

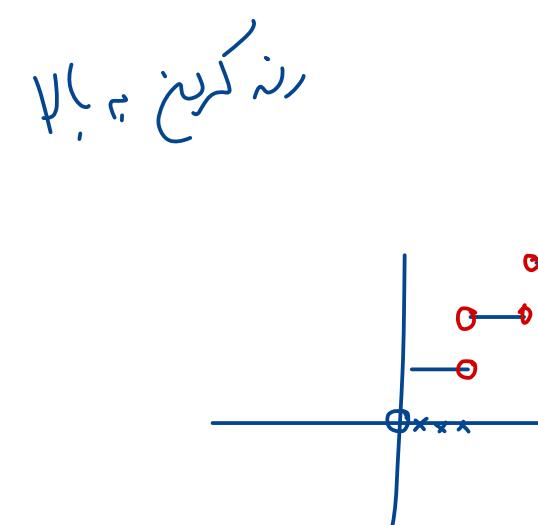
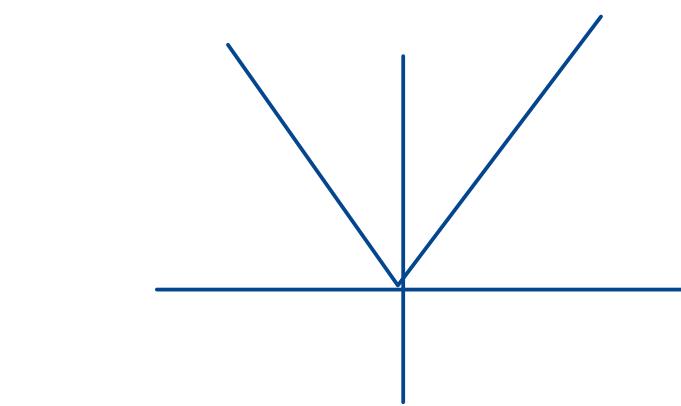
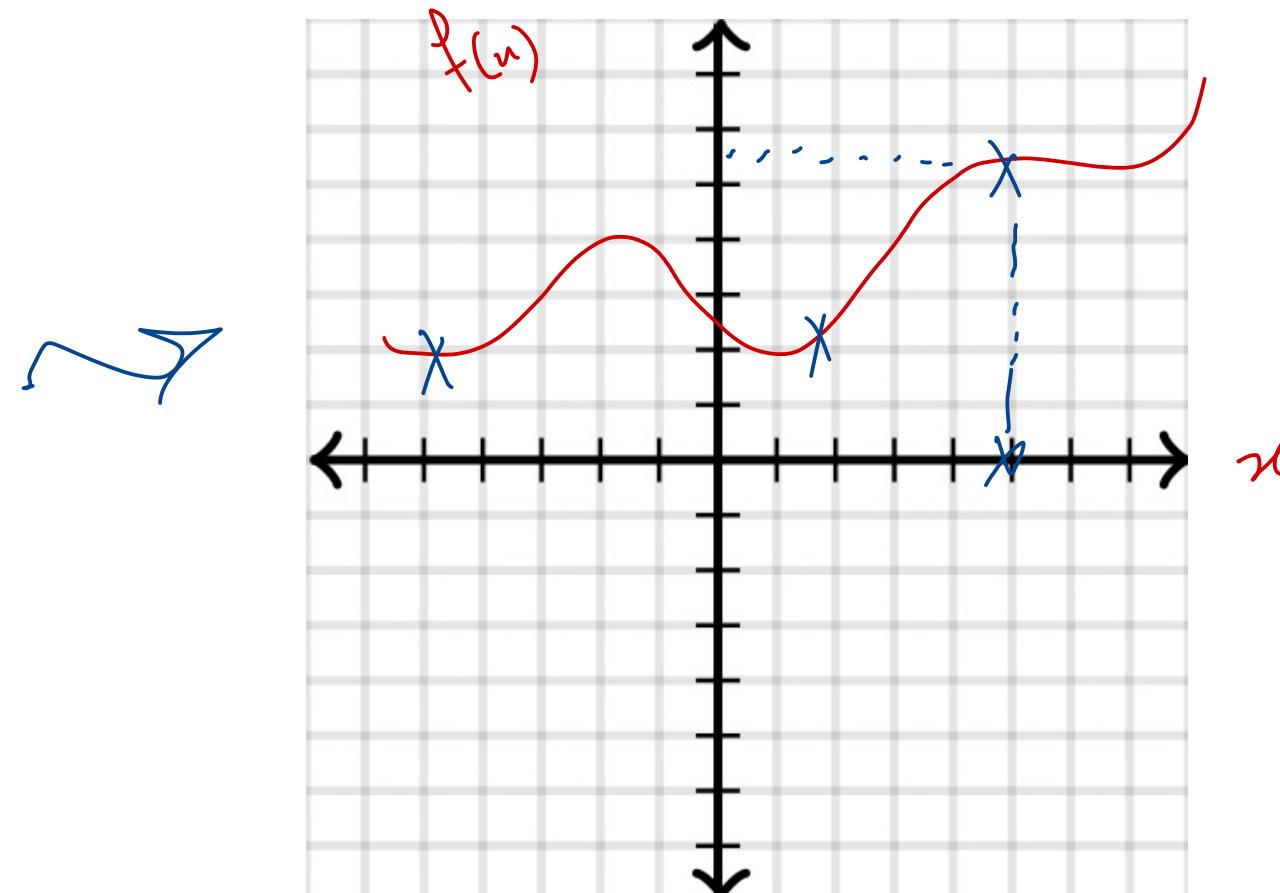
٤. حسابان

۴. حسابان

$$\forall \epsilon > 0 \exists \delta > 0$$

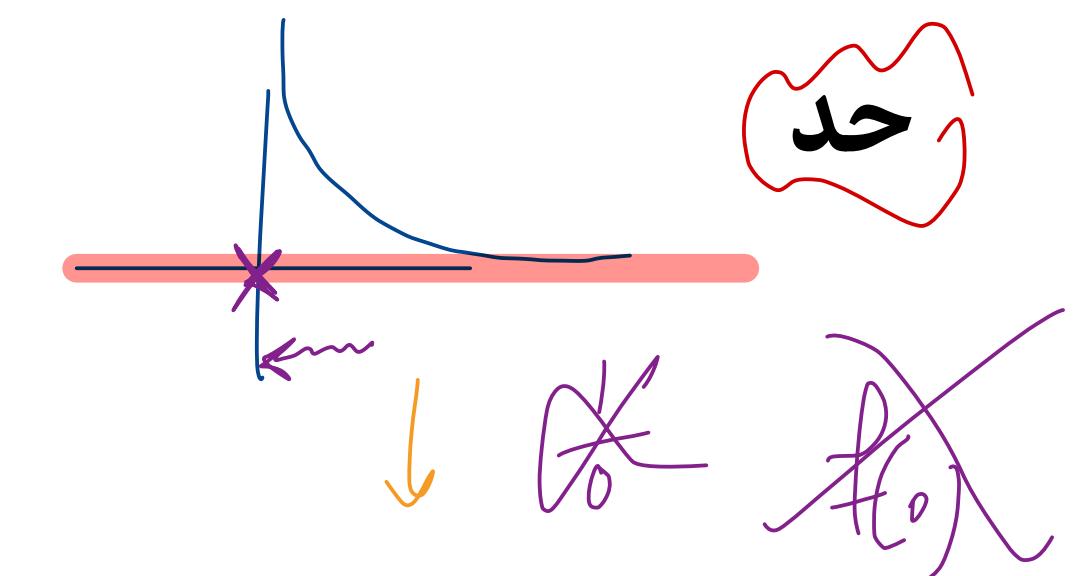
$$0 < |x - a| < \delta \implies |f(x) - L| < \epsilon$$

