Operations with vectors =

1- $[2,7]+[3,1]=[2+3,7+1]=[5,8]$

2- $2[-2,5]+6[2,-8]=[-4,10]+[12,-48]=[8,-38]$

3- $2\vec{u}+3\vec{v} \rightarrow \vec{u}=[2,-3], \vec{v}=[-1,2] \rightarrow 2\vec{u}=[4,-6], 3\vec{v}=[-3,6] \rightarrow [4,-6]+[-3,6]=[1,0]$

4-$[1,-2,4]+[6,-1,1]=[1+6,-2-1,4+1]=[7,-3,5]$

5- $-2[0,-2,1]+3[1,-1,0]=[0,4,-2]+[3,-3,0]=[3,1,-2]$

6- $[2,7]-[3,1]=[2-3, 7-1]=[-1,6]$

7- $3[2,7]-2[3,5]=[6,21]-[6,10]=[0,11]$

8- $5\vec{u}-8\vec{v} \rightarrow u=[3,0], v=[2,-5] \rightarrow 5u=[15,0], 8v=[16,-40] \rightarrow [15,0]-[16,-40]=[-1,40]$

9-$[2,-7,3]-[0,4,-1]=[2,-11,4]$

10-$3u-\frac{1}{2}v \rightarrow u=[a,2b,3c], v=[-4a,b,-2c] \rightarrow [3a,6b,9c]-\left[-2a,\frac{1}{2}b,-1c\right]=[5a,\frac{11}{2}b,10c]$

ضرب داخلی دو بردار (dot product) =

1-$[3,-2].[4,5]=3\times 4+(-2)\times 5=2$

2- $\vec{u}.\vec{v} \rightarrow u=2i+j, v=5i-6j \rightarrow [2,1].[5,-6]=10-6=4$

3- $[2,0].[1,1]=2\times 1+1\times 0=2$

4-$[3,4].[-2,-1]=-6-4=-10$

5-$[2,5,0].\left[\frac{1}{2},-1,10\right]=2\times\frac{1}{2}+5\times(-1)+0\times 10=-4$

6-$(2\mathrm{i}-3\mathrm{j}-\mathrm{k}).(-\mathrm{i}+2\mathrm{j}+8\mathrm{k})=[2,-3,-1].[-1,2,8]=-2-6-8=-16$

7-$\vec{u}.\vec{v} \rightarrow u=6i-4j-2k, v=\frac{5}{6}i+\frac{3}{2}j-k \rightarrow [6,-4,-2].\left[\frac{5}{6},+\frac{3}{2},-1\right]=5-6+2=1$

8-$[-1,2,3].[6,5,-1]=-6+10-3=1$

پیدا کردن زاویه بین دو بردار =

1- find the angle between the vectors $u=[2,5]$ and $v=[4,-3]$:

$$dot\ product=u.v=[2,5].[4,-3]=8-15=-7$$

$$|u|\times|v|\times\cos\Theta=-7$$

$$|u|=\sqrt{2^2+5^2}=\sqrt{29}, |v|=\sqrt{4^2+(-3)^2}=5 \rightarrow |u|\times|v|=5\sqrt{29}$$

$$\cos\Theta=-\frac{7}{5\sqrt{29}} \rightarrow \Theta=\cos^{-1}-\frac{7}{5\sqrt{29}}\approx 105.1°$$

2- find the angle between the vectors $u = [2,7]$ and $v = [3,1]$:

$$dot\ product = u.v = [2,7].[3,1] = 6 + 7 = 13$$

$$|u| \times |v| \cos \Theta = 13$$

$$|u| = \sqrt{2^2 + 7^2} = \sqrt{53}, \qquad |v| = \sqrt{3^2 + 1^2} = \sqrt{10} \to |u| \times |v| = \sqrt{530}$$

$$\cos \Theta = \frac{13}{\sqrt{530}} \to \Theta = \cos^{-1}\frac{13}{\sqrt{530}} \approx 55.6196°$$

3- find the angle between the vectors $u = [0,-5]$ and $v = [-1,-\sqrt{3}]$:

$$dot\ product = u.v = [0,-5].[-1,-\sqrt{3}] = 5\sqrt{3}$$

$$|u| \times |v| \cos \Theta = 5\sqrt{3}$$

$$|u| = \sqrt{(-5)^2} = 5, \qquad |v| = \sqrt{(-1)^2 + \left(-\sqrt{3}\right)^2} = 2 \to \ |u| \times |v| = 10$$

$$10 \cos \Theta = 5\sqrt{3} \to \Theta = \cos^{-1}\frac{5\sqrt{3}}{10} = \cos^{-1}\frac{\sqrt{3}}{2} = 30°$$

4- find the angle between the vectors $u = i + 3j$ and $v = 4i - j$:

$$dot\ product = u.v = [1,3].[4,-1] = 4 - 3 = 1$$

$$|u| \times |v| \cos \Theta = 1$$

$$|u| = \sqrt{1 + 3^2} = \sqrt{10}\ , |v| = \sqrt{4^2 + 1} = \sqrt{17} \to |u| \times |v| = \sqrt{170}$$

$$\cos \Theta = \frac{1}{\sqrt{170}} \to \cos^{-1}\frac{1}{\sqrt{170}} \approx 85.6°$$

