Sum of complex numbers =

1- $(3+5i)+(4-2i) = 3+4+5i-2i = 7+3i$

2- $(5-3i)+(-4-7i) = 1-10i$

3- $(-6+6i)+(9-i) = 3+5i$

4- $(3-2i)+\left(-5-\frac{1}{3}i\right) = -2-\frac{7}{3}i$

Differences of complex numbers =

1- $(7-\frac{1}{2}i)-\left(5+\frac{3}{2}i\right) = 2-2i$

2- $(-12+8i)-(7+4i) = -19+4i$

3- $(3+5i)-(4-2i) = 3-4+5i+2i = -1+7i$

4- $(-3+4i)-(2-5i) = -5+9i$

5- $(-4+i)-(2-5i) = -6+6i$

Multiplying complex numbers =

1- $(3+5i)(4-2i) = 12-6i+20i-10i^2 = 22+14i$

2- $i^{23} = i(i^{22}) = i(i^2)^{11} = i(-1)^{11} = -i$

3- $4(-1+2i) = -4+8i$

4- $(7-i)(4+2i) = 28+14i-4i-2i^2 = 30+10i$

5- $(6+5i)(2-3i) = 12-18i+10i-15i^2 = 27-8i$

6- $(2+5i)(2-5i) = 4-10i+10i-25i^2 = 25+4 = 29$

7- $(3-7i)^2 = (3-7i)(3-7i) = 9-21i-21i+49i^2 = -40-42i$

8- $(2+5i)^2 = (2+5i)(2+5i) = 4+10i+10i+25i^2 = -21+20i$

Dividing complex numbers=

$$\frac{a+bi}{c+di} = \frac{a+bi}{c+di}.\frac{c-di}{c-di} = \frac{ac+bi.c-di.a+bd}{c^2+c.di-c.di+d^2} = \frac{ac+bd+(bc-da)i}{c^2+d^2}$$

1- $\frac{3+5i}{1-2i} = \frac{3+5i}{1-2i}.\frac{1+2i}{1+2i} = \frac{3+5i+6i-10}{1+4} = \frac{-7+11i}{5} = -\frac{7}{5}+\frac{11}{5}i$

2- $\frac{7+3i}{4i} = \frac{7+3i}{4i}.\frac{-4i}{-4i} = \frac{-28i+12}{16} = \frac{3}{4}-\frac{7}{4}i$

3- $\frac{1}{i} = \frac{1}{i}.\frac{-i}{-i} = \frac{-i}{-i^2} = -i$

4- $\frac{2-3i}{1-2i}=\frac{2-3i}{1-2i}.\frac{1+2i}{1+2i}=\frac{2+4i-3i-6i^2}{1-2i+2i-4i^2}=\frac{8+i}{5}=\frac{8}{5}+\frac{1}{5}i$

5- $\frac{10i}{1-2i}=\frac{10i}{1-2i}.\frac{1+2i}{1+2i}=\frac{10i+20i^2}{1-2i+2i-4i^2}=\frac{-20+10i}{5}=-4+2i$

6- $\frac{4+6i}{3i}=\frac{4+6i}{3i}.\frac{-3i}{-3i}=\frac{-12i-18i^2}{-9i^2}=\frac{18-12i}{9}=2-\frac{4}{3}i$

7- $\frac{1}{1+i}-\frac{1}{1-i}=\frac{1}{1+i}.\frac{1-i}{1-i}-\frac{1}{1-i}.\frac{1+i}{1+i}=\frac{1-i}{1-i^2}-\frac{1+i}{1-i^2}=\frac{1-i-1-i}{2}=\frac{-2i}{2}=-i$

8- $\frac{(1+2i)(3-i)}{2+i}=\frac{3+6i-i-2i^2}{2+i}=\frac{5+5i}{2+i}=\frac{5+5i}{2+i}.\frac{2-i}{2-i}=\frac{10-5i+10i-5i^2}{4-2i+2i-i^2}=\frac{15+5i}{5}=3+i$

9- $(2-3i)^{-1}=\frac{1}{2-3i}.\frac{2+3i}{2+3i}=\frac{2+3i}{4-9i^2}=\frac{2+3i}{13}=\frac{2}{13}+\frac{3}{13}i$

