۲. جبر

۲. جبر

قوانین توان و رادیکال

$$x^m x^n = x^{m+n}$$

$$(x^m)^n = x^{mn}$$

$$(xy)^n = x^n y^n$$

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

$$\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}$$

$$\sqrt[m]{\sqrt[n]{x}} = \sqrt[n]{\sqrt[m]{x}} = \sqrt[mn]{x}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

$$x^{-n} = \frac{1}{x^n}$$

$$(\frac{x}{y})^n = \frac{x^n}{y^n}$$

$$x^{m/n} = \sqrt[n]{x^m}$$

$$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

۲. جبر

قوانین لگاریتم

$y = \log_a x \to a^y = x$

$\log_a a^x = x$

$\log_a 1 = 0$

$\log_a xy = \log_a x + \log_a y$

$\log_a x^b = b \log_a x$

$a^{\log_a x} = x$

$\log_a(\frac{x}{y}) = \log_a x - \log_a y$

$\log_b x = \frac{\log_a x}{\log_a b}$

۲. جبر

اتحاد

$$(a+b)(c+d) = ac + ad + bc + bd$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$(a+b)(a-b) = a^2 - b^2$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

۲. جبر

توابع مهم

$$f(x) = b$$

$$f(x) = ax + b \qquad f(x) = ax^2 + bx + c \qquad f(x) = ax^3 + bx^2 + cx + d$$

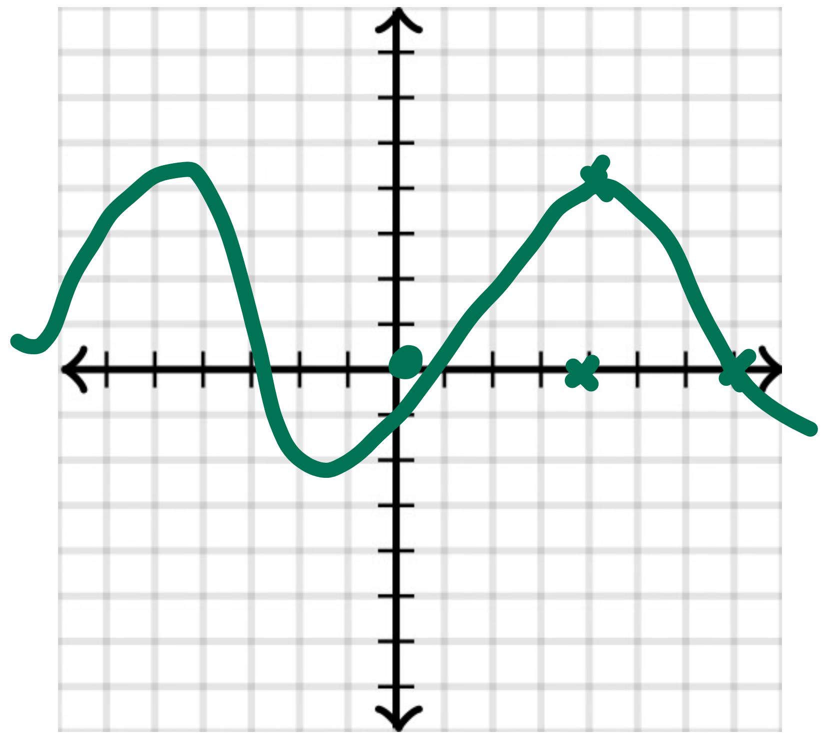
$$f(x) = \sqrt[n]{x}$$

$$f(x) = \frac{1}{x^n}$$

$$f(x) = b^x$$

$$f(x) = \log_b x$$

$$f(x) = \sin x \qquad f(x) = \cos x$$



۲. جبر

تغییرات تابع

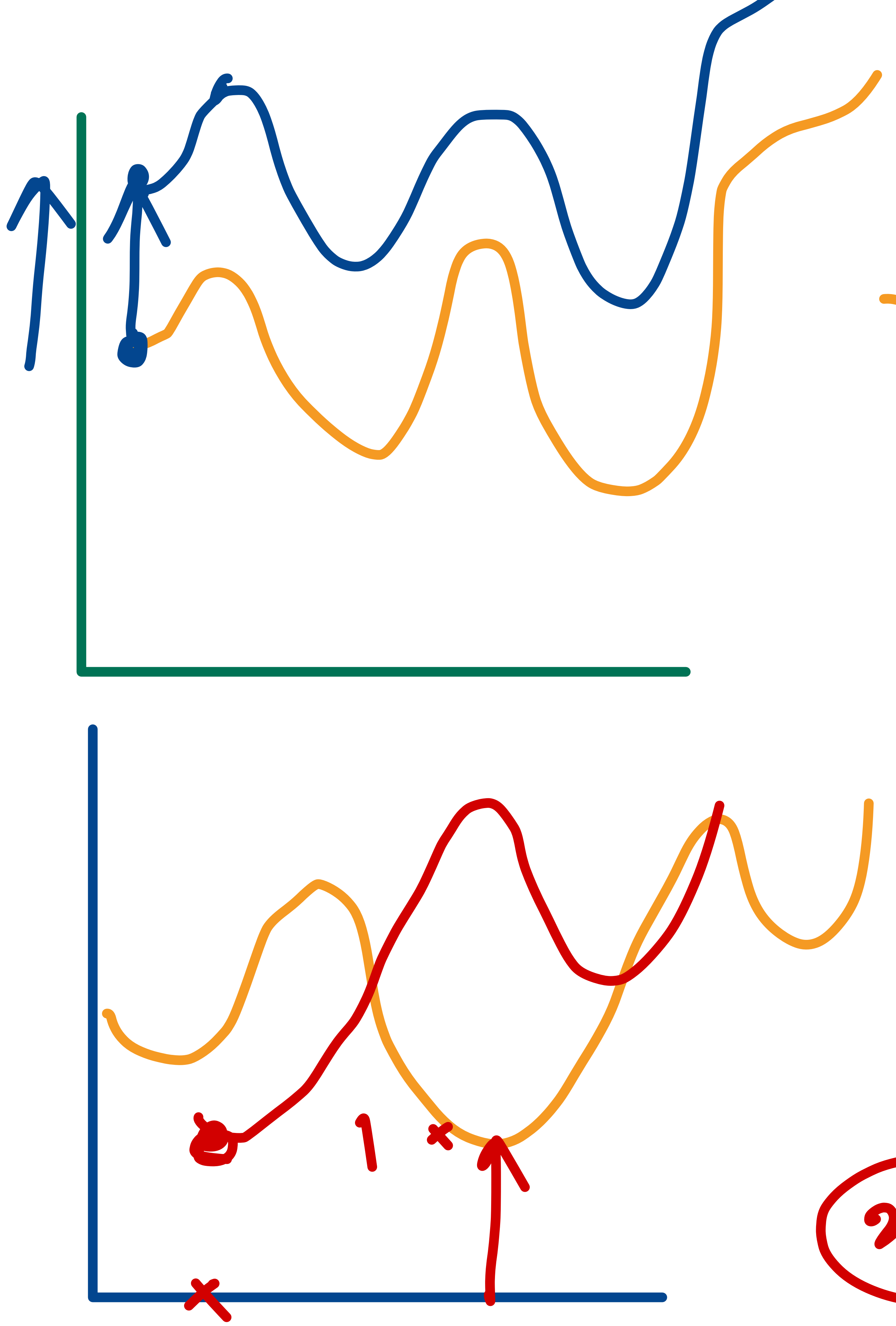
transformation

$$g(x) = f(x) + c$$

$$g(x) = f(x + c)$$

$$g(x) = cf(x)$$

$$g(x) = f(cx)$$



$f(x)$

$g(x) = f(x) + 2$

$g(x)$

$+2$

$x+2$

$g(x)$

$f(x+c)$

$(x+2)$

c

Poly nominal $c_0x^0 + c_1x^1 + c_2x^2 + \dots$

۲. جبر

ریشه‌های چندجمله‌ای

$ax = -b$

$x = \frac{-b}{a}$

$$f(x) = ax + b \implies x = -\frac{b}{a}$$

$\Delta = 0$ ⟶ ۱ جواب

$\Delta > 0$ ۲ جواب

$\Delta < 0$ صفر جواب

$$f(x) = ax^2 + bx + c \implies x = \frac{1}{2a}\left(-b \pm \sqrt{b^2 - 4ac}\right)$$

$$f(x) = x^3 + px + q \implies x = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}$$

۲. جبر

ابوجعفر محمد بن موسی الخوارزمی (۷۸۰-۸۵۰ میلادی)

1. Squares equal to roots.
2. Squares equal to numbers.
3. Roots equal to numbers.
4. Squares and roots equal to numbers
5. Squares and numbers equal to roots
6. Roots and numbers equal to squares

