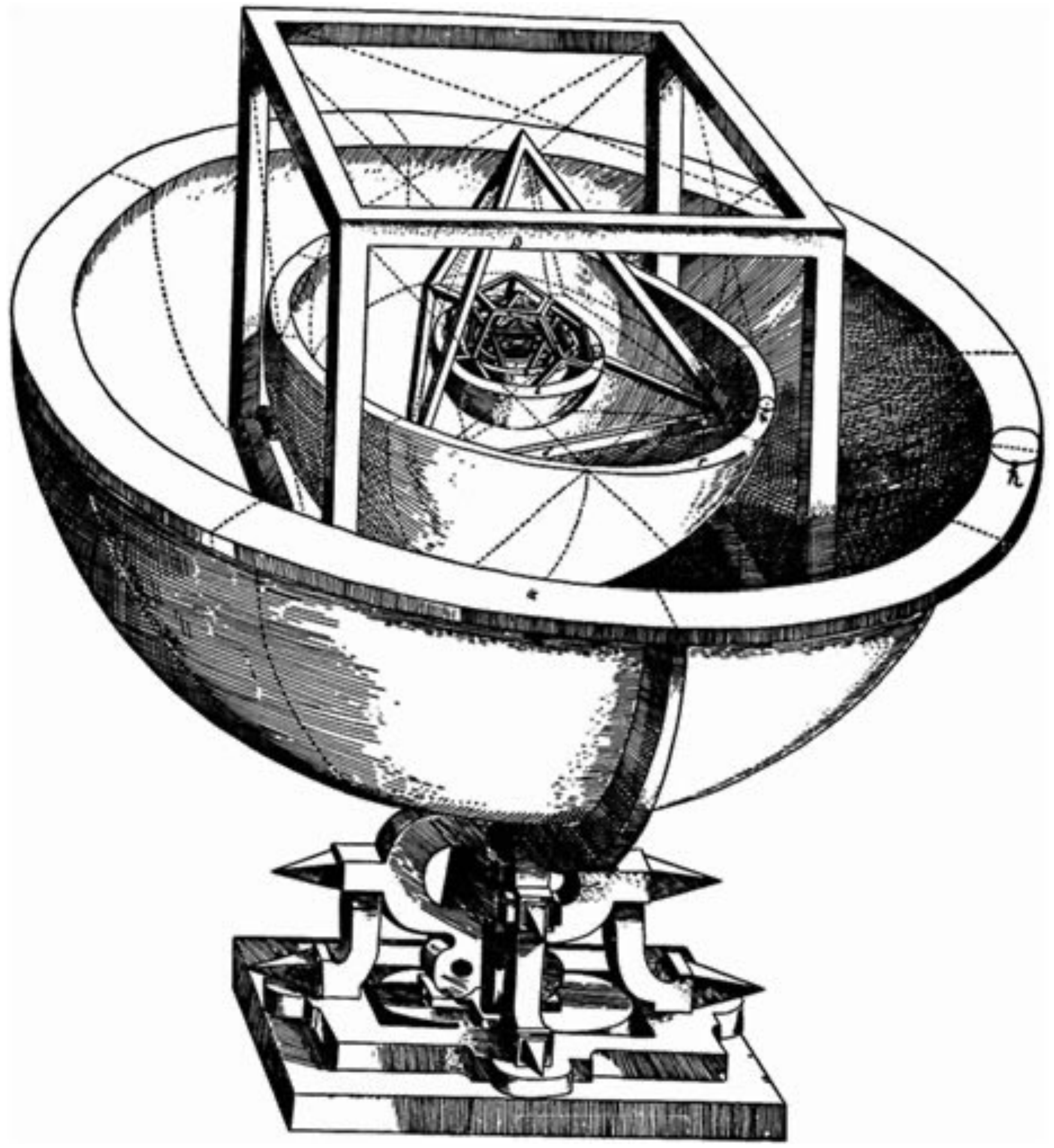


مدلسازی مقدماتی

امیرحسین زارع مهذبیه

تابستان ۰۰

اهداف

- مدل‌ها در علم؛ راهی برای شناخت
- انواع مدل در علوم زیستی
- چگونه پدیده‌ها را مدل کنیم
- چگونه مدل را تحلیل کنیم

فهرست

۱. مقدمه

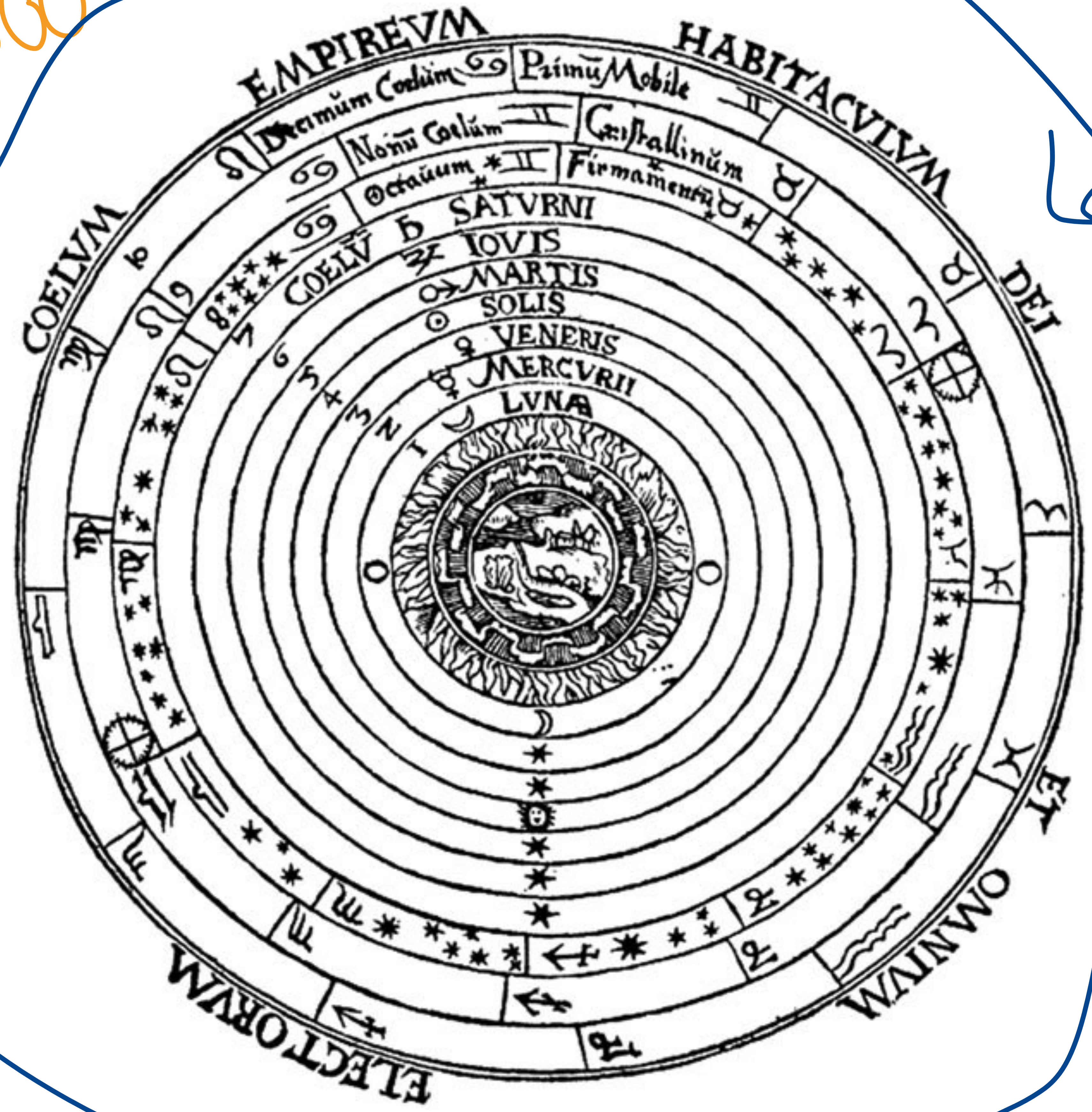
۱. مقدمه

۱.۱ مدل چیست

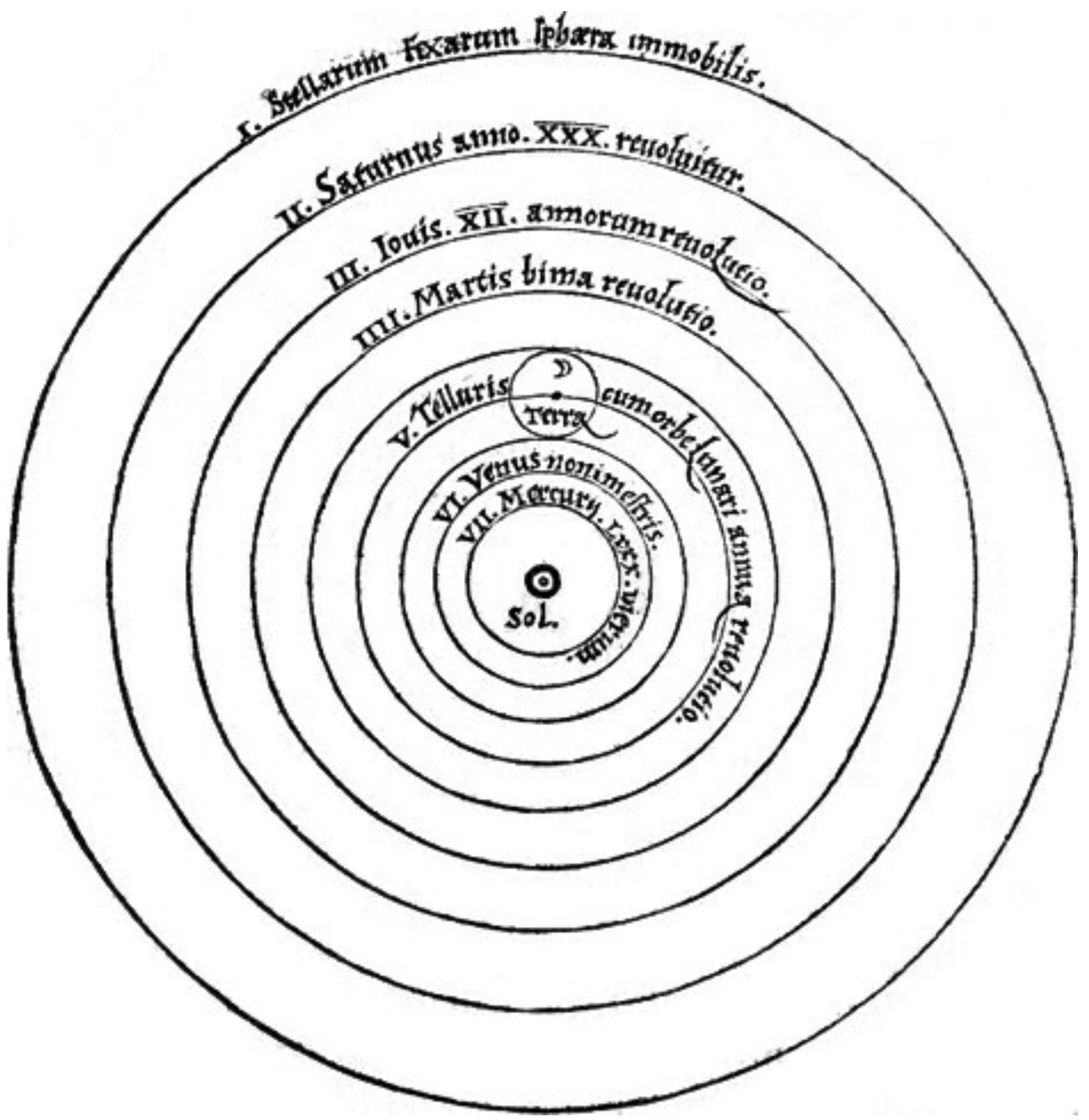
۱.۲ مدل در زیست‌شناسی

۱.۱ مدل چیست

1500



۱.۱ مدل چیست


I. Stellarum fixarum sphaera immobilis.
II. Saturnus anno. XXX. reuoluitur.
III. Iouis. XII. annorum reuolutio.
IIII. Martis bima reuolutio.
V. Telluris cum orbe lunari annua reuolutio.
Terra
VI. Venus nonimestris.
VII. Mercury. LXXX. dierum.
Sol.

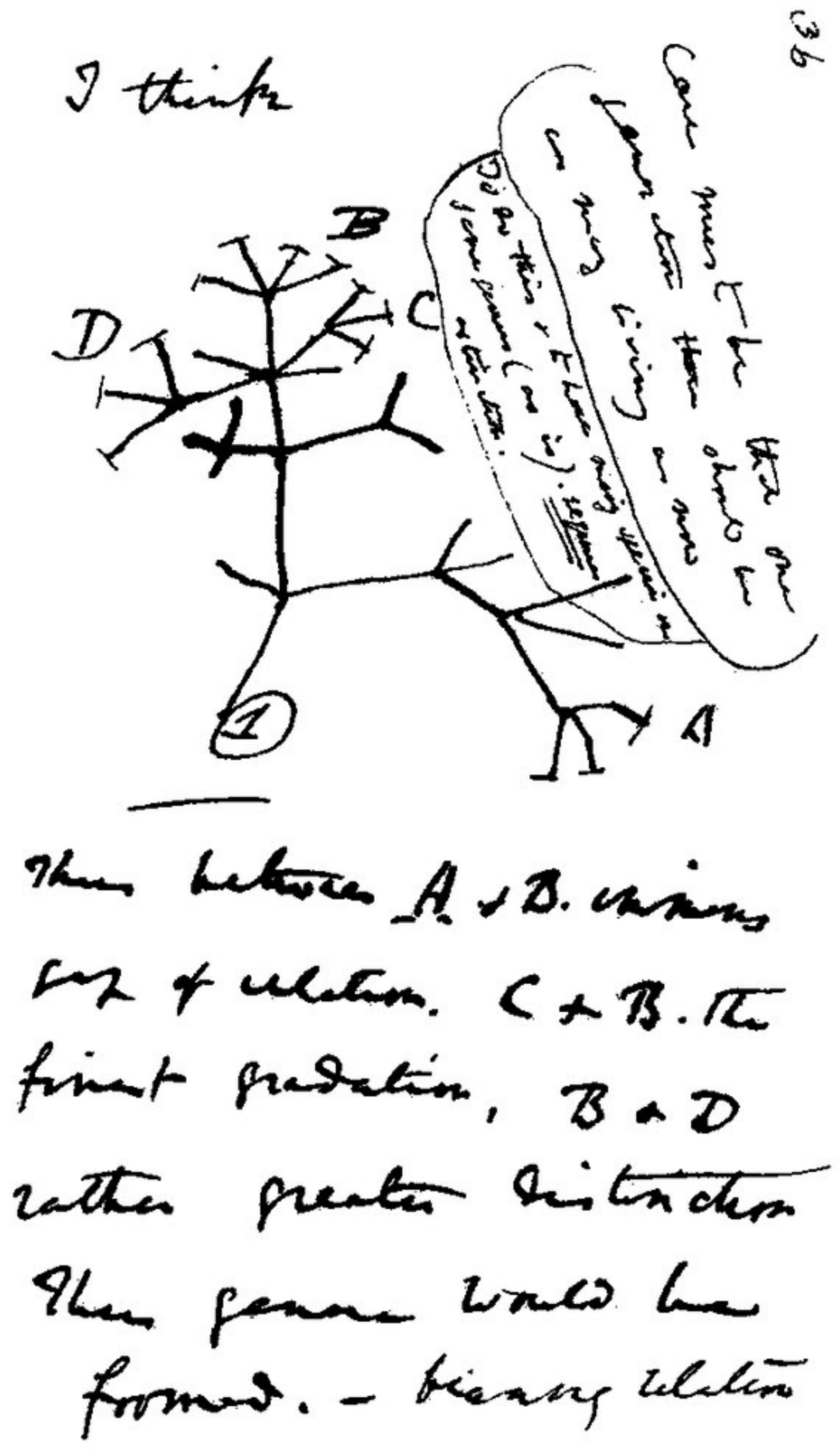
۱.۱ مدل چیست

۱.۱ مدل چیست

اهداف مدل

- یکپارچه‌سازی داده‌های متفاوت در قالب یک ایده ساختارمند
- توجیه مشاهدات
- پیشنهاد سازوکار
- پیش‌بینی آینده

۱.۲ مدل در زیست‌شناسی



۱.۲ مدل در زیست‌شناسی

میکائیلیس-منتن

$p \to q$

$\neg q \to \neg p$

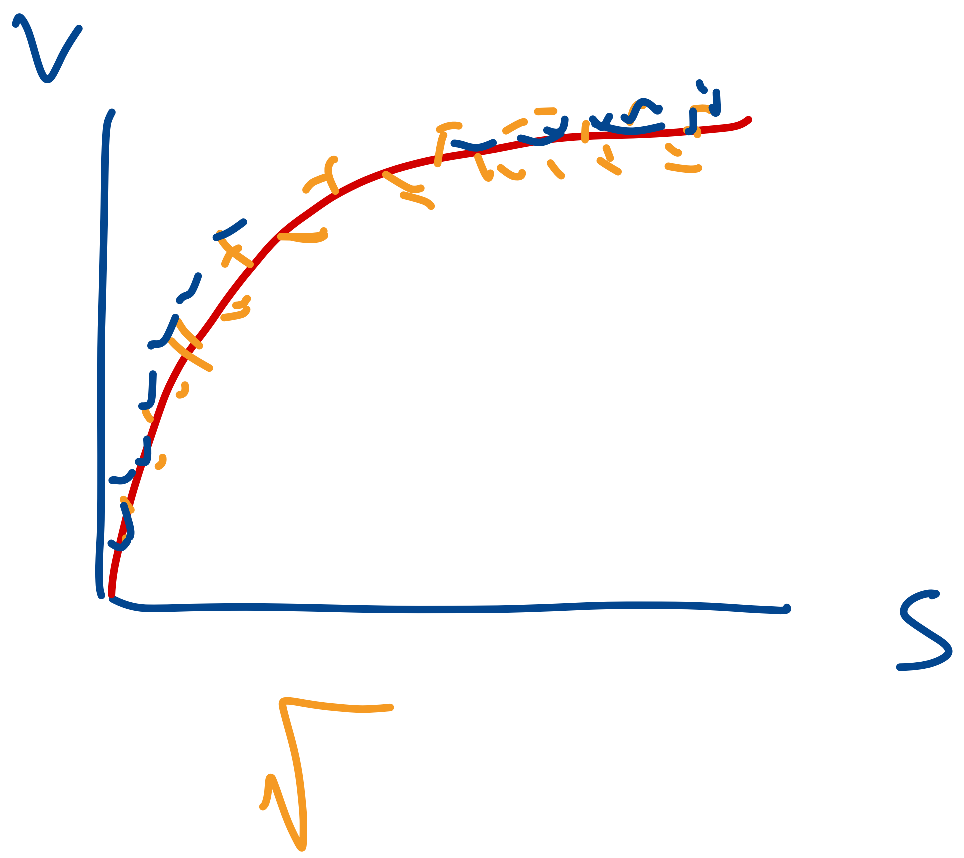
$$E + S \rightleftharpoons ES \to E + P$$

$$V = \frac{V_{max} \cdot S}{K_m + S}$$

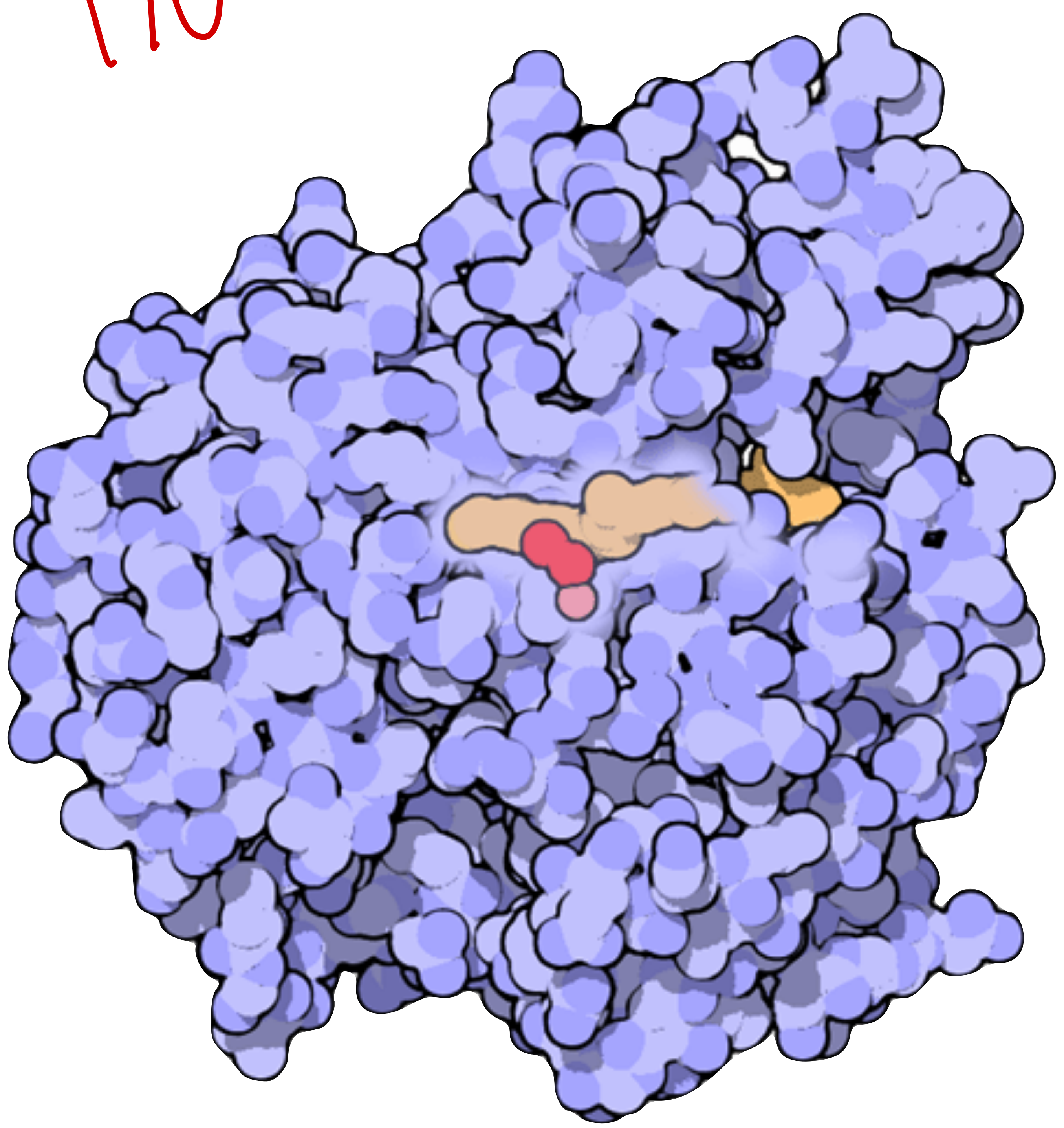
پیش‌فرض

$$E + S \rightleftharpoons ES \to E + P$$

$$V = \frac{V_{max}[S]}{K_m + [S]}$$



1902

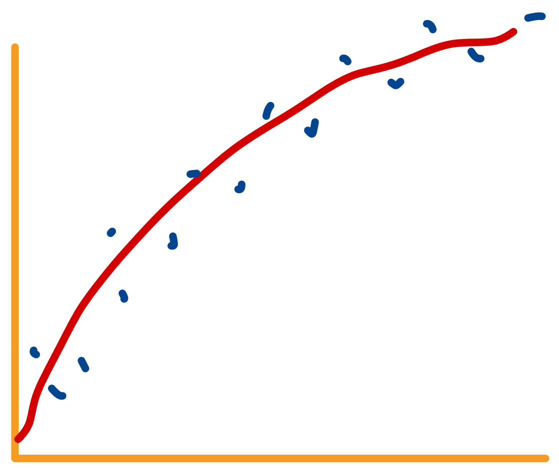


۱.۲ مدل در زیست‌شناسی

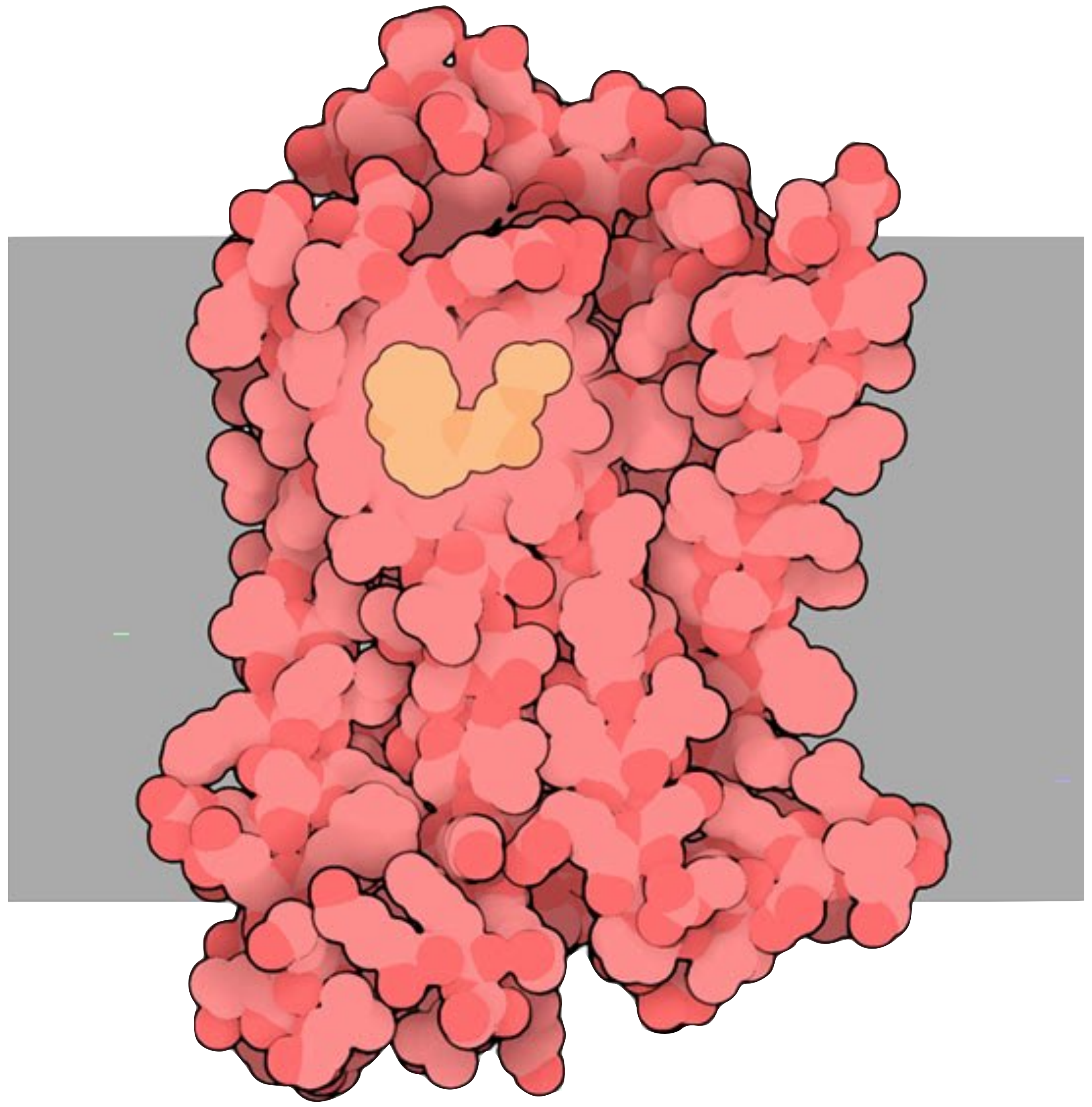
لانگلی-هیل

$$R + D \rightleftharpoons DR \rightarrow response$$

$$\frac{[DR]}{R_{total}} = \frac{[D]}{K_d + [D]}$$



Receptive



۲. مبانی ریاضیات

۲. مبانی ریاضیات

۲.۱ جبر

۲.۲ جبر خطی

۲.۳ حسابان

۲.۴ معادلات دیفرانسیل

۲.۱ جبر

اتحاد

$$(a+b)(c+d) = ac + ad + bc + bd$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$(a+b)(a-b) = a^2 - b^2$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

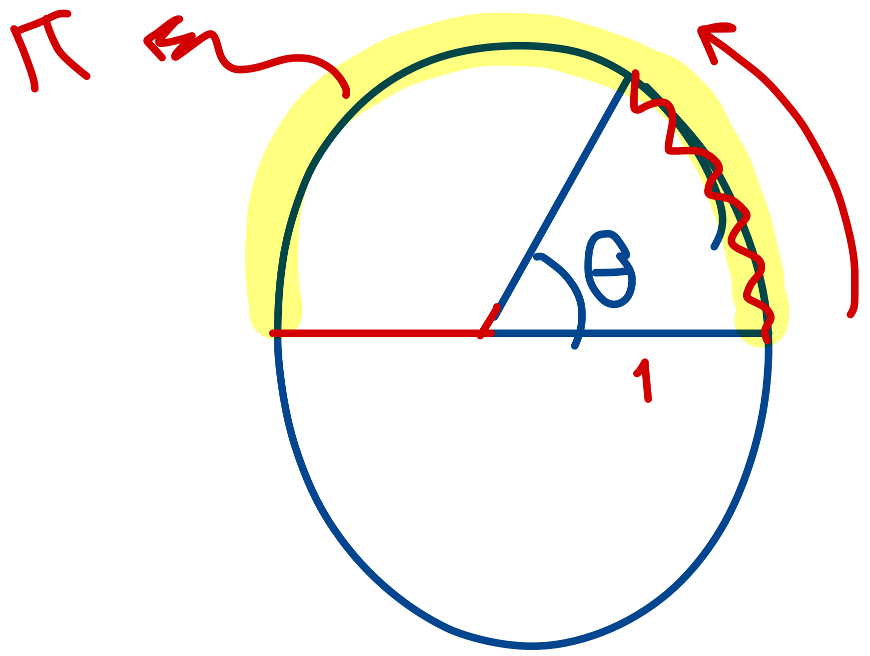
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

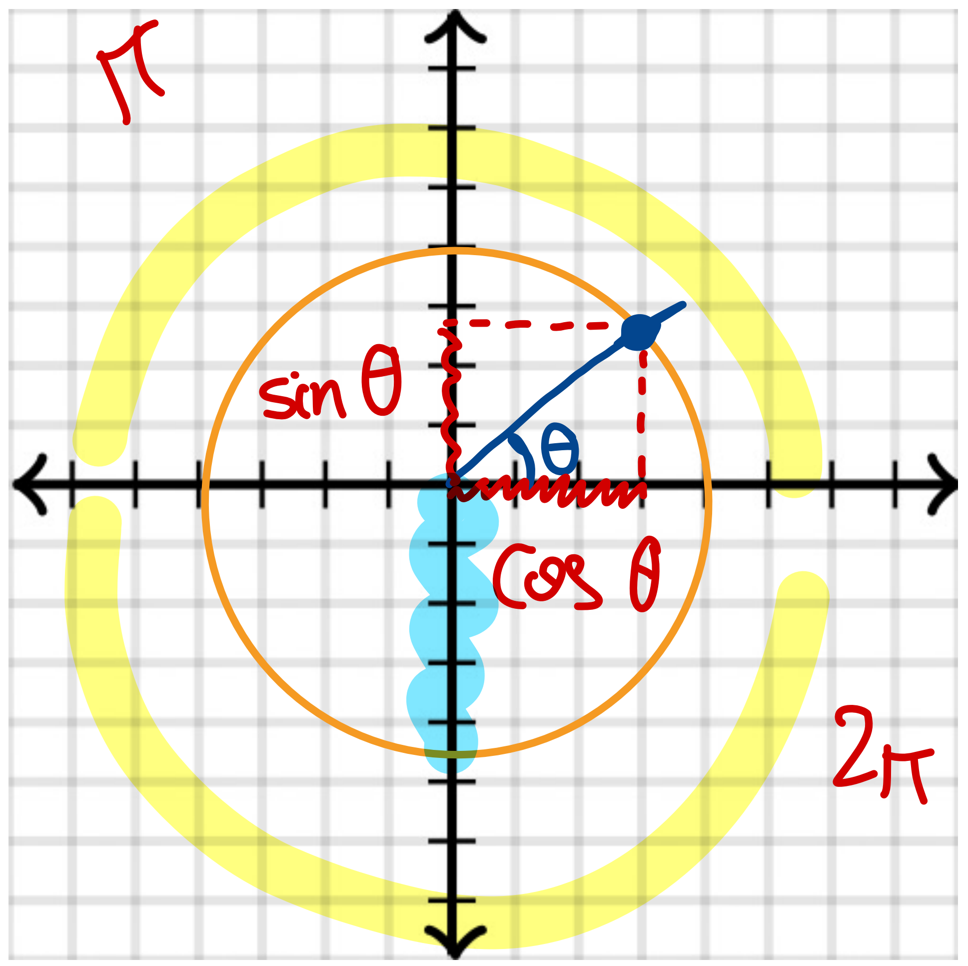
۲.۱ جبر

توابع مهم

$\sin \frac{3\pi}{2} = -1$

$\cos \frac{3\pi}{2} = 0$





ثابت

$f(x) = b$

$f(x) = ax + b$ $\qquad f(x) = ax^2 + bx + c$ $\qquad f(x) = ax^3 + bx^2 + cx + d$

