1- مشتق توابع زیر را پیدا کنید .

1- $(3x-2x^2)(5+4x)=$

$$\frac{d}{dx}(3x-2x^2)(5+4x)=$$

$$\frac{d}{dx}(3x-2x^2)\,.(5+4x)+\frac{d}{dx}(5+4x).(3x-2x^2)=(3-4x)(5+4x)+4(3x-2x^2)=$$

$$15-20x+12x-16x^2+12x-8x^2=-24x^2+4x^2+15$$

2- $\sqrt[3]{x^2}(2x-x^2)=$

$$\frac{d}{dx}\sqrt[3]{x^2}(2x-x^2)=$$

$$\frac{d}{dx}\,x^{\frac{2}{3}}\,.(\,2x-x^2)+\frac{d}{dx}(2x-x^2).\left(x^{\frac{2}{3}}\right)=$$

$$\frac{2}{3}x^{-\frac{1}{3}}(2x-x^2)+(2-2x)\left(x^{\frac{2}{3}}\right)=\frac{4}{3}x^{\frac{2}{3}}-\frac{2}{3}x^{\frac{5}{3}}+2x^{\frac{2}{3}}-2x^{\frac{5}{3}}=\frac{10}{3}x^{\frac{2}{3}}-\frac{8}{3}x^{\frac{5}{3}}$$

3- $\left(1+\sqrt{x^3}\right)\left(x^{-3}-2\sqrt[3]{x}\right)=$

$$\frac{d}{dx}\left(1+\sqrt{x^3}\right)\left(x^{-3}-2\sqrt[3]{x}\right)=$$

$$\frac{d}{dx}\left(1+x^{\frac{3}{2}}\right).\left(x^{-3}-2x^{\frac{1}{3}}\right)+\frac{d}{dx}\left(x^{-3}-2x^{\frac{1}{3}}\right).\left(1+x^{\frac{3}{2}}\right)=$$

$$\frac{3}{2}x^{\frac{1}{2}}\left(x^{-3}-2x^{\frac{1}{3}}\right)+\left(-3x^{-4}-\frac{2}{3}x^{-\frac{2}{3}}\right).\left(1+x^{\frac{3}{2}}\right)=$$

$$\frac{3}{2}x^{-\frac{5}{2}}-3x^{\frac{5}{6}}-3x^{-4}-\frac{2}{3}x^{-\frac{2}{3}}-3x^{-\frac{5}{2}}-\frac{2}{3}x^{\frac{5}{6}}=-\frac{11}{3}x^{\frac{5}{6}}-\frac{2}{3}x^{-\frac{2}{3}}-\frac{3}{2}x^{-\frac{5}{2}}-3x^{-4}$$

4- $\left(\frac{1}{x}+1\right)(\,x-1\,)=$

$$\frac{d}{dx}(x^{-1}+1).(x-1)+\frac{d}{dx}(x-1).(x^{-1}+1\,)=$$

$$-x^{-2}(x-1)+(x^{-1}+1)=-x^{-1}+x^{-2}+x^{-1}+1=x^{-2}+1$$

5- $2x(x^2+3x)=$

$$\frac{d}{dx}(2x).(x^2+3x)+\frac{d}{dx}(x^2+3x).(2x)=2x^3+6x^2+(2x+3).(2x)=$$

$$2x^3+6x^2+4x^2+6x=2x^3+10x^2+6x$$

6- $\sin x \cos x =$

$$\frac{d}{dx}\sin x \cos x = \frac{d}{dx}\sin x \,.\cos x + \frac{d}{dx}\cos x \,.\sin x =$$

$$\cos x \,.\cos x - \sin x \,.\sin x = \cos^2 x - \sin^2 x$$

7-$x^2 \sin x =$

$$\frac{d}{dx}x^2 \sin x = \frac{d}{dx}x^2 .\sin x + \frac{d}{dx}\sin x \,.x^2 = 2x.\sin x + x^2 \cos x$$