۲. جبر

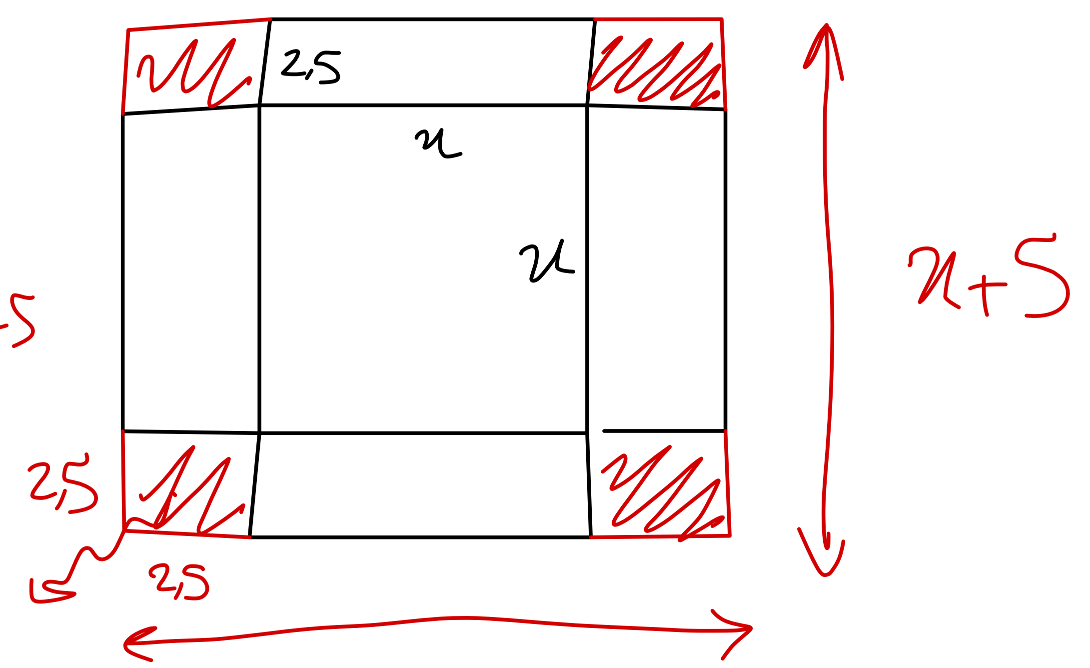
الجبر و المقابله

$x^2 + 10x = 39$

$x^2 + 10x + 25 = 39 + 25 = 64$

$(x+5)^2 = 64$

$x = 3$

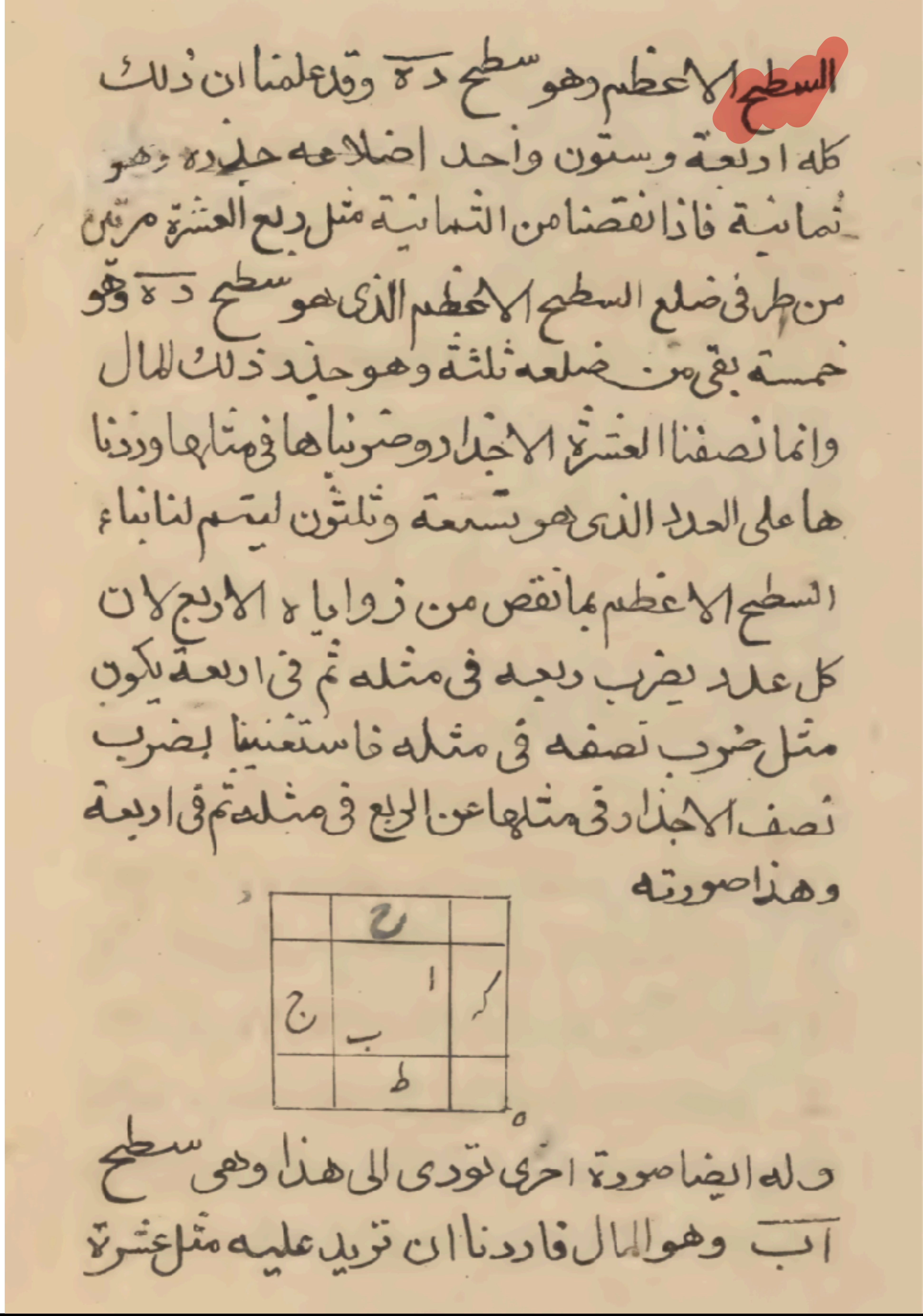


$$\underbrace{x^2 + 10x}_{} = 39$$

$$x^2 + 10x + 4\left(\frac{10}{4}\right)^2 = 39 + 25$$

$$(x+5)^2 = 8^2$$

$$x = 3$$


السطح الاعظم وهو سطح ده وقد علمنا ان ذلك
كله اربعة وستون واحد اضلاعه جذره وهو
ثمانية فاذا نقصنا من الثمانية مثل ربع العشرة مرتين
من طرفي ضلع السطح الاعظم الذي هو سطح ده وهو
خمسة بقي من ضلعه ثلثة وهو جذر ذلك المال
وانما نصفنا العشرة الاجذار وضربناها في مثلها وزدنا
ها على العدد الذي هو تسعة وثلثون ليتم لنا بناء
السطح الاعظم بما نقص من زوايا ه الاربع لان
كل عدد يضرب ربعه في مثله ثم في اربعة يكون
مثل ضرب نصفه في مثله فاستغنينا بضرب
نصف الاجذار في مثلها عن الربع في مثله ثم في اربعة
وهذا صورته

وله ايضا صورة اخرى تؤدي الى هذا وهي سطح
اب وهو المال فاردنا ان نزيد عليه مثل عشرة

۲. جبر

غیاث‌الدین ابوالفتح عُمَر بن ابراهیم خَیّام نیشابوری (۱۰۴۸-۱۱۴۳ میلادی)