5- find the angle between the vectors u=$[2,-2,-1]$and v=$[1,2,2]$:

$$dot\ product = [2,-2,-1].[1,2,2] = 2 - 4 - 2 = -4$$

$$|u| \times |v| \cos \Theta = -4$$

$$|u| = \sqrt{2^2 + (-2)^2 + (-1)^2} = \sqrt{9} = 3 \quad |v| = \sqrt{1^2 + 2^2 + 2^2} = 3 \to |u| \times |v| = 9$$

$$9 \cos \Theta = -4 \to \cos \Theta = -\frac{4}{9} \to \ \Theta = \cos^{-1} -\frac{4}{9} = 116.3878°$$

6- find the angle between the vectors $u = j + k\ , v = i + 2j - 3k$:

$$dot\ product = [0,1,1].[1,2,-3] = 0 + 2 - 3 = -1$$

$$|u| \times |v| \cos \Theta = -1$$

$$|u| = \sqrt{2}\ \ , \qquad |v| = \sqrt{1^2 + 2^2 + (-3)^2} = \sqrt{14} \to |u| \times |v| = \sqrt{280} = 2\sqrt{7}$$

$$2\sqrt{7} \cos \Theta = -1 \to \cos \Theta = -\frac{1}{2\sqrt{7}} \to \ \Theta = \cos^{-1} -\frac{1}{2\sqrt{7}} \approx 100.89°$$

ضرب خارجی دو بردار(cross product)=

$$det\begin{bmatrix} i & j & k \\ a1 & a2 & a3 \\ b1 & b2 & b3 \end{bmatrix} = i.det\begin{bmatrix} a2 & a3 \\ b2 & b3 \end{bmatrix} - j.det\begin{bmatrix} a1 & a3 \\ b1 & b3 \end{bmatrix} + k.det\begin{bmatrix} a1 & a2 \\ b1 & b2 \end{bmatrix} =$$

$$(a2b3 - a3b2)i - (a1b3 - a3b1)j + (a1b2 - a2b1)k$$

$$u \times v = [a2b3 - a3b2, a3b1 - a1b3, a1b2 - a2b1]$$

1-$[0,-1,3] \times [2,0,-1] \to \det\begin{bmatrix} i & j & k \\ 0 & -1 & 3 \\ 2 & 0 & -1 \end{bmatrix} = i.\det\begin{bmatrix} -1 & 3 \\ 0 & -1 \end{bmatrix} - j.\det\begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix} + k.\det\begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix} =$

$i + 6j + 2k = [1,6,2]$

2-$[1,0,-3] \times [2,3,0] \to \det\begin{bmatrix} i & j & k \\ 1 & 0 & -3 \\ 2 & 3 & 0 \end{bmatrix} = i.\det\begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} - j.\det\begin{bmatrix} 1 & -3 \\ 2 & 0 \end{bmatrix} + k.\det\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} =$

$9i - 6j + 3k = [9,-6,3]$

3-$[6,-2,8] \times [-9,3,-12] \to \det\begin{bmatrix} i & j & k \\ 6 & -2 & 8 \\ -9 & 3 & -12 \end{bmatrix} = i.\det\begin{bmatrix} -2 & 8 \\ 3 & -12 \end{bmatrix} - j.\det\begin{bmatrix} 6 & 8 \\ -9 & -12 \end{bmatrix} +$

$k.\det\begin{bmatrix} 6 & -2 \\ -9 & 3 \end{bmatrix} = 0$

4- $u \times v \to u = i + j + k \ , v = 3i - 4k \to \det\begin{bmatrix} i & j & k \\ 1 & 1 & 1 \\ 3 & 0 & -4 \end{bmatrix} = i.\det\begin{bmatrix} 1 & 1 \\ 0 & -4 \end{bmatrix} - j.\det\begin{bmatrix} 1 & 1 \\ 3 & -4 \end{bmatrix} +$

$k.\det\begin{bmatrix} 1 & 1 \\ 3 & 0 \end{bmatrix} = -4i + 7j - 3k = [-4,7,-3]$

5-$[2,-3,1] \times [4,-1,5] \to \det\begin{bmatrix} i & j & k \\ 2 & -3 & 1 \\ 4 & -1 & 5 \end{bmatrix} = i.\det\begin{bmatrix} -3 & 1 \\ -1 & 5 \end{bmatrix} - j.\det\begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix} + k.\det\begin{bmatrix} 2 & -3 \\ 4 & -1 \end{bmatrix} =$

$-14i - 6j + 10k = [16,-6,10]$

6-$[1,3,4] \times [2,7,-5] \to \det\begin{bmatrix} i & j & k \\ 1 & 3 & 4 \\ 2 & 7 & -5 \end{bmatrix} = i.\det\begin{bmatrix} 3 & 4 \\ 7 & -5 \end{bmatrix} - j.\det\begin{bmatrix} 1 & 4 \\ 2 & -5 \end{bmatrix} + k.\det\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} =$

$-43i + 13j + k = [-43,13,1]$

7-$u \times v \to u = [2,-3,1], v = [-2,1,1] \to \det\begin{bmatrix} i & j & k \\ 2 & -3 & 1 \\ -2 & 1 & 1 \end{bmatrix} \to i.\det\begin{bmatrix} -3 & 1 \\ 1 & 1 \end{bmatrix} - j.\det\begin{bmatrix} 2 & 1 \\ -2 & 1 \end{bmatrix} +$