10 - $\frac{1}{1+i}=\frac{1}{1+i}.\frac{1-i}{1-i}=\frac{1-i}{2}=\frac{1}{2}-\frac{1}{2}i$

Quadratic equations with complex solutions=

تجزیه:

1- $x^2+49=0 \rightarrow x^2=-49 \rightarrow x=\sqrt{-49} \rightarrow x=i\sqrt{49} \rightarrow x=\pm 7i$

2- $x^2-x+2=0 \rightarrow x^2-x+\frac{1}{4}+\frac{7}{4}=\left(x-\frac{1}{2}\right)^2=-\frac{7}{4} \rightarrow x-\frac{1}{2}=\sqrt{-\frac{7}{4}} \rightarrow x=\frac{1}{2}+\sqrt{\frac{7}{4}\,i^2} \rightarrow$

$x=\frac{1}{2}\pm\frac{\sqrt{7}}{2}i$

3- $x^2+3x+7=0 \rightarrow x^2+3x+\frac{9}{4}+\frac{19}{4}=0 \rightarrow \left(x+\frac{3}{2}\right)^2=-\frac{19}{4} \rightarrow x+\frac{3}{2}=\sqrt{-\frac{19}{4}}=x=-\frac{3}{2}\pm\frac{\sqrt{19}}{2}i$

4- $6x^2+12x+7=0 \rightarrow 6x^2+12x=-7 \rightarrow x^2+2x=-\frac{7}{6} \rightarrow x^2+2x+1=-\frac{1}{6} \rightarrow (x+1)^2=-\frac{1}{6}$

$x+1=\sqrt{-\frac{1}{6}} \rightarrow x=-1\pm\frac{\sqrt{6}}{6}i$

5- $2x^2-2x+1=0 \rightarrow 2x^2-2x=-1 \rightarrow x^2-x=-\frac{1}{2} \rightarrow x^2-x+\frac{1}{4}=-\frac{1}{4} \rightarrow \left(x-\frac{1}{2}\right)^2=-\frac{1}{4}$

$x-\frac{1}{2}=\sqrt{-\frac{1}{4}} \rightarrow x=\frac{1}{2}\pm\frac{1}{2}i$

روش هندسی:

1- $z^2 = 4\sqrt{3} + 4i \to \quad z = r(\cos\Theta + i.\sin\Theta) \to z^2 = r^2(\cos 2\,\Theta + i.\sin 2\,\Theta)$

$$r^2\cos 2\Theta = 4\sqrt{3}\,, r^2\sin 2\Theta = 4$$

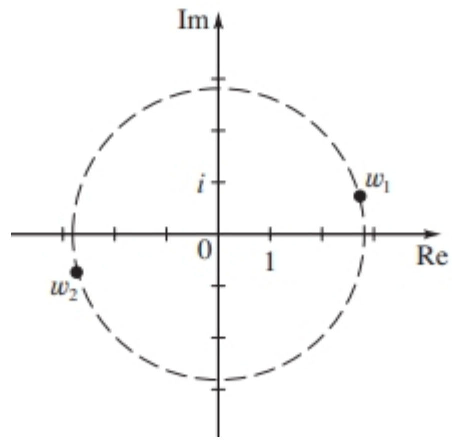
$$\frac{\sin 2\Theta}{\cos 2\Theta} = \frac{1}{\sqrt{3}} \to \tan 2\Theta = \frac{1}{\sqrt{3}} \to 2\Theta = \frac{\pi}{6} + 2k\pi$$

$$k = 0 \to \ \Theta = \frac{\pi}{12}\,, k = 1 \to \ \Theta = \frac{13\pi}{12}$$

$$r^2\cos 2\Theta = 4\sqrt{3} \to \cos 2\Theta = \frac{\sqrt{3}}{2} \to \frac{\sqrt{3}}{2}r^2 = 4\sqrt{3} \to r^2 = 8\,, r = \sqrt{8} = 2\sqrt{2}$$

