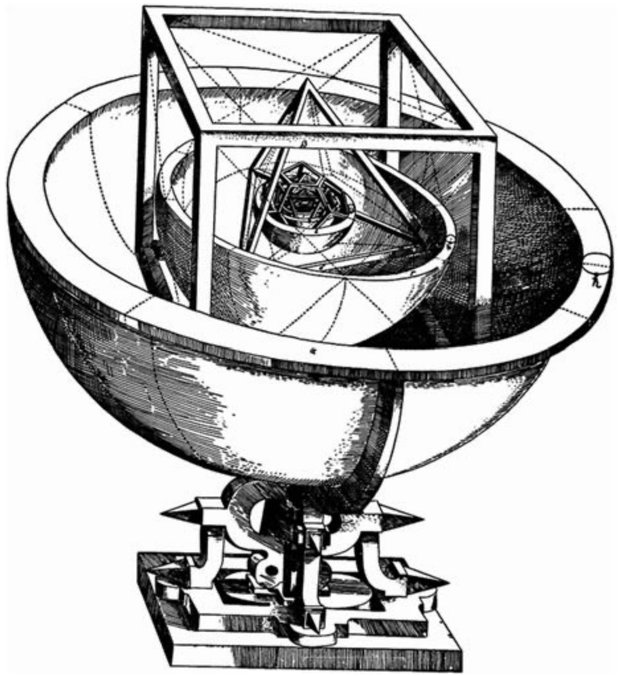


ریاضیات برای زیست‌شناسان

امیرحسین زارع مهذبیه

تابستان ۱۴۰۳

فهرست

۱. مقدمه، در باب مدل‌سازی در روش علمی

۲. جبر

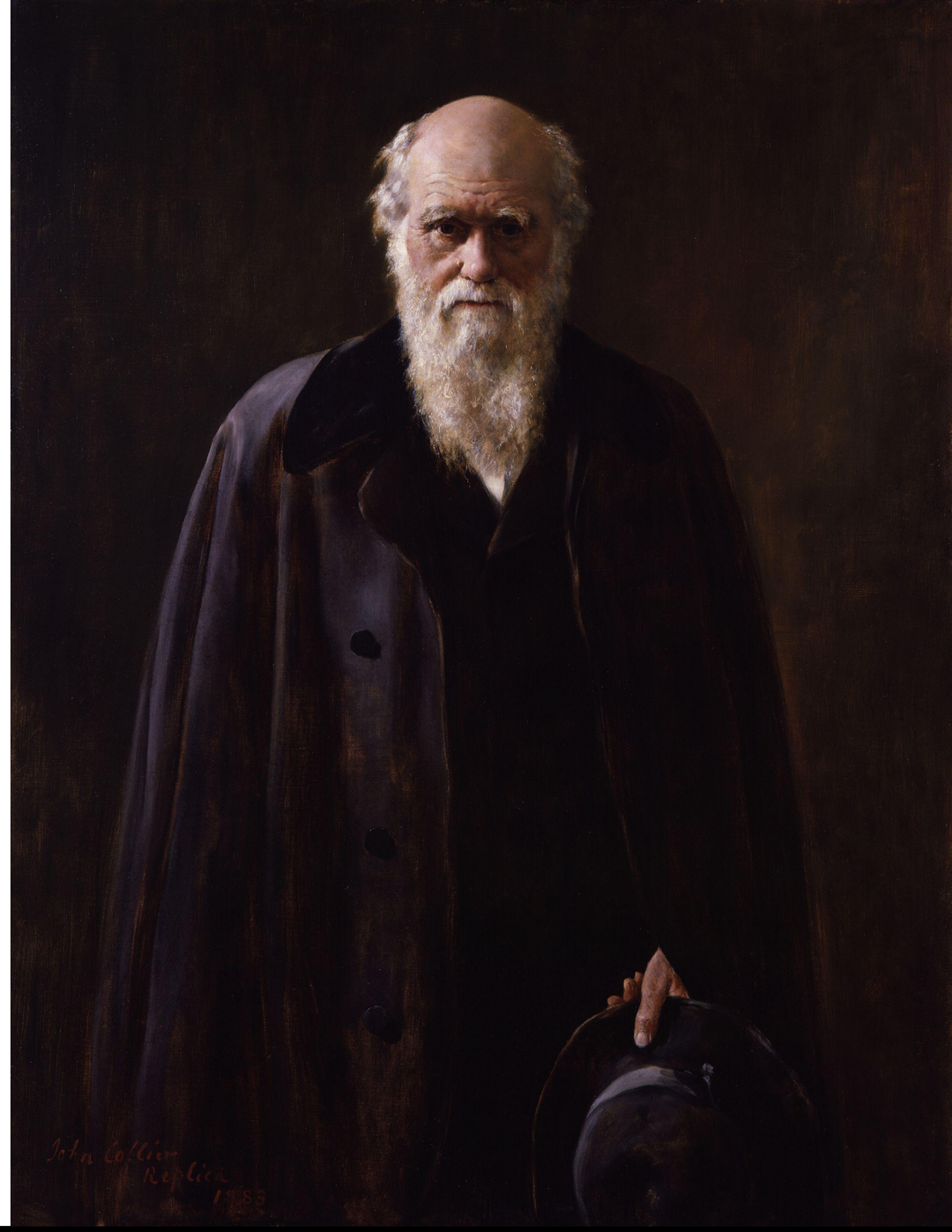
۳. جبر خطی

۴. حسابان

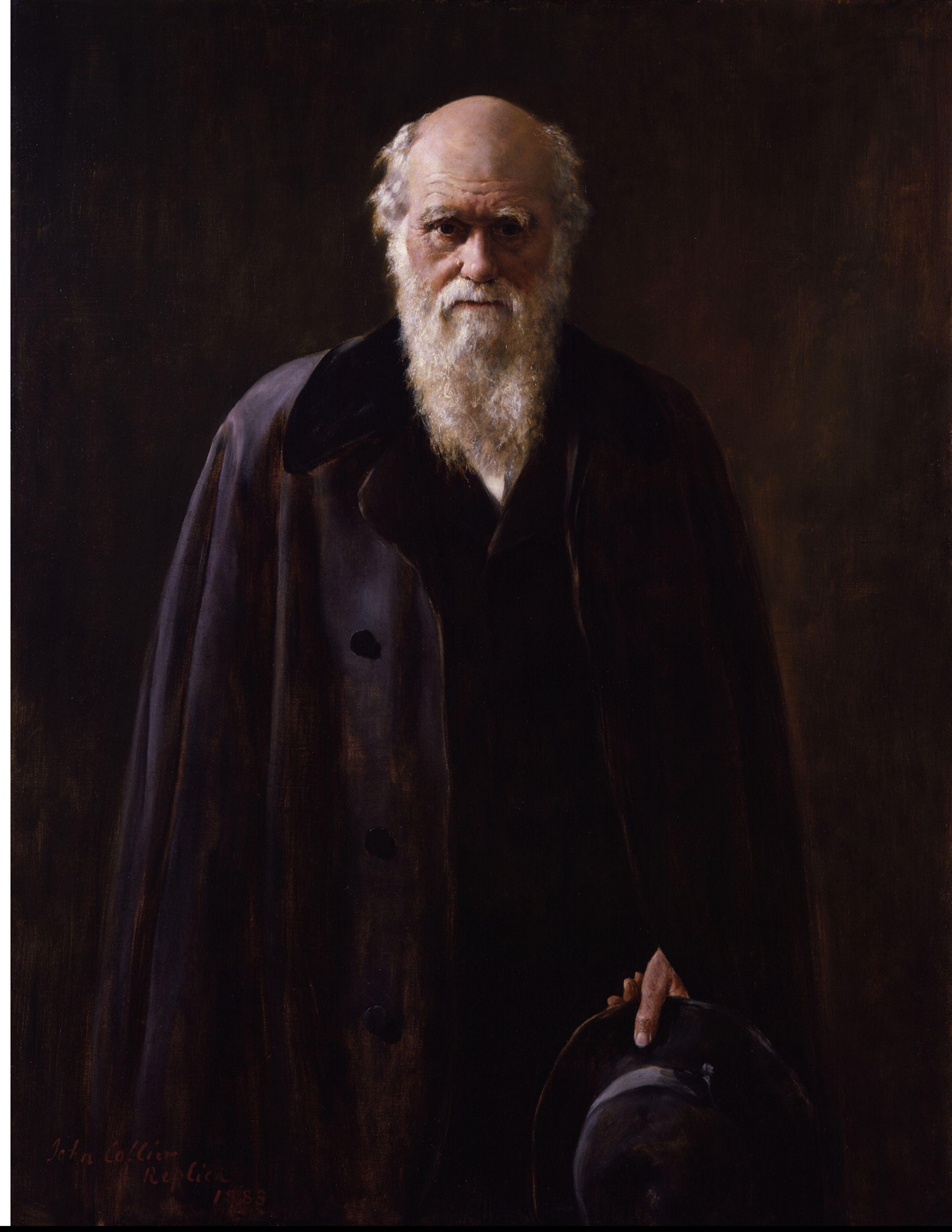
۵. معادلات دیفرانسیل

۶. احتمال و اندکی آمار

“I have deeply regretted that I did not proceed far enough at least to understand something of the great leading principles of mathematics, for men thus endowed seem to have an extra sense.”



“A mathematician is like a blind man in a dark room looking for a black cat that isn't there.”



۱. مقدمه

۱. مقدمه

۱.۱ مدل چیست

۱.۲ مدل در زیست‌شناسی

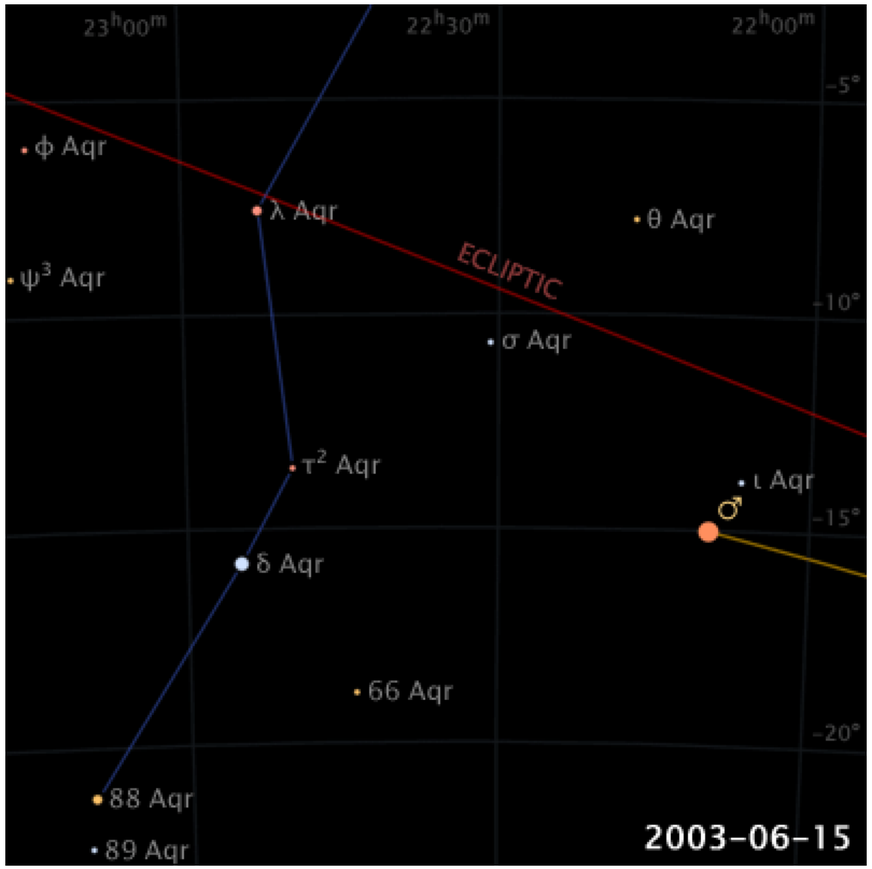
۱.۱ مدل چیست

۱.۱ مدل چیست

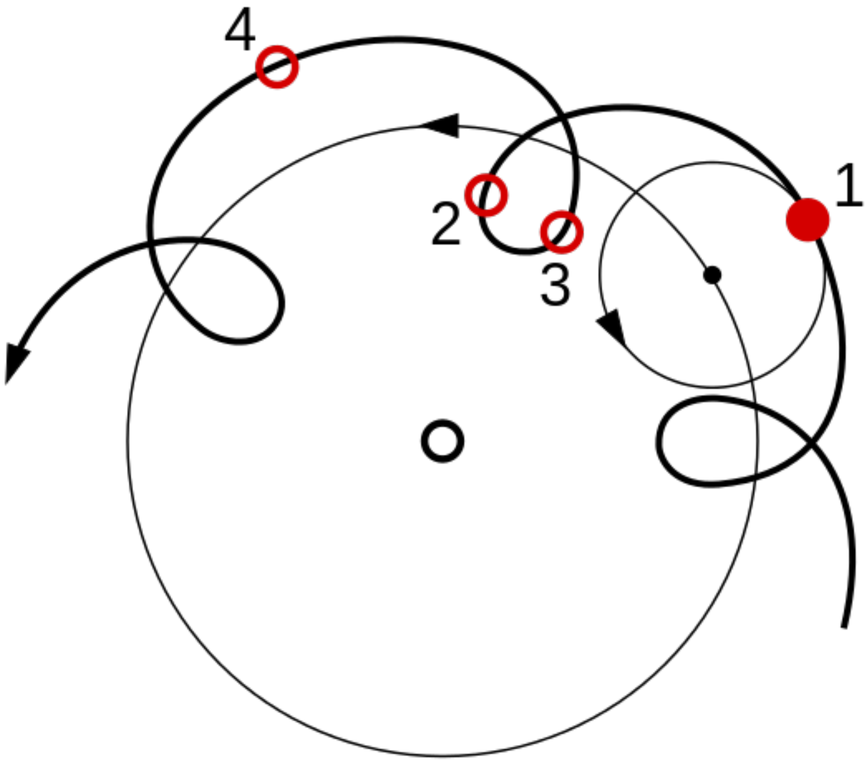
EMPIREVM
HABITACVLVM
DEI
ET
OMNIVM
ELECTORVM
COELVM
Primũ Mobile
Decimũm Coelũm
Nonũ Coelũm
Octauũm
Firmamentũ
Crystallinũm
COELV
SATVRNI
IOVIS
MARTIS
SOLIS
VENERIS
MERCVRII
LVNA