$k.\det\begin{bmatrix} 2 & -3 \\ -2 & 1 \end{bmatrix} = -4i - 4j + 8k = [-4,-4,8]$

حل سیستم معادله با ماتریس و روش گاوسی :

1- $\begin{cases} x-y+3z=4 \\ x+2y-2z=10 \\ 3x-y+5z=14 \end{cases} \rightarrow \begin{bmatrix} 1 & -1 & 3 & 4 \\ 1 & 2 & -2 & 10 \\ 3 & -1 & 5 & 14 \end{bmatrix} \rightarrow R2 = R2 - R1 = [\,1-1 \quad 2-(-1) \quad -2-3 \quad 10-4]$

$\rightarrow \begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 3 & -5 & 6 \\ 3 & -1 & 5 & 14 \end{bmatrix} \rightarrow R3 = R3 - 3R1 = [\,3-3 \quad -1-(-3) \quad 5-9 \quad 14-12\,]$

$\rightarrow \begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 3 & -5 & 6 \\ 0 & 2 & -4 & 2 \end{bmatrix} \rightarrow R3 = 3R3 - 2R2 = [0-0 \quad 6-6 \quad -12-(-10) \quad 6-12]$

$\rightarrow \begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 3 & -5 & 6 \\ 0 & 0 & -2 & -6 \end{bmatrix} \rightarrow R3 = -\frac{1}{2}R3 \rightarrow \begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 3 & -5 & 6 \\ 0 & 0 & 1 & 3 \end{bmatrix} \rightarrow$

$z=3 \;\; 3y-15=6 \rightarrow y=7 \; x=2 \rightarrow (2,7,3)$

2- $\begin{cases} x+y+z=2 \\ 2x-3y+2z=4 \\ 4x+y-3z=1 \end{cases} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & -3 & 2 & 4 \\ 4 & 1 & -3 & 1 \end{bmatrix} \rightarrow R2 = R2 - 2R1 \rightarrow \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -5 & 0 & 0 \\ 4 & 1 & -3 & 1 \end{bmatrix} \rightarrow$

$R3 = R3 - 4R1 \rightarrow \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -5 & 0 & 0 \\ 0 & -3 & -7 & -7 \end{bmatrix} \rightarrow R3 = 5R3 - 3R2 \rightarrow \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -5 & 0 & 0 \\ 0 & 0 & -35 & -35 \end{bmatrix} \rightarrow$

$R3 = -\frac{1}{35}\,R3 \rightarrow \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -5 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \rightarrow z=1, y=0 \; x+z=2 \rightarrow x=1 \rightarrow (1,0,1)$

3- $\begin{cases} x-2y+z=1 \\ \quad y+2z=5 \\ x+y+3z=8 \end{cases} \rightarrow \begin{bmatrix} 1 & -2 & 1 & 1 \\ 0 & 1 & 2 & 5 \\ 1 & 1 & 3 & 8 \end{bmatrix} \rightarrow R3 = R3 - R1 \rightarrow \begin{bmatrix} 1 & -2 & 1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 3 & 2 & 7 \end{bmatrix} \rightarrow R3 = R3 - 3R2$

$\rightarrow \begin{bmatrix} 1 & -2 & 1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & -4 & -8 \end{bmatrix} \rightarrow R3 = -\frac{1}{4}R3 \rightarrow \begin{bmatrix} 1 & -2 & 1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 2 \end{bmatrix} \rightarrow z=2\,, y+4=5 \rightarrow y=1\;\,, x=1 \rightarrow (1,1,2)$

4- $\begin{cases} 2x+y-2z=12 \\ -x-\frac{1}{2}y+z=-6 \\ 3x+\frac{3}{2}y-3z=18 \end{cases} \rightarrow \begin{bmatrix} 2 & 1 & -2 & 12 \\ -1 & -0.5 & 1 & -6 \\ 3 & 1.5 & -3 & 18 \end{bmatrix} \rightarrow R2=2R2+R1 \rightarrow \begin{bmatrix} 2 & 1 & -2 & 12 \\ 0 & 0 & 0 & 0 \\ 3 & 1.5 & -3 & 18 \end{bmatrix} \rightarrow$

$R3=2R3-3R1 \rightarrow \begin{bmatrix} 2 & 1 & -2 & 12 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow y\ and\ z\ can\ take\ any\ number \rightarrow y=s\ ,z=t$

$\rightarrow infinite\ solutions \rightarrow 2x+s-2t=12 \rightarrow 2x=-s+2t+12 \rightarrow x=-0.5s+t+6$

5- $\begin{cases} x+4y-2z=-3 \\ 2x-y+5z=12 \\ 8x+5y+11z=30 \end{cases} \rightarrow \begin{bmatrix} 1 & 4 & -2 & -3 \\ 2 & -1 & 5 & 12 \\ 8 & 5 & 11 & 30 \end{bmatrix} \rightarrow R2=R2-2R1 \rightarrow \begin{bmatrix} 1 & 4 & -2 & -3 \\ 0 & -9 & 9 & 18 \\ 8 & 5 & 11 & 30 \end{bmatrix} \rightarrow$