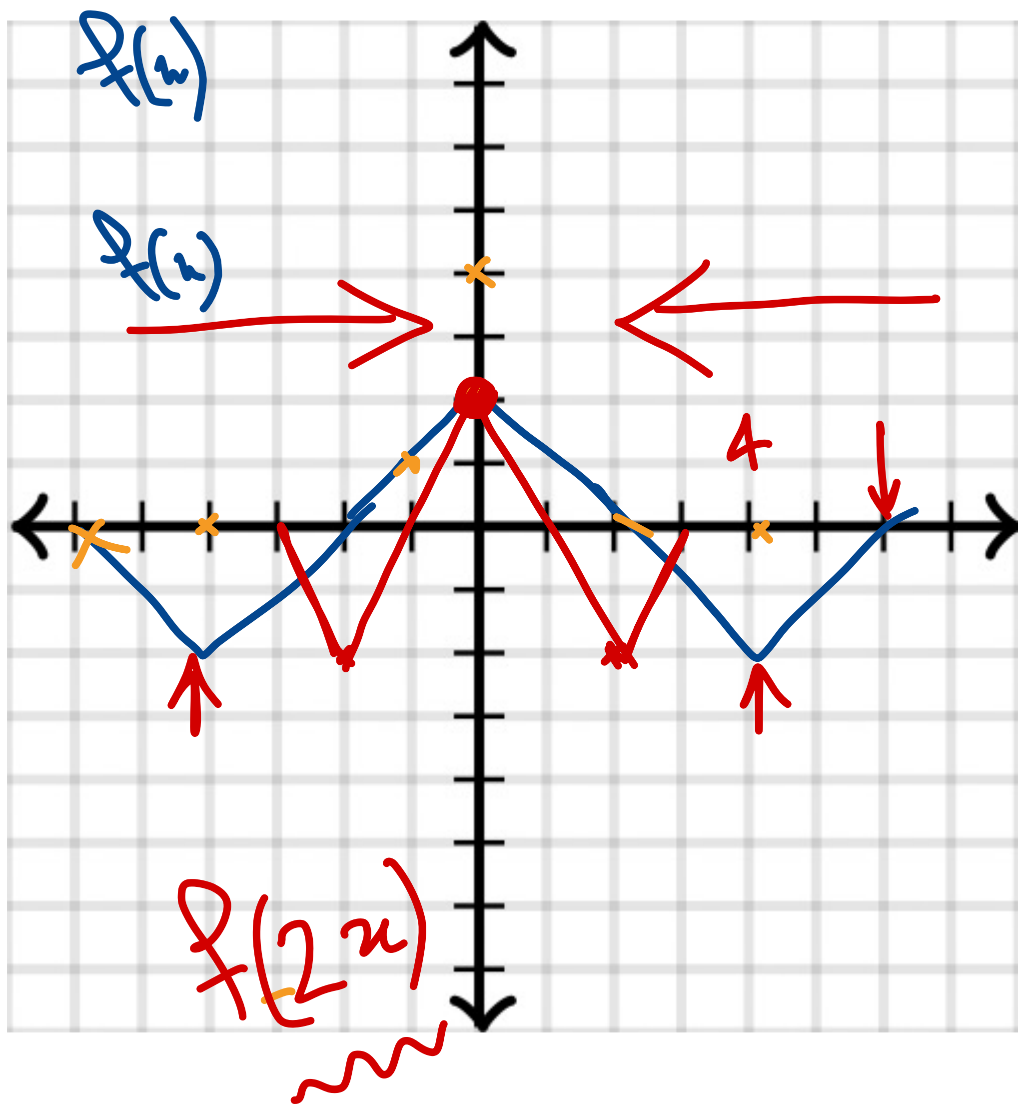
$f(x) = \sqrt[n]{x}$

$f(x) = \dfrac{1}{x^n}$

$f(x) = b^x$

$f(x) = \log_b x$

$f(x) = \sin x$ $\qquad f(x) = \cos x$



transformation

$c > 0$

$f(x) + c$ ↑ شیفت عمودی

$f(x) - c$ ↓

$c > 0$

$f(x+c)$ ← شیفت افقی

$f(x-c)$ →

$c > 1$

$c\,f(x)$ کشش عمودی در همان جهت

$\frac{1}{c} f(x)$ هل دادن عمودی

$f(cx)$ هل دادن افقی

$\frac{1}{c} f(x)$ کشش افقی

۲.۱ جبر

ریشه‌های چندجمله‌ای‌ها

ax^2

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$\Delta > 0$ 2 ریشه

$\Delta = 0$ 1 ریشه

$\Delta < 0$ ریشه ندارد

$$f(x) = ax + b \implies x = -\frac{b}{a}$$

$$\frac{-b \pm \sqrt{\boxed{b^2-4ac}}}{2a}$$

$\Delta >$ 2 ریشه

$= 0$ 1 ریشه

$<$

$$f(x) = ax^2 + bx + c \implies x = \frac{1}{2a}\left(-b \pm \sqrt{b^2-4ac}\right)$$

ax^3+bx^2+cx+d

دستور کاردانو

$$f(x) = x^3 + px^{\cancel{2}} + q \implies x = \sqrt[3]{-\frac{q}{2}+\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}} + \sqrt[3]{-\frac{q}{2}-\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}}$$

$\boxed{x^3+px+q}$

$2 \quad x-2$

$(x^2 + 2x - 2)$

$(x - 2)$

۲.۱ جبر

ریشه‌های چندجمله‌ای‌ها

$\cancel{1+\sqrt{-1}} + \cancel{1-\sqrt{-1}}$

$$x^3 - 6x + 4$$

$$\sqrt[3]{\frac{1}{2}(-4+\sqrt{-16})} + \sqrt[3]{\frac{1}{2}(-4-\sqrt{-16})} = 2$$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$$(1+\sqrt{-1})^3 = -2 + 2\sqrt{-1}$$

$1^3 + 3 \times 1^2 \times \sqrt{-1} + 3 \times 1 \times \sqrt{-1}^2 + \sqrt{-1}^3$

$$(1-\sqrt{-1})^3 = -2 - 2\sqrt{-1}$$

$1 + 3\sqrt{-1} - 3 - \sqrt{-1} \qquad -2+2\sqrt{-1}$

$$x = 2$$

۲.۱ جبر

اعداد موهومی

$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = +1$$

$$i^5 = i$$

Complex

$1+i$

$-3+\sqrt{2}i$

1

$\mathbb{R}$

$1/2$ $1/3$ $2/3$ $\sqrt{2}$ π -1 -2 0 1 2 3 $1/3$ $\sqrt{2}$ $1/2$

$1+i$

$2i$ $-3i$ $\sqrt{2}i$ موهومی i

۲.۱ جبر

اعداد مختلط

$$\frac{3+i}{3+i} \times \frac{-2+i}{3-i} = \frac{-6+3i-2i+i^2}{9+1}$$

-1

$$\frac{-7+i}{10}$$

$$-\frac{7}{10}+\frac{1}{10}i$$

$a + bi$

$(a + bi) + (c + di) = (a + c) + (b + d)i$

$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$

$$\frac{c-di}{c-di} \times \frac{(a + bi)}{(c + di)} = \frac{(ac + bd) + (bc - ad)i}{(c^2 + d^2)}$$

$c^2 + \cancel{cdi} - \cancel{dic} + d i d i$

$a+bi$, $a-bi$ } جفت مزدوج مختلط

۲.۱ جبر

اعداد مختلط

$z_1 = r_1(\cos\theta_1 + \sin\theta_1 i)$

$z_2 = r_2(\cos\theta_2 + \sin\theta_2 i)$

$z_1 z_2 = r_1 r_2(\cos\theta_1 + \sin\theta_1 i)(\cos\theta_2 + \sin\theta_2 i)$ $\quad - \sin\theta_1 \sin\theta_2$

$r_1 r_2(\cos\theta_1\cos\theta_2 + \cos\theta_1 \sin\theta_2 i + \sin\theta_1 i \cos\theta_2 + \sin\theta_1 i \sin\theta_2 i)$

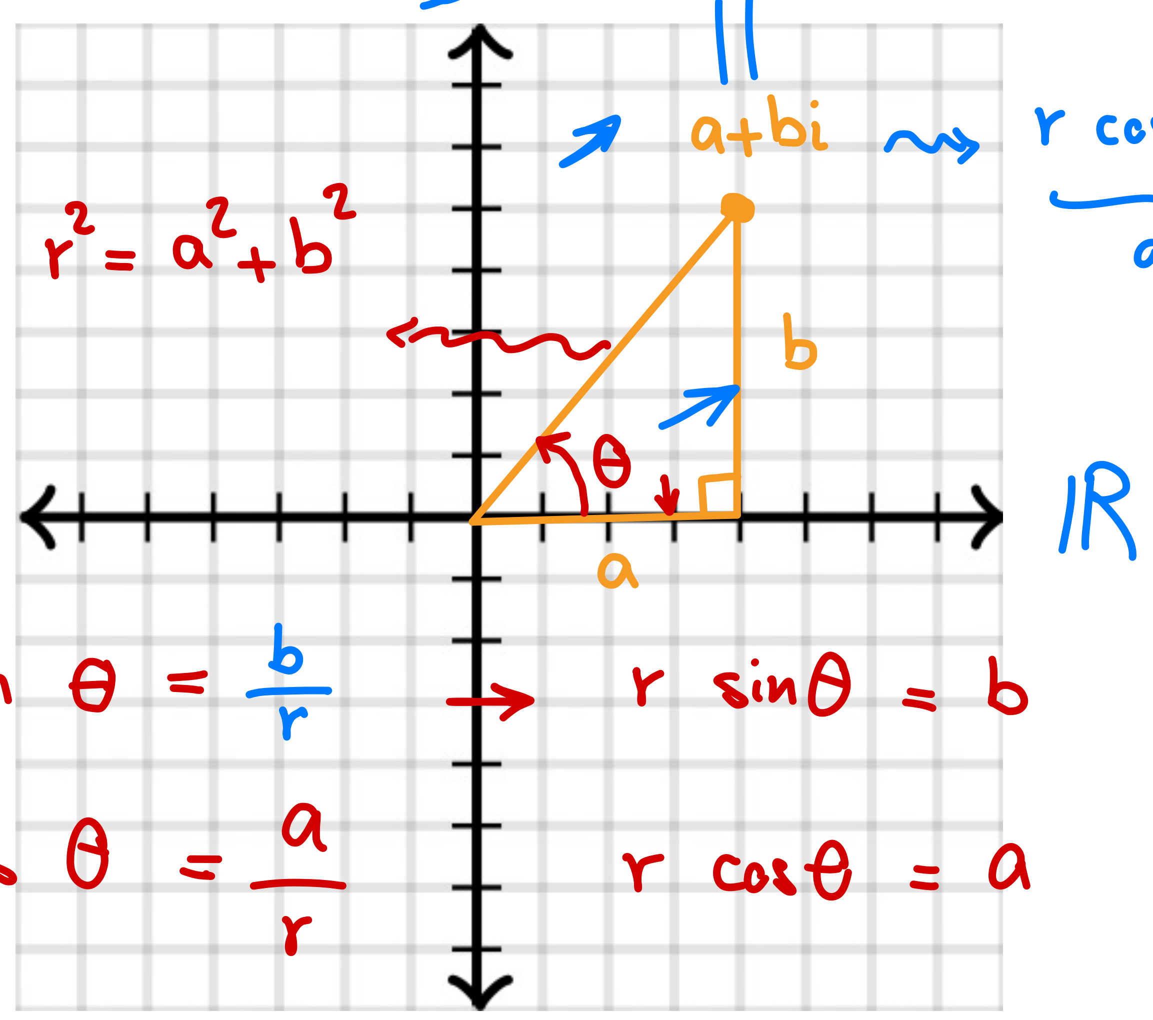
$\boxed{r_1 r_2}\left(\boxed{\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2} + \boxed{(\cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2)}\, i\right)$

r' $\quad \cos\theta'$ $\quad \sin\theta'$

$$a + bi \to r = \sqrt{a^2 + b^2}$$

$\cos(\theta_1 + \theta_2)$ $\quad \sin(\theta_1 + \theta_2)$

$$a + bi = r(\cos\theta + \sin\theta i)$$



$r(\cos\theta + \sin\theta i)$

$a+bi \rightsquigarrow \underbrace{r\cos\theta}_{a} + \underbrace{r\sin\theta}_{b}\, i$

$r^2 = a^2 + b^2$

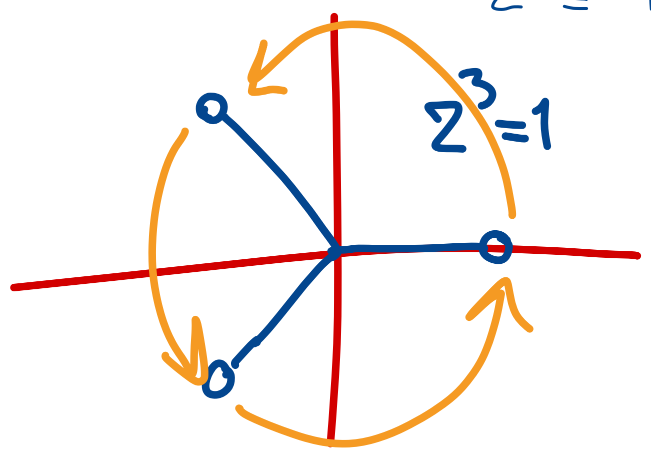
$\sin\theta = \frac{b}{r} \to r\sin\theta = b$

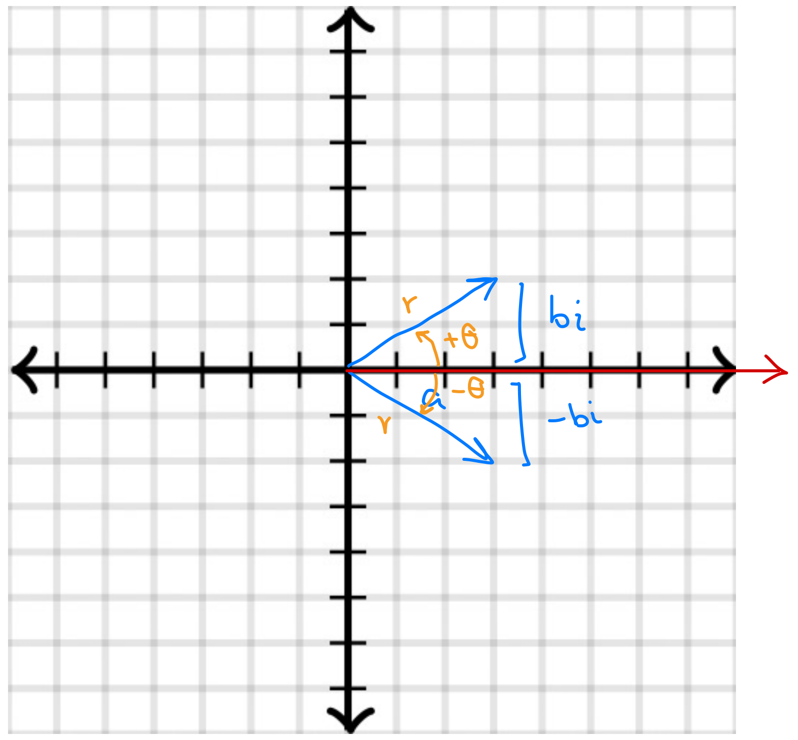
$\cos\theta = \frac{a}{r}$ $\quad r\cos\theta = a$

$$r_1(\cos\theta_1 + \sin\theta_1 i) r_2(\cos\theta_2 + \sin\theta_2 i) = \boxed{r_1 r_2}[\cos\boxed{(\theta_1 + \theta_2)} + \sin\boxed{(\theta_1 + \theta_2)}i]$$

$$z^n = 1 \implies \theta = \frac{2k\pi}{n}$$

$z^n = 1$





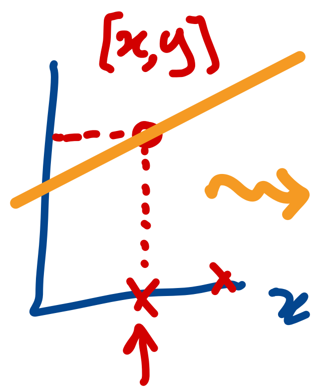
۲.۲ جبر خطی

بردار

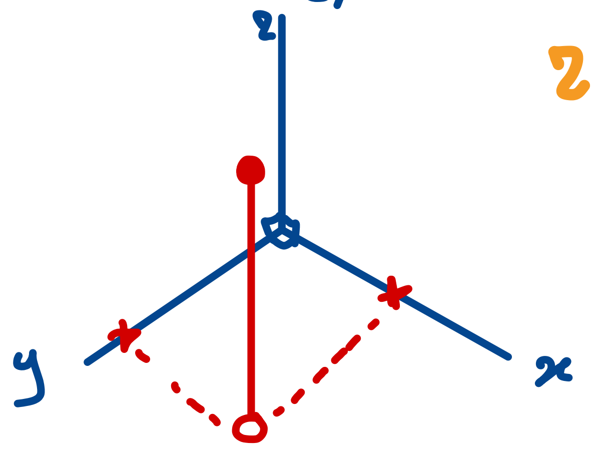
x [1,4,5,2]

وزن → x ... دیابت

$f(x) = y$ $\quad$ $[x, y]$ $\quad$ $f(x,y) = z$



$y = ax + b$



$z = ax + by + c$

$$\vec{u} = [x, y]$$ [3,2]

$$\vec{u} = x\hat{i} + y\hat{j}$$

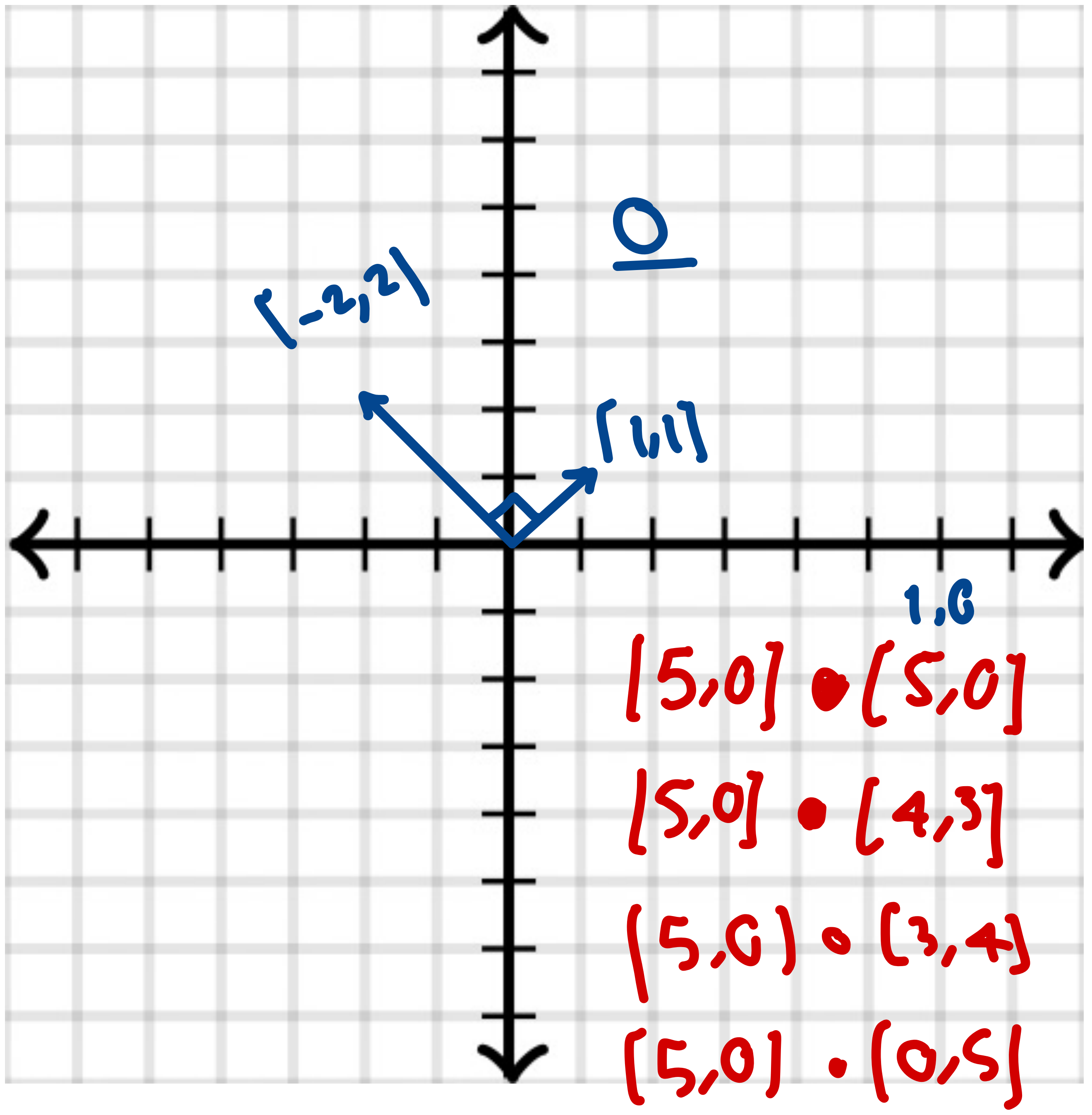
$$|\vec{u}| = \sqrt{u_x^2 + u_y^2}$$

$$c\vec{u} = cx\hat{i} + cy\hat{j}$$

$$\vec{u} + \vec{v} = (u_x + v_x)\hat{x} + (u_y + v_y)\hat{y}$$

$$\vec{u} \bullet \vec{v} = u_x v_x + u_y v_y = \boxed{|\vec{u}|\,|\vec{v}|\cos\theta}$$

dot product



= 5

[5,0] • [5,0] = 25

[5,0] • [4,3] = 20

[5,0] • [3,4] = 15

[5,0] • [0,5] = 0

[5,0] • [-3,4] = -15

[5,0] • [-5,0] = -25

max → $\theta = 0$ $\quad$ $|\vec{u}||\vec{v}|$

0 → ۹۰

min → $\theta = \pi$ $\quad$ $-|\vec{u}||\vec{v}|$

۲.۲ جبر خطی

ماتریس

$$\begin{array}{c|cc} & 1 & 2 \\ \hline 1 & 1 & 2 \\ 2 & 3 & \boxed{1} \\ 3 & 3 & 4 \end{array} \rightarrow 2,2$$

M : 3×2

$$2\begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{trace} \quad a+d \qquad \det \quad ad-bc$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1+0 & 2+3 \\ 3+1 & 4+2 \end{bmatrix}$$

$$\det\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} = 2 \times 3 - 1 \times 0 = 6 - 0 = 6$$

$$x_{ij} \qquad R_i \bullet C_j$$

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow i \times j$$

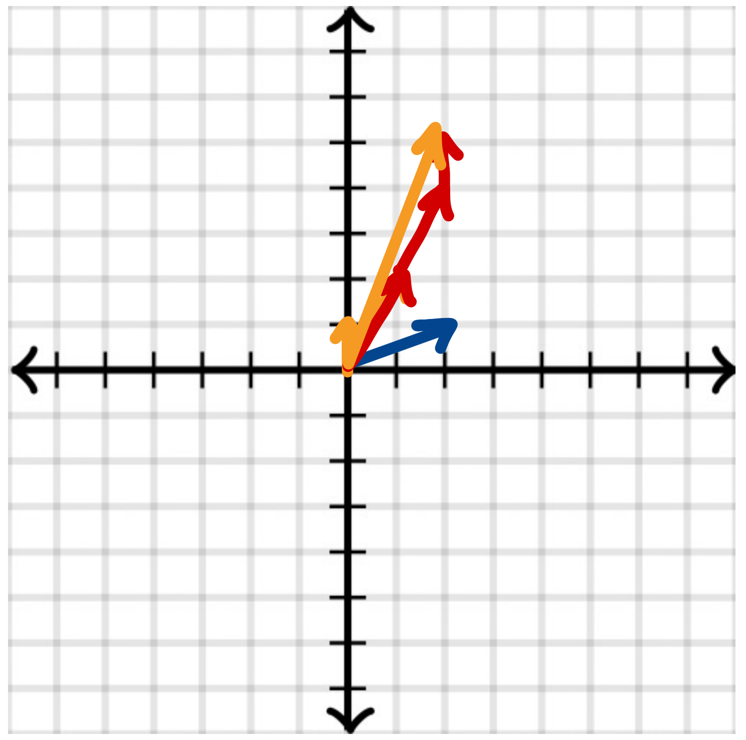
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax+by \\ cx+dy \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 4 \\ 3 & 5 & -2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ -1 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 16 \\ -1 & 2 \end{bmatrix}$$

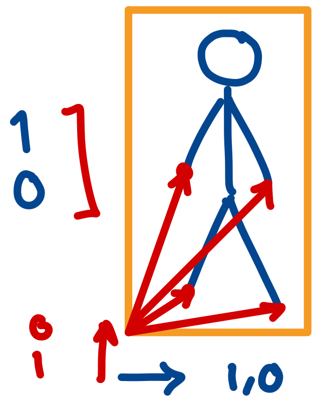
$$\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

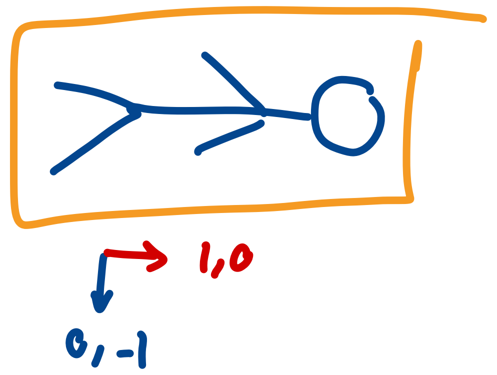


$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix} = \begin{bmatrix} ax \\ cx \end{bmatrix} + \begin{bmatrix} by \\ dy \end{bmatrix} = x\begin{bmatrix} a \\ c \end{bmatrix} + y\begin{bmatrix} b \\ d \end{bmatrix}$$

$$\rightarrow x\begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$





$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$x \rightarrow 1,0,0$

$y \rightarrow 0,1,0$

$z \rightarrow 0,0,1$

۲.۲ جبر خطی

ماتریس

trace $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $a+d$

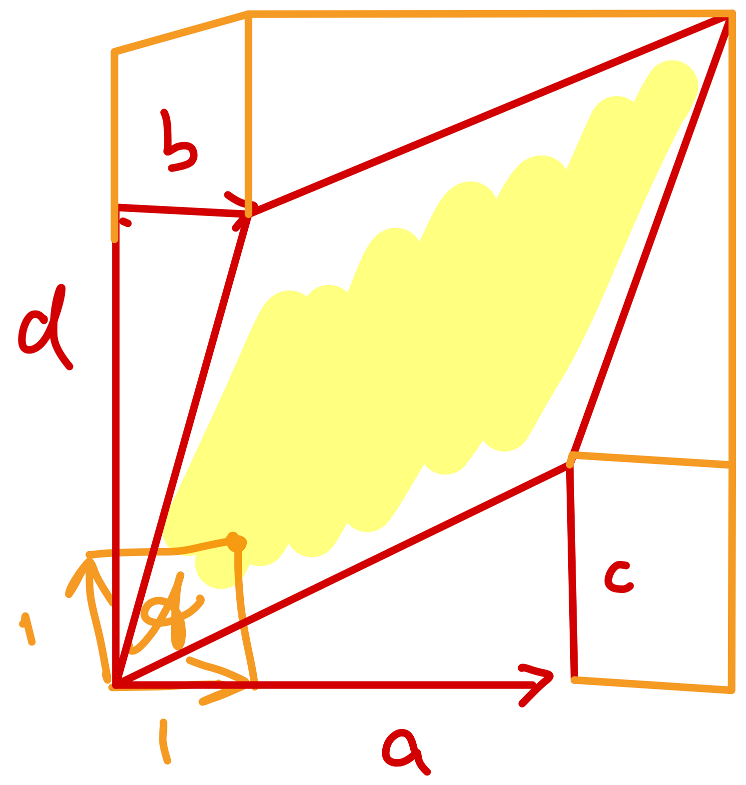
det $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $ad-bc$

(دترمینان)

$\det \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} = 2\times 3 - 1\times 1 \quad 5$

$\det \begin{bmatrix} -2 & 0 \\ 1 & 3 \end{bmatrix} = -2\times 3 - 0\times 1 \quad -6$

$\det \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} = 1\times 3 - \cancel{(-1)(-2)}^{\,2} \quad 1$



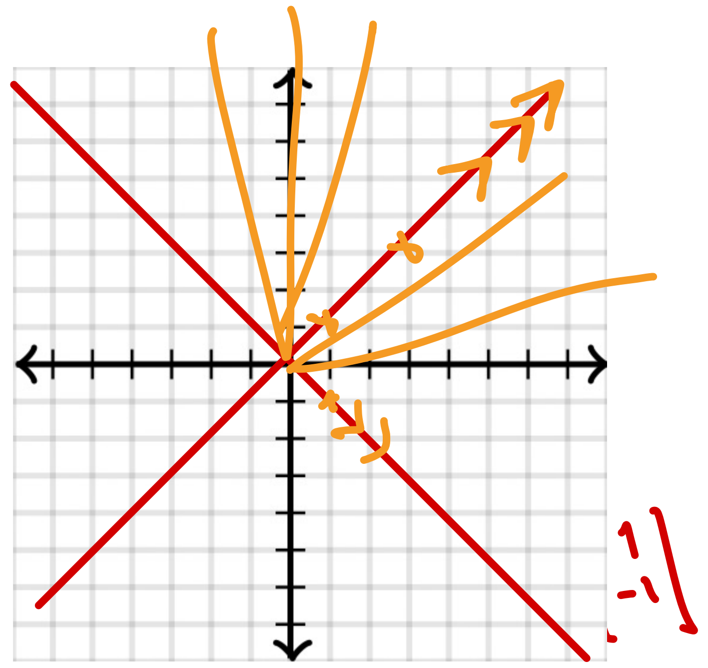
$$\det \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = ad - bc$$

$$A\vec{x} = \lambda \vec{x}$$

$$\det(A - \lambda I) = 0$$

$$\det \left(\begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix} \right) = 0$$

$$\lambda^2 - \tau\lambda \boxed{+\Delta} = 0$$



$0\cdot 0-(-1)1$

$\lambda^2 + \Delta = 0$

$\lambda^2+1=0 \quad \lambda^2=-1 \rightsquigarrow \lambda\ \underset{-i}{i}$

$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad \begin{matrix} 0-\lambda & -1 \\ 1 & 0-\lambda \end{matrix}$

$\begin{bmatrix} \text{—} \\ \text{—} \\ \text{—} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$

$\begin{vmatrix} 1 \\ -1 \end{vmatrix} \ \lambda=1$

$\lambda=1 \quad y=-x$

① $\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} \begin{vmatrix} x \\ y \end{vmatrix} = \begin{vmatrix} x \\ y \end{vmatrix}$

$x+y \quad y=-x$

$2x+y = x$

$y=x \quad \lambda=3$

$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$\lambda^2 - 4\lambda + 3 = 0$

$(\lambda-3)(\lambda-1)$ ③ ①

$$A\vec{x} = \lambda\vec{x}$$

$$A\vec{x} = \lambda I \vec{x}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \vec{x} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \vec{x}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \vec{x} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \vec{x} = 0$$

$$\begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix} \vec{x} = 0$$

eigenVector بردار ها ویژه

برداری ناصفر که بعد از انجام ضرب در راستای قبلی خود باقی بمانه

eigenValue مقدار ویژه

ضریب تغییرات طول هر بردار ویژه

$$\det \begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix} = 0$$

$$\lambda^2 - \underbrace{(a+d)}_{\text{trace}}\lambda + \underbrace{ad-bc}_{\det} = 0$$

$$(a-\lambda)(d-\lambda) - bc = 0$$

$$ad - (a+d)\lambda + \lambda^2 - bc = 0$$

① $\begin{bmatrix}3 & 1\\ 0 & 2\end{bmatrix}\vec{x} = \lambda\vec{x}$

② $\begin{bmatrix}3 & 1\\ 0 & 2\end{bmatrix}\vec{x} = \begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}\vec{x}$