۱.۱ مدل چیست

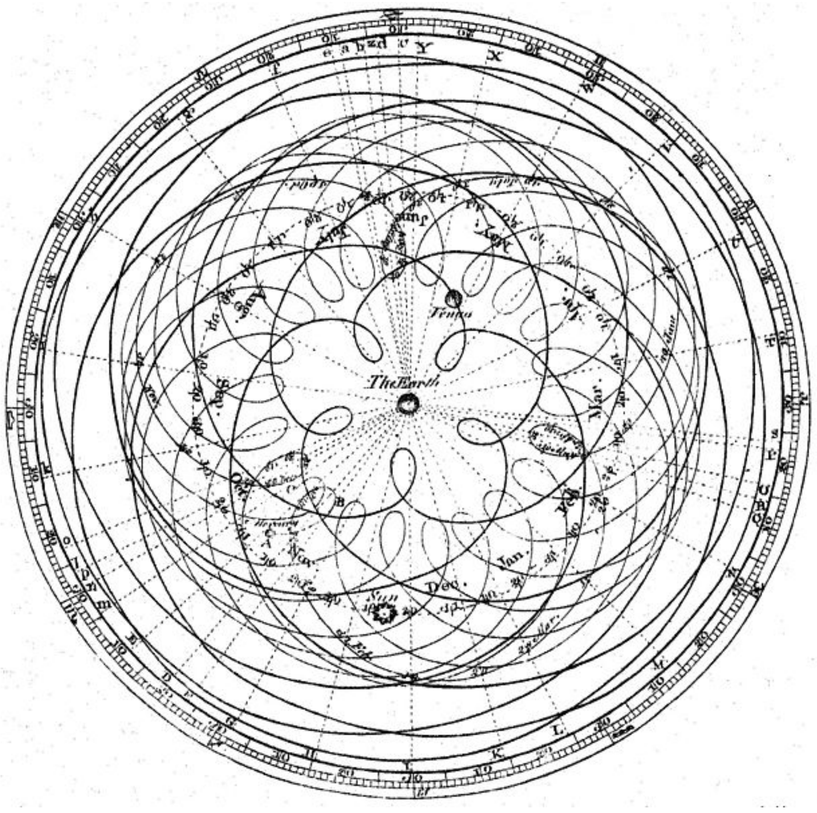
۱.۱ مدل چیست


23h00m
22h30m
22h00m
-5°
-10°
-15°
-20°
ϕ Aqr
λ Aqr
θ Aqr
ψ3 Aqr
ECLIPTIC
σ Aqr
τ2 Aqr
ι Aqr
♂
δ Aqr
66 Aqr
88 Aqr
89 Aqr
2003-06-15

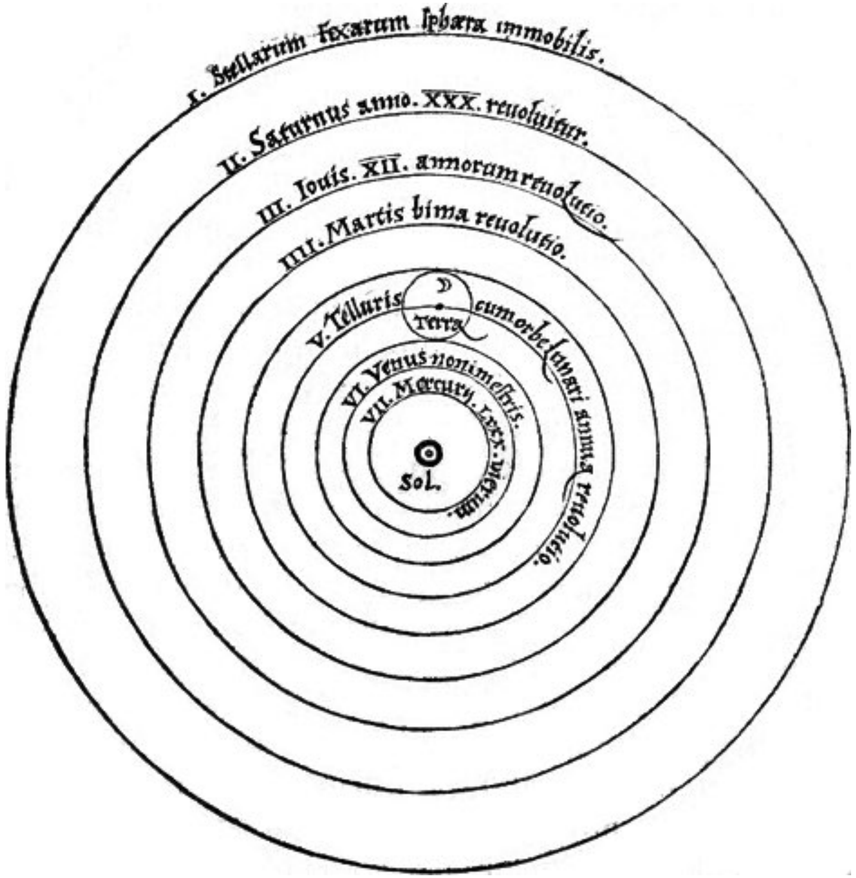
۱.۱ مدل چیست



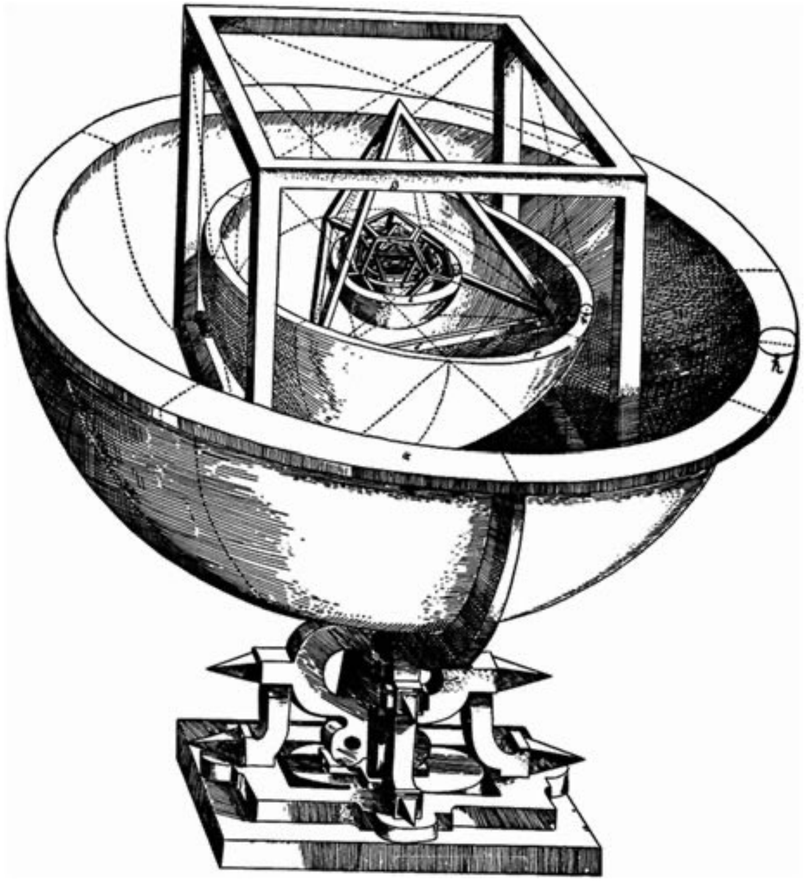
۱.۱ مدل چیست


Jupiter
Venus
The Earth
Mar.
Sat.
Dec.
Jan.
Feb.
Nov.
Oct.
Sept.
Aug.
Mar.
Apr.
May
Jun.
Jul.

۱.۱ مدل چیست


I. Stellarum fixarum sphæra immobilis.
II. Saturnus anno. XXX. reuoluitur.
III. Iouis. XII. annorum reuolutio.
IIII. Martis bima reuolutio.
V. Tellaris cum orbe lunari annua reuolutio.
terra
VI. Venus nonimestris.
VII. Mercurius. LXXX. dierum.
Sol.

۱.۱ مدل چیست



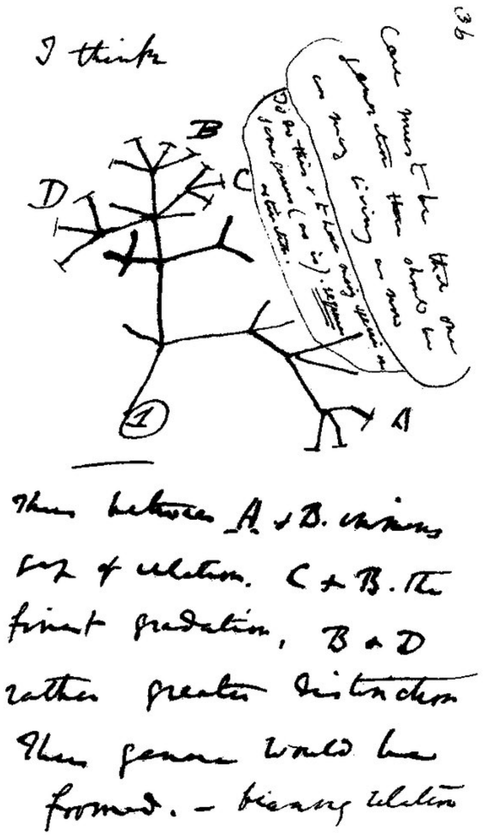
۱.۱ مدل چیست

۱.۱ مدل چیست

اهداف مدل

- یکپارچه‌سازی داده‌های متفاوت در قالب یک ایده ساختارمند
- توجیه مشاهدات
- پیش‌بینی آینده
- پیشنهاد سازوکار

۱.۲ مدل در زیست‌شناسی

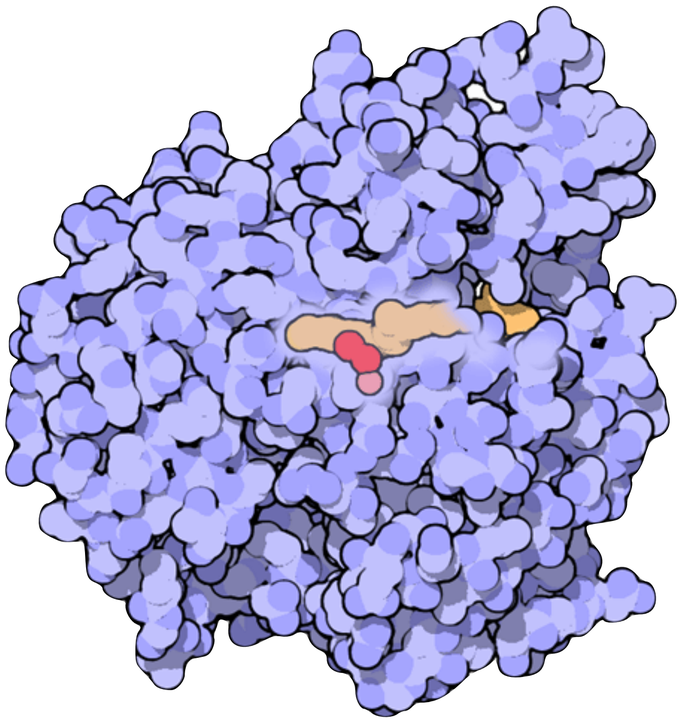


۱.۲ مدل در زیست‌شناسی

میکائیلیس-منتن

$$E + S \rightleftharpoons ES \rightarrow E + P$$

$$V = \frac{V_{max} \cdot S}{K_m + S}$$

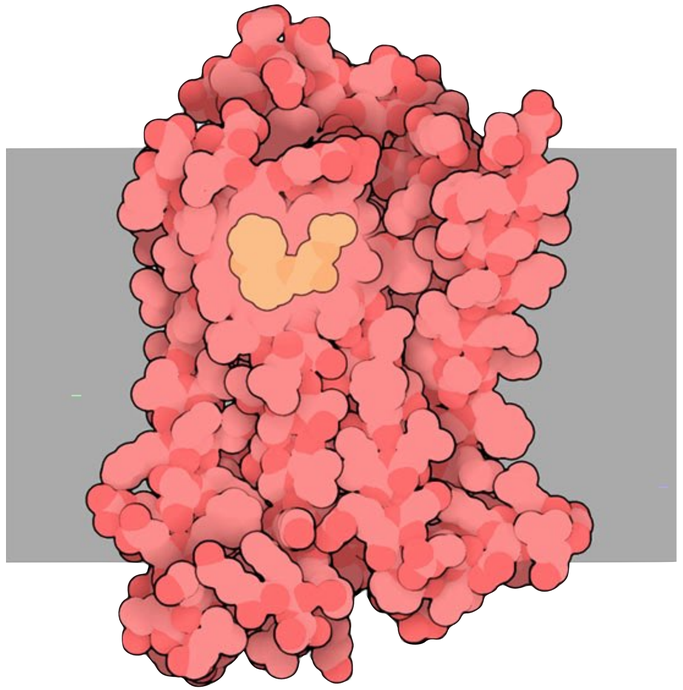


۱.۲ مدل در زیست‌شناسی

لانگلی-هیل

$$R + D \rightleftharpoons DR \rightarrow response$$

$$\frac{[DR]}{R_{total}} = \frac{[D]}{K_d + [D]}$$



۲. جبر

۲. جبر

قوانین توان و رادیکال

$$x^m x^n = x^{m+n}$$

$$(x^m)^n = x^{mn}$$

$$(xy)^n = x^n y^n$$

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

$$\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}$$

$$\sqrt[m]{\sqrt[n]{x}} = \sqrt[n]{\sqrt[m]{x}} = \sqrt[mn]{x}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

$$x^{-n} = \frac{1}{x^n}$$

$$(\frac{x}{y})^n = \frac{x^n}{y^n}$$

$$x^{m/n} = \sqrt[n]{x^m}$$

$$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

۲. جبر

قوانین لگاریتم

$$y = \log_a x \rightarrow a^y = x$$

$$\log_a a^x = x$$

$$\log_a 1 = 0$$

$$\log_a xy = \log_a x + \log_a y$$

$$\log_a x^b = b \log_a x$$

$$a^{\log_a x} = x$$

$$\log_a(\frac{x}{y}) = \log_a x - \log_a y$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

۲. جبر

اتحاد

$$(a+b)(c+d) = ac + ad + bc + bd$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$(a+b)(a-b) = a^2 - b^2$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

۲. جبر

توابع مهم

$$f(x) = b$$

$$f(x) = ax + b \qquad f(x) = ax^2 + bx + c \qquad f(x) = ax^3 + bx^2 + cx + d$$

$$f(x) = \sqrt[n]{x}$$

$$f(x) = \frac{1}{x^n}$$

$$f(x) = b^x$$

$$f(x) = \log_b x$$

$$f(x) = \sin x \qquad f(x) = \cos x$$

۲. جبر

تغییرات تابع

$$g(x) = f(x) + c$$

$$g(x) = f(x + c)$$

$$g(x) = cf(x)$$

$$g(x) = f(cx)$$

۲. جبر

ریشه‌های چندجمله‌ای

$$f(x) = ax + b \implies x = -\frac{b}{a}$$

$$f(x) = ax^2 + bx + c \implies x = \frac{1}{2a}(-b \pm \sqrt{b^2 - 4ac})$$

$$f(x) = x^3 + px + q \implies x = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}$$

