CS4431: Advanced Computer Architecture

Assignment #01

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Problem 1 According to Amdahl`s law:

$S(enhanced) = \frac{1}{(1-F)+\frac{F}{S}}$ where: S(enhhanced) is the Speedup obtained by the enhancement
and S is the speedup obtained by the *boosted* code
and F is the fraction of code that has been enhanced.

In this case, The CPU can perform Floating Point operations 'x' times faster. Therefore:

(a) Given that overall speedup is 1.7, we can directly substitute it in Amdahl`s law to get:

$1.7 = \frac{1}{(1-F)+\frac{F}{x}}$

therefore: $(1-F)+\frac{F}{x} = \frac{1}{1.7}$

therefore: $-F+\frac{F}{x} = \frac{1}{1.7} - 1$

therefore: $F(-1+\frac{1}{x}) = \frac{1-1.7}{1.7}$

therefore: $F(\frac{-x+1}{x}) = \frac{1-1.7}{1.7}$

therefore: $F = \frac{x(1-1.7)}{1.7(-x+1)}$

on reducting the above equation, we get: $F = \frac{0.7x}{1.7x-1.7}$

Hence, we can conclude that, as a function of 'x', the above mentioned CPU will perform with a speedup of 1.7, if the Frequency of floating point instructions in the code is: $\mathbf{F} = \frac{\mathbf{0.7x}}{\mathbf{1.7x - 1.7}}$

(b) The Frequency of floating point operation is given to be 50%. Therefore, in Amdahl`s law, we can substitute F = 50% = 0.5. Now, expected overall speedup is 1.25. Therefore, S(enhanced) = 1.25

We have: $1.25 = \dfrac{1}{(1-0.5)+\frac{0.5}{x}}$

therefore $1.25 = \dfrac{1}{0.5+\frac{0.5}{x}}$

therefore $0.5 + \dfrac{0.5}{x} = \dfrac{1}{1.25}$

therefore $\dfrac{0.5}{x} = \dfrac{1}{1.25} - 0.5$

therefore $\dfrac{0.5}{x} = \dfrac{1-0.5(1.25)}{1.25}$

therefore $\dfrac{0.5}{x} = \dfrac{0.375}{1.25}$

therefore $x = \dfrac{1.25 \times 0.5}{0.375}$

therefore $x = \dfrac{0.625}{0.375} = 1.667$

Hence, we can conclude that if a code has a 50% frequency of Floating point operations and we want to get an overall Speedup of 1.25, we need to have $\mathbf{x = 1.667}$

Problem 2

An enhancement to a CPU enables 50% of the code to perform 10 times faster. Therefore in Amdahl`s law:

$S(enhanced) = \frac{1}{(1-0.5)+\frac{0.5}{10}}$

(a) $S(enhanced) = \frac{1}{0.5+0.05}$

therefore $S(enhanced) = \frac{1}{0.55}$ =1.81

Hence, we can conclude that with a speedup of 10 times for 50% of the code, we can get an overall Speedup of 1.81

(b) To calculate the execution time, from Amdah`s law we can use :

$T(exewithE) = T(exewithoutE)[(1-F)+\frac{F}{S}]$

therefore $T(exewithE) = T(exewithoutE)[(1-0.5)+\frac{0.5}{10}]$

therefore $T(exewithE) = T(exewithoutE)[0.5+0.05]$

therefore $T(exewithE) = T(exewithoutE)[0.55]$

Since after enhancement, the code takes 0.55 times the execution time as without enhancement, We can say that the time enhancement is (1 - 0.55) which is 0.45 or as percentage 45%

Therefore, we can conclude that we have converted **45%** of the code to be executed in fast mode.

Problem 3

Assume two positive rates : 'r' and 's'. || Since they are positive, r >0 and s >0

The **arithmetic mean** (AM) of these rates is given by: $\frac{r+s}{2}$

The **harmonic mean** (HM) of these rates is given by: $\frac{2}{\frac{1}{r}+\frac{1}{s}}$

The HM can be reduced to: $\frac{2(r*s)}{r+s}$

Now, since both r and s are positive, lets assume the worst case: r = 1; s = 2

In this case, we get : $\quad AM = \frac{1+2}{2}$

$$= \frac{3}{2} \qquad = 1.5$$

and $\quad HM = \frac{2(1*2)}{1+2}$

$$= \frac{4}{3} \qquad = 1.334$$

Therefore, we can see that the AM is grreater than HM even in the worst case lowest values. Any value higher than this, results in a greater difference between the values of AM and HM, with AM always greater than HM.

However, AM can be **equal** to HM if and only if **r = s**

In such a case: $AM = \frac{r+r}{2}$

$$= \frac{2r}{2}$$

$$= r$$

and $\quad HM = \frac{2(r*r)}{r+r}$

$$= \frac{2r^2}{2r}$$

$$= r$$

Therefore, **when r = s**, the **arithmetic mean (AM) = harmonic mean (HM)**