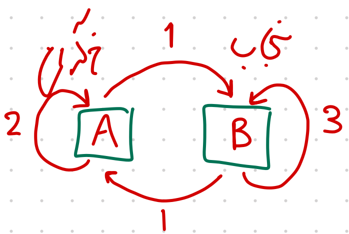
③ $\begin{bmatrix}3-\lambda & 1-0\\ 0-0 & 2-\lambda\end{bmatrix}\vec{x} = 0$

$$\det\begin{bmatrix}3-\lambda & 1\\ 0 & 2-\lambda\end{bmatrix} = 0$$

$$(3-\lambda)(2-\lambda) - 1\times 0 = 0$$

$$(3-\lambda)(2-\lambda) = 0$$

$\downarrow$ 3 $\quad\downarrow$ 2



$$A_{t+1} = 2A_t + 1B_t$$

$$B_{t+1} = 1A_t + 3B_t$$

$\lambda_1 = 3$

$$\begin{bmatrix}3 & 1\\ 0 & 2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = 3\begin{bmatrix}x\\ y\end{bmatrix}$$

$$\begin{bmatrix}3x+y\\ 2y\end{bmatrix} = \begin{bmatrix}3x\\ 3y\end{bmatrix}$$

$$\cancel{3x} + y = \cancel{3x}$$

$y = 0 \quad \begin{bmatrix}x\\ 0\end{bmatrix} \quad \begin{bmatrix}1\\ 0\end{bmatrix}$

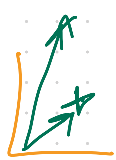
$\lambda_2 = 2$

$$\begin{bmatrix}3 & 1\\ 0 & 2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = 2\begin{bmatrix}x\\ y\end{bmatrix}$$

$$3x + y = 2x$$

$$x + y = 0$$

$y = -x \quad \begin{bmatrix}1\\ -1\end{bmatrix}$



$$\begin{bmatrix}A\\ B\end{bmatrix}_{t+1} = \begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}\boxed{\begin{bmatrix}A\\ B\end{bmatrix}_t}$$

$$\begin{bmatrix}3&1\\0&2\end{bmatrix} \rightarrow$$

1- $\begin{bmatrix}3&1\\0&2\end{bmatrix}\vec{x} = \lambda\vec{x}$

2- $\begin{bmatrix}3&1\\0&2\end{bmatrix}\vec{x} = \lambda\begin{bmatrix}1&0\\0&1\end{bmatrix}\vec{x}$

3- $\begin{bmatrix}3&1\\0&2\end{bmatrix}\vec{x} = \begin{bmatrix}\lambda&0\\0&\lambda\end{bmatrix}\vec{x}$

4- $\left(\begin{bmatrix}3&1\\0&2\end{bmatrix} - \begin{bmatrix}\lambda&0\\0&\lambda\end{bmatrix}\right)\vec{x} = 0$

5- $\begin{bmatrix}3-\lambda&1\\0&2-\lambda\end{bmatrix}\vec{x} = 0$

$\star$ $\det\begin{bmatrix}3-\lambda&1\\0&2-\lambda\end{bmatrix} = 0$

6- $(3-\lambda)(2-\lambda) - 1\times 0 = 0$

7- $(3-\lambda)(2-\lambda) = 0$

$\lambda_1 = 3 \quad \lambda_2 = 2$

$\lambda_1 = 3$

$$\begin{bmatrix}3&1\\0&2\end{bmatrix}\vec{x} = 3\vec{x}$$

$$\begin{bmatrix}3&1\\0&2\end{bmatrix}\vec{x} = \begin{bmatrix}3&0\\0&3\end{bmatrix}\vec{x}$$

$$\begin{bmatrix}3-3&1\\0&2-3\end{bmatrix}\vec{x} = \begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix}0&1\\0&-1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$\rightarrow 0\times x + 1\times y = 0$

$0 + y = 0$

$\rightarrow 0\times x + (-1)y = 0$

$0 - y = 0$

$0 + y = 0$

$\downarrow$

$y = 0$

$x = 3?$

$[1, 0]$

$\lambda_2 = 2$

$$\begin{bmatrix}3&1\\0&2\end{bmatrix}\vec{x} = 2\vec{x} \qquad \begin{bmatrix}3&1\\0&2\end{bmatrix}\vec{x} = \begin{bmatrix}2&0\\0&2\end{bmatrix}\vec{x}$$

$$\begin{bmatrix}3-2&1\\0&2-2\end{bmatrix}\vec{x} = \begin{bmatrix}0\\0\end{bmatrix} \qquad \begin{bmatrix}\boxed{1 \quad 1}\\0 \quad 0\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$\rightarrow 1\times x + 1\times y = 0 \rightarrow x + y = 0 \qquad y = -x$

$[-1, 1] \quad [1, -1]$

$x + y = 0$

$y = -x$

$[1, -1]$ بردار ویژه

$\times 2$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$T = 4$

$\Delta = 3$

$\lambda^2 - T\lambda + \Delta = 0$ $\quad [\quad]\begin{bmatrix} x \\ y \end{bmatrix}$

$\lambda^2 - 4\lambda + 3 = 0$

$\underbrace{(\lambda-3)}_{3}\underbrace{(\lambda-1)}_{1} = 0$

x خرگوش 3

$$\begin{bmatrix} x \\ y \end{bmatrix}_{t+1} = \begin{bmatrix} 3 & \cdot \\ \cdot & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}_t$$

$x_{t+1} = 3\boxed{x_t}$

y روباه 2

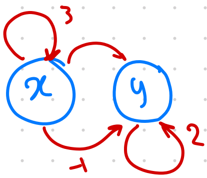
$y_{t+1} = 2y_t$

$x_{t+1} = 3x + \cdot y$

$y_{t+1} = \cdot x + 2y$

$\lambda_1 \; 3$

$\lambda_2 = \begin{bmatrix} 1 & \cdot \\ \cdot & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \end{bmatrix} \quad \begin{bmatrix} x \\ y \end{bmatrix} \quad \begin{bmatrix} 0 \\ 4 \end{bmatrix}$



$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\vec{x} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\vec{x}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\vec{x} = \begin{bmatrix} 1 & \cdot \\ \cdot & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 3 & -1 \\ +1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 2-3 & 1 \\ 1 & 2-3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2-1 & 1 \\ 1 & 2-1 \end{bmatrix}$$

$$\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix}\begin{vmatrix} x \\ y \end{vmatrix} = \begin{bmatrix} \cdot \\ \cdot \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cdot \\ \cdot \end{bmatrix}$$

$-x + y = 0$

$x + y = 0$

$\boxed{y = x}$ 3

$y = -x$

$[1, 1]$ $\lambda = 3$

$(1, -1)$ $\lambda = 1$

$$\begin{bmatrix} \bullet & -1 \\ 1 & \bullet \end{bmatrix}$$

$$\det\begin{vmatrix} 0-\lambda & -1 \\ 1 & 0-\lambda \end{vmatrix} = 0$$

$(-\lambda)(-\lambda) - -1 \times 1$

$\lambda^2 + 1 = 0$

$\lambda^2 = -1$

۲.۳ حسابان

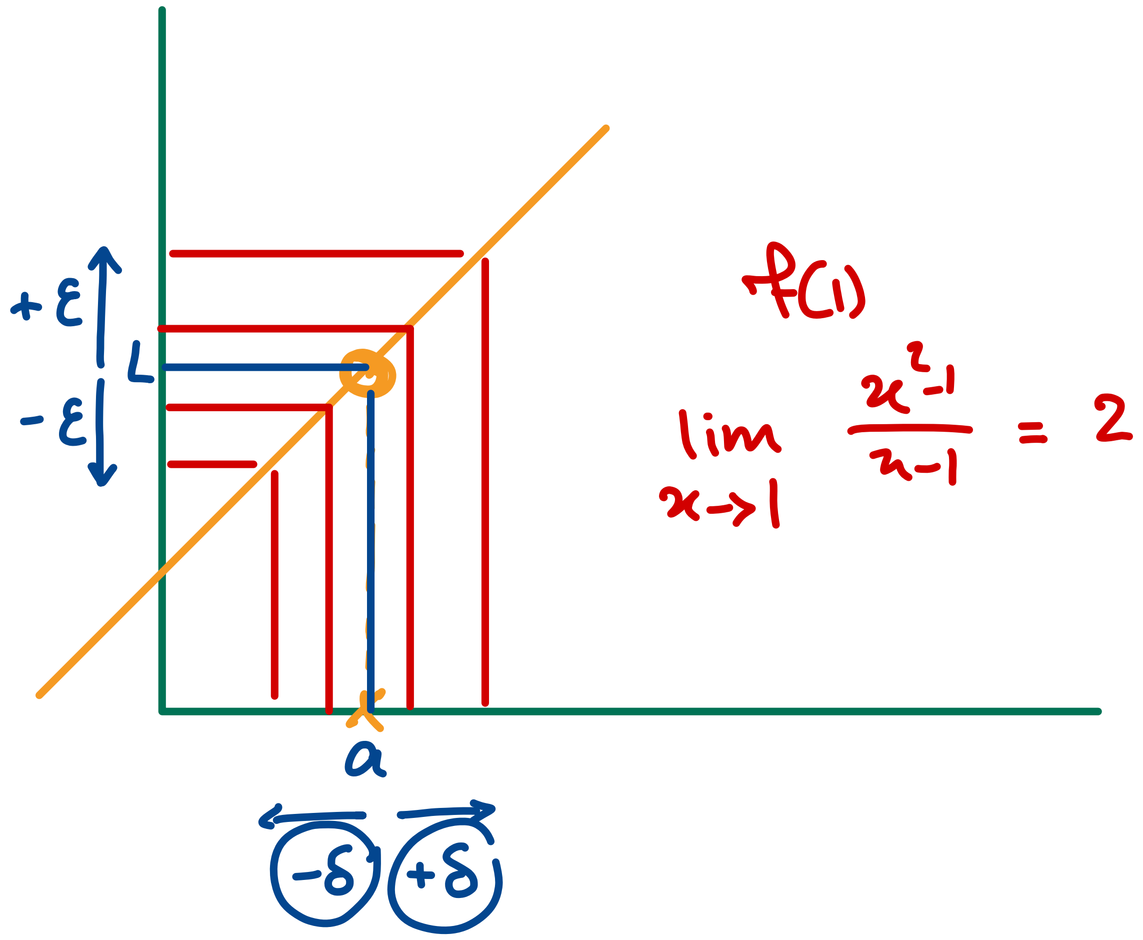
حد

Calculus

Infinitisimal

$\frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}}$

$f(x) = \frac{x^2-1}{x-1}$



f(1)

$\lim_{x\to 1} \frac{x^2-1}{x-1} = 2$

$\boxed{\forall}\ \epsilon > 0\ \boxed{\exists}\ \delta > 0$

$$0 < |x-a| < \delta \implies |f(x) - L| < \epsilon$$

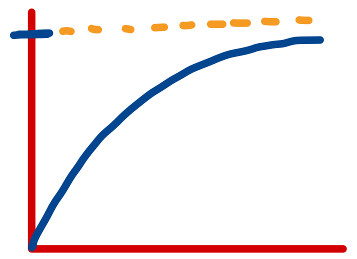
$$\lim_{x\to a} f(x) = L$$

$\cancel{f(a)}$

$\lim_{x\to a} f(x) = L$

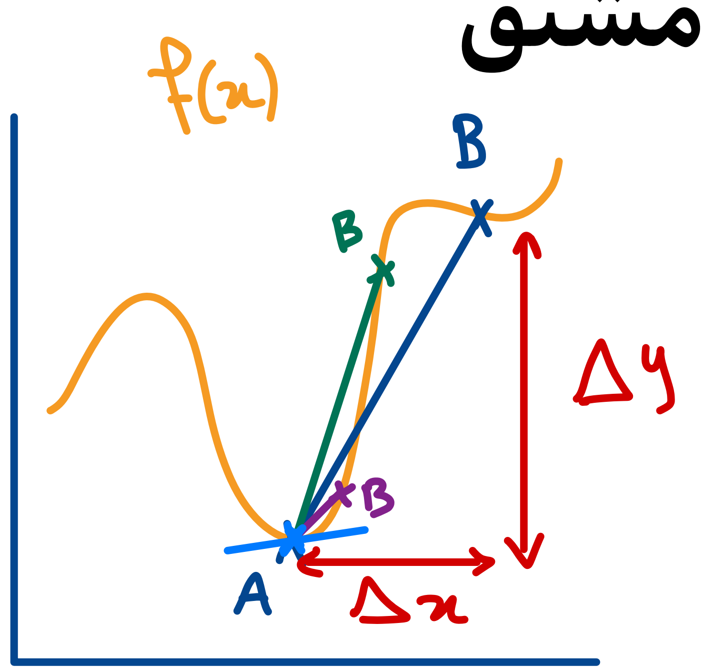
$\lim_{x\to\infty} \frac{1}{x} = 0$

$\lim_{x\to\infty} \frac{x}{x+1} = 1$



۲.۳ حسابان

مشتق



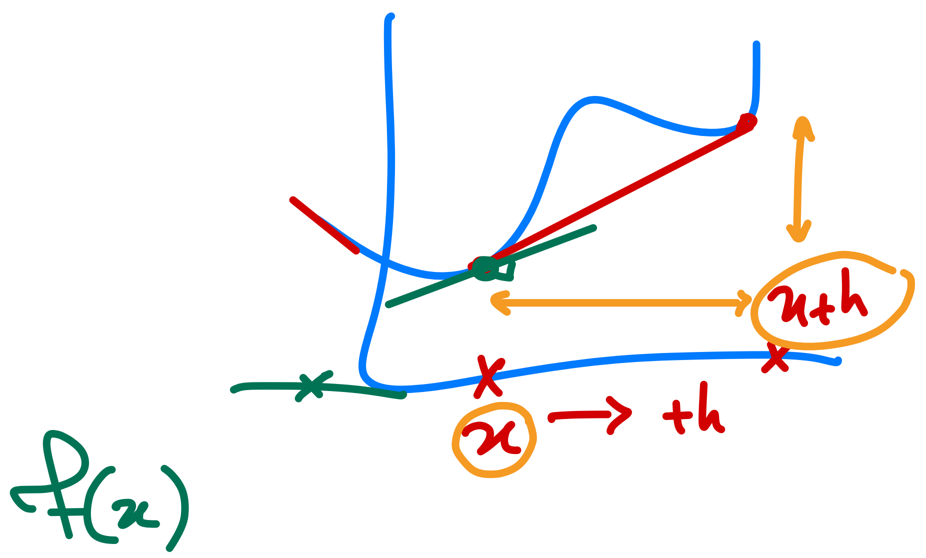
$$(A \to B) \quad \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

سرعت متوسط

$$\downarrow$$

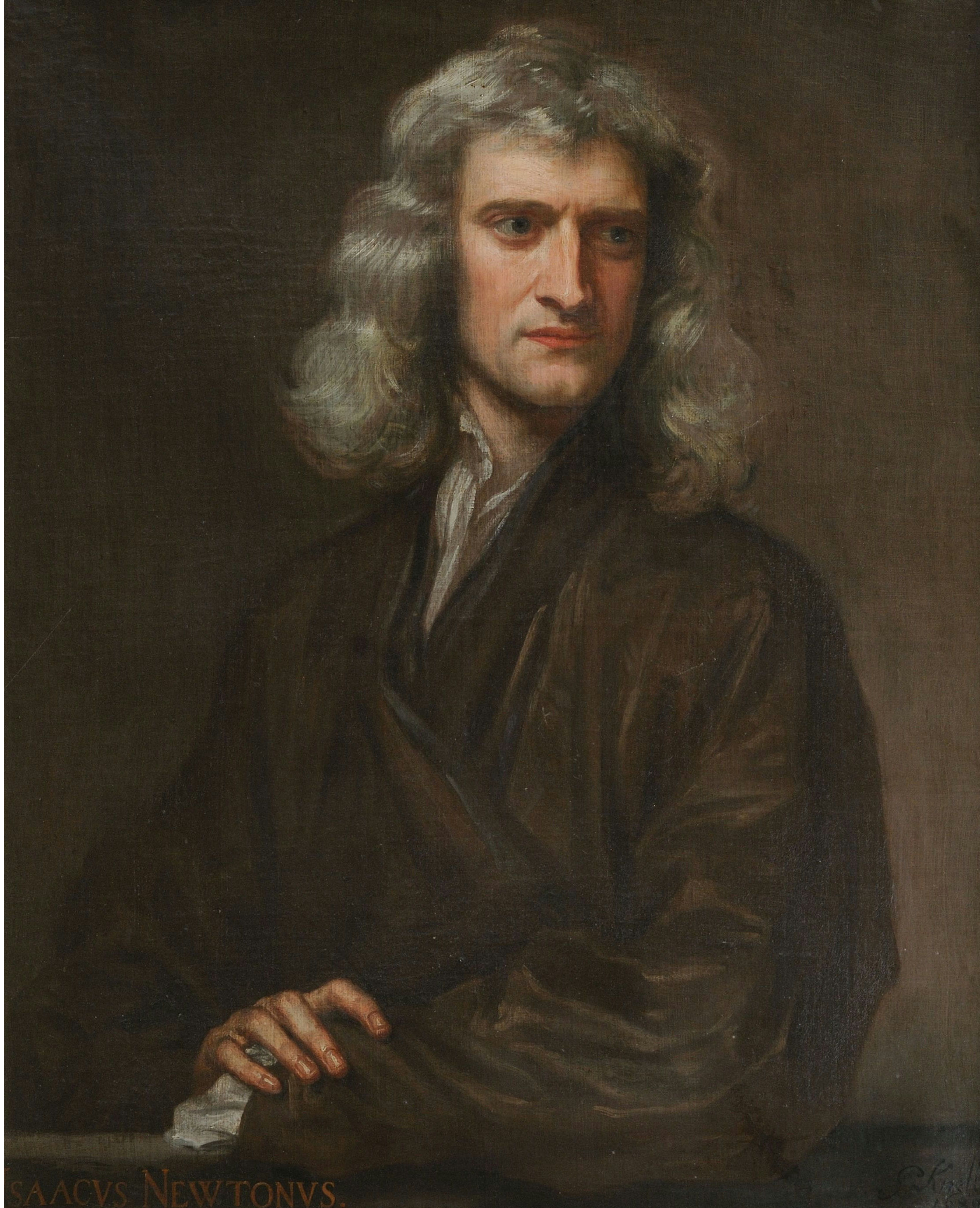
$$\frac{dy}{dx} =$$

سرعت لحظه‌ای



$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

$$f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h}$$



۲.۳ حسابان

مشتق

$$\frac{\mathrm{d}f}{\mathrm{d}x} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

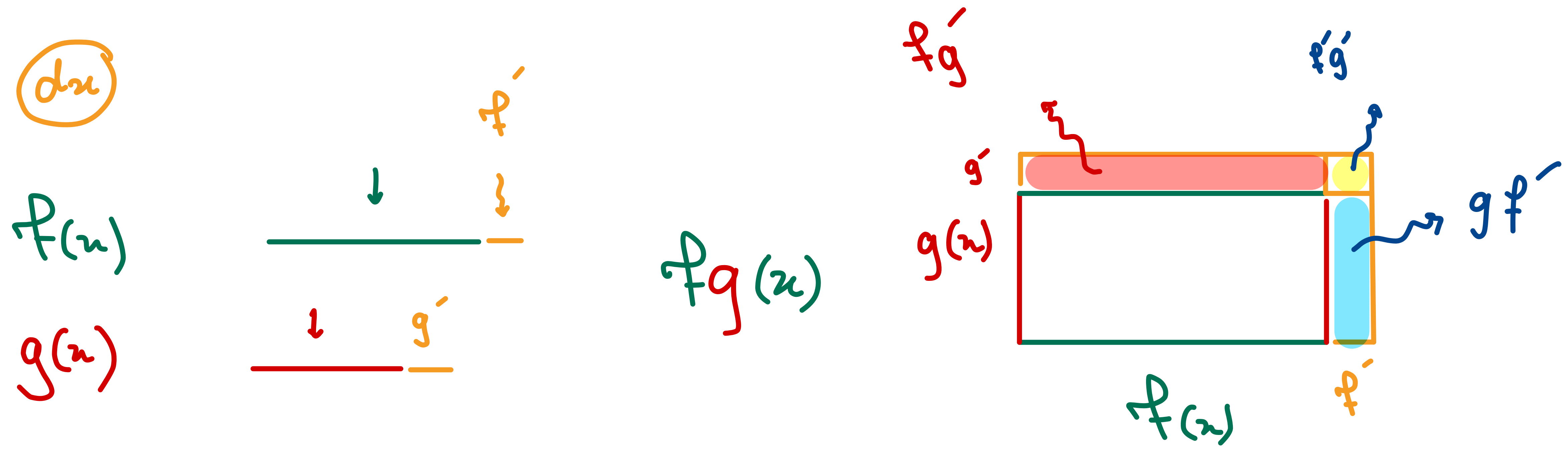
$$\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} = \lim_{h\to 0} \frac{\left.\frac{\mathrm{d}f}{\mathrm{d}x}\right|_{x+h} - \left.\frac{\mathrm{d}f}{\mathrm{d}x}\right|_{x}}{h}$$

۲.۳ حسابان

مشتق

$\lim_{h\to 0} \frac{cf(x+h)-cf(x)}{h}$

$c \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$



$$\frac{d\,fg}{dx} = fg' + gf'$$

$$(cf)' = cf'$$

$$(f+g)' = f' + g'$$

$$(f-g)' = f' - g'$$

قانون ضرب

$$(fg)' = fg' + gf'$$

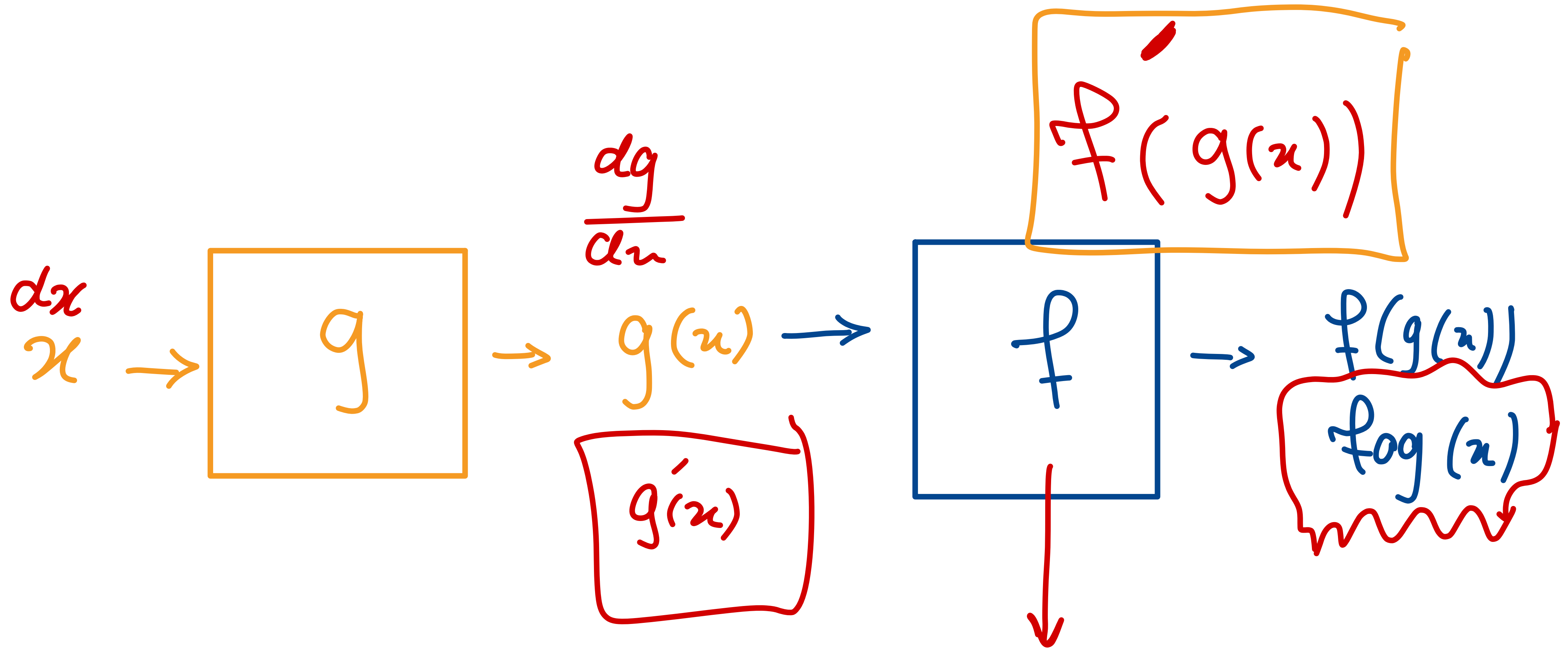
تقسیم

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

قانون زنجیره‌ای   chain rule

$$(fog)'(x) = f'(g(x))\,.\,g'(x)$$

$f(g(x))$



$$\frac{d\,fog}{dx} = g'(x) \times f'(g(x))$$

$$\frac{d\,fog}{dx} = \frac{df}{dg}\frac{dg}{dx} = \frac{df}{dx}$$

۲.۳ حسابان

مشتق

$$\frac{d}{dx} c = 0$$

$$\frac{d}{dx} x^n = nx^{n-1}$$

$\ln x$

$$\frac{d}{dx} b^x = \ln b \,.\, b^x$$

$$\frac{d}{dx} \boxed{\log_b x} = \boxed{\frac{1}{\ln b}}\boxed{\frac{1}{x}}$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx}\boxed{\sqrt{\boxed{\sin x}}} = \quad (f\circ g)' = f'(g(x)).g'(x)$$

$g(x) = \sin x \rightarrow \cos x$

$f(u) = \sqrt{u} \rightarrow \frac{1}{2\sqrt{\sin x}}$

$$\boxed{\frac{\cos x}{2\sqrt{\sin x}}}$$

$21x^6 - 3x^5 + 10x^4 - 4x^3 + 1$

$f(x) = 3x^7 - \frac{x^6}{2} + 2x^5 - x^4 + x - 2$

$f'(x) = \frac{d}{dx} 3x^7 + \frac{d}{dx} -\frac{x^6}{2} + \dots$

$f(x) = \sqrt{x} = x^{\frac{1}{2}}$

$f' = \frac{1}{2\sqrt{x}} \qquad \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2}\boxed{x^{-\frac{1}{2}}}$

$\frac{1}{2} \frac{1}{\sqrt{x}}$

$f(x) = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}} \qquad -\frac{1}{2} x^{-\frac{1}{2}-1}$

$\boxed{-\frac{1}{2x\sqrt{x}}} = -\frac{1}{2} \cdot \frac{1}{\sqrt{x^3}} = \frac{1}{x^{\frac{3}{2}}} = -\frac{1}{2} x^{-\frac{3}{2}}$

$f'(g(x)) \quad g$

$\frac{d}{dx} e^{-x} = -e^{-x}$

$e^{-x} \cdot -1$

$e^{-x}(\cos x - \sin x)$

$f \quad g$

$\frac{d}{dx} \boxed{e^{\boxed{-x}}}\boxed{\sin x}$

$f'g + fg'$

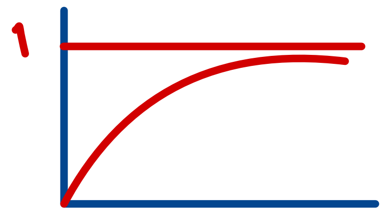
$e^{-x} \cos x + -e^{-x} \sin x$

$f(x) = \sqrt{\boxed{3x^2 - x + 4}}$

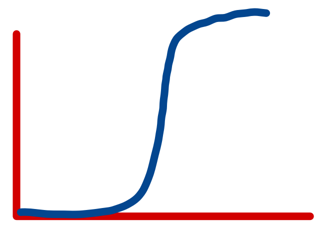
$g(x) = 3x^2 - x + 4$

$f(u) = \sqrt{u} \rightarrow \frac{1}{2\sqrt{u}}$

$\frac{ax}{x+b}$



$\frac{ax^n}{x^n + b^n}$ hill function



$\sqrt{\sin x}$

$g(x) = \sin x$

$f(u) = \sqrt{u}$

$f'(\boxed{g(x)}) \,.\, g'(x)$

$\frac{1}{2\sqrt{\sin x}} \cdot \cos(x)$

$f'(x) = f'(g(x)).g'(x)$

$\boxed{\frac{1}{2\sqrt{3x^2 - x + 4}} \cdot (6x-1)} = \boxed{\frac{6x-1}{2\sqrt{3x^2 - x + 4}}}$

$f \qquad g$

$f(x) = \frac{2x}{x^2 - 3x + 4} = \left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$

$= \frac{(x^2 - 3x + 4)(2) - (2x)(2x - 3)}{(x^2 - 3x + 4)^2}$

$2x^2 - 6x + 8 - (4x^2 - 6x)$

$2x^2 - \cancel{6x} + 8 - 4x^2 + \cancel{6x}$

$\frac{(-2x^2 + 8)}{(x^2 - 3x + 4)^2}$

$f \qquad g$

$f(x) = (-x^2 + 3x + 4)(\sin x) \qquad f'g + gf'$

$f'(x) = (-x^2 + 3x + 4)(\cos x) + (\sin x)(-2x + 3)$

o

1 $\frac{1}{x} \rightarrow \frac{d}{dx} x^{-1} \rightarrow -1\, x^{-1-1} = -1\, x^{-2} = -\frac{1}{x^2}$

2 $\sqrt{x} \rightarrow \frac{d}{dx} x^{\frac{1}{2}} \rightarrow \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$

3 $\frac{1}{\sqrt{x}} \rightarrow \frac{d}{dx} x^{-\frac{1}{2}} \rightarrow -\frac{1}{2} x^{-\frac{1}{2}-1} = -\frac{1}{2} x^{-\frac{3}{2}} = -\frac{1}{2x^{\frac{3}{2}}} = -\frac{1}{2\sqrt{x^3}}$

4 $3x^2+4x+1 \rightarrow 2 \times 3x^{2-1} + 4 = 6x+4$

o Product rule $\rightsquigarrow$ $(fg)' = f'g + g'f$

7 $\cos x \,.\, e^x \rightarrow (\cos x)' e^x + (e^x)' \cos x$

$(-\sin x)\, e^x + e^x \cos x \rightarrow e^x(\cos x - \sin x)$

10 $\log_{10} x \,.\, 10^x \rightarrow (\log_{10} x)'\, 10^x + (10^x)' \log_{10} x$

$\frac{1}{x \ln 10}\, 10^x + 10^x \ln 10 \,\log_{10} x$

o Quotient rule $\rightsquigarrow \left(\frac{f}{g}\right)' \quad \frac{gf' - fg'}{g^2}$

5 $\frac{2x^2+3}{3x+1} \rightsquigarrow \frac{3x+1 \,.\, (2x^2+3)' - 2x^2+3 \;(3x+1)'}{(3x+1)^2}$

$= \frac{3x+1 \,.\, 4x - 2x^2+3 \,.\, 3}{(3x+1)^2} = \frac{12x^2+4x-(6x^2+9)}{(3x+1)^2}$

$= \frac{6x^2+4x-9}{9x^2+6x+1}$

6 $\frac{\sin x}{x^3+x^2+x} \rightsquigarrow \frac{(x^3+x^2+x)(\sin x)' - (\sin x)(x^3+x^2+x)'}{(x^3+x^2+x)^2}$

$= \frac{(x^3+x^2+x)(\cos x) - (\sin x)(3x^2+2x+1)}{(x^3+x^2+x)^2}$

○ Chain rule $(f \circ g)' = f'(g(x))\, g'(x)$

8 $\sqrt{3x^2 - x + 5} \rightsquigarrow f(x) = \sqrt{x} \rightsquigarrow f'(x) = \frac{1}{2\sqrt{x}}$

$g(x) = 3x^2 - x + 5 \rightsquigarrow g'(x) = 6x - 1$

$\frac{1}{2\sqrt{3x^2 - x + 5}} \cdot 6x - 1 = \frac{6x - 1}{2\sqrt{3x^2 - x + 5}}$

11 $\sin(3x^2 + x) \rightsquigarrow f(x) = \sin x \rightsquigarrow f'(x) = \cos x$