۲. جبر

ابوجعفر محمد بن موسی الخوارزمی (۸۵۰-۷۸۰ میلادی)

1. Squares equal to roots.
2. Squares equal to numbers.
3. Roots equal to numbers.
4. Squares and roots equal to numbers
5. Squares and numbers equal to roots
6. Roots and numbers equal to squares

۲. جبر

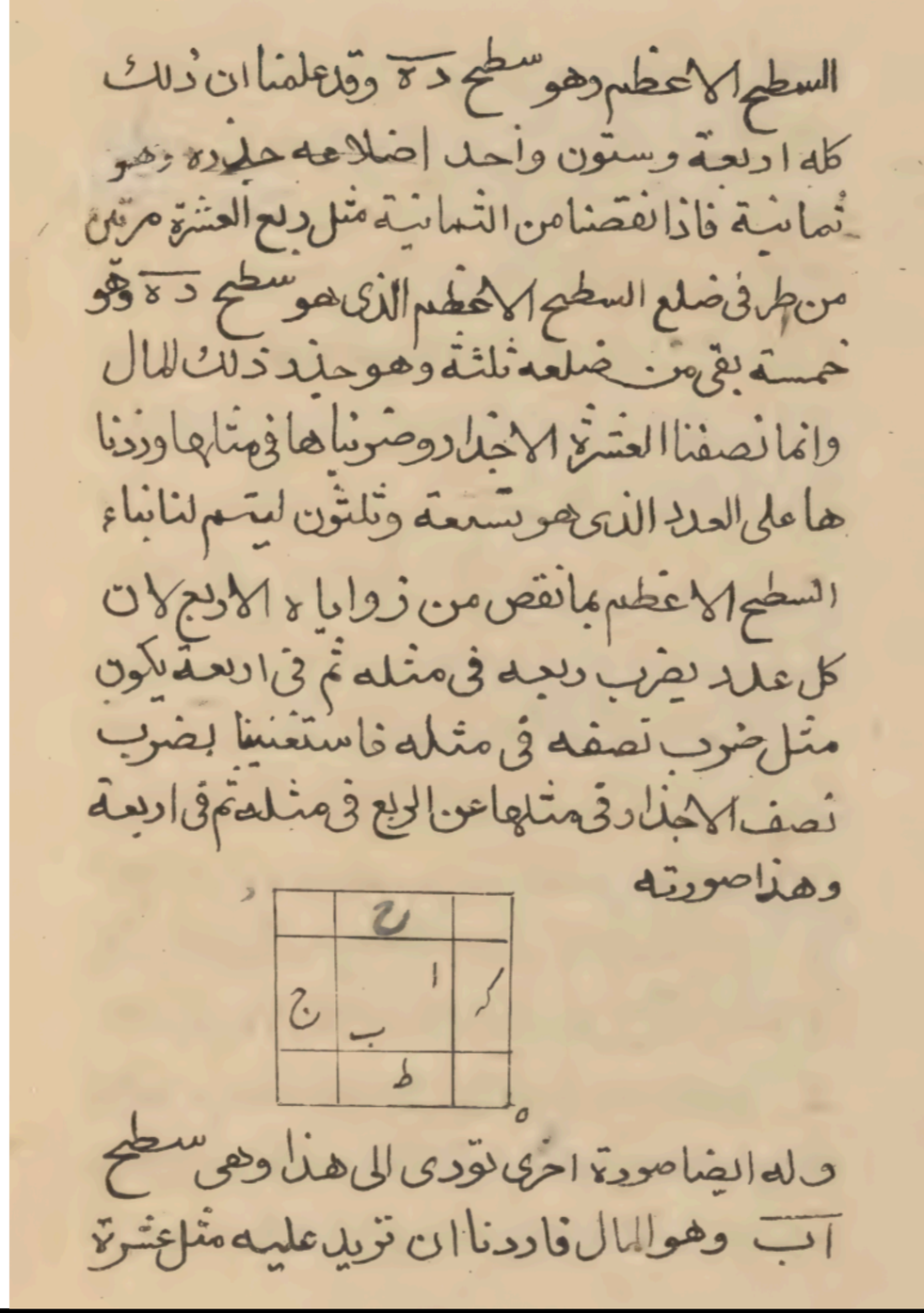
الجبر و المقابله

$$x^2 + 10x = 39$$

$$x^2 + 10x + 4(\frac{10}{4})^2 = 39 + 25$$

$$(x + 5)^2 = 8^2$$

$$x = 3$$


السطح الاعظم وهو سطح ده وقد علمنا ان ذلك
كله اربعة وستون واحد اضلاعه جذره وهو
ثمانية فاذا نقصنا من الثمانية مثل ربع العشرة مرتين
من طرفي ضلع السطح الاعظم الذي هو سطح ده وهو
خمسة بقي من ضلعه ثلثة وهو جذر ذلك المال
وانما نصفنا العشرة الاجذار وضربناها في مثلها وزدنا
ها على العدد الذي هو تسعة وثلثون ليتم لنا بناء
السطح الاعظم بما نقص من زواياه الاربع لان
كل عدد يضرب ربعه في مثله ثم في اربعة يكون
مثل ضرب نصفه في مثله فاستغنينا بضرب
نصف الاجذار في مثلها عن الربع في مثله ثم في اربعة
وهذا صورته

وله ايضا صورة اخرى تودي الى هذا وهي سطح
اب وهو المال فاردنا ان نزيد عليه مثل عشرة

۲. جبر

غیاث‌الدین ابوالفتح عُمَر بن ابراهیم خَیّام نیشابوری (۱۱۴۳-۱۰۴۸ میلادی)

$x^3 = d$

$x^3 + cx = d$

$x^3 + d = cx$

$x^3 = cx + d$

$x^3 + bx^2 = d$

$x^3 + d = bx^2$

$x^3 = bx^2 + d$

$x^3 + bx^2 + cx = d$

$x^3 + bx^2 + d = cx$

$x^3 + cx + d = bx^2$

$x^3 = bx^2 + cx + d$

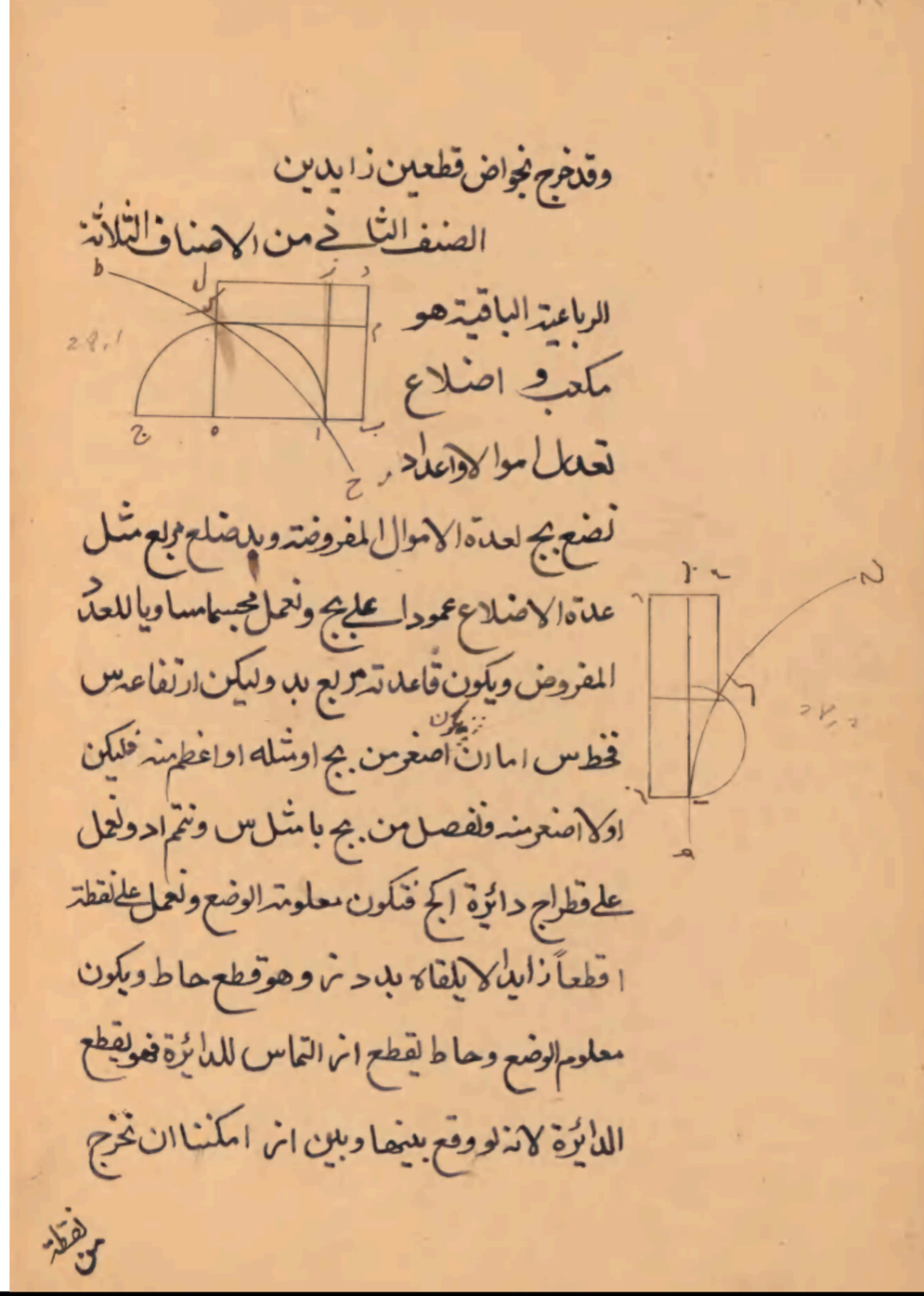
$x^3 + bx^2 = cx + d$

$x^3 + cx = bx^2 + d$

$x^3 + d = bx^2 + cx$

۲. جبر

غیاث‌الدین ابوالفتح عُمَر بن ابراهیم خَیّام نیشابوری (۱۱۴۳-۱۰۴۸ میلادی)


وقد خرج بخواص قطعين زايدين
الصنف الثاني من الاصناف الثلاثة
الرباعية الباقية هو
مكعب و اضلاع
تعدل اموالا واعداد
نضع بج لعدة الاموال المفروضة وبد ضلع مربع مثل
عدة الاضلاع عمودا على بج ونعمل مجسما مساويا للعدد
المفروض ويكون قاعدته مربع بد وليكن ارتفاعه س
فخط س اما ان يكون اصغر من بج او مثله او اعظم منه فليكن
اولا اصغر منه ونفصل من بج با مثل س ونتم اد ونعمل
على قطر اج دائرة اهج فتكون معلومة الوضع ونعمل على نقطة
ا قطعا زايدا لا يلقاه بد د ز وهو قطع حاط ويكون
معلوم الوضع وحاط يقطع از التماس للدائرة فهو يقطع
الدائرة لانه لو وقع بينها وبين از امكننا ان نخرج
من نقطة

“Maybe one of those who will come after us will succeed in finding it.”

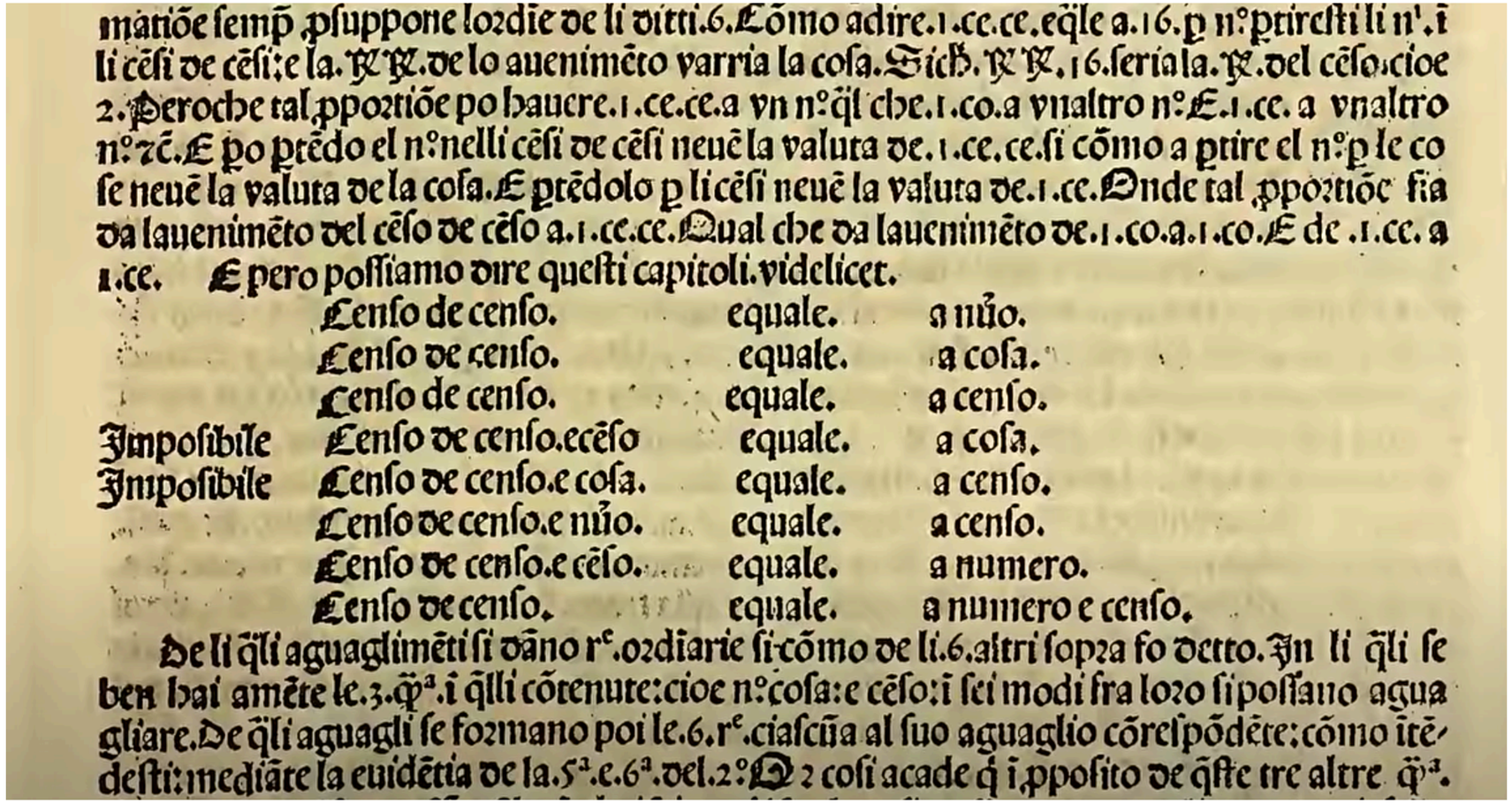
۲. جبر

Luca Paccioli (1447-1517)

“Impossible to solve cubic equations.”