$x^3 = d$

$x^3 + cx = d$

$x^3 + d = cx$

$x^3 = cx + d$

$x^3 + bx^2 = d$

$x^3 + d = bx^2$

$x^3 = bx^2 + d$

$x^3 + bx^2 + cx = d$

$x^3 + bx^2 + d = cx$

$x^3 + cx + d = bx^2$

$x^3 = bx^2 + cx + d$

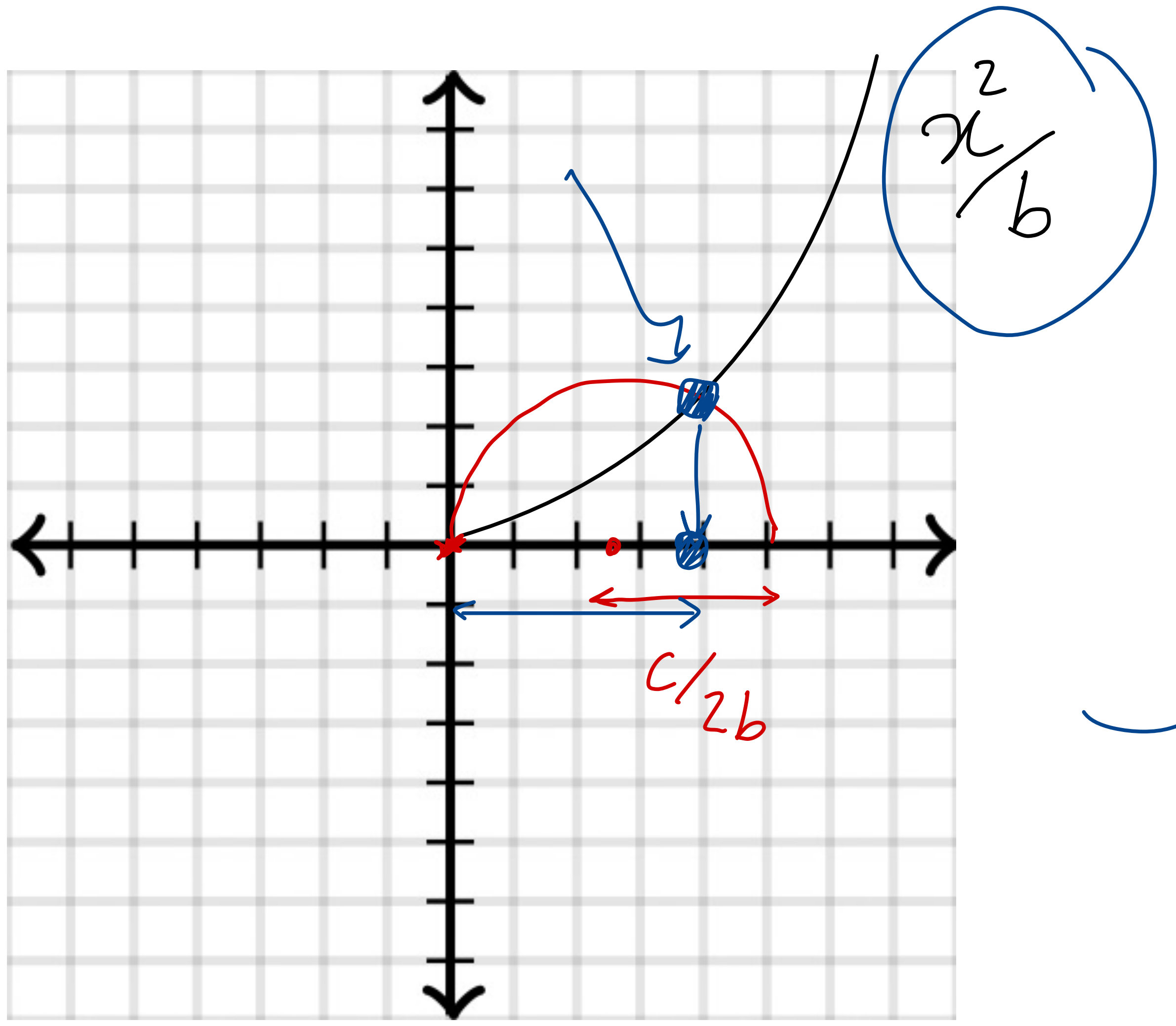
$x^3 + bx^2 = cx + d$

$x^3 + cx = bx^2 + d$

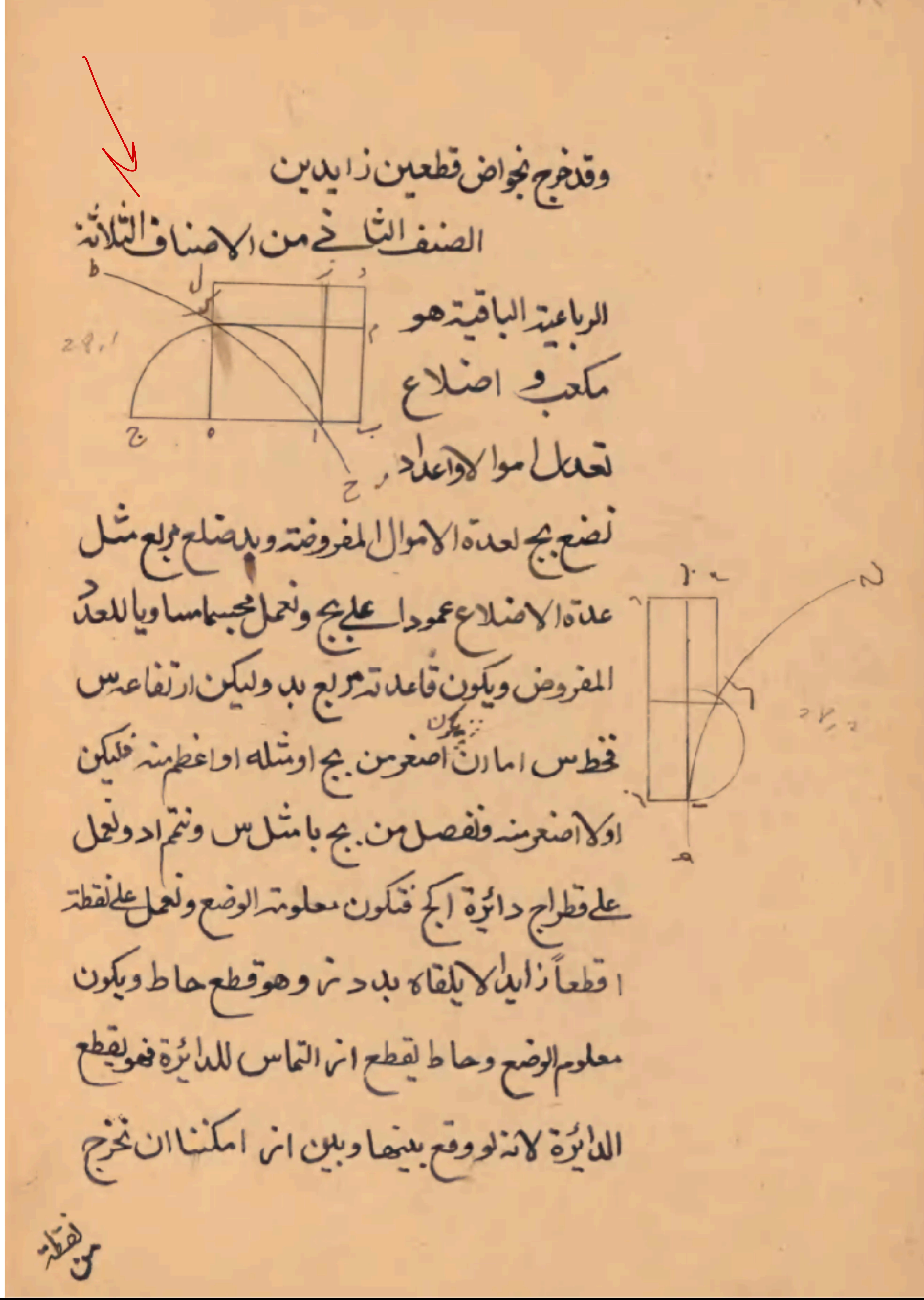