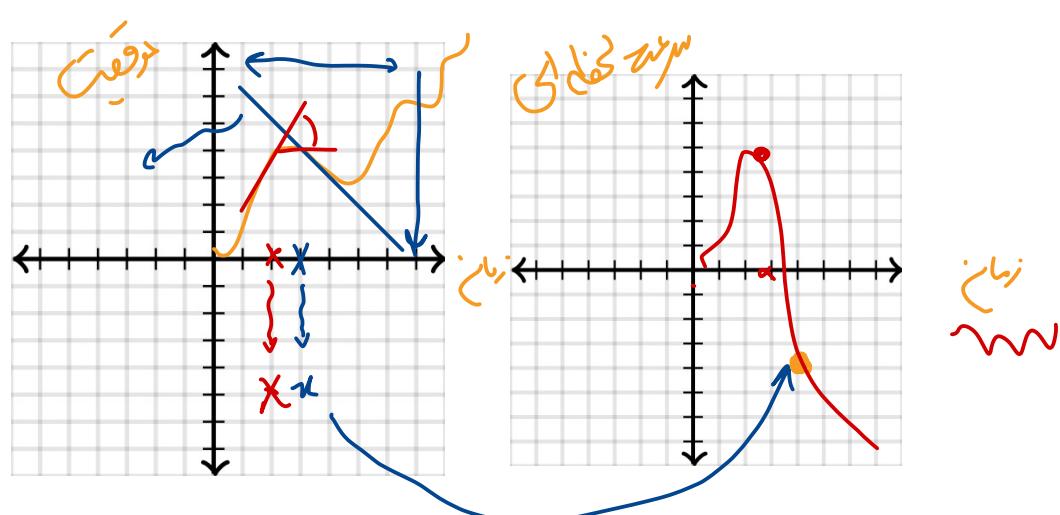
$$\lim_{x \rightarrow a} f(x) = L$$



$$\lim_{x \rightarrow a} f(x) = L$$

$$\lim_{x \rightarrow 0} \frac{1}{x} = \infty$$

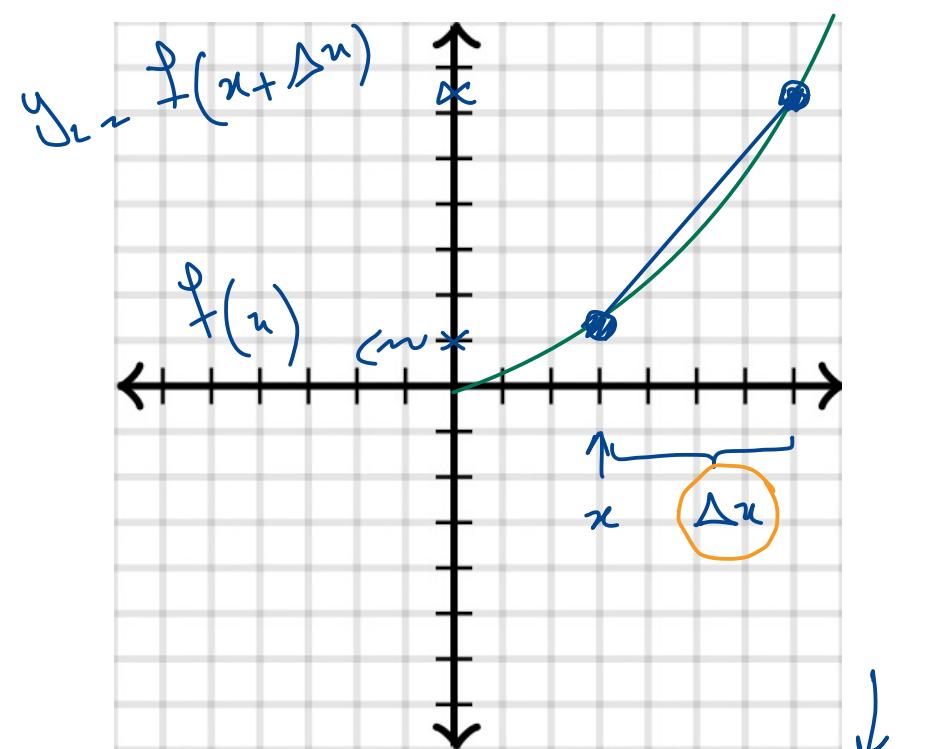




٤. حسابان Isaac Newton (1643-1727)

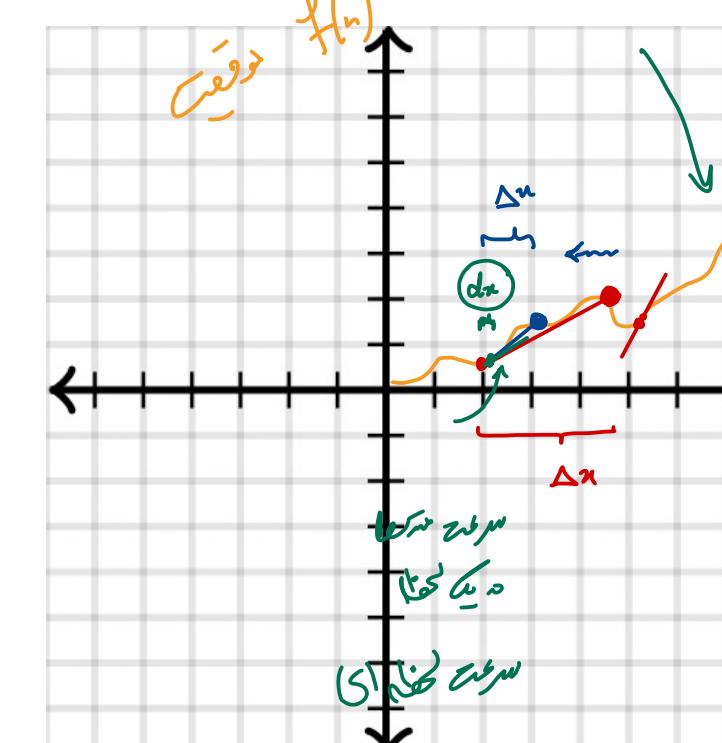
تابع $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$f''(x) = \lim_{h \rightarrow 0} \frac{f'(x+h) - f'(x)}{h}$$

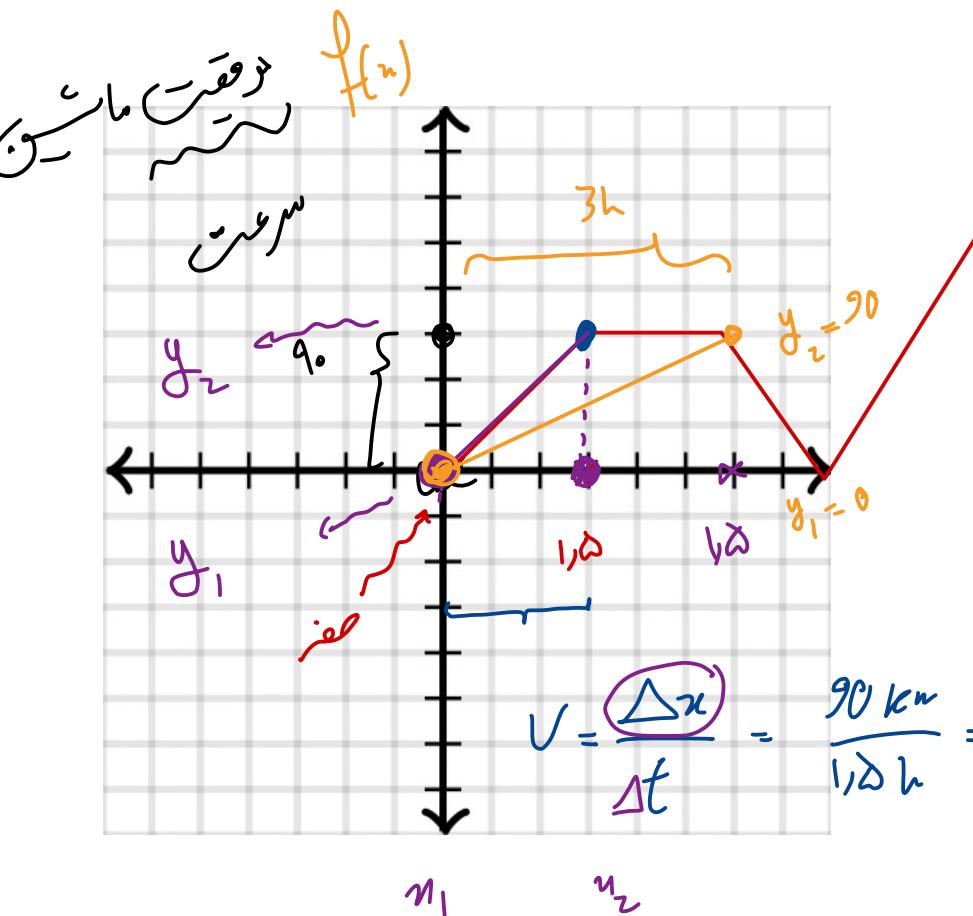


$$\frac{\Delta f}{\Delta x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$



$$\frac{\Delta f}{\Delta x} = \frac{\Delta y}{\Delta x}$$



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مشتق

Isaac Newton (1643-1727)

مشتق مارش

مشتق

٤. حسابان

Gottfried Wilhelm Leibniz (1646-1716) مشتق

$$\frac{df}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{\Delta f}{\Delta x} \xrightarrow{\Delta x \rightarrow 0} \frac{df}{dx}$$

$$\frac{d^2f}{dx^2} = \lim_{h \rightarrow 0} \frac{\frac{df}{dx} \Big|_{x+h} - \frac{df}{dx} \Big|_x}{h}$$