۲. جبر

Sũma De Arithmetica (1494)



matiõe semp̃ ꝓsuppone lordie de li ditti.6.Cõmo adire.1.ce.ce.eqle a.16.p n°ꝓtircesti li n'.i li cẽsi de cẽsi:e la.℞℞.de lo auenimẽto varria la cosa.Sich.℞℞.16.seria la.℞.del cẽso:cioe 2.Peroche tal ꝓportiõe po hauere.1.ce.ce.a vn n°ql che.1.co.a vnaltro n°.E.1.ce. a vnaltro n°.7c̃.E po ptẽdo el n°nelli cẽsi de cẽsi neuẽ la valuta de.1.ce.ce.si cõmo a ptire el n°p le co se neuẽ la valuta de la cosa.E ptẽdolo p li cẽsi neuẽ la valuta de.1.ce.Onde tal ꝓportiõe fia da lauenimẽto del cẽso de cẽso a.1.ce.ce.Qual che da lauenimẽto de.1.co.a.1.co.E de .1.ce. a 1.ce. E pero possiamo dire questi capitoli.videlicet.

	Censo de censo.	equale.	a nũo.
	Censo de censo.	equale.	a cosa.
	Censo de censo.	equale.	a censo.
Impossibile	Censo de censo.ecẽso	equale.	a cosa.
Impossibile	Censo de censo.e cosa.	equale.	a censo.
	Censo de censo.e nũo.	equale.	a censo.
	Censo de censo.e cẽso.	equale.	a numero.
	Censo de censo.	equale.	a numero e censo.

De li q̃li aguaglimẽti si dãno r^e.ordiarie si cõmo de li.6.altri sopra fo detto.In li q̃li se ben hai amẽte le.3.q̃^a.i q̃lli cõtenute:cioe n°cosa:e cẽso:i sei modi fra loro si possano agua gliare.De q̃li aguagli se formano poi le.6.r^e.ciascũa al suo aguaglio cõrespõdẽte:cõmo itẽ- desti:mediãte la euidẽtia de la.5^a.e.6^a.del.2°.Q 2 cosi acade q̃ i ꝓposito de q̃ste tre altre q̃^a.

۲. جبر

Scipione Del ferro (1465-1526)

$$x^3 + px = q$$

$$\sqrt[3]{\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \sqrt[3]{\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}$$

Finds it around 1510

۲. جبر

Niccolo Fontana Tartaglia (1500-1557)

Quando chel cubo con le cose appresso
Se aqquaglia ´a qualche numero discreto
Trouan duo altri differenti in esso
Dapoi terrai questo per consueto
Che”llor productto sempre sia equale
Alterzo cubo delle cose neto,
El residuo poi suo generale
Delli lor lati cubi ben sottrati
Varra la tua cosa principale.
In el secondo de cotestiatti
Quando chel cubo restasse lui solo
Tu osseruarai questaltri contratti,
Del numer farai due tal part``a uolo
Che luna in laltra si produca schietto
El terzo cubo delle cose in stolo
Delle qual poi, per communprecetto
Torrai li lati cubi insieme gionti
Et cotal somma sara il tuo concetto.
El terzo poi de questi nostri conti
Se solue col secondo se ben guardi
Che per natura son quasi congionti.
Questi trouai, non con passi tardi
Nel mille cinquecent`e, quatroe trenta
Con fondamenti ben sald`e gagliardi
Nella citta dal marintorno centa.

۲. جبر

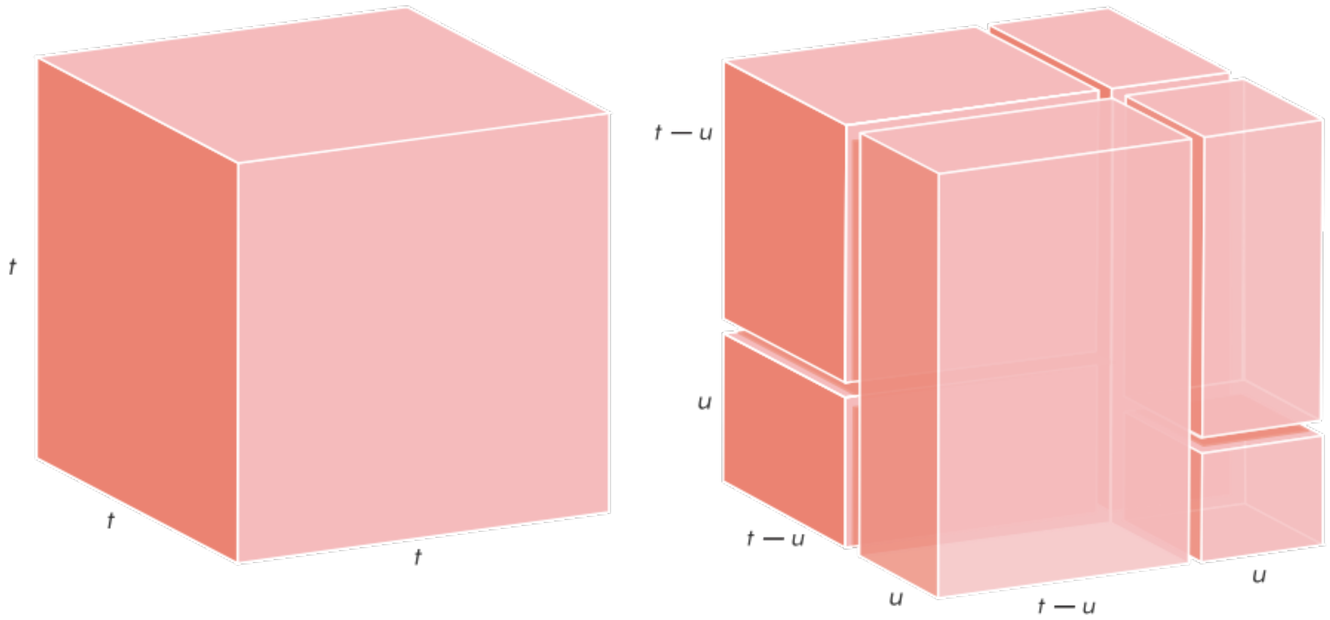
Niccolo Fontana Tartaglia (1500-1557)

When the cube with the things together
Are set equal to some discrete number
Find other two that by it differ
You will get into the habit
Of making their product always equal
To the third of the things cubed extactly
The result then
Of their cubic roots properly subtracted
Will be equal to your principal thing.
In the second of these cases
When the cube remains alone
You will take these other steps
From the number you will make twe two parts
So that one in the other yields
The third of the cube of the number of things
Of these two parts by known precept
You'll joint together their cubic roots
And this sum will be your answer.
Then the third one of your calculations
It's solved with the second,
If you look carefully
Because by nature they're related
All these I found,
and not with slow steps
In fifteen hundred thirty four,
With firm and vigorous foundations
In the city surrounded by the sea

۲. جبر

12th February 1535

$$x^3 + 9x = 26$$



۲. جبر

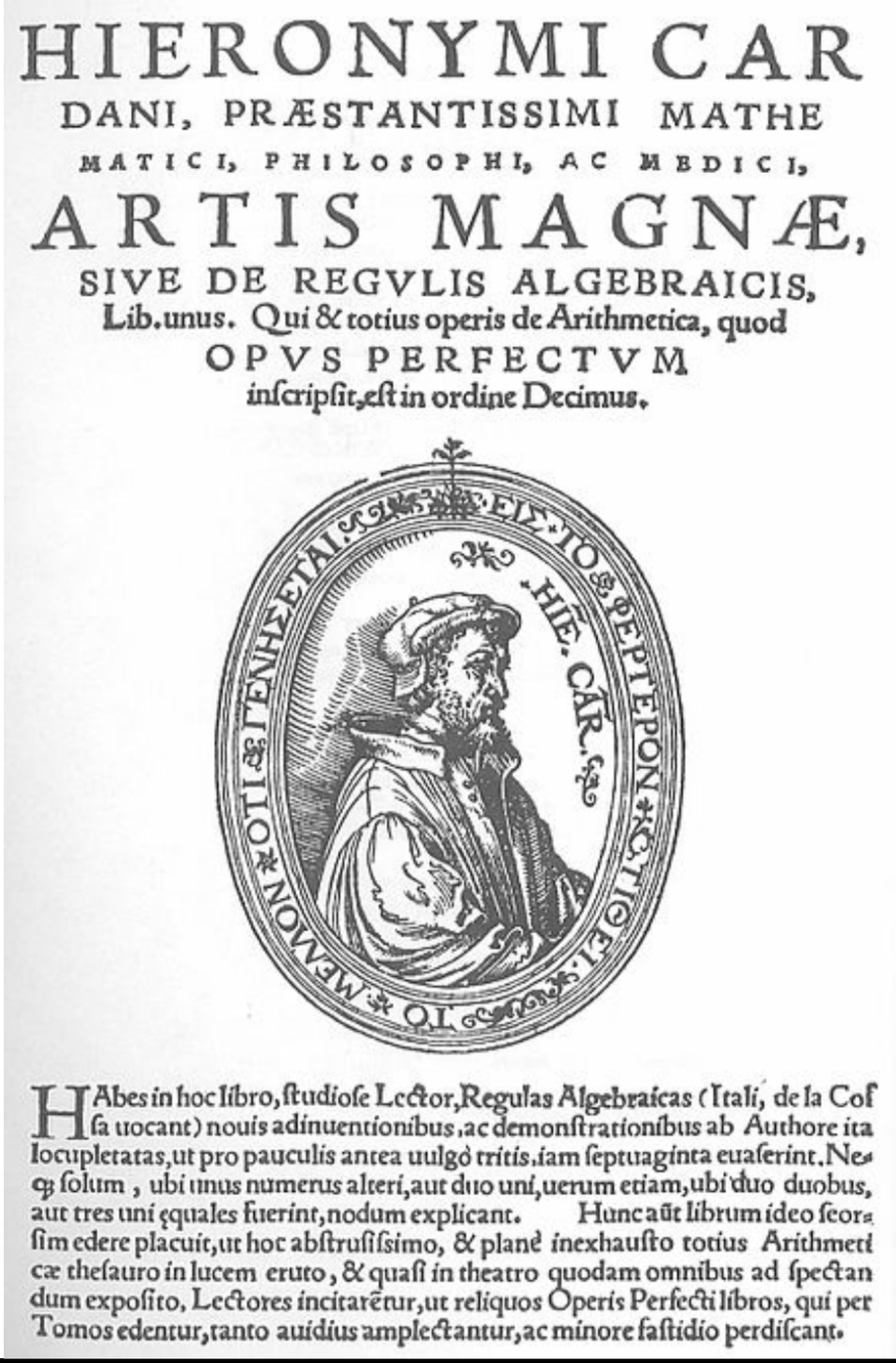
Geloramo Cardano (1501-1576)

۲. جبر

Artis Magnae (1545)

$x + y = 10$

$xy = 40$



HIERONYMI CAR
DANI, PRÆSTANTISSIMI MATHE
MATICI, PHILOSOPHI, AC MEDICI,
ARTIS MAGNÆ,
SIVE DE REGVLIS ALGEBRAICIS,
Lib. unus. Qui & totius operis de Arithmetica, quod
OPVS PERFECTVM
inscripsit, est in ordine Decimus.

HAbes in hoc libro, studiose Lector, Regulas Algebraicas (Itali, de la Cos sa uocant) nouis adinuentionibus, ac demonstrationibus ab Authore ita locupletatas, ut pro pauculis antea uulgò tritis, iam septuaginta euaserint. Neq; solum, ubi unus numerus alteri, aut duo uni, uerum etiam, ubi duo duobus, aut tres uni æquales fuerint, nodum explicant. Hunc aũt librum ideo seorsim edere placuit, ut hoc abstrusissimo, & planè inexhausto totius Arithmeticæ thesauro in lucem eruto, & quasi in theatro quodam omnibus ad spectandum exposito, Lectores incitarẽtur, ut reliquos Operis Perfecti libros, qui per Tomos edentur, tanto auidius amplectantur, ac minore fastidio perdiscant.

“Pitiful, A man of no substance. A very stupid man. An ignoramus in mathematical matters”

۲. جبر

Rafael Bombelli (1526-1572)

$$x^3 - 6x + 4$$

$$\sqrt[3]{\frac{1}{2}(-4+\sqrt{-16})} + \sqrt[3]{\frac{1}{2}(-4-\sqrt{-16})}$$

$$(1+\sqrt{-1})^3 = -2 + 2\sqrt{-1}$$

$$(1-\sqrt{-1})^3 = -2 - 2\sqrt{-1}$$

$$x = 2$$

۲. جبر

Rafael Bombelli (1526-1572)

"Neither positive nor negative"

$2 + 3i = 2\ p\ di\ m\ 3$

$2 - 3i = 2\ m\ di\ m\ 3$

۲. جبر

René Decartes (1596-1659)

“Imaginary numbers”

۲. جبر

Leonhard Euler (1707-1783)

$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = +1$$

$$i^5 = i$$

۲. جبر

اعداد مختلط

$$a + bi$$

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$$

$$\frac{(a + bi)}{(c + di)} = \frac{(ac + bd) + (bc - ad)i}{(c^2 + d^2)}$$

۲. جبر

اعداد مختلط

$$a + bi \to r = \sqrt{a^2 + b^2}$$

$$a + bi = r(\cos\theta + \sin\theta i)$$

$$r_1(\cos\theta_1 + \sin\theta_1 i)r_2(\cos\theta_2 + \sin\theta_2 i) = r_1 r_2[\cos(\theta_1 + \theta_2) + \sin(\theta_1 + \theta_2)i]$$

$$z^n = 1 \implies \theta = \frac{2k\pi}{n}$$

۳. جبر خطی

۳. جبر خطی

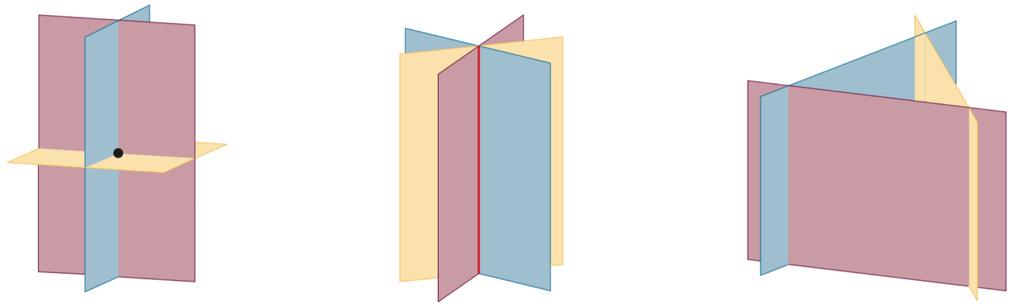
دستگاه معادله خطی

A system of linear equations

$$\begin{cases} x - 2y - z = 1 \\ -x + 3y + 3z = 4 \\ 2x - 3y + z = 10 \end{cases}$$

A system in triangular form

$$\begin{cases} x - 2y - z = 1 \\ y + 2z = 5 \\ z = 3 \end{cases}$$



(a) The three planes intersect at a single point. The system has one solution.

