Sasaki - Einstein metrics on spheres
(j. w. Yuchen liv and Taro Sero) , troduction $S^{n} = \int (x_{1}, -, x_{1}) \in \mathbb{R}^{n+1} \times_{1}^{2} + - + \times_{n}^{2} = 1$ Standard sphere: Homotopy sphere. a (real smooth) diff. manifold homotopy equivalent to 5 Princaré no phic

to so, these are called existic sylveres Rnk Metrics (M, g) Riensmian manifold din M even If there exist, a complex structure J on M compatible with a , then (M, g) is Kähler. Projective complex manifold.

(M, 3) din M odd Cone ((M):= Mx Rso, = 12g + dr2) where r is the coordinate on Rs. (M, g) is Sasakism if ((M), g)
admits a Kähler complex structure, (M, 5) is Souski- Firstein if (C(M), 5) is Kibler and g in Einstein (Ricz= 23)

String theory (S2n-1, gr) is Sasaki - Einstein Ex: $(C(S^{2n-1}), \overline{5}) \simeq (C^n \cdot 10), 9c^n)$ & Results Recall. A nonifold is parallelisable it its tongent bundle is brivial. S' bounds a parall. nonifold (disk)

Boyer - Galicki - Kollán 105 I SE netries on any homotopy sphere 5,4n+1 (n=1) Hist bounds parall. manitolds. Conje True also in din 4n-1. Conj: 7 00-nany SE netrics on every standard S2n-2, nz Than (Liv-Sano-T.) Any homotopy sphere E. 2a-1 that bounds parall manifolds admits to -nany SE metrics.

> The construction 123, a= (a0, ..., an) & Z 1, ai>1. y(a):- 1 2. + 2. + ... + 2. = 0 4 € C Bries Kon-Phom
1509. The link L (a):= /(a) , 5 is a snowth Milnor '68: compact symply connected (2n-1) -nanifold that bounds a parall, manifold.

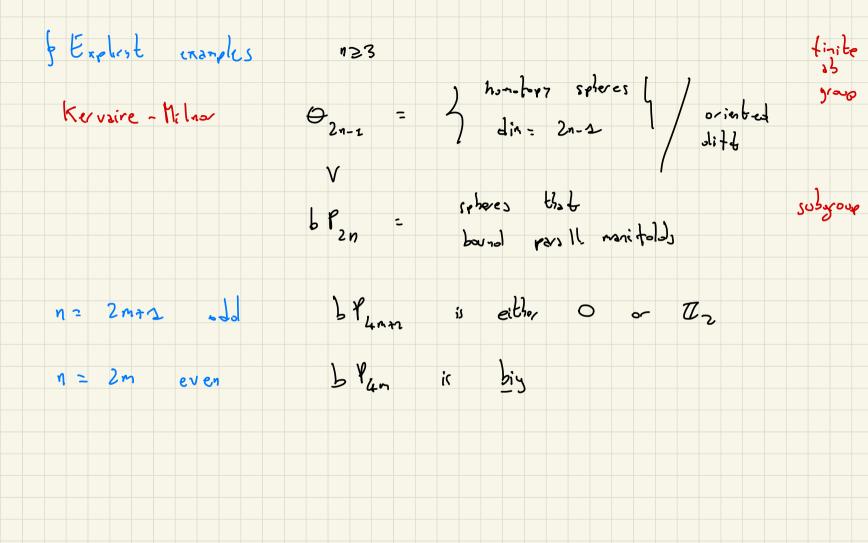
Then A (-) Assume $e_0 \in e_2 \in ... \in e_n$. L(e)

adoits a St-netric if $f_1 \in e_2 \in e_1$. L(e) die landail, die de C^* 2 $Y(x) \subseteq C^*$ $\lambda \cdot (\lambda, \dots, \lambda) = (\lambda \cdot \lambda_0, \dots, \lambda \cdot \lambda_n)$ $X^* \in \mathbb{P}(J_0, \dots, J_n)$ weighted hyp. Boyer - Galicki: L(R) adnik a St netric itt

X drite a Fara KE-netric. Note: X 5 Fino (=) - K x 5 ii ingle (=1 9. 8 1 20, Roughly speaking X adnits & Kt-netric itt (deep)

X ord is K-polystable

It's easy to slow that X x ord is K-polystable.



$$y(a) = \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{4}$$

Note: there is a combinatorial torold tor Z. Brieskun spheres: Q= (2,2,.,2,3,6K-1) & 2" 1 = 2m 1 = 2m 1 = 2m 1 = (-1) + (-1) + 1 = (-1) + 1 = (-1) + 1 = (-1) + 1 = (-1) + 1 = (-1) + 1 = (-1) + 1 = (-1) + 1 = (-1) + 1 = (-1) + 1 = (-1) + 1 = (-1) + (-1) + 1 = (-1) + (-1) + 1 = (-1) + (-1) + (-1) + (-1) + (-1) + (-1) + (-1) + (-1) + (-1) + (-1) + (-1) + (-1) + (-1) + (-1) + (so all exotic spheres re constructed. We generalised such examples.