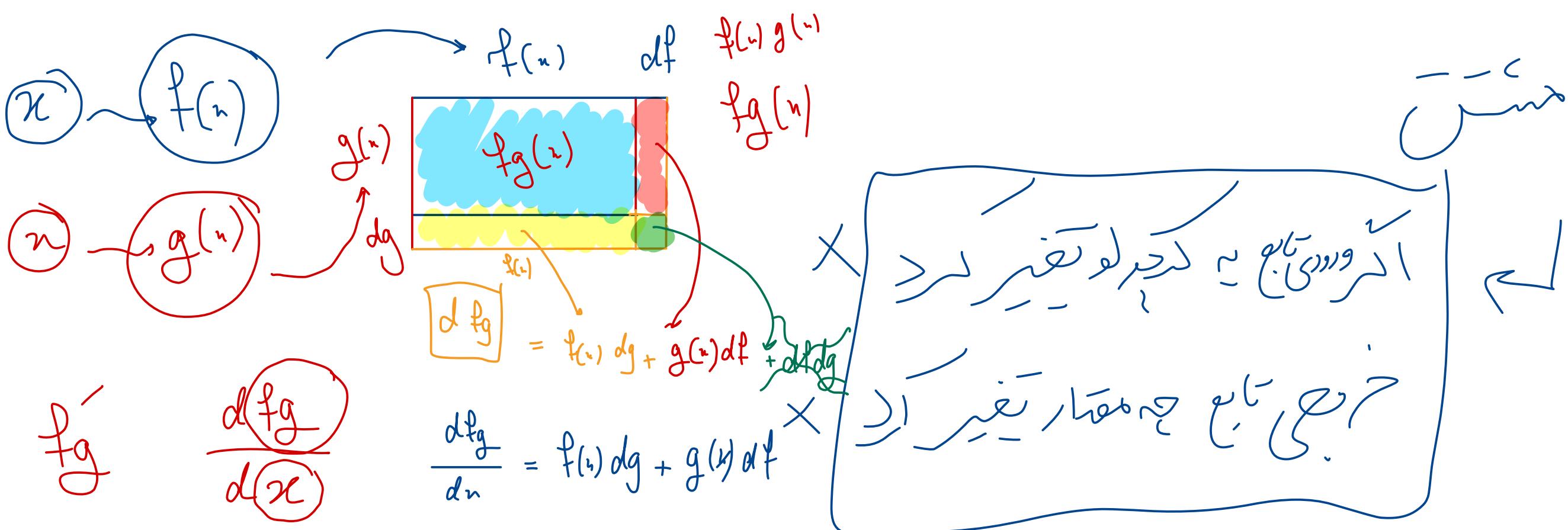
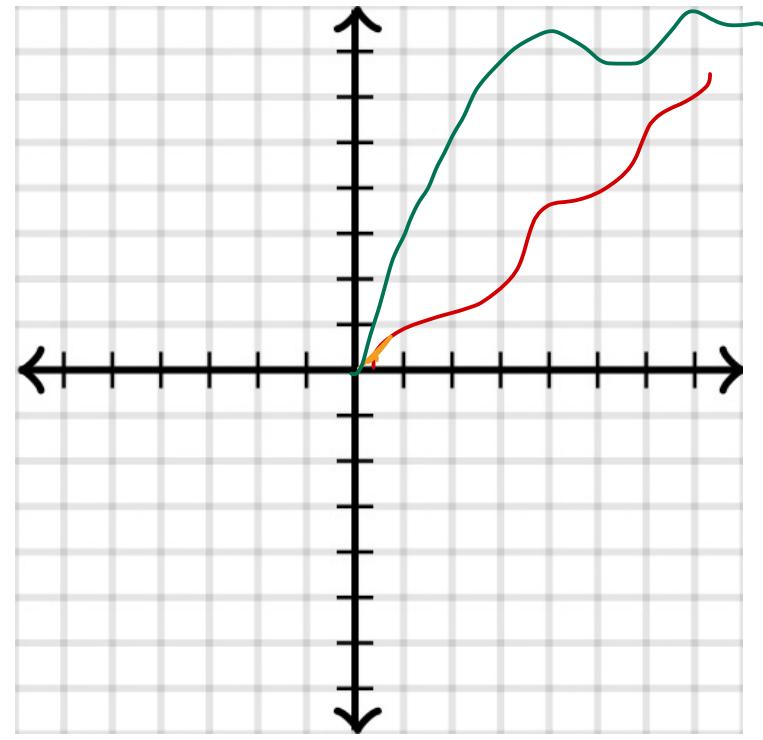
$$\frac{df}{du} \Big|_{u=2} = f''(2)$$

$$\frac{df}{du}$$



۴. حسابان

مشتق



$$(cf)' = cf'$$

$$(f+g)' = f' + g'$$

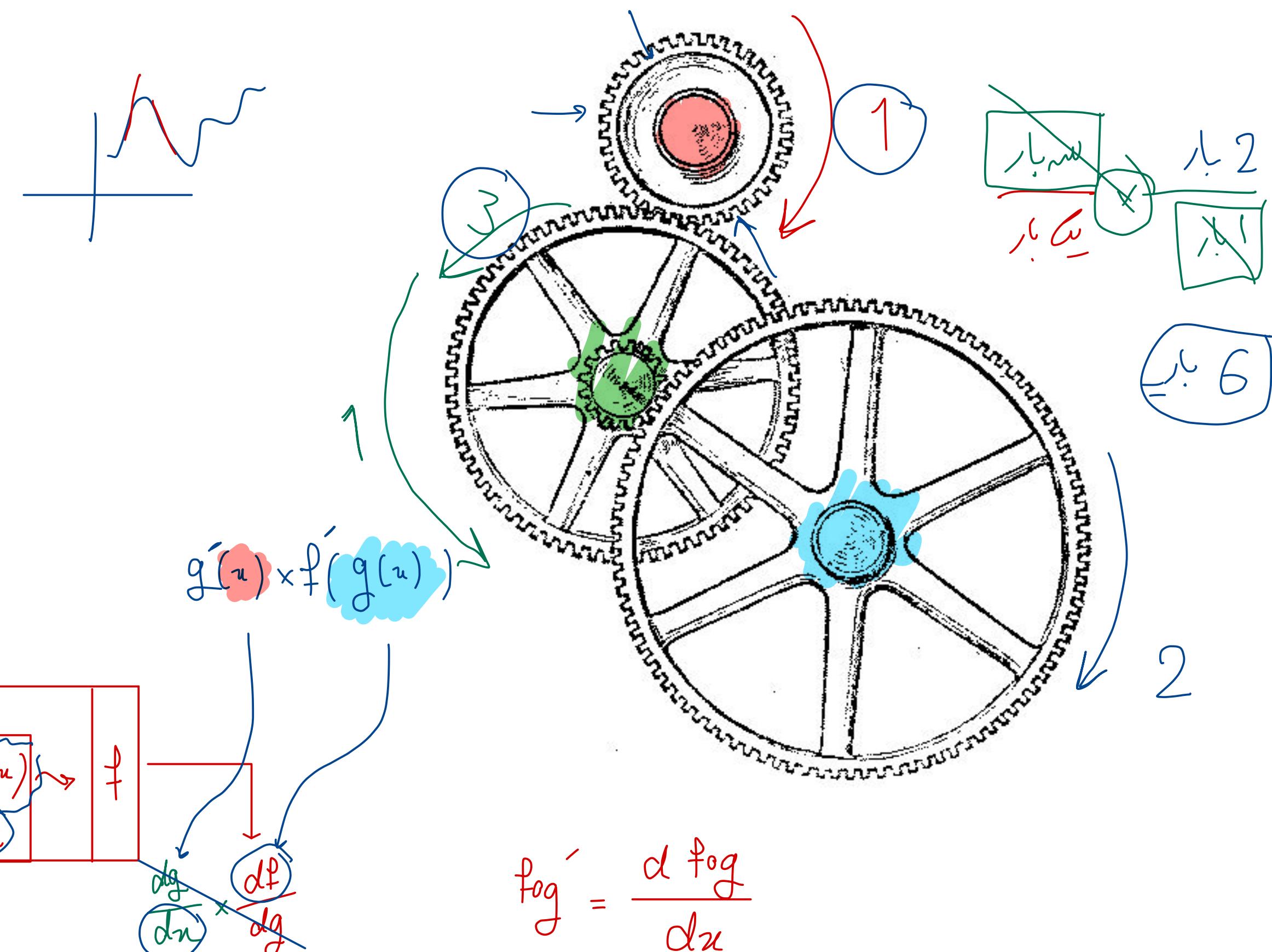
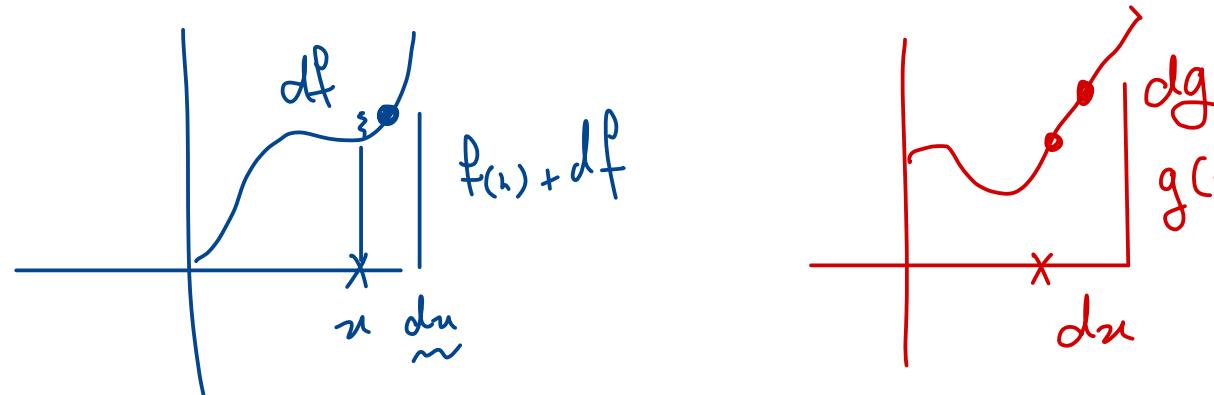
$$(f-g)' = f' - g'$$

