۳. جبر خطی

۳. جبر خطی

ماتریس

Augmented

ماتریس حالت

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ -1 & +3 & +3 & 4 \\ 2 & -3 & +1 & 10 \end{array}\right]$$

$[x \quad y \quad z]$

دستگاه معادلات خطی

$x^2 \quad \sqrt{\ } \quad \cancel{\sin}$

A system of linear equations

$$\begin{cases} x - 2y - z = 1 \\ -x + 3y + 3z = 4 \\ 2x - 3y + z = 10 \end{cases}$$

A system in triangular form

$$\begin{cases} x - 2y - z = 1 \\ y + 2z = 5 \\ z = 3 \end{cases}$$

معادله ~ مجهول

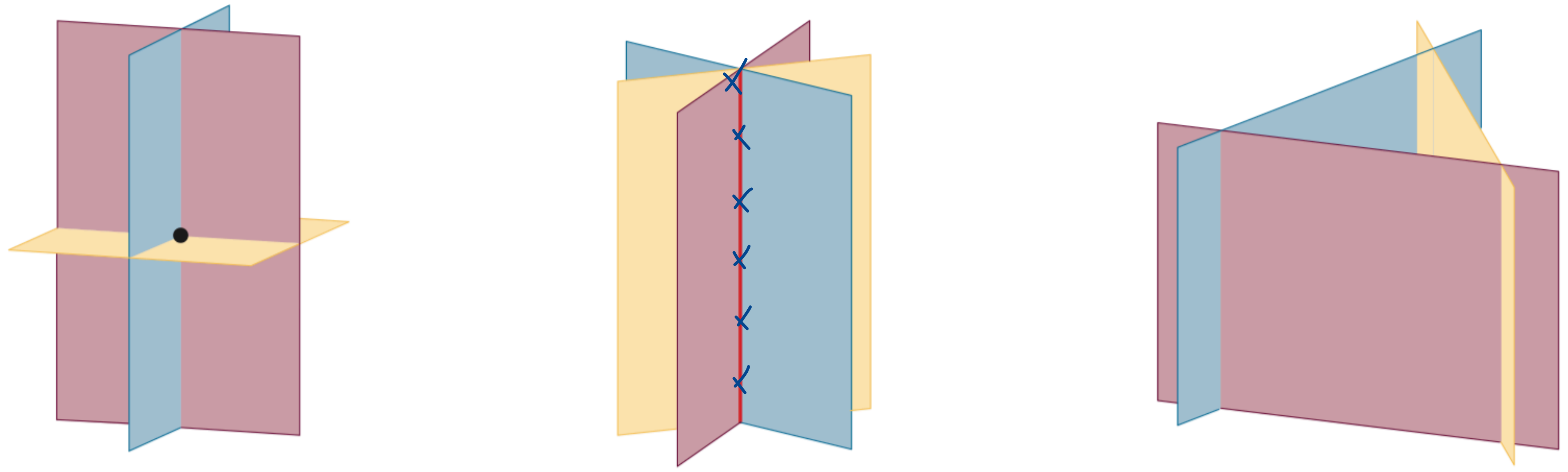
$[0\ 1\ 0]$ $\vec{u} = [1\ 0\ 0]$

$$u \times v = \det \begin{bmatrix} i & 1 & 0 \\ j & 0 & -1 \\ k & 0 & 0 \end{bmatrix} = +i \times 0 - j0 + k\,{-1}$$

$[0\ 0\ {-1}]$

$$\begin{cases} x - 2y - z = 1 \\ -x + 3y + 3z = 4 \\ 2x - 3y + z = 10 \end{cases} \rightarrow \begin{cases} 2x - 4y - 2z = 2 \\ -x + 3y + 3z = 4 \\ 2x - 3y + z = 10 \end{cases}$$

$$\begin{bmatrix} 1 & -2 & -1 & 1 \\ -1 & +3 & +3 & 4 \\ 2 & -3 & +1 & 10 \end{bmatrix} \qquad \begin{bmatrix} 2 & -4 & -2 & 2 \\ -1 & +3 & +3 & 4 \\ 2 & -3 & +1 & 10 \end{bmatrix}$$



(a) The three planes intersect at a single point. The system has one solution.

(b) The three planes intersect at more than one point. The system has infinitely many solutions.

(c) The three planes have no point in common. The system has no solution.

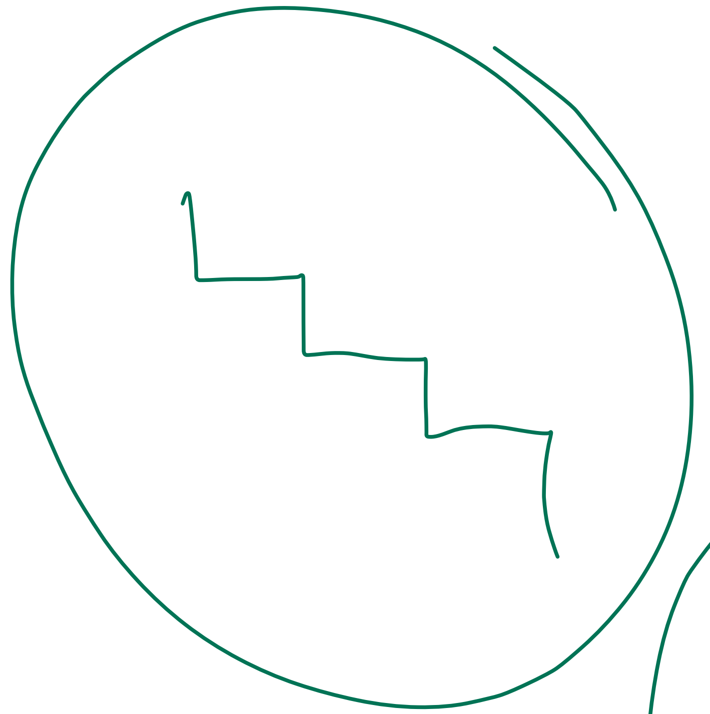
$$\begin{cases} -x + 3y + 3z = 4 \\ 2x - 4y - 2z = 2 \\ 2x - 3y + z = 10 \end{cases} \qquad \begin{bmatrix} -1 & +3 & +3 & 4 \\ 2 & -4 & -2 & 2 \\ 2 & -3 & +1 & 10 \end{bmatrix}$$

۳. جبر خطی

ماتریس

ردیف پلکانی

ردیفی echelon



elementary row operation

1 ردیف هارو جابجا کنیم

2 همه اعداد هر ردیف در عدد ثابت ضرب

3 ردیف هارو جمع کنیم

Not in row-echelon form ردیف

$$\begin{bmatrix} 0 & 1 & -\frac{1}{2} & 0 & 6 \\ 1 & 0 & 3 & 4 & -5 \\ 0 & 0 & 0 & 1 & 0.4 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Leading 1's do *not* shift to the right in successive rows

Row-echelon form

$$\begin{bmatrix} 1 & 3 & -6 & 10 & 0 \\ 0 & 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Leading 1's shift to the right in successive rows

