

Ray Tracing Introduction

- Rays
 - Origin and direction
- Ray tracing concept
 - Find a point along a ray
- Camera and viewport setup
- Ray intersection-sphere tests
 - Use a quadratic solution
- Diffuse lighting
 - Dot product surface normal with light direction
- Multiple spheres

- A ray is a mathematical abstraction, a model, of a ray of light
- It has an origin (the vector o) and a direction (the vector d)
- lacktriangle Any point p along the calculated with the scalar t

$$p = o + dt$$

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 $\begin{array}{cc} p \\ d & t = 0.8 \end{array}$

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p t=2

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$$p = o + dt$$

$$t = -0.5$$

LZ Quiz

■ L10 Q1 Ray Point At

- A ray is a mathematical abstraction, a model, of a ray of light
- It has an origin (the vector o) and a direction (the vector d)
- \blacksquare Any point p along the calculated with the scalar t

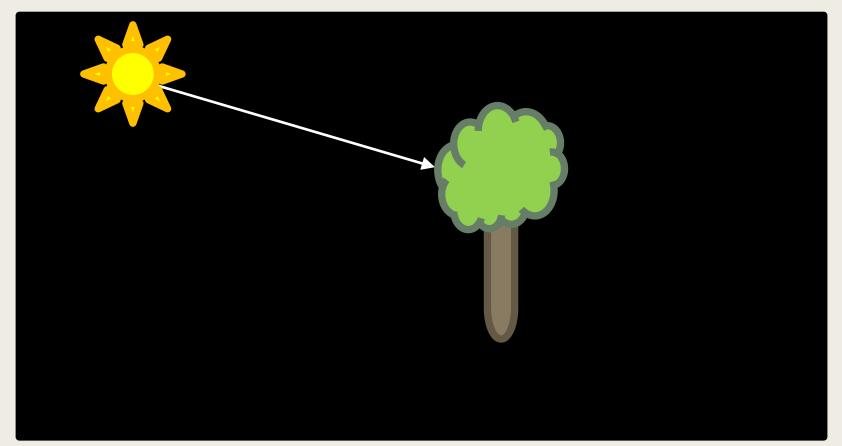
$$p = o + dt$$

My ray class code (you need to finish pointAt):

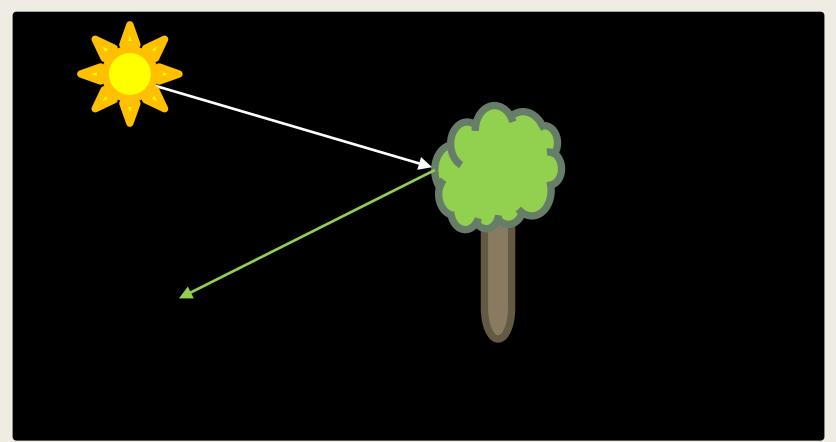
```
// Ray which has an origin and direction, both are Vec3s
class Ray
{
    constructor (origin, direction)
    {
        this.origin = origin
            this.direction = direction
    }

    // Calculate and return the point in space (a Vec3) for this ray for the given value of t pointAt(t) {}
}
```