$R3=R3-8R1 \rightarrow \begin{bmatrix} 1 & 4 & -2 & -3 \\ 0 & -9 & 9 & 18 \\ 0 & -27 & 27 & 54 \end{bmatrix} \rightarrow R3=R3-3R2 \rightarrow \begin{bmatrix} 1 & 4 & -2 & -3 \\ 0 & -9 & 9 & 18 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow$

$R2=\frac{1}{9}R2 \rightarrow \begin{bmatrix} 1 & 4 & -2 & -3 \\ 0 & -1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow z\ can\ take\ any\ number(t) \rightarrow infinite\ solutions$

$-y+t=2 \rightarrow y=t-2\ , \qquad x+4y-2t=-3 \rightarrow x+2t=5 \rightarrow x=5-2t$

6- $\begin{cases} 2x-3y-9z=-5 \\ x \qquad +3z=2 \\ -3x+y-4z=-3 \end{cases} \rightarrow \begin{bmatrix} 2 & -3 & -9 & -5 \\ 1 & 0 & 3 & 2 \\ -3 & 1 & -4 & -3 \end{bmatrix} \rightarrow R2=2R2-R1 \rightarrow \begin{bmatrix} 2 & -3 & -9 & -5 \\ 0 & 3 & 15 & 9 \\ -3 & 1 & -4 & -3 \end{bmatrix} \rightarrow$

$R3=2R3+3R1 \rightarrow \begin{bmatrix} 2 & -3 & -9 & -5 \\ 0 & 3 & 15 & 9 \\ 0 & -7 & -35 & -21 \end{bmatrix} \rightarrow R3=\frac{1}{7}R3\ , R2=\frac{1}{3}R2 \rightarrow$

$\begin{bmatrix} 2 & -3 & -9 & -5 \\ 0 & 1 & 5 & 3 \\ 0 & 1 & 5 & 3 \end{bmatrix} \rightarrow R3=R3-R2 \rightarrow \begin{bmatrix} 2 & -3 & -9 & -5 \\ 0 & 1 & 5 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow z\ can\ take\ any\ number(t)$

$\rightarrow infinite\ solutions \rightarrow y+5t=3 \rightarrow y=3-5t$

$2x-3y-9t=-5 \rightarrow 2x-3(3-5t)-9t=-5 \rightarrow 2x+6t=4 \rightarrow x=-3t+2$

$$7\text{-}\begin{cases} x+y+z=2 \\ y-3z=1 \\ 2x+y+5z=0 \end{cases} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & -3 & 1 \\ 2 & 1 & 5 & 0 \end{bmatrix} \rightarrow R3-2R1 \rightarrow \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & -3 & 1 \\ 0 & -1 & 3 & -4 \end{bmatrix} \rightarrow R3=R3+R2$$

$$\rightarrow \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & -3 \end{bmatrix} \rightarrow 0z=-3 \rightarrow 0=-3!! \rightarrow no\ solution!$$

$$8\text{-}\begin{cases} x-y+3z=3 \\ 4x-8y+32z=24 \\ 2x-3y+11z=4 \end{cases} \rightarrow \begin{bmatrix} 1 & -1 & 3 & 3 \\ 4 & -8 & 32 & 24 \\ 2 & -3 & 11 & 4 \end{bmatrix} \rightarrow R2=\tfrac{1}{4}R2 \rightarrow \begin{bmatrix} 1 & -1 & 3 & 3 \\ 1 & -2 & 8 & 6 \\ 2 & -3 & 11 & 4 \end{bmatrix} \rightarrow R2=R2-R1$$

$$\begin{bmatrix} 1 & -1 & 3 & 3 \\ 0 & -1 & 5 & 3 \\ 2 & -3 & 11 & 4 \end{bmatrix} \rightarrow R3=R3-2R1 \rightarrow \begin{bmatrix} 1 & -1 & 3 & 3 \\ 0 & -1 & 5 & 3 \\ 0 & -1 & 5 & -2 \end{bmatrix} \rightarrow R3=R3-R2$$

$$\begin{bmatrix} 1 & -1 & 3 & 3 \\ 0 & -1 & 5 & 3 \\ 0 & 0 & 0 & -5 \end{bmatrix} \rightarrow\ 0=-5! \rightarrow no\ solution!$$

$$9\text{-}\begin{cases} x-3y+2z=12 \\ 2x-5y+5z=14 \\ x-2y+3z=20 \end{cases} \rightarrow \begin{bmatrix} 1 & -3 & 2 & 12 \\ 2 & -5 & 5 & 14 \\ 1 & -2 & 3 & 20 \end{bmatrix} = R2=R2-2R1 \rightarrow \begin{bmatrix} 1 & -3 & 2 & 12 \\ 0 & 1 & 1 & -10 \\ 1 & -2 & 3 & 20 \end{bmatrix} \rightarrow R3=R3-R1$$

$$\begin{bmatrix} 1 & -3 & 2 & 12 \\ 0 & 1 & 1 & -10 \\ 0 & 1 & 1 & 8 \end{bmatrix} \rightarrow R3=R3-R2 \rightarrow \begin{bmatrix} 1 & -3 & 2 & 12 \\ 0 & 1 & 1 & -10 \\ 0 & 0 & 0 & 18 \end{bmatrix} \rightarrow 0=18\,! \rightarrow no\ solution!$$