Reduced row-echelon form

$$\begin{bmatrix} 1 & 3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Leading 1's have 0's above and below them

$$\begin{cases} -x + 3y + 3z = 4 \\ 2x - 4y - 2z = 2 \\ 2x - 3y + z = 10 \end{cases}$$

$$R_1 + R_2: \quad \boxed{-x}\ \boxed{+3y}\ \boxed{+3z} = \boxed{4}$$
$$\boxed{2x}\ \boxed{-4y}\ \boxed{-2z} = \boxed{2}$$
$$\Downarrow \; R_1 \quad = x - y + z = 6$$

$$\begin{cases} x - y + z = 6 \\ 2x - 4y - 2z = 2 \\ 2x - 3y + z = 10 \end{cases}$$

$$\begin{bmatrix} 1 & -1 & 1 & 6 \\ 2 & -4 & -2 & 2 \\ 2 & -3 & +1 & 10 \end{bmatrix}$$

This is how the process might work for a 3×4 matrix:

$$\begin{bmatrix} 1 & \blacksquare & \blacksquare & \blacksquare \\ 0 & \blacksquare & \blacksquare & \blacksquare \\ 0 & \blacksquare & \blacksquare & \blacksquare \end{bmatrix} \quad \begin{bmatrix} 1 & \blacksquare & \blacksquare & \blacksquare \\ 0 & 1 & \blacksquare & \blacksquare \\ 0 & 0 & \blacksquare & \blacksquare \end{bmatrix} \quad \begin{bmatrix} 1 & \blacksquare & \blacksquare & \blacksquare \\ 0 & 1 & \blacksquare & \blacksquare \\ 0 & 0 & 1 & \blacksquare \end{bmatrix}$$

$$\left\{\begin{array}{l} -x + 3y + 3z = 4 \\ 2x - 4y - 2z = 2 \\ 2x - 3y + z = 10 \end{array}\right\}$$

$R_2 + 2R_1 \rightsquigarrow R_2$

$$2(-x + 3y + 3z = 4)$$

$$-2x + 6y + 6z = 8$$

$$\left\{\begin{array}{l} -x + 3y + 3z = 4 \\ 2x - 4y - 2z = 2 \\ 2x - 3y + z = 10 \end{array}\right.$$

$$+ \quad 2x - 4y - 2z = 2$$

$$\rightsquigarrow \cancel{0} + 2y + 4z = 10$$

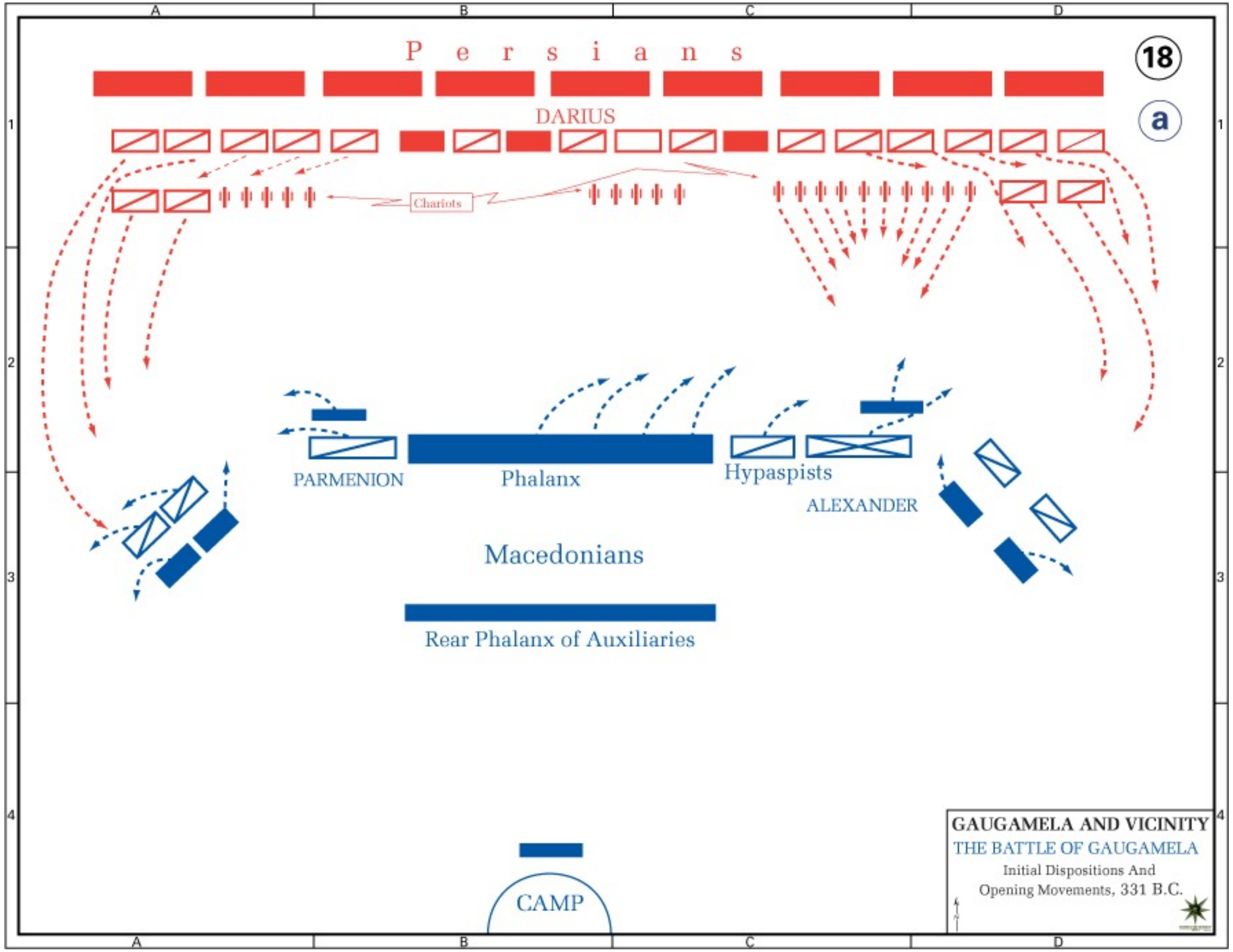
$$\left\{\begin{array}{l} -x + 3y + 3z = 4 \\ 0x + 2y + 4z = 10 \\ 2x - 3y + z = 10 \end{array}\right.$$

$$\begin{bmatrix} -1 & +3 & +3 & 4 \\ 0 & +2 & +4 & 10 \\ 2 & -3 & +1 & 10 \end{bmatrix}$$