$$w1 = 2\sqrt{2}(\cos\frac{\pi}{12} + i.\sin\frac{\pi}{12})$$

$$w2 = 2\sqrt{2}(\cos\frac{13\pi}{12} + i.\sin\frac{13\pi}{12})$$



2-$z^4 = -81\,i \to r^4(\cos 4\Theta + i.\sin 4\Theta) = -81i$

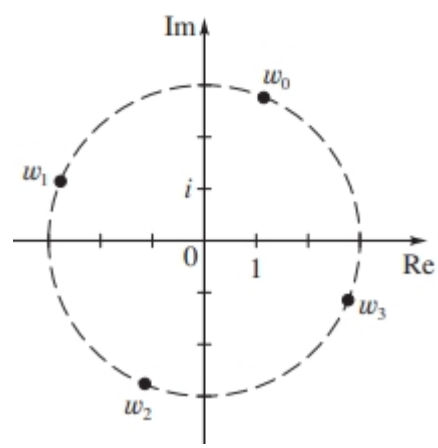
$$r^4 cos4\Theta = 0\,, r^4\sin 4\Theta = -81$$

$$\cos 4\Theta = 0 \to \sin 4\Theta = \pm 1 \to \ -1\,, r^4 = 81 \to r = \sqrt[4]{81} = 3$$

$$4\Theta = \frac{3\pi}{2} + 2k\pi \to \quad k = 0\ \Theta = \frac{3\pi}{8} \quad , k = 1\ \ \Theta = \frac{7\pi}{8} \quad , k = 2\ \Theta = \frac{11\pi}{8} \quad , k = 3\ \Theta = \frac{15\pi}{8}$$

$w1 = 3(\cos\frac{3\pi}{8} + i.\sin\frac{3\pi}{8})$ $, w2 = 3(\cos\frac{7\pi}{8} + i.\sin\frac{7\pi}{8})$,

$w3 = 3(\cos\frac{11\pi}{8} + i.\sin\frac{11\pi}{8})$ $\quad w4 = 3(\cos\frac{15\pi}{8} + i.\sin\frac{15\pi}{8})$



3- $z^8 = 1 \to z^8 = r^8(\cos 8\Theta + i \sin 8\Theta) = 1 + 0i$

$$r^8 \sin 8\Theta = 0 \to \cos 8\Theta = \pm 1 \to 1 \to 8\Theta = 2k\pi \,, r^8 = 1 \to r = 1$$

$$8\Theta = 2k\pi$$

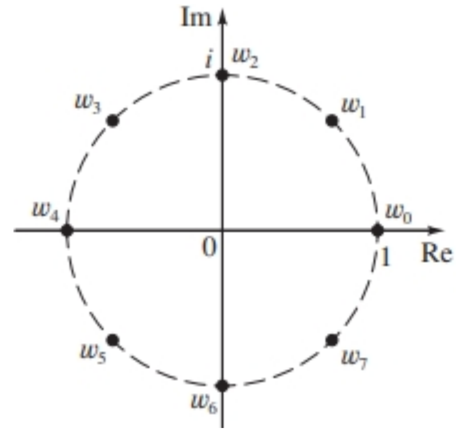
$k = 0 \to \ \Theta = 0 \quad , k = 1 \to \ \Theta = \frac{\pi}{4} \quad , k = 2 \to \ \Theta = \frac{\pi}{2} \quad , k = 3 \to = \frac{3\pi}{4}$

$k = 4 \to \ \Theta = \pi \quad , k = 5 \to \ \Theta = \frac{5\pi}{4} \quad , k = 6 \to \ \Theta = \frac{3\pi}{2} \quad , k = 7 \to \ \Theta = \frac{7\pi}{4}$

$w0 = 1 \qquad w1 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i \qquad w2 = i \,,$

$w3 = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i \quad w4 = -1$

$w5 = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i, \quad w6 = -i \quad w7 = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i$



4- $z^3 = i \to r^3(\cos 3\Theta + i.\sin 3\Theta) = i$

$r^3 \cos 3\Theta = 0 \to \cos 3\Theta = 0 \to \sin 3\Theta = \pm 1 \to 1 \,, r^3 \sin 3\Theta = 1 \to r^3 = 1 \to r = \sqrt[3]{1} = 1$

$$3\Theta = \frac{\pi}{2} + 2k\pi$$

$$k = 0 \,, 3\Theta = \frac{\pi}{2} \to \ \Theta = \frac{\pi}{6}$$

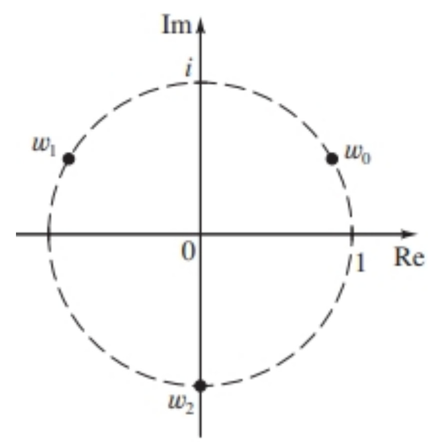
$$k = 1 \,, 3\Theta = \frac{5\pi}{2} \to \ \Theta = \frac{5\pi}{6}$$

$$k = 2 \,, 3\Theta = \frac{9\pi}{2} \to \ \Theta = \frac{9\pi}{6} = \frac{3\pi}{2}$$