$$(fg)' = fg' + gf'$$

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

$$(fog)'(x) = f'(g(x)) \cdot g'(x)$$

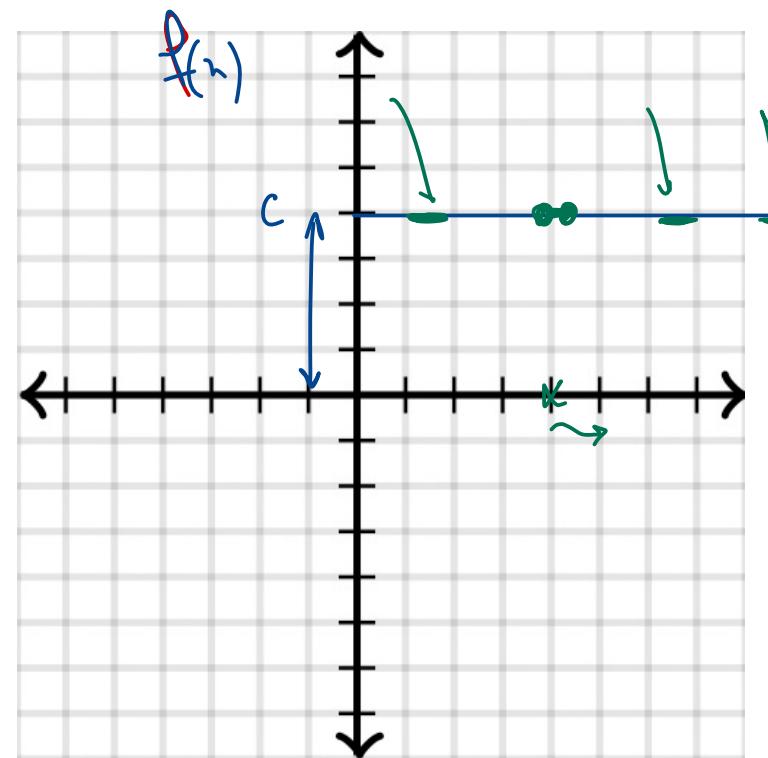
chain rule



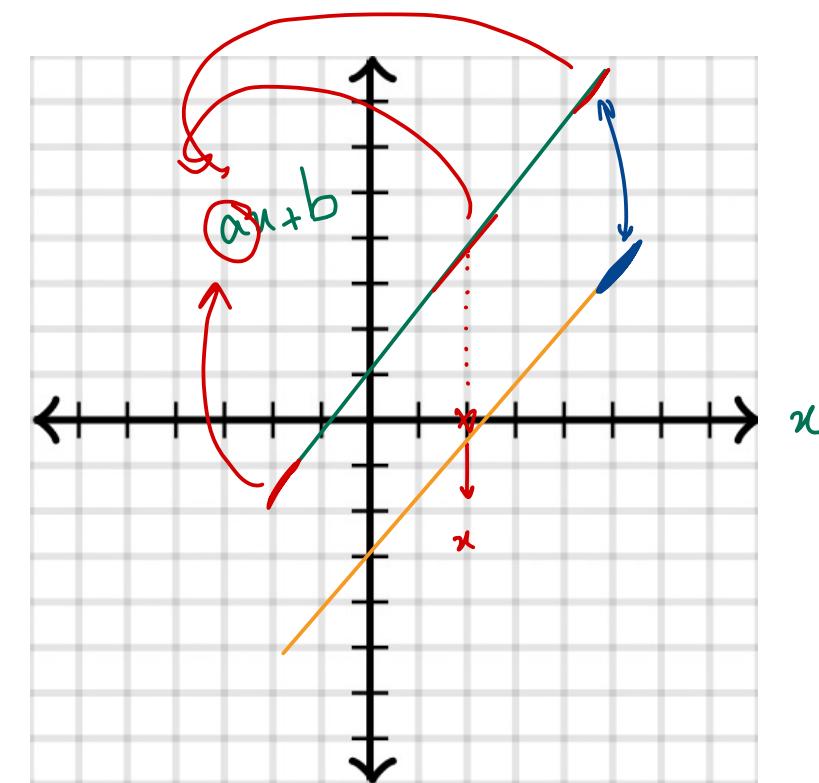
$$(fog)' = \frac{d(fog)}{dx}$$

٤. حسابان

مشتق



$$\frac{d}{dx} c = 0$$



$$\frac{d}{dx} ax + b = \frac{d}{dx} ax + \cancel{\frac{d}{dx} b}$$

$$\frac{d}{dx} x^n = nx^{n-1}$$

$$\frac{d}{dx} x^7 = 7x^{7-1} = 7x^6 = a$$

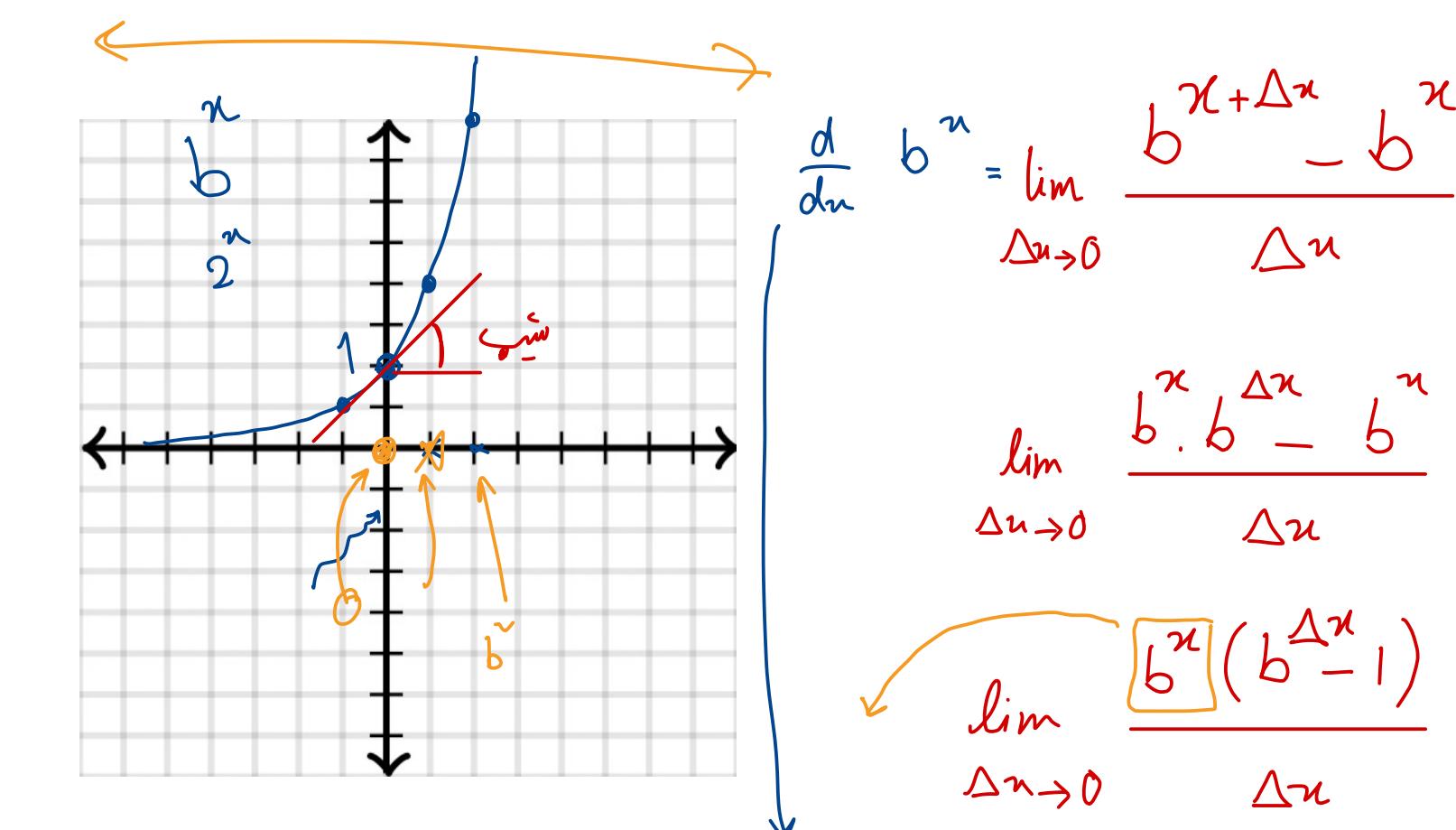
$$\frac{d}{dx} b^x = \ln b \cdot b^x$$

$$\frac{d}{dx} \log_b x = \frac{1}{\ln b} \cdot \frac{1}{x}$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \sqrt{n} = \frac{d}{dx} n^{\frac{1}{2}} = \frac{1}{2} n^{-\frac{1}{2}} = \frac{1}{2} \frac{1}{n^{\frac{1}{2}}} = \frac{1}{2} \frac{1}{\sqrt{n}}$$



$$\frac{d}{dx} b^x = \lim_{\Delta x \rightarrow 0} \frac{b^{x+\Delta x} - b^x}{\Delta x}$$

$$\lim_{\Delta x \rightarrow 0} \frac{b^x \cdot b^{\Delta x} - b^x}{\Delta x}$$

$$\lim_{\Delta x \rightarrow 0} \frac{b^x(b^{\Delta x} - 1)}{\Delta x}$$

$$\frac{d}{dx} b^x = b^x \cdot \lim_{\Delta x \rightarrow 0} \frac{b^{\Delta x} - 1}{\Delta x}$$

$$\left. \frac{d}{dx} b^x \right|_{x=0} = 1 \times \lim_{\Delta x \rightarrow 0} \frac{b^{\Delta x} - 1}{\Delta x} \rightarrow \ln b$$

