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Design and Implementation of a 3D Dynamic Geometric Global Coordinate System

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***Abstract*—The three-dimensional dynamic geometry system（3D DGS）can intuitively present three-dimensional spatial and is widely used in solid geometry teaching. The design and implementation of the coordinate system in three-dimensional space is its essential foundation. The coordinate system of the existing 3D DGS can't be moved, rotated, zoomed, which make the DGS not meet teaching needs. In this paper, we analyze the transformation requirements and the design goal of the coordinate system. And we propose a method of the coordinate transformation of DGS and introduce the transformation process among the pixel coordinate system, the image coordinate system, the camera coordinate system and the world coordinate system. Based on the above coordinate transformation method, the movement, rotation, and zoom of the coordinate system are implemented in NetPad 3D. Finally, the comparison of a teaching case demonstrates that users develop resources conveniently with NetPad 3D.**

***Keywords*—DGS; global coordinate system; NetPad 3D**

I. Introduction

As an essential teaching aid for mathematics education, the dynamic geometry system provides a portable and straightforward interactive operating environment for teachers and students. Dynamic geometry system intuitive and dynamic characteristics are widely used in mathematics teaching [1-3]. Solid geometry teaching is one of the keys and difficult points in mathematics teaching. The three-dimensional dynamic geometry system (3D DGS) displays solid geometry and can observe geometry from multiple angles to enhance mathematics teaching's interactive effect. A large number of studies show that using a 3D DGS can improve students' understanding of solid geometry, reduce the burden on teachers and students and increase students' interest in learning [4-7]. At present, the 3D dynamic geometry software as GeoGebra[8-9], NetPad[10-12], Cabri 3D[13-14] and Super Sketchpad[15-16] are commonly used at home and abroad.

In the 3D DGS, reasonable coordinate system construction gives users an intuitive understanding of geometry and achieves the effect of helping to teach. The construction of the coordinate system is an essential and vital issue. It is necessary to observe geometry in space from multiple angles according to actual teaching habits and requirements. This method is an effective transformation strategy and has been widely used. However, those methods still hard to meet the actual needs for the construction and operation of teaching resources. Because those methods set the camera view in a fixed point, which means that the origin of the 3D coordinate cannot be moved, it results in a limited viewing perspective. Moreover, after a serial change of perspective included rotation and zoom, the system is still visually relative to the screen's center.

This paper proposes a global coordinate system design method for the 3D DGS. This method can effectively solve the shortcomings of the existing technology used to implement the perspective transformation. Its makeup for the current lack of solid geometry software makes resource layout more natural and reasonable and helps teachers and students better carry out solid geometry teaching activities. This method is implemented on NetPad 3D. NetPad 3D is a dynamic mathematical system based on the Web. In the next section, we achieved the goals according to the design requirements. In the third section, we introduced the primary process of constructing the global coordinate system of the 3D DGS. The fourth section gives details on the implemented method of the global coordinate system function. In the fifth section, we show the effect of the global coordinate system and compare the effects of related software through specific teaching cases. The sixth section concludes the value of the global coordinate system function and future improvement.

II. Design Goals

At present, most existing 3D DGS implement the perspective transformation technology by moving the origin of the 3D coordinate system. Moving the system origin have been used in the past to realize the user perspective's transformation. Besides, adjusting the camera's distance to the origin is one of the most common methods for zooming in and out of the user's perspective. However, the immovable coordinate system is the consequence of the camera view being fixed. The main disadvantage of methods is hardly meet the resource layout and use needs of geometry teaching.

Based on this problem, the three points that this design needs to meet are the movement, rotation, and zoom of the global coordinate system.

A. Moving the coordinate system

Users can move the coordinate system arbitrarily by dragging the origin of the coordinate system. Users can place the coordinates at any position on the screen, which freely layout the coordinate system—breaking the limitation that the

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coordinate origin can only be fixed at the center of the screen and more suitable for different teaching needs.

B. Rotating the coordinate system

Users can modify the world coordinate system's rotation angle by dragging the blank space on the screen. To implement the viewing angle change and keep the rotation center at the origin of the coordinate system, making the observation angle of the geometry more diversified.

C. Zooming the coordinate system

The system can be zoom in and out of scene depth by mouse wheel. The scene depth zoom is centered on the origin of the system, making the coordinate system display more diverse.