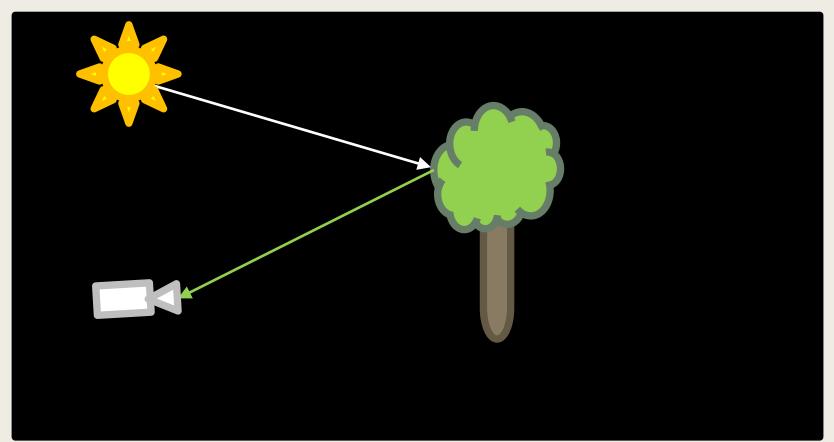
■ Light sources emit rays of light



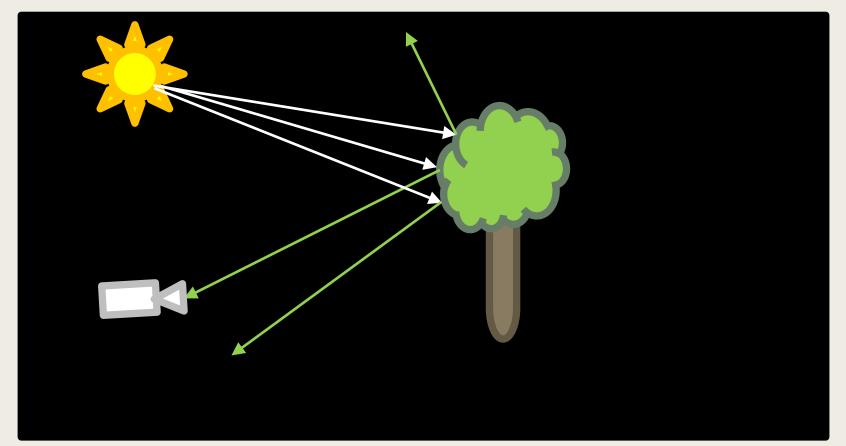
■ The rays reflect of surfaces, some light is absorbed giving colour



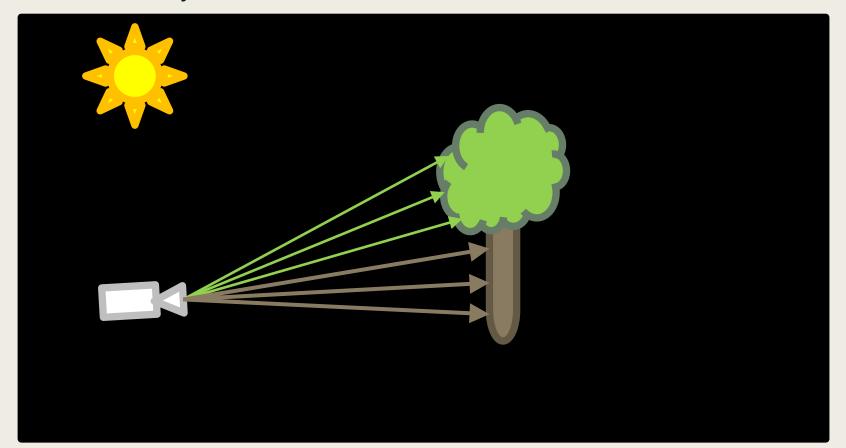
■ Some of these rays travel to an eye or camera and form a picture



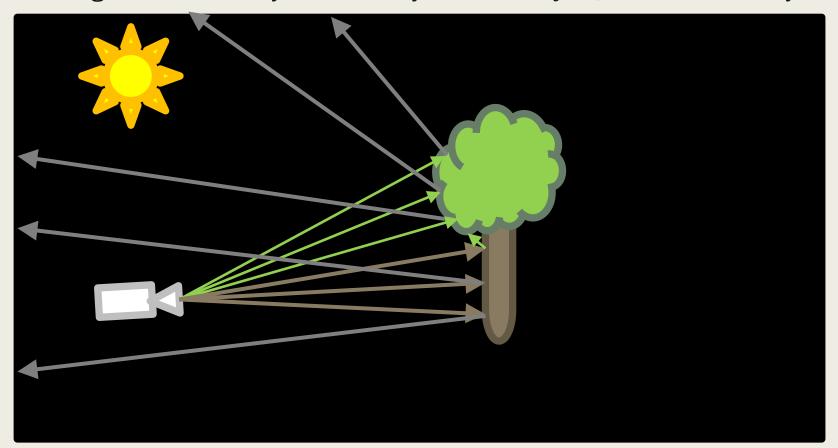
Of course many more rays don't travel to an eye or camera



■ So instead we fire rays from the camera to the scene



■ We can also generate new rays when a ray hits and object, reflections or ray bounces



- Usually we'd use matrices for our camera
- Here we'll use a simplified version without matrices
- We have a canvas we will render too and we'll find it's size:

```
let imageWidth = document.getElementById("canvas").width
let imageHeight = document.getElementById("canvas").height
```

This also gives an aspect ratio:

```
let aspectRatio = document.getElementById("canvas").height / document.getElementById("canvas").width
```

We then convert these to parameters in [0, 1] called uv co-ordinates per pixel

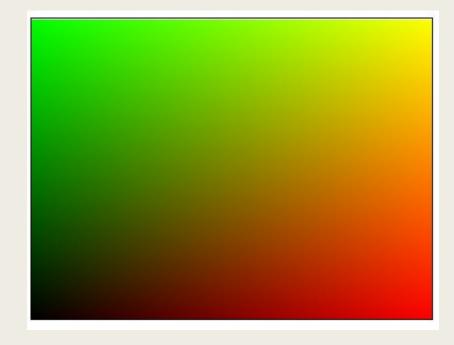
```
for (let i = 0; i < imageWidth; i++)
{
    for (let j = 0; j <= imageHeight; j++)
        {
        let u = i / (imageWidth-1)
        let v = j / (imageHeight-1)
        }
}</pre>
```