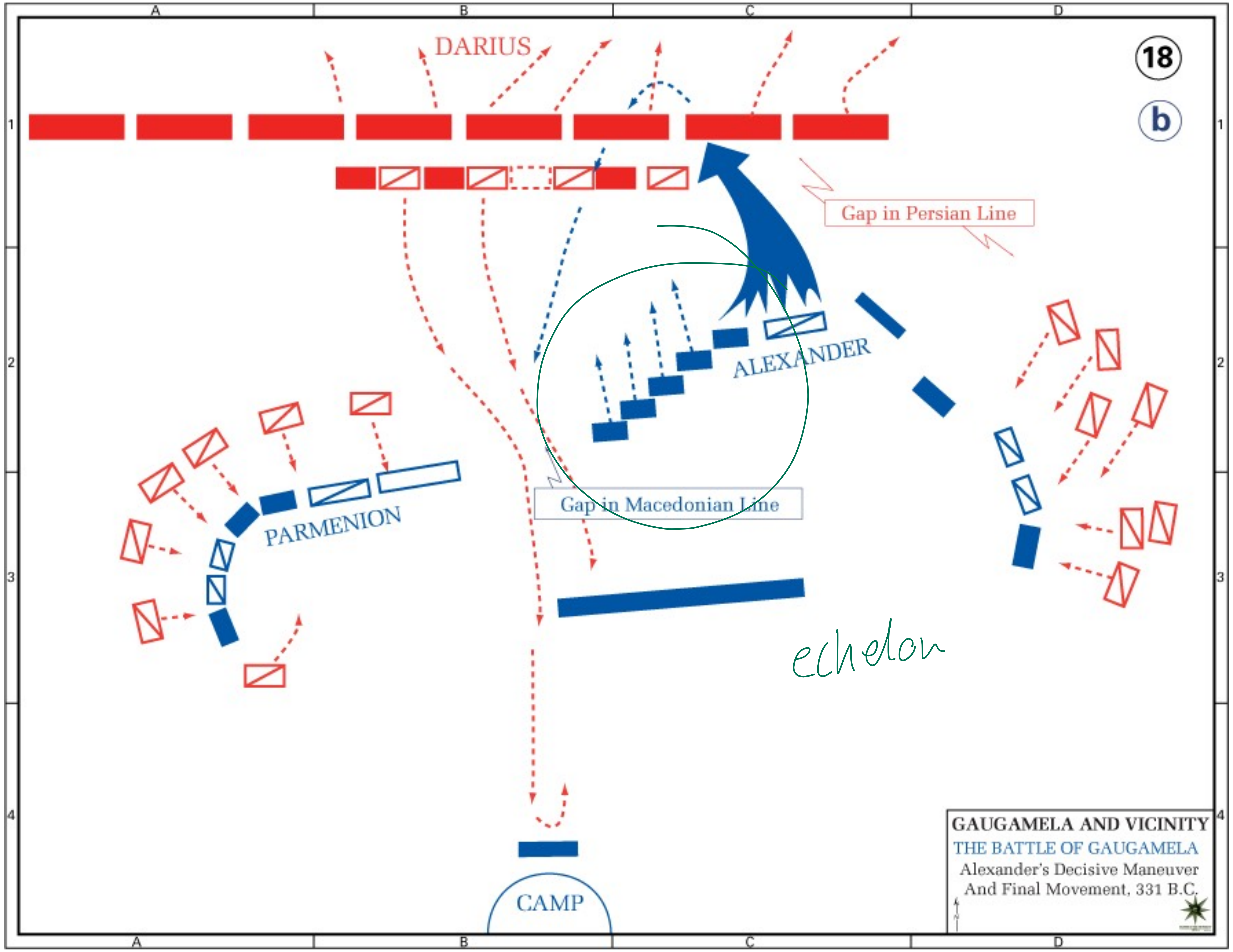
$R_3 + 2R_1 \rightsquigarrow R_3$

$$\begin{bmatrix} -1 & +3 & +3 & 4 \\ 0 & +2 & +4 & 10 \\ 0 & +3 & 7 & 18 \end{bmatrix}$$

اربل



A
B
C
D
1
2
3
4
P e r s i a n s
18
a
DARIUS
Chariots
PARMENION
Phalanx
Hypaspists
ALEXANDER
Macedonians
Rear Phalanx of Auxiliaries
CAMP
GAUGAMELA AND VICINITY
THE BATTLE OF GAUGAMELA
Initial Dispositions And
Opening Movements, 331 B.C.



A
B
C
D
1
2
3
4
18
b
DARIUS
Gap in Persian Line
ALEXANDER
PARMENION
Gap in Macedonian Line
CAMP
GAUGAMELA AND VICINITY
THE BATTLE OF GAUGAMELA
Alexander's Decisive Maneuver
And Final Movement, 331 B.C.

echelon

۳. جبر خطی

ماتریس

Linear system

$$\begin{cases} 3x - 2y + z = 5 \\ x + 3y - z = 0 \\ -x + 4z = 11 \end{cases}$$

Augmented matrix

$$\begin{bmatrix} 3 & -2 & 1 & 5 \\ 1 & 3 & -1 & 0 \\ -1 & 0 & 4 & 11 \end{bmatrix}$$

Symbol	Description
$R_i + kR_j \to R_i$	Change the ith row by adding k times row j to it, and then put the result back in row i.
kR_i	Multiply the ith row by k.
$R_i \leftrightarrow R_j$	Interchange the ith and jth rows.

System		Augmented matrix
$\begin{cases} x - y + 3z = 4 \\ x + 2y - 2z = 10 \\ 3x - y + 5z = 14 \end{cases}$		$\begin{bmatrix} 1 & -1 & 3 & 4 \\ 1 & 2 & -2 & 10 \\ 3 & -1 & 5 & 14 \end{bmatrix}$
$\begin{cases} x - y + 3z = 4 \\ 3y - 5z = 6 \\ 2y - 4z = 2 \end{cases}$	$\xrightarrow{R_2 - R_1 \to R_2}$ $\xrightarrow{R_3 - 3R_1 \to R_3}$	$\begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 3 & -5 & 6 \\ 0 & 2 & -4 & 2 \end{bmatrix}$
$\begin{cases} x - y + 3z = 4 \\ 3y - 5z = 6 \\ y - 2z = 1 \end{cases}$	$\xrightarrow{\frac{1}{2}R_3}$	$\begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 3 & -5 & 6 \\ 0 & 1 & -2 & 1 \end{bmatrix}$
$\begin{cases} x - y + 3z = 4 \\ z = 3 \\ y - 2z = 1 \end{cases}$	$\xrightarrow{R_2 - 3R_3 \to R_2}$	$\begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & -2 & 1 \end{bmatrix}$
$\begin{cases} x - y + 3z = 4 \\ y - 2z = 1 \\ z = 3 \end{cases}$	$\xrightarrow{R_2 \leftrightarrow R_3}$	$\begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

۳. جبر خطی

ماتریس

$$\begin{cases} 4x + 8y - 4z = 4 \\ 3x + 8y + 5z = -11 \\ -2x + y + 12z = -17 \end{cases}$$

Augmented matrix:

$$\begin{bmatrix} 4 & 8 & -4 & 4 \\ 3 & 8 & 5 & -11 \\ -2 & 1 & 12 & -17 \end{bmatrix}$$

Need a 1 here

$$\xrightarrow{\frac{1}{4}R_1} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 3 & 8 & 5 & -11 \\ -2 & 1 & 12 & -17 \end{bmatrix}$$