Gauss Jordan elimination: (using reduced row-echelon form)

$$1\text{-}\begin{cases} 4x+8y-4z=4 \\ 3x+8y+5z=-11 \\ -2x+y+12z=-17 \end{cases} \rightarrow \begin{bmatrix} 4 & 8 & -4 & 4 \\ 3 & 8 & 5 & -11 \\ -2 & 1 & 12 & -17 \end{bmatrix} \rightarrow R1=\tfrac{1}{4}R1 \rightarrow \begin{bmatrix} 1 & 2 & -1 & 1 \\ 3 & 8 & 5 & -11 \\ -2 & 1 & 12 & -17 \end{bmatrix} \rightarrow$$

$$R2=R2-3R1 \rightarrow \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 2 & 8 & -14 \\ -2 & 1 & 12 & -17 \end{bmatrix} \rightarrow R2=\frac{1}{2}R2 \quad , \quad R3=R3+2R1 \rightarrow$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 5 & 5 & -15 \end{bmatrix} \rightarrow R3=\frac{1}{5}R3 \rightarrow \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 1 & 2 & -3 \end{bmatrix} \rightarrow R3=R3-R2 \rightarrow$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & -2 & 4 \end{bmatrix} \rightarrow R3=-\frac{1}{2}R3 \rightarrow \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 1 & -2 \end{bmatrix} \rightarrow R1=R1+R3 \rightarrow$$

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 1 & -2 \end{bmatrix} \rightarrow R2=R2-4R3 \rightarrow \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} \rightarrow\ R1=R1-2R2 \rightarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} \rightarrow x=-3\,, y=1, z=-2 \rightarrow (-3,1,-2)$$

$$2-\begin{cases} x+y+z=5 \\ 2x+3y+5z=8 \\ 4x \qquad +5z=2 \end{cases} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 5 \\ 2 & 3 & 5 & 8 \\ 4 & 0 & 5 & 2 \end{bmatrix} \rightarrow R2-2R1 \rightarrow \begin{bmatrix} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 4 & 0 & 5 & 2 \end{bmatrix} \rightarrow R3-4R1 \rightarrow$$

$$\begin{bmatrix} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 0 & -4 & 1 & -18 \end{bmatrix} \rightarrow R3+4R2 \rightarrow \begin{bmatrix} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 13 & -26 \end{bmatrix} \rightarrow R3=\frac{1}{13}R3 \rightarrow \begin{bmatrix} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 1 & -2 \end{bmatrix} \rightarrow$$

$$R1=R1-R2 \rightarrow \begin{bmatrix} 1 & 0 & -2 & 7 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 1 & -2 \end{bmatrix} \rightarrow R1=R1+2R3 \rightarrow \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 1 & -2 \end{bmatrix} \rightarrow R2=R2-3R3 \rightarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -2 \end{bmatrix} \rightarrow$$

$$x=3\,, y=4\,, z=-2 \rightarrow (3,4,-2)$$

$$3\text{-}\begin{cases} 3x-y+z=-4 \\ x+y+z=2 \\ 2x+3y+4z=8 \end{cases} \rightarrow \begin{bmatrix} 3 & -1 & 1 & -4 \\ 1 & 1 & 1 & 2 \\ 2 & 3 & 4 & 8 \end{bmatrix} \rightarrow R2=3R2-R1 \rightarrow \begin{bmatrix} 3 & -1 & 1 & -4 \\ 0 & 4 & 2 & 10 \\ 2 & 3 & 4 & 8 \end{bmatrix} \rightarrow R2=\frac{1}{2}R2$$

$$, R3=3R3-2R1 \rightarrow \begin{bmatrix} 3 & -1 & 1 & -4 \\ 0 & 2 & 1 & 5 \\ 0 & 11 & 10 & 32 \end{bmatrix} \rightarrow R3=2R3-11R2 \rightarrow \begin{bmatrix} 3 & -1 & 1 & -4 \\ 0 & 2 & 1 & 5 \\ 0 & 0 & 9 & 9 \end{bmatrix} \rightarrow R3=\frac{1}{9}R3$$

$$\begin{bmatrix} 3 & -1 & 1 & -4 \\ 0 & 2 & 1 & 5 \\ 0 & 0 & 1 & 1 \end{bmatrix} \rightarrow R1=2R1+R2 \rightarrow \begin{bmatrix} 6 & 0 & 3 & -3 \\ 0 & 2 & 1 & 5 \\ 0 & 0 & 1 & 1 \end{bmatrix} \rightarrow R1=R1-3R3 \rightarrow \begin{bmatrix} 6 & 0 & 0 & -6 \\ 0 & 2 & 1 & 5 \\ 0 & 0 & 1 & 1 \end{bmatrix} \rightarrow$$

$$R1=\frac{1}{6}R1 \rightarrow \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 2 & 1 & 5 \\ 0 & 0 & 1 & 1 \end{bmatrix} \rightarrow R2=R2-R3 \rightarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix} \rightarrow R2=\frac{1}{2}R2 \rightarrow \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$x=-1\,, y=2 \quad , z=1 \rightarrow (-1\,, 2\,, 1)$$

Matrix operations:

1- $\begin{bmatrix} 2 & -3 \\ 0 & 5 \\ 7 & -5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -3 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ -3 & 6 \\ 9 & -3 \end{bmatrix}$

2- $\begin{bmatrix} 7 & -3 & 0 \\ 0 & 1 & 5 \end{bmatrix} - \begin{bmatrix} 6 & 0 & -6 \\ 8 & 1 & 9 \end{bmatrix} = \begin{bmatrix} 1 & -3 & 6 \\ -8 & 0 & -4 \end{bmatrix}$

3- $\begin{bmatrix} 2 & 6 \\ -5 & 3 \end{bmatrix} + \begin{bmatrix} -1 & -3 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 1 & 5 \end{bmatrix}$

4- $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & -2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 2 \\ 0 & -2 & 2 \end{bmatrix}$

5- $3\begin{bmatrix} 1 & 2 \\ 4 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 12 & -3 \\ 3 & 0 \end{bmatrix}$

6- $2\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} + 3\begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 5 \\ 3 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 0 & 2 \\ 0 & 2 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & 0 \\ 6 & 3 & 15 \\ 9 & 3 & -6 \end{bmatrix} = \begin{bmatrix} 5 & 5 & 0 \\ 8 & 3 & 17 \\ 9 & 5 & -4 \end{bmatrix}$

7- $\begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 17 & 23 \\ 1 & -5 & -2 \end{bmatrix}$

8- $\begin{bmatrix} 5 & 7 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 9 & -1 \end{bmatrix} = \begin{bmatrix} 68 & 3 \\ -3 & -6 \end{bmatrix}$

9- $\begin{bmatrix} 1 & 2 \\ 9 & -1 \end{bmatrix} \begin{bmatrix} 5 & 7 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 7 \\ 48 & 63 \end{bmatrix}$

10- $\begin{bmatrix} 2 & 6 \\ 1 & 3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} 5 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ -2 \end{bmatrix}$

11- $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 \\ 2 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 2 & 1 \\ 7 & 10 & -7 \end{bmatrix}$

12- $\begin{bmatrix} 2 & -3 \\ 0 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 7 \end{bmatrix}$

13- $5\begin{bmatrix} 2 & -5 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 10 & -25 \\ 0 & 35 \end{bmatrix}$

14- $\begin{bmatrix} 2 & -5 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} 3 & 0.5 & 5 \\ 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 6 & -5 \\ 7 & -7 & 21 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 13 & -7 \end{bmatrix}$

15- $5\begin{bmatrix} 2 & -1 & 8 \\ -2 & 3 & 1 \end{bmatrix} . 2\begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 10 & -5 & 40 \\ -10 & 15 & 5 \end{bmatrix} . \begin{bmatrix} 8 \\ -4 \\ 6 \end{bmatrix} = \begin{bmatrix} 340 \\ -110 \end{bmatrix}$

Determinant of matrices =

1- $\det\begin{pmatrix}6 & -3\\2 & 3\end{pmatrix} = 6\times 3-(-3)\times 2 = 24$

2-$\det\begin{pmatrix}2 & 0\\0 & 3\end{pmatrix} = 6$

3-$\det\begin{pmatrix}\frac{3}{2} & 1\\-1 & -\frac{2}{3}\end{pmatrix} = \frac{3}{2}\times\left(-\frac{2}{3}\right)-(-1) = 0$

4-$\det\begin{pmatrix}4 & 5\\0 & -1\end{pmatrix} = 4\times(-1) = -4$

5-$\det\begin{pmatrix}2 & 3 & -1\\0 & 2 & 4\\-2 & 5 & 6\end{pmatrix} = 2.\det\begin{pmatrix}2 & 4\\5 & 6\end{pmatrix}-3.\det\begin{pmatrix}0 & 4\\-2 & 6\end{pmatrix}-\det\begin{pmatrix}0 & 2\\-2 & 5\end{pmatrix} = -44$

6-$\det\begin{pmatrix}2 & 1 & 0\\0 & -2 & 4\\0 & 1 & -3\end{pmatrix} = 2.\det\begin{pmatrix}-2 & 4\\1 & -3\end{pmatrix}-1.\det\begin{pmatrix}0 & 4\\0 & -3\end{pmatrix} = 4$

7-$\det\begin{pmatrix}30 & 0 & 20\\0 & -10 & -20\\40 & 0 & 10\end{pmatrix} = 30.\det\begin{pmatrix}-10 & -20\\0 & 10\end{pmatrix}+20.\det\begin{pmatrix}0 & -10\\40 & 0\end{pmatrix} = 5000$

8-$\det\begin{pmatrix}1 & 3 & 7\\2 & 0 & 8\\0 & 2 & 2\end{pmatrix} = \det\begin{pmatrix}0 & 8\\2 & 2\end{pmatrix}-3.\det\begin{pmatrix}2 & 8\\0 & 2\end{pmatrix}+7.\det\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix} = -16-12+28 = 0$

Finding the inverse of a matrix =

$$for\ a\ 2\times 2\ matrix \rightarrow \begin{bmatrix}a & b\\c & d\end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix}d & -b\\-c & a\end{bmatrix}$$