■ We can test this by drawing the uv co-ordinates

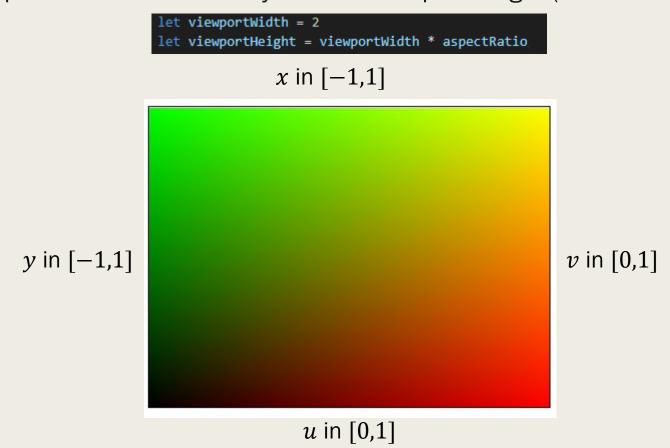
```
let colour = new Vec3(0,0,0)

let imageWidth = document.getElementById("canvas").width
let imageHeight = document.getElementById("canvas").height

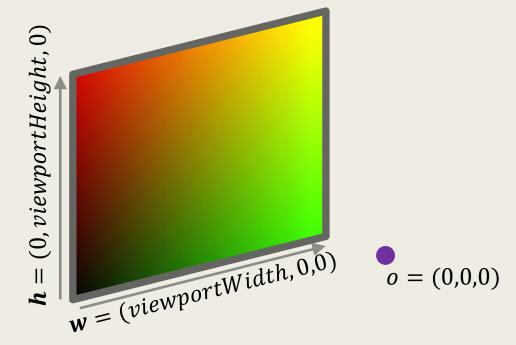
for (let i = 0; i < imageWidth; i++)
{
    for (let j = 0; j <= imageHeight; j++)
    {
        let u = i / (imageWidth-1)
        let v = j / (imageHeight-1)
        colour.x = u * 255
        colour.y = v * 255
        setPixel(i,j,colour)
    }
}</pre>
```



■ Given a viewport width we can easily find the viewport height (I've used NDC)

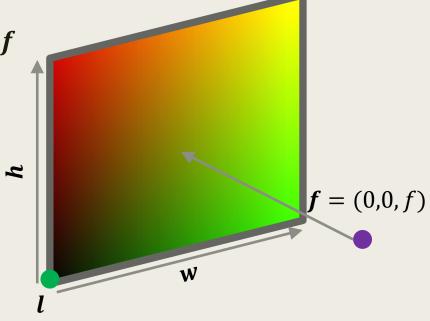


lacktriangle Casting rays in this space from camera origin o with focal length f



- lacktriangle Casting rays in this space from camera origin $oldsymbol{o}$ with focal length f
- lacktriangle The lower left corner is the position l

$$l = -(\mathbf{w} \times \mathbf{0.5}) - (\mathbf{h} \times \mathbf{0.5}) - \mathbf{f}$$



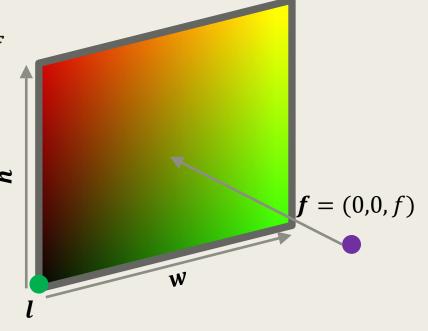
- lacktriangle Casting rays in this space from camera origin o with focal length f
- lacktriangle The lower left corner is the position l

$$l = -(\mathbf{w} \times \mathbf{0.5}) - (\mathbf{h} \times \mathbf{0.5}) - \mathbf{f}$$

- We now cast a ray for each pixel in our image
 - Ray origin is the camera origin o
 - Ray direction is

$$l + h \times u + w \times v - o$$

- Basically left corner plus scale height and width
- Lets see this in code



Ray casting loop

```
let imageWidth = document.getElementById("canvas").width
let imageHeight = document.getElementById("canvas").height
let aspectRatio = imageHeight / imageWidth
let viewportWidth = 2
let viewportHeight = viewportWidth * aspectRatio
let focalLength = 1.0
let camPosition = new Vec3(0,0,0)
let horizontal = new Vec3(viewportWidth, 0, 0)
let vertical = new Vec3(0, viewportHeight, 0)
let lowerLeftCorner = camPosition.minus(horizontal.scale(0.5)).minus(vertical.scale(0.5)).minus(new Vec3(0, 0, focalLength))
let colour = new Vec3(0,0,0)
for (let i = 0; i < imageWidth; i++)</pre>
    for (let j = 0; j <= imageHeight; j++)</pre>
       let u = i / (imageWidth-1)
       let v = j / (imageHeight-1)
        let ray = new Ray(camPosition, lowerLeftCorner.add(horizontal.scale(u)).add(vertical.scale(v)).minus(camPosition))
        colour = rayColor(ray).scale(255)
       setPixel(i,j,colour)
```