$$\left. \frac{d}{dx} b^x \right|_{x=1} = b^1 = b$$

$$(2-3)^x$$

۴. حسابان

عدد اویلر

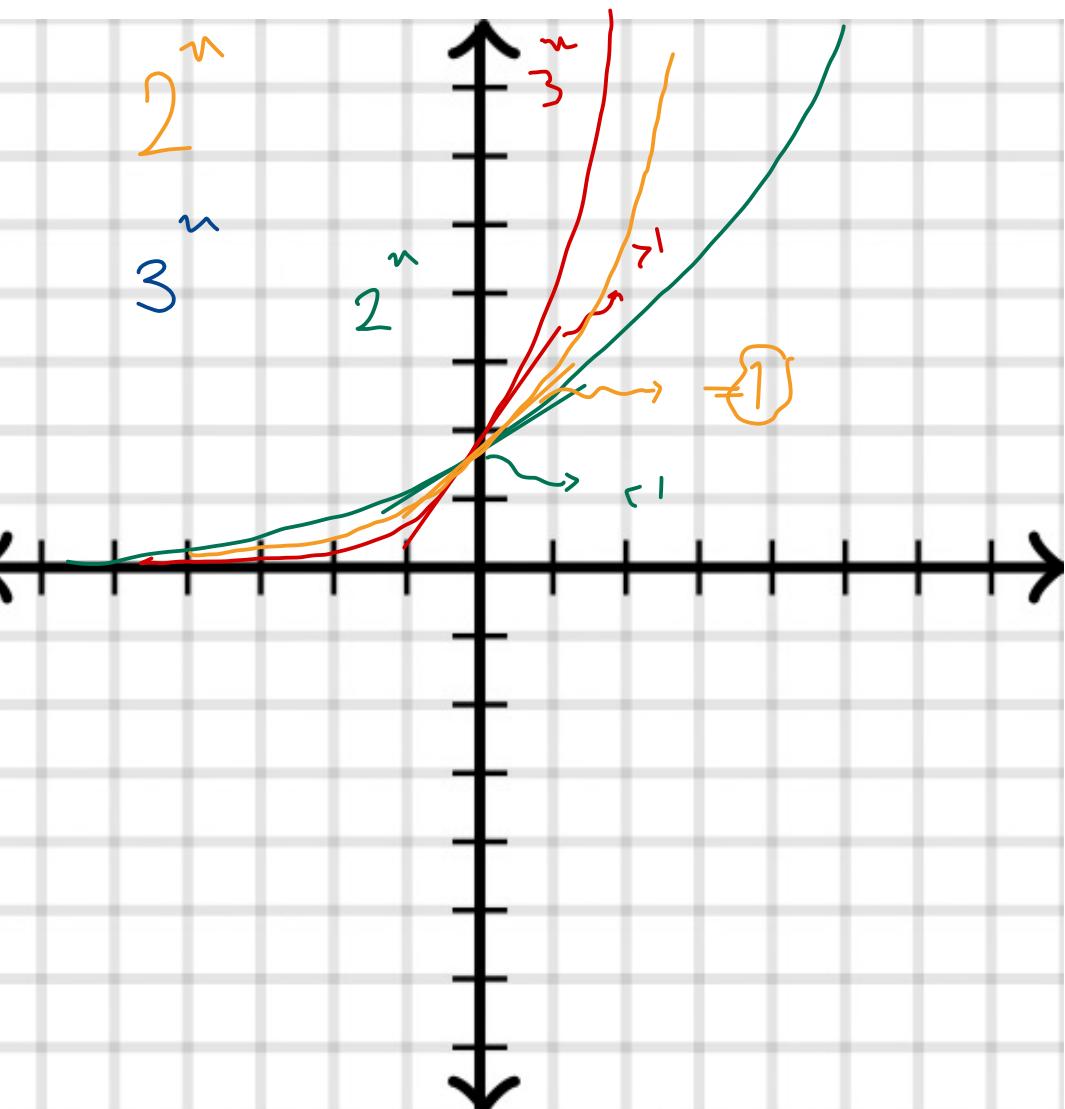
$$\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1$$

2 → ~

$$\ln e = 1$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$



$$\frac{d}{du} 2^u \Big|_{u=0} \approx \boxed{69}$$

$$\frac{d}{du} 3^u \Big|_{u=0} \approx \boxed{11}$$



$$\frac{d}{dx} \sqrt{3x^2 - x + 5} = \frac{(6x-1+0)}{2\sqrt{3x^2 - x + 5}}$$

$\log(u)$

$$g(u) = 3x^2 - x + 5$$

$$f(u) = \sqrt{u}$$

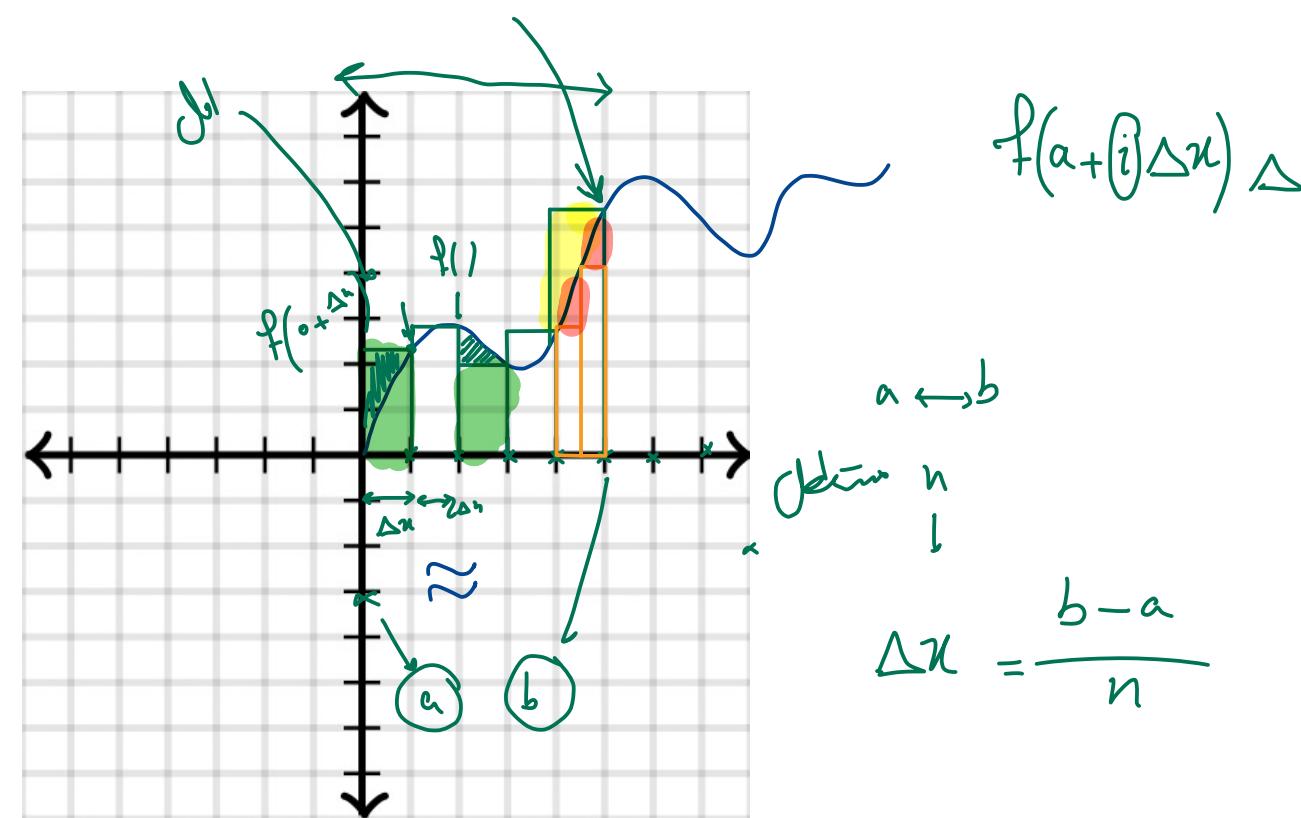
$$g'(x) = 6x - 1 + 0$$

$$f'(u) = \frac{1}{2\sqrt{u}}$$

$$= \frac{6x-1}{2\sqrt{3x^2 - x + 5}}$$



۴. حسابان انتگرال



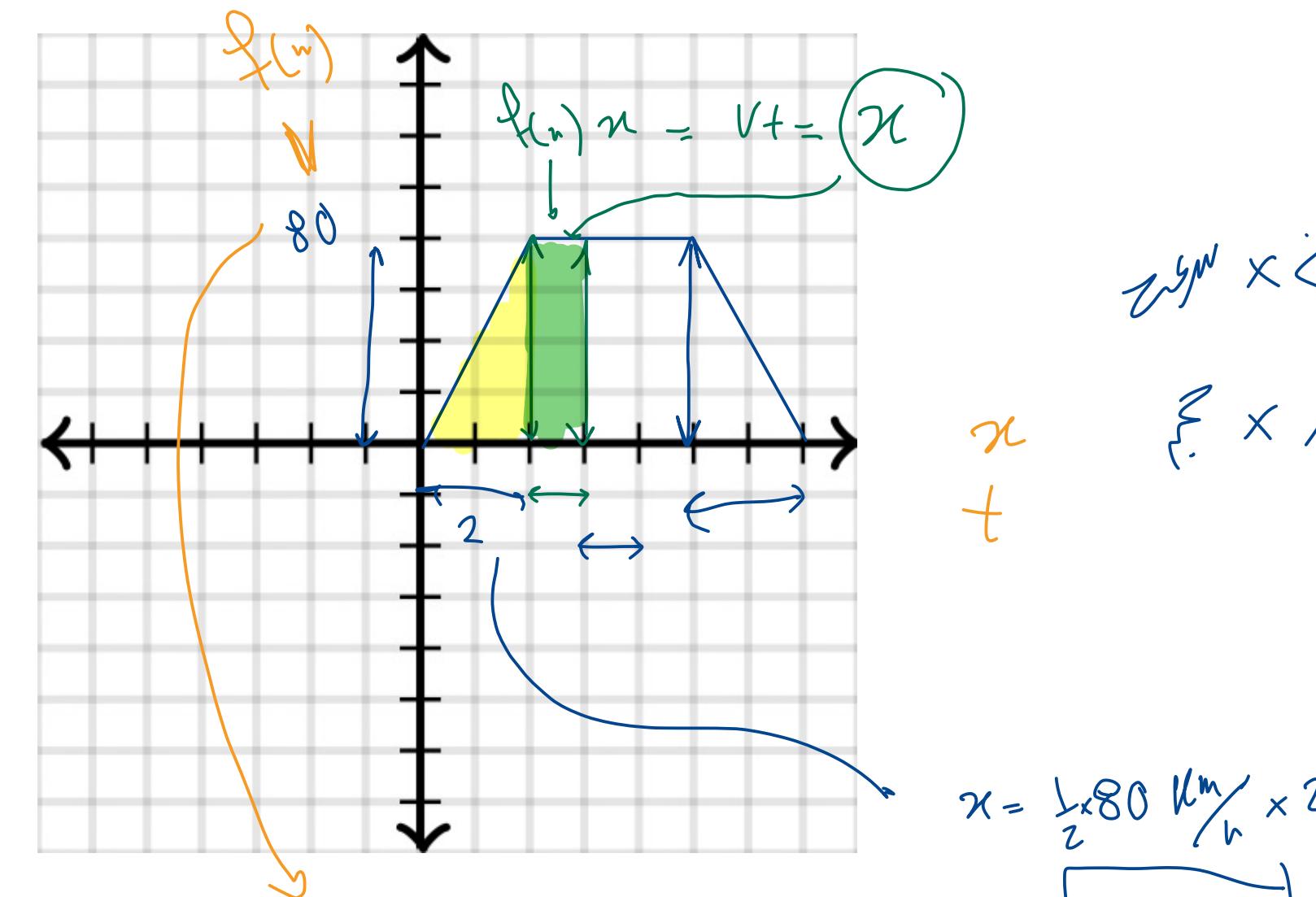
$$\Delta x = \frac{(b-a)}{n}, \quad x_i = a + i\Delta x$$