$x^3 + d = bx^2 + cx$

۲. جبر

غیاث‌الدین ابوالفتح عُمَر بن ابراهیم خَیّام نیشابوری (۱۰۴۸-۱۱۴۳ میلادی)



$$x^3 + bx = c$$



“Maybe one of those who will come after us will succeed in finding it.”

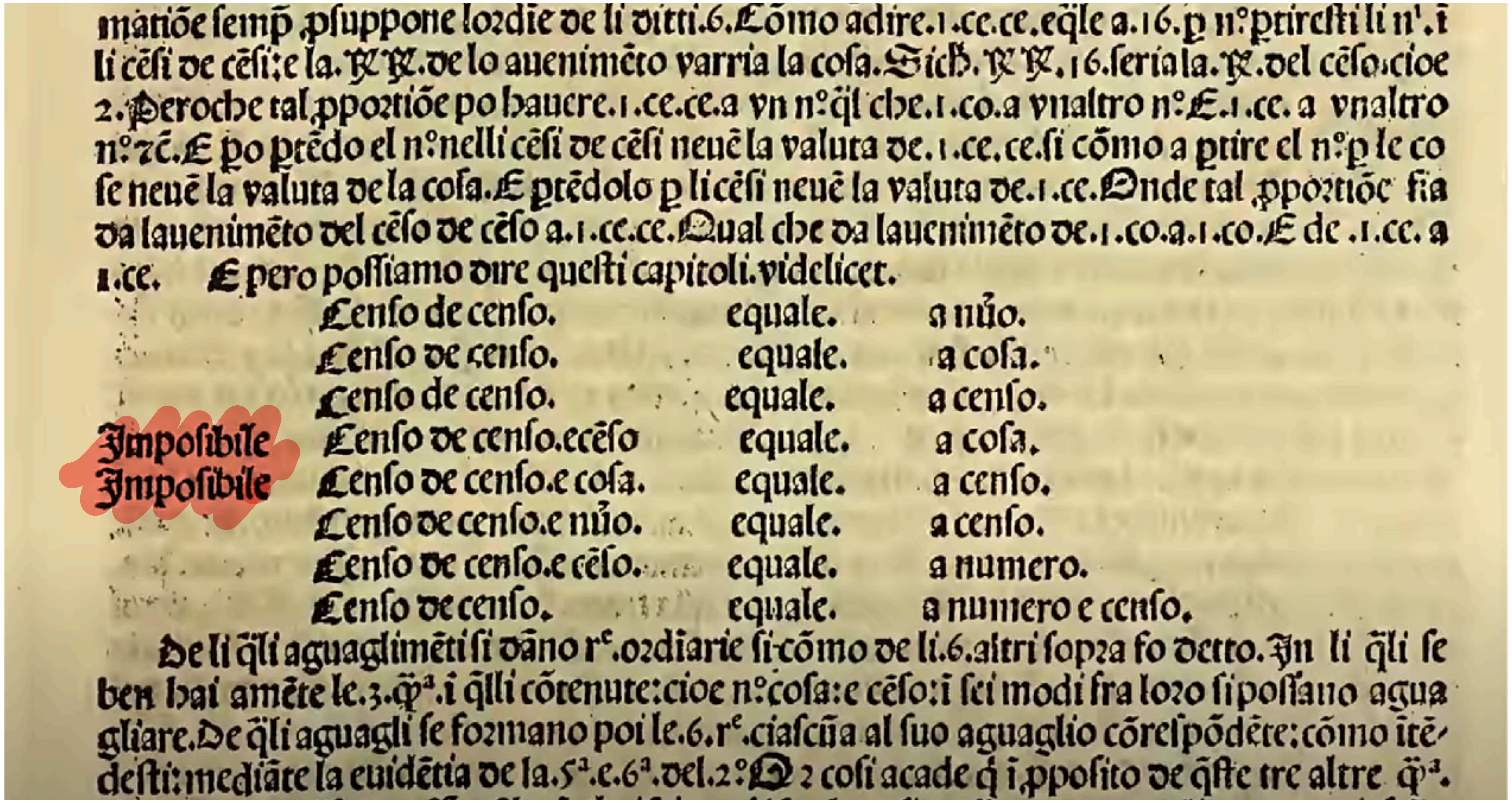
۲. جبر

Luca Paccioli (1447-1517)

“Impossible to solve cubic equations.”