Need 0's here

$$\xrightarrow[R_3 + 2R_1 \to R_3]{R_2 - 3R_1 \to R_2} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 2 & 8 & -14 \\ 0 & 5 & 10 & -15 \end{bmatrix}$$

Need a 1 here

$$\xrightarrow{\frac{1}{2}R_2} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 5 & 10 & -15 \end{bmatrix}$$

Need a 0 here

$$\xrightarrow{R_3 - 5R_2 \to R_3} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & -10 & 20 \end{bmatrix}$$

Need a 1 here

Row-echelon form:

$$\xrightarrow{-\frac{1}{10}R_3} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

۳. جبر خطی

ماتریس

$$\begin{cases} 4x + 8y - 4z = 4 \\ 3x + 8y + 5z = -11 \\ -2x + y + 12z = -17 \end{cases}$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

Need 0's here

$$\xrightarrow[R_1 + R_3 \to R_1]{R_2 - 4R_3 \to R_2} \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

Need a 0 here

$$\xrightarrow{R_1 - 2R_2 \to R_1} \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$$1x + \cancel{0y} + \cancel{0z} = -3$$
$$\cancel{0x} + 1y + \cancel{0z} = 1$$
$$\cancel{0x} + \cancel{0y} + 1z = -2$$

۳. جبر خطی

ماتریس

No solution

$$\begin{bmatrix} 1 & 2 & 5 & 7 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Last equation says ~~0 = 1~~

$\cancel{0x + 0y + 0z = 1}$

One solution

$$\begin{bmatrix} 1 & 6 & -1 & 3 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & 1 & 8 \end{bmatrix}$$

Each variable is a leading variable

Infinitely many solutions

$$\begin{bmatrix} 1 & 2 & -3 & 1 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

z is not a leading variable

$0 = 0$

$$\begin{cases} 4x + 8y - 4z = 4 \\ 3x + 8y + 5z = -11 \\ -2x + y + 12z = -17 \end{cases} \sim \left|\begin{matrix} 4 & 8 & -4 & 4 \\ 3 & 8 & 5 & -11 \\ -2 & 1 & 12 & -17 \end{matrix}\right|$$

1 باید بشه

$R_1 \times \frac{1}{4}$

$$x + 2y - z = 1$$

$$= \left|\begin{matrix} 1 & 2 & -1 & 1 \\ 3 & 8 & 5 & -11 \\ -2 & 1 & 12 & -17 \end{matrix}\right|$$

0 باید بشه

$R_2 - 3R_1 \rightsquigarrow R_2$

$$\begin{matrix} & 3 & 8 & 5 & -11 \\ -3(& 1 & 2 & -1 & 1 \;) \end{matrix}$$

$$\begin{matrix} 3 & 8 & 5 & -11 \\ -3 & 6 & +3 & -3 \\ \hline 0 & 2 & 8 & -14 \end{matrix}$$

$$\left|\begin{matrix} 1 & 2 & -1 & 1 \\ 0 & 2 & 8 & -14 \\ -2 & 1 & 12 & -17 \end{matrix}\right|$$

0 بشه

$R_3 + 2R_1$

$$\begin{matrix} & -2 & 1 & 12 & -17 \\ +2(& 1 & 2 & -1 & 1 \;) \\ & +2 & 4 & -2 & 2 \\ \hline & 0 & 5 & 10 & -15 \end{matrix}$$

$$\left|\begin{matrix} 1 & 2 & -1 & 1 \\ 0 & 2 & 8 & -14 \\ 0 & 5 & 10 & -15 \end{matrix}\right|$$

1 بشه

$R_2 \; \frac{1}{2}$

$$\left|\begin{matrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 5 & 10 & -15 \end{matrix}\right|$$

0 بشه

$R_3 - 5R_2$

$$\begin{matrix} & 0 & 5 & 10 & -15 \\ -5(& 0 & 1 & 4 & -7 \;) \\ & 0 & -5 & -20 & +35 \\ \hline & 0 & 0 & -10 & +20 \end{matrix}$$

$$\left|\begin{matrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & -10 & 20 \end{matrix}\right|$$

1 بشه

$R_3 \times -\frac{1}{10}$

$$\left|\begin{matrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 1 & -2 \end{matrix}\right|$$

$$\left|\begin{array}{cccc} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 1 & -2 \end{array}\right| \rightarrow \begin{array}{l} x + 2y - z = 1 \\ y + 4z = -7 \\ z = -2 \end{array}$$

$$x + 2y - z = 1$$

$$y + 4(-2) = -7$$

$$y - 8 = -7$$

$$y = 1$$

$$x + 2(1) - (-2) = 1$$

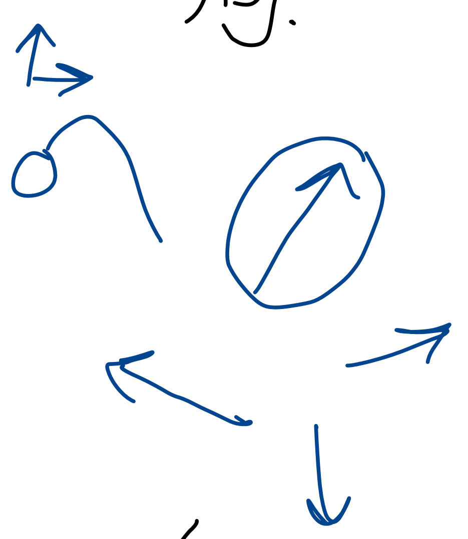
$$x + 2 + 2 = 1$$

$$x = -3$$

۳. جبر خطی

Vector

بردار



بردار

$$\begin{bmatrix} 73kg & 182cm & 1 \end{bmatrix}$$
$$\begin{bmatrix} 2\frac{m}{s} & 3\frac{m}{s} \end{bmatrix}$$
$$\begin{bmatrix} 127m^2 & 3 \end{bmatrix}$$