III. COORDINATE SYSTEM AND ITS TRANSFORMATION

The design of 3D DGS is based on the camera model to realize camera calibration and viewing angle correction functions. Its core part comprises four coordinate systems: the pixel coordinate system, the image coordinate system, the camera coordinate system, and the world coordinate system.

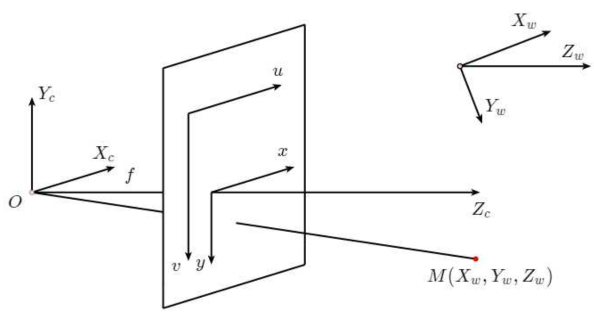

Fig. 1. Four coordinate system relationship diagram

By default, the screen coordinate system is defined as the pixel coordinate system. The image coordinate system and the pixel coordinate system are two-dimensional coordinates located in the initial screen plane. The difference between the two is the coordinate origin position and the pixel.

$$u = \frac{x}{dx} + u_0 \tag{1}$$

$$v = \frac{y}{dx} + v_0 \tag{2}$$

Therefore, the transformation between the two coordinate systems satisfies (1) and (2).

As shown in Figure 2, where dx and dy represent the length and width of each pixel in the pixel coordinate system, and the horizontal and vertical coordinates of the origin of the image coordinate system in the pixel coordinate system are u_0,v_0.

The camera coordinate system projects the 3D space object onto the 2D screen. Viewpoint is the origin of the world coordinate system and the center of the projection. This method fixes the viewport in the positive z-axis direction of the world coordinate system and the negative direction of the x-axis upwards on the screen. As shown in Figure 3, the position of the camera in the world coordinate system is: $(0,0,cameraZ)$. Because the camera is placed in a three-dimensional space, we need the world coordinate system, a reference coordinate system, to describe the position of the camera and use it to describe the position of any other objects placed in this 3D environment.

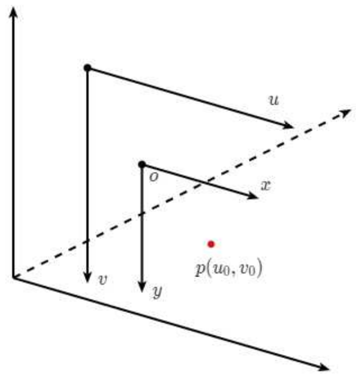

Fig. 2. Image coordinate system and pixel coordinate system

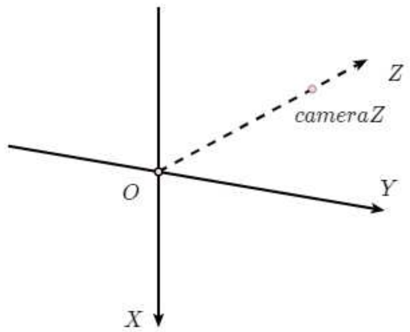

Fig. 3. Camera position and world coordinate system

The world coordinate system is used to describe the position of the camera and the object. The camera coordinate system is the coordinate system where the XOY plane is parallel to the screen. The world coordinate system can be moved and rotated to obtain the camera coordinate system. As shown in Figure 4, set the distance between the XOY plane of the camera coordinate system and the screen; the world coordinate system's z-axis is perpendicular to the screen. The camera coordinate system places the camera, and the position remains unchanged.

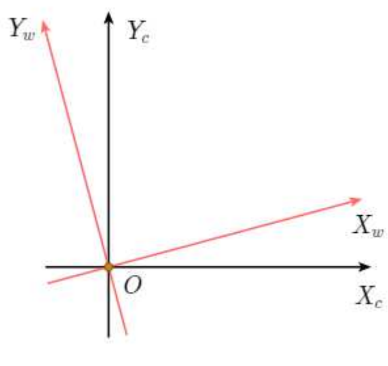

Fig. 4. World coordinate system rotation model

IV. THE IMPLEMENTATION OF WORLD COORDINATE SYSTEM

The world coordinate system is constructed in a 3D DGS, and 3D geometry is drawn in the world coordinate system.

A. Movement

This paper designs a point coordinate transformation method and does the following processing for any point A.