1- $A = \begin{bmatrix}4 & 5\\2 & 3\end{bmatrix} \rightarrow find\ A^{-1}: \frac{1}{2}\begin{bmatrix}3 & -5\\-2 & 4\end{bmatrix} = \begin{bmatrix}1.5 & -2.5\\-1 & 2\end{bmatrix}$

2- $A = \begin{bmatrix}-3 & -5\\2 & 3\end{bmatrix} \rightarrow find\ A^{-1}: \frac{1}{-9-(-10)}\begin{bmatrix}3 & 5\\-2 & -3\end{bmatrix} = \begin{bmatrix}3 & 5\\-2 & -3\end{bmatrix}$

3- $A = \begin{bmatrix}2 & 5\\-5 & -13\end{bmatrix} \rightarrow find\ A^{-1}: \frac{1}{-26-(-25)}\begin{bmatrix}-13 & -5\\5 & 2\end{bmatrix} = \begin{bmatrix}13 & 5\\-5 & -2\end{bmatrix}$

4-$A = \begin{bmatrix}6 & -3\\-8 & 4\end{bmatrix} \rightarrow find\ A^{-1}: \frac{1}{24-24}\begin{bmatrix}4 & 3\\8 & 6\end{bmatrix} \rightarrow no\ inverse$

5- $A = \begin{bmatrix}0.4 & -1.2\\0.3 & 0.6\end{bmatrix} \rightarrow find\ A^{-1}: \frac{1}{0.24-(-0.36)}\begin{bmatrix}0.6 & 1.2\\-0.3 & 0.4\end{bmatrix} = \begin{bmatrix}1 & 2\\-0.5 & 0.66\end{bmatrix}$

Eigenvalues and eigenvectors =

1- find eigenvalues and eigenvectors of $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$:

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} v = \lambda v$$

$$\left(\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) v = 0$$

$$\det\begin{pmatrix} 2-\lambda & 1 \\ 3 & 4-\lambda \end{pmatrix} = 0 \rightarrow (2-\lambda)(4-\lambda) - 3 = 0$$

$$\lambda^2 - 6\lambda + 5 = 0 \rightarrow \lambda = 5\,,1$$

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5x \\ 5y \end{bmatrix} \rightarrow \begin{bmatrix} 2x+y \\ 3x+4y \end{bmatrix} = \begin{bmatrix} 5x \\ 5y \end{bmatrix} \rightarrow \begin{cases} 5x = 2x + y \\ 5y = 3x + 4y \end{cases}$$

$$y = 3x \rightarrow v = \begin{bmatrix} x \\ 3x \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} 2x+y \\ 3x+4y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{cases} x = 2x + y \\ y = 3x + 4y \end{cases}$$

$$y = -x \rightarrow v = \begin{bmatrix} x \\ -x \end{bmatrix}$$

2- find eigenvalues and eigenvectors of $\begin{bmatrix} 8 & -2 \\ -3 & 3 \end{bmatrix}$:

$$\begin{bmatrix} 8 & -2 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\left(\begin{bmatrix} 8 & -2 \\ -3 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\det\begin{bmatrix} 8-\lambda & -2 \\ -3 & 3-\lambda \end{bmatrix} = 0 \rightarrow (8-\lambda)(3-\lambda) + 6 = 0$$

$$\lambda^2 - 11\lambda + 18 = 0 \rightarrow \lambda = 9\,,2$$

$$\begin{bmatrix} 8 & -2 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9x \\ 9y \end{bmatrix} = \begin{bmatrix} 8x-2y \\ -3x+3y \end{bmatrix} \rightarrow \begin{cases} 9x = 8x - 2y \\ 9y = -3x + 3y \end{cases}$$

$$x = -2y \;\rightarrow\; v = \begin{bmatrix} -2y \\ y \end{bmatrix}$$

$$\begin{bmatrix} 8 & -2 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix} = \begin{bmatrix} 8x-2y \\ -3x+3y \end{bmatrix} \rightarrow \begin{cases} 2x = 8x - 2y \\ 2y = -3x + 3y \end{cases}$$

$$y = 3x \rightarrow v = \begin{bmatrix} x \\ 3x \end{bmatrix}$$

3- find eigenvalues and eigenvectors of $\begin{bmatrix} 3 & 0 \\ 7 & 2 \end{bmatrix}$:

$$(3-\lambda)(2-\lambda)=0$$

$$\lambda = 2\,,3$$

$$\begin{bmatrix} 3 & 0 \\ 7 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix} = \begin{bmatrix} 3x \\ 7x+2y \end{bmatrix} \to 2x = 3x \to x = 0 \to v = \begin{bmatrix} 0 \\ y \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 \\ 7 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3x \\ 3y \end{bmatrix} = \begin{bmatrix} 3x \\ 7x+2y \end{bmatrix} \to 3y = 7x + 2y \to y = 7x \to v = \begin{bmatrix} x \\ 7x \end{bmatrix}$$