- We are now casting a single ray from our camera through each pixel
- We now need to know if it hits anything in our scene or not
- Our scene currently contains three spheres held in an array

- Each ray fired will either hit a sphere or not
- We colour the pixel depending on the this result

Ray colour cast the ray out into our scene and return the correct colour

```
// Returns the colour the ray should have as a Vec3 with RGB values in [0,1]
function rayColor(ray)
{
   let castResult = traceRay(ray)
   if(castResult.t < 0) return backgroundColour(ray)
   return new Vec3(1,0,0) // Red
}</pre>
```

- Information about the result of each ray cast is held in a RayCastResult
 - Position, point where ray hits something
 - normal, the normal of the surface at position
 - t, the point along the ray at position
 - This is positive if there is a hit
 - It is zero if the ray misses everything
 - sphereIndex, the array index of the sphere hit

```
class RayCastResult
{
    constructor(position, normal, t, sphereIndex)
    {
        this.position = position
        this.normal = normal
        this.t = t
        this.sphereIndex = sphereIndex
    }
}
```

- Trace ray cast the ray our into the scene gives back a result
 - Currently we always miss everything

```
// Check whether a ray hits anything in the scene and return a RayCast Result
function traceRay(ray)
{
    return miss()
}
```

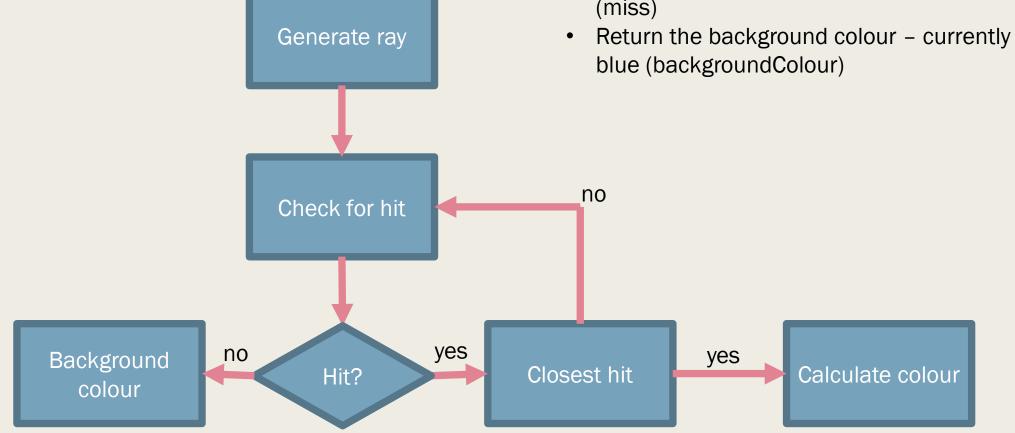
A miss is reported below

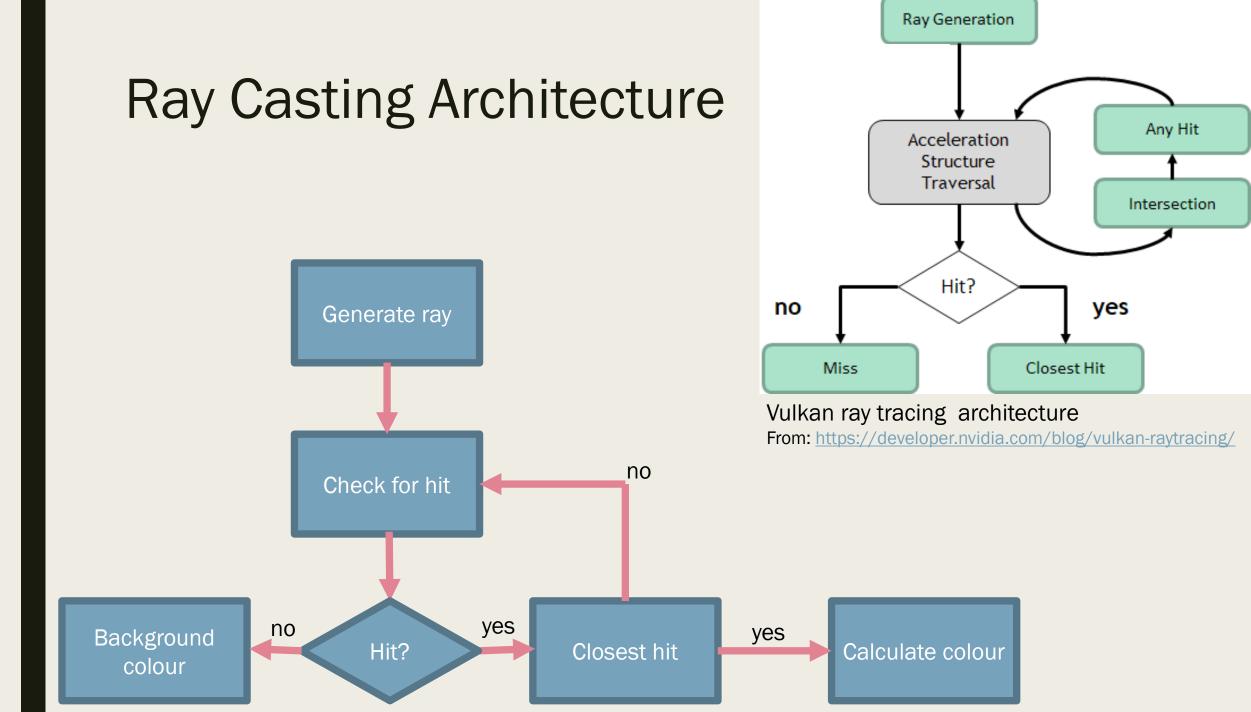
```
// Return a RayCastResult when a ray misses everything in the scene
function miss()
{
    return new RayCastResult(new Vec3(0,0,0), new Vec3(0,0,0), -1, -1)
}
```

- Stepping through the current setup
- For each pixel
 - Fire a ray from the camera through the pixel (rayColour)
 - Check if the ray hits a sphere it doesn't yet (traceRay)
 - If not every pixel for now record a miss (miss)
 - Return the background colour currently blue (backgroundColour)

For each pixel