Step1:Calculate the shooting angle vector.

$$DR2 = unproject（DR1, C）- OG - DR1 \quad (3)$$

In (3), C is a coordinate of the camera in the world coordinate system, and OG is a vector from the origin of the image coordinate system to the origin of the world coordinate system. $DR1$ is the vector from point A to G in the standard coordinate system, and $unproject\ (DR1, C)$ represents the projection of the $DR1$ vector in the standard coordinate system to the world coordinate system with the C point as the light source point.

Step2: Calculate the shooting angle unit vector.

$$e = DR2/|DR2| \quad (4)$$

Among them, |*| means to take the modulus of *.

Step3: Calculate the vector angle.

$$\text{Cos}(\theta) = \frac{OG \cdot e}{|OG| \cdot |e|} \quad (5)$$

Step4: *Calculate the distance from the camera along the unit vector to the world coordinate plane.*

$$dis = cos(\theta) \cdot |OG| \quad (6)$$

Step5: *Calculate the new coordinates of point A in the world coordinate system.*

$$OA'' = OG + dis \cdot e \quad (7)$$

In the traversal world coordinate system, this method transforms every point on the three-dimensional geometric figure. Then a new figure in the screen coordinate system under the camera's perspective change can be obtained.

B. Rotation

In the design for the world coordinate system rotation function. In Figure 3, X_w, Y_w *and* Z_w are $X, Y,$ and Z axes of the world coordinate system, respectively. After the world coordinate system rotated, X_c,Y_c and Z_c are the $X, Y, and\ Z$ axes. Furthermore, X_w, Y_w and Z_w are X, Y and Z axes.

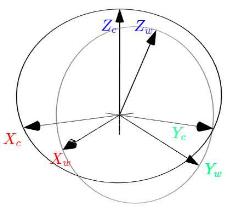

Fig. 5. Schematic diagram of the rotation world coordinate system

The conversion relationship between the image coordinate system and the camera coordinate system is (8) and (9).

$$x = \frac{fX_c}{Z_c} \quad (8)$$

$$y = \frac{fY_c}{Z_c} \quad (9)$$

f is the distance $cameraZ$ from the camera to the origin of the image coordinate system. It is expressed as a homogeneous coordinate system and matrix.

$$Z_c \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & f & 0 \end{pmatrix} \begin{pmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{pmatrix} \quad (10)$$

Equation (11) is the transformation between the camera coordinate system and the world coordinate system.

$$\begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} = R \begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix} + T \quad (11)$$

Among them, R is a 3×3 orthogonal rotation matrix, $T = \begin{pmatrix} t_1 \\ t_2 \\ t_3 \end{pmatrix}$ represent a transformation matrix amount added by three dimensions. It is expressed as a homogeneous coordinate system and matrix.

$$\begin{pmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{pmatrix} = \begin{pmatrix} R & T \\ 0^T & 1 \end{pmatrix} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix} \quad (12)$$

The operation can be expressed as converting the vertical component of the mouse movement into the y-axis rotation angle of the world coordinate system around the world coordinate system. Then, convert the horizontal component of the mouse movement into the z-axis rotation angle of the world coordinate system. Because the entire rotation process is to change the rotation angle of the camera coordinate system relative to the world coordinate system, the center position of the camera coordinate system has been fixed, so the rotation world coordinate system is realized simultaneously. The rotation process always determines the rotation center as the origin of the world coordinate system.

C. Zoom

The specific operation process of the zoom function is defined as: scroll the mouse firstly and get a zoom ratio according to the distance of the mouse scroll. Secondly, change the zoomed value of the world coordinate system to the value of the current world coordinate system divide by radio. For example, the original length of a line segment is 10, and the zoom ratio is 2. The world coordinate system is converted to the camera coordinate system after scaling. Besides, its unit length is reduced by half. When scaling down, the zoom center always remains as the coordinate origin of the world coordinate system.

V. Application

According to the above method's main points, we have implemented the global coordinate system in the NetPad. Further analysis showed that this method makes the entire page's resource layout more natural and beautiful. We take the NetPad (https://www.netpad.net.cn/) to design solid geometry word problems as an example to explore the main steps involved and the comparison with other software.

A. Constructing the resource

1) Open the NetPad editor and switch to 3D mode. (https://www.netpad.net.cn/svg.html).

2) Construct several points in the coordinate; select the construction points in turn with the mouse, and construct the geometric figure(Figure 6).