4- find eigenvalues and eigenvectors of $\begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}$:

$$\begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\left(\begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\begin{bmatrix} 0-\lambda & 2 \\ 2 & 3-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\det\begin{bmatrix} 0-\lambda & 2 \\ 2 & 3-\lambda \end{bmatrix} = 0 \to -\lambda(3-\lambda) - 4 = 0$$

$$\lambda^2 - 3\lambda - 4 = 0 \to \ \lambda = 4\ ,-1$$

$$\begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4x \\ 4y \end{bmatrix} = \begin{bmatrix} 2y \\ 2x+3y \end{bmatrix} \to \begin{cases} 4x = 2y \\ 4y = 2x + 3y \end{cases}$$

$$y = 2x \to v = \begin{bmatrix} x \\ 2x \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ -y \end{bmatrix} = \begin{bmatrix} 2y \\ 2x+3y \end{bmatrix} \to \begin{cases} -x = 2y \\ -y = 2x + 3y \end{cases}$$

$$y = -\frac{1}{2}x \to \ \begin{bmatrix} 2x \\ -x \end{bmatrix}$$

5- find eigenvalues and eigenvectors of $\begin{bmatrix}-2 & 1\\ -8 & 2\end{bmatrix} =$

$$\begin{bmatrix}-2 & 1\\ -8 & 2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \lambda\begin{bmatrix}x\\ y\end{bmatrix}$$

$$\begin{bmatrix}-2-\lambda & 1\\ -8 & 2-\lambda\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = 0$$

$$\det\begin{bmatrix}-2-\lambda & 1\\ -8 & 2-\lambda\end{bmatrix} = 0 \to (-2-\lambda)(2-\lambda)+8=0$$

$$\lambda^2-4+8=0 \to \lambda^2=-4 \to \lambda = \pm 2i$$

$$\lambda = 2i \to \begin{bmatrix}-2 & 1\\ -8 & 2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}2ix\\ 2iy\end{bmatrix} = \begin{bmatrix}-2x+y\\ -8x+2y\end{bmatrix} \to \begin{cases}2ix=-2x+y\\ 2iy=-8x+2y\end{cases}$$

$$y=(2+2i)x \ , x=\frac{y-iy}{4} \to v = \begin{bmatrix}1\\ (2+2i)x\end{bmatrix}$$

$$\lambda = -2i \to \begin{bmatrix}-2 & 1\\ -8 & 2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}-2ix\\ -2iy\end{bmatrix} = \begin{bmatrix}-2x+y\\ -8x+2y\end{bmatrix} \to \begin{cases}-2ix=-2x+y\\ -2iy=-8x+2y\end{cases}$$

$$y=(2-2i)x \ , x=\frac{y+iy}{4} \to v = \begin{bmatrix}1\\ (2-2i)x\end{bmatrix}$$

6- find eigenvalues and eigenvectors of $\begin{bmatrix}-8 & 4\\ -5 & 0\end{bmatrix} =$

$$\begin{bmatrix}-8 & 4\\ -5 & 0\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \lambda\begin{bmatrix}x\\ y\end{bmatrix}$$

$$\begin{bmatrix}-8-\lambda & 4\\ -5 & -\lambda\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = 0$$

$$\det\begin{bmatrix}-8-\lambda & 4\\ -5 & -\lambda\end{bmatrix} = 0 \to (-8-\lambda)(-\lambda)+20=0$$

$$\lambda^2+8\lambda+20=0 \to \lambda=-4\pm 2i$$

$$\lambda=-4+2i \to \begin{bmatrix}-8 & 4\\ -5 & 0\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}-4x+2ix\\ -4y+2iy\end{bmatrix} = \begin{bmatrix}-8x+4y\\ -5x\end{bmatrix} \to \begin{cases}-4x+2ix=-8x+4y\\ -4y+2iy=-5x\end{cases}$$

$$x=\frac{4-2i}{5}y \to v = \begin{bmatrix}\frac{4-2i}{5}y\\ y\end{bmatrix}$$

$$\lambda=-4-2i \to \begin{bmatrix}-8 & 4\\ -5 & 0\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}-4x-2ix\\ -4y-2iy\end{bmatrix} = \begin{bmatrix}-8x+4y\\ -5x\end{bmatrix} \to \begin{cases}-4x-2ix=-8x+4y\\ -4y-2iy=-5x\end{cases}$$

$$x=\frac{4+2i}{5}y \to v = \begin{bmatrix}\frac{4+2i}{5}y\\ y\end{bmatrix}$$

7- find eigenvalues and eigenvectors of $\begin{bmatrix} 2 & 1 \\ -5 & 4 \end{bmatrix} =$

$$\det \begin{bmatrix} 2-\lambda & 1 \\ -5 & 4-\lambda \end{bmatrix} = 0 \rightarrow (2-\lambda)(4-\lambda) + 5 = 0$$

$$\lambda^2 - 6\lambda + 13 = 0 \rightarrow \lambda = 3 \pm 2i$$

$$\lambda = 3 + 2i \rightarrow \begin{bmatrix} 2 & 1 \\ -5 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x + y \\ -5x + 4y \end{bmatrix} = \begin{bmatrix} 3x + 2ix \\ 3y + 2iy \end{bmatrix} \rightarrow \begin{cases} 2x + y = 3x + 2ix \\ -5x + 4y = 3y + 2iy \end{cases}$$

$$x = \frac{y + 2iy}{5} \rightarrow v = \begin{bmatrix} \frac{y + 2iy}{5} \\ y \end{bmatrix}$$

$$\lambda = 3 - 2i \rightarrow v = \begin{bmatrix} \frac{y - 2iy}{5} \\ y \end{bmatrix}$$