۲. جبر

Sũma De Arithmetica (1494)


matiõe semp ꝓsuppone lordie de li ditti.6.Cõmo adire.1.ce.ce.eq̃le a.16.p n°ptircsti li n°.i li cẽsi de cẽsi:e la.RR.de lo auenimẽto varria la cosa.Sich.RR.16.seria la.R.del cẽso.cioe 2.Peroche tal ꝓportiõe po hauere.1.ce.ce.a vn n°q̃l che.1.co.a vnaltro n°.E.1.ce.a vnaltro n°.7c.E po ptẽdo el n°nelli cẽsi de cẽsi neuẽ la valuta de.1.ce.ce.si cõmo a ptire el n°p le co se neuẽ la valuta de la cosa.E ptẽdolo p li cẽsi neuẽ la valuta de.1.ce.Onde tal ꝓportiõe fia da lauenimẽto del cẽso de cẽso a.1.ce.ce.Qual che da lauenimẽto de.1.co.a.1.co.E de.1.ce.a 1.ce. E pero possiamo dire questi capitoli.videlicet.

	Censo de censo.	equale.	a nũo.
	Censo de censo.	equale.	a cosa.
	Censo de censo.	equale.	a censo.
Impossibile	Censo de censo.e cẽso	equale.	a cosa.
Impossibile	Censo de censo.e cosa.	equale.	a censo.
	Censo de censo.e nũo.	equale.	a censo.
	Censo de censo.e cẽso.	equale.	a numero.
	Censo de censo.	equale.	a numero e censo.

De li q̃li aguaglimẽti si dãno r^e.ordiarie si cõmo de li.6.altri sopra fo detto.In li q̃li se ben hai amẽte le.3.q̃^a.i q̃lli cõtenute:cioe n°cosa:e cẽso:i sei modi fra loro si possano agua gliare.De q̃li aguagli se formano poi le.6.r^e.ciascũa al suo aguaglio cõrespõdẽte:cõmo itẽ desti:mediãte la euidẽtia de la.5^a.e.6^a.del.2°.D 2 cosi acade q̃ i ꝓposito de q̃ste tre altre q̃^a.

۲. جبر

Scipione Del ferro (1465-1526)

$$x^3 + px = q$$

$$\sqrt[3]{\frac{q}{2}+\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}}+\sqrt[3]{\frac{q}{2}-\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}}$$

Finds it around 1510

Antonio Fior

۲. جبر

Niccolo Fontana Tartaglia (1500-1557)

Quando chel cubo con le cose appresso
Se aqquaglia ´a qualche numero discreto
Trouan duo altri differenti in esso
Dapoi terrai questo per consueto
Che"llor productto sempre sia equale
Alterzo cubo delle cose neto,
El residuo poi suo generale
Delli lor lati cubi ben sottrati
Varra la tua cosa principale.
In el secondo de cotestiatti
Quando chel cubo restasse lui solo
Tu osseruarai questaltri contratti,

Del numer farai due tal part'`a uolo
Che luna in laltra si produca schietto
El terzo cubo delle cose in stolo
Delle qual poi, per communprecetto
Torrai li lati cubi insieme gionti
Et cotal somma sara il tuo concetto.
El terzo poi de questi nostri conti
Se solue col secondo se ben guardi
Che per natura son quasi congionti.
Questi trouai, non con passi tardi
Nel mille cinquecent`e, quatroe trenta
Con fondamenti ben sald`e gagliardi
Nella citta dal marintorno centa.

۲. جبر

Niccolo Fontana Tartaglia (1500-1557)

When the cube with the things together
Are set equal to some discrete number
Find other two that by it differ
You will get into the habit
Of making their product always equal
To the third of the things cubed extactly
The result then
Of their cubic roots properly subtracted
Will be equal to your principal thing.
In the second of these cases
When the cube remains alone
You will take these other steps
From the number you will make twe two parts
So that one in the other yields
The third of the cube of the number of things
Of these two parts by known precept
You'll joint together their cubic roots
And this sum will be your answer.
Then the third one of your calculations
It's solved with the second,
If you look carefully
Because by nature they're related
All these I found,
and not with slow steps
In fifteen hundred thirty four,
With firm and vigorous foundations
In the city surrounded by the sea