8- $(2x-3)^{-2} \times (4x+3)^{-2} =$

$$\frac{d}{dx}(2x-3)^{-2} \times (4x+3)^{-2} + \frac{d}{dx}(4x+3)^{-2} \times (2x-3)^{-2} =$$

$$-2(2x-3)^{-3} \times 2 \times (4x+3)^{-2} - 2(4x+3)^{-3} \times 4 \times (2x-3)^{-2} =$$

$$\frac{-4}{(2x-3)^3 \times (4x+3)^2} + \frac{-8}{(2x-3)^2 \times (4x+3)^3} =$$

$$\frac{-4(4x+3)}{(2x-3)^3 \times (4x+3)^3} + \frac{-8(2x-3)}{(2x-3)^3 \times (4x+3)^3} =$$

$$\frac{-16x-12-16x+24}{(2x-3)^3 \times (4x+3)^3} = \frac{-32x+12}{(2x-3)^3 \times (4x+3)^3} = \frac{-4(8x-3)}{(2x-3)^3 \times (4x+3)^3}$$

9-$\cos(\,2x+1) =$

$$2x+1 \rightarrow u \rightarrow \frac{d}{dx}\cos\,(\,2x+1) = \frac{d}{dx}\cos u \;\times \frac{d}{dx}u =$$

$$-\sin u \times \frac{d}{dx}(2x+1) = -2\sin(2x+1)$$

10-$(4x-3)^5 =$

$$4x-3 \rightarrow u \rightarrow \frac{d}{dx}(4x-3)^5 = \frac{d}{dx}u^5 \times \frac{d}{dx}u$$

$$5u^4 \times \frac{d}{dx}(4x-3) = 5(4x-3)^4 \times 4 = 20(4x-3)^4$$

11- $(x^2+1)^3 =$

$$u = x^2+1 \rightarrow \frac{d}{dx}(x^2+1)^3 = \frac{d}{dx}u^3 \times \frac{d}{dx}u =$$

$$3u^2 \times \frac{d}{dx}(x^2+1) = 3(x^2+1)^2 \times 2x = 6x(x^2+1)^2$$

12- $-3\sin(4x^2+5) =$

$$4x^2+5 \to u \ , \frac{d}{dx} - 3\sin(4x^2+5) = \frac{d}{dx} - 3\sin u \times \frac{d}{dx} u =$$

$$-3\cos u \times \frac{d}{dx}(4x^2+5) = -3\cos(4x^2+5) \times 8x = -24x\cos(4x^2+5)$$

13-5 $\ln(x^4) =$

$$u = x^4, \frac{d}{dx} 5\ln(x^4) = \frac{d}{dx} 5\ln u \times \frac{d}{dx} u =$$

$$\frac{5}{u} \times \frac{d}{dx}(x^4) = \frac{5}{x^4} \times 4x^3 = \frac{20x^3}{x^4} = \frac{20}{x}$$

14-$\ln\sin\sqrt{1+x^2} =$

$$f = \sin\sqrt{1+x^2}\,, g = \sqrt{1+x^2}\,, h = 1+x^2$$

$$\frac{d}{dx}\ln\sin\sqrt{1+x^2} = \frac{d}{dx}\ln f \times \frac{d}{dx}\sin g \times \frac{d}{dx}\sqrt{h} \times \frac{d}{dx} h =$$

$$\frac{1}{f} \times \cos g \times \frac{1}{2\sqrt{h}} \times \frac{d}{dx}(1+x^2) =$$

$$\frac{1}{\sin\sqrt{1+x^2}} \times \cos\sqrt{1+x^2} \times \frac{1}{2\sqrt{1+x^2}} \times 2x =$$

$$\cot\sqrt{1+x^2} \times \frac{x}{\sqrt{1+x^2}} = \frac{x\ \cot\sqrt{1+x^2}}{\sqrt{1+x^2}}$$