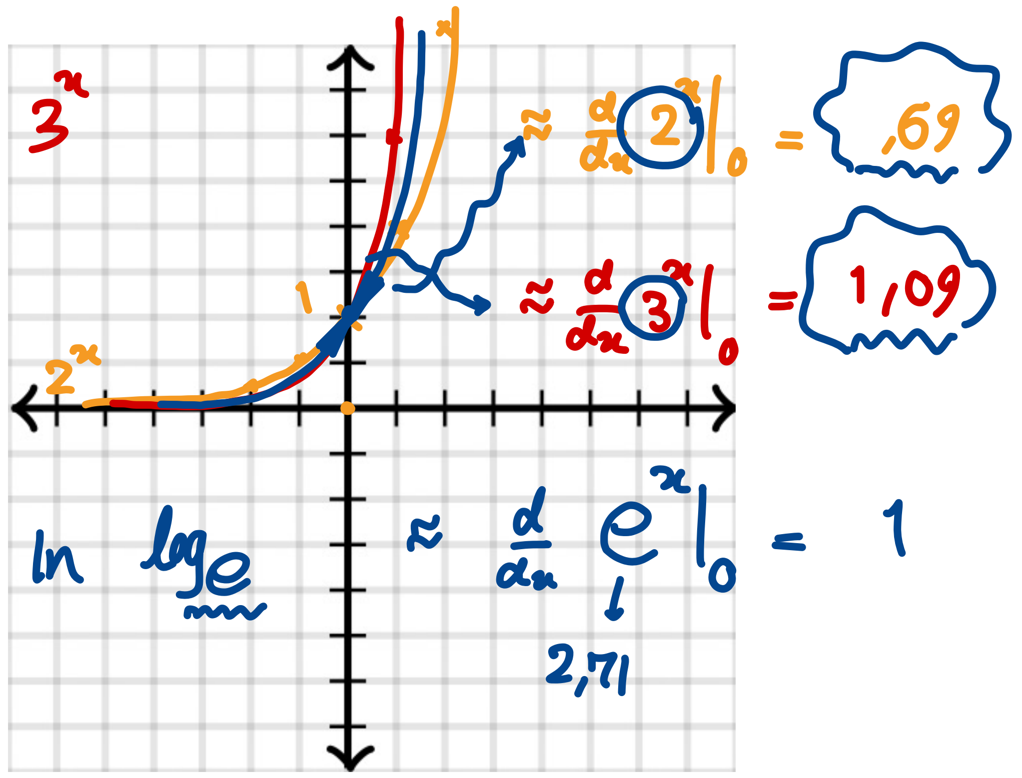
$g(x) = 3x^2 + x \rightsquigarrow g'(x) = 6x + 1$

$\cos(3x^2 + x) \cdot 6x + 1$

13 $e^{\ln x} \rightsquigarrow 1$

۲.۳ حسابان

عدد اویلر



$$\frac{d}{dx} 2^x \rightarrow \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{2^{x+h}-2^x}{h}$$

$$= \lim_{h\to 0} \frac{2^x \cdot 2^h - 2^x}{h}$$

$$= \lim_{h\to 0} \frac{\boxed{2^x}(2^h-1)}{h}$$

$$\frac{d}{dx} 2^x = 2^x \boxed{\lim_{h\to 0} \frac{2^h-1}{h}} \quad \ln 2$$

$$\frac{d}{dx} 2^x \Big|_{x=0} = \lim_{h\to 0} \frac{2^h-1}{h}$$

$$\lim_{h\to 0} \frac{e^h-1}{h} = 1$$

$$\ln e = 1$$

$$\frac{\mathrm{d}}{\mathrm{d}x} e^x = e^x$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \ln x = \frac{1}{x}$$

۲.۳ حسابان

انتگرال

$$\int_a^b f(x)\, dx$$

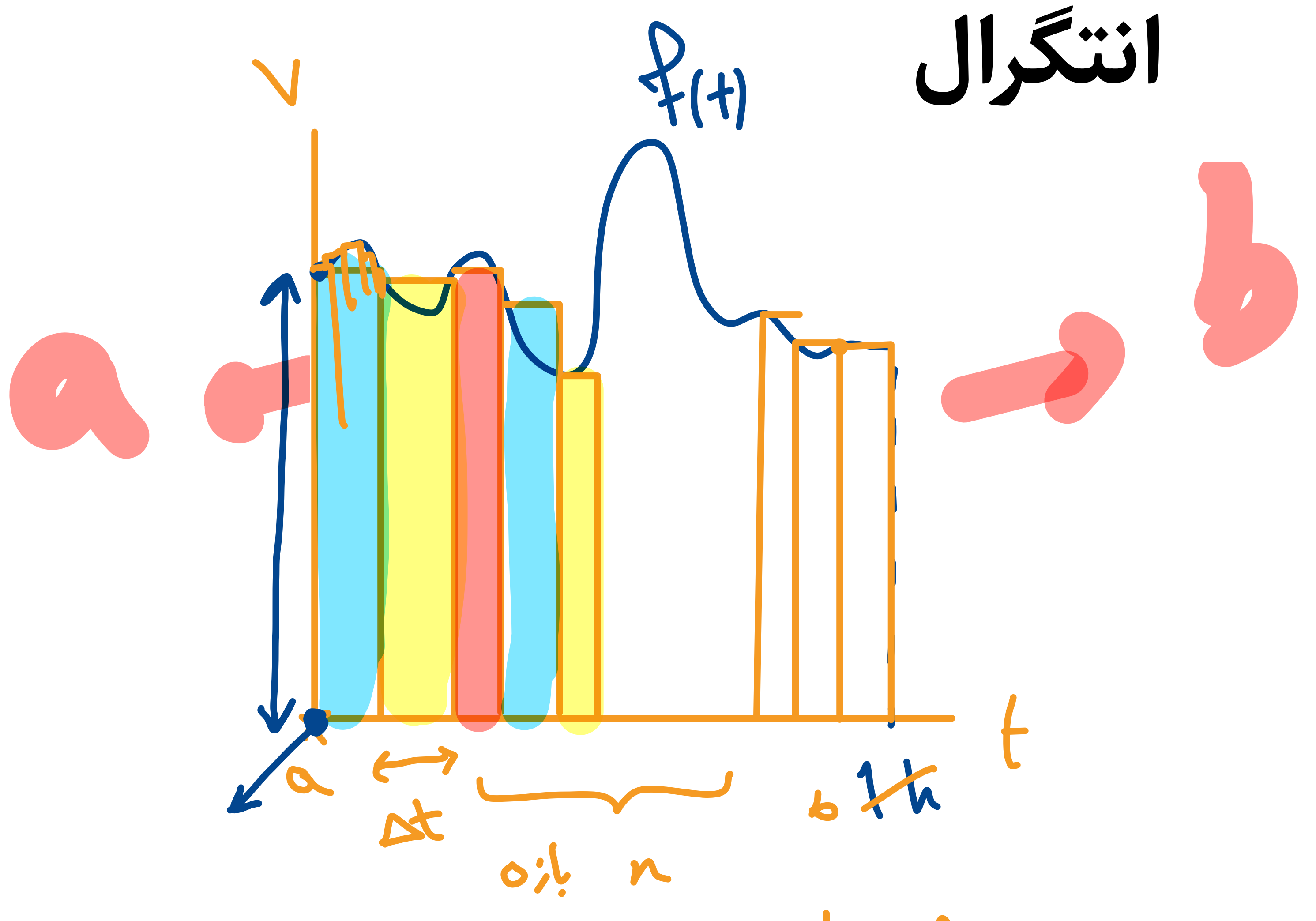
$$80 \frac{km}{h} \times 1^h$$

$80 \frac{km}{h}$: $0 \rightarrow 1$ → $80\ km$

$100 \frac{km}{h}$: $1 \rightarrow 1.5$ → $50\ km$

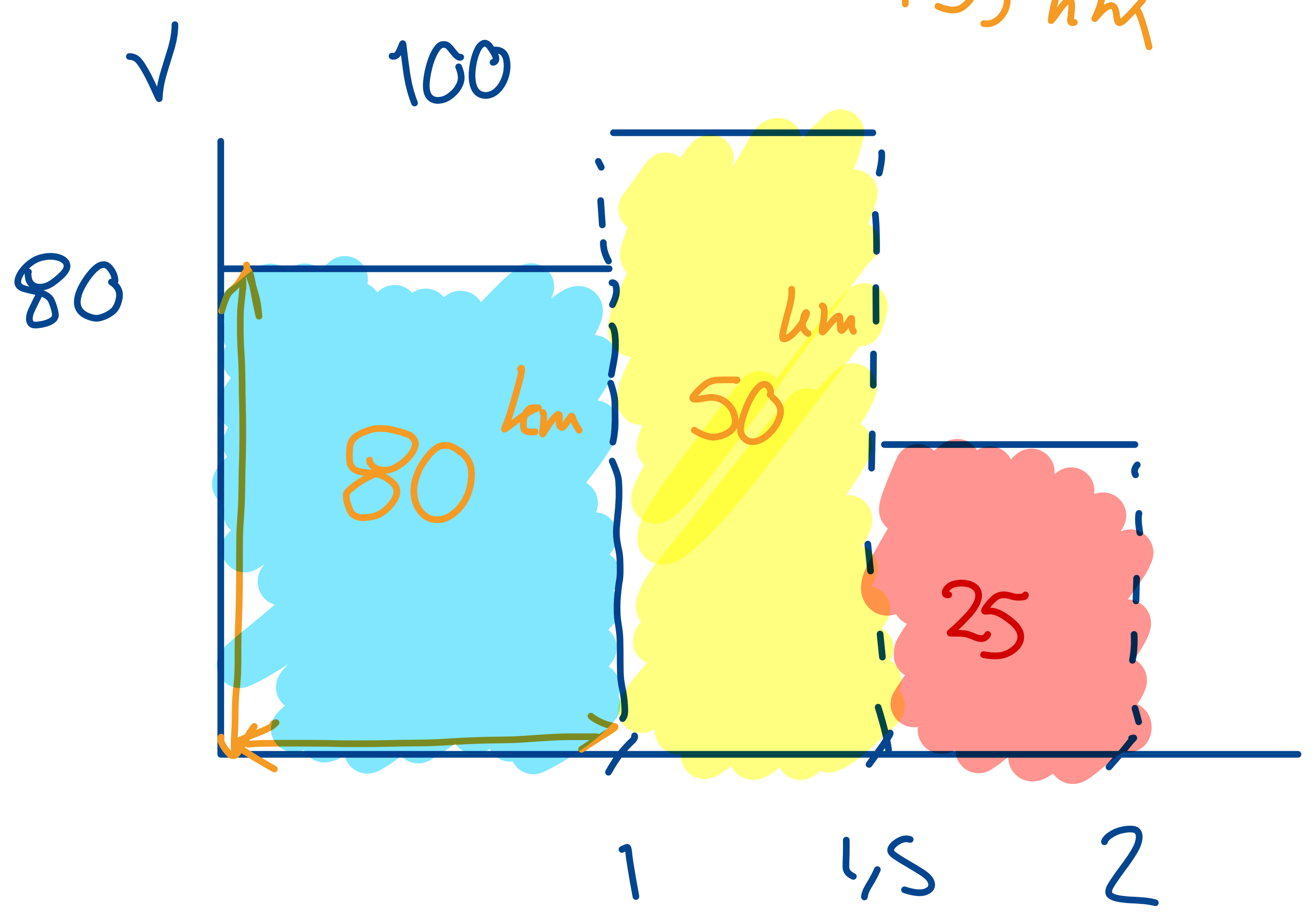
$50 \frac{km}{h}$: $1.5 \rightarrow 2$ → $25\ km$

$155\ km$

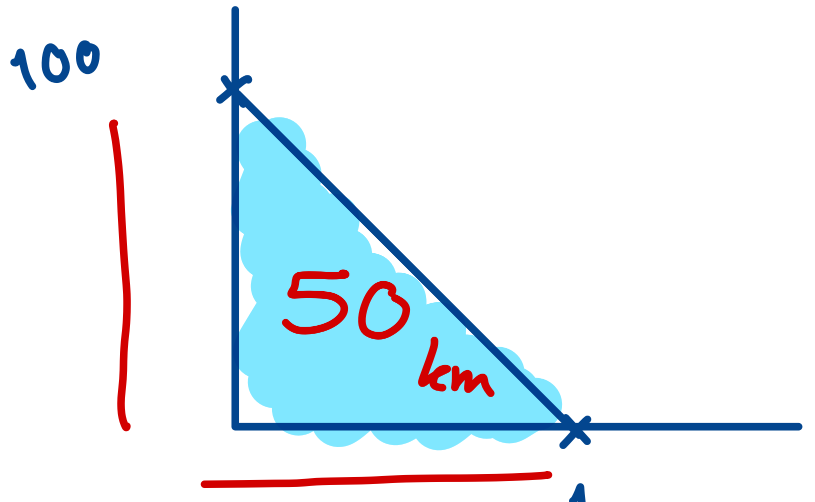


$$\rightarrow \Delta t = \frac{b-a}{n}$$

$$x_i = a + i\Delta t$$



$$\text{مساحت} \approx \sum_{i=1}^{n} f(x_i)\,\Delta x$$



$$\Delta x = \frac{(b-a)}{n}\ ,\ x_i = a + i\Delta x$$

$$\int_a^b f(x)\ \mathrm{d}x = \lim_{n\to\infty} \sum_{i=1}^{\infty} f(x_i)\Delta x$$

$$\boxed{\int_a^b f(x)\, dx}$$

$$\text{مساحت دقیق} = \lim_{n\to\infty} \sum_{i}^{n} f(x_i)\,\Delta x$$

$$= \int_a^b f(x)\, dx \rightarrow \text{عدد}$$

انتگرال معین

۲.۳ حسابان

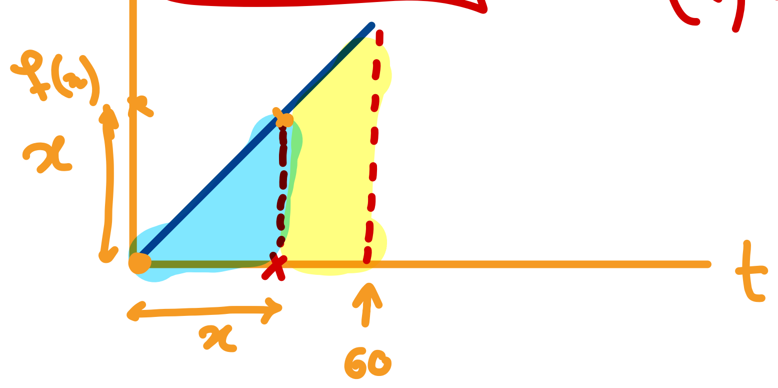
انتگرال

F

$\int f(t)\,dt$

$$= F(b) - F(a)$$
$$= \int f(t)\,dt \Big|_a^b$$
$$\int_a^b f(t)\,dt$$

$\boxed{f(t) = t}$ $\quad f(x) = x$



$f(t) = t$

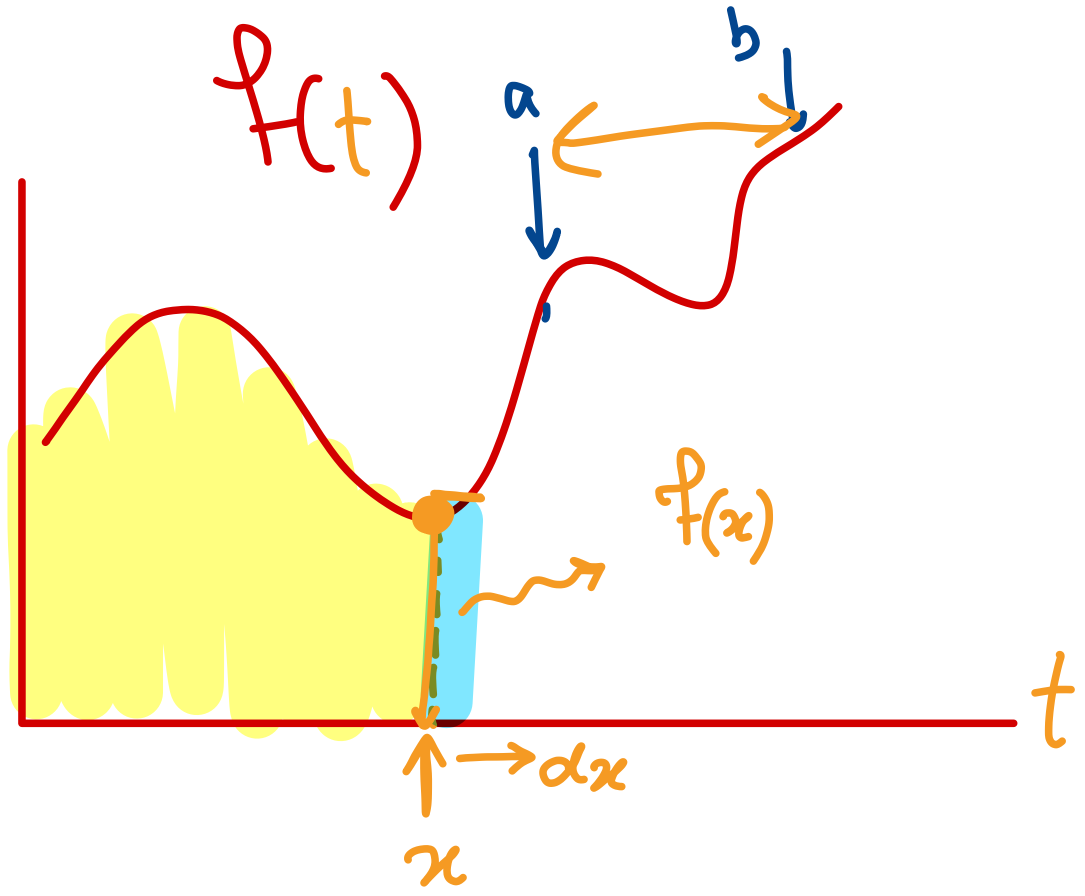
$F(x) = \int f(t)\,dt = \frac{1}{2}x^2$

$\int_{30}^{60} f(t)\,dt = F(60) - F(30)$

$= \int f(t)\,dt \Big|_{30}^{60}$

$F(x) = \frac{1}{2} x x \quad \boxed{\frac{1}{2}x^2} \rightarrow \frac{1}{2}x^2 \Big|_{30}^{60}$ = کل مسافت $\boxed{45}$ km

$F'(x) = \frac{d}{dx}\frac{1}{2}x^2 = x$



$$F(x) = \int_a^x f(t)\,\mathrm{d}t \implies F'(x) = f(x)$$

$$F(x) = \int_0^x f(t)\,dt$$

$$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a)$$

نامعین

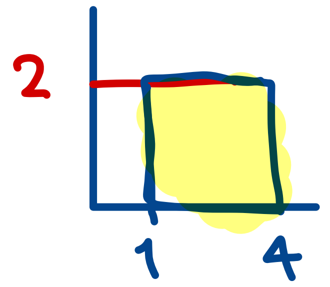
$\int f(t)\,dt$ ⟶ تابع

$$\int_a^b f(x)\,\mathrm{d}x = \int f(x)\,\mathrm{d}x \Big|_a^b$$

یادمشتق

$$F'(x) = f(x)$$

$f(t) = 2$



$\int f(t)\,dt = 2t$

$\int_1^4 f(t)\,dt = \int f(t)\,dt \Big|_1^4 = 2\times4 - 2\times1$

$8 - 2 = \boxed{6}$

۲.۳ حسابان

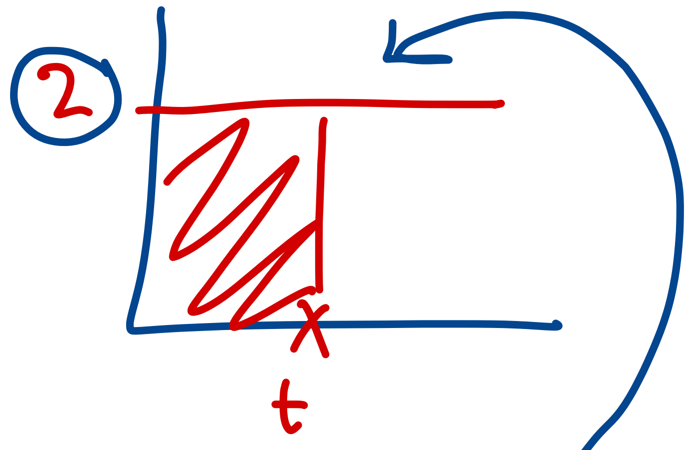
انتگرال

$$\int cf(x)\ \mathrm{d}x = c\int f(x)\ \mathrm{d}x$$

$$\int [f(x) + g(x)]\ \mathrm{d}x = \int f(x)\ \mathrm{d}x + \int g(x)\ \mathrm{d}x$$

$$\int [f(x) - g(x)]\ \mathrm{d}x = \int f(x)\ \mathrm{d}x - \int g(x)\ \mathrm{d}x$$

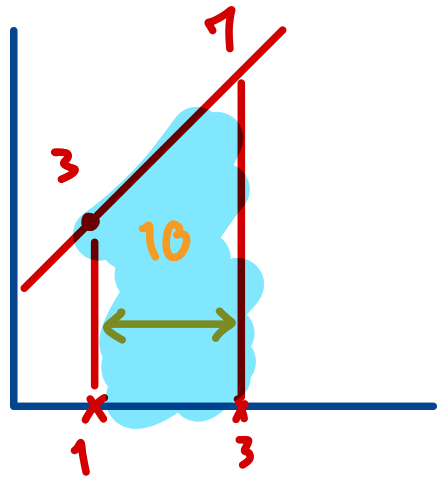
$f(t) = 2$



$\int f(t)\ dt = \boxed{2t + C}$

$\int_1^4 f(t)\,dt = \int f(t)\,dt \Big]_1^4$

$f(x) = 2x + 1$



$\int f(x)\ dx = x^2 + x + C$

$\int_1^3 f(x)\ dx = \int f(x)\,dx \Big|_1^3 = 10$

۲.۳ حسابان

انتگرال

$$\int k \ \mathrm{d}x = kx + C$$

$$\int x^n \ \mathrm{d}x = \frac{x^{n+1}}{n+1} + C$$

$(b^x)' \to b^x \ln b$

$? \to b^x$

$$\int b^x \ \mathrm{d}x = \frac{b^x}{\ln b} + C$$

$$\int \frac{1}{x} \ \mathrm{d}x = \ln|x| + C$$

$$\int \sin x \ \mathrm{d}x = -\cos x + C$$

$$\int \cos x \ \mathrm{d}x = \sin x + C$$

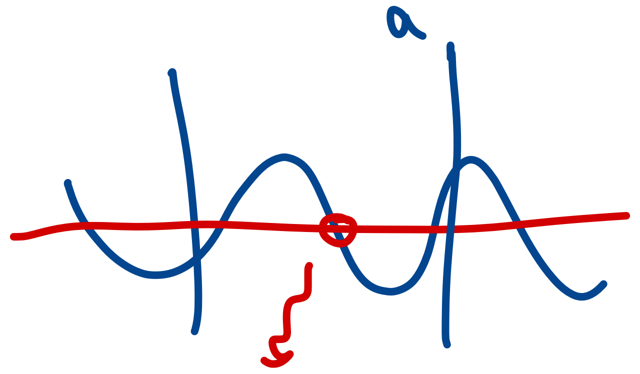
$$\int 3x^5 \ dx =$$

$$\int \boxed{3x^5 - 2x^4 + x^3 - 2} \ dx$$

$$= \boxed{3\frac{x^6}{6} - 2\frac{x^5}{5} + \frac{x^4}{4} - 2x} + C$$

۲.۳ حسابان

بسط تیلور



$f(a) + f'(a)(x-a)$

$f(x) \approx f(0) + f'(0)x$ تقریب خطی

$f(\pi) = 0$

$f'(\pi) = -1$

$e^0 \quad 1$

$a=0$

$g(x)$

taylor series

$c_0 + c_1x + c_2x^2 + c_3x^3 + \dots$

$f(x)$

$\sin x$

مشتق n ام $\quad a = \pi$

$$f(a) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

① $g(0) = f(0)$

$c_0 = f(0) = \sin 0 = 0 = c_0$

$0 + x + 0 - \frac{x^3}{6}$

② $g'(0) = f'(0)$

$0 + c_1 + 2c_2x + 3c_3x^2 + 4c_4x^3 + \dots$

$c_1 = f'(0) = \cos 0 = 1 = c_1$

$\cos x \quad 0$

③ $g''(0) = f''(0)$

$=$

$0 + 0 + \boxed{2c_2} + 6c_3x + 12c_4x^2 + \dots$

$2c_2 = f''(0) = -\sin 0 \quad 0 = c_2$

④ $g'''(0) = f'''(0)$

$0 + 0 + 0 + 6c_3 + 24c_4x$

$-\frac{1}{6} = c_3$

$6c_3 = f'''(0) = -\cos 0 = -1$

$-1 = 6c_3$

$n=0$

$c_0 + c_1x + c_2x^2 + c_3x^3 + c_4x^4$

$\frac{\cos(0)}{0!} \quad \frac{-\sin(0)}{1!} \quad \frac{-\cos(0)}{2!} \quad \frac{\sin(0)}{3!} \quad \frac{\cos(0)}{4!}$

$1 + 0x + -\frac{1}{2}x^2 + 0x^3 + \frac{1}{4!}x^4$

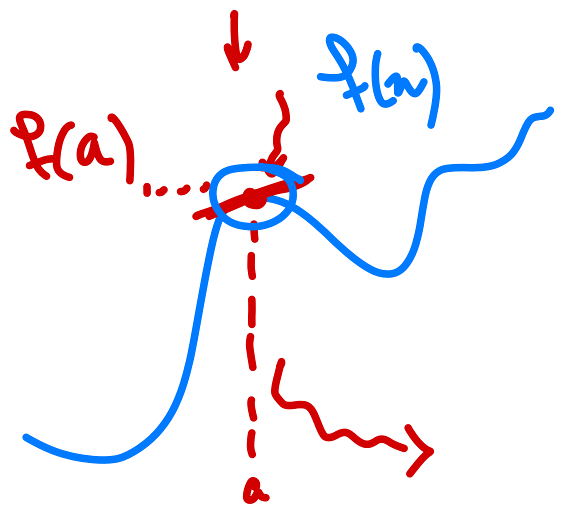
$1 - \frac{1}{2}x^2 + \frac{1}{24}x^4$

$c_0 \; 1 \qquad c_1 \qquad c_2 \; 0$

$\frac{e^0}{0!} \qquad \frac{e^0}{1!} \qquad \frac{e^0}{2!}$

۲.۳ حسابان

بسط تیلور



$$\boxed{f(a) + f'(a)(x-a)}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

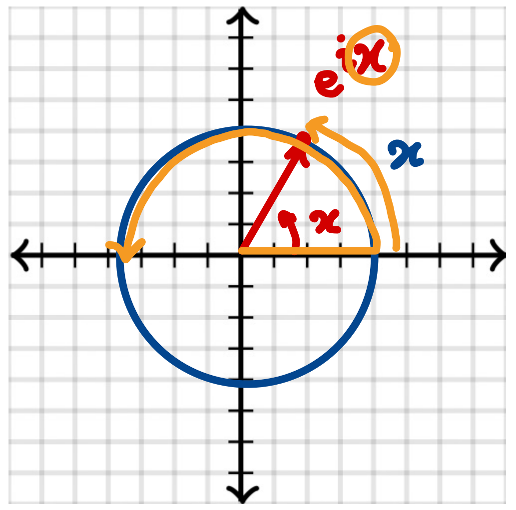
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$e^{ix} = \cos x + i \sin x$$

$r=1$

$\theta = x$



$$e^{i\pi} = -1$$

$$e^{ix} = 1 + ix + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \cdots$$

$$1 + ix + \frac{\overset{-1}{i^2} x^2}{2!} + \frac{i^3 x^3}{3!} + \frac{i^4 x^4}{4!} + \cdots$$

$$1 + ix - \frac{x^2}{2!} - \frac{ix^3}{3!} + \frac{x^4}{4!}$$

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} + ix - i\frac{x^3}{3!} + i\frac{x^5}{5!}$$

$\cos x$ $\sin x$

$$\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!}\right) + i\left(x - \frac{x^3}{3!} + \frac{x^5}{5!}\right)$$

$f(x) \leftarrow f'(x)$

۲.۴ معادلات دیفرانسیل

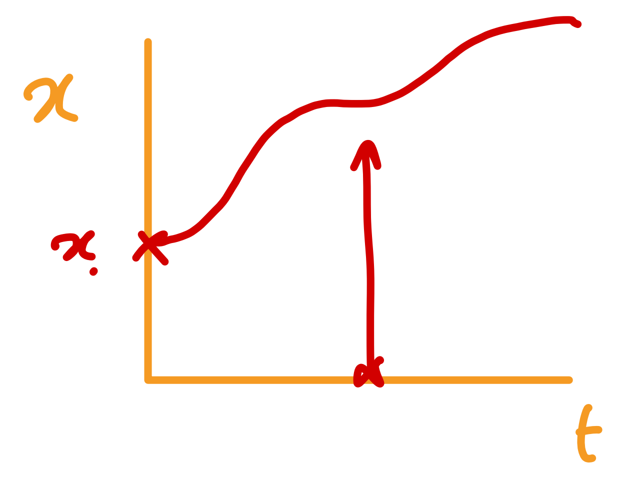
$t \quad t_1$

$x \rightsquigarrow \boxed{f} \quad f \quad \frac{df}{dt} = f(x, p)$

$\dot{x}$

$f' \to f$

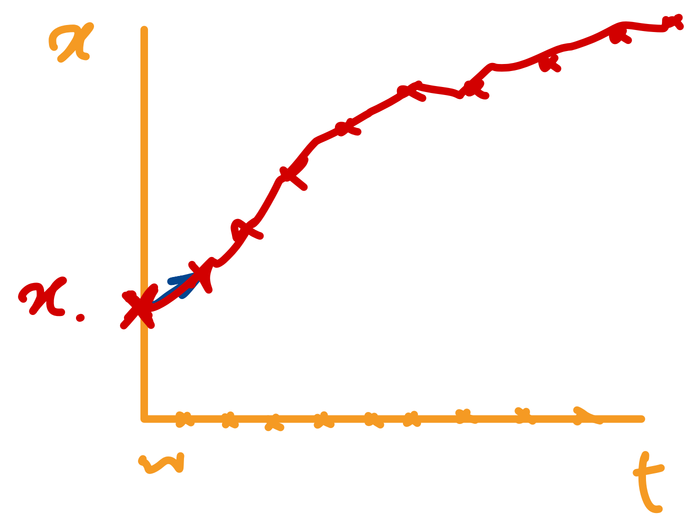
x_0



$$\dot{x} = \frac{\mathrm{d}x}{\mathrm{d}t} = f(x)$$

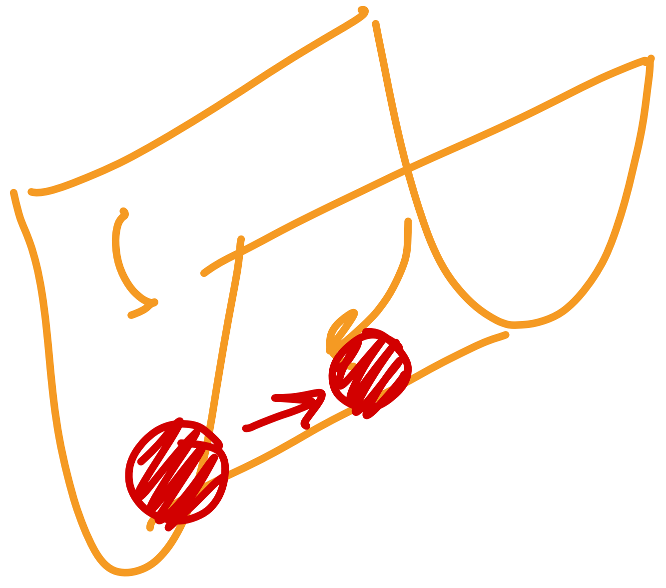
$\dot{x} = f(x)$

$\dot{x} = f(x_0)$



$$\boxed{x_{t+\Delta t}} = x_t + \dot{x}(x_t)\,\Delta t$$

$$\begin{cases} \dot{x}(x^*) = 0 \to x^* \text{ is an equilibrium point} \\ \ddot{x}(x^*) < 0 \to x^* \text{ is a stable equilibrium point} \\ \ddot{x}(x^*) > 0 \to x^* \text{ is a unstable equilibrium point} \end{cases}$$