- Fire a ray from the camera through the pixel (rayColour)
- Check if the ray hits a sphere it doesn't yet (traceRay)
- If not every pixel for now record a miss (miss)



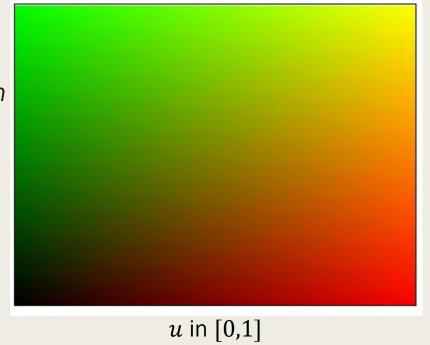


- Updating background colour to return a gradient colour
- x in [-1,1]

- Background colour takes in a ray
- We need to get v back from the ray
- Let y be the y component of the ray direction

Use this to calculate v

- y in [-1,1]
- Use v to interpolate between two colours



v in [0,1]

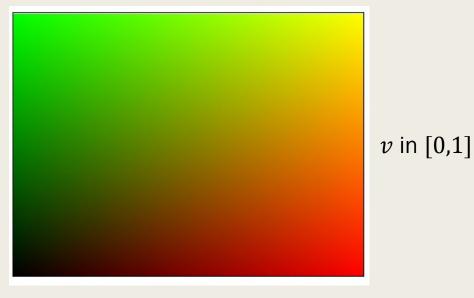
```
function backgroundColour(ray)
{
    let white = new Vec3(1, 1, 1)
    let blue = new Vec3(0.3,0.5,0.9)
    t = 0.5*(ray.direction.y + 1.0)
    return white.scale(1-t).add(blue.scale(t))
}
```

- Updating background colour to return a gradient colour
 - Background colour takes in a ray
 - We need to get *v* back from the ray
 - Let y be the y component of the ray direction

y in [-1,1]

- Use this to calculate v
- Use v to interpolate between two colours

x in [-1,1]



u in [0,1]

- But what about hitting something?
- We need to test whether our ray intersects with each of the spheres
- If it does
 - Find our which sphere is closest to the camera
 - Construct and return a RayCastResult
- First we need to test for ray sphere intersects
 - Use a quadratic solution

- Ray-sphere intersection is very similar to line-circle intersection
- lacktriangle All points p on the sphere with a centre c and a radius r is defined by the equation

$$(p_x - c_x)^2 + (p_y - c_y)^2 + (p_z - c_z)^2 = r^2$$

Think of p and c as vectors and we get

$$(\boldsymbol{p} - \boldsymbol{c}) \cdot (\boldsymbol{p} - \boldsymbol{c}) = r^2$$

 \blacksquare Recall our ray gives a point given a parametric value t

$$p = o + dt = \boldsymbol{p}(t)$$

 \blacksquare So for we are looking for a value of t where

$$(p(t) - c) \cdot (p(t) - c) = r^2$$

Our sphere equation

$$(\boldsymbol{p} - \boldsymbol{c}) \cdot (\boldsymbol{p} - \boldsymbol{c}) = r^2$$