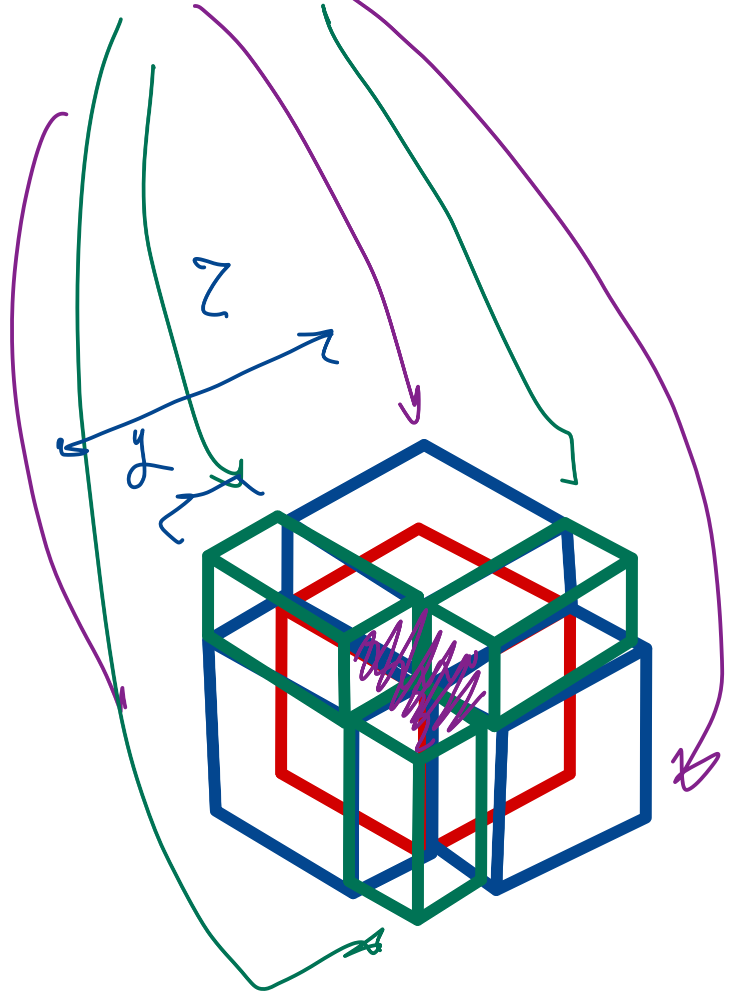
۲. جبر

12th February 1535

$+y^3 \quad +y^3$

$$x^3 + 9x = 26$$

z^3



$$3x^2y + 3xy^2$$

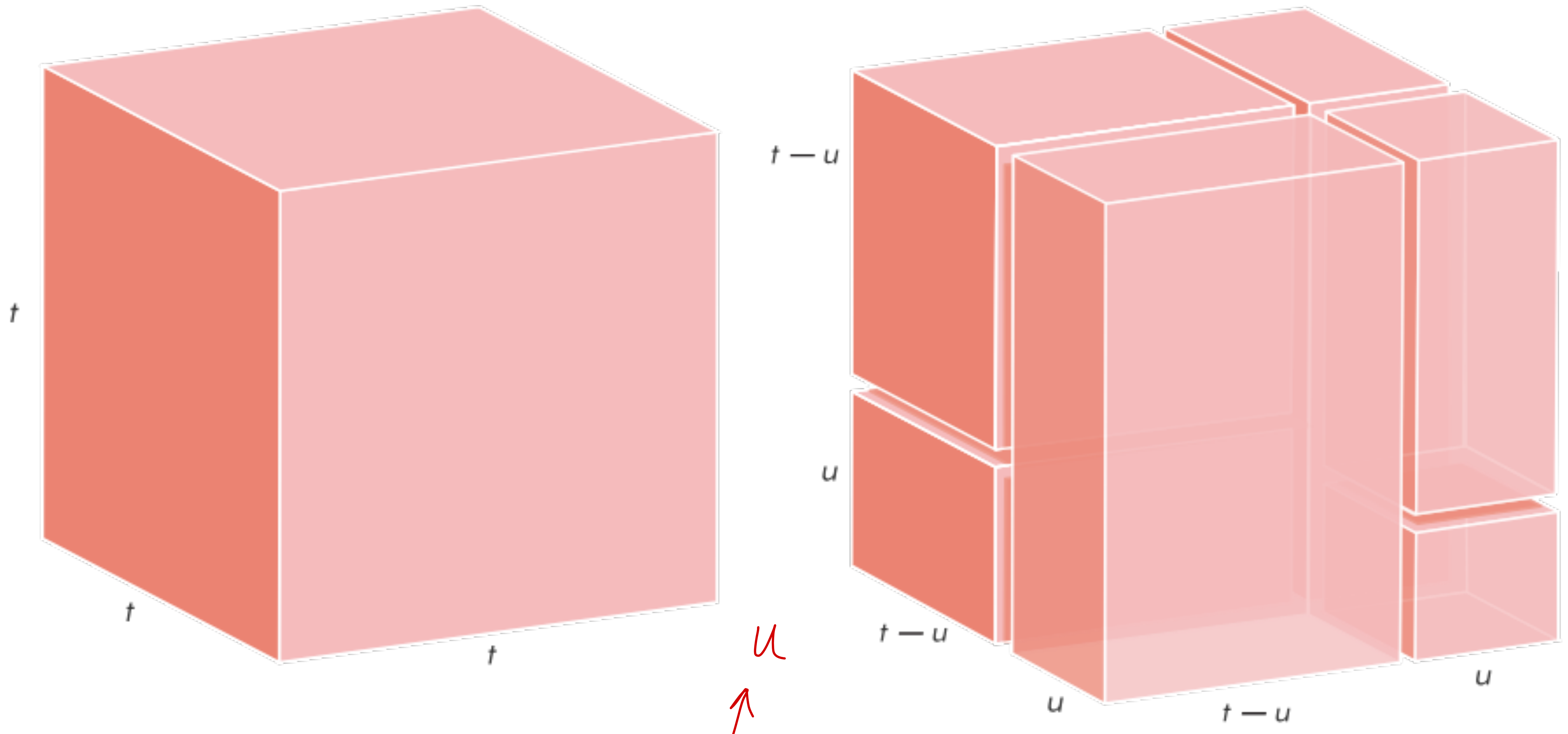
$$3xy(x+y) = 9x$$

$$3xyz = 9x$$

$$yz = 3$$

$$z = \frac{3}{y}$$

x^3



u

$$z^3 = \frac{27}{y^3} = 26 + y^3 \Rightarrow 27 = 26y^3 + y^6 \qquad 27 = 26u + u^2$$

۲. جبر

Geloramo Cardano (1501-1576)