۲.۴ معادلات دیفرانسیل

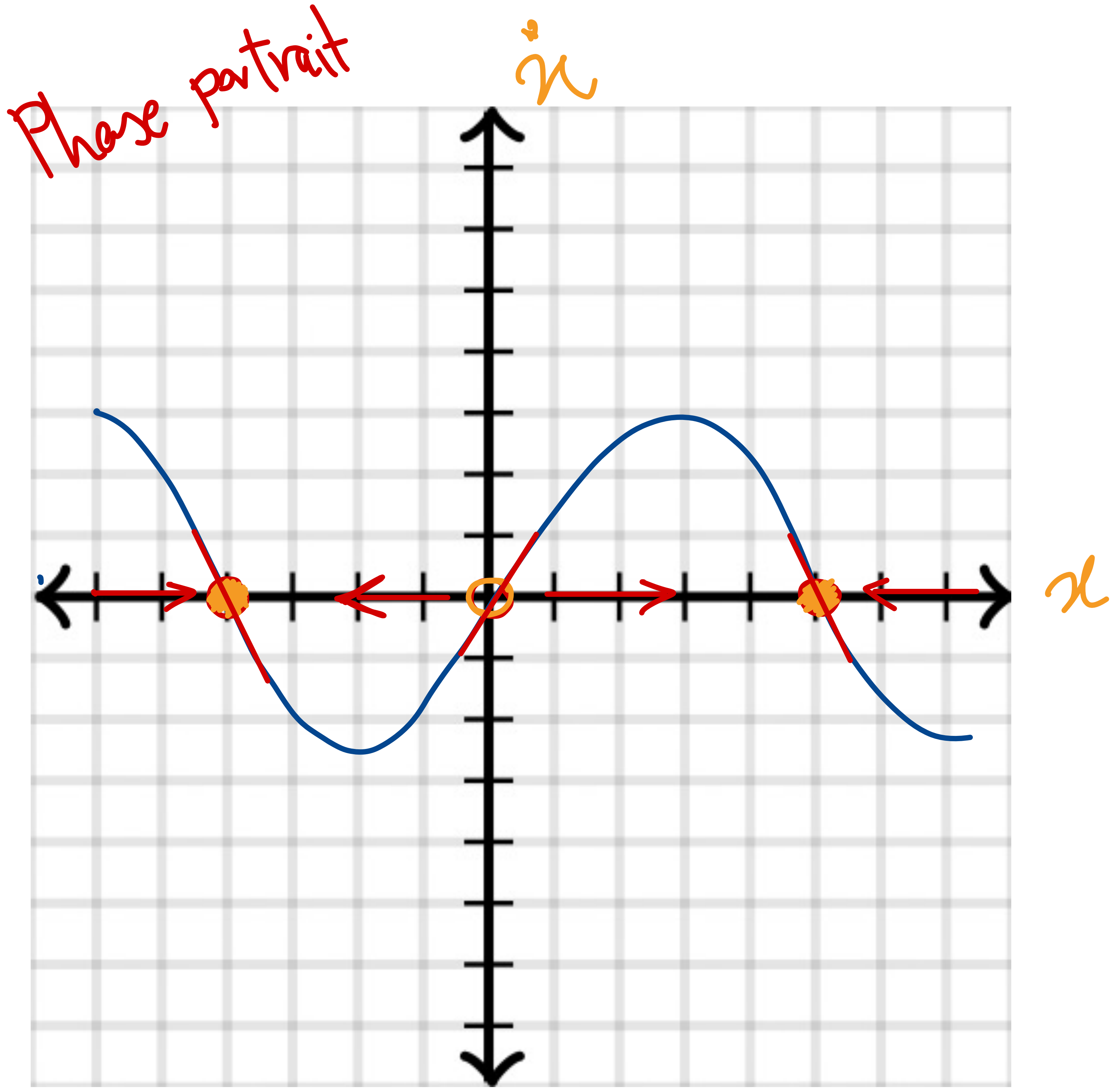
$$\dot{x} = \sin x$$

$$\frac{dx}{dt}$$

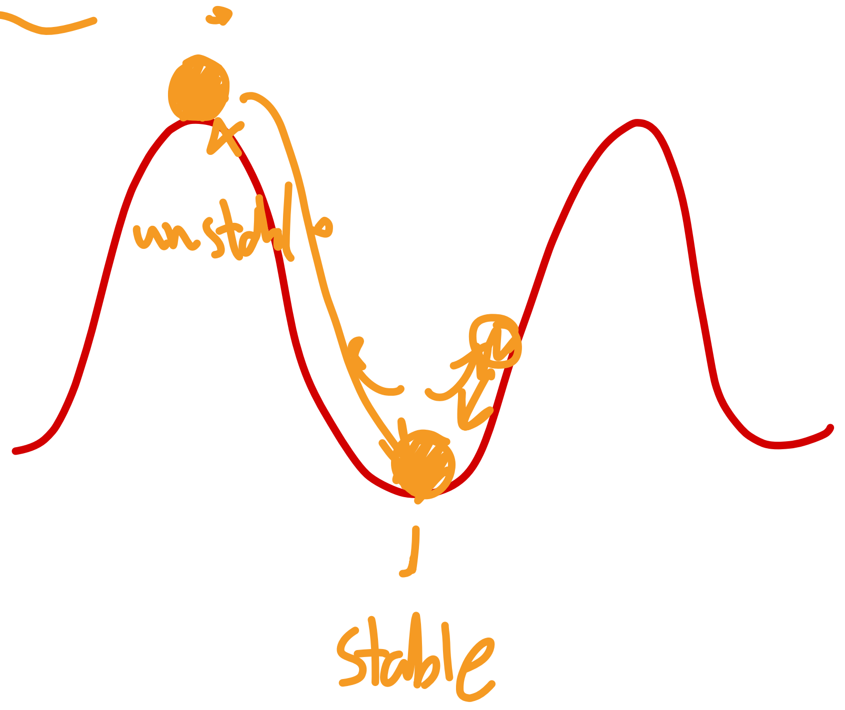
$t \to x$

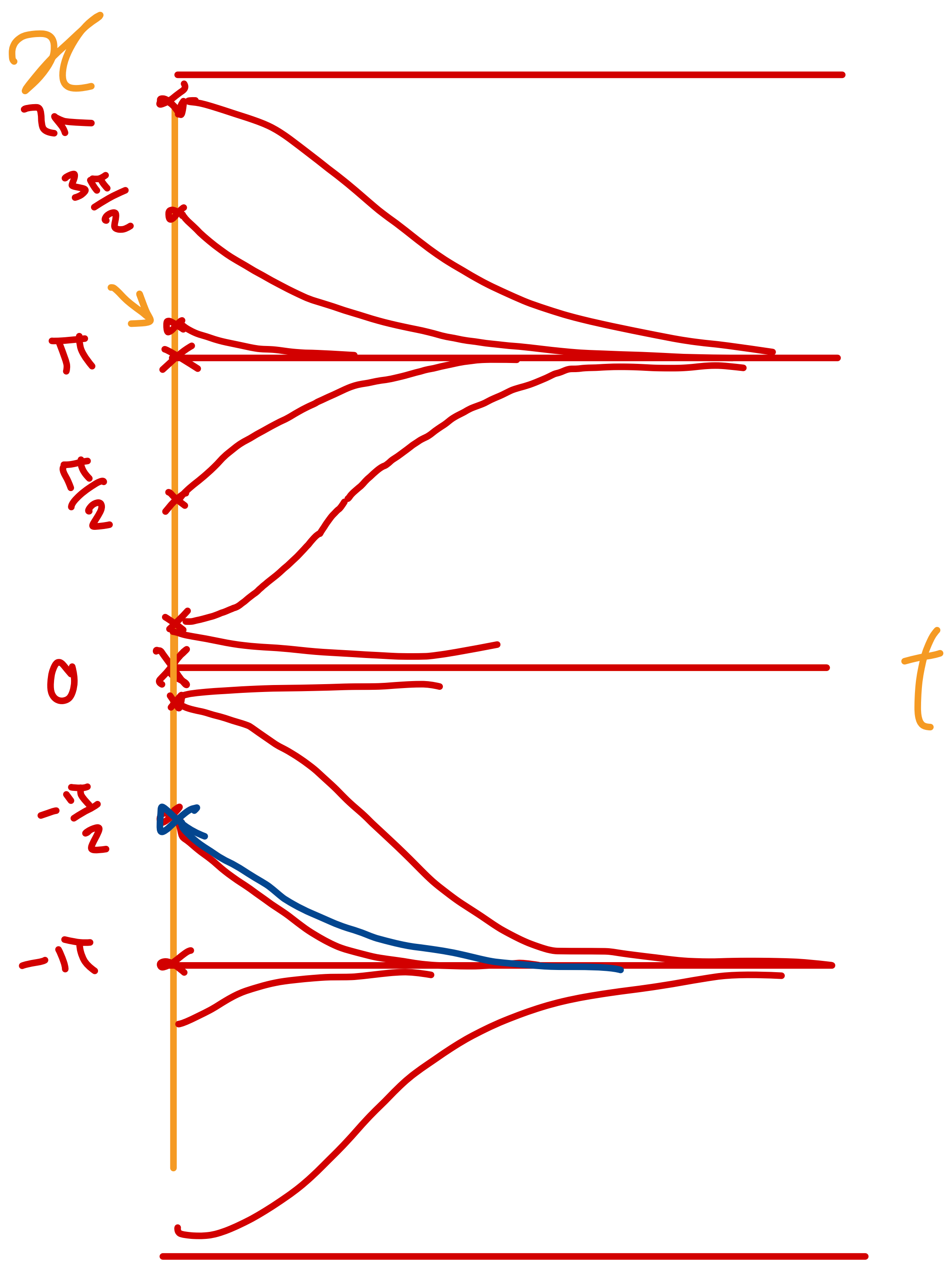
$\boxed{x(t)}$

$$\frac{d}{dt}x(t) = \sin x(t)$$



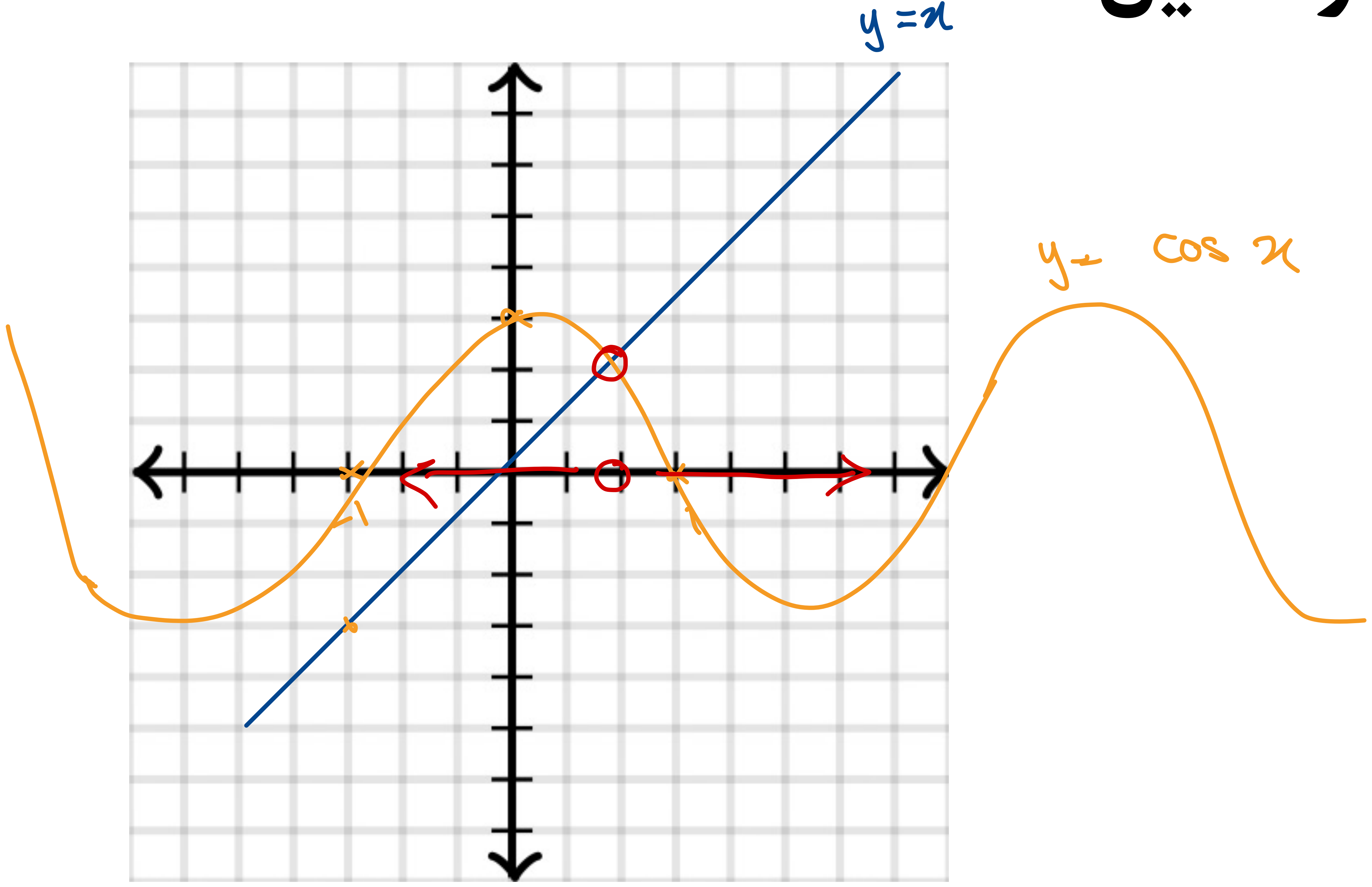
≈





۲.۴ معادلات دیفرانسیل

$$\dot{x} = x - \cos x$$



$x = \cos x \qquad \dot{x} = 0$

$x > \cos x \qquad \dot{x} > 0$

$x < \cos x \qquad \dot{x} < 0$

۳. پدیده‌ها

۳. پدیده‌ها

- بر اساس متغیر در حال تغییر
  - گسسته (نسل)
  - پیوسته (زمان)

۳. پدیده‌ها

- بر اساس تعداد متغیرهای مورد بررسی
  - یک بعدی
  - دو بعدی
  - ...

جمعیت خرگوش

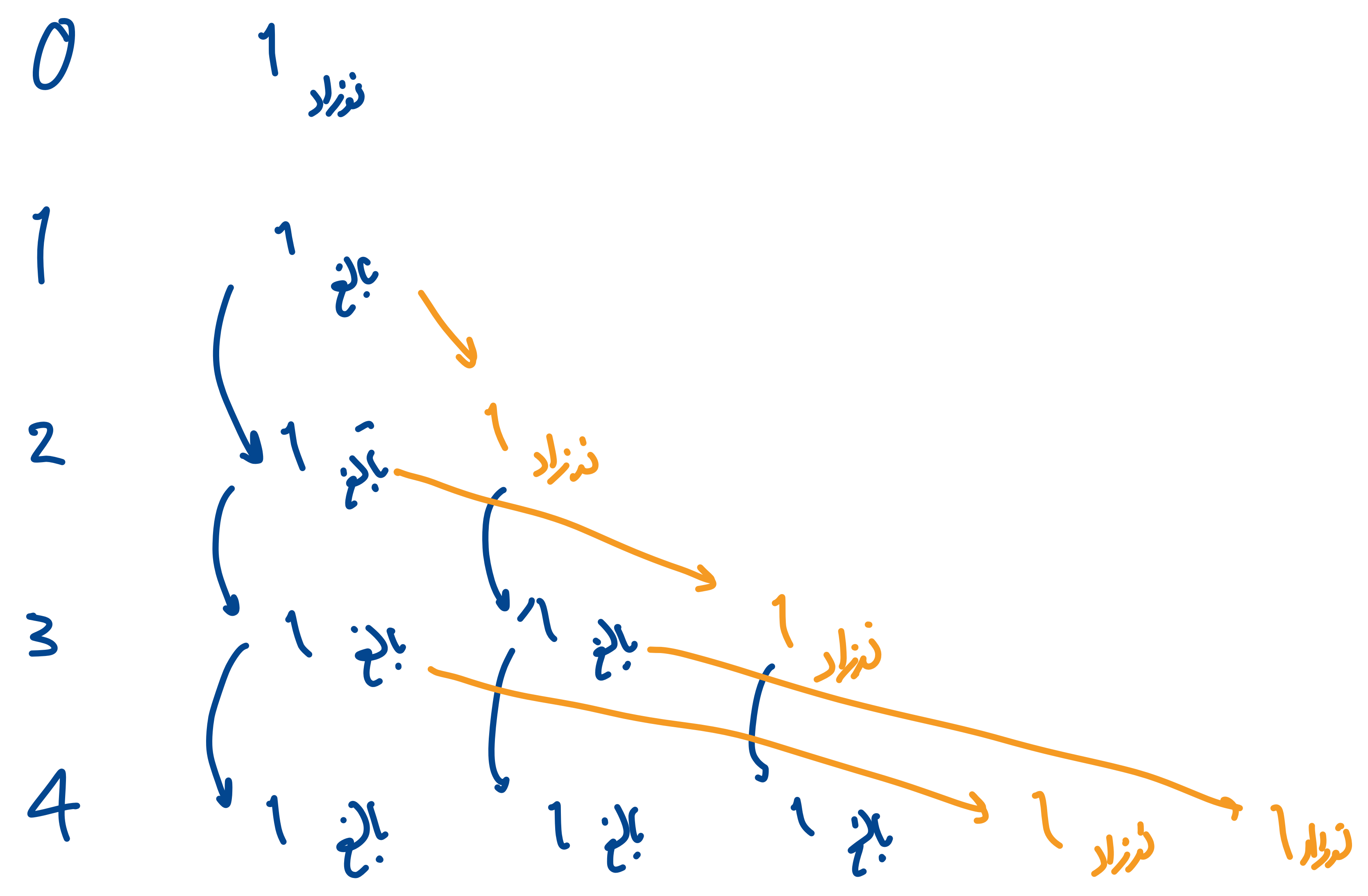
خرگوش ۱ ماهه بالغ

هر جفت خرگوش در هر ماه
یک جفت خرگوش تولید می‌کنه
(دختر و پسر)

نمی‌میرن

یک جفت نر و ماده نوزاد
در محیط جعبه

۴. مدل‌های گسسته



۴. مدل‌های گسسته

۴.۱ خرگوش‌های فیبوناچی

۴.۲ رشد جمعیت در زمان گسسته

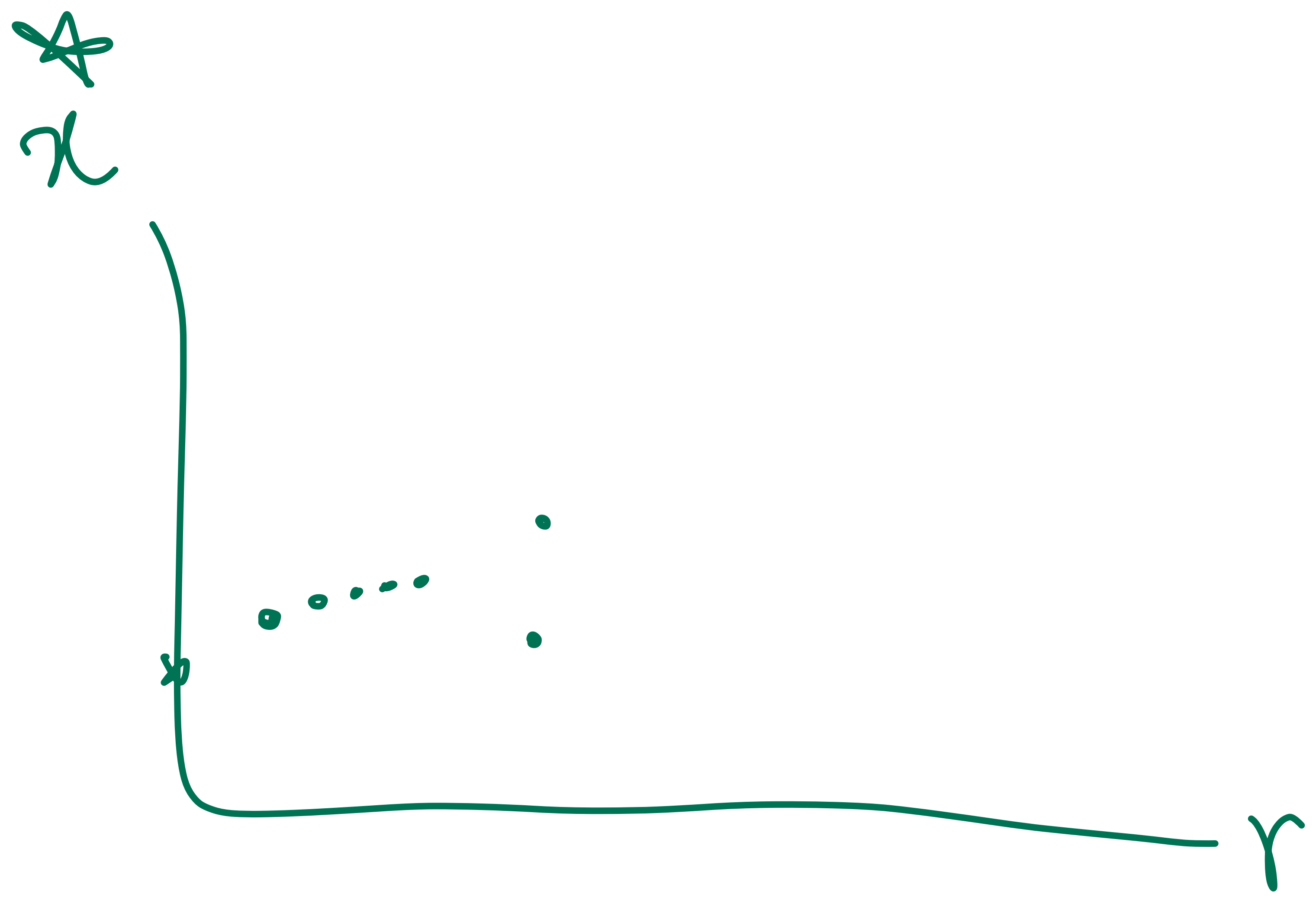
۴.۱ خرگوش‌های فیبوناچی

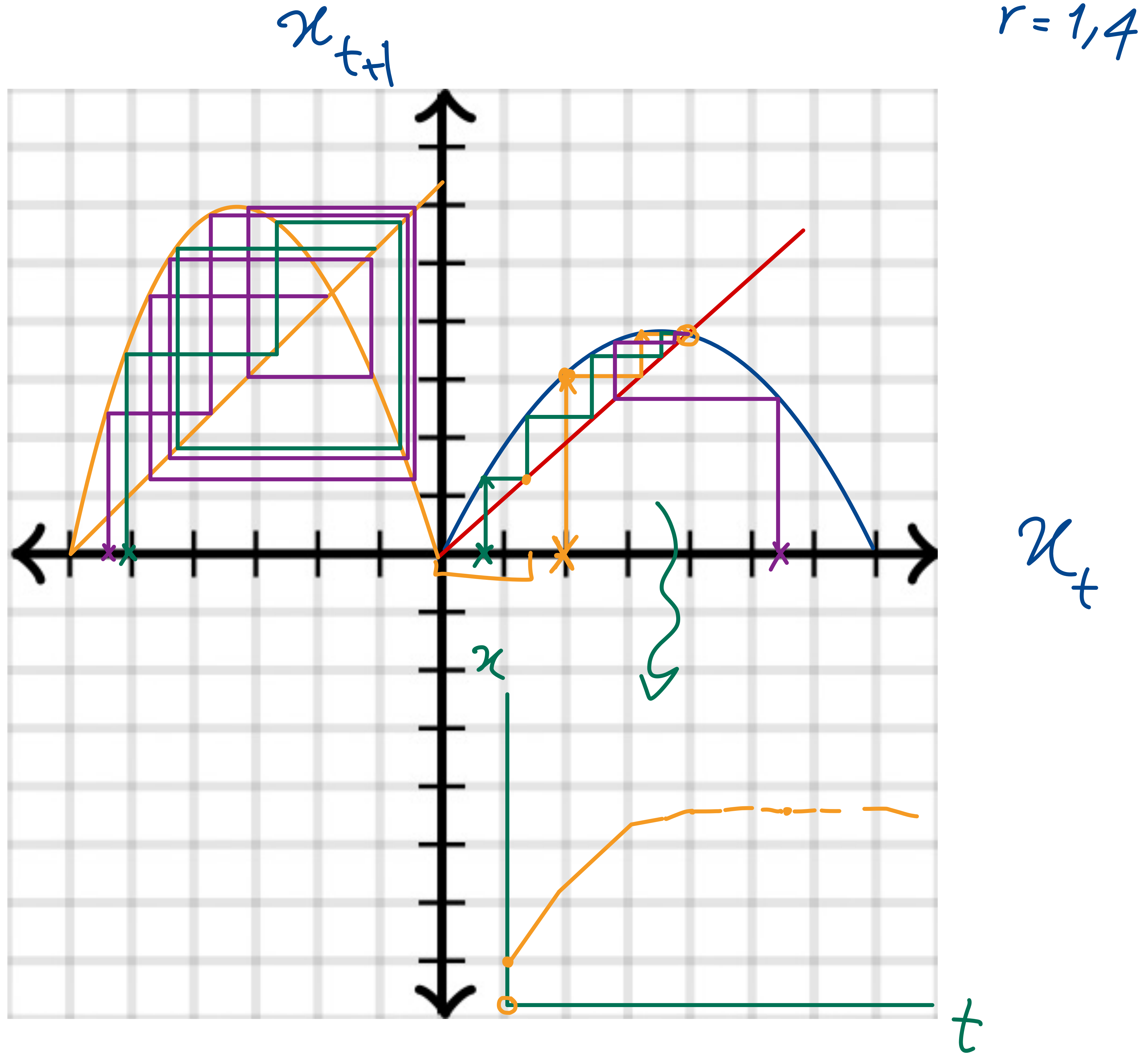
۴.۲ رشد جمعیت در زمان گسسته

logistic map

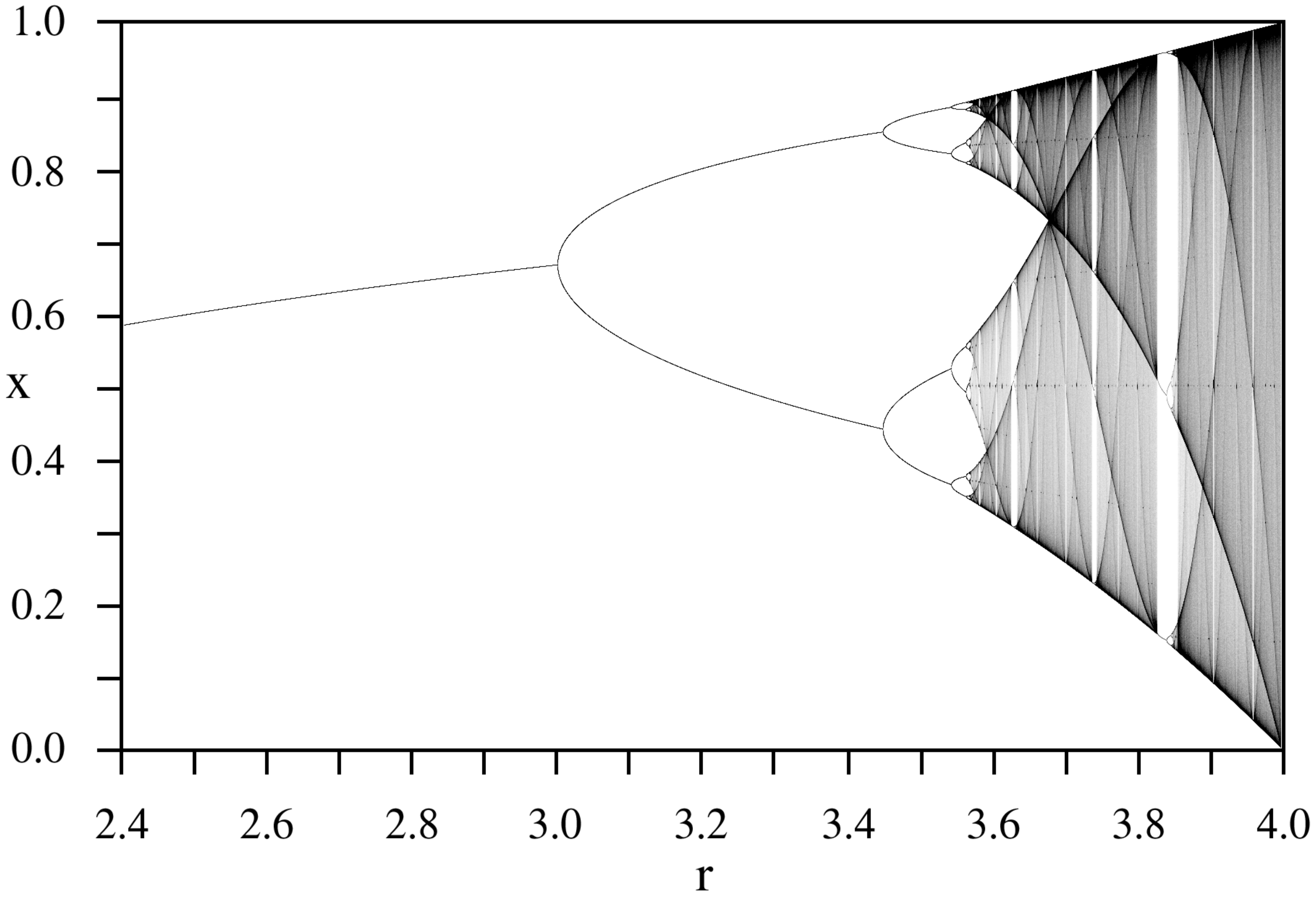
$$x_{t+1} = rx_t(1 - x_t)$$

deterministic chaos





۴.۲ رشد جمعیت در زمان گسسته



۵. مدل‌های پیوسته

۵. مدل‌های پیوسته

۵.۱ در یک بعد

رشد جمعیت در زمان پیوسته

کینتیک آنزیم

رشد تومور

بیان ژن

خودتنظیمی منفی در بیان ژن

خودتنظیمی مثبت در بیان ژن

۵.۲ در دو بعد

جمعیت‌های مستقل

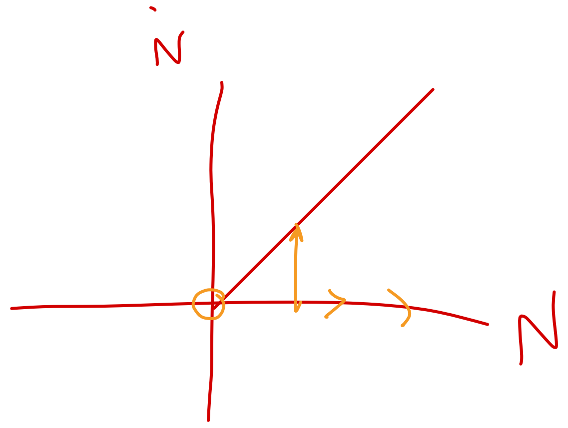
مدار تنظیمی گلوکز-انسولین

رقابت جمعیت‌ها

شکار و شکارچی

۵.۱ مدل‌های پیوسته در یک بعد

رشد جمعیت در زمان پیوسته (رشد نمایی)



$N(t)$?