15- $\sin^2 5x =$

$$\frac{d}{dx}\sin^2 5x = \frac{d}{dx}(\sin 5x)^2 \to f = \sin 5x \ , g = 5x$$

$$\frac{d}{dx}\sin^2 5x = \frac{d}{dx} f^2 \times \frac{d}{dx}\sin g \times \frac{d}{dx} 5x =$$

$$2f \times \cos g \times 5 = 10\ \sin 5x \times \cos 5x$$

16-$(3x^2-4x+1)^8 =$

$$u \to 3x^2-4x+1 \to \frac{d}{dx}(3x^2-4x+1)^8 = \frac{d}{dx} u^8 \times \frac{d}{dx} u =$$

$$8u^7 \times \frac{d}{dx}(3x^2-4x+1) = 8(3x^2-4x+1)^7 \times (6x-4) = 8(6x-4)(3x^2-4x+1)^7$$

$17 = \sqrt{\frac{a^2 - x^2}{a^2 + x^2}} =$

$$h = \frac{f}{g} \to f = a^2 - x^2\,, \quad g = a^2 + x^2$$

$$\frac{d}{dx}\sqrt{\frac{a^2 - x^2}{a^2 + x^2}} = \frac{d}{dx}\sqrt{h} \times \frac{d}{dx}h = \frac{1}{2\sqrt{h}} \times \frac{\acute{f}g - \acute{g}f}{g^2} \to$$

$$\acute{f} = \frac{d}{dx}(a^2 - x^2) = -2x \quad , \qquad \acute{g} = \frac{d}{dx}(a^2 + x^2) = 2x$$

$$\frac{1}{2\sqrt{\frac{a^2 - x^2}{a^2 + x^2}}} \times \frac{-2x(a^2 + x^2) - 2x(a^2 - x^2)}{(a^2 + x^2)^2} =$$

$$\frac{-x(a^2 + x^2 + a^2 - x^2)}{\sqrt{\frac{a^2 - x^2}{a^2 + x^2}} \times (a^2 + x^2)^2} = \frac{-2a^2x}{\sqrt{a^2 - x^2} \times (a^2 + x^2)^{-\frac{1}{2}} \times (a^2 + x^2)^2} =$$

$$\frac{-2a^2x}{\sqrt{a^2 - x^2} \times (a^2 + x^2)^{\frac{3}{2}}}$$

18-$3e^{4x} =$

$$u = 4x \to \frac{d}{dx}3e^{4x} = \frac{d}{dx}3e^{u} \times \frac{d}{dx}u$$

$$3e^{u} \times \frac{d}{dx}4x = 12e^{u} = 12e^{4x}$$

2- انتگرال توابع زیر را پیدا کنید .

1- $\int_0^3 (x^3 - 6x)dx =$

$$\int (x^3 - 6x)dx \,\Big|_0^3 = F(3) - F(0)$$

$$F = \int (x^3 - 6x\,)\,dx = \frac{1}{4}x^4 - 3x^2 + c$$

$$\frac{1}{4}(3)^4 - 3(3)^2 - \frac{1}{4}(0)^4 + 3(0)^2 = \frac{1}{4}81 - 27 = -6.75$$

2- $\int_0^1 (4 + 3x^2)dx =$

$$\int (4 + 3x^2)dx \Big|_0^1 = F(1) - F(0) =$$

$$F = \int (4 + 3x^2)dx \rightarrow \acute{F} = (4 + 3x^2) \rightarrow F = 4x + x^3 + c$$

$$4(1) + (1)^3 = 5$$

3- $\int_0^2 (2 - x^2)dx =$

$$\int (2 - x^2)dx \Big|_0^2 = F(2) - F(0)$$

$$F = \int (2 - x^2)dx \rightarrow \acute{F} = (2 - x^2) \rightarrow F = 2x - \frac{1}{3}x^3 + c$$