(b) The three planes intersect at more than one point. The system has infinitely many solutions.

(c) The three planes have no point in common. The system has no solution.

۳. جبر خطی

دستگاه معادله خطی

Not in row-echelon form

$$\begin{bmatrix} 0 & 1 & -\frac{1}{2} & 0 & 6 \\ 1 & 0 & 3 & 4 & -5 \\ 0 & 0 & 0 & 1 & 0.4 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Leading 1's do *not* shift to the right in successive rows

Row-echelon form

$$\begin{bmatrix} 1 & 3 & -6 & 10 & 0 \\ 0 & 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Leading 1's shift to the right in successive rows

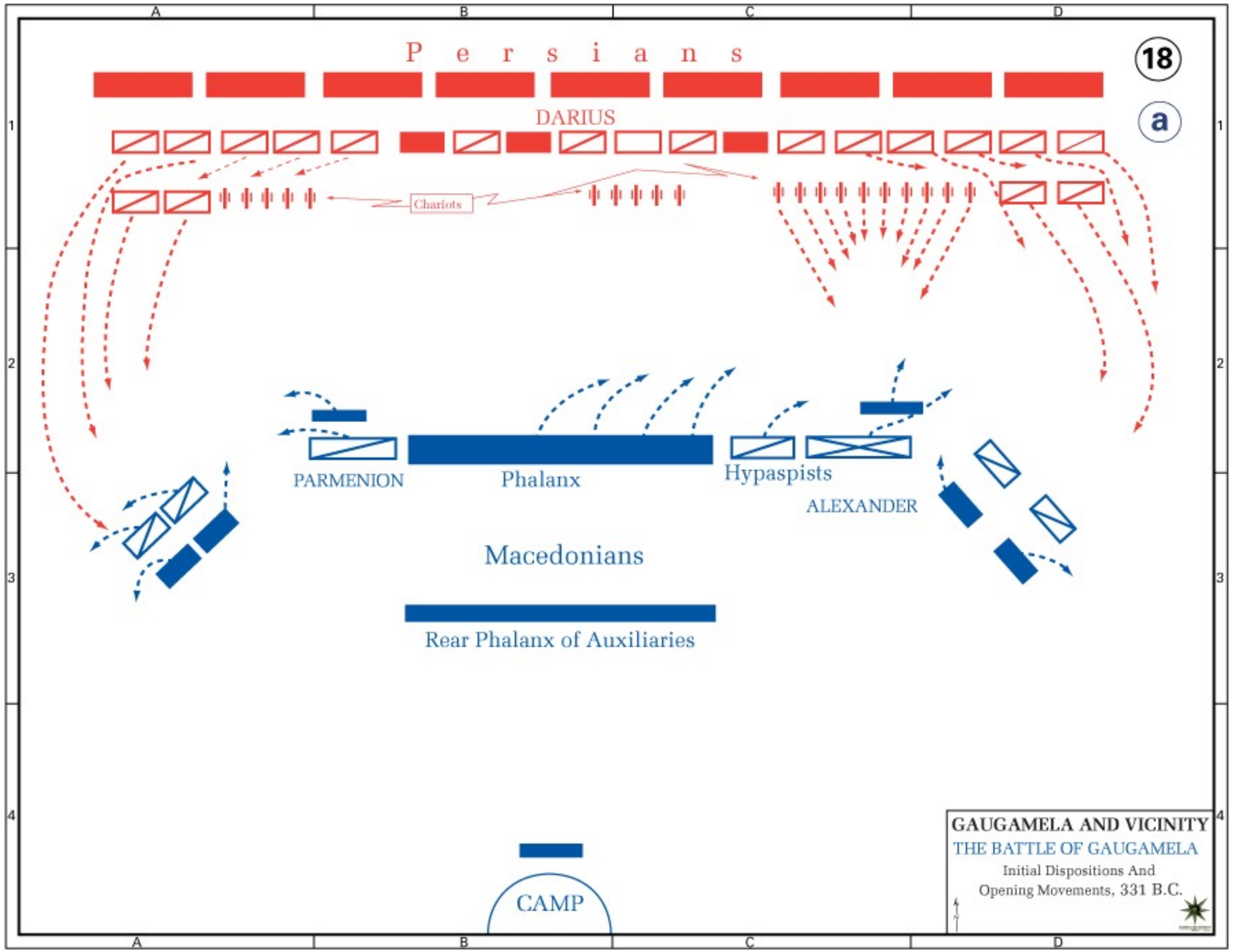
Reduced row-echelon form

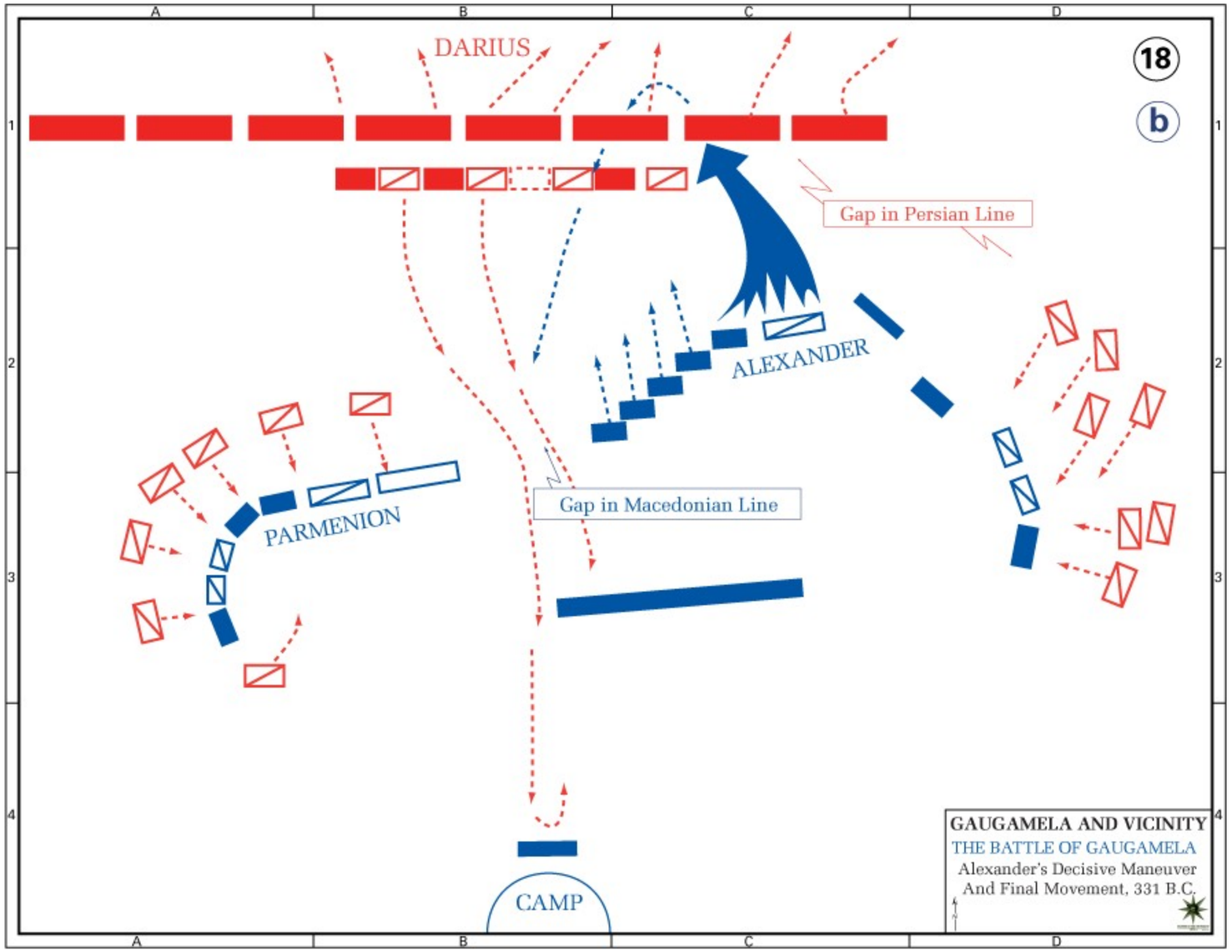
$$\begin{bmatrix} 1 & 3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Leading 1's have 0's above and below them

This is how the process might work for a 3×4 matrix:

$$\begin{bmatrix} 1 & \blacksquare & \blacksquare & \blacksquare \\ 0 & \blacksquare & \blacksquare & \blacksquare \\ 0 & \blacksquare & \blacksquare & \blacksquare \end{bmatrix} \qquad \begin{bmatrix} 1 & \blacksquare & \blacksquare & \blacksquare \\ 0 & 1 & \blacksquare & \blacksquare \\ 0 & 0 & \blacksquare & \blacksquare \end{bmatrix} \qquad \begin{bmatrix} 1 & \blacksquare & \blacksquare & \blacksquare \\ 0 & 1 & \blacksquare & \blacksquare \\ 0 & 0 & 1 & \blacksquare \end{bmatrix}$$


Persians
18
a
DARIUS
Chariots
PARMENION
Phalanx
Hypaspists
ALEXANDER
Macedonians
Rear Phalanx of Auxiliaries
CAMP
GAUGAMELA AND VICINITY
THE BATTLE OF GAUGAMELA
Initial Dispositions And
Opening Movements, 331 B.C.


DARIUS
18
b
Gap in Persian Line
ALEXANDER
PARMENION
Gap in Macedonian Line
CAMP
GAUGAMELA AND VICINITY
THE BATTLE OF GAUGAMELA
Alexander's Decisive Maneuver
And Final Movement, 331 B.C.

۳. جبر خطی

دستگاه معادله خطی

Linear system

$$\begin{cases} 3x - 2y + z = 5 \\ x + 3y - z = 0 \\ -x + 4z = 11 \end{cases}$$

Augmented matrix

$$\begin{bmatrix} 3 & -2 & 1 & 5 \\ 1 & 3 & -1 & 0 \\ -1 & 0 & 4 & 11 \end{bmatrix}$$

Symbol	Description
$R_i + kR_j \to R_i$	Change the ith row by adding k times row j to it, and then put the result back in row i.
kR_i	Multiply the ith row by k.
$R_i \leftrightarrow R_j$	Interchange the ith and jth rows.

System

$$\begin{cases} x - y + 3z = 4 \\ x + 2y - 2z = 10 \\ 3x - y + 5z = 14 \end{cases}$$

$$\begin{cases} x - y + 3z = 4 \\ 3y - 5z = 6 \\ 2y - 4z = 2 \end{cases}$$

$$\begin{cases} x - y + 3z = 4 \\ 3y - 5z = 6 \\ y - 2z = 1 \end{cases}$$

$$\begin{cases} x - y + 3z = 4 \\ z = 3 \\ y - 2z = 1 \end{cases}$$

$$\begin{cases} x - y + 3z = 4 \\ y - 2z = 1 \\ z = 3 \end{cases}$$

Augmented matrix

$$\begin{bmatrix} 1 & -1 & 3 & 4 \\ 1 & 2 & -2 & 10 \\ 3 & -1 & 5 & 14 \end{bmatrix}$$

$$\xrightarrow[R_3 - 3R_1 \to R_3]{R_2 - R_1 \to R_2} \begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 3 & -5 & 6 \\ 0 & 2 & -4 & 2 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{2}R_3} \begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 3 & -5 & 6 \\ 0 & 1 & -2 & 1 \end{bmatrix}$$

$$\xrightarrow{R_2 - 3R_3 \to R_2} \begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & -2 & 1 \end{bmatrix}$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

۳. جبر خطی

دستگاه معادله خطی

$$\begin{cases} 4x + 8y - 4z = 4 \\ 3x + 8y + 5z = -11 \\ -2x + y + 12z = -17 \end{cases}$$

Augmented matrix:

$$\begin{bmatrix} 4 & 8 & -4 & 4 \\ 3 & 8 & 5 & -11 \\ -2 & 1 & 12 & -17 \end{bmatrix}$$

Need a 1 here

$$\xrightarrow{\frac{1}{4}R_1} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 3 & 8 & 5 & -11 \\ -2 & 1 & 12 & -17 \end{bmatrix}$$

Need 0's here

$$\xrightarrow[R_3 + 2R_1 \to R_3]{R_2 - 3R_1 \to R_2} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 2 & 8 & -14 \\ 0 & 5 & 10 & -15 \end{bmatrix}$$

Need a 1 here

$$\xrightarrow{\frac{1}{2}R_2} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 5 & 10 & -15 \end{bmatrix}$$

Need a 0 here

$$\xrightarrow{R_3 - 5R_2 \to R_3} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & -10 & 20 \end{bmatrix}$$

Need a 1 here

Row-echelon form:

$$\xrightarrow{-\frac{1}{10}R_3} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

۳. جبر خطی

دستگاه معادله خطی

$$\begin{cases} 4x + 8y - 4z = 4 \\ 3x + 8y + 5z = -11 \\ -2x + y + 12z = -17 \end{cases}$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

Need 0's here

Need a 0 here

$$\xrightarrow[R_1 + R_3 \to R_1]{R_2 - 4R_3 \to R_2} \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$$\xrightarrow{R_1 - 2R_2 \to R_1} \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

۳. جبر خطی

دستگاه معادله خطی

No solution

$$\begin{bmatrix} 1 & 2 & 5 & 7 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Last equation says 0 = 1

One solution

$$\begin{bmatrix} 1 & 6 & -1 & 3 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & 1 & 8 \end{bmatrix}$$

Each variable is a leading variable

Infinitely many solutions

$$\begin{bmatrix} 1 & 2 & -3 & 1 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

z is not a leading variable

۳. جبر خطی

بردار

$$\overrightarrow{u} = [x, y]$$

$$\overrightarrow{u} = x\hat{i} + y\hat{j}$$

$$|\overrightarrow{u}| = \sqrt{u_x^2 + u_y^2}$$

$$c\overrightarrow{u} = cx\hat{i} + cy\hat{j}$$

$$\overrightarrow{u} + \overrightarrow{v} = (u_x + v_x)\hat{x} + (u_y + v_y)\hat{y}$$

$$\overrightarrow{u} \bullet \overrightarrow{v} = u_x v_x + u_y v_y = |\overrightarrow{u}|\ |\overrightarrow{v}| \cos\theta$$

۳. جبر خطی

ماتریس

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \to i \times j$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax+by \\ cx+dy \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

۳. جبر خطی

ماتریس

$$A = \begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix}$$

Entry	Inner product of:	Value	Product matrix
c_{12}	$\begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix}$	$1\cdot 5 + 3\cdot 4 = 17$	$\begin{bmatrix} -1 & 17 & \\ & & \end{bmatrix}$
c_{13}	$\begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix}$	$1\cdot 2 + 3\cdot 7 = 23$	$\begin{bmatrix} -1 & 17 & 23 \\ & & \end{bmatrix}$
c_{21}	$\begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix}$	$(-1)\cdot(-1) + 0\cdot 0 = 1$	$\begin{bmatrix} -1 & 17 & 23 \\ 1 & & \end{bmatrix}$
c_{22}	$\begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix}$	$(-1)\cdot 5 + 0\cdot 4 = -5$	$\begin{bmatrix} -1 & 17 & 23 \\ 1 & -5 & \end{bmatrix}$
c_{23}	$\begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix}$	$(-1)\cdot 2 + 0\cdot 7 = -2$	$\begin{bmatrix} -1 & 17 & 23 \\ 1 & -5 & -2 \end{bmatrix}$

Thus we have $$AB = \begin{bmatrix} -1 & 17 & 23 \\ 1 & -5 & -2 \end{bmatrix}$$

۳. جبر خطی

ماتریس

$$det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$$

$$[A^T]_{ij} = [A]_{ji}$$

$$AA^{-1} = A^{-1}A = I_n$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{(ad - bc)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

۳. جبر خطی

ماتریس

THE DETERMINANT OF A SQUARE MATRIX

If A is an $n \times n$ matrix, then the **determinant** of A is obtained by multiplying each element of the first row by its cofactor and then adding the results. In symbols,

$$\det(A) = |A| = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = a_{11}A_{11} + a_{12}A_{12} + \cdots + a_{1n}A_{1n}$$

Where $A_{ij} = (-1)^{i+j}M_{ij}$ and M_{ij} is the determinant of matrix M obtained by removing ith row and jth column.