$\frac{d}{dt}N(t) = rN(t)$

$N(t) = \boxed{N_0 e^{rt}} = r\boxed{N_0 e^{rt}}$

$$\dot N = rN \rightsquigarrow N(t) = c_0 e^{ct}$$

$$\boxed{\left.\frac{dN}{dt}\right|_t = rN(t)}$$

$N(t=0) = c_0 \cancel{e}^{\,1}$

c_0 ✓

$e^x \rightsquigarrow e^x$

$\frac{d}{dx}\boxed{e^{cx}} \rightsquigarrow \boxed{ce^{cx}}$

$$\frac{d}{dx}b^x = \lim_{h\to 0}\frac{b^{x+h}-b^x}{h}$$

$$\lim_{h\to 0}\frac{b^x\cdot b^h - b^x}{h}$$

$$\lim_{h\to 0} b^x\left(\frac{b^h-1}{h}\right)$$

$$\frac{d}{dx}b^x = b^x\boxed{\lim_{h\to 0}\left(\frac{b^h-1}{h}\right)}$$

$\dot N = 0 \rightsquigarrow$ eq

$\ddot N(eq) \rightsquigarrow > ? <$

۵.۱ مدل‌های پیوسته در یک بعد

رشد جمعیت در زمان پیوسته (رشد لجستیک)

$$\frac{d\dot N}{dt} = \ddot N = \frac{d}{dt}\, r\,\underbrace{N}_{f}\,(\underbrace{1-N}_{g}) = r\left(\overset{f}{N}\times\overset{g'}{-1} + \overset{f'}{1}\,(\overset{g}{1-N})\right)$$

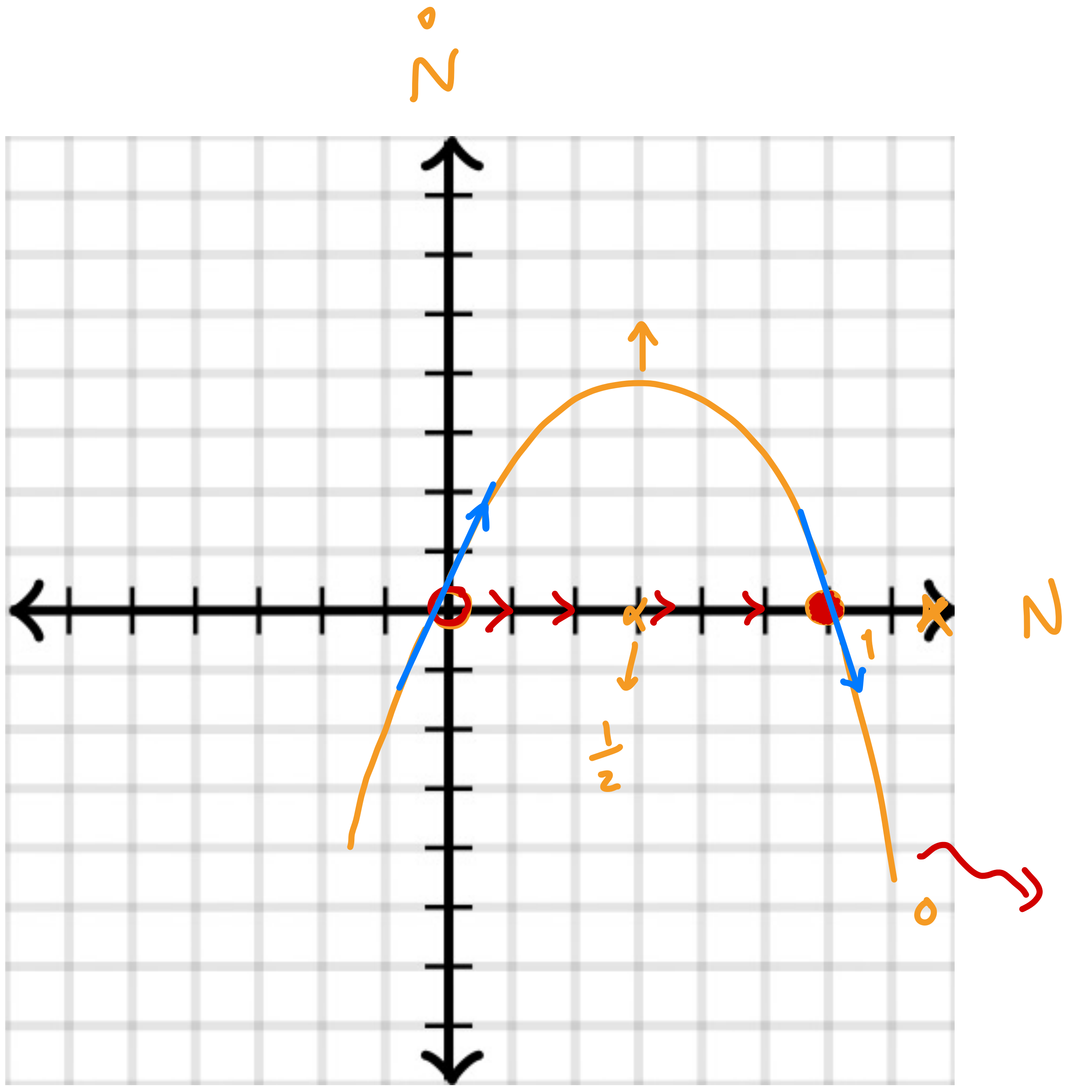
$$-N+1-N \to 1-2N \rightsquigarrow \ddot N = r(1-2N)$$

$$\dot N = rN(1-N)$$

$N(t)$

$$\left.\frac{dN}{dt}\right|_t = r\;N(t)\;\;1-N(t)$$

$= 0 \rightsquigarrow$ eq point

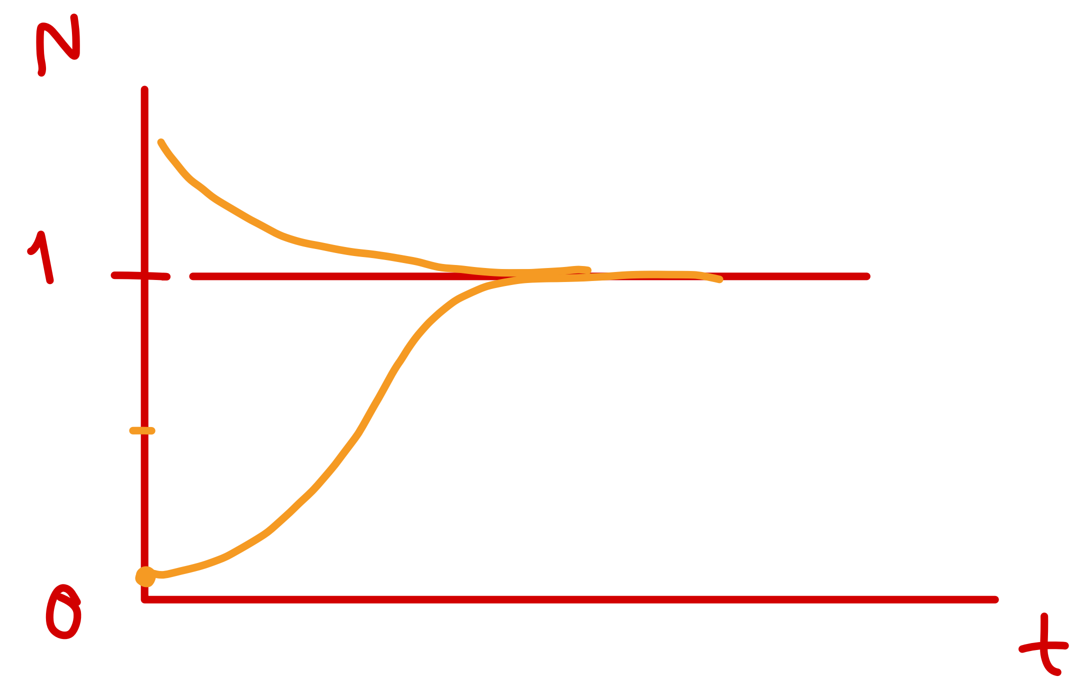


$$\left.\frac{d^2N}{dt^2}\right|_{eq=0} = r(1-2N) \to r \rightsquigarrow \text{unstable}$$

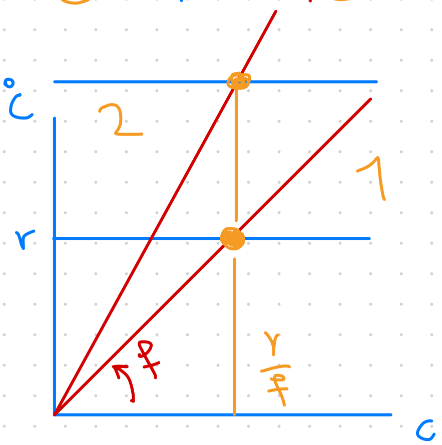
$$\left.\frac{d^2N}{dt^2}\right|_{eq=1} = r(1-2\times1) \to -r \rightsquigarrow \text{stable}$$

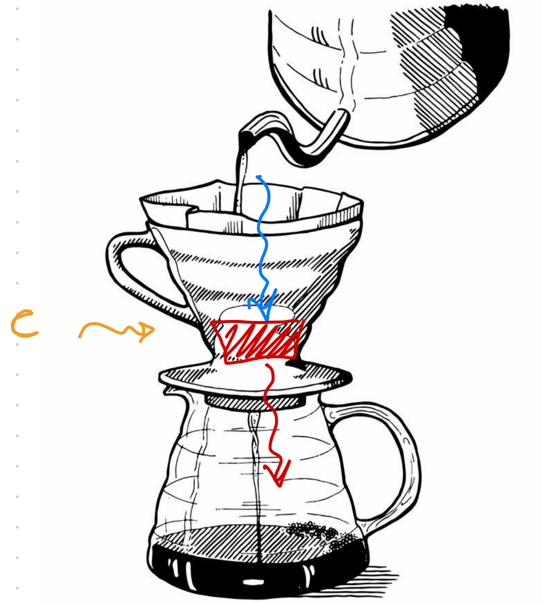
$$\left.\frac{d^2N}{dt^2}\right|_{eq}$$

- $\rightsquigarrow 0$ eq is neutral
- $\rightsquigarrow <0$ eq is stable
- $\rightsquigarrow >0$ eq is unstable

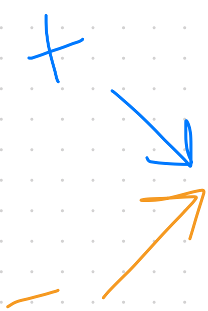


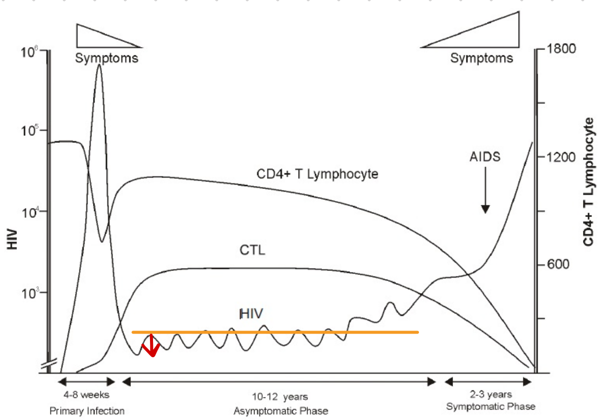
$$\dot{c} = +r - fc$$





$$c = r - fc = 0$$
$$r = fc$$
$$\frac{r}{f} = c$$





۵.۱ مدل‌های پیوسته در یک بعد

کینتیک آنزیم

$$X + A \underset{k_2}{\overset{k_1}{\rightleftharpoons}} 2X$$

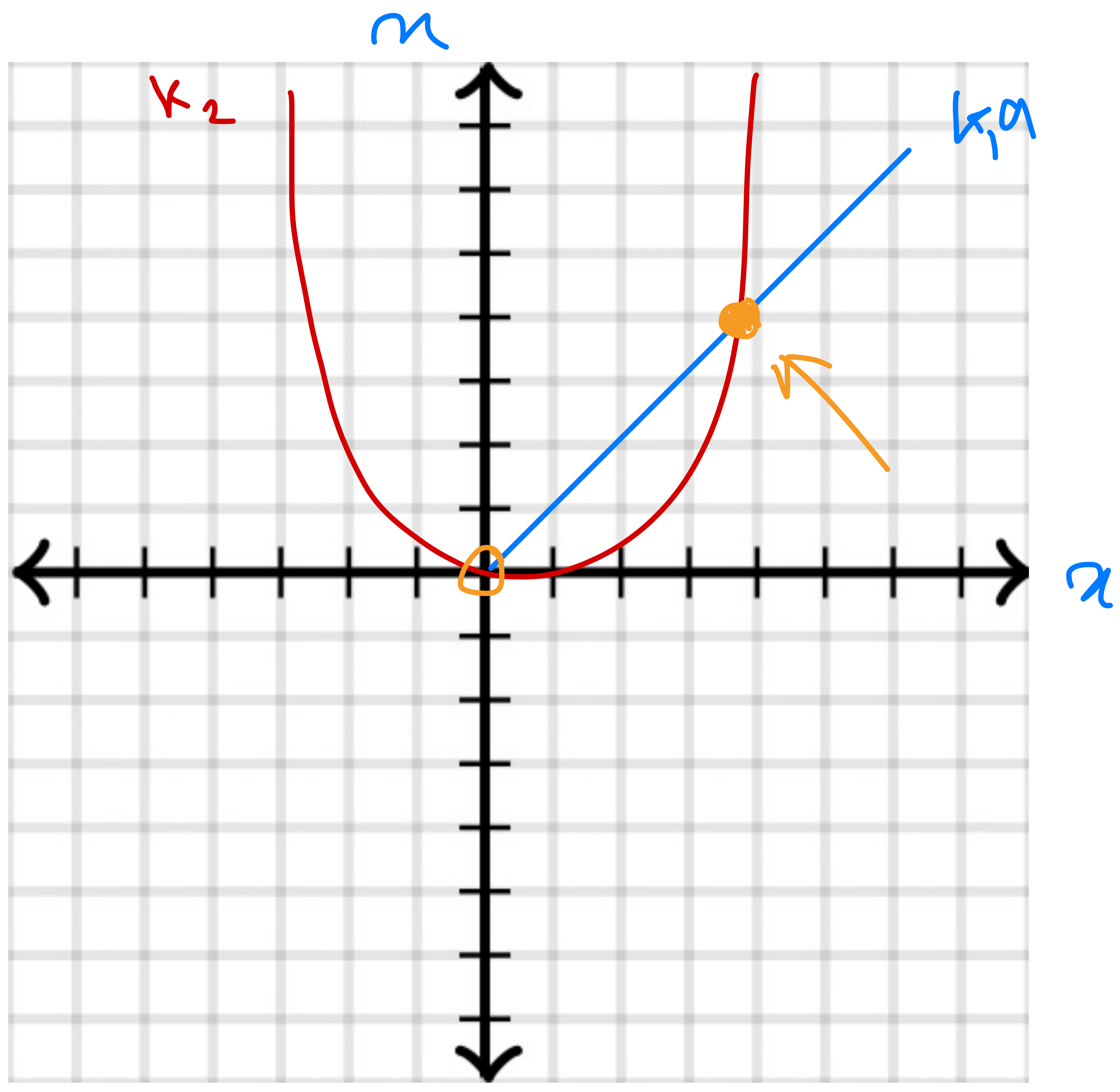
$$a \gg x$$

$$\dot{x} = \underbrace{k_1 a x} - \underbrace{k_2 x^2}$$

$x = 0 \qquad k_1 a x = k_2 x^2$

$- x \qquad k_1 a = k_2 x$

$$\frac{k_1}{k_2} a = x$$



۵.۱ مدل‌های پیوسته در یک بعد

رشد تومور

Gompertz

$-a(1+\ln bS)$

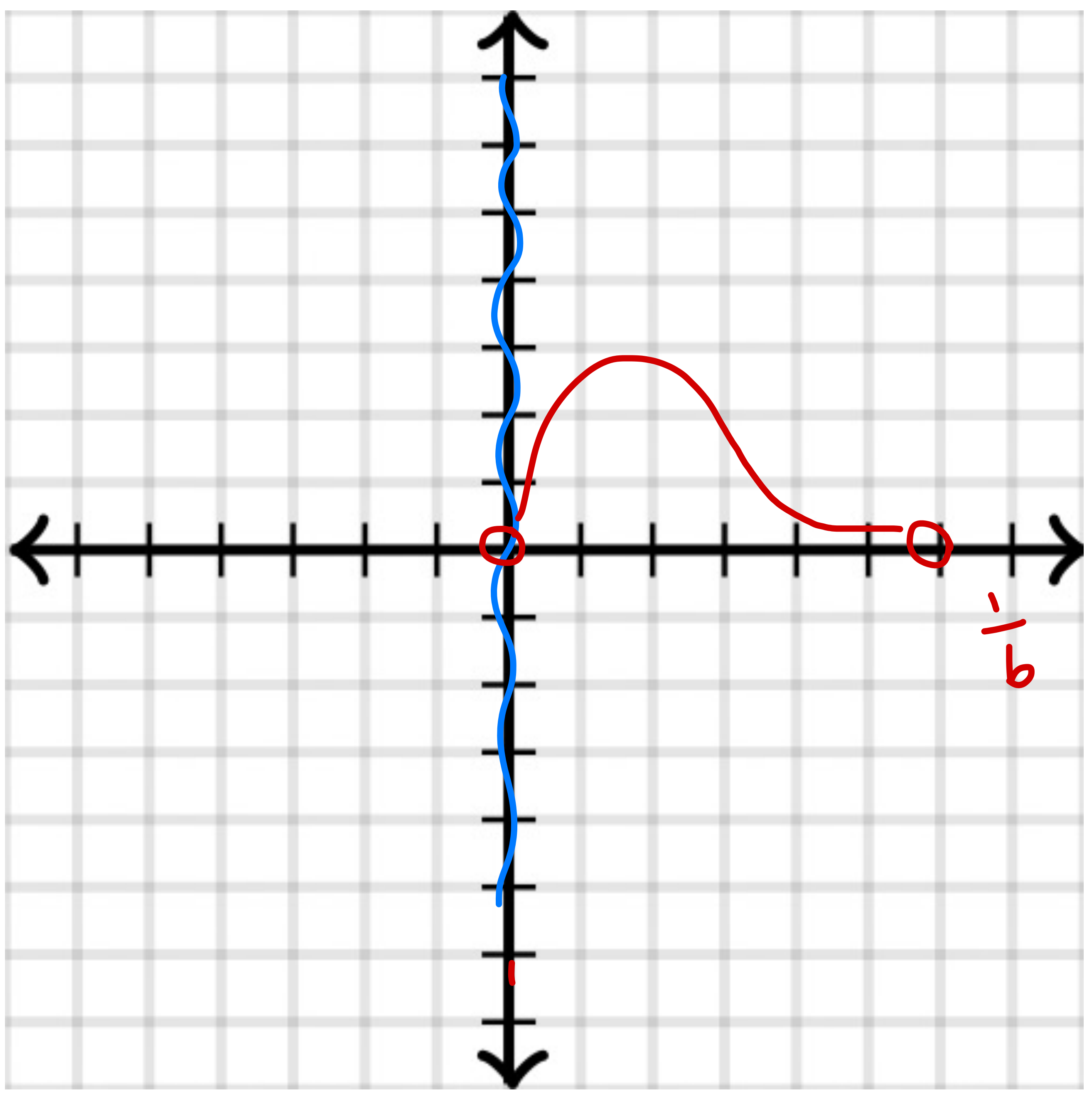
$S = -a\left(\frac{d}{dt} S \ln bS\right) = S\frac{b}{bS} + \ln bS$

$b = 0{,}01$

$$\dot{S} = \boxed{-a}\boxed{S}\boxed{\ln(bS)}$$

$S=0$

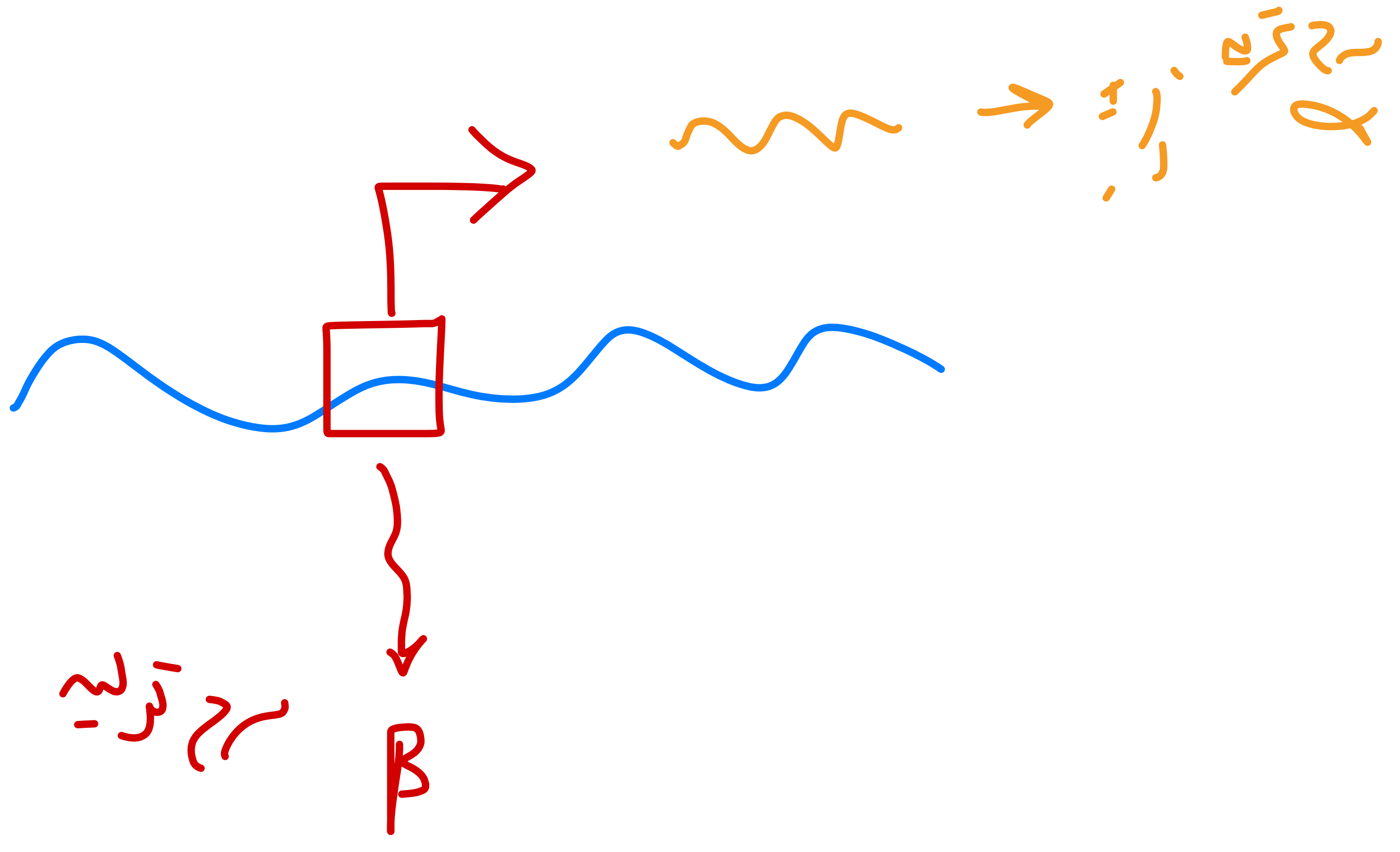
$S=\frac{1}{b}$



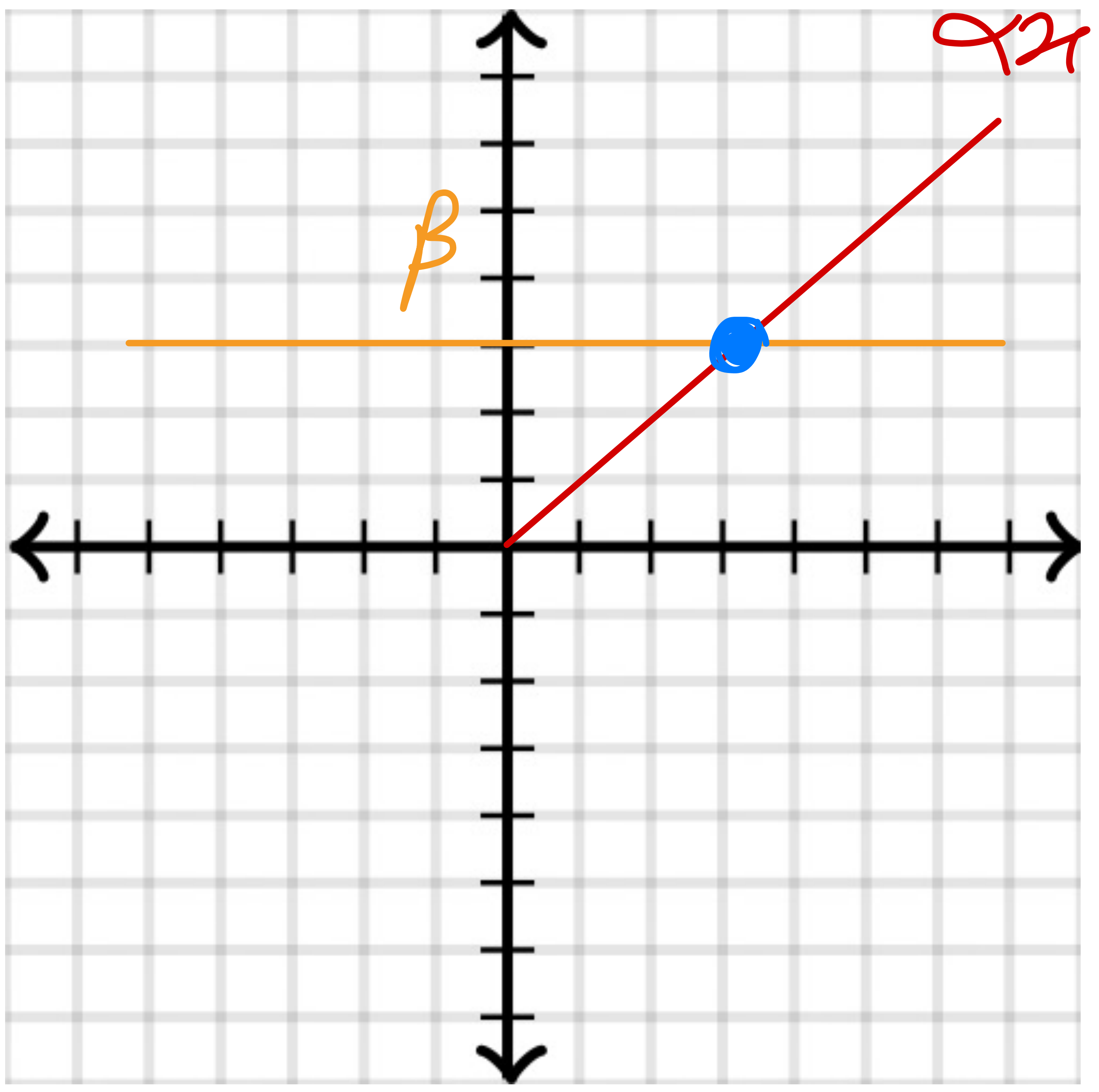
$S \ll \frac{1}{b}$ $\boxed{-a}$ O $-$ $+a$