انتزاعی

کلی

فیزیکدان

آمار دان / علم کامپیوتر

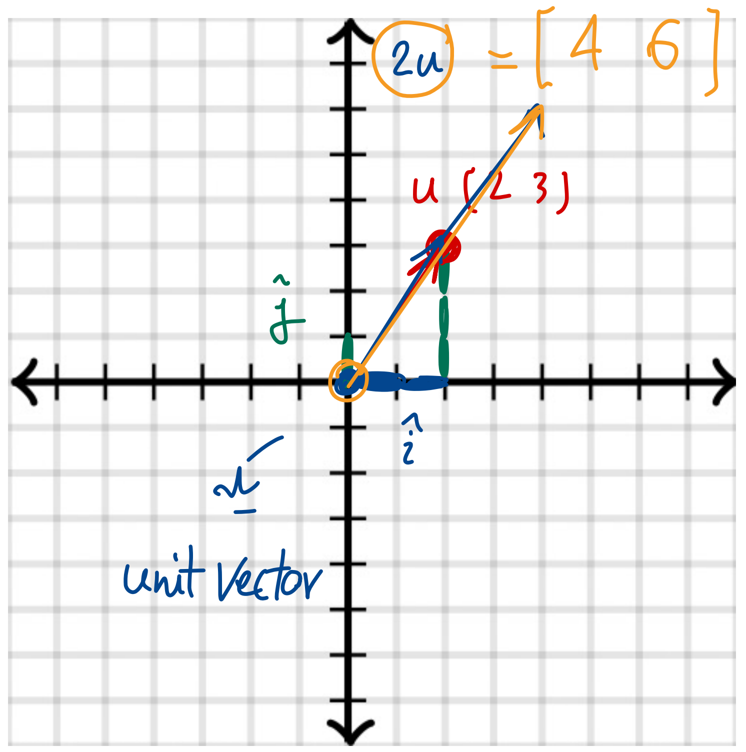
ریاضی دان

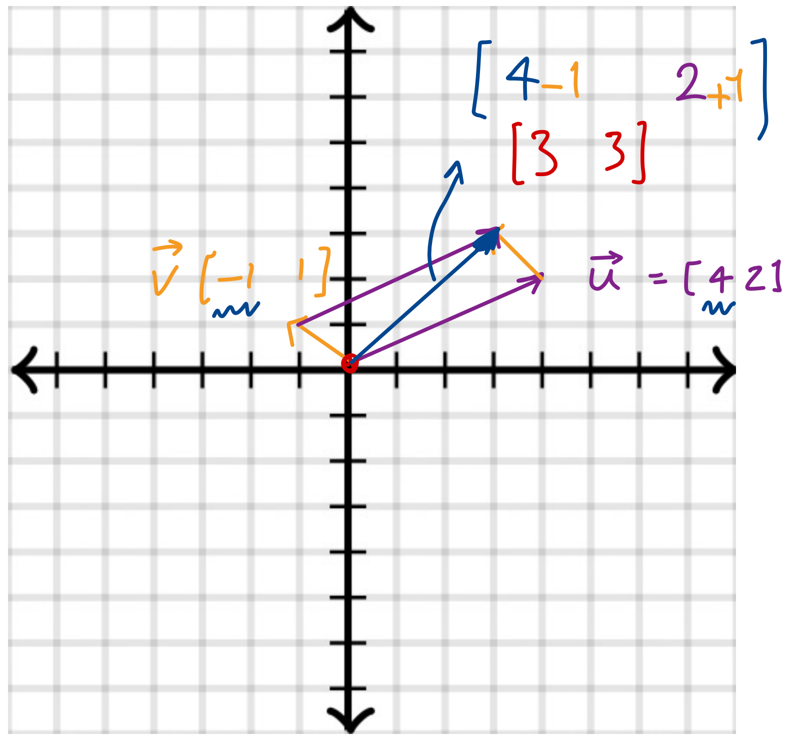
$[2 \; 3 \; 1]$

$\vec{u} = [2 \; 3]$

$x \in \mathbb{R}$

$x \in \mathbb{R}^2$





$$\vec{u} = [\overset{\times}{x}, \overset{\times}{y}]$$

$$\vec{u} = x\hat{i} + y\hat{j}$$

$$|\vec{u}| = \sqrt{u_x^2 + u_y^2}$$

Scalor

$$c\vec{u} = cx\hat{i} + cy\hat{j}$$

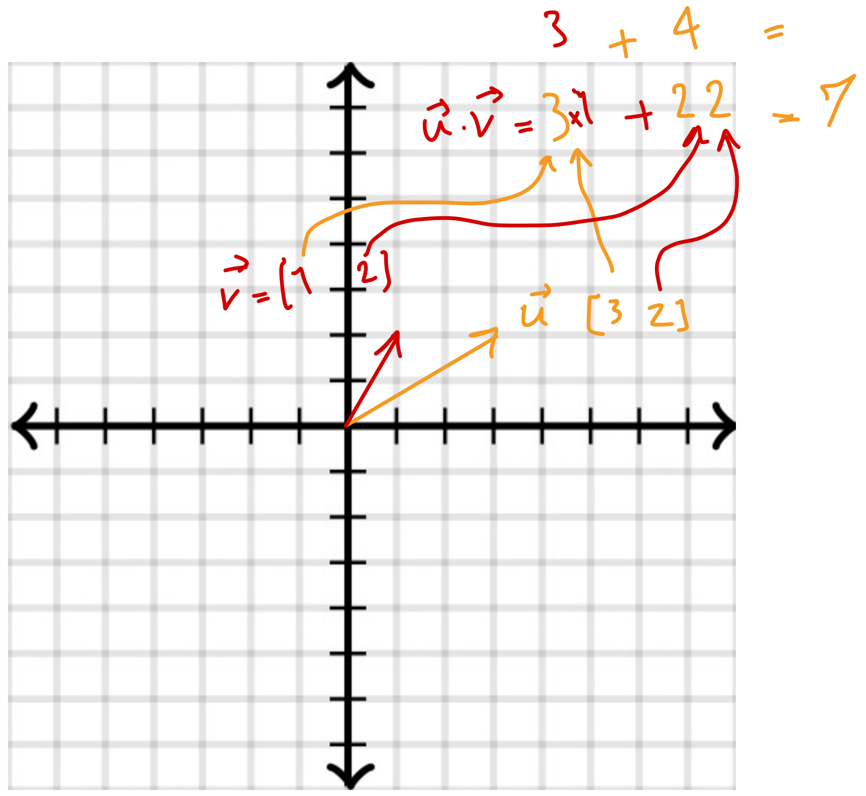
$\vec{u} = [u_x \; u_y]$ $\quad \vec{V} = [V_x \; V_y]$

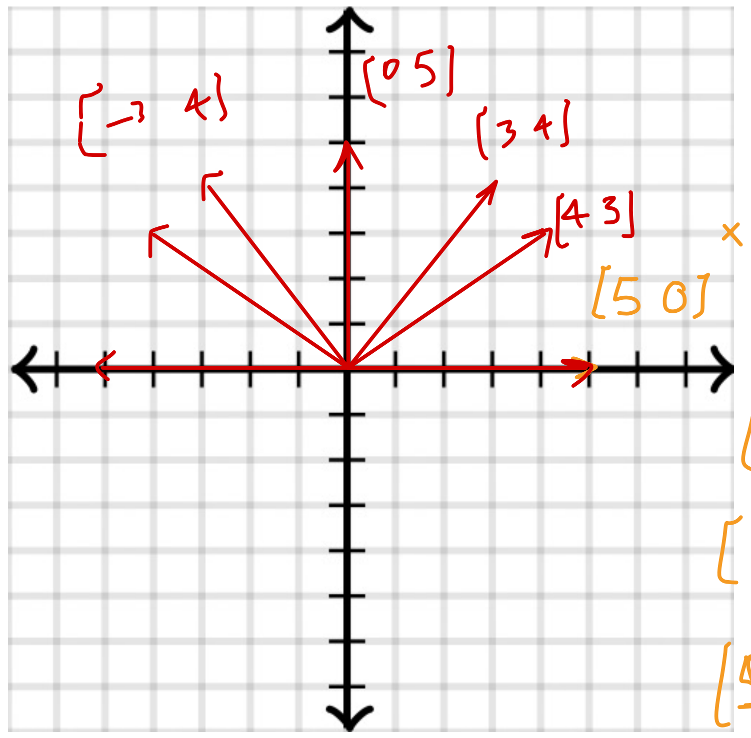
$$\vec{u} + \vec{v} = (u_x + v_x)\hat{x} + (u_y + v_y)\hat{y}$$