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^{\infty} f(x_i) \Delta x$$

طبع

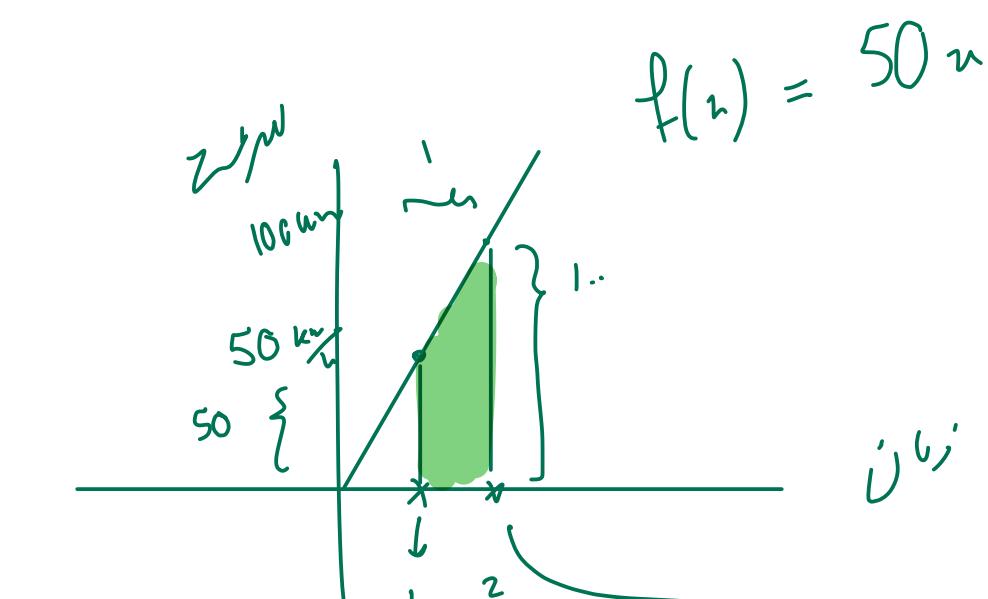
$$\lim_{\substack{n \rightarrow \infty \\ \Delta x \rightarrow 0}} \left(\sum_{i=1}^n f(a+i\Delta x) \Delta x \right) = \text{مقدار}$$

$\int_a^b f(x) dx = \text{مقدار}$



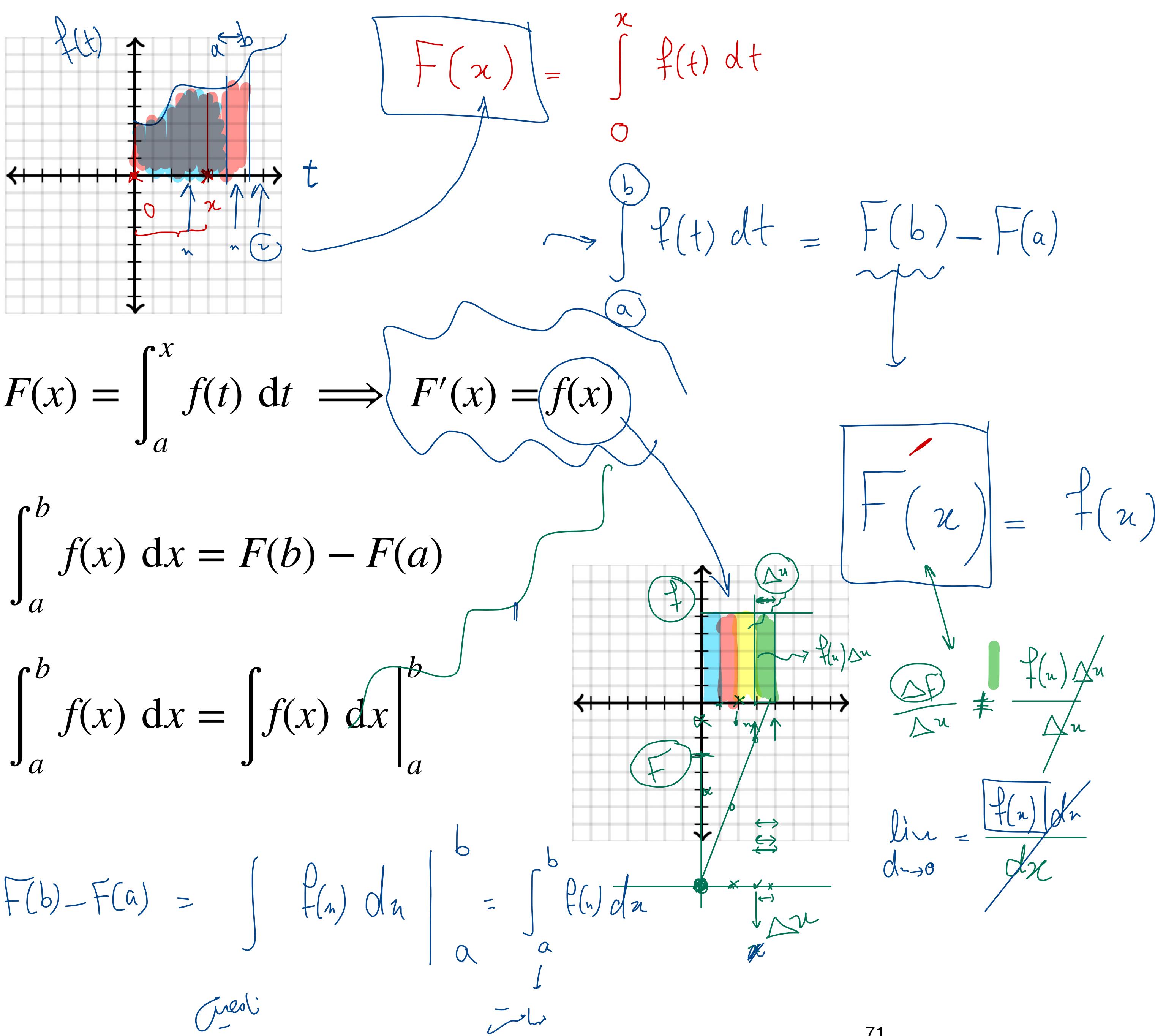
$$\text{مسافت} = x = v \Delta t$$

$$120 = 60 \frac{\text{كم}}{\text{h}} \times 2 \text{h}$$



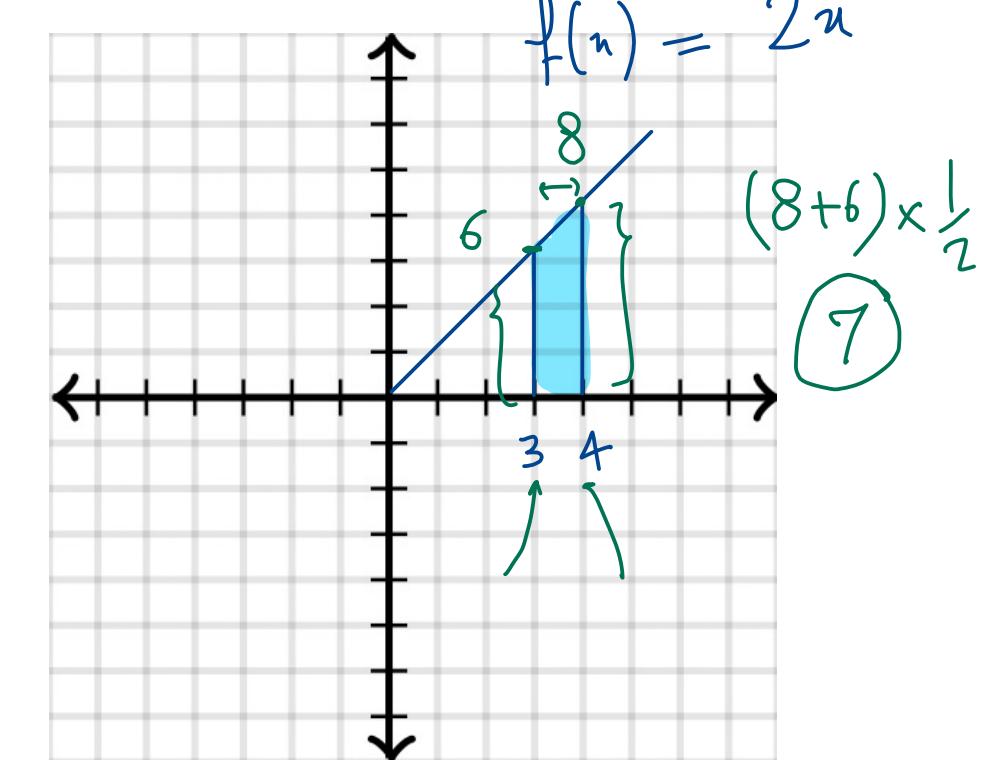
$$\int_1^2 f(x) dx = \int_1^2 50x dx = 75 \text{ km}$$