 \blacksquare Recall our ray gives a point given a parametric value t

$$p = o + dt = \boldsymbol{p}(t)$$

 \blacksquare So for we are looking for a value of t where

$$(\mathbf{p}(\mathbf{t}) - \mathbf{c}) \cdot (\mathbf{p}(\mathbf{t}) - \mathbf{c}) = r^2$$

Substitute in and we get

$$(o+dt-\mathbf{c})\cdot(o+dt-\mathbf{c})=r^2$$

Which can be rearranged to give the quadratic equation

$$t^{2}d \cdot d + 2td \cdot (o - c) + (o - c) \cdot (o - c) - r^{2} = 0$$

Quadratic equation for ray-sphere intersection

$$t^{2}d \cdot d + 2td \cdot (o - c) + (o - c) \cdot (o - c) - r^{2} = 0$$

■ Let's match the terms to the standard quadratic formula

$$ax^2 + bx + c = 0$$

Quadratic equation for ray-sphere intersection

$$t^{2}d \cdot d + 2td \cdot (o - c) + (o - c) \cdot (o - c) - r^{2} = 0$$

■ Let's match the terms to the standard quadratic formula

$$ax^2 + bx + c = 0$$

- $\mathbf{x} = t$
- $a = d \cdot d$
- $b = 2d \cdot (o c)$
- $c = (o c) \cdot (o c) r^2$

Quadratic equation for ray-sphere intersection

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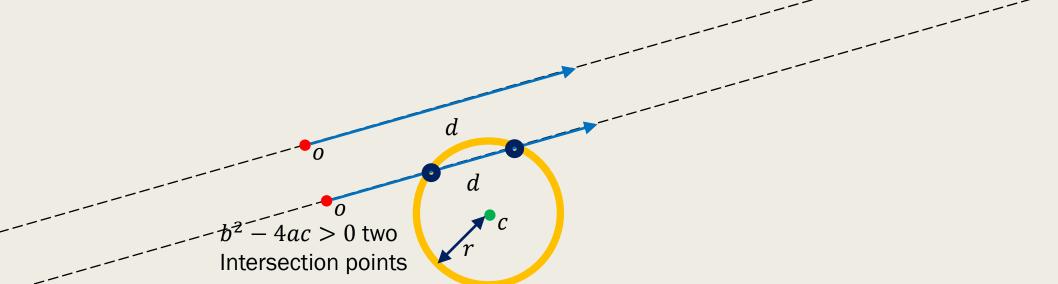
- $\mathbf{x} = t$
- $a = d \cdot d$
- $\bullet b = 2d \cdot (o c)$
- $c = (o c) \cdot (o c) r^2$
- We can now find t using the quadratic formula $x = \frac{-b \pm \sqrt{b^2 4ac}}{2a}$

- Recall the quadratic equation has a discriminant which tells use the number of roots b^2-4ac
- For a ray-sphere intersection this say where the ray intersects the sphere 0, 1 or 2 times

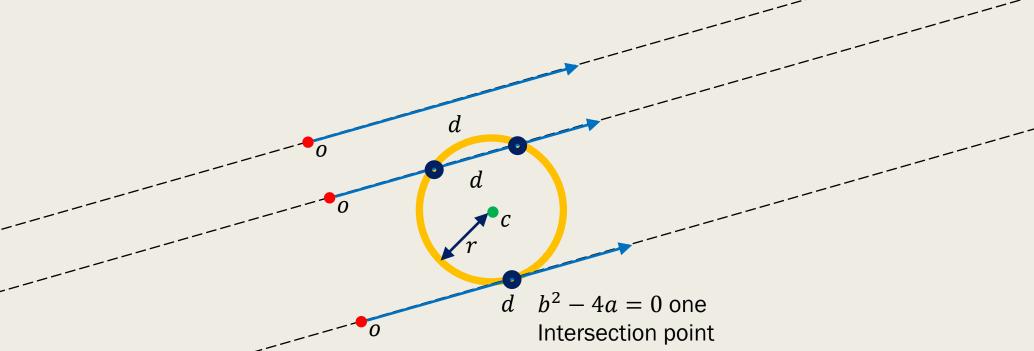
 $b^2 - 4ac < 0$ no intersection

d r

- Recall the quadratic equation has a discriminant which tells use the number of roots $b^2 4ac$
- For a ray-sphere intersection this say where the ray intersects the sphere 0, 1 or 2 times.