$w1 = \cos\frac{\pi}{6} + i.\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} + \frac{1}{2} i$

$w2 = \cos\frac{5\pi}{6} + i.\sin\frac{5\pi}{6} = -\frac{\sqrt{3}}{2} + \frac{1}{2} i$

$w3 = \cos\frac{3\pi}{2} + i.\sin\frac{3\pi}{2} = \ -i$



5- $z^4 = -1 \rightarrow r^4 \cos 4\Theta + r^4 \sin 4\Theta = -1 + 0i$

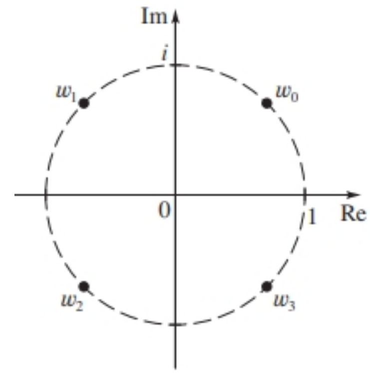
$$r^4 \cos 4\Theta = -1 \,, r^4 \sin 4\Theta = 0 \rightarrow \cos 4\Theta = -1 \,, \sin 4\Theta = 0 \rightarrow r^4 = 1 \rightarrow r = 1$$

$$4\Theta = \pi + 2k\pi$$

$$k = 0 \rightarrow 4\Theta = \pi \rightarrow \Theta = \frac{\pi}{4} \quad , \quad k = 1 \rightarrow 4\Theta = 3\pi \rightarrow \Theta = \frac{3\pi}{4}$$

$$k = 2 \rightarrow 4\Theta = 5\pi \rightarrow \Theta = \frac{5\pi}{4} \quad , \quad k = 3 \rightarrow 4\Theta = 7\pi \rightarrow \Theta = \frac{7\pi}{4}$$

$$w1 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \quad , \quad w2 = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \quad , \quad w3 = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i \quad , w4 = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$$



6- $z^3 = 2 + 2i \rightarrow r^3(\cos 3\Theta + i.\sin 3\Theta) = 2 + 2i$

$$r^3 \cos 3\Theta = 2 \quad , \qquad r^3 \sin 3\Theta = 2$$

$$\frac{r^3 \sin 3\Theta}{r^3 \cos 3\Theta} = 1 \rightarrow \tan 3\Theta = 1 \rightarrow \tan^{-1} 1 = \frac{\pi}{4}$$

$$\cos 3\Theta = \frac{\sqrt{2}}{2} \rightarrow r^3 = \frac{2}{\frac{\sqrt{2}}{2}} \rightarrow \frac{4}{\sqrt{2}} = \sqrt{8} \rightarrow r = \sqrt[6]{8} = \sqrt{2}$$

$$3\Theta = \frac{\pi}{4} + 2k\pi$$

$$k = 0 \rightarrow 3\Theta = \frac{\pi}{4} \rightarrow \Theta = \frac{\pi}{12} \quad , \quad k = 1 \rightarrow 3\Theta = \frac{9\pi}{4} \rightarrow \Theta = \frac{9\pi}{12} = \frac{3\pi}{4}, k = 2 \rightarrow 3\Theta = \frac{17\pi}{4} \rightarrow \Theta = \frac{17\pi}{12}$$

$$w1 = \sqrt{2}(\cos\frac{\pi}{12} + i.\sin\frac{\pi}{12}) \,, w2 = \sqrt{2}(\cos\frac{3\pi}{4} + i.\sin\frac{3\pi}{4}) = -1 + i \,, w3 = \sqrt{2}(\cos\frac{17\pi}{12} + i.\sin\frac{17\pi}{12})$$