۵.۱ مدل‌های پیوسته در یک بعد

بیان ژن



$$\dot{x} = \beta - \alpha x$$

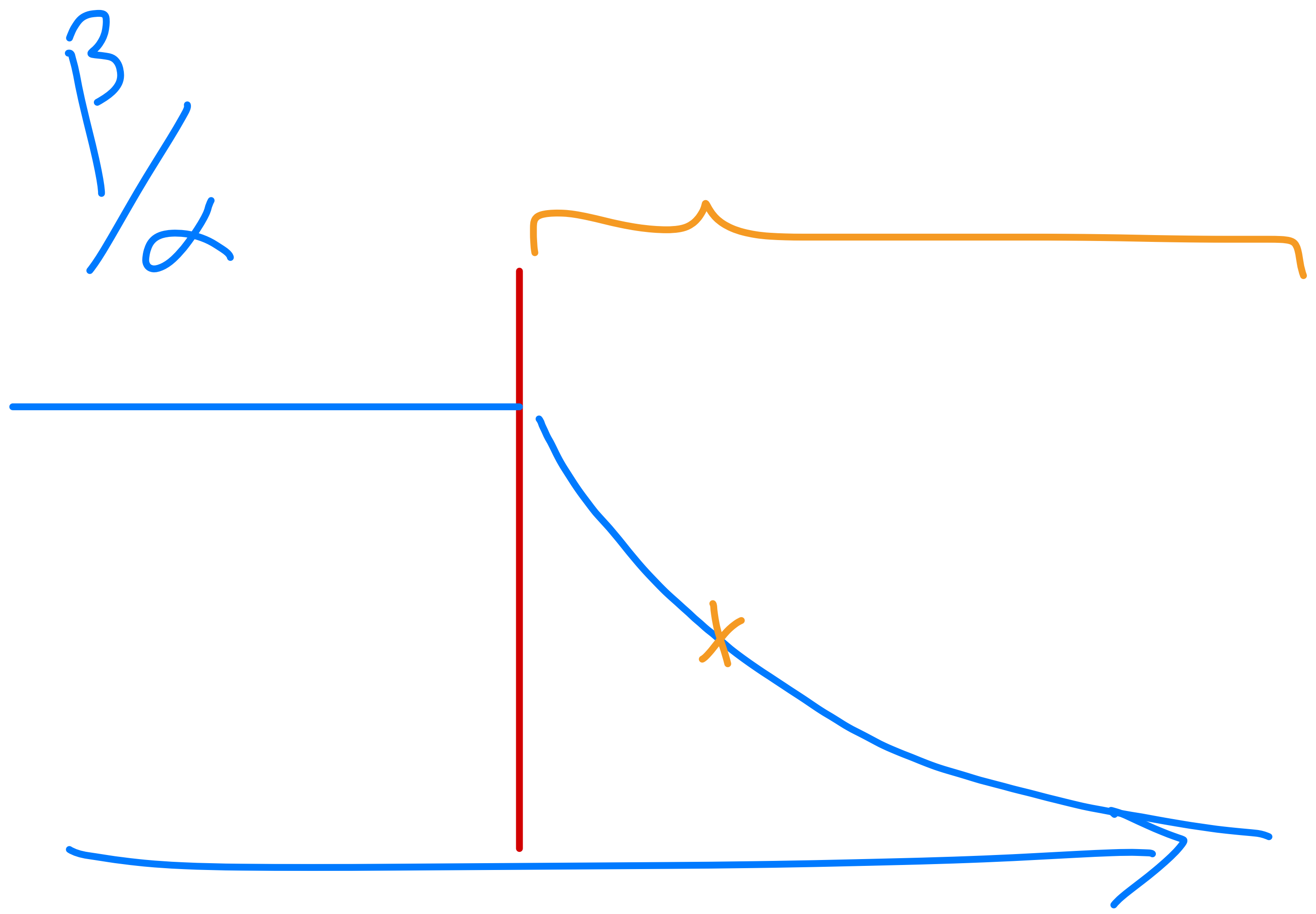


$$x_{eq} = \frac{\beta}{\alpha}$$

$$\dot{x} = -\alpha x$$

$$x_{eq}\, e^{-\alpha t}$$

$$t_{\frac{1}{2}}$$



$$x_{eq}\, e^{-\alpha t_{1/2}} = \frac{1}{2} x_{eq}$$

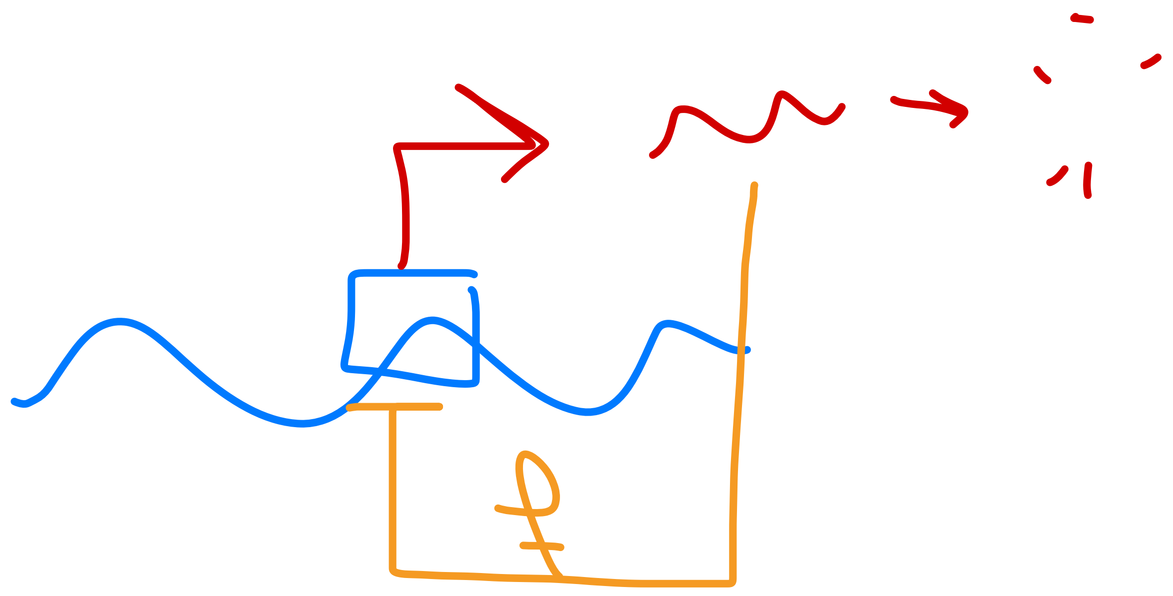
$$e^{-\alpha t_{1/2}} = \frac{1}{2}$$

$$-\alpha t_{\frac{1}{2}} = -\ln 2$$

$$t_{\frac{1}{2}} = \frac{\ln 2}{\alpha}$$

۵.۱ مدل‌های پیوسته در یک بعد

خودتنظیمی منفی در بیان ژن



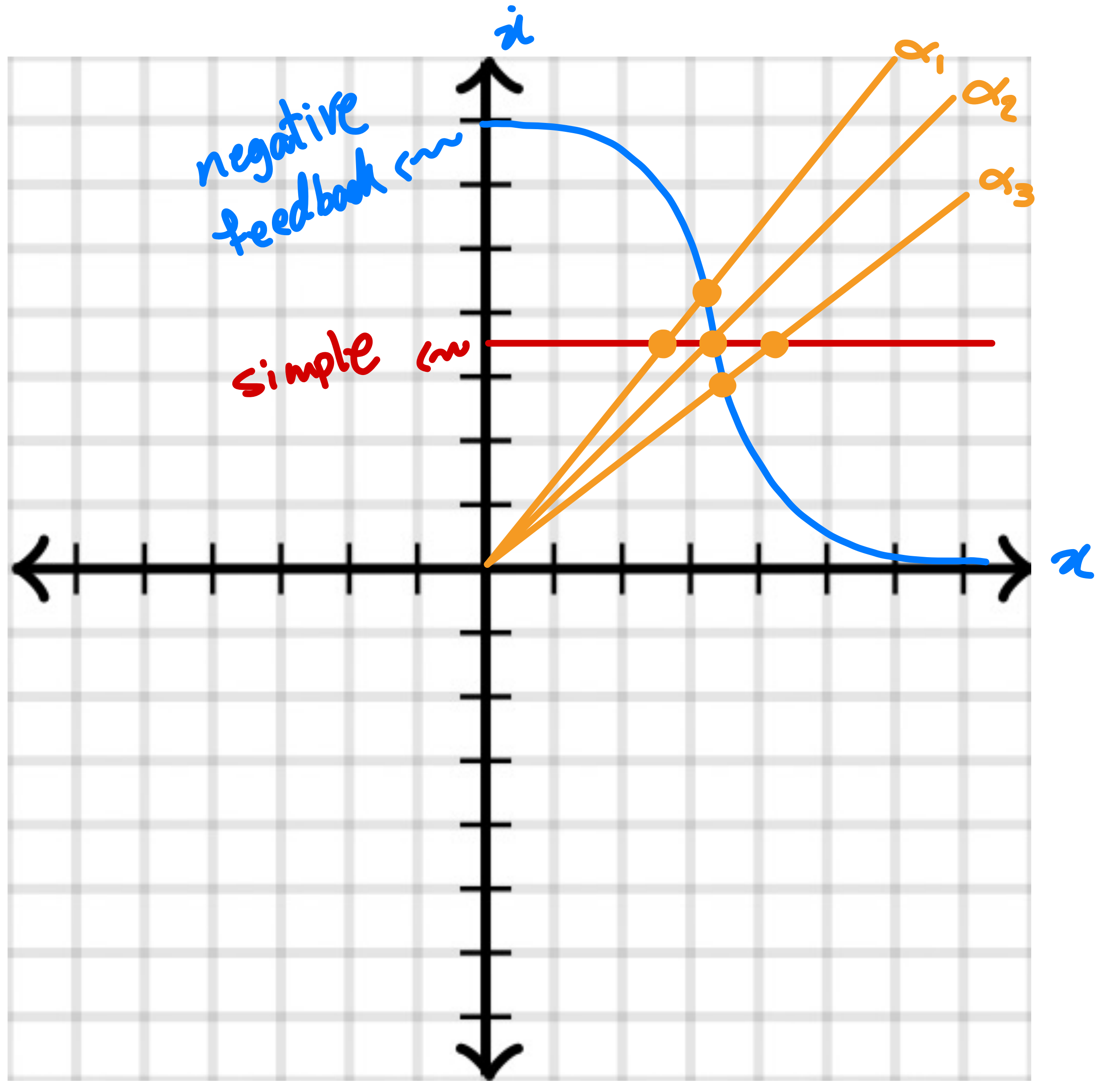
پاسخ دهی سریع به محیط

Robustness

$$\dot{x} = \beta \boxed{\frac{K^n}{K^n + x^n}} - \boxed{\alpha x}$$

$$x = f(x) - \alpha x$$

hill function



صعودی

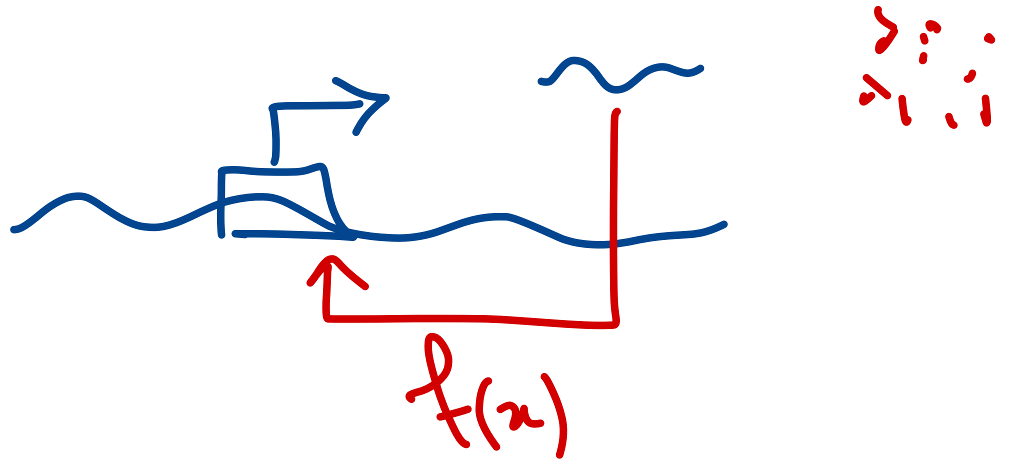
$$\frac{x^n}{x^n + K^n}$$

نزولی

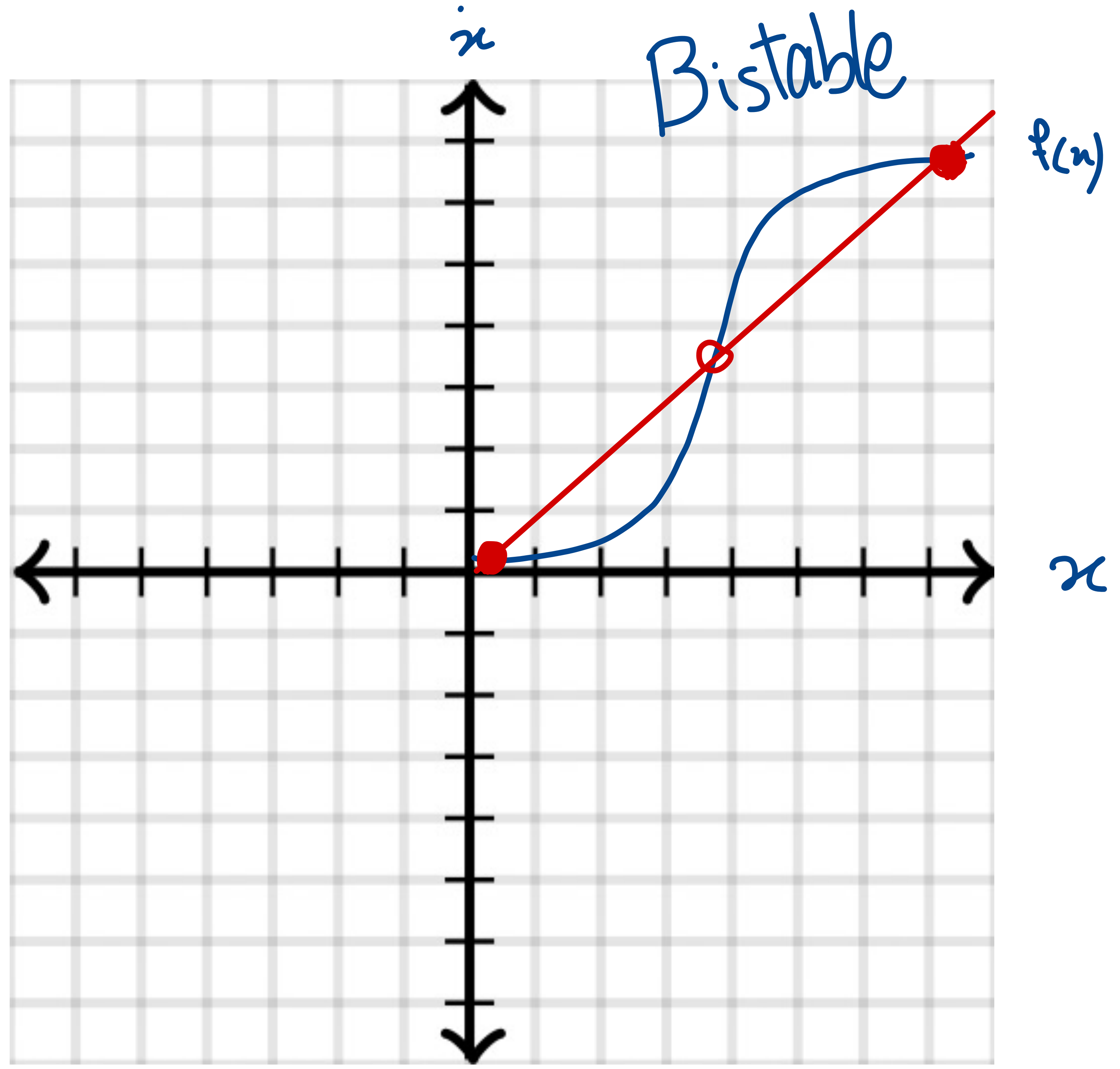
$$1 - \frac{x^n}{x^n + k^n} \qquad \frac{K^n}{x^n + k^n}$$

۵.۱ مدل‌های پیوسته در یک بعد

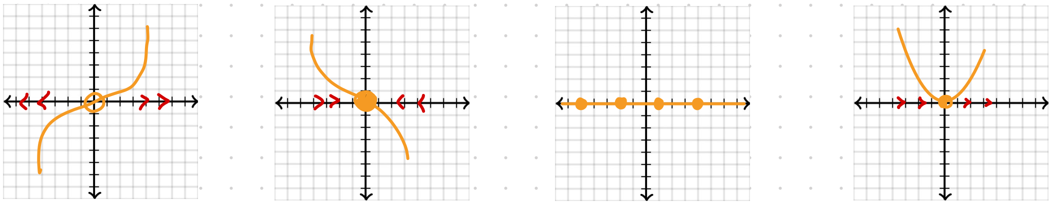
خودتنظیمی مثبت در بیان ژن



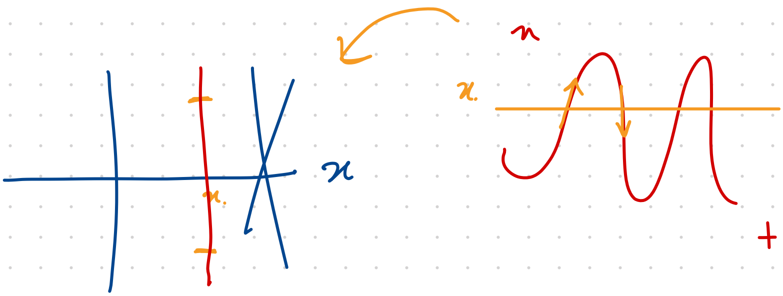
$$\dot{x} = \beta \frac{x^n}{K^n + x^n} - \alpha x$$



$\dot{x} = f(x)$ ⇝ One dimensional



unstable node — stable node — neutral — saddle point



impossibility of oscillation

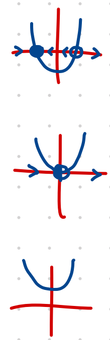
$\dot{x} = f(x, p)$

↳ $\dot{x} = x^2 + p$

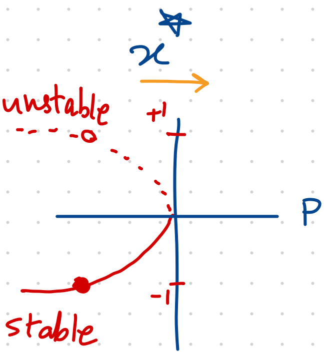
$p < 0$

$p = 0$

$p > 0$



Bifurcation

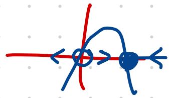


saddle-node bifurcation

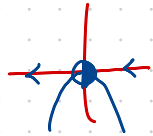
$$\dot{x} = px - x^2$$

$$x(p-x)$$

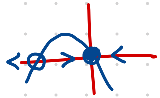
$p > 0$

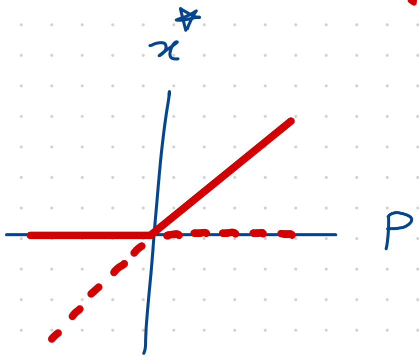


$p = 0$



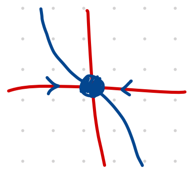
$p < 0$





transcritical Bifurcation

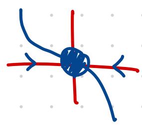
$p < 0$



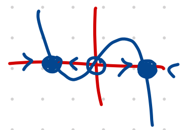
$$\dot{x} = px - x^3$$

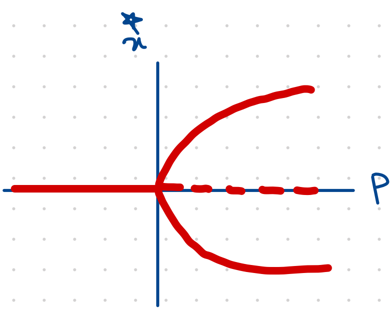
$$x(p - x^2)$$

$p = 0$



$p > 0$



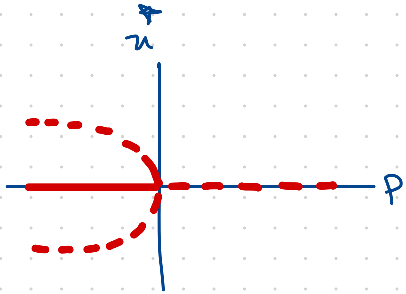


Pitchfork Bifurcation

Supercritical

$\dot{x} = px + x^3$

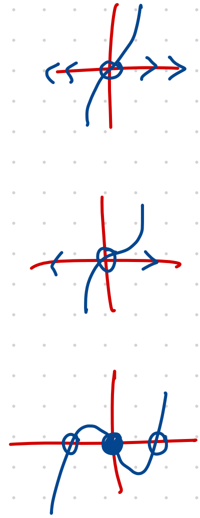
$x(p + x^2)$



$P > 0$

$P = 0$

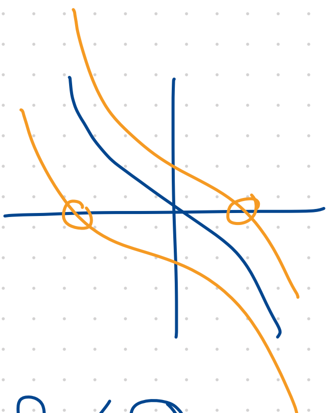
$P < 0$

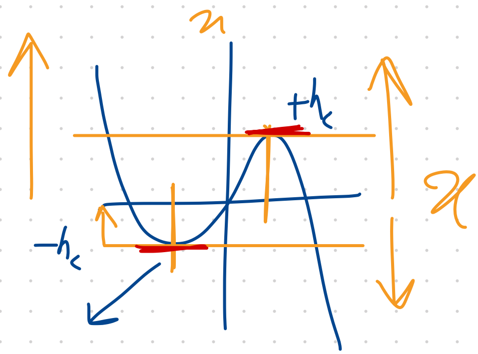


Subcritical pitchfork

$\dot{x} = h + px - x^3$

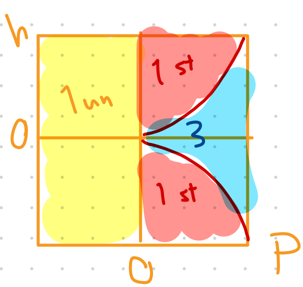
$px - x^3$





$P < 0$

$P > 0$



$\frac{d}{dx}\, px - x^3 = 0$

$P - 3x^2 = 0$

$P = 3x^2$

$x = \sqrt{\frac{P}{3}}$

$px - x^3$

$= P\sqrt{\frac{P}{3}} - \sqrt{\frac{P}{3}}^3$

$\frac{2P}{3}\sqrt{\frac{P}{3}}$

$$\dot N = RN\left(1-\frac{N}{K}\right) - p(N)$$

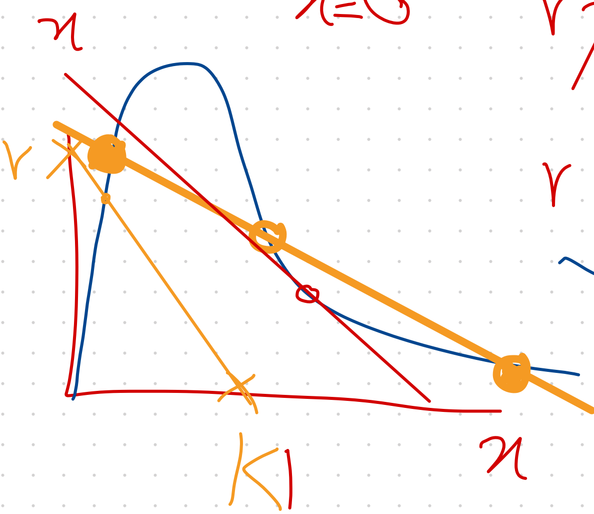
$$B\frac{N^2}{A^2+N^2}$$

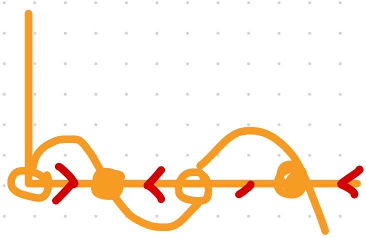
$$\boxed{\dot x = rx\left(1-\frac{x}{k}\right) - \frac{x^2}{1+x^2}}$$

$$x=0$$

$$r\cancel{x}\left(1-\frac{x}{K}\right) = \frac{\cancel{x^2}^{\,x}}{1+x^2}$$

$$r\left(1-\frac{x}{k}\right) = \frac{x}{1+x^2}$$

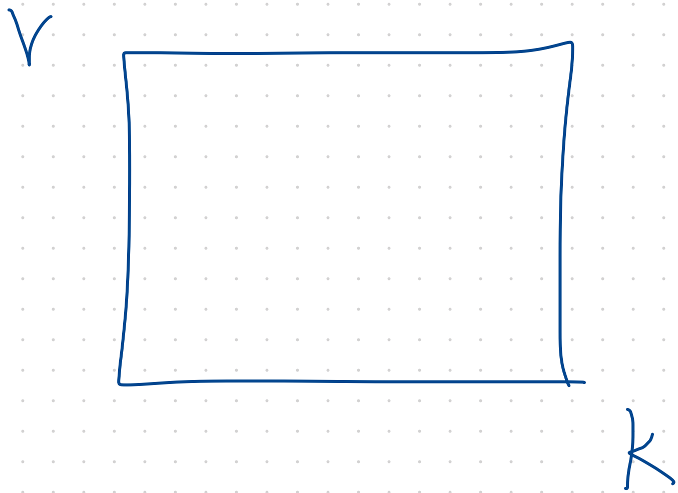




$$r\left(1-\frac{x}{k}\right) = \frac{x}{1+x^2}$$

&

$$\frac{d}{dx}\left[r\left(1-\frac{x}{k}\right)\right] = \frac{d}{dx}\left[\frac{x}{1+x^2}\right]$$



$$\dot{x} = f(x,y)$$
$$\dot{y} = f(x,y)$$

↓

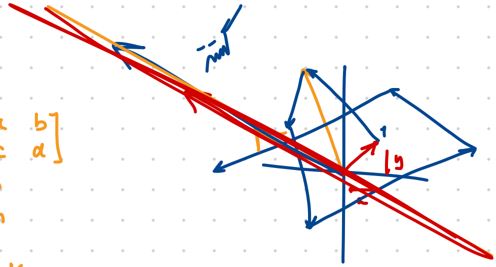
سیستم خطی

$$\dot{x} = ax + by$$
$$\dot{y} = cx + dy$$

$$\rightsquigarrow \quad \dot{X} = AX, \qquad \begin{bmatrix} x \\ y \end{bmatrix} \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

(دوبعدی)

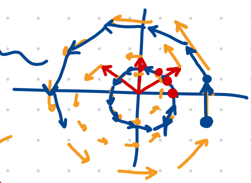
گسسته

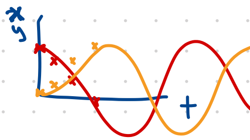


پیوسته

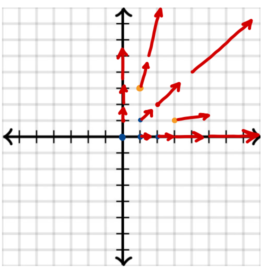
Phase plot

trajectory





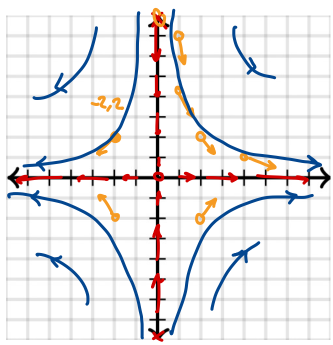
$$\dot{X} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} X$$



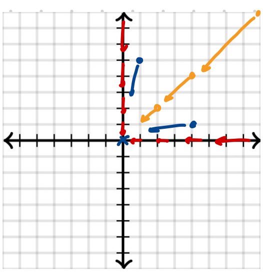
$$\dot{x} = 2x$$
$$\dot{y} = 2y$$

$$\begin{bmatrix} -4 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

$$\dot{X} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} X$$



$$\dot{X} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} X$$



$\dot{x} = f(x,y)$

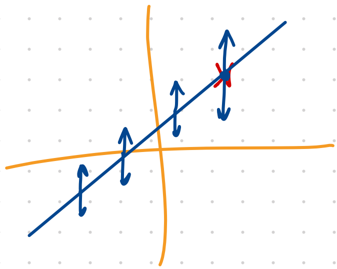
$\dot{y} = f(x,y)$

$\dot{x} \,\&\, \dot{y} = 0 \to$ eq points

$\left.\begin{array}{l}\dot{x} = 0 \\ \dot{y} \neq 0\end{array}\right\}$

nullcline

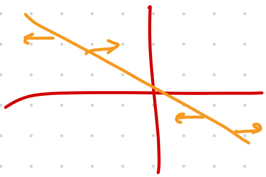
x-nullcline



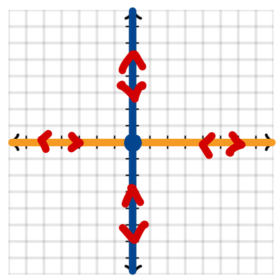
$\dot{x} \neq 0$

$\dot{y} = 0$

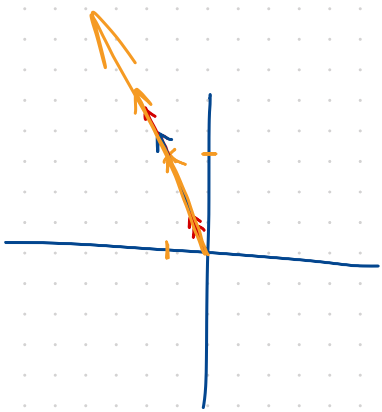
y-nullcline



$\dot{x} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} X$



$\to \begin{array}{l}\dot{x} = 0 \\ \dot{y} = 0\end{array} \to \begin{array}{l}2x + 0y = 0 \\ 2y + 0x = 0\end{array} \to \begin{array}{l}x = 0 \\ y = 0\end{array}$



$$\dot{x} = Ax$$

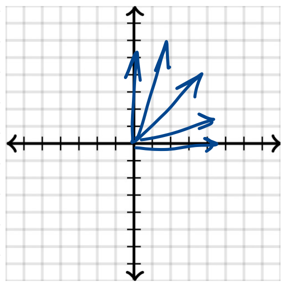
$$Ax = \lambda x$$

$$\det\begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix} = 0 \quad \rightarrow \quad \lambda^2 - T\lambda + \Delta = 0$$

$$(a-\lambda)(d-\lambda) - bc = 0$$

---

1 $\quad x = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} x$



$T = 4$

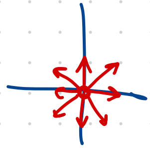
$\Delta = 4$

$$\lambda^2 - 4\lambda + 4 = 0$$

$$(\lambda-2)(\lambda-2) \quad \rightarrow \quad \lambda_1 \, \lambda_2 > 0 \quad \in R$$

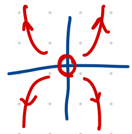
2 $\quad x = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} x$

① $\dot{x} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} x$ $\lambda_1 = \lambda_2 = 2$



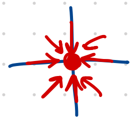
unstable node

② $\dot{x} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} x$ $\lambda_1 = 2$, $\lambda_2 = 3$



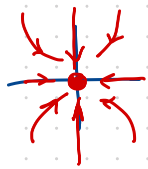
unstable node

③ $\dot{x} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} x$ $\lambda_1 = \lambda_2 = -2$



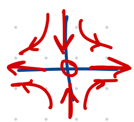
stable node

④ $\dot{x} = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} x$ $\lambda_1 = -2$, $\lambda_2 = -3$



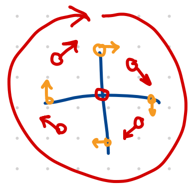
stable node

⑤ $\dot{x} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} x$ $\lambda_1 = 2$, $\lambda_2 = -2$



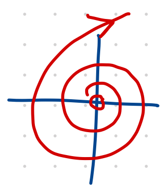
saddle point

⑥ $\dot{x} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} x$ λ_1 $2i$, λ_2 $-2i$



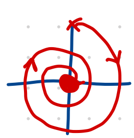
Center

⑦ $\dot{x} = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} x$ $\lambda_1 = 1+2i$, $\lambda_2 = 1-2i$

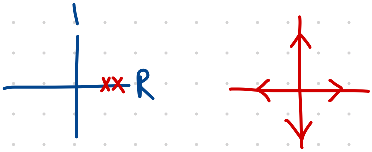


unstable spiral

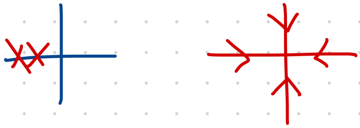
⑧ $\dot{x} = \begin{bmatrix} -1 & 2 \\ -2 & -1 \end{bmatrix} x$ λ_1 $-1+2i$, λ_2 $-1-2i$



Stable spiral



unstable node



stable node

Saddle

center

unstable spiral

stable spiral

$\dot{x} = Ax \quad \rightsquigarrow \quad x_0 e^{Ax}$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

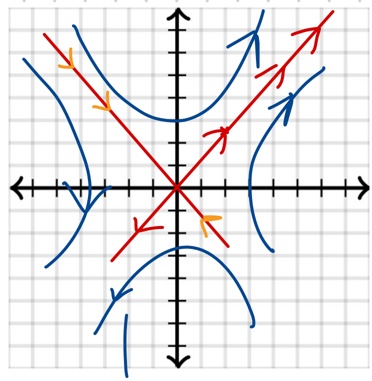
$x = rx \quad \rightsquigarrow \quad c \cdot e^{rx}$

$$\underbrace{1 + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + \frac{1}{2!}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}^2 + \cdots}$$

$$\begin{cases} \dot{x} = f(x,y) \\ \dot{y} = g(x,y) \end{cases} \qquad \dot{\begin{bmatrix} x \\ y \end{bmatrix}} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\lambda^2 - T\lambda + \Delta = 0$$

$2 \qquad \Delta \quad -5$



$$x = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} x$$

$\lambda -$ $\begin{bmatrix} 1 \\ -\frac{\sqrt{6}}{2} \end{bmatrix}$ 'nıl $+$ $\begin{bmatrix} 1 \\ \sqrt{6}/2 \end{bmatrix}$

$Ax = \lambda x$

$\lambda_1 = 1 + \sqrt{6}$

$$\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1+\sqrt{6} & \\ & 1+\sqrt{6} \end{bmatrix} x$$

$\lambda_2 = 1 - \sqrt{6}$

1,22

$$\begin{bmatrix} 1-(1+\sqrt{6}) & 2 \\ 3 & 1-(1+\sqrt{6}) \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \ \end{bmatrix}$$

$\sqrt{6}x + 2y =$

$2y = -\sqrt{6}x$

$$\begin{bmatrix} \sqrt{6} & 2 \\ 3 & \sqrt{6} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \ \end{bmatrix} \qquad y = \frac{-\sqrt{6}}{2}x$$

$$\begin{bmatrix} -\sqrt{6} & 2 \\ 3 & -\sqrt{6} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$-\sqrt{6}x + 2y =$

$2y = \sqrt{6}x$

$y = \frac{\sqrt{6}}{2}x$

$\begin{bmatrix} 1 \\ \sqrt{6}/2 \end{bmatrix}$

$$\begin{bmatrix} -1 & 2 \\ -3 & -2 \end{bmatrix}$$

$T = -3$

$\Delta = (-1)(-2) - (2)(-3)$

8

$$\lambda^2 - (-3)\lambda + 8 = 0$$

$$\lambda^2 + 3\lambda + 8 = 0$$

$$\tfrac{1}{2} \quad -3 \pm \sqrt{9 - 4 \times 8}$$

$$\tfrac{1}{2} \quad -3 \pm \sqrt{-1}\sqrt{23}$$

$$\left\{\begin{array}{ll} \lambda & a+bi \\ v & \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix} i = p + qi \end{array}\right.$$

$$y(t) = e^{at}\left(p \sin bt + q \cos bt\right)$$

$$\left.\begin{array}{ll} \lambda_1 & -1{,}5 + \frac{\sqrt{23}}{2} i \\ \lambda_2 & -1{,}5 - \frac{\sqrt{23}}{2} i \end{array}\right\} \text{stable spiral}$$

$$\begin{bmatrix} 1 \\ -0{,}25 \end{bmatrix} \pm \begin{bmatrix} 0 \\ \frac{\sqrt{23}}{4} \end{bmatrix} i$$

$$\left(-1{,}5 + \frac{\sqrt{23}}{2} i\right) X = \begin{bmatrix} -1 & 2 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -0{,}25 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\sqrt{23}}{4} i \end{bmatrix}$$

$$\begin{bmatrix} -1 - \left(-1{,}5 + \frac{\sqrt{23}}{2} i\right) & 2 \\ -3 & -2 - \left(-1{,}5 + \frac{\sqrt{23}}{2} i\right) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -0{,}25 + \frac{\sqrt{23}}{4} i \end{bmatrix}$$

$$\left(0{,}5 - \frac{\sqrt{23}}{2} i\right) x + 2y = 0$$

$$2y = \left(\frac{\sqrt{23}}{2} i - 0{,}5\right) x$$

$$y = \left(-0{,}25 - \frac{\sqrt{23}}{4} i\right) x$$

$$y = \left(-0{,}25 + \frac{\sqrt{23}}{4} i\right) x$$

$$\begin{cases} \lambda & a+bi \\ v & \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix} i = p + qi \end{cases}$$

$$y(t) = e^{at}\left(p \sin bt + q \cos bt\right)$$

$$p = \begin{bmatrix} 1 \\ -0{,}25 \end{bmatrix} \qquad q = \begin{bmatrix} 0 \\ \frac{\sqrt{23}}{4} \end{bmatrix} \simeq \begin{bmatrix} 0 \\ 1{,}2 \end{bmatrix}$$

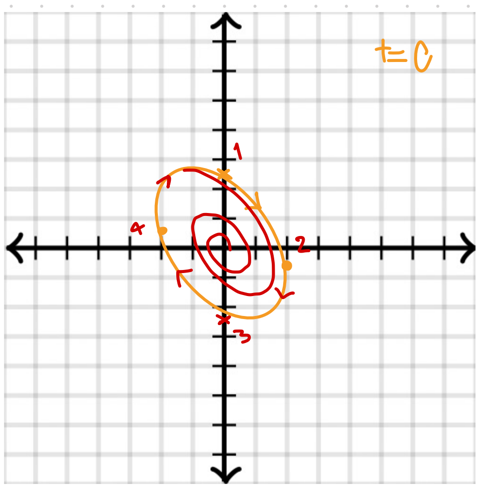
$$a = -1{,}5 \qquad b = \frac{\sqrt{23}}{2}$$

$t = 0$ $\qquad p \overset{0}{\sin 0} + q \overset{1}{\cos 0}$ $\qquad q$ $\qquad t=0$ $\qquad \begin{bmatrix} 0 \\ 1{,}2 \end{bmatrix}$

$bt = \frac{\pi}{2}$ $\qquad p \overset{1}{\sin \frac{\pi}{2}} + q \overset{0}{\cos \frac{\pi}{2}}$ $\qquad p$ $\qquad + \qquad \begin{bmatrix} 1 \\ -0{,}25 \end{bmatrix}$