ضرب داخلی

dot product

$$\vec{u} \bullet \vec{v} = u_x v_x + u_y v_y = |\vec{u}|\,|\vec{v}|\cos\theta$$



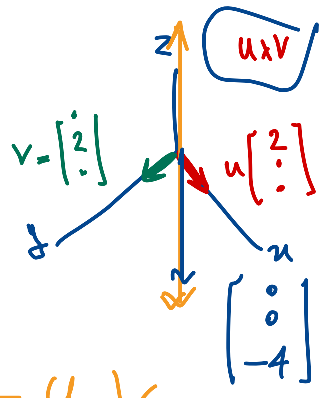


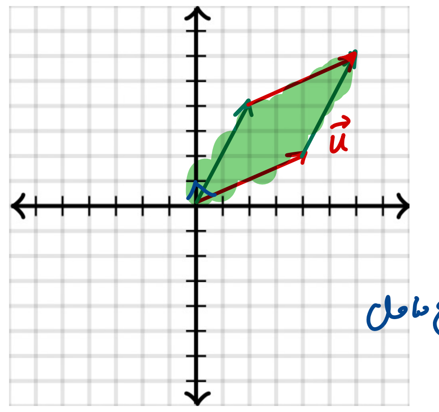
$[5 \; 0] \cdot [5 \; 0] = 25$
$[5 \; 0] \cdot [4 \; 3] = 20$
$[5 \; 0] \cdot [3 \; 4] = 15$
$[5 \; 0] \cdot [0 \; 5] = 0$
$[5 \; 0] \cdot [-3 \; 4] = -15$
$[5 \; 0] \cdot [-4 \; 3] = -20$
$[5 \; 0] \cdot [-5 \; 0] = -25$

دو بردار

$\vec{u} \circ \vec{v} = u_x V_x + u_y V_y$

یک عدد





دو بردار

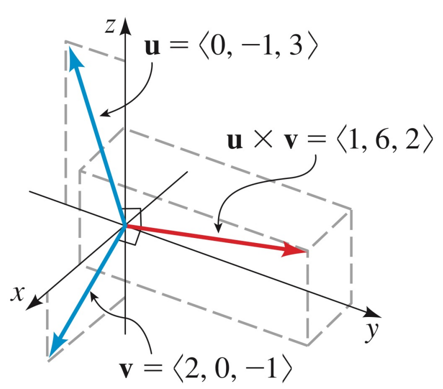
cross product

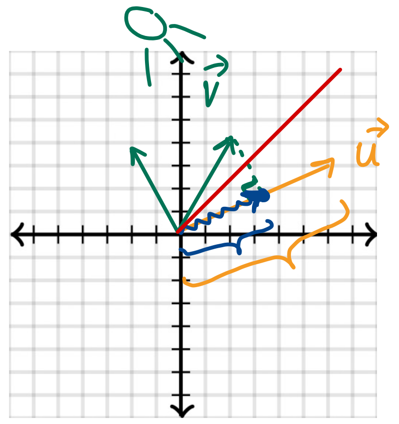
یک بردار

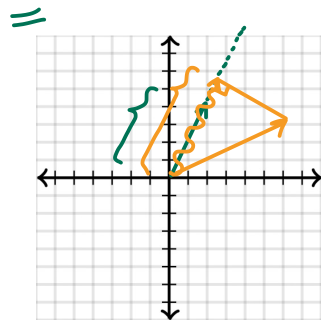
طول هم اندازه با

مساحت متوازی الاضلاع حاصل

با هر دو u و v عمود



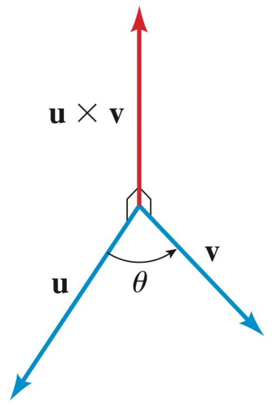




$u \times v$

اشاره

حلقه



ضرب خارجی

Cross product $\vec{u} \times \vec{V} = \begin{vmatrix} x_1 \\ y_1 \\ z_1 \end{vmatrix} \times \begin{vmatrix} x_2 \\ y_2 \\ z_2 \end{vmatrix} = \det \begin{vmatrix} \hat{i} & x_1 & x_2 \\ \hat{j} & y_1 & y_2 \\ \hat{k} & z_1 & z_2 \end{vmatrix} = \hat{i}(y_1 z_2 - y_2 z_1) - \hat{j}(\text{——}$

۳. جبر خطی

ماتریس

$x \in \mathbb{R}^2$

لیست از ابعاد

مستطیلی

ستون ۲

سطر ۳

$$3 \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$$

$1,2 \quad x_{ij} - x_{1,2} = b$

3×2

$x \in \mathbb{R}$

$$M = \boxed{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} \rightarrow i \times j \qquad 2M = \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix}$$

$\rightsquigarrow cM$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

عضو ij ام ماتریس حاصل جمع

را با جمع درایه‌های نظیر ij از ماتریس‌ها به دست می‌آوریم

$ij = M_{ij} + N_{ij}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax+by \\ cx+dy \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix}$$

$$\begin{array}{cc} 1 \rightarrow \\ \text{دوم} \rightarrow \end{array} \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

۱- ابعاد هر ماتریس

$2\times 2 \quad 2\times 2$

۲- آیا مساوی؟

۳-

2×2

ابعاد ماتریس حاصل ضرب

۳. جبر خطی

ماتریس

$C = AB$

$$A = \begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix}$$

AB $2 \times \boxed{2 \checkmark 2} \times 3$

$\cancel{BA}$ $2 \times \boxed{3 \quad 2} \times 2$

2×3

-2

$2,3$

Entry	Inner product of:	Value	Product matrix
c_{12}	$\begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix}$	$1 \cdot 5 + 3 \cdot 4 = 17$	$\begin{bmatrix} -1 & 17 & \\ & & \end{bmatrix}$
c_{13}	$\begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix}$	$1 \cdot 2 + 3 \cdot 7 = 23$	$\begin{bmatrix} -1 & 17 & 23 \\ & & \end{bmatrix}$
c_{21}	$\begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix}$	$(-1) \cdot (-1) + 0 \cdot 0 = 1$	$\begin{bmatrix} -1 & 17 & 23 \\ 1 & & \end{bmatrix}$
c_{22}	$\begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix}$	$(-1) \cdot 5 + 0 \cdot 4 = -5$	$\begin{bmatrix} -1 & 17 & 23 \\ 1 & -5 & \end{bmatrix}$
c_{23}	$\begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix}$	$(-1) \cdot 2 + 0 \cdot 7 = -2$	$\begin{bmatrix} -1 & 17 & 23 \\ 1 & -5 & -2 \end{bmatrix}$