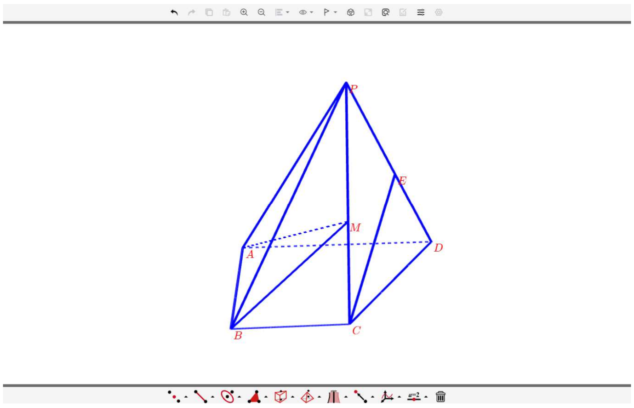

Fig. 6. Schematic diagram of geometry structure

3) By dragging the coordinate origin, move the geometric figure to the right side of the screen(Figure 7).

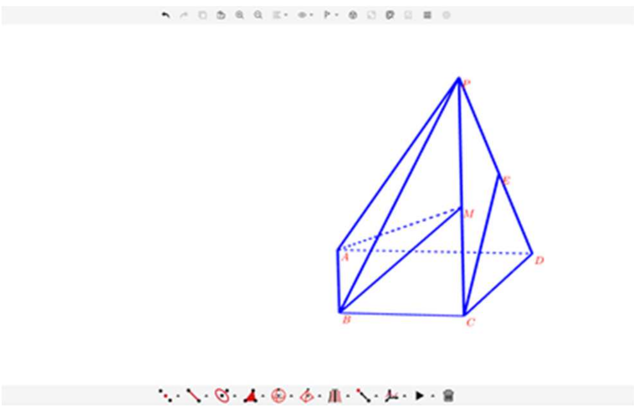

Fig. 7. Schematic diagram of geometric movement

4) Add text information and function buttons to construct a complete resource page(Figure 8).

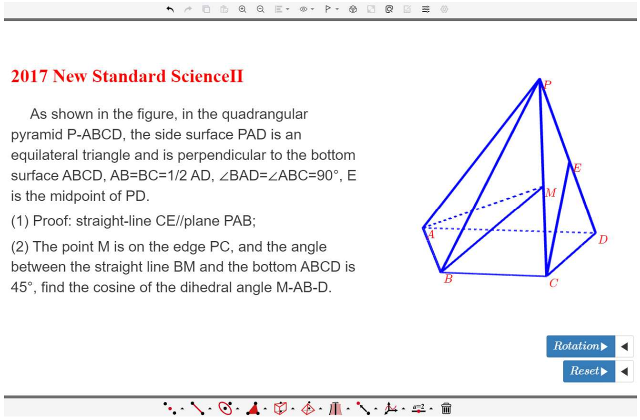

Fig. 8. The interface of the resource

5) Also, scroll the mouse can perform zooming(Figure 9); drag the blank space can perform rotating.

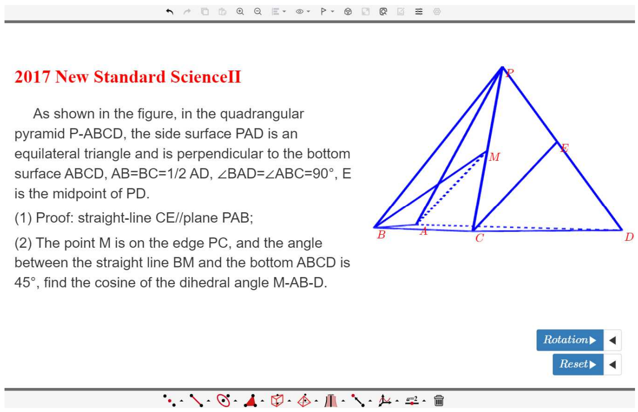

Fig. 9. Geometric figure rotation and zoom effect

B. Comparison

As shown in Figure 10, we use the same geometry word problem in GeoGebra to construct a teaching resource. Due to the coordinate system's position being limited to the center of the screen, the position of the text and the operation button are limited. So the overall page is not neat and natural. After using the 3D DGS constructed under the global coordinate system proposed in this paper, the layout of the components is more flexible and diverse, and the viewing angle is in line with the user's perception.