$bt = \pi$ $\qquad -q$ $\qquad -\begin{bmatrix} 0 \\ 1{,}2 \end{bmatrix}$

$\frac{3}{2}\pi$ $\qquad -p$ $\qquad \begin{bmatrix} -1 \\ 0{,}25 \end{bmatrix}$



$$\lambda^2 - T\lambda + \Delta = 0$$

$$-b \pm \sqrt{b^2 - 4ac}$$

$$T \pm \sqrt{T^2 - 4\Delta} = \lambda$$

$$T^2 - 4\Delta > 0 \qquad T^2 > 4\Delta$$

$$T^2 - 4\Delta = 0$$

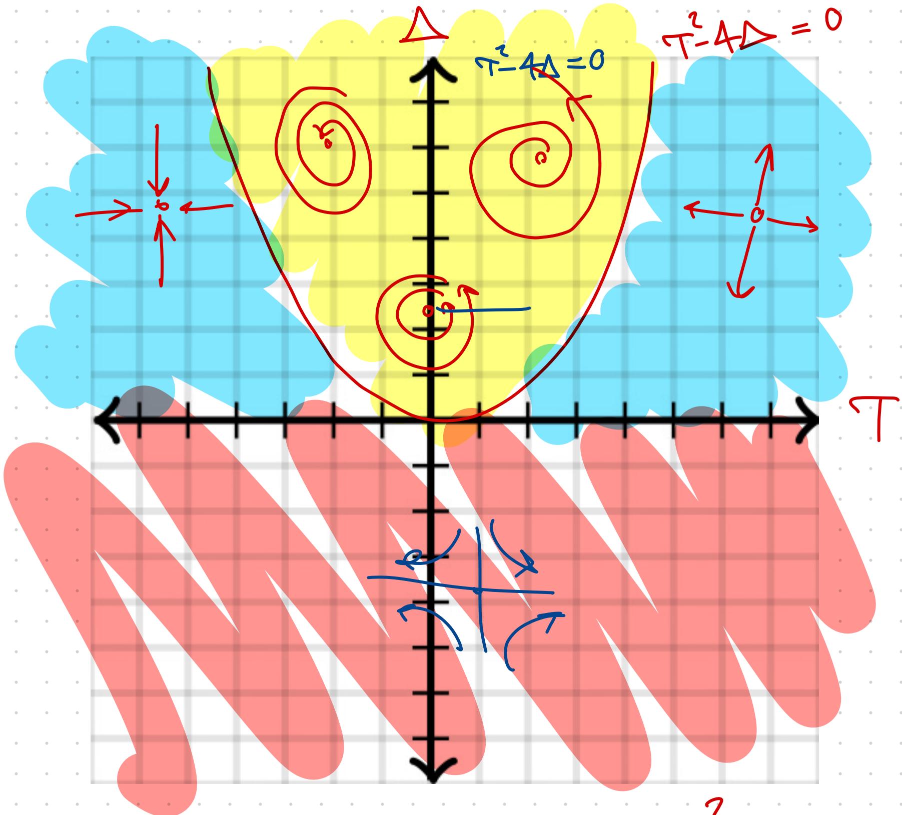
$$T^2 - 4\Delta < 0 \qquad T^2 < 4\Delta$$

$$\left.\begin{array}{l} T^2 < 4\Delta \\ T = 0 \end{array}\right\} \text{Center}$$

$$\left.\begin{array}{l} T^2 < 4\Delta \\ T < 0 \end{array}\right\} \text{stable spiral}$$

$$\left.\begin{array}{l} T^2 < 4\Delta \\ T > 0 \end{array}\right\} \text{unstable spiral}$$

$$\boxed{T^2 - 4\Delta}$$



$$x^2-4y=0$$

$$4y=x^2$$

$$y=\frac{x^2}{4}$$

$$\begin{bmatrix} \dot{G} \\ \dot{I} \end{bmatrix} = \begin{bmatrix} -3 & -4 \\ ,2 & -,8 \end{bmatrix} \begin{bmatrix} G \\ I \end{bmatrix}$$

$\rightarrow \quad \dot{G} = 0$ گلوکز nullcline

سرعت انسولین

$\rightarrow \quad \dot{I} = 0$ انسولین nullcline

$\rightarrow \quad \begin{vmatrix} \dot{G} \\ \dot{I} \end{vmatrix} = 0$ eq

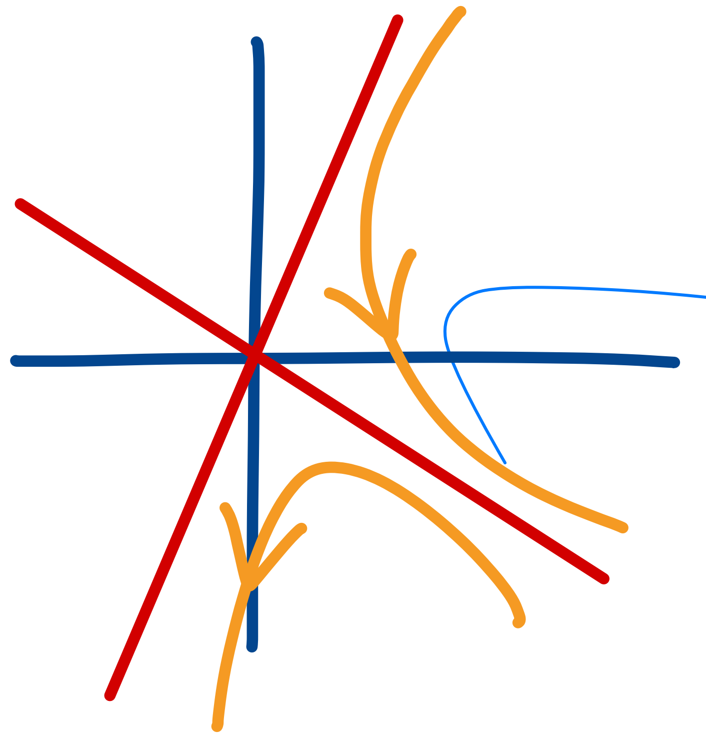
$\rightarrow$ eigenvalue eq نوع تعادل

$\rightarrow$ eigenvector phaseplane محورها

$\rightarrow$ plane تخمین

۵.۱ مدل‌های پیوسته در دو بعد

رشد نمایی دو جمعیت مستقل



$$\dot{x} = r_1 x$$

$$\dot{y} = r_2 y$$

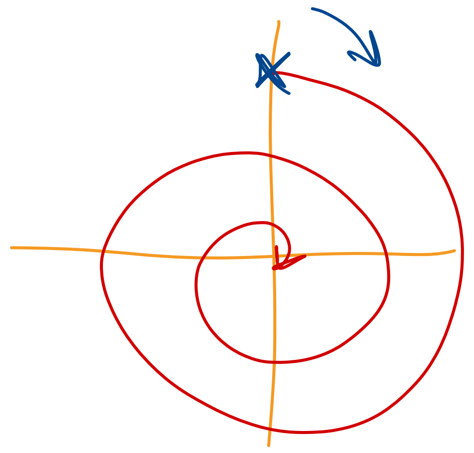
$$\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \overset{A}{\begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix}} \begin{bmatrix} x \\ y \end{bmatrix}$$

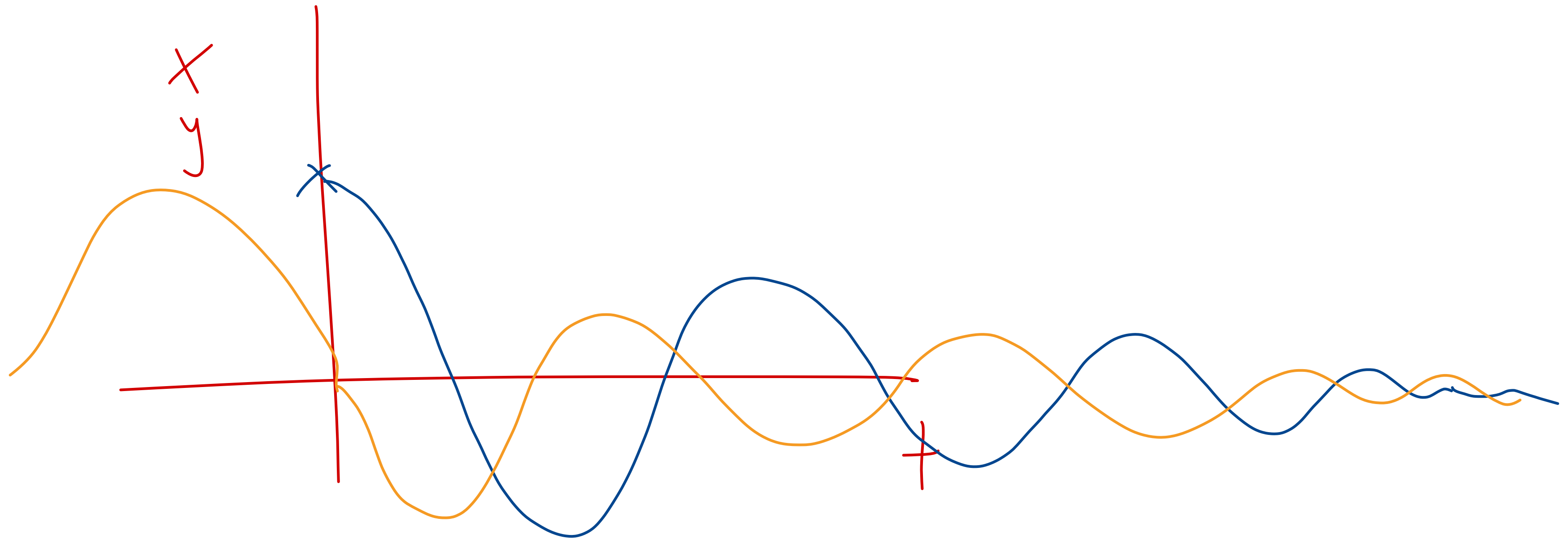
$$\dot{x} = r_1 x + 0y$$

$$\dot{y} = 0y + r_2 y$$

$$\dot{x} = F(x, y)$$

$$\dot{y} = G(x, y)$$





۵.۱ مدل‌های پیوسته در دو بعد

مدار تنظیمی گلوکز-انسولین

$$\dot{G} = -aG - rI$$

$$\dot{I} = sG - dI$$

$\begin{bmatrix} -a & -r \\ s & -d \end{bmatrix}\begin{bmatrix} G \\ I \end{bmatrix}$ $\begin{bmatrix} -3 & -4 \\ ,2 & -,8 \end{bmatrix}$

$\begin{bmatrix} -3 & -4 \\ 0,2 & -0,8 \end{bmatrix}$

$\det\begin{bmatrix} -3-\lambda & -4 \\ 0,2 & -0,8-\lambda \end{bmatrix} = 0$

$\lambda^2 - (T)\lambda + (\Delta) = 0$

$\lambda^2 - (-3 + -0,8)\lambda + 3,2$

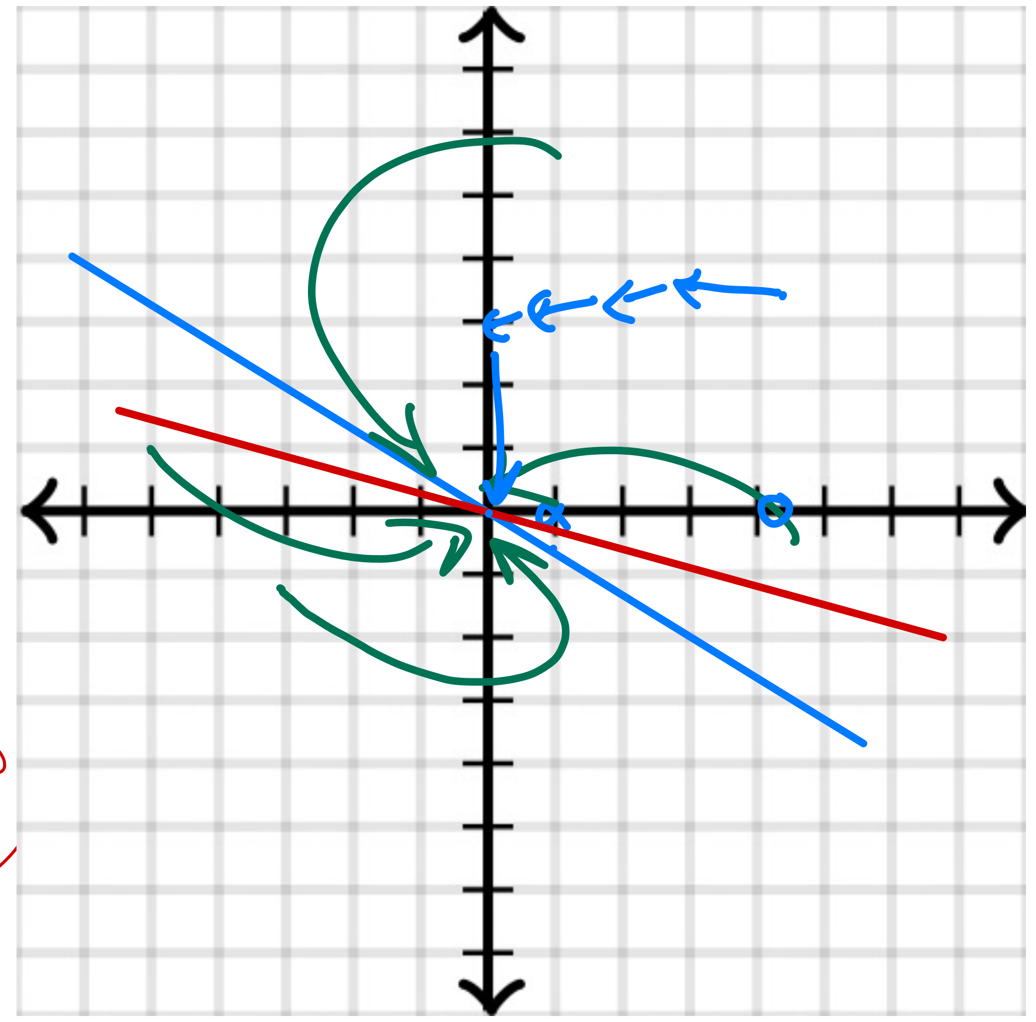
$\boxed{\lambda^2 + 3,8\lambda + 3,2 = 0}$

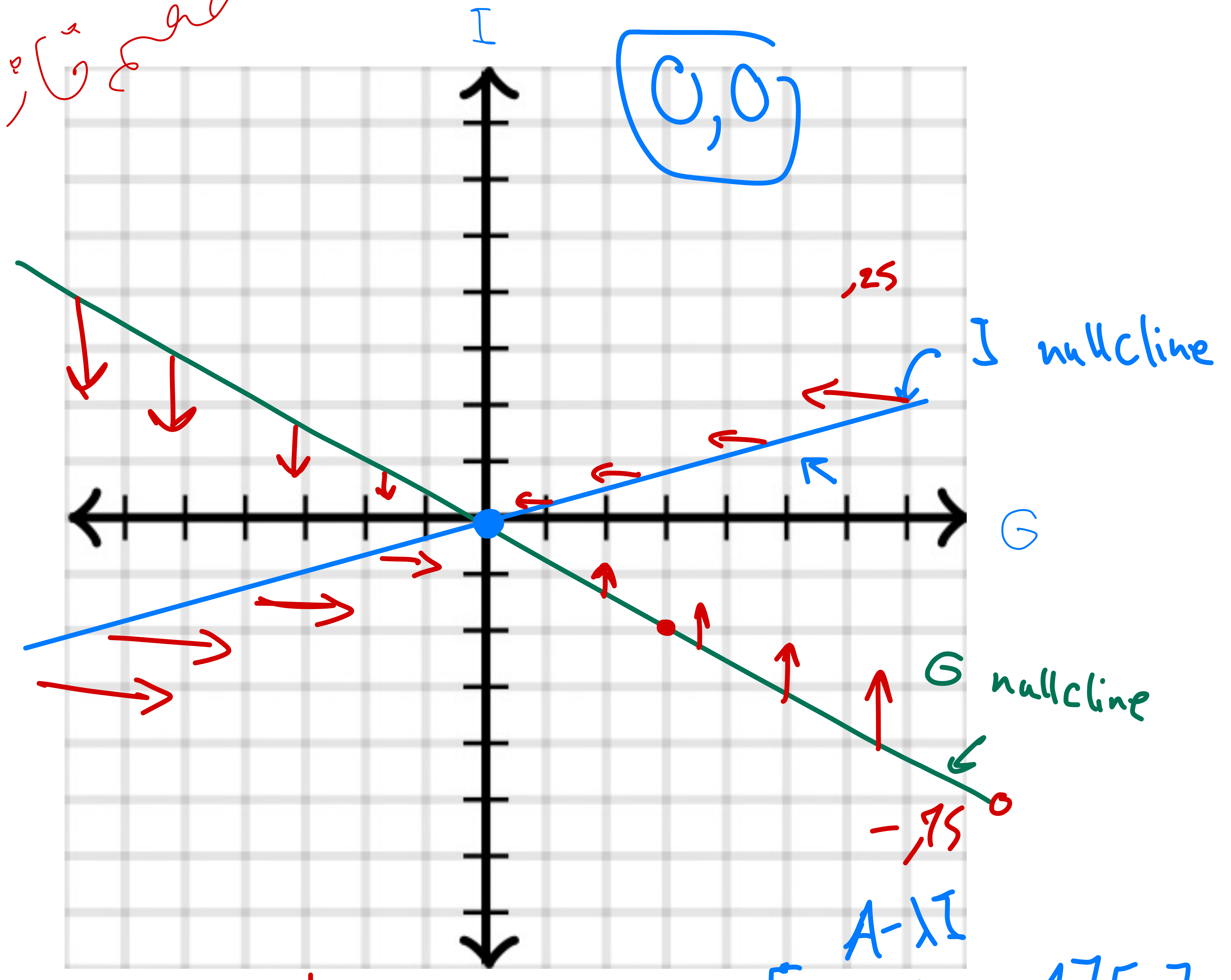
$\frac{1}{2}x - 3,8 \pm \sqrt{3,8^2 - 4 \times 3,2}$

$-3,8 \pm \sqrt{1,64}$

$\frac{1}{2}(-3,8 \pm 1,28)$ $-1,26$, $-2,54$

صفحه فاز





$G=1$

$\dot{G} = -3 \times 1 - 4 \times 1 = -7$

$I=1$

$\dot{I} = 0,2 \times 1 - 0,8 \times 1 = -0,6$

$\begin{bmatrix} \dot{G} \\ \dot{I} \end{bmatrix}_{1,1} = -7, -0,6$

$\dot{I} = 0$ $0,2G - 0,8I = 0$ $0,8I = ,2G$

$I = \frac{1}{4}G$

$A - \lambda I$

$\lambda_1 = -1,26 \begin{bmatrix} : \end{bmatrix} = \begin{bmatrix} -1,74 & -4 \\ 0,2 & ,46 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

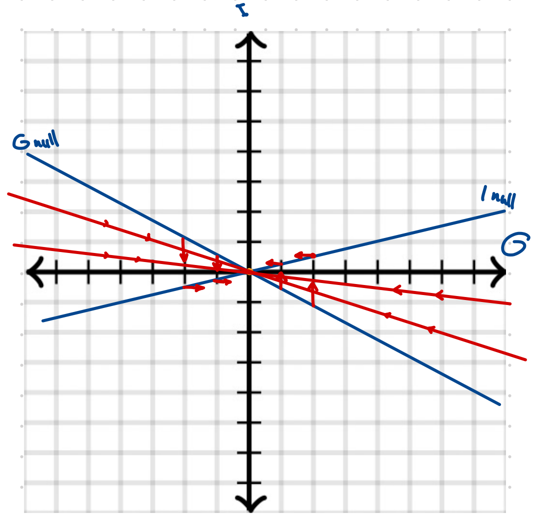
$-1,74x - 4y = 0$

$4y = -1,74x$ $y = -0,435$

$\lambda = -1,26 \rightarrow [1, -,435]$

$\lambda = -2,54$ $[1, -,75]$

$$\begin{bmatrix}\dot G\\ \dot I\end{bmatrix} = \begin{bmatrix}-3 & -4\\ ,2 & -,8\end{bmatrix}\begin{bmatrix}G\\ I\end{bmatrix}$$



① G nullcline

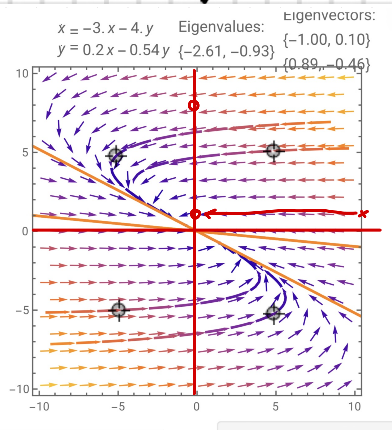
$\dot G = 0 \qquad -3G - 4I = 0$

$4I = -3G \qquad \boxed{I = \frac{-3}{4} G}$

② I nullcline

$\dot I = 0 \qquad ,2G - ,8I = 0$

$,8I = ,2G \qquad \boxed{I = \frac{1}{4} G}$



③ SS

$\dot G = 0,\ \dot I = 0 \rightarrow \boxed{[0,0]}$

④ stability analysis

$T = -3,8 \qquad \Delta = 2,4 - (\cancel{-4} \times ,2) \quad -,8 \quad 3,2$

$\lambda^2 + 3,8\lambda + 3,2 = 0 \qquad \frac{1}{2} \cdot -3,8 \pm \sqrt{3,8^2 - 4 \cdot 3,2}$

$-3,8 - 1,28 \quad -2,5$

$-3,8 + 1,28 \quad -1,25$

$\sqrt{14,44 - 12,8}$

$-3,8 \pm \sqrt{1,64}$

→ stable node

⑤ eigenvector

$\lambda_1 = -2,5$

$$\begin{bmatrix}-3-(-2,5) & -4\\ ,2 & -,8-(-2,5)\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}\cdot\\ \cdot\end{bmatrix}$$

$,5x - 4y = 0$

$4y = ,5x$

$y = \frac{1}{8}x$

$\lambda_2 = -1,25$

$$\begin{bmatrix}-3(-\ -1,25) & -4\\ & \end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}\cdot\\ \cdot\end{bmatrix}$$

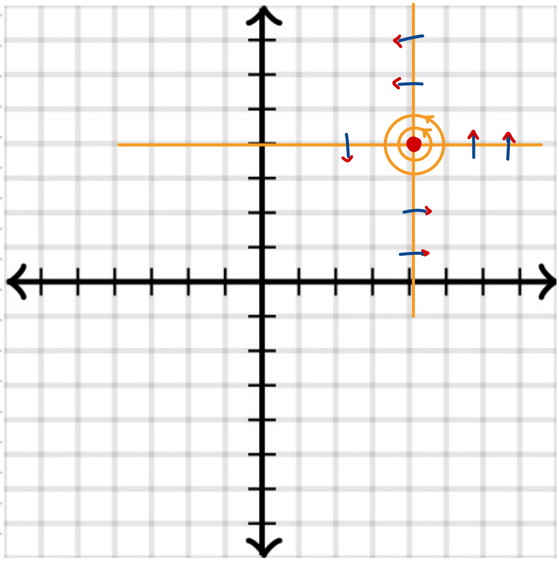
$1,75x - 4y = \cdot$

$4y = 1,75x$

$y = 0,43x$

$$\dot{x} = \alpha x - \beta x y$$

$$\dot{y} = \delta x y - \gamma y$$



① x null $\rightarrow$ $\dot{x} = 0$

$$x - xy = 0$$

$$x = xy$$

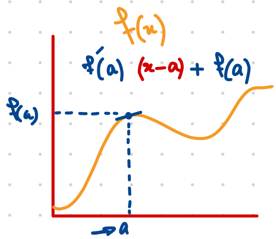
$$y = 1$$

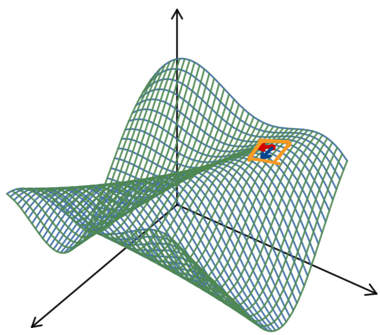
② y null $\rightarrow$ $\dot{y} = 0$

$$xy - y = 0$$

$$xy = y$$

$$x = 1$$





$ax + by + c$

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + f(x, y)$$

$$\frac{\partial f}{\partial y} \qquad \frac{\partial f(x)}{\partial x}$$

مشتق
جزئی

$$\begin{bmatrix} \dot{x} \\ y \end{bmatrix}$$

$\dot{x} = f(x,y)$ $\quad f(x^*) = 0$

$\dot{y} = g(x,y)$ $\quad g(x^*) = 0$

$\rightsquigarrow \quad x + \quad y + \quad \cancel{\overset{\circ}{f(x) + g(x)}}$

jacobian

$\dot{x} = f(x,y)$

$\dot{y} = g(x,y)$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \approx \begin{bmatrix} \frac{\partial f}{\partial x}\Big|_{x^*} & \frac{\partial f}{\partial y}\Big|_{x^*} \\ \frac{\partial g}{\partial x}\Big|_{x^*} & \frac{\partial g}{\partial y}\Big|_{x^*} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$\dot{x} = x - xy$

$\dot{y} = \boxed{xy - y}$

eq = [1,1]

$\frac{\partial f}{\partial x} \quad 1 - y$

$\frac{\partial f}{\partial y} \quad -x$

$\frac{\partial g}{\partial x} \quad y$

$\frac{\partial g}{\partial y} \quad x - 1$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 1-y & -x \\ y & x-1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad 1,1$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$\tau = 0 \quad \Delta = 1$

$\lambda^2 + 1 = 0 \quad \boxed{\lambda = \pm i}$

۵.۱ مدل‌های پیوسته در دو بعد

رقابت جمعیت‌ها

$$\dot{x} = rx[1 - (x + \alpha y)]$$

$$\dot{y} = ry[1 - (y + \beta x)]$$

$\lambda_1 \ \lambda_2$

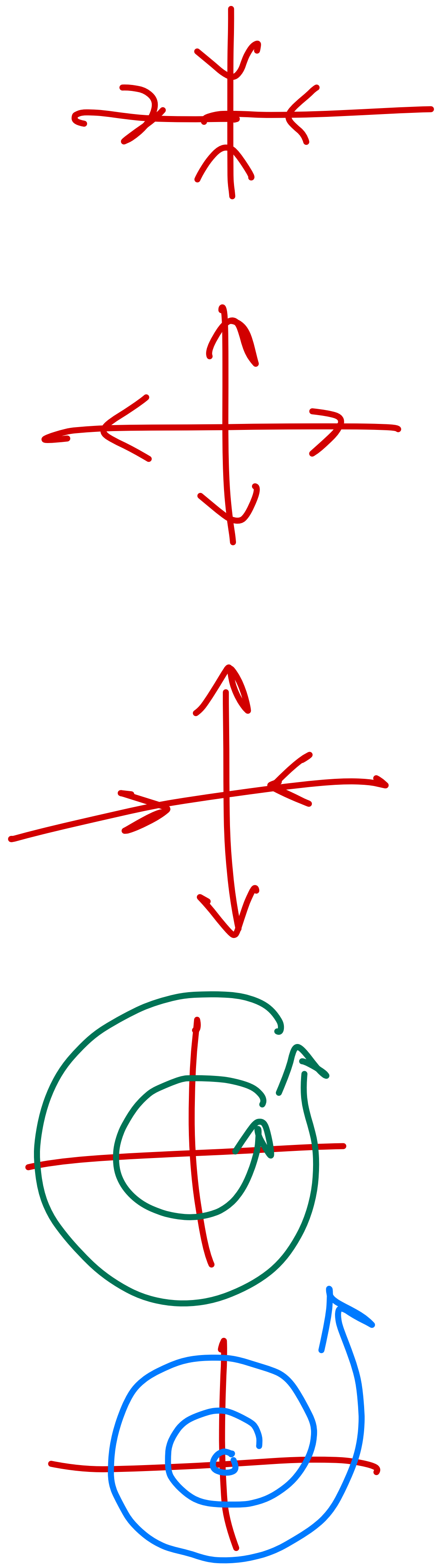
$-a \ -b$

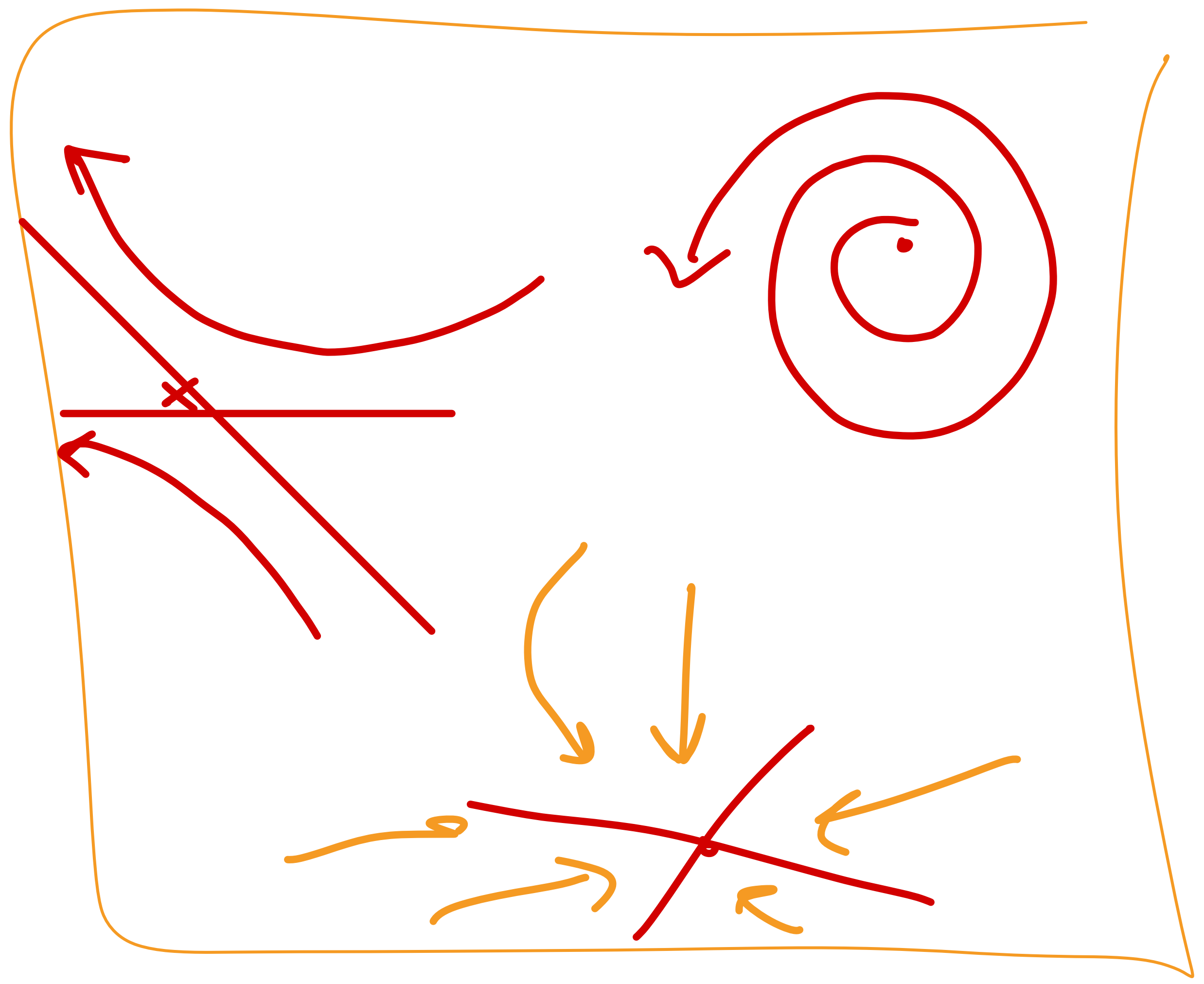
$+a \ +b$

$-a \ +b$

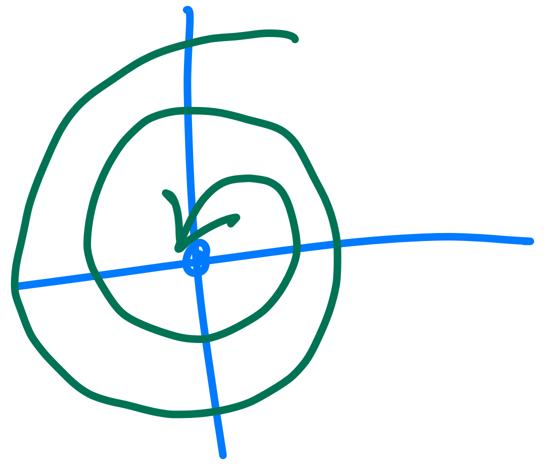
$\pm i$

$+a + i$





$-a + i$



۵.۱ مدل‌های پیوسته در دو بعد

شکار و شکارچی

$$\dot{R} = \alpha R - \beta RF$$

$$\dot{F} = \delta RF - \gamma F$$