

HW 10: Sum of Products and Karnaugh Maps (CS220-03)

For problems 1 through 3, write the (a) complete Sum of Products expression and (b) Sum of Products Minterm form expression equivalent to the logic in the truth table provided.

1)

A	B		X
0	0		1
0	1		0
1	0		1
1	1		1

$$X(A,B) = \text{sum}\{0,2,3\} =$$

$$\overline{A}\overline{B} + \overline{A}B + AB$$

2)

A	B	C		X
0	0	0		1
0	0	1		0
0	1	0		1
0	1	1		1
1	0	0		0
1	0	1		1
1	1	0		1
1	1	1		1

$$X(A,B,C) = \text{sum}\{0,2,3,5,6,7\} =$$

$$\overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}\overline{C} + A\overline{B}C + ABC$$

3)

A	B	C		X
0	0	0		0
0	0	1		1
0	1	0		1
0	1	1		0
1	0	0		1
1	0	1		1
1	1	0		1
1	1	1		1

$$X(A,B,C) = \text{sum}\{1,2,4,5,6,7\} =$$

$$\overline{A}\overline{B}C + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}\overline{C} + A\overline{B}C + ABC$$

Name: Anna Kurchenko

For problems 4 through 5, complete the truth table to be equivalent to the Sum of Products minterm form expression provided.

4) $X(A,B,C) = \sum_m\{0, 1, 3, 4, 6, 7\}$

A	B	C	X
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

5) $X(A,B,C,D) = \sum_m\{1, 3, 5, 7, 10, 11, 13, 14, 15\}$

A	B	C	D	X
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

For the following logical functions (6 through 12), use the Karnaugh map and 1s looping to find a simplified equivalent Boolean expression

6)

A	B	X
0	0	1
0	1	0
1	0	1
1	1	1

	0	1
0	1	0
1	1	1

$$x(A,B) = \bar{B} + A$$

7)

A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

	BC 00	01	11	10
A 0	1	0	1	1
1	0	1	1	1

$$X(A,B,C) = \bar{A}\bar{C} + AC + B$$

8)

A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

	BC 00	01	11	10
A 0	0	1	0	1
1	1	1	1	1

$$X(A,B,C) = A + \bar{B}\bar{C} + \bar{B}C$$

9)

A	B	C	X
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

BC	00	01	11	10
A	1	1	1	0
1	1	1	1	0

$$X(A,B,C) = \bar{B} + BC$$

10)

A	B	C	D	X
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1

A	B	C	D	X
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

CD	00	01	11	10
AB	0	1	1	0
00	0	1	1	0
01	0	1	1	0
11	0	1	1	1
10	0	0	1	1

$$X(A,B,C,D) = \bar{A}D + ABD + AC$$

11)

A	B	C	D	X
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0

A	B	C	D	X
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

CD	00	01	11	10
AB	0	1	0	0
00	0	1	0	0
01	1	1	0	1
11	1	1	0	1
10	0	1	0	0

$$X(A,B,C,D) = \bar{B}\bar{C} + \bar{B}CD + B\bar{C}$$

11) 'X' as output means that input minterm is impossible

A	B	C	D	Z	A	B	C	D	Z
0	0	0	0	1	1	0	0	0	1
0	0	0	1	0	1	0	0	1	0
0	0	1	0	X	1	0	1	0	0
0	0	1	1	0	1	0	1	1	0
0	1	0	0	0	1	1	0	0	1
0	1	0	1	0	1	1	0	1	X
0	1	1	0	1	1	1	1	0	X
0	1	1	1	1	1	1	1	1	1

AB \ CD	00	01	11	10
00	1	0	0	X
01	0	0	1	1
11	1	X	1	X
10	1	0	0	0

$$X(A,B,C,D) = \overline{A}CD + AB + BC$$

12) $Z(A,B,C,D)$ outputs a 1 for all prime 4-bit inputs. All multiples of 4 (including zero) and minterms 6, 10, and 14 are impossible.

AB \ CD	00	01	11	10
00	X	1	1	1
01	X	1	1	X
11	1	1	X	1
10	1	1	1	X

6 = 0110
 10 = 1010
 14 = 1111
 0 = 0000
 4 = 0100
 8 = 0110

$X(A,B,C,D) = n/a$
 since everything cancels out