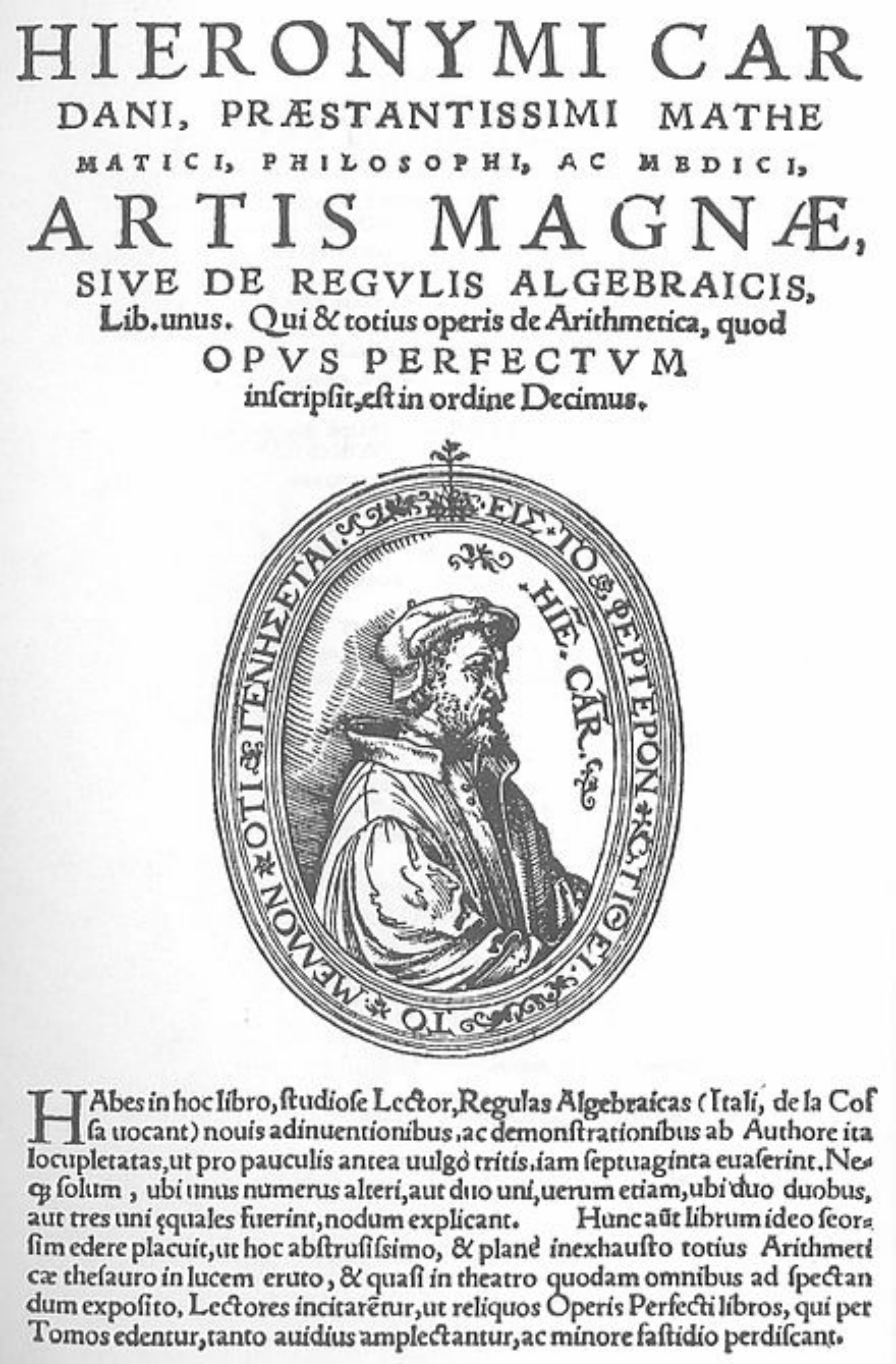
Ferrari

۲. جبر

Artis Magnae (1545)

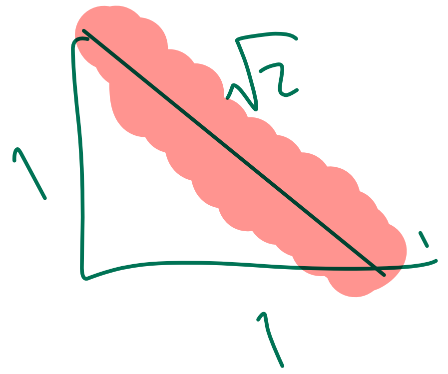
$x + y = 10$

$xy = 40$



“Pitiful, A man of no substance. A very stupid man. An ignoramus in mathematical matters”

۲. جبر



Rafael Bombelli (1526-1572)

2

$$x = (1+\cancel{\sqrt{-1}}) + (1-\cancel{\sqrt{-1}}) = 2$$

$x^3 - 6x + 4$

$$\sqrt[3]{\frac{1}{2}(-4+\sqrt{-16})} + \sqrt[3]{\frac{1}{2}(-4-\sqrt{-16})}$$

$$(1+\sqrt{-1})^3 = \boxed{-2+2\sqrt{-1}}$$

$$(1+\sqrt{-1})^3 = 1^3 + 3\times 1^2 \times \sqrt{-1} + 3\times 1 \times (\sqrt{-1})^2 + (\sqrt{-1})^3$$

$$1 + 3\sqrt{-1} - 3 - \sqrt{-1}$$

$$-2 + 2\sqrt{-1}$$

$$(1-\sqrt{-1})^3 = -2-2\sqrt{-1}$$

$x = 2$

۲. جبر

Rafael Bombelli (1526-1572)

$\sqrt{-1}$

"Neither positive nor negative"

plus

$2 + 3i = 2\ p\ di\ m\ 3$

$x^2 + 1 = 0$

$\boxed{i}$

$\sqrt{-1}$

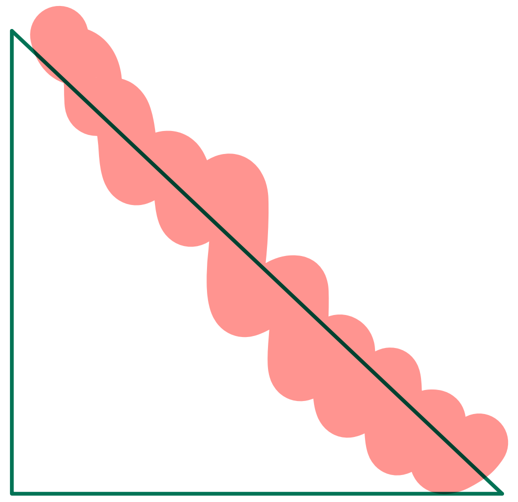
$2 - 3i = 2\ m\ di\ m\ 3$

۲. جبر

René Decartes (1596-1659)

"Imaginary numbers"

$\boxed{\sqrt{-1}}$



۲. جبر

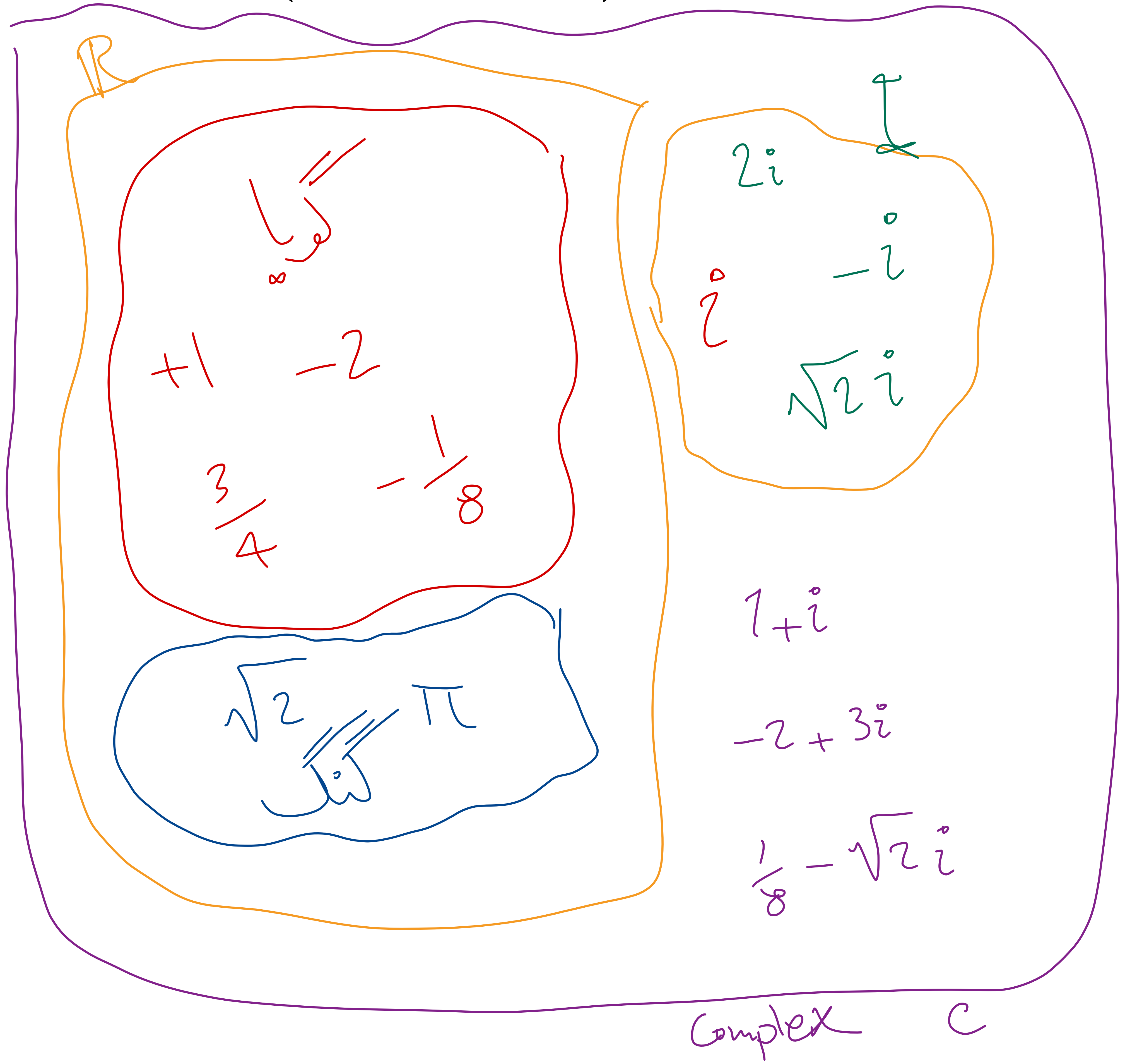
Leonhard Euler (1707-1783)