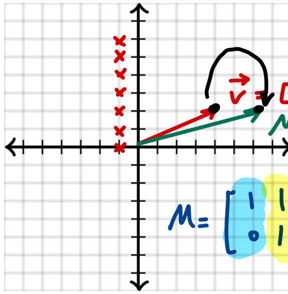
Thus we have $$AB = \begin{bmatrix} -1 & 17 & 23 \\ 1 & -5 & -2 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

$$2 \times \boxed{2 \checkmark 2} \times 1$$

$$2 \times 1$$

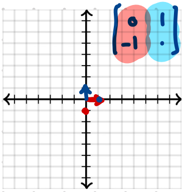
$$V = [4,2] = 4\hat{i} + 2\hat{j}$$
$$= 4\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$



$\vec{v} = [4,2] = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

$Mv = [6\ 2]$

$M = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$



$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$Mv = \begin{bmatrix} 1 \times 4 + 1 \times 2 \\ 0 \times 4 + 1 \times 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

Mv

$$\begin{bmatrix} ax + by \\ cx + dy \end{bmatrix} = \begin{bmatrix} ax \\ cx \end{bmatrix} + \begin{bmatrix} by \\ dy \end{bmatrix}$$

$$x\begin{bmatrix} a \\ c \end{bmatrix} + y\begin{bmatrix} b \\ d \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$i \begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\hat{j} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$V = 4\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$Mv = \begin{bmatrix} 6 \\ 2 \end{bmatrix} \quad 4\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 1 \\ 1 \end{bmatrix} = [6\ 2]$$

۳. جبر خطی

ماتریس

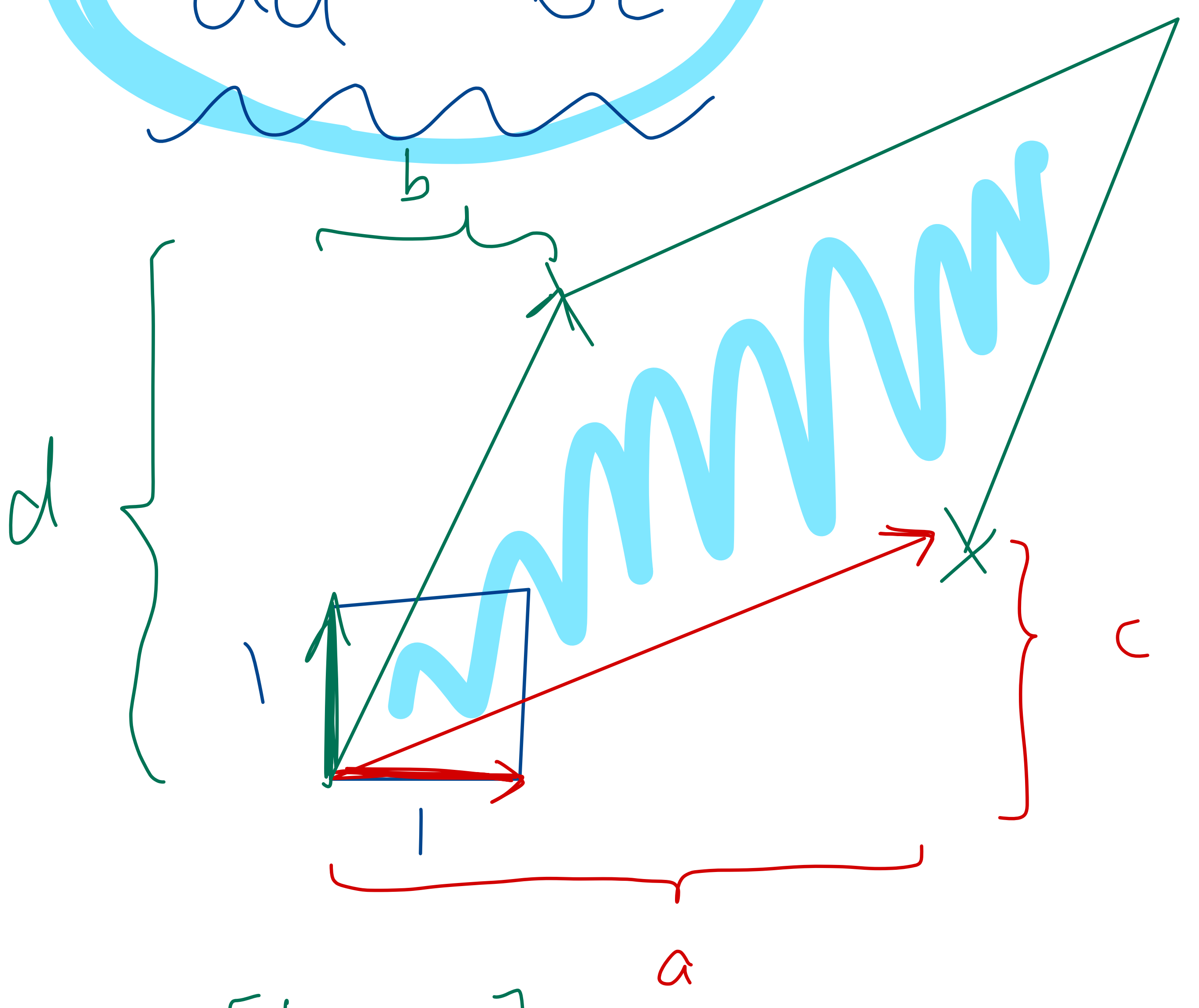
A^{T} transpose

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}^{T} = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$$

$i,j \quad j,i$

$$det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$$

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad ad - bc$$



$$[A^{T}]_{ij} = [A]_{ji}$$

$$AA^{-1} = A^{-1}A = I_n$$

A^{-1} inverse معکوس

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{(ad - bc)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A^{-1}A = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad M^{-1} \quad \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

۳. جبر خطی

ماتریس

$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$

$\det \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(+(ei - fh)) + b((-1)(di - fg)) + c((-1)^4(dh - eg))$

$(-1)^{i+j}[M]$

علامت   مقدار

$-1 \quad +1$

$+a(ei - fh) - b(di - fg) + c(dh - eg)$

$\det \begin{bmatrix} e & f \\ h & i \end{bmatrix}$

$\det \begin{bmatrix} d & f \\ g & i \end{bmatrix}$

$ei - fh \qquad (di - fg)$

$\det \begin{bmatrix} d & e \\ g & h \end{bmatrix} \qquad (dh - eg)$

THE DETERMINANT OF A SQUARE MATRIX

If A is an $n \times n$ matrix, then the **determinant** of A is obtained by multiplying each element of the first row by its cofactor and then adding the results. In symbols,

$$\det(A) = |A| = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = a_{11}A_{11} + a_{12}A_{12} + \cdots + a_{1n}A_{1n}$$

Where $A_{ij} = (-1)^{i+j}M_{ij}$ and M_{ij} is the determinant of matrix M obtained by removing ith row and jth column.