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- What to we need to do in code?
- The sphere class has a rayIntesects method to complete

```
// Calculate the point on the sphere where the ray intersects using
// a quadratic equation and return the t value of the ray for that point
// If two solutions exist return the minus solution
// If no solutions exist return -1
rayIntersects(ray) {}
```

- With reference to the quadratic equation
 - If discriminate is positive
 - Return $\frac{-b-\sqrt{b^2-4ac}}{2a}$ which is the point along the ray the intersection happened
 - Because our camera is facing negative z and doesn't move the -ve solution is always closest
 - Else
 - Return -1

- What are the terms of the quadratic equation in code?
 - $a = d \cdot d$ so dot product of ray direction with itself
 - $b = 2d \cdot (o c)$ so (o c) is ray origin minus sphere centre, dot product with ray direction times 2
 - $c = (o c) \cdot (o c) r^2$ so ray origin minus sphere centre dot product with itself minus sphere radius squared
- These all give scalars (numbers) not vectors
- The rayIntersects method simply needs to implement these then
 - Calculate the determinant
 - If it's negative return -1
 - If it's positive return $\frac{-b-\sqrt{b^2-4ac}}{2a}$

- So rayIntersects gives us *t* value where the ray intersects the sphere
- We already have three spheres

We can now test if our ray hits the first sphere at index 0

- We can now test if our ray hits the first sphere at index 0
- Update rayTrace

```
// Check whether a ray hits anything in the scene and return a RayCast Result
function traceRay(ray)
{
   let sphere = spheres[0];
   let t = sphere.rayIntersects(ray)
   if(t < 0)return miss()
   else return hit(ray, t, 0)
}</pre>
```

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function traceRay(ray)
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    let t = sphere.rayIntersects(ray)
    if(t < 0)return miss()
    else return hit(ray, t, 0)
}</pre>
```

And update hit so it returns t

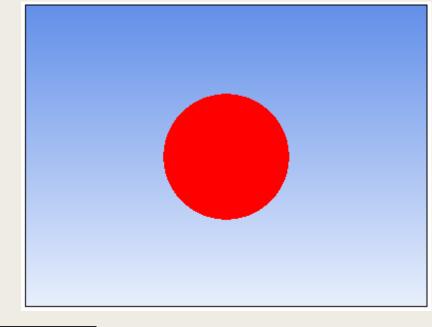
```
// Calculate the intersection point and normal when a ray hits a sphere. Returns a RayCastResult.
function hit(ray, t, sphereIndex)
{
    return new RayCastResult(intersectionPoint, intersectionNormal, t, sphereIndex)
}
```

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}
```



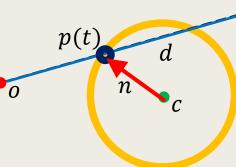
■ The sphere is a flat red colour because that's what rayColour returns

```
function rayColour(ray)
{
   let castResult = traceRay(ray)
   if(castResult.t < 0) return backgroundColour(ray)
   return new Vec3(1,0,0) // Red
}</pre>
```

- This needs to be updated, first we'll add diffuse light
- First step is to calculate the intersection point and normal
- Do this in the hit function and return via the RayCastResult

- To shade the sphere with simulated light we need the normal on the sphere
- We can use our ray to find the point where ray hit the sphere
- Take the sphere centre from this and we get the normal

$$\hat{n} = \|(o+dt) - c\|$$



LZ Quiz

■ L10 Q2 Ray-Sphere Intersection Normal

LZ Quiz

- A hit has been detected between a ray and sphere with a t value of 0.6. The rays origin is (0,0,-1) and it's direction is (-0.6, 0.4, 1). The sphere's centre is at (-0.25,0.1,-0.6). Calculate the normal of the ray-sphere intersection. Give your answer using 2 decimal places.
- $\blacksquare \quad \text{Hint: use } \hat{n} = \|(o + dt) c\|$
- 1. Calculate intersection point (o + dt):

$$(0 - 0.6 \times 0.6, 0 + 0.4 \times 0.6, -1 + 1 \times 0.6)$$

 $(-0.36, 0.25, -0.4)$

2. Take away sphere centre:

$$(-0.36, 0.25, -0.4) - (-0.25, 0.1, 0.6) = (-0.11, 0.14, 0.2)$$

3. Normalise

$$|\hat{n}| = \sqrt{(-0.11^2 + 0.14^2 + 0.2^2} = 0.267769$$

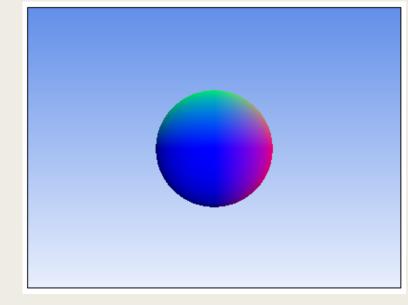
$$\left(\frac{-0.11}{|\hat{n}|}, \frac{0.14}{|\hat{n}|}, \frac{0.2}{|\hat{n}|}\right) = (-0.41, 0.52, 0.75)$$

- So hit now returns
 - The calculated intersection point
 - The calculated normal
 - The value t
 - The index of the sphere hit (0 at the moment)
- Update ray colour to return the intersection normal on hit

```
// Returns the colour the ray should have as a Vec3 with RGB values in [0,1]
function rayColor(ray)
{
    let castResult = traceRay(ray)
    if(castResult.t < 0) return backgroundColour(ray)
    return castResult.normal
}</pre>
```