۴. حسابان انتگرال



۴. حسابان

انتگرال

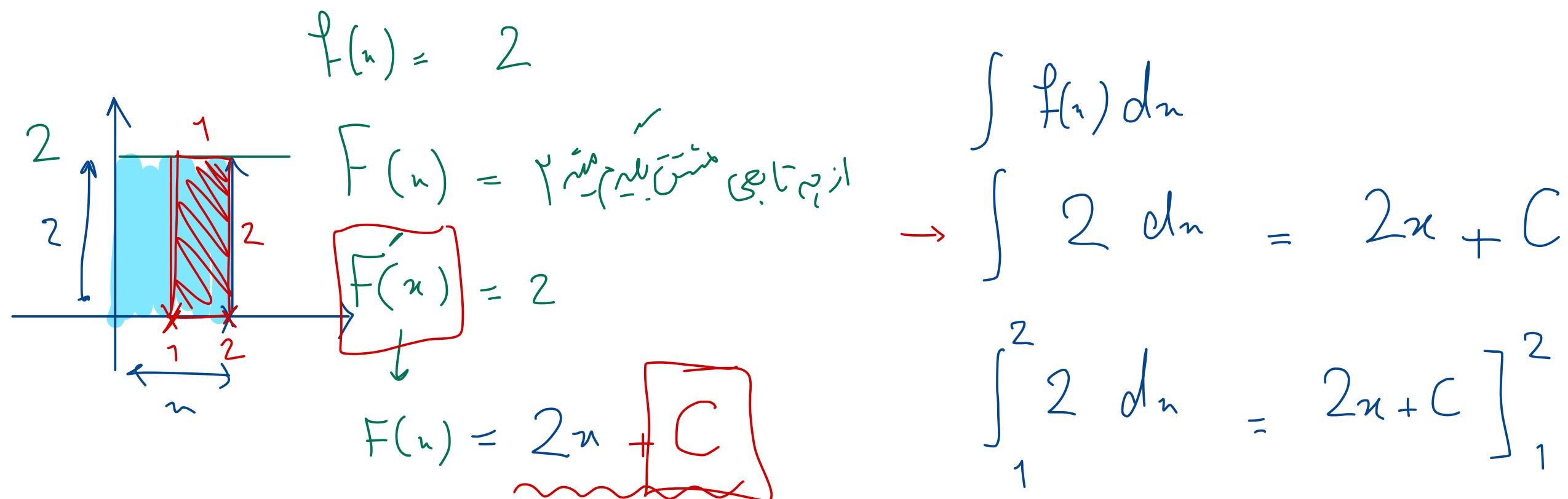


$$c\int f(x) \, dx = cF$$

$$\int cf(x) \, dx = c \int f(x) \, dx$$

$$\int [f(x) + g(x)] \, dx = \int f(x) \, dx + \int g(x) \, dx$$

$$\int [f(x) - g(x)] \, dx = \int f(x) \, dx - \int g(x) \, dx$$



۴. حسابان انتگرال

$$\int k \, dx = kx + C$$

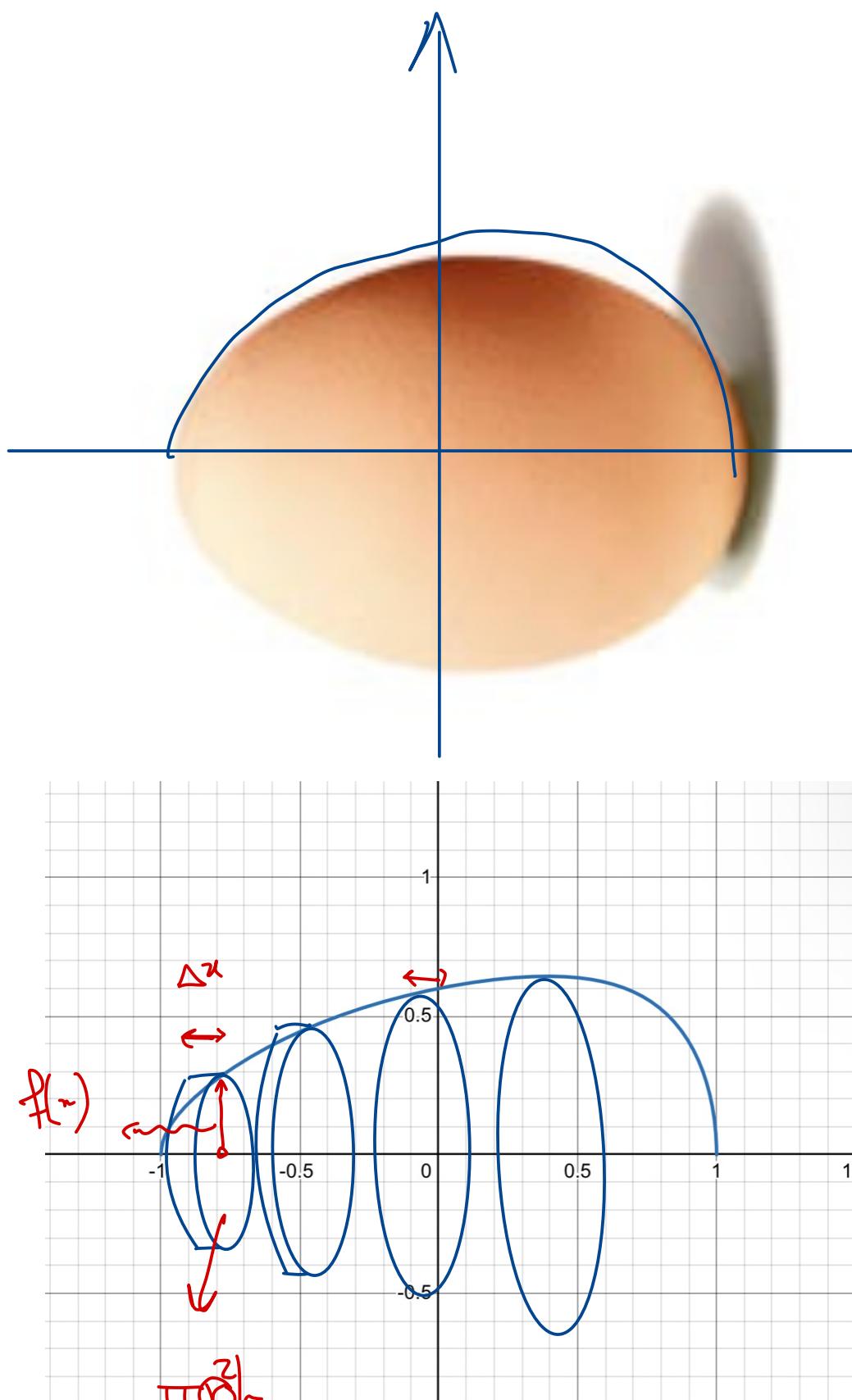
$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

$$\int b^x \, dx = \frac{b^x}{\ln b} + C$$

$$\int \frac{1}{x} \, dx = \ln|x| + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$



$$\lim_{\Delta x \rightarrow 0} \left(\sum \pi f(u)^2 \Delta x \right) \approx \int_{-1}^1 \pi f(u)^2 \, du$$

۴. حسابان

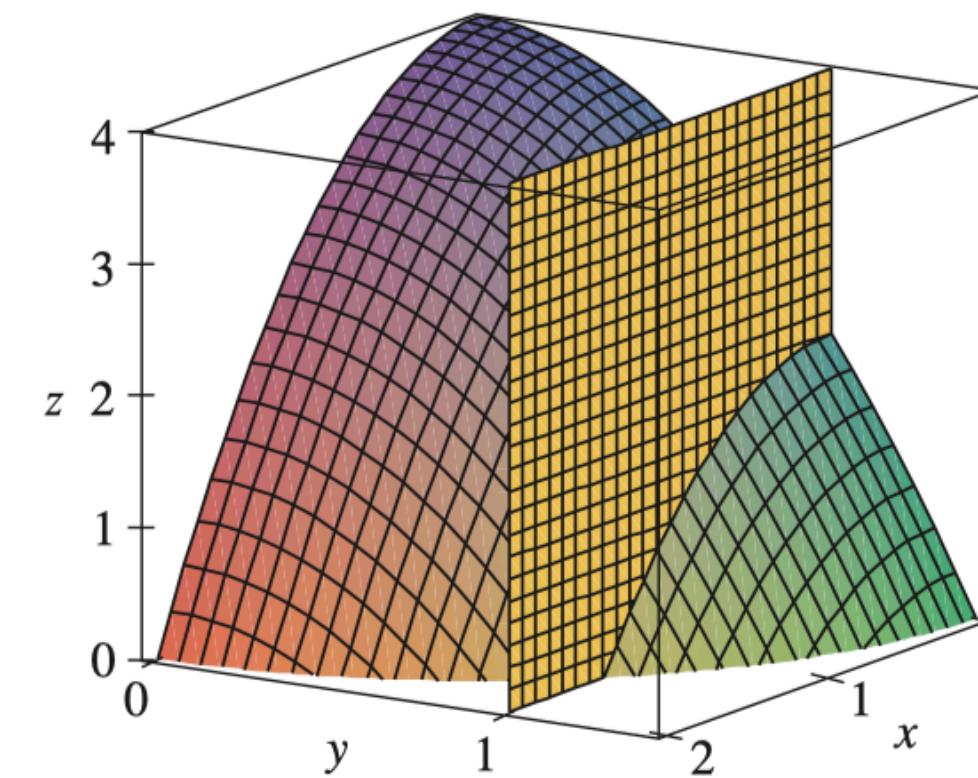
مشتق جزئی

$$f_x(x, y) = \frac{\partial f}{\partial x} = \lim_{h \rightarrow 0} \frac{f(x + h, y) - f(x, y)}{h}$$