۳. جبر خطی

ماتریس

To find the inverse of an $n \times n$ matrix, first construct an $n \times 2n$ matrix:

$$\left[\begin{array}{cccc|cccc} a_{11} & a_{12} & \cdots & a_{1n} & 1 & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & a_{2n} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} & 0 & 0 & \cdots & 1 \end{array}\right]$$

Then use elementary row operations to turn the left side to the identity matrix (change the whole matrix to reduced row-echelon form). The right side is automatically transformed into the inverse matrix.

۳. جبر خطی

ماتریس

$$A\overrightarrow{x} = \lambda\overrightarrow{x}$$

$$det(A - \lambda I) = 0$$

$$det\ (\begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix}) = 0$$

$$\lambda^2 - \tau\lambda + \Delta = 0$$

۴. حسابان

۴. حسابان

حد

$$\forall\ \epsilon > 0\ \exists\ \delta > 0$$

$$0 < |x - a| < \delta \implies |f(x) - L| < \epsilon$$

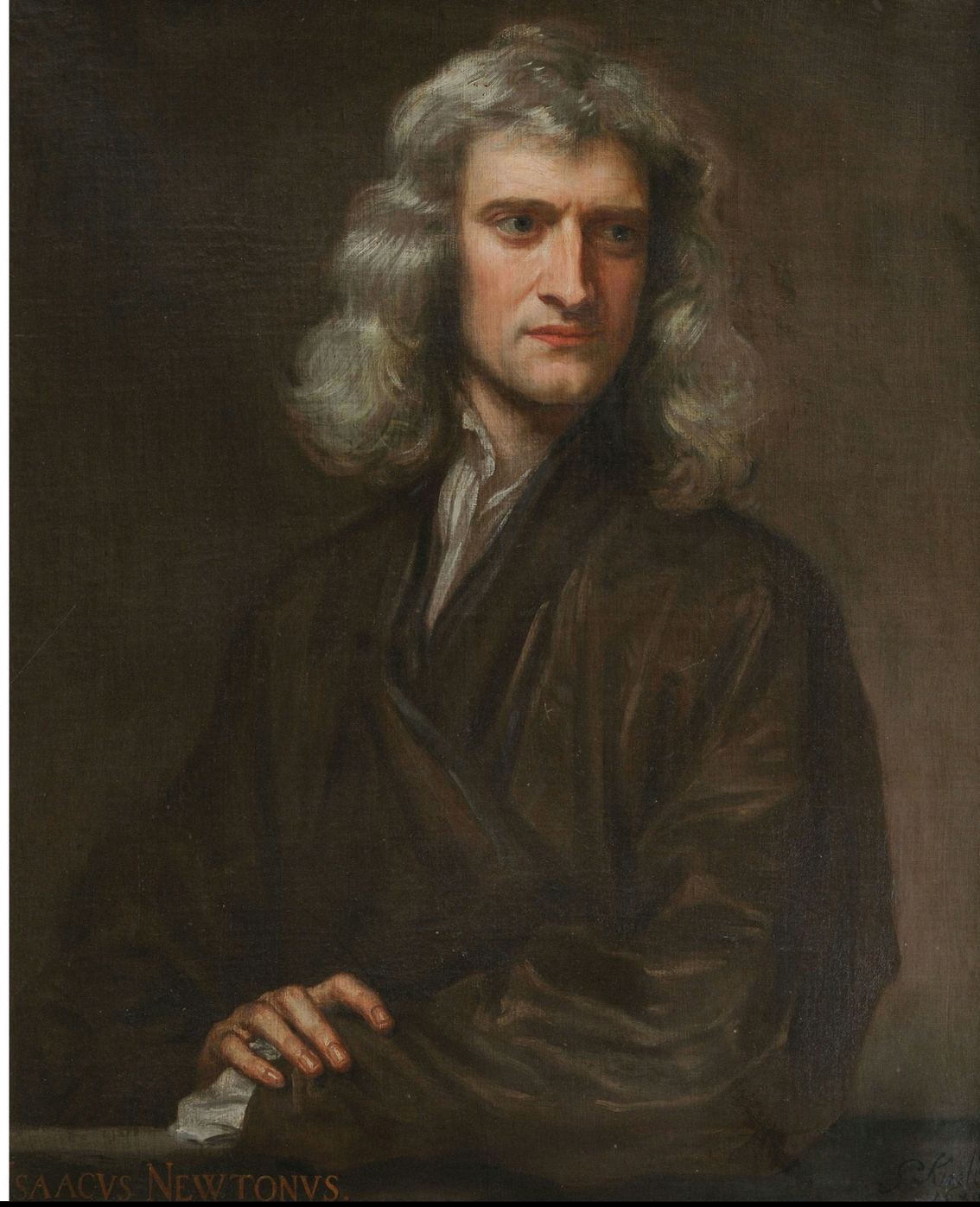
$$\lim_{x \to a} f(x) = L$$

۴. حسابان

Isaac Newton (1643-1727)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h}$$



۴. حسابان

Gottfried Wilhelm Leibniz (1646-1716)

$$\frac{\mathrm{d}f}{\mathrm{d}x} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} = \lim_{h\to 0} \frac{\left.\frac{\mathrm{d}f}{\mathrm{d}x}\right|_{x+h} - \left.\frac{\mathrm{d}f}{\mathrm{d}x}\right|_{x}}{h}$$

۴. حسابان

مشتق

$$(cf)' = cf'$$

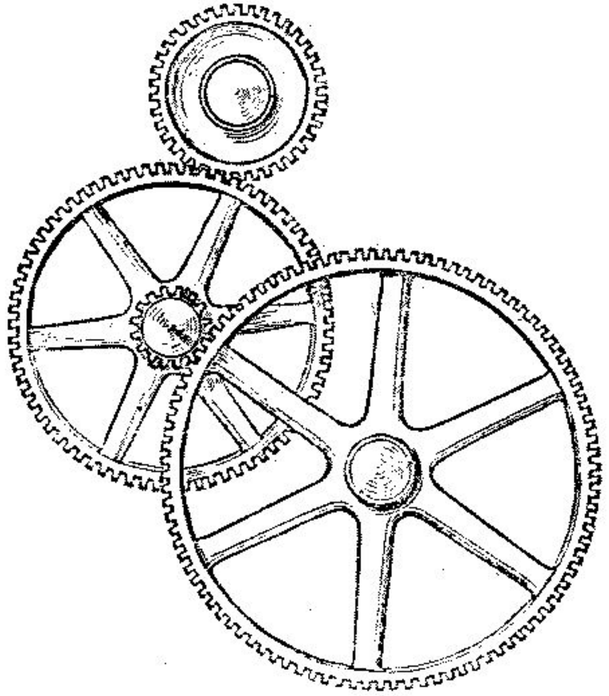
$$(f+g)' = f' + g'$$

$$(f-g)' = f' - g'$$

$$(fg)' = fg' + gf'$$

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

$$(fog)'(x) = f'(g(x)) \,.\, g'(x)$$



۴. حسابان

مشتق

$$\frac{\mathrm{d}}{\mathrm{d}x}\, c = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\, x^n = nx^{n-1}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\, b^x = \ln b \,.\, b^x$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \log_b x = \frac{1}{\ln b}\frac{1}{x}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\, \sin x = \cos x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\, \cos x = -\sin x$$

۴. حسابان

عدد اویلر

$$\lim_{h\to 0}\frac{e^h-1}{h}=1$$

$$\ln e = 1$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\,e^x = e^x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\ln x = \frac{1}{x}$$

۴. حسابان

انتگرال

$$\Delta x = \frac{(b-a)}{n} \ , \ x_i = a + i\Delta x$$

$$\int_a^b f(x)\ \mathrm{d}x = \lim_{n\to\infty} \sum_{i=1}^{\infty} f(x_i)\Delta x$$

۴. حسابان

انتگرال

$$F(x) = \int_a^x f(t)\ \mathrm{d}t \implies F'(x) = f(x)$$

$$\int_a^b f(x)\ \mathrm{d}x = F(b) - F(a)$$

$$\int_a^b f(x)\ \mathrm{d}x = \int f(x)\ \mathrm{d}x \Big|_a^b$$

۴. حسابان

انتگرال

$$\int cf(x)\ \mathrm{d}x = c\int f(x)\ \mathrm{d}x$$

$$\int [f(x)+g(x)]\ \mathrm{d}x = \int f(x)\ \mathrm{d}x + \int g(x)\ \mathrm{d}x$$

$$\int [f(x)-g(x)]\ \mathrm{d}x = \int f(x)\ \mathrm{d}x - \int g(x)\ \mathrm{d}x$$

۴. حسابان

انتگرال

$$\int k \, \mathrm{d}x = kx + C$$

$$\int x^n \, \mathrm{d}x = \frac{x^{n+1}}{n+1} + C$$

$$\int b^x \, \mathrm{d}x = \frac{b^x}{\ln b} + C$$

$$\int \frac{1}{x} \, \mathrm{d}x = \ln|x| + C$$

$$\int \sin x \, \mathrm{d}x = -\cos x + C$$

$$\int \cos x \, \mathrm{d}x = \sin x + C$$

۴. حسابان

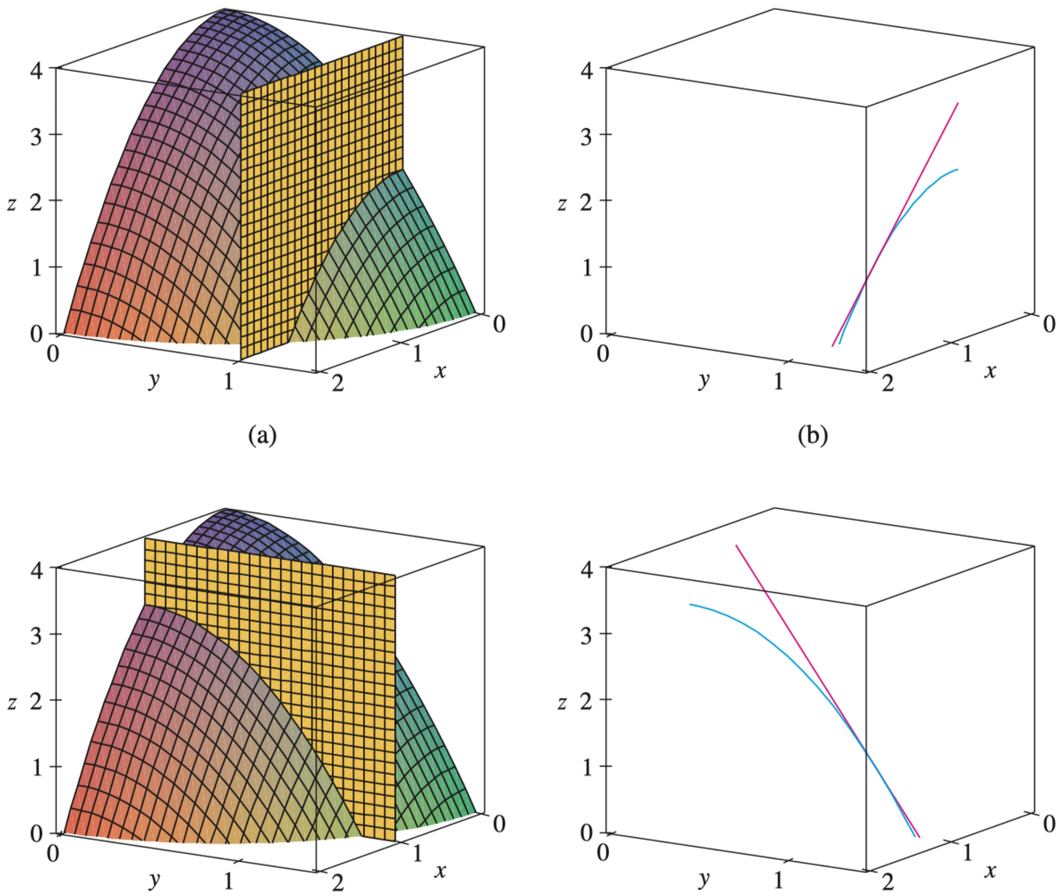
مشتق جزئی

$$f_x(x,y) = \frac{\partial f}{\partial x} = \lim_{h\to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

$$f_y(x,y) = \frac{\partial f}{\partial y} = \lim_{h\to 0} \frac{f(x,y+h) - f(x,y)}{h}$$

۴. حسابان

مشتق جزئی



۴. حسابان

بسط تیلور

$$f(a) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

۴. حسابان

بسط تیلور

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

۵. معادله دیفرانسیل

۲.۴ معادلات دیفرانسیل

$f' \to f$

$$\dot{x} = \frac{\mathrm{d}x}{\mathrm{d}t} = f(x)$$

$$x_{t+\Delta t} = x_t + \dot{x}(x_t)$$

$\dot{x}(x^*) = 0 \to x^*$ is an equilibrium point

$\ddot{x}(x^*) < 0 \to x^*$ is a stable equilibrium point

$\ddot{x}(x^*) > 0 \to x^*$ is a unstable equilibrium point

۲.۴ معادلات دیفرانسیل

$$\dot{x} = \sin x$$

۲.۴ معادلات دیفرانسیل

$$\dot{x} = x - \cos x$$

۶. احتمال و اندکی آمار

Applications
19th Century
1900
1908
1925
2016b
2000
2016a
1933
1937
1979
1995
2001
1972
1962
1950
Mathematics
Computation

Pierre de Fermat (1607-1665)

Monsieur,

If I undertake to make a point with a single die in eight throws, and if we agree after the money is put at stake, that I shall not cast the first throw, it is necessary by my theory that I take 1/6 of the total sum to he impartial because of the aforesaid first throw.