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$\det \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

۳. جبر خطی

ماتریس

$$
\begin{array}{c} \\ n \end{array}
\left[\begin{array}{ccc:ccc}
a & b & c & 1 & 0 & 0 \\
d & e & f & 0 & 1 & 0 \\
g & h & i & 0 & 0 & 1
\end{array}\right]
\sim
\left[\begin{array}{ccc:c}
1 & 0 & 0 & \\
0 & 1 & 0 & \\
0 & 0 & 1 &
\end{array}\right]
$$

elementary row operation

To find the inverse of an $n \times n$ matrix, first construct an $n \times 2n$ matrix:

$$
\left[\begin{array}{cccc:cccc}
a_{11} & a_{12} & \cdots & a_{1n} & 1 & 0 & \cdots & 0 \\
a_{21} & a_{22} & \cdots & a_{2n} & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
a_{n1} & a_{n2} & \cdots & a_{nn} & 0 & 0 & \cdots & 1
\end{array}\right]
\sim n
\overbrace{\left[\begin{array}{ccc:c}
1 & 0 & 0 & \\
0 & 1 & 0 & \\
0 & 0 & 1 &
\end{array}\right]}^{2n}
$$

Then use elementary row operations to turn the left side to the identity matrix (change the whole matrix to reduced row-echelon form). The right side is automatically transformed into the inverse matrix.

۳. جبر خطی

ماتریس

$$A\vec{x} = \lambda\vec{x}$$

$$det(A - \lambda I) = 0$$

$$det\left(\begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix}\right) = 0$$

$$\lambda^2 - \tau\lambda + \Delta = 0$$

مقدار ویژه eigenvalue

بردار ویژه eigenvector

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

$A_{t+1} = 2A_t + 1B_t$

$B_{t+1} = 1A_t + 3B_t$

$V = \begin{bmatrix} A \\ B \end{bmatrix}$ $\quad V_{t+1} = MV$

$$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} =$$

$$\begin{bmatrix} \; \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \; \end{bmatrix}_{t+1}$$

$Ax = \lambda x$ (مقدار ویژه، بردار ویژه)

$Ax - \lambda x = 0$

$x(A - \lambda I) = 0$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$x\begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix} \stackrel{\checkmark}{=} 0 \quad det\begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix} = 0$$

$(a-\lambda)(d-\lambda) - bc = 0$

$ad - a\lambda - d\lambda + \lambda^2 - bc = 0$

معادله درجه ۲ $\quad \lambda^2 - (a+d)\lambda + (ad-bc) = 0$

$tr\begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad det\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\rightarrow \hat{i} \rightarrow [1 \;\; 0]$

$\hat{j} \quad [0 \;\; 1]$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$A=\begin{bmatrix}3&1\\0&2\end{bmatrix}$ ⇝ بردار ویژه A را پیدا کنید $\vec{x}$

$A\vec{x}=\lambda\vec{x}$ $\qquad A\vec{x}-\lambda\vec{x}=0$ $\qquad \vec{x}(A-\lambda)=0$

$\det \underbrace{A-\lambda I} = 0$

$\det\begin{bmatrix}3-\lambda & 1\\ 0 & 2-\lambda\end{bmatrix}=0$ $\qquad (3-\lambda)(2-\lambda)-\cancel{1\times 0}=0$

$\boxed{\lambda=3}$ $\quad \lambda=2$ $\quad$ مقدار ویژه

$\boxed{\lambda=3}$ $\quad A\vec{x}=3\vec{x}$ $\quad =\begin{bmatrix}3&1\\0&2\end{bmatrix}\vec{x}=3\vec{x}$

$\vec{x}=\begin{bmatrix}1\\0\end{bmatrix}$ $\lambda=3$

$=\begin{bmatrix}3&1\\0&2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=3\begin{bmatrix}x\\y\end{bmatrix}$

$\begin{bmatrix}3x+y\\0x+2y\end{bmatrix}=\begin{bmatrix}3x\\3y\end{bmatrix}$

$0x+2y=3y$

$3x+y=3x$

$y=0$

$y=0$ $\quad \begin{bmatrix}1\\0\end{bmatrix}$

② $\begin{bmatrix}3 & 1\\ 0 & 2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = 2\begin{bmatrix}x\\ y\end{bmatrix}$

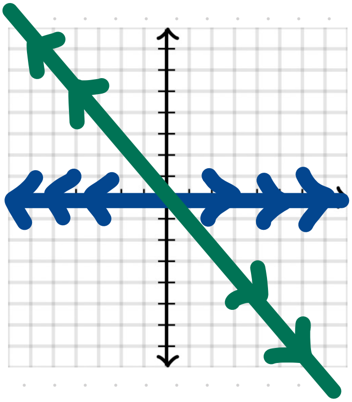
$\begin{bmatrix}3x+y\\ 0x+2y\end{bmatrix} = \begin{bmatrix}2x\\ 2y\end{bmatrix}$

$3x+y = \cancel{2}x$

$x+y=0$

$\begin{bmatrix}1 & -1\end{bmatrix}$

$y=-x$



$M = \begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$

مقدار ویژه

بردار ویژه

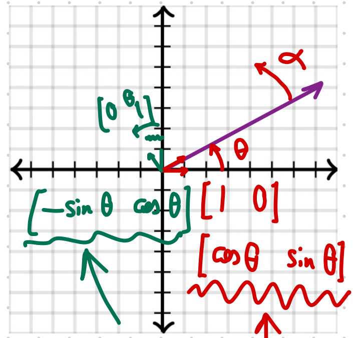
$\det(M-\lambda)=0$

$\det\begin{vmatrix}0-\lambda & -1\\ 1 & 0-\lambda\end{vmatrix} = 0 \quad \underbrace{(-\lambda)(-\lambda)-(-1)(1)}=0$

$\lambda^2+1=0$

$\lambda=\pm i$

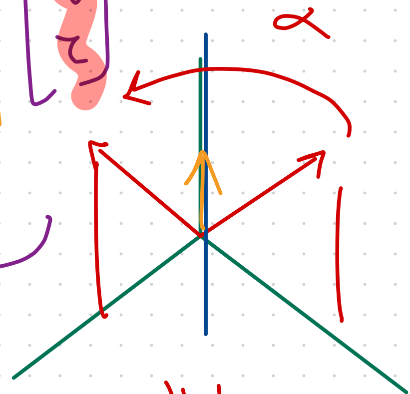
$$\begin{bmatrix} \cos\theta_1 \\ \sin\theta_1 \end{bmatrix} \rightarrow \begin{bmatrix} \cos(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) \end{bmatrix}$$



$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

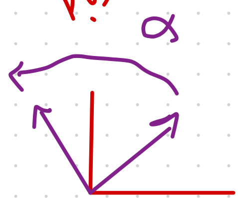
$\hat{j}$



$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{array}{c} a_1 \\ a_2 \\ a_3 \end{array} \begin{bmatrix} \cos\alpha\, x - \sin\alpha\, y + 0z \\ \sin\alpha\, x + \cos\alpha\, y + 0z \\ 0x + 0y + 1z \end{bmatrix}$$

(b_1, c_1; b_2, c_2; b_3, c_3)

new



$$P_{new} \begin{bmatrix} \quad \end{bmatrix}$$