$i^2 = -1$

$i^3 = -i$

$i^4 = +1$

$i^5 = i$



۲. جبر

اعداد مختلط

حقیقی ↑ موهومی ↑

$a + bi$

$(a+c)+(b+d)i$

$a+bi+c+di$

جمع $(a + bi) + (c + di) = (a + c) + (b + d)i$

ضرب $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$

$(a+bi)(c+di) = ac + adi + bic + \boxed{bidi}$

bdi^2

$-bd$

$\boxed{(ac-bd)} + \boxed{(ad+bc)}i$

$+bd$

$-bdi^2$

$$\frac{(a + bi)}{(c + di)} = \frac{(ac + bd) + (bc - ad)i}{(c^2 + d^2)}$$

$$\frac{a+bi}{c+di} \times \frac{c-di}{c-di} = \frac{\boxed{ac} - adi + bic \boxed{-bidi}}{c^2 - \cancel{cdi} + \cancel{dic} - didi} = \frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$$

$-d^2i^2$

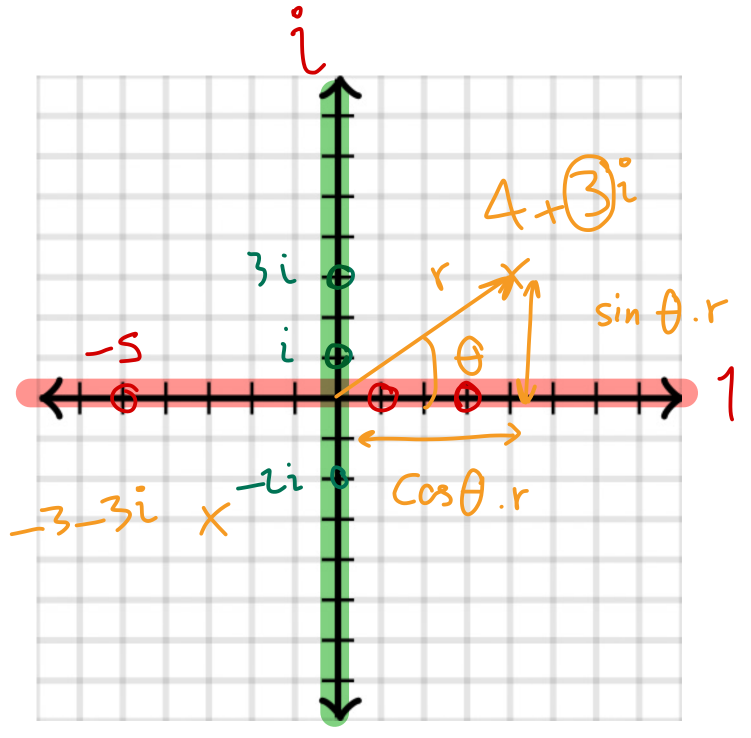
$+d^2$

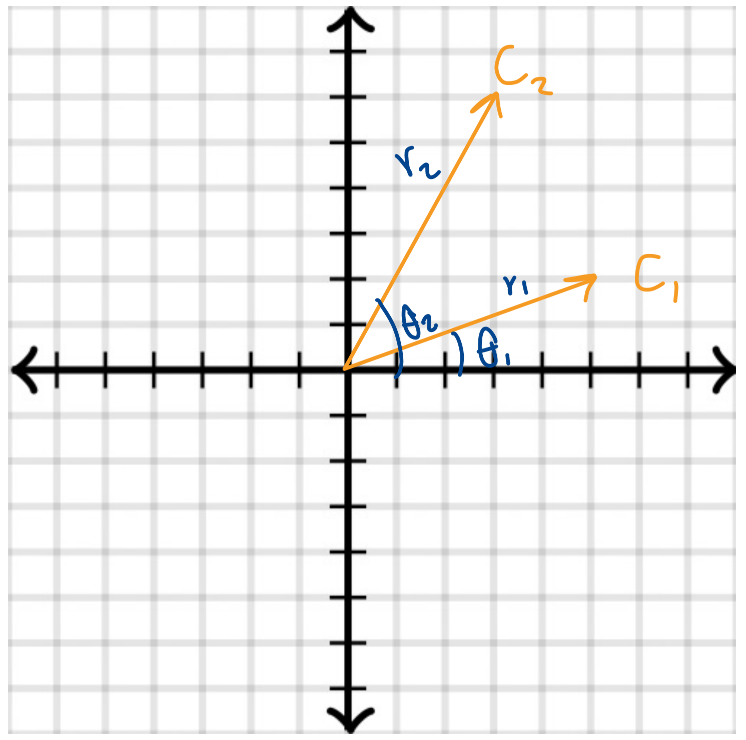
$$= \left(\frac{ac+bd}{c^2+d^2}\right) + \left(\frac{bc-ad}{c^2+d^2}\right)i$$

Conjugate pair

۲. جبر

اعداد مختلط





$(a+bi)(c+di)$

θ_1, r_1

$r_1(\cos\theta_1 + \sin\theta_1 i) \times r_2(\cos\theta_2 + \sin\theta_2 i)$

$r_1 r_2 \left[(\cos\theta_1 + \sin\theta_1 i)(\cos\theta_2 + \sin\theta_2 i)\right]$

$\left[\cos\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2 i + \sin\theta_1 i \cos\theta_2 + \sin\theta_1 i \sin\theta_2 i\right]$

$r_1 r_2 \left[(\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2) + (\cos\theta_1 \sin\theta_2 + \sin\theta_1 \cos\theta_2) i\right]$

$r_1 r_2 \left(\cos(\theta_1+\theta_2) + (\sin(\theta_1+\theta_2) i)\right)$

$$a + bi \to r = \sqrt{a^2 + b^2}$$

$$a + bi = r(\cos\theta + \sin\theta i)$$

$$r_1(\cos\theta_1 + \sin\theta_1 i) r_2(\cos\theta_2 + \sin\theta_2 i) = r_1 r_2[\cos(\theta_1 + \theta_2) + \sin(\theta_1 + \theta_2) i]$$

$$z^n = 1 \implies \theta = \frac{2k\pi}{n}$$