Blaise Pascal (1623-1669)

Monsieur,

1. Impatience has seized me as well as it has you, and although I am still abed, I cannot refrain from telling you that I received your letter in regard to the problem of the points 1 yesterday evening from the hands of M. Carcavi, and that I admire it more than I can tell you. I do not have the leisure to write at length, but, in a word, you have found the two divisions of the points and of the dice with perfect justice. I am thoroughly satisfied as I can no longer doubt that I was wrong, seeing the admirable accord in which I find myself with you.

7. i have no time to send you the proof of a difficult point which astonished M. (de M´er´e) so greatly, for he has ability but he is not a geometer (which is, as you know, a great defect) and he does not even comprehend that a mathematical line is infinitely divisible and he is firmly convinced that it is composed of a finite number of points. I have never been able to get him out of it. If you could do so, it would make him perfect.

He tells me then that he has found an error in the numbers for this reason .

If one undertakes to throw a six with a die, the advantage of undertaking to do it in 4 is as 671 is to 625.

If one undertakes to throw double sixes with two dice the disadvantage of the undertaking is 24.

But nonetheless, 24 is to 36 (which is the number of faces of two dice) 2 as 4 is to 6 (which is the number of faces of one die).

This is what was his great scandal which made him say haughtily that the theorems were not consistent and that arithmetic was demented. But you will easily see the reason by the principles which you have.

Monsieur,

1. Our interchange of blows still continues, and I am well pleased that our thoughts are in such complete adjustment as it seems since they have taken the same direction and followed the same road.

3. For the rest, there is nothing that I will not write you in the future with all frankness. Meditate however, if you find it convenient, on this theorem: The squared powers of 2 augmented by unity 11 are always prime numbers. [That is,] The square of 2 augmented by unity makes 5 which is a prime number;
The square of the square makes 16 which, when unity is added makes 17, a prime number;
The square of 16 makes 256 which, when unity is added, makes 257, a prime number;
The square of 256 makes 65536 which, when unity is added, makes 65537, a prime number;
and so to infinity.
This is a property whose truth I will answer to you. The proof of it is very difficult and I assure you that I have not yet been able to find it fully. I shall not set it for you to find unless I come to the end of it.

Leonhard Euler (1707-1783)

No

A game of chance

- A game of chance
- Sample space
- Event

- $\Omega = \{s_1, s_2 \ldots, s_n\}$
- Mutually exclusive and homogenous
- $A \subset \Omega$

$$\mathbb{P}(A) = \frac{|A|}{|\Omega|}$$

Axioms of probability

- non-zero
- Certainty
- Disjoint events

- $\mathbb{P}(A) \geq 0$
- $\mathbb{P}(\Omega) = 1$
- $\mathbb{P}(A) + \mathbb{P}(B)$

Results of axioms of probability

- Complement

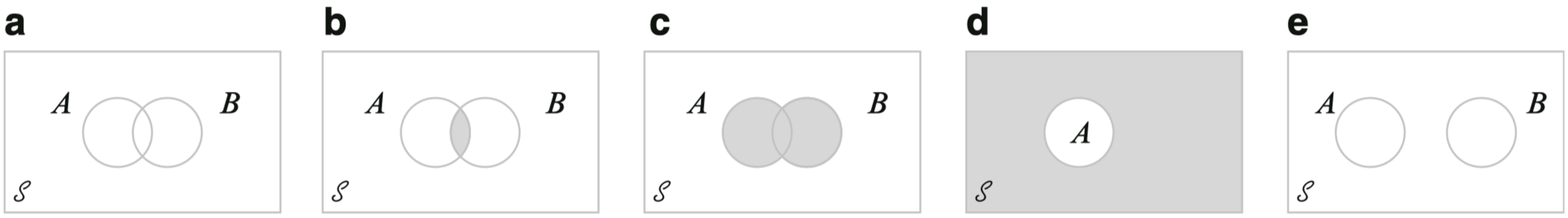
- $\mathbb{P}(A^C) = 1 - \mathbb{P}(A)$
- $\mathbb{P}(A) \leq 1$
- $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$

Conditional probability

- A certain event has happened
- What is the probability in this new space

- $\Omega_{new} = B$

Conditional probability



Venn diagram of events A and B

Shaded region is $A \cap B$

Shaded region is $A \cup B$

Shaded region is A'

Mutually exclusive events

$$\mathbb{P}(A \mid B) = \frac{|A \cap B|}{|B|}$$

Thomas Bayes (1701-1761)

LII. *An Eſſay towards ſolving a Problem in the Doctrine of Chances. By the late Rev. Mr.* Bayes, *F. R. S. communicated by Mr.* Price, *in a Letter to* John Canton, *A. M. F. R. S.*

Dear Sir,

Read Dec. 23, 1763.

I Now ſend you an eſſay which I have found among the papers of our deceaſed friend Mr. Bayes, and which, in my opinion, has great merit, and well deſerves to be preſerved. Experimental philoſophy, you will find, is nearly intereſted in the ſubject of it; and on this account there ſeems to be particular reaſon for thinking that a communication of it to the Royal Society cannot be improper.

Conditional probability

- Multiplication

- $\mathbb{P}(A \cap B) = \mathbb{P}(A \mid B) \cdot \mathbb{P}(B)$

Bayes' theorom

- Symmetry

- $\mathbb{P}(A \cap B) = \mathbb{P}(B \cap A)$
- $\mathbb{P}(A \mid B) \cdot \mathbb{P}(B) = \mathbb{P}(B \mid A) \cdot \mathbb{P}(A)$

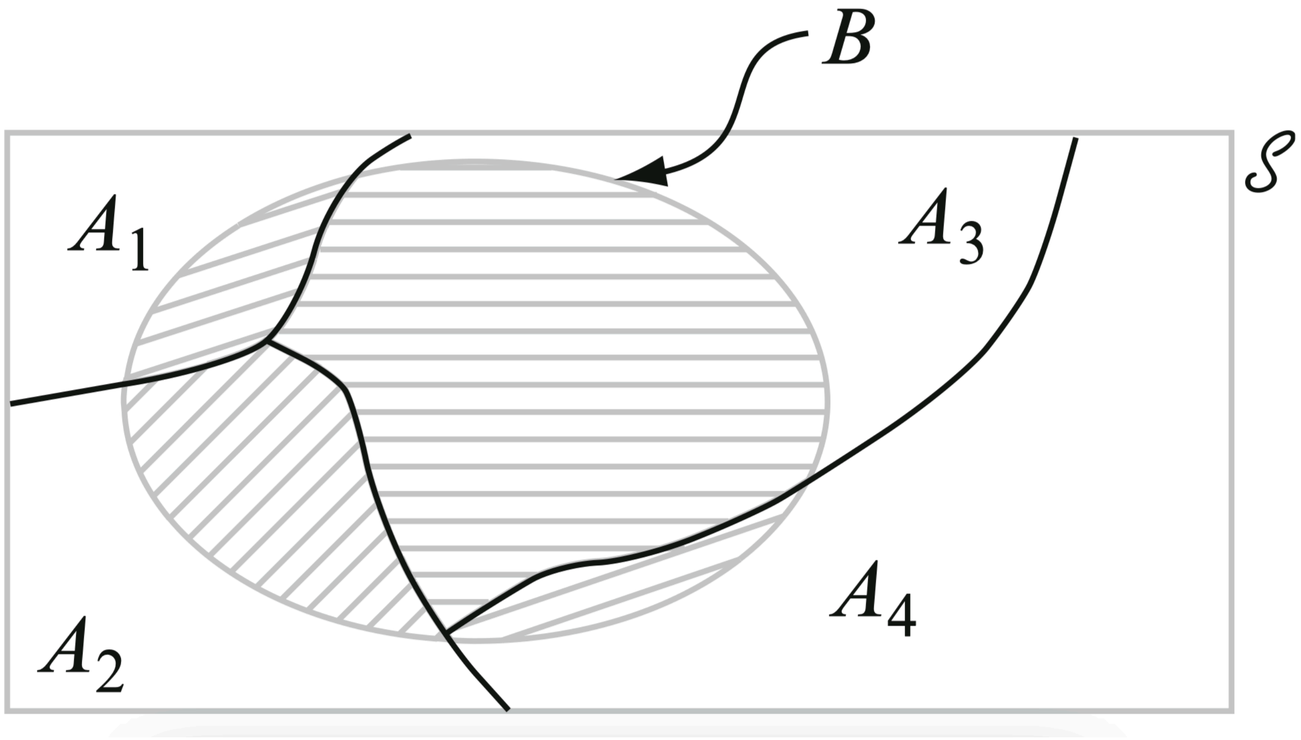
$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(B \mid A) \cdot \mathbb{P}(A)}{\mathbb{P}(B)}$$

Law of total probability

- Mutually exclusive and exhaustive chunks of the sample space

- $A_1 \cup A_2 \cup \ldots \cup A_k = \Omega$
- $\mathbb{P}(B) = \mathbb{P}(B \cap A_1) + \ldots + \mathbb{P}(B \cap A_k)$

Law of total probability



$$\mathbb{P}(B) = \sum_{1}^{k} \mathbb{P}(B \mid A_i) \cdot \mathbb{P}(A_i)$$

Independence

- Definition of independence
- Multiplication rule

- $\mathbb{P}(A \mid B) = \mathbb{P}(A)$
- $\mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B)$

How to systematically update our beliefs

- Prior
- Likelihood
- Evidence
- Posterior

- $\mathbb{P}(H)$
- $\mathbb{P}(D \mid H)$
- $\mathbb{P}(D)$
- $\mathbb{P}(H \mid D)$

$$\mathbb{P}(H_1 \mid D) = \frac{\mathbb{P}(D \mid H_1) \cdot \mathbb{P}(H_1)}{\sum_1^k \mathbb{P}(D \mid H_i) \cdot \mathbb{P}(H_i)}$$

Random variable

- A mapping from sample space to real number
- $\Omega \to \mathbb{R}$

Discrete Random Variables

- Uniform
- Bernouli
- Binominal
- Poisson
- Negative Binominal
- Geometric
- Hypergeometric

Siméon Denis Poisson (1781-1840)

$$\lim_{n\to\infty} \binom{n}{k} \cdot p^k \cdot (1-p)^{(n-k)}$$

$$n \to \infty$$
$$p \to 0$$
$$np \to \lambda$$

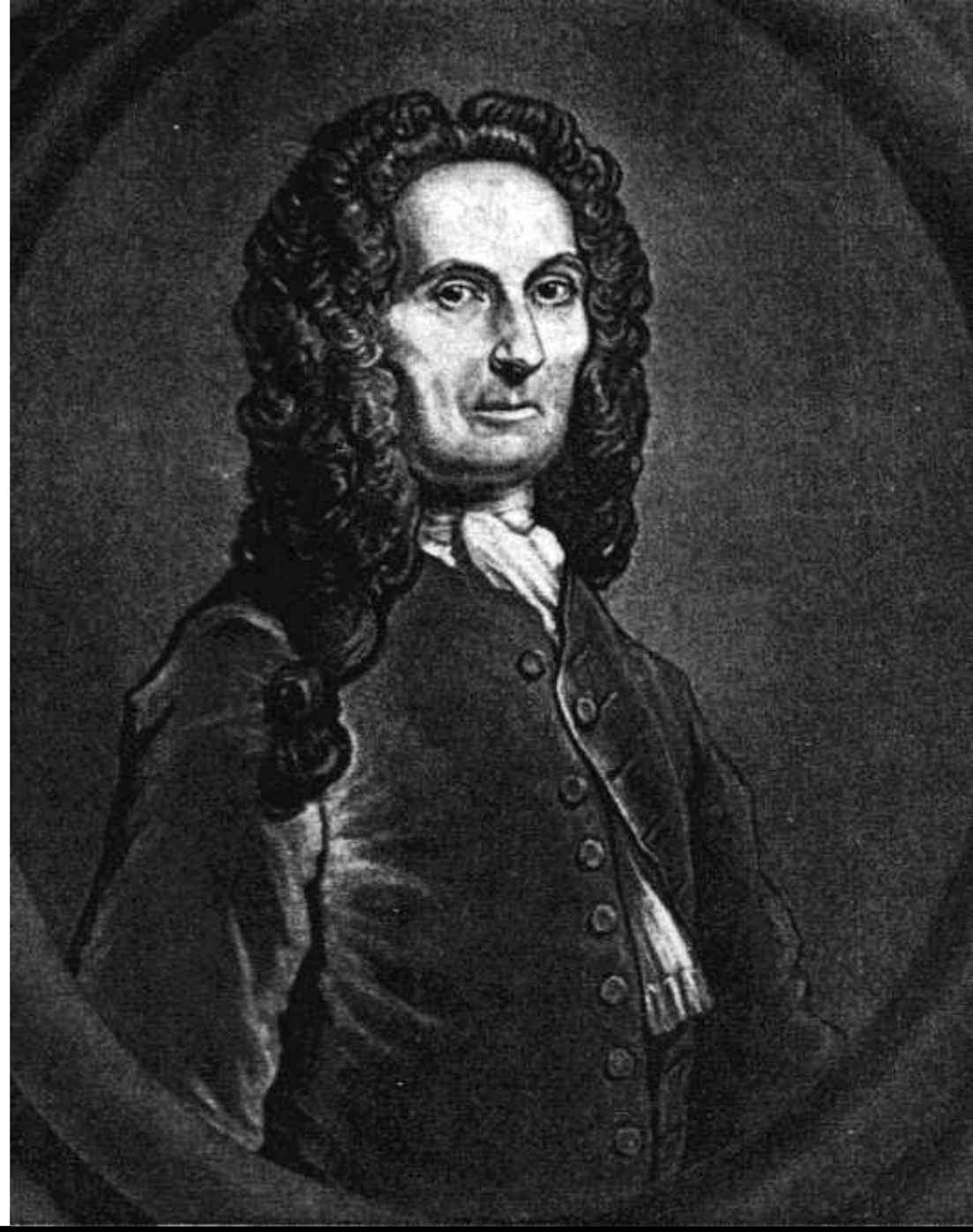
$$\lim_{n\to\infty} (1 - \frac{a}{n})^n = e^{-a}$$