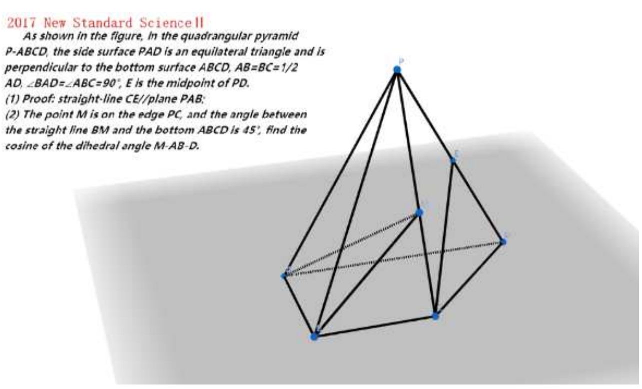

Fig. 10. Other 3D software construct word problem

VI. Conclusions and Prospects

The focus of 3D dynamic geometry software development is the resources layout constructing a page suitable for teaching. The difficulty of the resources layout is to construct a reasonable perspective transformation. At present, much 3D dynamic geometry software at home and abroad, such as Super Sketchpad. They can already support the viewing angle transformation function. However, its transformation effect is not good, which multi-angle observation cannot be performed and the layout of the resource is unnatural. A good perspective transformation of the coordinate system plays a significant role in improving the solid geometry teaching quality.

In this paper, we comprehensively analyzed the shortcomings of the existing 3D DGS and proposed a 3D dynamic geometric global coordinate system. This method brings many new features to the existing 3D DGS. One of the most important is the implementation of the movement of the coordinate system. This feature is very conducive to the multi-resource layout of the teaching page. The disadvantage of the method is that the rotation of the coordinate system must be rotated according to the established angle; otherwise, the entire coordinate system cannot be successfully rotated. Although this shortcoming does not affect the use of the function, continued efforts are needed to improve the rotation rules as much as possible. We look forward to providing users with more teaching convenience.

References

[1] Wang J, Rao Y, Chen R, et al. The Iteration Function on NetPad[C]. 2019 14th International Conference on Computer Science & Education (ICCSE). IEEE, 2019: 439-443.

[2] Chen B, Huang Y, Rao Y, et al. The Beauty of Geometry: A touch-operation-based DGS for mathematics education[C]. 2016 11th International Conference on Computer Science & Education (ICCSE). IEEE, 2016: 443-447.

[3] Zhang J, Ge Q, Peng X. Educational Technology Research Should be in-depth Discipline[J]. Journal of E-Education Research, 2010 (2): 8-13. (in Chinese)

[4] Aliyev Y N. Use of dynamic geometry software in teaching and research[C]. 2011 5th International Conference on Application of Information and Communication Technologies (AICT). IEEE, 2011: 1-3.

[5] Zhang J, Jiang C, Peng X. The value of dynamic geometry course in mathematics teaching and learning[J]. Journal of Mathematics Education, 2007,16(3):1-5. (in Chinese)

[6] Güven B, Kosa T. The effect of dynamic geometry software on student mathematics teachers' spatial visualization skills[J]. Turkish Online Journal of Educational Technology-TOJET, 2008, 7(4): 100-107.

[7] Straesser R. Cabri-Geometre: Does dynamic geometry software (DGS) change geometry and its teaching and learning?[J]. International Journal of Computers for Mathematical Learning, 2002, 6(3): 319-333.

[8] GeoGebra, http://www.geogebra.org.

[9] Saha R A, Ayub A F M, Tarmizi R A. The effects of GeoGebra on mathematics achievement: enlightening coordinate geometry learning[J]. Procedia-Social and Behavioral Sciences, 2010, 8: 686-693.

[10] NetPad, https://www.netpad.net.cn.

[11] Wang Y, Rao Y, Guan H, et al. NetPad: an online DGS for mathematics education[C]. 2017 12th International Conference on Computer Science and Education (ICCSE). IEEE, 2017: 305-309.

[12] Rao Y, Guan H, Chen R, et al. A Novel Dynamic Mathematics System Based on the Internet[C]. International Congress on Mathematical Software. Springer, Cham, 2018: 389-396.

[13] Cabri 3D, http://www.cabri.com.

[14] Kösa T, Karakuş F. Using dynamic geometry software Cabri 3D for teaching analytic geometry[J]. Procedia-Social and Behavioral Sciences, 2010, 2(2): 1385-1389.

[15] The Super Sketchpad, http://ssp.gzhu.edu.cn.

[16] Hou X, Fan Y, Liu F. Application of Super-sketchpad in Improving Mathematics Prospective Teachers Quality [J]. Journal of Mathematics Education, 2011, 6.