$$2(2) - \frac{1}{3}(2)^3 = 4 - \frac{8}{3} = \frac{4}{3}$$

4- $\int_{-1}^5 (1 + 3x)dx =$

$$\int (1 + 3x)dx \Big|_{-1}^{5} = F(5) - F(-1)$$

$$F = \int (1 + 3x)dx \rightarrow \acute{F} = 1 + 3x \rightarrow F = x + \frac{3}{2}x^2 + c$$

$$5 + \frac{3}{2}(5)^2 - (-1) - \frac{3}{2}(-1)^2 = 5 + \frac{75}{2} + 1 - \frac{3}{2} = \frac{85}{2} - \frac{1}{2} = \frac{84}{2} = 42$$

5-$\int_0^1 (5 - 6x^2)dx =$

$$\int (5 - 6x^2)\, dx \Big|_0^1 = F(1) - f(0)$$

$$F = \int (5 - 6x^2)\, dx \rightarrow \acute{F} = 5 - 6x^2 \rightarrow F = 5x - 2x^3 + c$$

$$5(1) - 2(1)^3 = 3$$

6- $\int_1^2 x^3 dx =$

$$\int x^3 dx \Big|_1^2 = F(2) - F(1)$$

$$F = \int x^3 = \frac{1}{4}x^4 + c \rightarrow \frac{1}{4}(2)^4 + c - \frac{1}{4}(1)^4 - c = 4 - \frac{1}{4} = \frac{15}{4}$$

7-$\int_0^1 10^x dx =$

$$\int 10^x\, dx \Big|_0^1 = F(1) - F(0)$$

$$F = \int 10^x\, dx = \frac{10^x}{\ln 10} + c$$

$$\frac{10}{\ln 10} + c - \frac{10^0}{\ln 10} - c = \frac{10-1}{\ln 10} = \frac{9}{\ln 10}$$

8-$\int_1^3 e^x\, dx =$

$$\int e^x dx \Big|_1^3 = F(3) - F(1)$$

$$F = \int e^x \rightarrow e^x \rightarrow e^3 - e^1 = e(e^2 - 1)$$

$9 - \int_3^6 \frac{dx}{x} =$

$$\int \frac{1}{x} dx \Big|_3^6 = F(6) - F(3)$$

$$F = \int \frac{1}{x} \rightarrow \ln x \rightarrow \ln 6 - \ln 3 \;= \ln 2$$

10- $\int_\pi^{2\pi} \cos\Theta\, d\Theta =$

$$\int \cos\Theta\, d\Theta \Big|_\pi^{2\pi} = F(2\pi) - F(\pi)$$

$$F = \sin\Theta \rightarrow \sin(2\pi) - \sin(\pi) = 0$$

11-$\int_{-1}^{0} (2x - e^x)\, dx =$

$$\int (2x - e^x) dx \Big|_{-1}^{0} = F(0) - F(-1)$$

$$F = x^2 - e^x \rightarrow\; -e^0 - (-1)^2 + e^{-1} = e^{-1} - 2 = \frac{1}{e} - 2$$

12-$\int_{-2}^{-1}\left(4y^3+\frac{2}{y^3}\right)dy =$

$$\int\left(4y^3+\frac{2}{y^3}\right)dy\Big|_{-2}^{-1} = F(-1)-F(-2)$$

$$\acute{F} = \left(4y^3+2\frac{1}{y^3}\right) \to F = \left(y^4+(-y^{-2})\right) = (y^4-\frac{1}{y^2})$$

$$(-1)^4-\frac{1}{(-1)^2}-(-2)^4+\frac{1}{(-2)^2} = -16+\frac{1}{4} = -\frac{63}{4}$$

13-$\int_{0}^{2}(6x^2-4x+5)dx =$

$$\int(6x^2-4x+5)\,dx\Big|_{0}^{2} = F(2)-F(0)$$

$$F = 2x^3-2x^2+5x \to 2(2^3)-2(2^2)+5(2) = 16-8+10 = 18$$