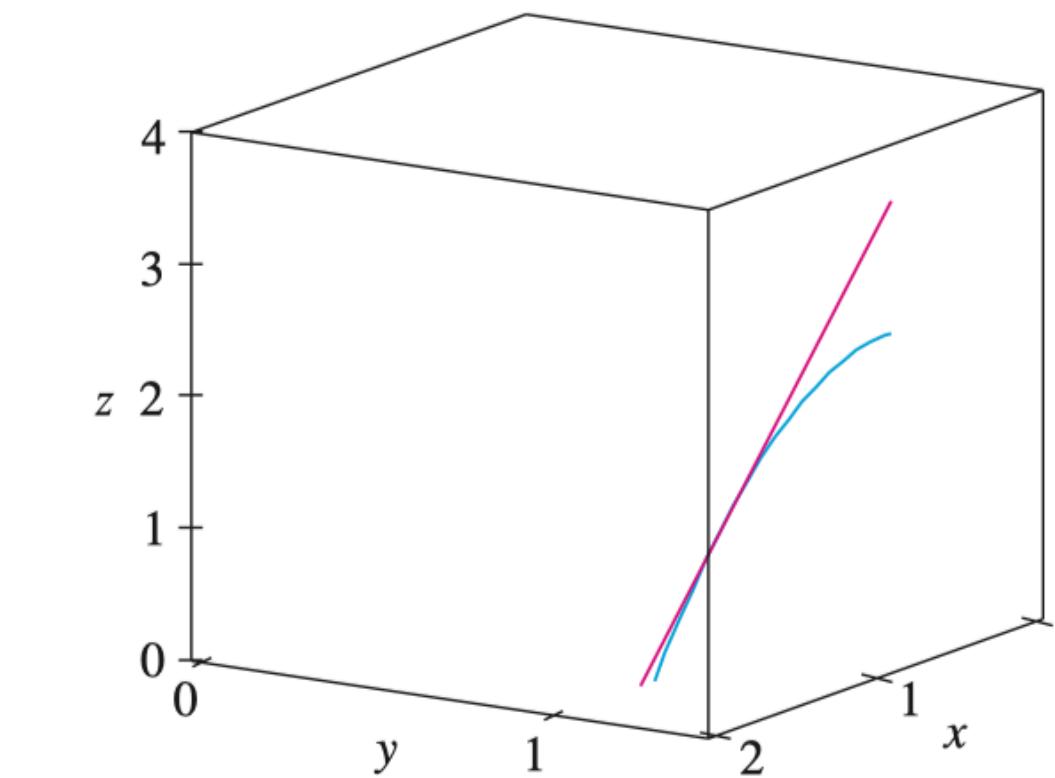
$$f_y(x, y) = \frac{\partial f}{\partial y} = \lim_{h \rightarrow 0} \frac{f(x, y + h) - f(x, y)}{h}$$

۴. حسابان

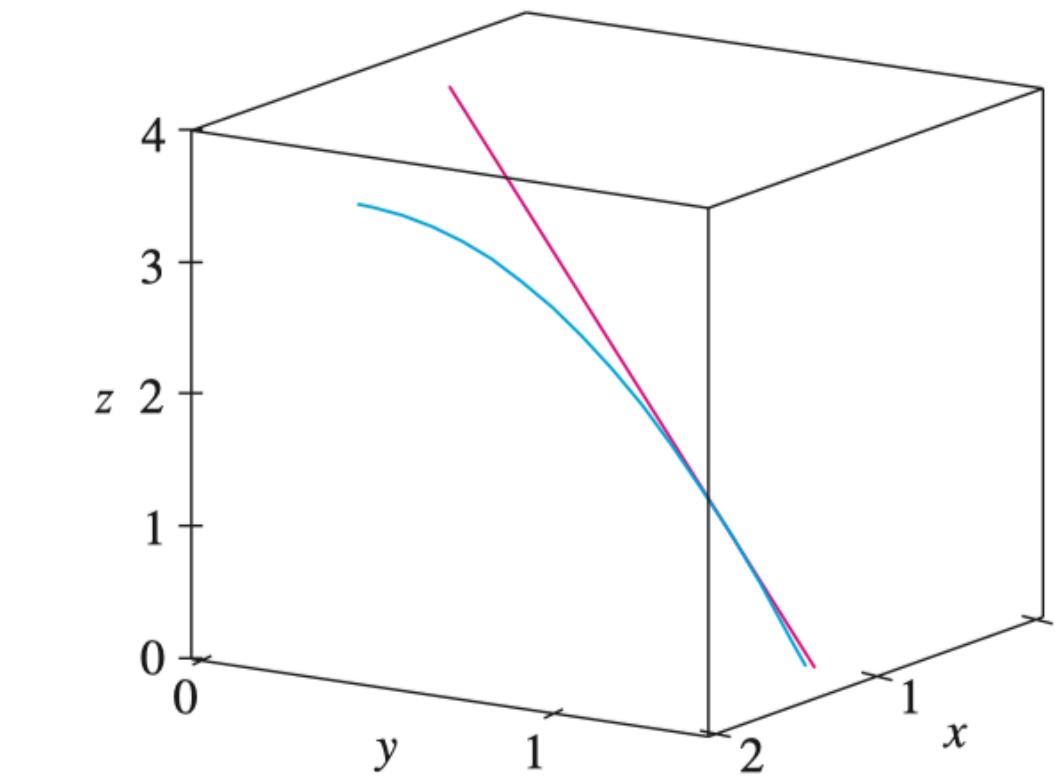
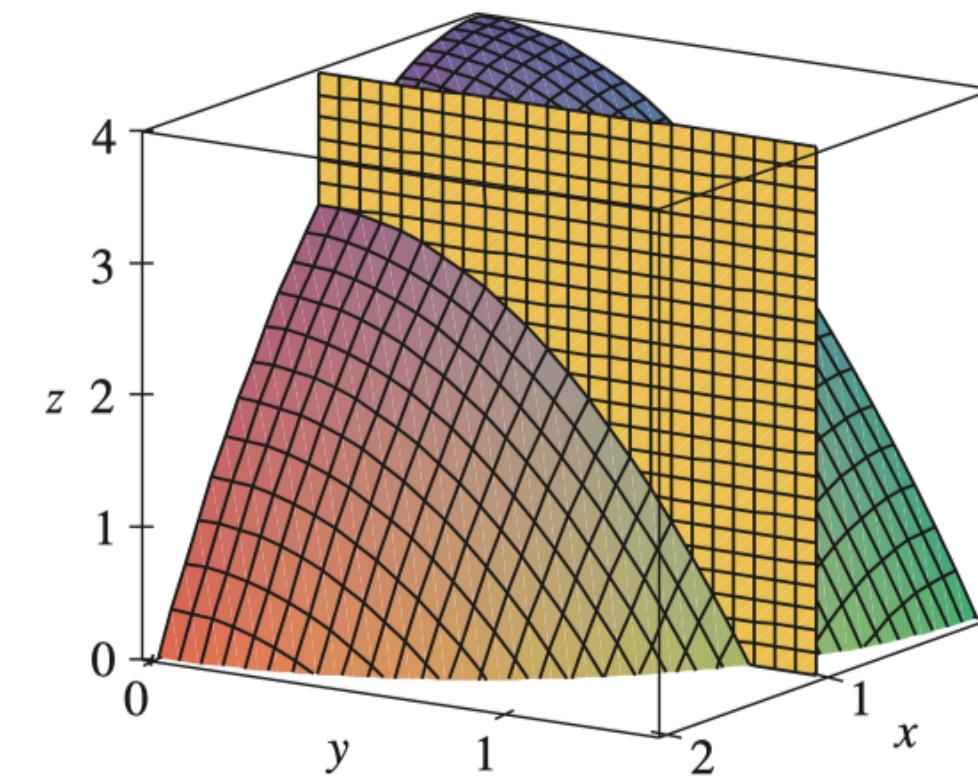
مشتق جزئی



(a)



(b)



۴. حسابان

بسط تیلور

$$f(a) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

۴. حسابان

بسط تیلور

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

٢.٤ معادلات دیفرانسیل

$$f' \rightarrow f$$

$$\dot{x} = \frac{dx}{dt} = f(x)$$

$$x_{t+\Delta t} = x_t + \dot{x}(x_t)$$

$\dot{x}(x^*) = 0 \rightarrow x^*$ is an equilibrium point

$\ddot{x}(x^*) < 0 \rightarrow x^*$ is a stable equilibrium point

$\ddot{x}(x^*) > 0 \rightarrow x^*$ is a unstable equilibrium point

٢.٤ معادلات دیفرانسیل

$$\dot{x} = \sin x$$

٢.٤ معادلات دیفرانسیل

$$\dot{x} = x - \cos x$$