- So hit now returns
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}</pre>
```



Add a global light direction and its opposite

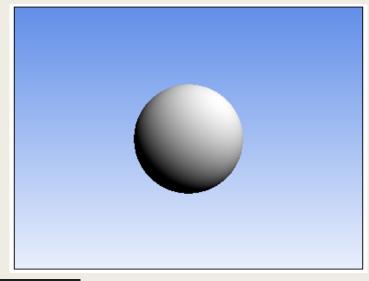
```
let lightDirection = new Vec3(-1.1,-1.3,-1.5).normalised();
let negLightDirection = new Vec3(-lightDirection.x,-lightDirection.y, -lightDirection.z)
```

Now take the dot product with the direction to the light

```
function rayColor(ray)
{
    let castResult = traceRay(ray)
    if(castResult.t < 0) return backgroundColour(ray)

let diffuse = Math.max(castResult.normal.dot(negLightDirection),0)
    let colour = new Vec3(diffuse, diffuse, diffuse);

return colour
}</pre>
```



Add a global light direction and its opposite

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let lightDirection = new Vec3(-1.1,-1.3,-1.5).normalised();
let negLightDirection = new Vec3(-lightDirection.x,-lightDirection.y, -lightDirection.z)
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return colour
}</pre>
```

- Now use the sphere colour instead
- Just scale the sphere colour (albedo) by the diffuse scalar

```
function rayColor(ray)
{
    let castResult = traceRay(ray)
    if(castResult.t < 0) return backgroundColour(ray)

let albedo = spheres[castResult.sphereIndex].colour
    let diffuse = Math.max(castResult.normal.dot(negLightDirection),0)
    let colour = albedo.scale(diffuse)

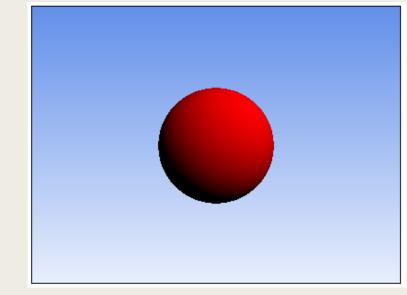
return colour
}</pre>
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return colour
}</pre>
```

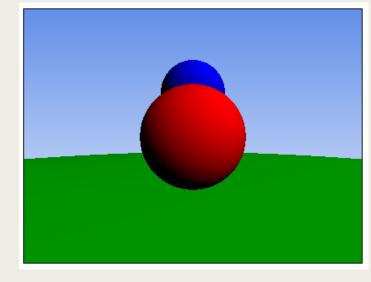


- Rather than check one sphere for intersect we need to check all
- We must do this in the trace ray function, just add a loop

```
function traceRay(ray)
{
    for (let i = 0; i < spheres.length; i++)
    {
        let sphere = spheres[i]
        let t = sphere.rayIntersects(ray)
        if(t >= 0)return hit(ray, t, i)
     }
    return miss()
}
```

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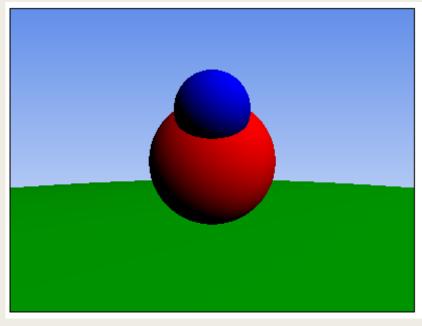
- This loop returns the first sphere the ray hits
- We want to check all spheres and return the closest
- Update the loop

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- We want to check all spheres and return the closest
- Update the loop

```
function traceRay(ray)
    let t = 1000000 // Set t to some high value
    let closestSphereIndex = -1
    for (let i = 0; i < spheres.length; i++)</pre>
        let current_t = spheres[i].rayIntersects(ray)
        if(current_t > 0 && current_t < t)</pre>
            t = current_t
            closestSphereIndex = i
    if(closestSphereIndex < 0) return miss()</pre>
    return hit(ray, t, closestSphereIndex)
```

- The this loop returns the first sphere the ray hits
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    let closestSphereIndex = -1
    for (let i = 0; i < spheres.length; i++)</pre>
        let current_t = spheres[i].rayIntersects(ray)
        if(current_t > 0 && current_t < t)</pre>
            t = current_t
            closestSphereIndex = i
    if(closestSphereIndex < 0) return miss()</pre>
    return hit(ray, t, closestSphereIndex)
```



Summary

- Rays
 - Origin and direction
- Ray tracing concept
 - Find a point along a ray
- Camera and viewport setup
- Ray intersection-sphere tests
 - Use a quadratic solution
- Diffuse lighting
 - Dot product surface normal with light direction
- Multiple spheres