Continous Random Variables

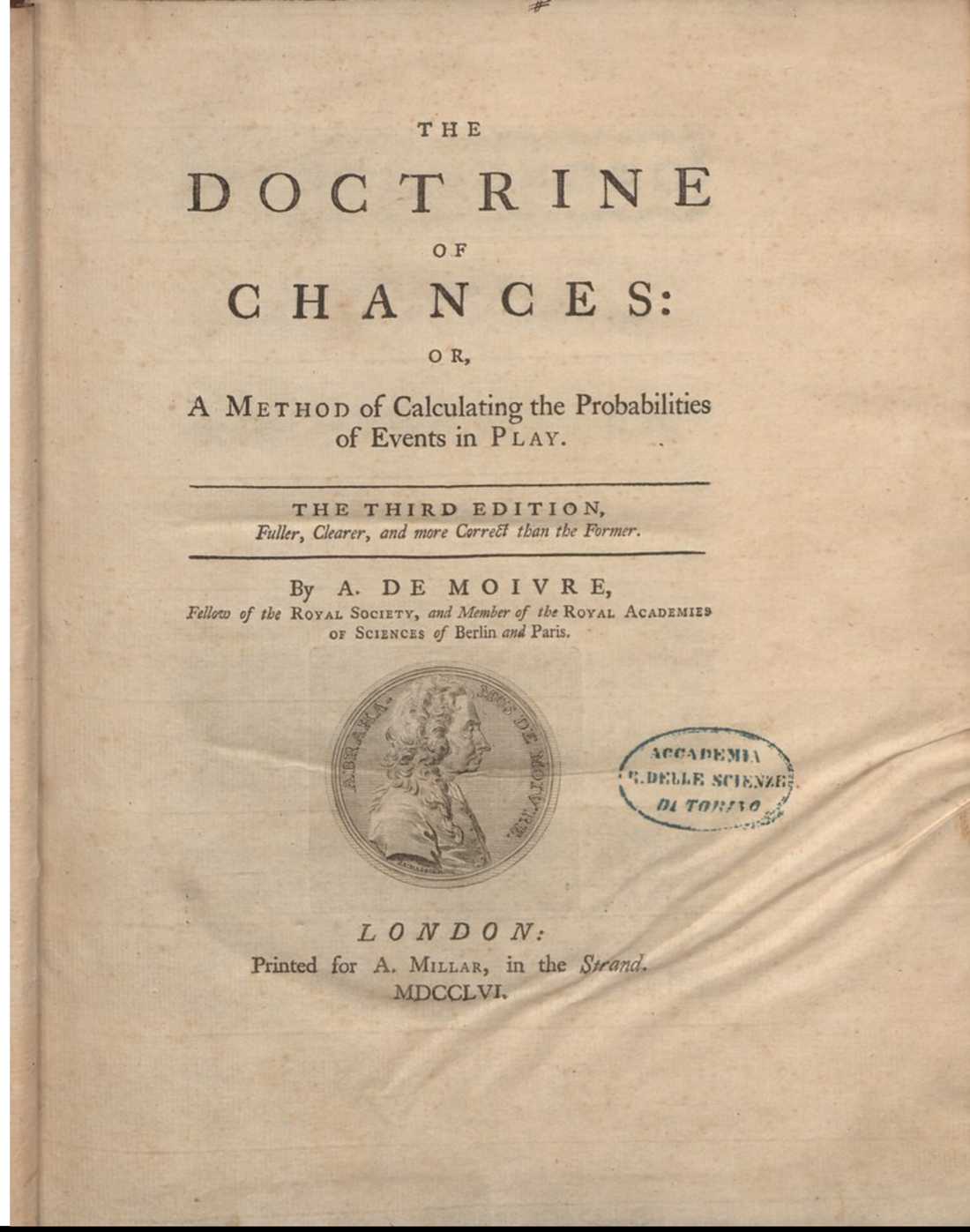
- Uniform
- Normal
- Geometric

Abraham de Moivre (1667-1754)

$$\binom{n}{\frac{n}{2}+d}(\frac{1}{2})^n \approx \frac{2}{\sqrt{2\pi n}} e^{\frac{-2d^2}{n}}$$



Doctorine of chances (1718)



THE

DOCTRINE

OF

CHANCES:

OR,

A METHOD of Calculating the Probabilities of Events in PLAY.

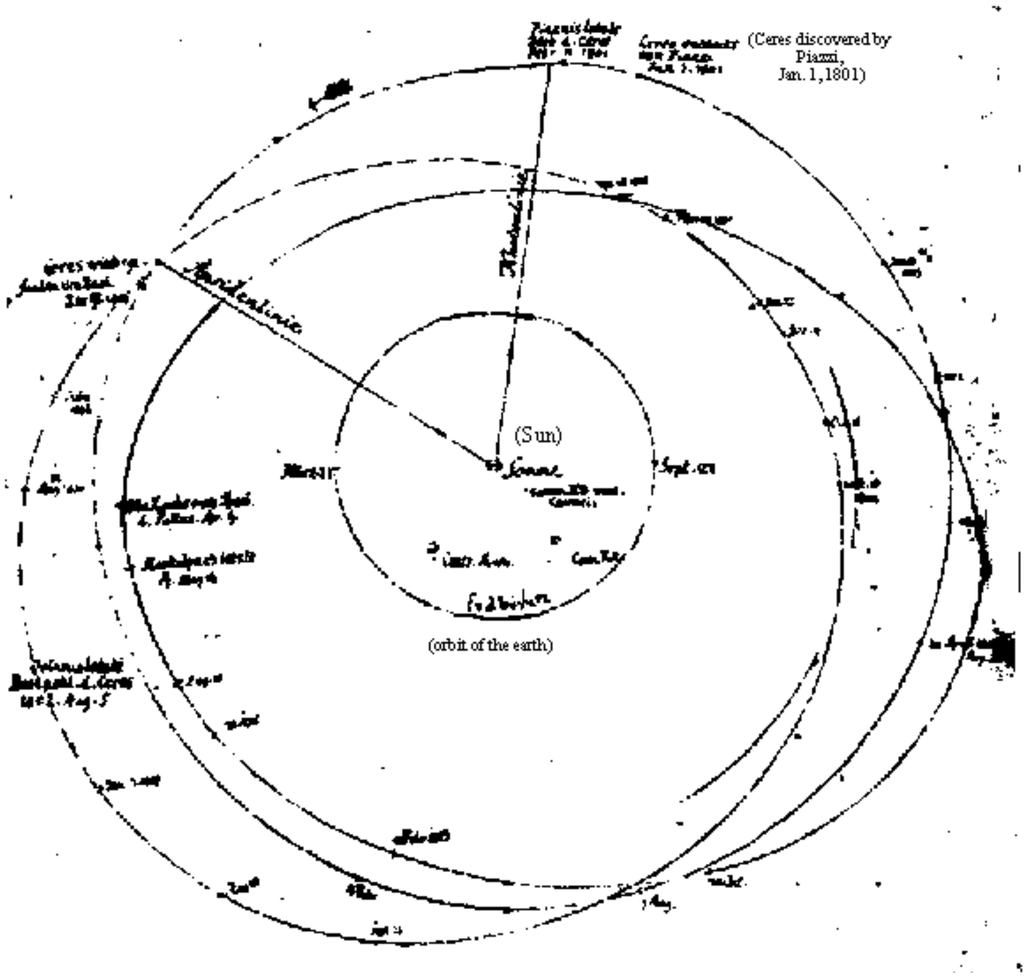
THE THIRD EDITION,
Fuller, Clearer, and more Correct than the Former.

By A. DE MOIVRE,
Fellow of the ROYAL SOCIETY, *and Member of the* ROYAL ACADEMIES OF SCIENCES *of* Berlin *and* Paris.

LONDON:
Printed for A. MILLAR, in the *Strand.*
MDCCLVI.

ACCADEMIA R. DELLE SCIENZE DI TORINO

The story of “the missing planet” and the celestial police lead to the development of error curves.



Galileo di Vincenzo Bonaiuti de' Galilei (1564-1642)

1. There is only one number which gives the distance of the star from the center of the earth, the true distance.
2. All observations are encumbered with errors, due to the observer, the instruments, and the other observational conditions.
3. The observations are distributed symmetrically about the true value; that is the errors are distributed symmetrically about zero.
4. Small errors occur more frequently than large errors.
5. The calculated distance is a function of the direct angular observations such that small adjustments of the observations may result in a large adjustment of the distance.

Pierre-Simon Laplace (1749-1827)

$$\phi(x) = \frac{m}{2} e^{-m|x|}$$

$$\phi(x) = \frac{1}{2a} \ln\left(\frac{a}{|x|}\right)$$

Johann Carl Friedrich Gauss (1777-1855)

$$g(\sqrt{x^2+y^2}) = g(x)g(y)$$

$$\phi(x) \propto e^{-h^2x^2}$$

$$\int_{\mathbb{R}} e^{-x^2} = \sqrt{\pi}$$

$$\phi(x) = \frac{h}{\sqrt{\pi}} e^{-h^2x^2}$$

Gauss had to estimate 6 parameters from only 19 data points. With scarce data, measurement error can substantially alter the prediction accuracy.

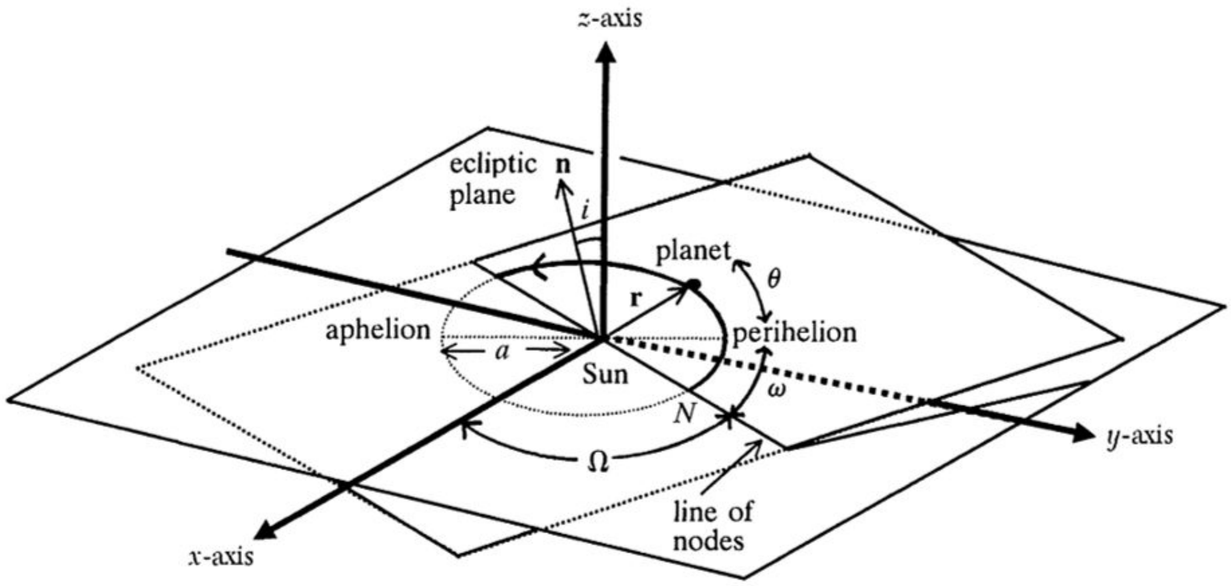
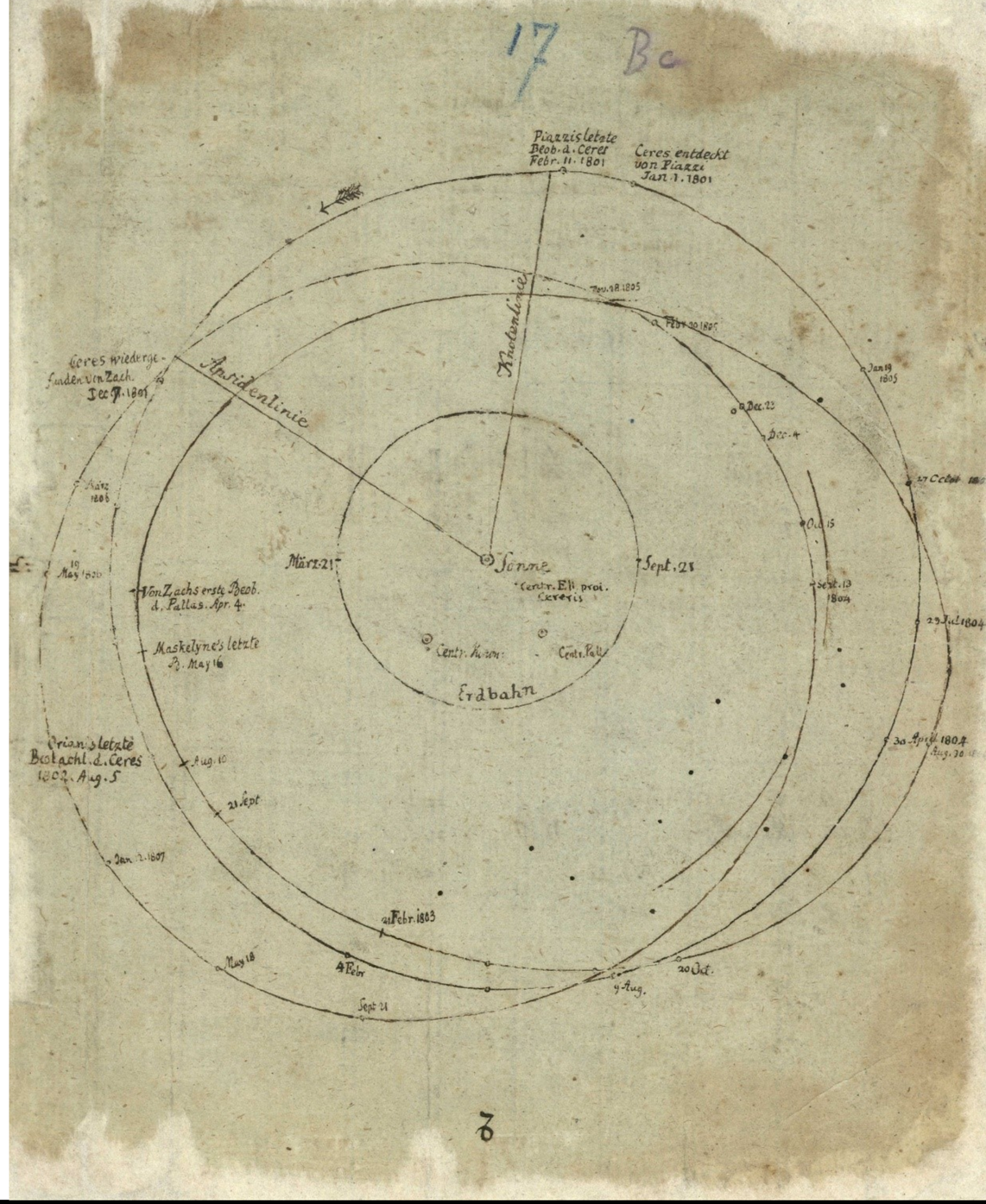


FIGURE 1
Parameters describing the planetary orbit

Gauss' original sketch of the orbit of Ceres